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$(a + b\mathbf{i}) + (c + d\mathbf{i}) = (a + c) + (b + d)\mathbf{i}$
$(a + b\mathbf{i})(x + y\mathbf{i}) = (ac - bd) + (ad + bc)\mathbf{i}$
$\bar{z} = x - y\mathbf{i}$
$|z|^2 = z\bar{z}$
$\xi = a + b\mathbf{i}$ and $z = x + y\mathbf{i}$. $\xi z + \overline{\xi z} = 2c \implies ax - by = c$
$z\bar{z} - b\bar{z} - \bar{b}z = c$ is equation of circle with center b and radius $\sqrt{c + |b|^2}$
$\mathrm{Arg}(z)$ is angle between positive real axis and z. $\arg(z) = \mathrm{Arg}(z) + 2\pi k$
Polar form: $z = |z|(\cos(\mathrm{Arg}(z)) + \mathbf{i}\sin(\mathrm{Arg}(z)))$
De Moivre's Formula: $z = |z|(\cos(\mathrm{Arg}(z)) + \mathbf{i}\sin(\mathrm{Arg}(z)))$. $z^n = |z|^n(\cos(n\mathrm{Arg}(z)) + \mathbf{i}\sin(n\mathrm{Arg}(z)))$
$\log(w) = \ln(|w|e^{\mathbf{i}\arg(w)}) = \ln|w| + \mathbf{i}\arg(w)$
Complex Dot Product: $(a + b\mathbf{i}) \cdot (x + y\mathbf{i}) = ax + by$
Real Dot Product: $(a + b\mathbf{i}, x + y\mathbf{i}) = (a + b\mathbf{i})\overline{(x + y\mathbf{i})}$
Schwarz Inequaltiy: $|z \cdot w| \leq |z||w|$
Complex Law of Cosines: $|z + w|^2 = |z|^2 + 2Re(z\bar{w}) + |w|^2$
Triangle Inequality, reverse: $|z + w| \leq |z| + |w|$. $|z| - |w| \leq |z - w|$.
Neighborhood: $N(p, \epsilon) = \{z : |z - p| < \epsilon\}$
Circle of radius ϵ centered at p: $C(p, \epsilon) = \{z : |z - p| = \epsilon\}$
Polygonal Path: A fininte number of line segments joined end to end beginning at $a \in \mathbb{C}$ and ending at $b \in \mathbb{C}$
Open set: A set $O \subset \mathbb{C}$ is an open set if each point in O has a neighborhood which is contained in O.
Domain: A domain $D \subset \mathbb{C}$ is an open set for wich it is possible to join any two points by a polygonal path in D
Boundary: A point p is a boundary point of S if each neighborhood of p contains a point in S and a point not in S. Written as ∂S. The Boundary of a domain is its edge.
Closure: The closure of S is $\bar{S} = S \cup \partial S$
Closed set: Set is closed if $S = \bar{S}$
Cluster Point: A point p is a cluster point of S if each neighborhood of p contains infinitly may points of S
Complement: The complement of S is S' which contains all points not in S.
Compact Set: A set S is compact if every infinite sequence of distinct points in S has a cluster point in S.
Open Cover: A collection O_i of open sets such that $S \subset \cup O_i$ is called an open cover of S.
Finite Subcover: A finite subset of the open cover that cover S
Continous at a point: $f : D \to \mathbb{C}$ is continous if $\lim_{w \to z} f(w) = f(z)$ or $\forall \epsilon > 0 \exists\ \delta > 0$ s.t. $|z - w| < \delta \implies |f(z) - f(w)| < \epsilon$.
Continous on D: Function is continous of D if continous on every point of D.
Uniform Continuity: f is uniformly continous on $K \subset D$ if $\forall \epsilon > 0 \exists\ \delta > 0$ s.t. $|z_1 - z_2| < \delta \implies |f(z_1) - f(z_2)| < \epsilon \forall z_1, z_2 \in K$.
$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$
Differentiable: f is differentiable at z if $f'(z)$ exists.
Analytic at z_0: f is analytic at z_0 if $f'(z)$ exists at all points in some neighborhood of z_0
Analytic on D: f is analytic on domain D if $f'(z)$ exits $\forall z \in D$.
Partial Derivative of f with respect to x: $f_x = \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(z + \Delta x) - f(z)}{\Delta x} = u_x + \mathbf{i}v_x$
Partial Derivative of f with respect to y: $f_y = \frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(z + \mathbf{i}\Delta y) - f(z)}{\Delta y} = u_y + \mathbf{i}v_y$
C^1 at z: if f_x and f_y exist and are continous on a neighborhod containing z, f is C^1 at z.
C^1 on D: if f_x and f_y exits and are continous on D, f is C^1 on D.
Complex Cauchy-Riemann Equations: If $f'(z)$ exists then $f_x(z) = \frac{1}{\mathbf{i}}f_y(z)$. If f is C^1 at z then $f_x(z) = \frac{1}{\mathbf{i}}f_y(z) \implies f'(z)$ exitsts. If f is C^1 on D, then f is analytic on $D \iff f_x = f_y/\mathbf{i}$ on D.
Real Cauch-Riemann Equations: Suppose $f = u + \mathbf{i}v$ is C^1 on domain D. Then f is analytic on $D \iff u_x(z) = v_y(z)$ and $u_y(z) = -v_x(z)$ on D.
Polar Cauchy-Riemann: If $f(z = re^{\mathbf{i}\theta}) = u(r, \theta) + \mathbf{i}v(r, \theta)$ is C^1 at z then $ru_r = v_\theta$ and $rv_r = -u_\theta$
$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\mathbf{i}\partial y}\right)$
$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - \frac{\partial f}{\mathbf{i}\partial y}\right)$

A C^1 function is analytic iff $\frac{\partial f}{\partial \bar{z}} = 0$
$z^\xi = exp(\xi \log(z)) = exp(\xi(\ln|z| + \mathbf{i} \arg(z)))$