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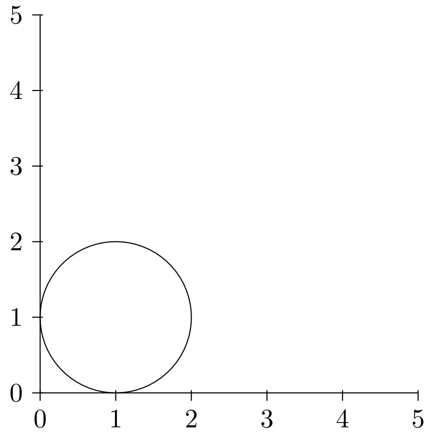
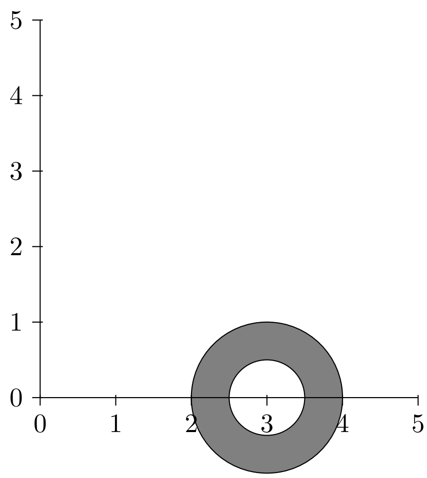
September 17, 2021

MATH 438: Introduction to Complex Variables
Assignment 2

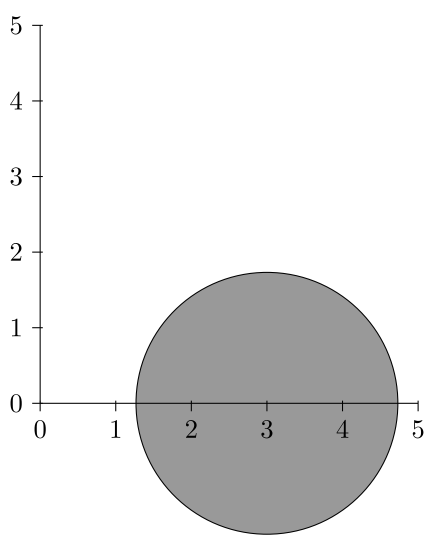
1. a



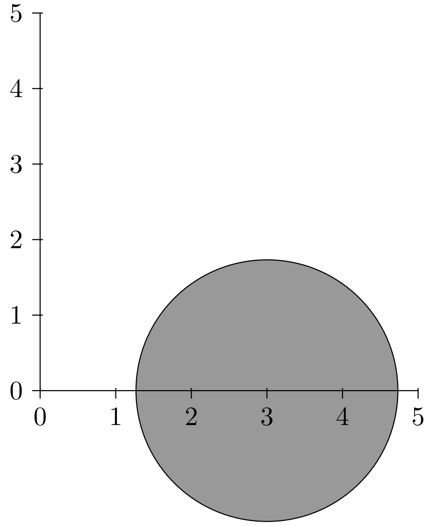
b



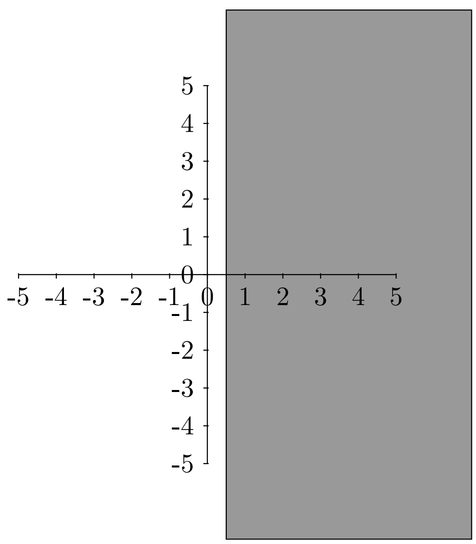
c



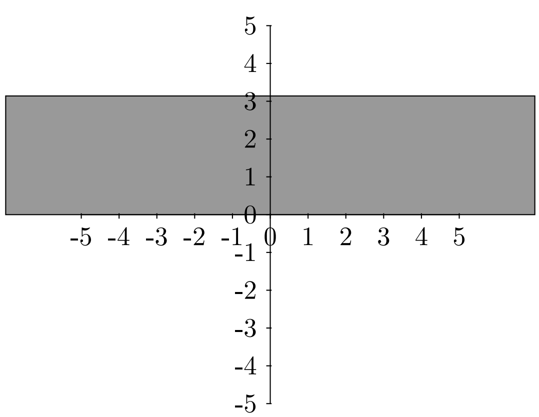
d



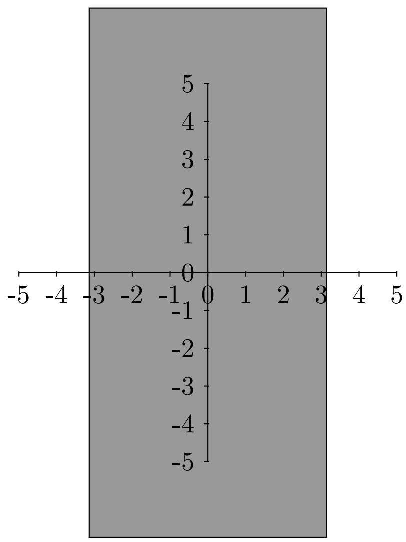
e $x > 1/2$



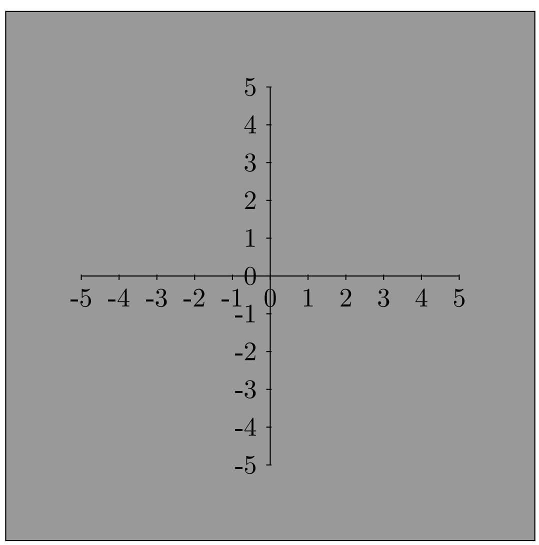
f $0 < y < \pi$



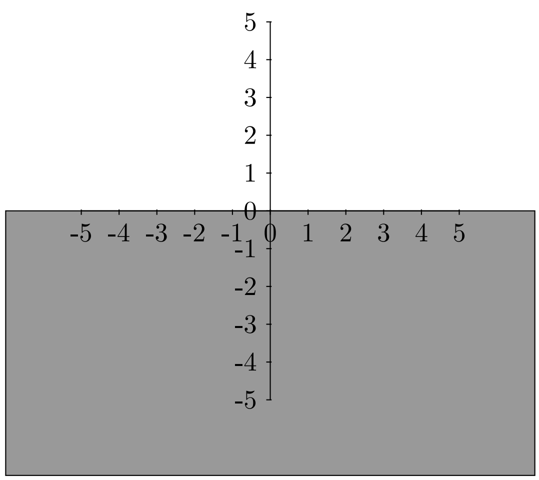
g $-\pi < x < \pi$



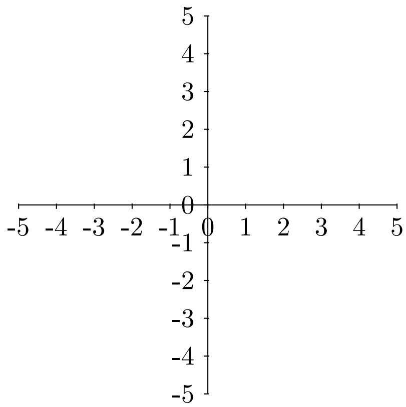
h *true*



i $y < 0$



j *false*



8. **Proof**

$$\begin{aligned} p(z) &= (z-z_0)h(z)+p(z_0) \\ p(z)-p(z_0) &= (z-z_0)h(z) \\ \frac{p(z)-p(z_0)}{z-z_0} &= h(x) \\ \text{Let} p(z_0) &= 0 \\ \frac{p(z)}{z-z_0} &= h(z) \end{aligned}$$

Since $p(z_0) = 0, z_0$ is a root of p and therefore it evenly devides p. Making $h(z)$ having degree of one less than $p(z)$
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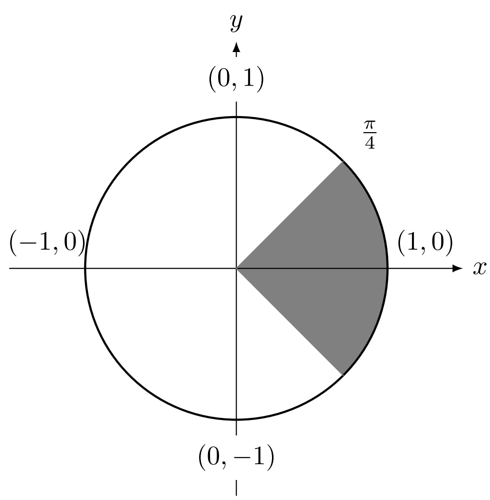
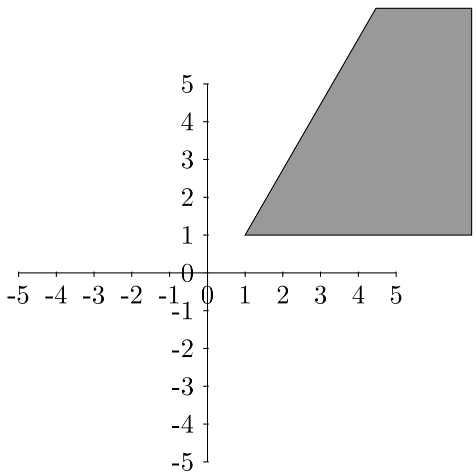
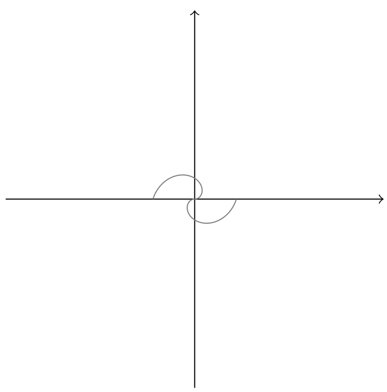
2. a $|arg(z)| < \pi/4$



b $1 < y < \sqrt{3}(x-1)+1$



c $|z| = arg(z)$



7. **Proof**

$$\begin{aligned}
\left|\frac{z^m}{z^n+1}\right| &\leq \frac{|z|^m}{|z|^n-1} \\
\frac{|z^m|}{|z^n+1|} &\leq \frac{|z|^m}{|z|^n-1} \\
\frac{1}{|z^n+1|} &\leq \frac{1}{|z|^n-1} \\
|z|^n-1 &\leq |z^n+1| \\
|z^n| &\leq |z^n+1| + 1\text{by the triangle inequality}
\end{aligned}$$

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