University of Waterloo

CS241 - Winter 2024 - Course Notes

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Lecture 1

- Definition: A bit is a binary digit. That is, a 0 or 1 (off or on)
- Definition: A nibble is 4 bits.
  - Example: 1001
- Definition: A byte is 8 bits.
  - Example: 10011101
- in C/C++:
  - Char: 8 bits
  - Unsigned char: 8 bits
  - Short: 2 bytes/16 bits
  - int: 4 bytes
  - longlong: 16 bytes
- Definition: A word is a machine-specific grouping of bytes. For us, a word will be 4 bytes (32-bit architecture) though 8-byte (or 64-bit architectures) words are more common now.
- Definition (Hexadecimal Notation): The base-16 representation system is called the hexadecimal system. It consists of the numbers from 0 to 9 and the letters a, b, c, d, e, f (which convert to the numbers from 10 to 15 in decimal notation)
  - Sometimes we denote the base with a subscript like 10011101_2 and $9d_{16}$.
  - Also, for hexadecimal,you will routinely see the notation `0x9d`. (The `0x` denotes a hexadecimal representation in computer science).
  - Note that each hexadecimal character is a nibble (4 bits).
- Conversion Table
  - Note: upper case letters are also used for hexadecimal notation. Context should make things clear.

Binary	Decimal	Hex
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7

Binary	Decimal	Hex
1000	8	8
1001	9	9
1010	10	a
1011	11	b
1100	12	c
1101	13	d
1110	14	e
1111	15	f

- Notation:
  - Binary: 0, 1
  - Decimal(Base 10): 0, ..., 9
  - Hexadecimal: 0, ..., 9, A(10), B(11), C(12), D(13), E(14), F(15)
- Examples:
  - 0000(base 2) -> 0x0(base 16)
  - 1111(base 2) -> 0xf(base 16)
- What do bytes represent?
  - Numbers
  - Characters
  - Garbage in memory
  - Instructions (Words, or 4 bytes, will correspond to a compute instruction in our computer system)
- Bytes as Binary Numbers
  - Unsigned (non-negative integers)
    - $b_7 ... b_0$ (base 2) = $b_7 \times 2^7 + ... + b_0 \times 2^0$ (base 10)
    - Example: $01010101 = 0 \times 2^7 + ... + 1 \times 2^0$
    - Converting to Binary:
      - One way: Take the largest power of 2 less than the unsigned integer, subtract and repeat
      - Another way is to constantly divide by 2, get the remainer for each division, reading *from the bottom to up* at the end, and that will be the binary representation of this unsigned integer

- Example: 38

Number	Quotient	Remainder
38	19	0
19	9	1
9	4	1
4	2	0
2	1	0
1	0	1

- Brief Explanation: Consider

$$N = b_0 + 2b_1 + 2^2 b_2 + \dots$$

The remainder when dividing N by 2 gives the b_0 value. After doing $\frac{N-b_0}{2}$, we end of with

$$\frac{N - b_0}{2} = b_1 + 2b_2 + 2^2 b_3 + \dots$$

and we can repeat the process. (This is why we have to read bottom-up as we get b_0 first, then b_1 ...)

- Signed integers
  - Attempt 1: make the first bit a signed bit. This is called the "sign-magnitude" representation
    - Problems:
      - Two representations of 0(wasteful and awkward)
      - Arithmetic is tricky. Is the sum of a positive number and a negative number positive or negative? It depends!
  - Attempt 2: Two's complement form
    - Similar to "sign-magnitude" representation in spirit, first bit is 0 if non-negative, 1 if negative
    - Negate a value by just subtracting from zero and *letting it overflow*.
    - Decimal to Two's Compliment:
      - A trick to the same thing of negating a value:
        - Take the complement of all bits (flip the 0 bits to 1 and 1 bits to 0)
        - Add 1
      - A slightly faster trick is to locate the rightmost 1 bit and flip all the bits to the left of it
        - Example: 11011010 Negating: 00100110 = 00100101 + 1
        - Note: Flipping the bits and adding 1 is the same as
          - subtracting 1 and flipping the bits for non-zero numbers
          - subtracting from 0
        - Example: compute -38_{10} using this notation in one byte of space:
          - Step 1: $38_{10} = 00100110_2$
          - Step 2: take the complment of all the bits: 11011001_2
          - Step 3: plus 1: 11011010_2
    - Two's Compliment to Decimal
      - Let's compute -38_{10} using one-byte Two's complement. First, write 38 in binary: $38_{10} = 00100110_2$. Next, take the complement of all the bits 11011001_2. Finally, add 1: 11011010_2. This last value is -38_{10}.
      - To convert 11011010_2, a number in Two's complement representation, to decimal, one method is to flip the bits and add 1: $00100110_2 = 2^5 + 2^2 + 2^1 = 38$. Thus, the corresponding positive number is 38 and so the original number is -38.
      - Another way to do this computation is to treat the original number 11011010_2 as an unsigned number, convert to decimal and subtract 28 from it (since we have 8 bits, and the first bit is a 1 meaning it should be a negative value). This also gives -38

$$\begin{aligned} 11011010_2 &= 2^7 + 2^6 + 2^4 + 2^3 + 2^1 - 2^8 \\ &= 128 + 64 + 16 + 8 + 2 - 256 \\ &= 218 - 256 \\ &= -38 \end{aligned}$$

- The idea behind [one byte] Two's Complement notation is based on the following observations:
  - The range for unsigned integers is 0 to 255. Recall that 255 is 11111111_2. If we add 1 to 255, then, after discarding overflow bits, we get the number 0.
  - Thus, let's treat 2^8 as 0, i.e., let's work modulo $2^8 = 256$. In this vein, we set up a correspondence between the positive integer k and the unsigned integer $2^8 - k$. Since we are working modulo 2^8, subtracting a positive integer k from 0 is the same as subtracting it from 2^8.
  - In this cse, note that $255 = 2^8 - 1 = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$ and in general

$$2^n - 1 = \sum_{i=0}^{n-1} 2^i$$

  - As an explicit example (which can be generalized naturally) take a number, say $38_{10} = 00100110_2 = 2^5 + 2^2 + 2^1$. What should the corresponding negative number be? Well, note that we've said subtracting a positive integer k from 0 is the same as subtracting it from 2^8 :

$$\begin{aligned} 2^8 - 1 &= 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 \\ 2^8 - 1 &= 38 + 2^7 + 2^6 + 2^4 + 2^3 + 2^0 \\ 2^8 - 38 &= 2^7 + 2^6 + 2^4 + 2^3 + 2^0 + 1 \qquad \text{(flip the bits and add 1)} \end{aligned}$$

  - We mentioned that another method of negating a two's complement number is to flip the bits to the left of the rightmost 1. Justify why this technique works.
    - Every bit to the right of the rightmost 1 is a 0. When we "flip the bits", these become 1s. When we "add 1", the carry propagates up until the position of the "rightmost 1" (the "rightmost 1" is 0 after flip and will stop propagating when the carry reaches this point, and everything on the left of the "rightmost 1" is flipped).

- The main difference between signed and unsigned binary arithmetic is that we are now working modulo 256 (or, more generally, 2n in the case of n-bit Two's complement numbers)
- When working in Two's complement, overflow occurs when adding numbers if the original two numbers have the same sign, but the result has a different sign.
- Arithmetic of Signed Integers
  - All of the arithmetic works by ignoring overflow precisely because arithmetic works in Z_{256}!
- What is the range of numbers expressible in one-byte Two's Complement notation?
  - $-128 \sim 127$
- Definitions: The Most Significant Bit (MSB) is the left- most bit (highest value/sign bit); The Least Significant Bit (LSB) is the right-most bit (lowest value).

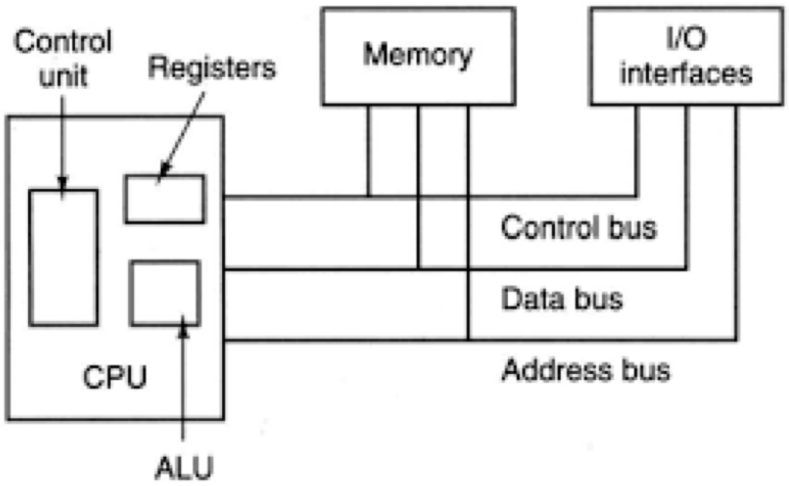
Lecture 2

- ASCII (American Standard Code for Information Interchange) uses 7 bits to represent characters.

```
char c = '0';
printf("%c", c);
```

```
// stdout: 0
printf("%d", (int)c);
// stdout: 48
```

- Bit-Wise Operators
  - Example: suppose we have `unsigned char a=5, b=3;`, which means $a = 00000101, b = 00000011$
    - Bitwise not $\sim a$, for example $c = \sim a;$ gives $c = 11111010$
    - Bitwise and $\boxed{\&}$, for example $\boxed{\texttt{c=a\&b;}}$ gives $c = 00000001$
    - Bitwise or $|$, for example $c = a|b;$ gives $c = 00000111$
    - Bitwise exclusive$\hat{}$(only true if the bit in a and b is different), for example $c = a\hat{}b;$ gives $c = 00000110$
    - Bitwise shift right or left $>>$ and $<<$, for example
      - $c = a >> 2;$ gives $c = 00000001$ and
      - $c = a << 3;$ gives $c = 00101000$
    - $a << 1$ equivalent to $a * 2$
    - $a >> 1$ equivalent to $a/2$
    - These can even be combined with the assignment operator: `c &= 5;`



- CPU with Memory
- MIPS has 32 registers that are called “general purpose”
  - Some general-purpose registers are special:
    - `$0` is always 0
    - `$31` is for return address
    - `$30` is our stack pointer
    - `$29` is our frame pointer
- Problem: We only know from context what bits have what meaning, and in particular, which are instructions.
  - Solution: Convention is to set memory address 0 in RAM to be an instruction. `0x0: instruction i1`
- Problem: How does MIPS know what to do next?
  - Solution: Have a special register called the Program Counter (or PC for short) to tell us what instruction to do next.
- Problem: How do we put our program into RAM?
  - Solution: A program called a loader puts our program into memory and sets the PC to be the first address.
- Algorithm 1 Fetch-Execute Cycle

```
PC=0
while true do
    IR = MEM[PC]
    PC += 4
    Decode and execute instruction in IR
endwhile
```

- Write a program in MIPS that adds the values of registers `$8` and `$9` and stores the result in register `$3`.

```
add $d, $s, $t
0000 00ss ssst tttt dddd d000 0010 0000
```

- Why 5 bits for each? Because there are 32 registers `$0 - $31` and $2^5 = 32$
- Adds registers `$s` and `$t` and stores the sum in register `$d`. Important! The order of `$d`, `$s` and `$t` are shifted in the encoding.

Lecture 3

Lecture 4

Lecture 5

Lecture 6

Lecture 7

Lecture 8

Lecture 9

Lecture 10

Lecture 11

Lecture 12

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Lecture 24

Final Review