University of Waterloo

CS241 - Winter 2024 - Course Notes

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Course Code: CS241

Course Name: Foundations of Sequential Programs

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Section: 001

Date Created: January 09, 2024

Final Exam Date: TBA

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Section 1: Lectures

Lecture 1

- Definition: A bit is a binary digit. That is, a 0 or 1 (off or on)
- Definition: A nibble is 4 bits.
    - Example: 1001
- Definition: A byte is 8 bits.
    - Example: 10011101
- in C/C++:
    - Char: 8 bits
    - Unsigned char: 8 bits
    - Short: 2 bytes/16 bits
    - int: 4 bytes
    - longlong: 16 bytes
- Definition: A word is a machine-specific grouping of bytes. For us, a word will be 4 bytes (32-bit architecture) though 8-byte (or 64-bit architectures) words are more common now.
- Definition (Hexadecimal Notation): The base-16 representation system is called the hexadecimal system. It consists of the numbers from 0 to 9 and the letters a, b, c, d, e, f (which convert to the numbers from 10 to 15 in decimal notation)
    - Sometimes we denote the base with a subscript like 10011101_2 and $9d_{16}$.
    - Also, for hexadecimal,you will routinely see the notation `0x9d`. (The `0x` denotes a hexadecimal representation in computer science).
    - Note that each hexadecimal character is a nibble (4 bits).
- Conversion Table
    - Note: upper case letters are also used for hexadecimal notation. Context should make things clear.

Binary	Decimal	Hex
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7

Binary	Decimal	Hex
1000	8	8
1001	9	9
1010	10	a
1011	11	b
1100	12	c
1101	13	d
1110	14	e
1111	15	f

- Notation:
    - Binary: 0, 1
    - Decimal(Base 10): 0, ..., 9
    - Hexadecimal: 0, ..., 9, A(10), B(11), C(12), D(13), E(14), F(15)
- Examples:
    - 0000(base 2) -> 0x0(base 16)
    - 1111(base 2) -> 0xf(base 16)
- What do bytes represent?
    - Numbers
    - Characters
    - Garbage in memory
    - Instructions (Words, or 4 bytes, will correspond to a compute instruction in our computer system)
- Bytes as Binary Numbers
    - Unsigned (non-negative integers)

- $b_7 ... b_0$ (base 2) = $b_7 \times 2^7 + ... + b_0 \times 2^0$ (base 10)
- Example: $01010101 = 0 \times 2^7 + ... + 1 \times 2^0$
- Converting to Binary:
    - One way: Take the largest power of 2 less than the unsigned integer, subtract and repeat
    - Another way is to constantly divide by 2, get the remainer for each division, reading *from the bottom to up* at the end, and that will be the binary representation of this unsigned integer
        - Example: 38

Number	Quotient	Remainder
38	19	0
19	9	1
9	4	1
4	2	0
2	1	0
1	0	1

        - Brief Explanation: Consider

$$N = b_0 + 2b_1 + 2^2 b_2 + \dots$$

The remainder when dividing N by 2 gives the b_0 value. After doing $\frac{N-b_0}{2}$, we end of with

$$\frac{N - b_0}{2} = b_1 + 2b_2 + 2^2 b_3 + \dots$$

and we can repeat the process. (This is why we have to read bottom-up as we get b_0 first, then b_1 ...)

- Signed integers
    - Attempt 1: make the first bit a signed bit. This is called the "sign-magnitude" representation
        - Problems:
            - Two representations of 0(wasteful and awkward)
            - Arithmetic is tricky. Is the sum of a positive number and a negative number positive or negative? It depends!
    - Attempt 2: Two's complement form
        - Similar to "sign-magnitude" representation in spirit, first bit is 0 if non-negative, 1 if negative
        - Negate a value by just subtracting from zero and *letting it overflow*.
        - Decimal to Two's Compliment:
            - A trick to the same thing of negating a value:
                - Take the complement of all bits (flip the 0 bits to 1 and 1 bits to 0)
                - Add 1
            - A slightly faster trick is to locate the rightmost 1 bit and flip all the bits to the left of it
                - Example: 11011010 Negating: 00100110 = 00100101 + 1
                - Note: Flipping the bits and adding 1 is the same as
                    - subtracting 1 and flipping the bits for non-zero numbers
                    - subtracting from 0
                - Example: compute -38_{10} using this notation in one byte of space:
                    - Step 1: $38_{10} = 00100110_2$
                    - Step 2: take the complment of all the bits: 11011001_2
                    - Step 3: plus 1: 11011010_2
        - Two's Compliment to Decimal
            - Let's compute -38_{10} using one-byte Two's complement. First, write 38 in binary: 38_{10} = 00100110_2. Next, take the complement of all the bits 11011001_2. Finally, add 1: 11011010_2. This last value is -38_{10}.

- To convert 11011010_2, a number in Two's complement representation, to decimal, one method is to flip the bits and add 1: 00100110_2 = $2^5 + 2^2 + 2^1 = 38$. Thus, the corresponding positive number is 38 and so the original number is -38.
- Another way to do this computation is to treat the original number 11011010_2 as an unsigned number, convert to decimal and subtract 28 from it (since we have 8 bits, and the first bit is a 1 meaning it should be a negative value). This also gives -38

$$\begin{aligned} 11011010_2 &= 2^7 + 2^6 + 2^4 + 2^3 + 2^1 - 2^8 \\ &= 128 + 64 + 16 + 8 + 2 - 256 \\ &= 218 - 256 \\ &= -38 \end{aligned}$$

- The idea behind [one byte] Two's Complement notation is based on the following observations:
  - The range for unsigned integers is 0 to 255. Recall that 255 is 11111111_2. If we add 1 to 255, then, after discarding overflow bits, we get the number 0.
  - Thus, let's treat 2^8 as 0, i.e., let's work modulo $2^8 = 256$. In this vein, we set up a correspondence between the positive integer k and the unsigned integer $2^8 - k$. Since we are working modulo 2^8, subtracting a positive integer k from 0 is the same as subtracting it from 2^8.
  - In this cse, note that $255 = 2^8 - 1 = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$ and in general

$$2^n - 1 = \sum_{i=0}^{n-1} 2^i$$

  - As an explicit example (which can be generalized naturally) take a number, say $38_{10} = 00100110_2 = 2^5 + 2^2 + 2^1$. What should the corresponding negative number be? Well, note that we've said subtracting a positive integer k from 0 is the same as subtracting it from 2^8 :

$$\begin{aligned} 2^8 - 1 &= 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 \\ 2^8 - 1 &= 38 + 2^7 + 2^6 + 2^4 + 2^3 + 2^0 \\ 2^8 - 38 &= 2^7 + 2^6 + 2^4 + 2^3 + 2^0 + 1 && \text{(flip the bits and add 1)} \end{aligned}$$

  - We mentioned that another method of negating a two's complement number is to flip the bits to the left of the rightmost 1. Justify why this technique works.
    - Every bit to the right of the rightmost 1 is a 0. When we "flip the bits", these become 1s. When we "add 1", the carry propagates up until the position of the "rightmost 1" (the "rightmost 1" is 0 after flip and will stop propagating when the carry reaches this point, and everything on the left of the "rightmost 1" is flipped).

- The main difference between signed and unsigned binary arithmetic is that we are now working modulo 256 (or, more generally, 2n in the case of n-bit Two's complement numbers)
- When working in Two's complement, overflow occurs when adding numbers if the original two numbers have the same sign, but the result has a different sign.
- Arithmetic of Signed Integers
  - All of the arithmetic works by ignoring overflow precisely because arithmetic works in Z_{256}!
- What is the range of numbers expressible in one-byte Two's Complement notation?
  - $-128 \sim 127$

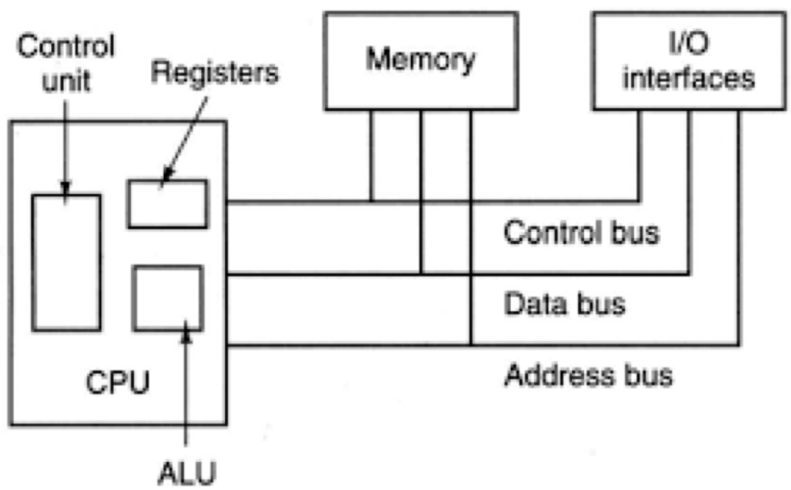
- Definitions: The Most Significant Bit (MSB) is the left- most bit (highest value/sign bit); The Least Significant Bit (LSB) is the right-most bit (lowest value).

Lecture 2

- ASCII (American Standard Code for Information Interchange) uses 7 bits to represent characters.

```
char c = '0';
printf("%c", c);
// stdout: 0
printf("%d", (int)c);
// stdout: 48
```

- Bit-Wise Operators
  - Example: suppose we have `unsigned char a=5, b=3;`, which means $a = 00000101, b = 00000011$
    - Bitwise not $\sim a$, for example $c = \sim a$; gives $c = 11111010$
    - Bitwise and $\boxed{\&}$, for example $\boxed{\text{c=a\&b;}}$ gives $c = 00000001$
    - Bitwise or $|$, for example $c = a|b$; gives $c = 00000111$
    - Bitwise exclusiveˆ(only true if the bit in a and b is different), for example $c = \hat{ab}$; gives $c = 00000110$
    - Bitwise shift right or left $>>$ and $<<$, for example
      - $c = a >> 2$; gives $c = 00000001$ and
      - $c = a << 3$; gives $c = 00101000$
    - $a << 1$ equivalent to $a * 2$
    - $a >> 1$ equivalent to $a/2$
    - These can even be combined with the assignment operator: `c &= 5;`



- CPU with Memory
- MIPS has 32 registers that are called “general purpose”
  - Some general-purpose registers are special:
    - `$0` is always 0
    - `$31` is for return address
    - `$30` is our stack pointer
    - `$29` is our frame pointer
- Problem: We only know from context what bits have what meaning, and in particular, which are instructions.
  - Solution: Convention is to set memory address 0 in RAM to be an instruction. `0x0: instruction i1`
- Problem: How does MIPS know what to do next?
  - Solution: Have a special register called the Program Counter (or PC for short) to tell us what instruction to do next.
- Problem: How do we put our program into RAM?
  - Solution: A program called a loader puts our program into memory and sets the PC to be the first address.
- Algorithm 1 Fetch-Execute Cycle

```
PC=0
while true do
    IR = MEM[PC]
```

```
    PC += 4
    Decode and execute instruction in IR
end while
```

- Write a program in MIPS that adds the values of registers `$8` and `$9` and stores the result in register `$3`.

```
add $d, $s, $t
0000 00ss ssst tttt dddd d000 0010 0000
```

- Why 5 bits for each? Because there are 32 registers `$0 - $31` and $2^5 = 32$
- Adds registers `$s` and `$t` and stores the sum in register `$d`. Important! The order of `$d`, `$s` and `$t` are shifted in the encoding.

Lecture 3

- **Putting values in registers** Load immediate and skip. This places the next value in RAM [an immediate] into $d and increments the program counter by 4 (it skips the next line which is usually not an instruction).

```
lis $d: 0000 0000 0000 0000 dddd d000 0001 0100
# What it really does is
$d = MEM[PC]
PC = PC + 4
```

- How do we get the value we care about into the next location in RAM?

```
.word i: iiii iiii iiii iiii iiii iiii iiii iiii
```

- The above is an assembler directive (not a MIPS instruction). The value i, as a two's complement integer, is placed in the correct memory location in RAM as it occurs in the code.
  - Can also use hexadecimal values: `0xi`
  - Decimal is also allowed.
- Example:

```
lis $1
.word 10
# At this moment, $1 = 10
add $1, $1, $0
```

- Example: Write a MIPS program that adds together 11 and 13 and stores the result in register $3.

```
lis $1
.word 11
list $2
.word 13
add $3, $1, $2
# Solution on the course notes
lis $8          0000 0000 0000 0000 0100 0000 0001 0100
.word 11        0000 0000 0000 0000 0000 0000 0000 1011
lis $9          0000 0000 0000 0000 0100 1000 0001 0100
.word 0xd       0000 0000 0000 0000 0000 0000 0000 1101
add $3,$8,$9    0000 0001 0000 1001 0001 1000 0010 0000
    # The code on the left is what we call Assembly Code.
    # The code on the right is what we call Machine Code.
```

- Jump Register. Sets the pc to be $s.

```
jr $s
0000 00ss sss0 0000 0000 0000 0000 1000
```

- For us, our return address will typically be in $31, so we will typically call the below. This command returns control to the loader.

```
jr $31
0000 0011 1110 0000 0000 0000 0000 1000
```

- So the complete example for the example is (so that the while loop will terminate)

```
lis $8          0000 0000 0000 0000 0100 0000 0001 0100
.word 11        0000 0000 0000 0000 0000 0000 0000 1011
lis $9          0000 0000 0000 0000 0100 1000 0001 0100
.word 0xd       0000 0000 0000 0000 0000 0000 0000 1101
add $3,$8,$9    0000 0001 0000 1001 0001 1000 0010 0000
jr $31          0000 0011 1110 0000 0000 0000 0000 1000
```

- To multiply two words, we need to use the two special registers `hi` and `lo`.
  - `hi` is most significant 4 bytes
  - `lo` is least significant 4 bytes

```
mult $s, $t
0000 00ss ssst tttt 0000 0000 0001 1000
```

- The above performs the multiplication and places the most significant word (largest 4 bytes) in `hi` and the least significant word in `lo`.
- `div $s, $t` performs integer division and places the quotient $s/$t in `lo` [lo quo] and the remainder st in `hi`. Note the sign of the remainder matches the sign of the divisor stored in $s.

```
div $s, $t
0000 00ss ssst tttt 0000 0000 0001 1010
```

- Multiplication and division happen on these special registers hi and lo. How can I access the data?
  - Move from register `hi` into register `$d`.

  ```
  mfhi $d
  0000 0000 0000 0000 dddd d000 0001 0000
  ```

  - Move from register `lo` into register `$d`.

  ```
  mflo $d
  0000 0000 0000 0000 dddd d000 0001 0010
  ```

- RAM
  - Large[r] amount of memory stored off the CPU.
  - RAM access is slower than register access (but is larger, as a tradeoff).
  - Data travels between RAM and the CPU via the bus.
  - Modern day RAM consists of in the neighbourhood of 10^{10} bytes.
  - Instructions occur in RAM starting with address 0 and increase by the word size (in our case 4).
    - But, this simplification will vanish later...
  - Each memory block in RAM has an address; say from 0 to $n-1$
  - Words occur every 4 bytes, starting with byte 0. Indexed by $0, 4, 8, ... n-4$.
  - Words are formed from consecutive, aligned (usually) bytes.
  - Cannot directly use the data in the RAM. Must transfer first to registers.
- Load word. Takes a word from RAM and places it into a register. Specifically, load the word in `MEM[$s + i]` and store in `$t`.

```
lw $t, i($s)
1000 11ss ssst tttt iiii iiii iiii iiii
# which is equivalent to
$t = MEM[$s + i]
# Example
lw $1, -4($30)
# which means
$1 <- MEM[$30 - 4]
```

- Store word. Takes a word from a register and stores it into RAM. Specifically, load the word in `$t` and store it in `MEM[$s + i]`.

```
sw $t, i($s)
1010 11ss ssst tttt iiii iiii iiii iiii
```

- Note that `i` must be an immediate, NOT another register! It is a 16-bit Two's complement immediate

- Example: Suppose that `$1` contains the address of an array of words, and `$2` takes the number of elements in this array (assume less than 220). Place the number 7 in the last possible spot in the array.

```
# First element in the array is arr, then the second element is arr + 4 ... the last element in the array is
arr + 4 * (length - 1)
lis $8 ; 7
.word 7
lis $9 ; 4
.word 4
mult $2, $9 ; length * 4
mflo $3 ; length * 4
add $3, $3, $1 ; arr + length 4
sw $8, -4($3) ; MEM[$3 - 4] = $8
jr $31
```

- Branch on equal.If `$s==$t` then `pc += i*4`. That is, skip ahead i many instructions if `$s` and `$t` are equal.

```
beq $s, $t, i
0001 00ss ssst tttt iiii iiii iiii iiii
# It is like
if ($s == $t) {
    PC += i*4
}
```

- Branch on not equal.If `$s!=$t` then `pc+=i*4`. That is, skip ahead i many instructions if `$s` and `$t` are not equal.

```
bne $s, $t, i
0001 01ss ssst tttt iiii iiii iiii iiii
# It is like
if ($t != $s) {
    PC += i*4
}
```

- Example:

```
beq $0, $0, 0 ; This executes the next instruction as PC has been updated to +4 already
beq $0, $0, 1 ; This executes the second next instruction
```

Lecture 4

- Write an assembly language MIPS program that places the value 3 in register `$2` if the signed number in register `$1` is odd and places the value 11 in register `$2` if the number is even.

```
lis $8 ; $8 = 2
.word 2
lis $9 ; $9 = 3
.word 3
lis $2 ; $2 = 11
.word 11
div $1, $8
mfhi $3
beq $3, $0, 1
add $2, $9, $0
jr $31
```

- Set Less Than. Sets the value of register `$d` to be 1 provided the value in register `$s` is less than the value in register `$t` and sets it to be 0 otherwise.

```
slt $d, $s, $t
0000 00ss ssst tttt dddd d000 0010 1010
# which basically means
if ($s < $t) {
    $d = 1 (true)
} else {
    $d = 0 (false)
}
```

- Example: Write an assembly language MIPS program that negates the value in register `$1` provided it is positive.

```
slt $2, $1, $0
bne $2, $0, 1
sub $1, $0, $1
jr $31
```

- Exercise: Write an assembly language MIPS program that places the absolute value of register $1inregister2$.

```
add $2, $1, $0 ; $2 = $1
slt $3, $0, $1 ; 0 < $1
bne $3, $0, 1
sub $2, $0, $2 ; $2 = 0 - $2
jr $31
```

- Looping exmaple: Write an assembly language MIPS program that adds together all even numbers from 1 to 20 inclusive. Store the answer in register `$3`.
    - Note: semicolons for comments in MIPS assembly

```
lis $2
.word 20
lis $1
.word 2
add $3, $0, $0
add $3, $3, $2 ; line -3
sub $2, $2, $1 ; line -2
bne $2, $0, -3 ; line -1 from here
jr $31
```

- Labels aren't machine code, so don't take words. That means that for `beq` and `bne`, *labels don't have "line numbers" on their own*. A label at the end of code is allowed. It has the address of what would be the first instruction after the program.

```
label: operation commands
```

- Example: `sample` has the address $0x4$, which is the location of `add $1, $0, $0`.

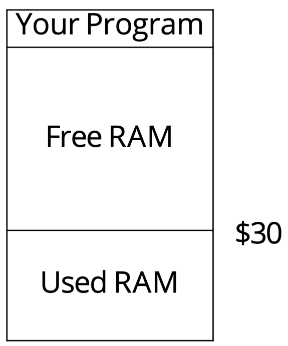
```
sub $3, $0, $0
sample:
add $1, $0, $0
```

- A better way to loop without hard-coding -3 in the previous example is (otherwise, if we were to, say, add a new instruction in between the lines specified by our branching, all our numbers would be incorrect.)
    - Note that top in bne is computed by the assembler to be the *difference between the program counter and top*. That is, here it computes $(top - PC)/4$ which is $(0\text{x}14 - 0\text{x}20)/4 = -3$
    - PC is the line number after the current line

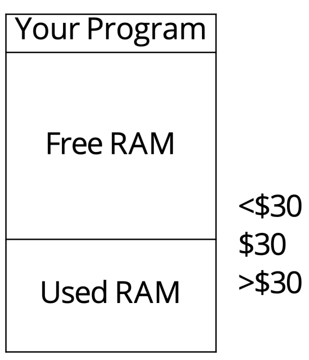
```
lis $2
.word 20
lis $1
.word 2
add $3, $0, $0
top:
    add $3, $3, $2
    sub $2, $2, $1
    bne $2, $0, top
jr $31
```

- RAM

- Register `$30` initially points to the very bottom of the free RAM. It can be used as a bookmark to separate the used and unused free RAM if we allocate from the one end, and push and pop things like a stack! In other words, we will use `$30` as a pointer to the top of a stack.
- Really, `$30` points to the top of the stack of memory in RAM.

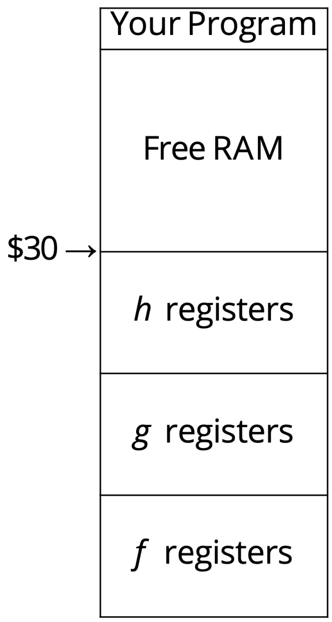


- Because our program is at zero, the stack grows from high memory to low memory, so pushing involves reducing the value of `$30`.



- Example: Suppose procedures f, g and h are such that:

```
f calls procedure g
    g calls procedure h
        h returns
    g returns
f returns.
```



- In the previous example:
  - Calling procedures pushes more registers onto the stack and returning pops them off.
  - This is a stack, and we call `$30` our **stack pointer**.
  - We can also use the stack for local storage if needed in procedures. Just reset `$30` before procedures return.
- Template for a procedure `f` that modifies registers `$1`and `$2`:

```
f:
sw $1, -4($30) ; Push registers we modify
sw $2, -8($30)
lis $2 ; Decrement stack pointer
.word 8
sub $30, $30, $2
    ; Insert procedure here
add $30, $30, $2 ; Assuming $2 is still 8
lw $2, -8($30) ; Pop = restore
lw $1, -4($30)
    ; Uh oh! How do we return?
```

- There is a problem with returning:

```
main:
lis $8
.word f ; Recall f is an address
jr $8 ; Jump to the first line of f
(NEXT LINE)
```

- Once f completes, we really want to jump back to the line labelled above as (NEXT LINE), i.e., set the program counter back to that line. How do we do that?
- Jump and Link Register. Sets `$31` to be the PC and then sets the PC to be `$s`. Accomplished by `temp = $s` then `$31 = PC` then `PC = temp`.

```
jalr $s
0000 00ss sss0 0000 0000 0000 0000 1001
```

- Main Changes

```
main:
    lis $8
    .word f
    sw $31, -4($30) ; Push $31 to stack
    lis $31 ; Use $31 since it’s been saved .word 4
    sub $30, $30, $31
    jalr $8 ; Overwrites $31
    lis $31 ; Use $31 since we’ll restore it .word 4
```

```
    add $30, $30, $31
    lw $31, -4($30) ; Pop $31 from stack
    jr $31 ; Return to loader
```

- Procedure Changes

```
f:
    sw $1, -4($30) ; Push registers we will modify
    sw $2, -8($30)
    lis $2
    .word 8
    sub $30, $30, $2 ; Decrement stack pointer
    ; Insert procedure here
    add $30, $30, $2 ; Assuming $2 is still 8
    lw $2, -8($30) ; Pop registers to restore
    lw $1, -4($30)
    jr $31 ; New line!
```

Lecture 5

- Note: there is NO default value to register, so remember to initialize it
- How do we pass parameters?

```
void f(int a, int b) {}
```

- Example: sumEvens1ToN adds all even numbers from 1 to N
  - `$1` Scratch Register (Should Save!)
  - `$2` Input Register (Should Save!)
  - `$3` Output Register (Do NOT Save!)

```
; The idea is:
; $3 = 0
; $1 = $2 % 2
; $2 = $2 - $1
; $1 = 2
; Top: $3 = $3 + $2
;      $2 = $2 - 2
;      if ($2 != 0) go to top
; jr $31

lis $1
.word 8
sw $1, -4($30)
sw $2, -8($30)
sub $30, $30, $1
add $3, $0, $0
lis $1
.word 2
div $2, $1
mfhi $1
sub $2, $2, $1
lis $1
.word 2
top:
add $3, $3, $2
sub $2, $2, $1
bne $2, $0, top
lis $1
.word 8
add $30, $30, $1
lw $2, -8($30)
lw $1, -4($30) ; Reload $1 and $2
jr $31 ; Back to caller
```

- Input and Output
  - We do this one byte at a time!
  - Output: Use `sw` to store words in location `0xffff000c`. Least significant byte will be printed.
  - Input: Use `lw` to load words in location `0xffff0004`. Least significant byte will be the next character from stdin.
  - Input/Output the ASICC character

```
lis $1
.word 0xffff000c
lis $2
.word 48 ; In ASCII code, 48 means 0!
sw $2, 0($1)
```

- Example: Printing CS241 to the screen followed by a newline character:

```
lis $1
.word 0xffff000c
lis $2
.word 67 ; C
sw $2, 0($1)
lis $2
.word 83 ; S
sw $2, 0($1)
lis $2
.word 50 ; 2
sw $2, 0($1)
lis $2
sw $2, 0($1)
lis $2
.word 49 ; 1
sw $2, 0($1)
lis $2
.word 10 ; \n
sw $2, 0($1)
jr $31
```

- Part of our long-term goal is to convert assembly code (our MIPS language) into machine code (bits).
    - Input: Assembly code
    - Output: Machine code
- Any such translation process involves two phases: Analysis and Synthesis.
    - Analysis: Understand what is meant by the input source
    - Synthesis: Output the equivalent target code in the new format
- What if a label is used before it is defined? We don't know the address when it's used!
    - Perform two passes:
        - Pass 1: Group tokens into instructions and record addresses of labels (data structure?).
        - Note: multiple labels are possible for the same line! For example, `f: g: add $1, $1, $1`.
        - Pass 2: translate each instructions into machine code. If it refers to a label, look up the associated address compute the value.
- A label at the end of code is allowed (it would be the address of the first line after your program).
- Our instruction (`bne`) can be broken down as follows:

Opcode (6 bits)	Register s (5 bits)	Register t (5 bits)	Offset (16 bits)

- Only the offset part `i` is signed, others are unsigned. That's why we need to do bit masking for `i` with `0xFFFF` (We do not want the leading 1 will overwrite our results if `i` is negative)!
- We can use bit shifting to put information into the correct position, and use a bitwise or to join them:

```
int instr = (5 << 26) | (2 << 21) | (0 << 16) | offset
```

- Recall in C++, ints are 4 bytes. We only want the last two bytes. First, we need to apply a "mask" to only get the last 16 bits:

```
offset = -3 & 0xffff
```

- Printing Bytes in C++

```
int instr = (5 << 26) | (2 << 21) | (0 << 16) | (-3 & 0xffff); unsigned char c = instr >> 24;
cout << c;
c = instr >> 16;
cout << c;
c = instr >> 8;
cout << c;
c = instr;
cout << c; // will output the least sinificant 8 bits
```

- Note: You can also mask here to get the 'last byte' by doing & 0xff if you're worried about which byte will get copied over.

Lecture 6

- Definition: An alphabet is a non-empty, finite set of symbols, often denoted by Σ (capital sigma).
- Definition: A string (or word) w is a finite sequence of symbols chosen from Σ. The set of all strings over an alphabet Σ is denoted by $\Sigma*$.
- Definition: A language is a set of strings.
- Definition: The length of a string w is denoted by $|w|$.
- Since an alphabet is a set of "symbols" (which is vague), and a language is a set of words, a language can be the alphabet of another language
- Examples - Alphabets:
  - $\Sigma = a, b, c, \ldots, z$, the Latin (English) alphabet.
  - $\Sigma = 0, 1$, the alphabet of binary digits.
- Examples - Strings:
  - ϵ (epsilon) is the empty string. It is in $\Sigma*$ for any Σ. $|\epsilon| = 0$
  - For $\Sigma 0, 1$, strings include $w = 011101$ or $x = 1111$. Note $|w| = 6$ and $|x| = 4$.
  - For our course, assume Σ will never contain the symbol ϵ. ϵ is just a notational convention; the actual string is empty.
- Examples - Languages:
  - $L = \emptyset$ or , the empty language
  - $L = \epsilon$, the language consisting of (only) the empty string
- Why are finite languages are easy to determine membership?
  - To determine membership in a language, just check for equality with all words in the language!
- Definition: A regular language over an alphabet Σ consists of one of the following:
  - The empty language and the language consisting of the empty word are regular.
  - All languages a for all $a \in \Sigma$ are regular.
  - The union, concatenation or Kleene star (pronounced klay-nee) of any two regular languages are regular.
  - Nothing else.
- Basically, if L is finite, L is regular.
- Let L, L_1 and L_2 be three regular languages. Then the following are regular languages
  - Union: $L1 \cup L2 = x : x \in L_1 \text{ or } x \in L_2$
  - Concatenation: $L_1 \cdot L_2 = L_1 L_2 = xy : x \in L_1, y \in L_2$
  - Kleenestar:$L* = \epsilon \cup xy : x \in L^*, y \in L = \cup_{n=0}^{\infty} L^n$
- Error States in CS 241: if a bubble does not have a valid arrow leaving it, we assume this will transition to an error state.
- Definition: A DFA(Deterministic Finite Automata) is a 5-tuple $(\Sigma, Q, q0, A, \delta)$:
  - Σ is a finite non-empty set (alphabet).
  - Q is a finite non-empty set of states.
  - $q0 \in Q$ is a start state
  - $A \subseteq Q$ is a set of accepting states
  - $\delta : (Q \times \Sigma) \to Q$ is our [total]transition function (givena state and a symbol of our alphabet, what state should we go to?).
- Rules for DFAs
  - States can have labels inside the bubble. This is how we refer to the states in Q.
  - For each character you see, follow the transition. If there is none, go to the (implicit) error state.
  - Once the input is exhausted, check if the final state is accepting. If so, accept. Otherwise reject.

Section 2: Tutorials

Tutorial 1

- What is Binary?
  - Binary - ways our machines encode info, and $b \in 0, 1$
- Eg. what is 1000 be
  - $2^3 = 8$ unsigned magnitude

- -8 2's complement
- $[T, F, F, F]$ array of bools
- "backspace char" in ASCII
- **Representation matters**

- Unsigned binary: n-bit binary number is represented as $b_{n-1}, b_{n-2}, \ldots, b_0$, where $b \in 0, 1$
- To convert to decimal: $2^{n-1} \times b_{n-1} + \ldots + 2^0 \times b_0$
- Decimal to binary
    - Idea 1: take the highest powers of 2 from the decimal (inefficient)
    - Idea 2: Repeatedly divide the number by 2, tracking the quotient & remainder
    - Eg. Convert 23 to binary

Number	Quotient	Remainder
23/2	11	1
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

        - and read from bottom to up, that will be $10111_2 = 23_{10}$
- 2's complement
    - Range of values for n-ary
        - Unsigned Binary $0 \sim 2^n - 1$
        - 2's complement $-2^{n-1} \sim 2^{n-1} - 1$
- Convert decimal to 2's complement
    - if number is ≥ 0: use the unsigned representation
    - if number is < 0:
        - Get the binary rep of the positive number
        - flip the bits
        - then add 1
- Convert from 2's complement to decimal
    - Method 1: if $b_{n-1} = 1$, then flip the bits, add 1 and negate the positive decimal
    - Method 2: Treat $b_{n-1} - > -2^{n-1} b_{n-1}$, and add the rest as unsigned representation
- Assembly
    - CS 241: MIPS
        - Runs programs $ stores its data all in MEM (RAM)
        - 32 bits system where instructions are encoded as 4 bytes (1 word)
        - Registers hold 1 word of info
        - Special registers
            - 0 = 0$, immutable
            - 31$, end address in RAM, `jr $31` means return address
            - 3, 29,30$
            - $iii \ldots$ <- 2's complement number
                - `divu, multu, addi ...` treats the register values as unsigned binary
    - **Programs live in the same sapce in MEM (RAM) as as the data they operate on**
        - PC cannot distinguish the two
    - Fetch-Execute Cycle

```
PC = 0x00
while True do:    // until PC = $31
    IR = MEM[PC]
    PC = PC + 4
    ... decode $ execute IR ...
done
```

- Constant Values
    - Use the Load Immediate Skip command (`lis $s`) followed by an instruction to save into `$s`.

- `lis $s` -> skip the next instruction -> store that instruction into `$s`
- Use with `.word i` to store `i` into `$s`
- Eg. Store 10 into `$5`

```
lis $5
.word 10
; the above two lines of commands skips the .word 10 & sets $5 = 10
```

- Machine Code (based on the order of machine code horizontally for `lis $5`)
  - 000000 (operating code)
  - 00000 (`$s`)
  - 00000 (`$t`)
  - 00101 (`$d`)
  - 00000 (dead code, always be 0)
  - 010100 (function code)
- Machine Code (`.word 10`)
  - `00000 ....... 000 001010`

Tutorial 2

- Recall: Fetch-Execute Cycle

```
PC = 0x00
while PC != $31 do
    IR = MEM[PC]
    PC = Pc + 4
    ... run IR's command ...
done
```

- Loops
  - Use `bne`/`beq`
  - such that, `bne $s, $t, i` and `beq $s, $t, i`, and $PC = PC + 4 * i$
  - i = 2's complement or label
- eg. write a program that a ≥ 0 # n in `$1` and store $n!$ into `$3`

```
lis $3
.word 1
loop: beq $1, $0, end
      mult $3, $1
      mflo $3 ; $3*$1
      lis $11
      .word 1
      sub $1, $1, $11
end: jr $31
```

MIPS Array

- mips.array
  - `$1` = address of the start of your array `Arr`
  - `$2` = length of `Arr`
- eg. `Arr = [1,2,3]`
  - `$1` $= 0 \times$ ..., `$2`$= 3$
  - element 1: `MEM[$1 + 4*1]`, element 2: `MEM[$1 + 4*2]`, element 3: `MEM[$1 + 4*3]`
  - the end of the array is `$1 + 4 * $2`
- eg. write a program that returns the product of all elements in `$1` to `$3`

```
lis $1
.word 1
lis $4
.word 4
mult $2, $4
mflo $2 ; $2 = $2 * 4
add $2, $1, $2 ; $2 = $1 + 4 * $2
```

```
loop: beq $1, $2, end
      lw $5, 0($1) ; Arr[i] = *Arr
      mult $3, $5
      mflo $3 ; $3 = $3 * $5
      add $1, $1, $4 ; i++
      beq $0, $0, loop
end: jr $31
```

Stack

- `$30` = Stack Pointer
- Initially, out of bounds address (since it is very last of the memory)
- grow backwards in MEM
- Idea: Preservation
    - Ensures the user/client that only the expected registers are mutated
- Push

```
sw $1, -4($30)
lis $1
.word $4
sub $30, $30, $1
.
. code program
.
```

- Pop

```
.
. code program
.
lis $1
.word 4
add $30, $30, $1
lw $1,  -4($30) ; $1 = old value
```

- Eg. write the factorial program (eg 1) but ensure all registers aside from $3 are preserved

```
sw $1, -4($30)
sw $11, -8($30)
sw $4, -12($30)
list $4
.word 12
sub $30, $30, $4
.
. factorial code from the previous example
.
lis $4
.word 12
add $30, $30, $4
lw $1, -4($30)
lw $11, -8($30)
lw $4, -12($30)
jr $31
```

Procedures & Recursion

- Procedure ≈ function names
    - Label with code & a `jr` often represent our function area
    - return value is equivalent to `$3`
- Recursion
    - Stack & `jalr`
    - `jalr $s`: `$31`=PC, PC=`$s`
    - Push `$31` & all parameters
    - `jalr` procedure
    - Popping `$31`
- eg. factorial with recursion

```
fact:
sw $1, -4($30)
sw $11, -8($30)
sw $31, -12($30)
lis $31
.word 12
sub $30, $30, $31

lis $11
.word 1

bne $1, $0, recur
add $3, $11, $0 ; base case: Set $3 = 1 & unwind
beq $0, $0, unwind ; base case

recur: ; call fact($1 - 1)
sub $1, $1, $11 ; $1 = $1 - 1
lis $31
.word fact
jalr $31 ; return to from the base case
add $1, $1, $11 ; return here after jr $31, restore old $1
mult $3, $1 ; Multiply previous answer by $1 to get new factorials
mflo $3

unwind:
lis $31
.word 12
add $30, $30, $31
lw $1, -4($30)
lw $11, -8($30)
lw $31, -12($30)
jr $31 ; jump to jalr or terminates
```

Section 3: Reviews