

# Computational Complexity Theory

**Brandt Bucher**

**June 22nd, 2024**









 On the first day   
 of Christmas... 











1780



18



18

**18, 22**



**18, 22, 25**

**18, 22, 25, 28**

**18, 22, 25, 28,  
31**

**18, 22, 25, 28,  
31, 34**

**18, 22, 25, 28,  
31, 34, 37**

**18, 22, 25, 28,  
31, 34, 37, 40**

18, 22, 25, 28,  
31, 34, 37, 40,  
43

**18, 22, 25, 28,  
31, 34, 37, 40,  
43, 46**



**18, 22, 25, 28,  
31, 34, 37, 40,  
43, 46, 49**

**18, 22, 25, 28,  
31, 34, 37, 40,  
43, 46, 49, 52!**

**verse = 52**

**verse = 52**  
**song = 425**

$$\text{verse}(x) = 3x + 16$$

$$\text{verse}(x) = 3x + 16$$

$$\text{song}(x) = \text{verse}(1)$$

$$\begin{aligned}\text{verse}(x) &= 3x + 16 \\ \text{song}(x) &= \text{verse}(1) + \\ &\quad \text{verse}(2)\end{aligned}$$

$$\begin{aligned}\text{verse}(x) &= 3x + 16 \\ \text{song}(x) &= \text{verse}(1) + \\ &\quad \text{verse}(2) + \text{verse}(3)\end{aligned}$$



$$\text{verse}(x) = 3x + 16$$

$$\text{song}(x) = \text{verse}(1) + \text{verse}(2) + \text{verse}(3) + \dots$$

$$\text{verse}(x) = 3x + 16$$

$$\text{song}(x) = \text{🧮}$$

$$\text{verse}(x) = 3x + 16$$

$$\text{song}(x) = 1.5x^2 + 17.5x - 1$$



$$\text{verse}(x) = 3x + 16$$

$$\text{song}(x) = 1.5x^2 + 17.5x - 1$$

$$\text{verse}(x) = mx + b$$

$$\text{song}(x) = 1.5x^2 + 17.5x - 1$$

$$\text{verse}(x) = \nearrow$$


$$\text{song}(x) = 1.5x^2 + 17.5x - 1$$

**verse(x) = ↗**

**song(x) =  $ax^2 + bx + c$**



**verse(x) = ↗**  
**song(x) = ↶**

verse(x) = ↗ =   
song(x) = ↶

verse(x) = ↗ = 🚶  
song(x) = ↶ = 🚶

verse(x) = ↗ = 🚶  
song(x) = ↶ = 🧗

**verse(365) = 1,111**

**verse(365) = 1,111**  
**song(365) = 206,224**

**verse(365) = 20× longer**  
**song(365) = 206,224**

**verse(365) = 20× longer**  
**song(365) = 500× longer!**










1780

1840







 **Verse-Chorus**   
 **Form!** 



Fixed-Size  
Verses!

verse(x) = ↗ = 🚶  
song(x) = ↶ = 🧗

verse(x) = → =   
song(x) = ↗ = 

verse(x) = → = 🧘  
song(x) = ↗ = 🧗

verse(x) = → = 🧘

song(x) = ↗ = 🧗

verse(x) = → = 🧘  
song(x) = ↗ = 🚶

**verse(365) = 20× longer**  
**song(365) = 500× longer!**

**verse(365) = same length**  
**song(365) = 500× longer!**



**verse(365) = same length**  
**song(365) = 30× longer**











verse(x)

= → =



song(x)

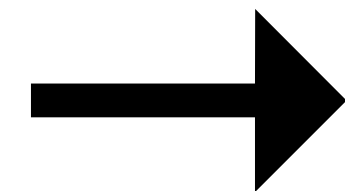
= ↗ =





verse(x)

=

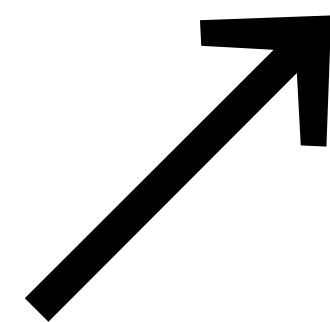


=



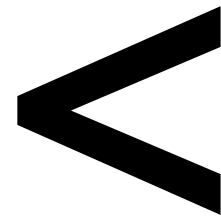
song(x)

=



=









1840

1890

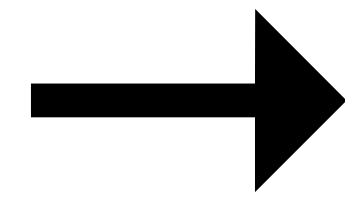


 99 bottles of beer   
 on the wall... 



verse(x)

=

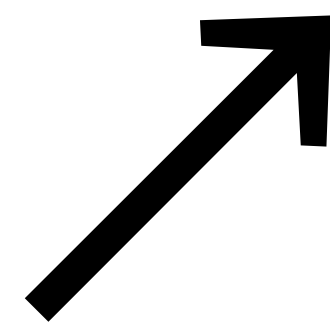


=





song(x)



=






=



 **verse(x) = → =** 

 **song(x) = → =** 

 **verse(x) = → =** 

 **song(x) = → =** 