Astron 98 Final Project: Determining a Dark-Matter Density Equation for the Milky Way and other Spiral Galaxies

1. Introduction:

In my project, I aim to develop a method of modeling the density equation of dark matter in a given galaxy, and apply this procedure to the Milky Way Galaxy to determine the equation parameters within a reasonable margin of error.

2. Chosen Phenomenon and Data Source:

Dark matter affects the velocity of stars in galaxies. Because mass is what holds a galaxy together, after calculating the expected mass of stars in the milky way, it can be found that there isn't enough mass holding it and other galaxies together from luminosity alone. Therefore, there is a distribution of "dark matter" throughout galaxies holding it together, and the amount corresponds to the missing mass to keep objects orbiting at specific velocities. I primarily used the Gaia Archive from the European Space Agency (https://gea.esac.esa.int/archive/) to collect information on the galactic coordinates and orbital velocity of tens of thousands of stars in the Milky Way. The source of data is millions large, so I had to parse through several different search features in order to generate a balanced collection of data from the entire set. I imported data on 20000 galaxies that were within 10kpc from the earth, and another 10000 that were between 10kpc and 25kpc.

3. Equation to Fit Data:

I will be fitting quite a few different equations in order to eventually solve one involving dark matter. First of all, however, my original data collected by data only includes the Parallax, Galactic Latitude and Longitude, Proper Motion, and Radial Motion. Using the following equation, I converted the Equatorial Coordinates into Galactic:

$$\vec{R}_{hel} = \begin{pmatrix} d\cos(b)\cos(l) \\ d\cos(b)\sin(l) \\ d\sin(b) \end{pmatrix}, \qquad \vec{R}_0 = \begin{pmatrix} R_0 \\ 0 \\ 0 \end{pmatrix}$$

Then, I will be fitting an equation between the Mass within the radius of each star's orbit, and the radius that it draws out. These values are linked through Newton's Laws, specifically gravity:

$$V_c \approx \sqrt{\frac{GM}{R}}$$

The equation I fit will be M(enc) as a function of the radius, and after plotting the data points for these values, I will likely consider a linear or quadratic relationship.

At which point I would have a solution for the equation of M(Renc).

Next, I will use an equation I derived in 7B for the Luminous Mass enclosed within a radius r in the galaxy:

$$M(R) = 21 M_\odot / L_\odot \left(205000 e^{-7.67(R/2000)^{1/4}}\right)$$

This is derived from an Approximate Luminosity Ratio : 21 SolarM / SolarL, an equation for the Brightness of an elliptical called the Valcouleurs Profile:

$$I(r) = I_0 \, e^{-(r/r_0)^{1/4}}.$$

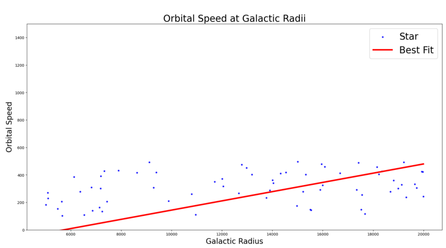
Based on this equation, I could create a Mass(Renc) equation that I can use to calculate the remaining missing mass.

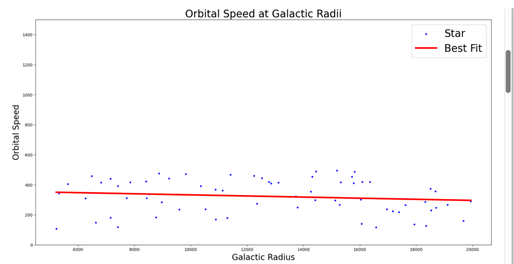
Next, I will subtract this mass from the total mass, graph the data, and match an equation to the remaining mass.

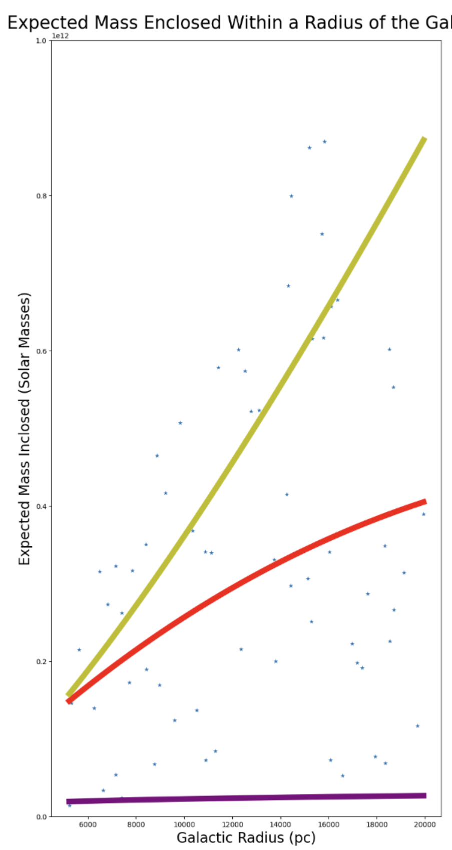
The most likely equation I could use to fit the data will be a simple coefficient and power relationship because of how I expect the velocity will be related to the radius in an earlier equation.

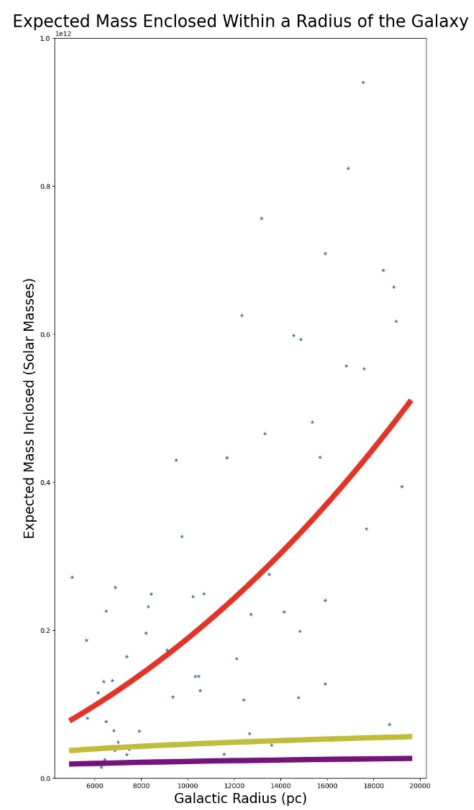
4.Data Generation for Testing

I tested all of my code initially with a random set of numbers within a reasonable range for the orbital velocity and galactic radius. Here are some examples:









5.Data Filtering:

Here are the steps I took to ensure my data was well collected:

A] limiting the range of my data set to where I had an actual significant amount of data.

- Ylim, Xlim function when necessary
- Cropping my data to farther than 5000 and closer than 20000 pc from the center of the galaxy.
- The behavior of the dark matter at the center of the galaxy is worth categorizing separately

due to more unique phenomena occurring (black holes, etc).

B] When initially collecting data from Gaia, I put it through several filters to make sure I avoided

collecting lopsided data.

Query:

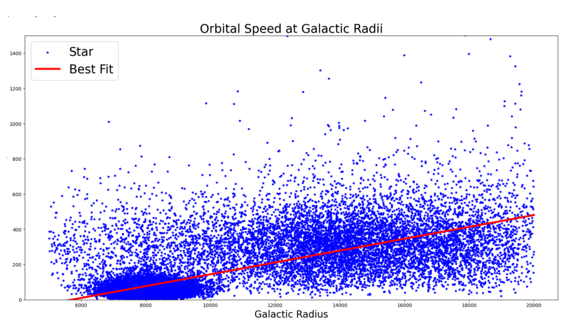
```
SELECT TOP 20000
gaia_source.source_id,gaia_source.ra,gaia_source.parallax,gaia_source.pm,gaia_source.radi
FROM gaiadr3.gaia_source WHERE (gaia_source.parallax_over_error BETWEEN 30 AND 60
AND gaiadr3.gaia_source.radial_velocity BETWEEN -1000 AND 1000 AND
gaiadr3.gaia_source.parallax BETWEEN -1 AND 16 AND gaiadr3.gaia_source.dec
BETWEEN -45 AND 45 AND gaiadr3.gaia_source.distance_gspphot BETWEEN 0 AND 16000
AND gaiadr3.gaia_source.ra BETWEEN 0 AND 360 AND gaiadr3.gaia_source.pm BETWEEN
-16000 AND 16000 AND gaiadr3.gaia_source.l BETWEEN 0 AND 360 AND
gaiadr3.gaia_source.b BETWEEN -60 AND 60)
```

C] Lastly, collecting such a large sample size of 30000 stars allowed me to let the calculations that I made create some outliers, but there was still a heavy presence of data at the center of my data fits.

6. Data Fitting:

Firstly, I had to fit the data for velocity to radius, because that would be quite revealing how the rest of the research would follow:

The plot was roughly linear, so I used a np.polyfit function to track the data. Now, I had an equation for the orbital velocity of the star in terms of its radius, which would be valuable for reducing the error in later parts of the fitting.



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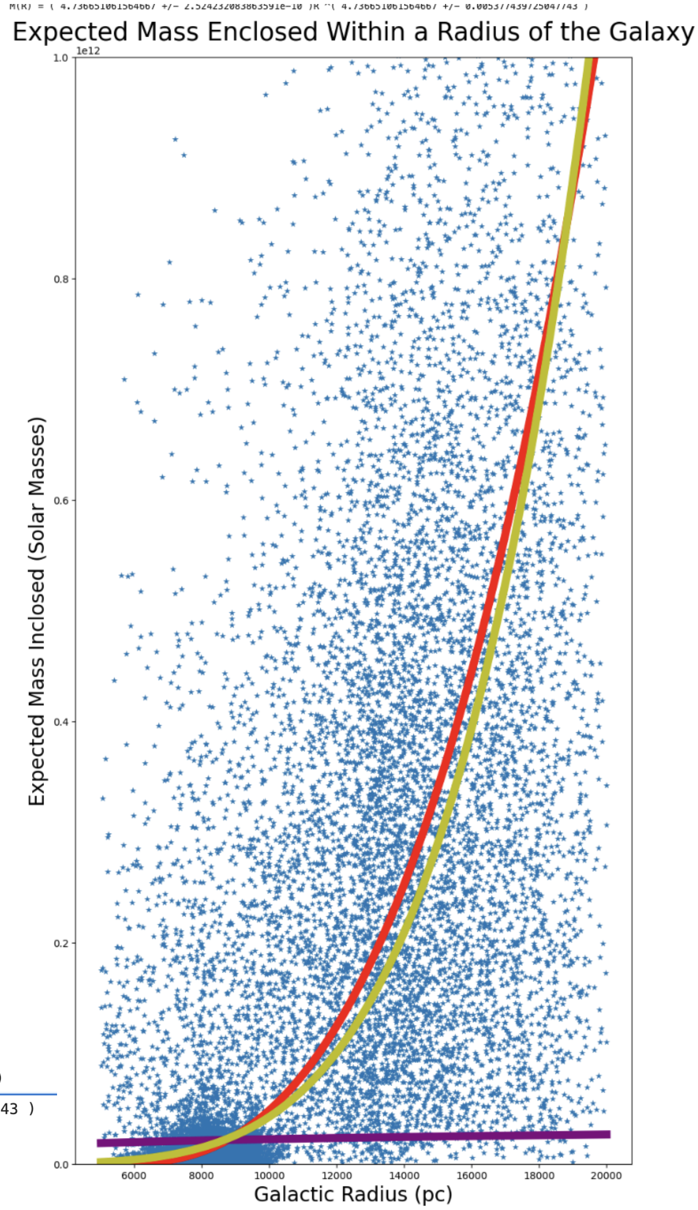
Next, I graphed the total mass enclosed based on proven numerical relationships, luminous mass enclosed, and started to approach a way to find the dark matter mass per radius.

Upon plotting the data, it was clearly exponential, so I decided to use scipy.optimize to determine a function for M(r) of dark matter in the form Pnaught * Radius ^ n power.

This is the result of plotting the red (totalmass), yellow(darkmatter), and purple(luminousmass). The total mass is an overwhelming amount more than the luminous mass, and this has to be made up for by a greater amount of dark matter. This could have been hypothesized earlier when the orbital velocity was essentially constant with orbital radius, Implying there was much more mass than we could not see making farther objects rotate faster. All of the blue stars in the backdrop represent the expected mass they have holding them in, however, despite it seeming spread, the axis are on very large scales. The equation I fit for mass with scipy was:M(R) = (4.736651061564667 +/- 2.524232083863591e-10)
* R ^(4.736651061564667 +/- 0.005377439725047743)

The plus minuses were determined using the cov feature in curve fit, and squaring the diagonals to find the possible error in the parameters. The error for both is extremely low, less than 1% for n and hardly a fraction of the coefficient.

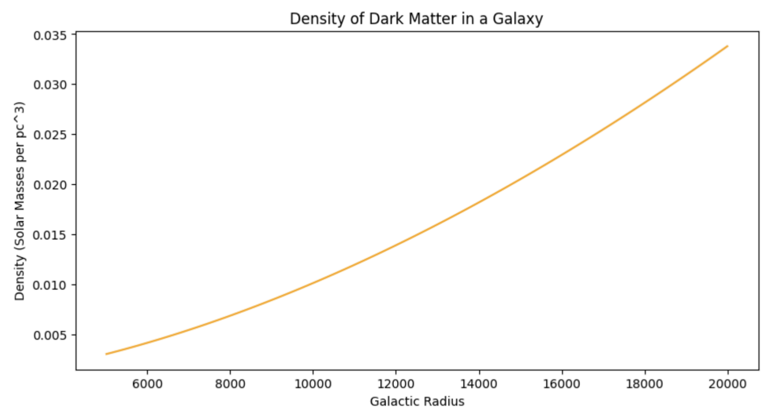



```
56 print('M(R) = (',str(popt[1]),'+/-',str(P0_error),')R ^(',str(popt[1]),'+/-',str(n_error),')')
```

```
M(R) = ( 4.736651061564667 +/- 2.524232083863591e-10 )R ^( 4.736651061564667 +/- 0.005377439725047743 )
```

Finally, I converted the Mass Equation into a density one because it simply means dividing by the volume of the sphere, 4pi/3 R^3:

It appears that the density of dark matter increases only marginally throughout the galaxy, hardly reaching a percent of solar mass for every cubic parsec.



7. Fitting Conclusions:

First off, orbital velocity is nearly constant with Galactic Radius, and at the least proportional beyond 5 kpc. The equation I fit showed that there was a direct proportion between the radius and the orbital velocity of the stars in our universe. Next, the total mass in the galaxy is mostly dark matter. Most of the mass in our galaxy is not even where we can see it. Specifically, the mass of dark matter scales proportionately with radius to the 4.5 power, so when dividing by volume, R^3, the density of dark matter nearly increases linearly with distance from the center of the galaxy, but closer to the 3/2 Power.

8.Final Conclusions

These simple relationships of dark matter distribution could just prove to be the key to unraveling the mystery around dark matter. We still know nothing about it, other than what it does, and exploring the patterns associated with it will be the key to further defining it into existence. This process could be easily repeated given the accurate data of stars in similar elliptical galaxies, and compared to contribute to our understanding of this relationship as a feature of dark matter in general, or just our galaxy.

Ultimately, my data and tests reveal one astrophysical relationship between regular and dark matter, however, the repetition of this test in various other conditions and galaxies will prove to be a greater pattern of insight into the behavior of dark matter.