

Midterm 1, STAT 620, Fall 2024

Instructions: You have 100 minutes to complete the exam. Write all solutions on your own paper or Markdown document, including all pertinent work for full credit. **Clearly label questions and parts** and make sure your name and netID is written on your solutions. Partial credit will only be given where work is shown. Please box/bold all answers and include any code used. This is an open book, open notes exam but there is to be no use of AI and no internet browsing other than course resources on Canvas. Once complete, scan your solutions or render to PDF and upload them to Canvas. Up to 10 points will be taken off if you fail to follow directions. Good luck!

1. [24 points] Consider 3 draws taken at random and without replacement from a standard deck of cards.
 - a) What is the probability that all three draws are the same suit?
 - b) What is the probability that the third draw is an Ace?
 - c) If the second and third draws are Aces, what is the probability that the first draw was an Ace?
 - d) Are the events “Draw a King on the first draw” and “Draw a Diamond on the second draw” independent?
2. [28 points] Consider a random variable (X) with the following probability mass function (PMF):

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 1 \\ 0.3, & \text{if } x = 2 \\ 0.4, & \text{if } x = 3 \\ 0.2, & \text{if } x = 4 \end{cases}$$

- a) Verify that this is a valid probability mass function.

- b) Calculate the expected value ($E[X]$) and variance ($V[X]$) of the random variable X using the definitions.
- c) Find the moment generating function (MGF) of X , $M_X(t)$.
- d) Use the moment generating to find the variance of X and compare this to what you calculated in part (b).
3. [24 points] A basketball player has a 70% chance of making a free throw. Assume that each free throw is independent of the others. The player takes 10 free throws.
- a) What is the probability that the player makes exactly 7 free throws out of 10?
- b) What is the probability that the player makes fewer than 5 free throws out of 10?
- c) What is the probability that the player makes at least 1 but no more than 4 successful free throws?
- d) Find the expected number and standard deviation of successful free throws the player will make out of 10.
4. [24 points] A bakery receives an average of 4 customers per hour.
- a) What is the probability that more than 5 customers enter the bakery in the next 2 hours?
- b) What is the probability that no customers enter the bakery in the next 30 minutes?
- c) If the bakery is open for 8 hours, what is the expected value and standard deviation for the number of customers for the entire day?
- d) Fully justify why the distribution you used for this problem is appropriate.