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STAT 620
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TTH - 8- 9:40pm

1.2-2

(a) E_1 = Experiment 1 with one combination of temperature, pressure, and catalyst

n_1: Temperature levels = 4
n_2: Pressure levels = 5

E_1 = Experiment 1 with $(\text{Temp}_1, \text{Pressure}_1, \text{Catalyst}_1)$

multiplication Principal $(T, P, C) = (5)(4)(2) = 40$

> There would need to be a total of 40 experiments conducted

(b) E_1 = Experiment 1 with (Temp, Pressure, Catalyst)

with 2 levels is $(2)(2)(2)$ or $2^3 = 8$

> There are 8 possible combinations with each factor at two levels

1.2-10 Pascal's triangle

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots
\end{array}
$$

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$

$(a+b)^{n-1}$

LHS

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

RHS

$$\binom{n-1}{r} + \binom{n-1}{r-1} = \frac{(n-1)!}{r!(n-1)-r!} + \frac{(n-1)!}{(r-1)!(n-1)-(r-1)!}$$

$$= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!}$$

$$= \frac{(n-1)!(n-r)}{r!(n-r)!} + \frac{(n-1)!\cdot r}{r!(n-r)!}$$

$$= \frac{(n-1)!(n-r) + (n-1)!\cdot r}{r!(n-r)!}$$

$$= \frac{(n-1)!\cdot[(n-r)+r]}{r!(n-r)!} = \frac{(n-1)!\cdot n}{r!(n-r)!}$$

$$\frac{n\cdot(n-1)!}{r!(n-r)!}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad \blacksquare$$

$$\boxed{\therefore \binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}}$$

{19 white, 10 tan, 7 pink, 3 purple, 5 yellow, 2 orange, 6 green}

1.2-16

(a) $S = \{$ All possible combinations of 9 hearts from 52 $\}$

P(9 hearts: 3W and 6 non-white)

$\binom{n}{r} = \binom{19}{3}$ white hearts

Must take into account 6 other colors and 33 other pieces

$\binom{n}{r} = \binom{33}{6}$ non-white hearts

$$P(E) = \frac{\binom{19}{3} \cdot \binom{33}{6}}{\binom{52}{9}} = \boxed{0.2917128015}$$

(b) P(9 hearts: 3white, 2 tan, 1 pink, 1 yellow, 2 green)

$$P(E) = \frac{\binom{19}{3} \cdot \binom{10}{2} \binom{7}{1} \cdot \binom{5}{1} \cdot \binom{6}{2}}{\binom{52}{9}}$$

$$= \boxed{0.0062223854}$$

1.4-2 Let $P(A) = 0.3$ and $P(B) = 0.6$

(a) Find $P(A \cup B)$ when A and B are independent

$P(A \cup B) = 1 - P[(A \cup B)^c]$ Theorem 1.1-1

$P(A^c) = 1 - P(A)$
$= 1 - 0.3 = 0.7$

$P(B^c) = 1 - P(B)$
$= 1 - 0.6 = 0.4$

$$P(A\cup B) = 1 - P[(A\cup B)^c]$$
$$= 1 - P(A^c)\cdot P(B^c)$$
$$= 1 - (0.7)(0.4)$$
$$= \boxed{0.72}$$

(b) Find $P(A|B)$ when A and B are mutually exclusive

Def 1.3-1 $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$ provided $P(B) > 0$

However due to them being mutually exclusive (disjoint)
$A_i \cap A_j = \emptyset$

$$\therefore \frac{P(A\cap B)}{P(B)} = \frac{0}{0.6} = \boxed{0}$$

1.4-15
(a) with replacement

Urn = {10 red balls, 10 white balls}

$P(4^{th}$ white ball drawn $4^{th})$

$$P(W_1W_2W_3W_4) = P(W_1\cap W_2\cap W_3\cap W_4) = P(W_1)\cdot P(W_2)\cdot P(W_3)\cdot P(W_4)$$
$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$$
$$= \boxed{\frac{1}{16}}$$

$P(4^{th}$ white ball drawn $5^{th}) = (R_1W_1W_2W_3W_4)$

$(W_1R_1W_2W_3W_4)(W_1W_2R_1W_3W_4)(W_1W_2W_3R_1W_4)$

$P(R_1 W_1 W_2 W_3 W_4)$ can have 4 other possible outcomes

$$4[P(R_1 W_1 W_2 W_3 W_4)] = 4 \cdot P(R_1)\, P(W_1)\, P(W_2)\, P(W_3)\, P(W_4)$$
$$= 4\left(\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\right)$$
$$= 4\left(\tfrac{1}{2^5}\right)$$
$$= \boxed{1/8}$$

$P(4^{th}$ white ball drawn $6^{th})$

$$P(R_1 R_2 W_1 W_2 W_3 W_4) = P(R_1 \cap R_2 \cap W_1 \cap W_2 W_3 \cap W_4)$$

number of distinct ways to arrange 2 red balls among first 5 positions is $\binom{5}{2} = 10$

$$10[P(R_1 \cap R_2 \cap W_1 \cap W_2 W_3 \cap W_4)] = 10[P(R_1)\cdot P(R_2)\cdot P(W_1)\cdot P(W_2)\cdot P(W_3)\cdot P(W_4)]$$
$$= 10\left[\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\right]$$
$$= 10\left[\tfrac{1}{2^6}\right]$$
$$= \boxed{\tfrac{5}{32}}$$

$P(4^{th}$ white ball drawn $7^{th})$

$$P(R_1 R_2 R_3 W_1 W_2 W_3 W_4\}$$

$$= P(R_1 \cap R_2 \cap R_3 \cap W_1 \cap W_2 \cap W_3 \cap W_4)$$

number of distinct ways to arrange 3 red balls among first 6 positions is $\binom{6}{3} = 20$

$20\left[\cdot P(R_1)\cdot P(R_2)\cdot P(R_3) P(W_1)\cdot P(W_2)\cdot P(W_3)\cdot P(W_4)\right]$

$= 20\left(\frac{1}{2^7}\right) = \boxed{\frac{5}{32}}$

(b) Without replacement

Urn = { 10 red balls, 10 white balls }

P(4th white ball drawn 4th)

$P(W_1 W_2 W_3 W_4) = \binom{10}{4}$ ways to draw 4 white balls

Drawing four balls out of 20 $\binom{20}{4}$

$$\frac{\binom{10}{4}}{\binom{20}{4}} = \boxed{\frac{14}{323}}$$

$P(4^{th}$ white ball drawn $5^{th}) = P(R_1 \cap W_1 \cap W_2 \cap W_3 \cap W_4)$

$4\left[P(R_1 W_1 W_2 W_3 W_4)\right]$ can have 4 other possible outcomes

$$= 4\left(\frac{10}{20}\cdot\frac{10}{19}\cdot\frac{9}{18}\cdot\frac{8}{17}\cdot\frac{7}{16}\right) = \boxed{\frac{35}{323}}$$

P(4th white ball drawn 6th)

$P(R_1 R_2 W_1 W_2 W_3 W_4) = P(R_1 \cap R_2 \cap W_1 \cap W_2 W_3 \cap W_4)$

number of distinct ways to arrange 2 red balls among first 5 positions is $\binom{5}{2} = 10$

$$= 10\left(\frac{10}{20}\cdot\frac{9}{19}\cdot\frac{10}{18}\cdot\frac{9}{17}\cdot\frac{8}{16}\cdot\frac{7}{15}\right) = \boxed{\frac{105}{646}}$$

$P(4^{th}$ white ball drawn $7^{th})$

$P(R_1 R_2 R_3 W_1 W_2 W_3 W_4) = P(R_1 \cap R_2 \cap R_3 \cap W_1 \cap W_2 \cap W_3 \cap W_4)$

number of distinct ways to arrange 3 red balls among first 6 positions is $\binom{6}{3} = 20$

$$20\left(\frac{10}{20} \cdot \frac{9}{19} \cdot \frac{8}{18} \cdot \frac{10}{17} \cdot \frac{9}{16} \cdot \frac{8}{15} \cdot \frac{7}{14}\right) = \boxed{\frac{60}{323}}$$

	4-Games	5-Games	6-Games	7-Games
(c)	$\frac{21}{109}$	$\frac{25}{109}$	$\frac{24}{109}$	$\frac{39}{109}$

No the urn model is not a good representation of this exercise. This is because the events are not equally likely. Since the number of games of differing length series are not equiprobable. For example observing a four game series is $\frac{21}{109}$ and observing a 5 game series is $\frac{25}{109}$.

1.5-11

15% heavy smokers, 30% light smokers, 55% non-smokers

Death Rate

heavy $\longrightarrow$ 5 X non-smokers

light $\longrightarrow$ 3 X non-smokers

non-smoker = n

died = d

heavy smoker = h

ligh smoker = L

solve for this

P(non-smoker | died)

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}.$$

Baye's Theorem

$P(\text{non-smoker} \mid \text{died}) =$

$$\frac{P(n) \cdot P(d \mid n)}{P(h) \cdot P(d \mid h) + P(L) \cdot P(d \mid L) + P(n) \cdot P(d \mid n)}$$

$$= \frac{.55 \cdot X}{(.15)(5X) + (.3)(3X) + (.55)(X)}$$

$$= \frac{.55X}{.75X + .9X + .55X}$$

$$= \frac{.55X}{2.2X} = 1/4$$

= The probability that the participant was a non-smoker is 1/4

2.1-3

(a) $f(x) = x/c, \quad x = 1, 2, 3, 4$

Definition 2.1-2 (a) $f(x) > 0, \quad x \in S;$

(b) $\sum_{x \in S} f(x) = 1;$

(c) $P(X \in A) = \sum_{x \in A} f(x)$, where $A \subset S$

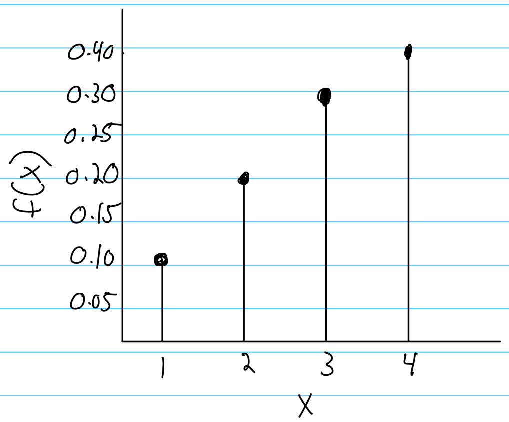
$$\sum_{x=1}^{4} f(x) = 1 \qquad \sum_{x=1}^{4} \frac{x}{c} = \frac{1}{c} + \frac{2}{c} + \frac{3}{c} + \frac{4}{c} = 1$$

$$= \frac{10}{c} = 1$$

$$c = 10$$

So now $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{10}{10} = 1$

$f(1) = \frac{1}{10}$
$f(2) = \frac{2}{10}$
$f(3) = \frac{3}{10}$
$f(4) = \frac{4}{10}$



(b) $f(x) = cx, \quad x = 1, 2, 3, \ldots, 10$

$$\sum_{x=1}^{10} f(x) = 1 = \sum_{x=1}^{10} cx$$

$$= c(1+2+3+4+5+6+7+8+9+10) = 1$$

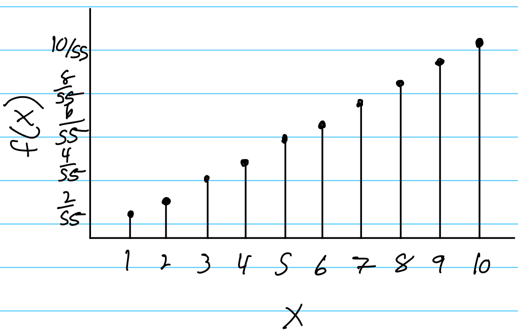
$$= c(55) = 1$$

$$= 55c = 1$$

$$c = \frac{1}{55}$$

$$\sum_{x=1}^{10} = \frac{1}{55} + \frac{2}{55} + \frac{3}{55} + \frac{4}{55} + \frac{6}{55} + \frac{7}{55} + \frac{8}{55} + \frac{9}{55} + \frac{10}{55} = \frac{55}{55} = 1$$

$f(1) = \frac{1}{55}$
$f(2) = \frac{2}{55}$
$\vdots$
$f(10) = \frac{10}{55}$



(c) $f(x) = c\left(\frac{1}{4}\right)^x \quad , \quad x = 1, 2, 3, \ldots$

$$\sum_{x=1}^{\infty} f(x) = 1$$

$$\sum_{x=1}^{\infty} c\left(\frac{1}{4}\right)^x = c \sum_{x=1}^{\infty} \left(\frac{1}{4}\right)^x = 1$$

$$= c \cdot \frac{\frac{1}{4}}{1-\frac{1}{4}} = 1$$

$$= c \cdot \frac{\frac{1}{4}}{\frac{3}{4}} = 1 \Rightarrow c\left(\frac{1}{3}\right) = 1 \Rightarrow c = 3$$

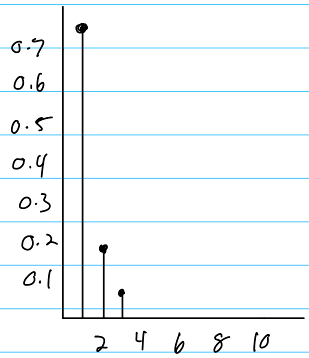
$$f(x) = 3\left(\frac{1}{4}\right)^x, \quad \text{for } x = 1, 2, 3, \ldots$$

$f(1) = 3\left(\frac{1}{4}\right)^1 = 3/4$

$f(2) = 3\left(\frac{1}{4}\right)^2 = 3/16$

$f(3) = 3\left(\frac{1}{4}\right)^3 = 3/64$

$\vdots$



(d) $f(x) = c(1+x)^2, \quad x = 0, 1, 2, 3.$

$$\sum_{x=1}^{\infty} c(1+x)^2$$

$$= c \sum_{x=1}^{\infty} (1+x)^2 = c[(0+1)^2 + (1+1)^2 + (2+1)^2 + (3+1)^2]$$

$$f(0) = (1+0)^2$$

$$= 1$$

$$f(1) = (1+1)^2$$
$$= (2)^2$$
$$= 4$$

$$f(2) = (2+1)^2$$
$$= (3)^2$$
$$= 9$$

$$f(3) = (3+1)^2$$
$$= (4)^2$$
$$= 16$$

$$c(1+4+9+16)$$
$$= 30c = 1$$
$$= c = \frac{1}{30}$$

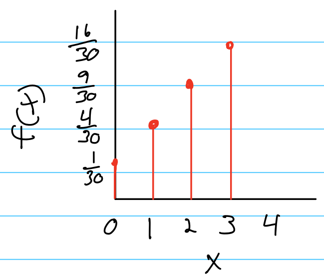
$$f(x) = \frac{(x+1)^2}{30}, \quad \text{for } x = 1, 2, 3.$$

$$f(0) = \frac{1^2}{30} = \frac{1}{30}$$

$$f(1) = \frac{2^2}{30} = \frac{4}{30}$$

$$f(2) = \frac{3^2}{30} = \frac{9}{30}$$

$$f(3) = \frac{4^2}{30} = \frac{16}{30}$$



(e) $f(x) = \frac{x}{c}, \qquad x = 1, 2, 3, \ldots, n$

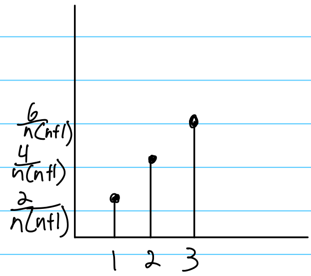
$$\sum_{x=1}^{n} f(x) = \frac{1}{c} \sum_{x=1}^{n} x$$

$$= \frac{1}{c} \cdot \frac{n(n+1)}{2} = 1$$

$$= \frac{n(n+1)}{2} = c$$

$$f(x) = \frac{2x}{n(n+1)} \qquad \text{for } x = 1, 2, 3, \ldots, n$$

$$f(1) = \frac{2}{n(n+1)}$$



$$f(2) = \frac{4}{n(n+1)}$$

$$f(3) = \frac{6}{n(n+1)}$$

(f) $f(x) = \frac{c}{(x+1)(x+2)}, \qquad x = 0, 1, 2, 3, \ldots$

$$\sum_{x=1}^{\infty} f(x) = \sum_{x=1}^{\infty} \frac{c}{(x+1)(x+2)}$$

$$= c \sum_{x=1}^{\infty} \frac{1}{(x+1)(x+2)}$$

$$= \left(\frac{1}{x+1} - \frac{1}{x+2}\right) \Rightarrow \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots$$

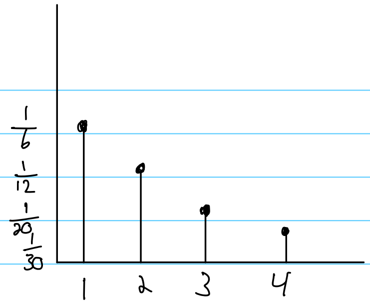
$$= 1 - \lim_{x \to \infty} \frac{1}{x+2} = 1 - 0 = 1$$

$$f(1) = \frac{1}{(1+1)(1+2)} = \frac{1}{6}$$

$$f(2) = \frac{1}{(2+1)(2+2)} = \frac{1}{12}$$

$$f(3) = \frac{1}{(3+1)(3+2)} = \frac{1}{20}$$

$$f(4) = \frac{1}{(4+1)(4+2)} = \frac{1}{30}$$



2.1-10 $W = X+Y$

(a) Determine the Pmf of W

Let X = die 1 and Y = die 2

$P(X=0) = \frac{2}{4}$, $P(X=2) = \frac{2}{4}$

$P(Y=0) = \frac{1}{4}$

$P(Y=1) = \frac{1}{4}$

$P(Y=4) = \frac{1}{4}$

$P(Y=5) = \frac{1}{4}$

$W = X+Y$

$(X=0, Y=0) \Rightarrow W = 0+0 = 0$

$(X=0, Y=1) \Rightarrow W = 0+1 = 1$

$(X=0, Y=4) \Rightarrow W = 0+4 = 4$

$(X=0, Y=5) \Rightarrow W = 0+5 = 5$

$(X=2,\ Y=0) = W = 2+0 = 2$

$(X=2,\ Y=1) = W = 2+1 = 3$

$(X=2,\ Y=4) = W = 2+4 = 6$

$(X=2,\ Y=5) = W = 2+5 = 7$

$P(W=0) = P(X=0)\cdot P(Y=0) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$P(W=1) = P(X=0)\cdot P(Y=1) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$P(W=2) = P(X=2)\cdot P(Y=0) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$P(W=3) = P(X=2)\cdot P(Y=1) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$P(W=4) = P(X=0)\cdot P(Y=4) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

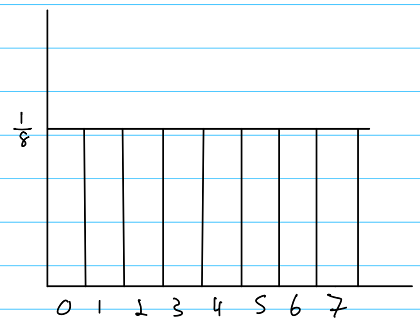
$P(W=5) = P(X=0)\cdot P(Y=5) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$P(W=6) = P(X=2)\ P(Y=4) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$P(W=7) = P(X=2)\ P(Y=5) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

$$\boxed{\therefore\ f(x) = \frac{1}{8},\ x \in \{0,1,2,3,4,5,6,7\}}$$

(b) Draw a probability histogram of W



2.1-11

$$f(x) = \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}, \quad x = 0, 1, 2, \ldots$$

Find $P(X \geq 4 \mid X \geq 1)$

$$P(X \geq 4 \mid X \geq 1) = \frac{P(X \geq 4 \cap X \geq 1)}{P(X \geq 1)}$$

conditional Probability

$$= \frac{P(X \geq 4)}{P(X \geq 1)}$$

might be easier to calculate complements

$$P(A^c) = 1 - P(A) = P(A) = 1 - P(A^c)$$

$$P(X \geq 1) = 1 - P(X = 0) \qquad \frac{1}{(0+1)(0+2)} = \frac{1}{2}$$

$$P(X \geq 4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

$$= 1 - \frac{1}{(0+1)(0+2)} - \frac{1}{(1+1)(1+2)} - \frac{1}{(2+1)(2+2)} - \frac{1}{(3+1)(3+2)}$$

$$= 1 - \frac{1}{2} - \frac{1}{6} - \frac{1}{12} - \frac{1}{20}$$

$$= \frac{1}{5}$$

$$P(X \geq 4 \mid X \geq 1) = \frac{P(X \geq 4 \cap X \geq 1)}{P(X \geq 1)}$$

$$= \frac{P(X \geq 4)}{P(X \geq 1}$$

$$= \frac{\frac{1}{5}}{\frac{1}{2}}$$

$$= \boxed{\frac{2}{5}}$$