HW 9

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Section 5.6

5.6-1

The formula for the mean of the uniform distribution is

$$\mu = \frac{a+b}{2}$$

$$\mu = \frac{0+1}{2} = \frac{1}{2}$$

The variance formula is

$$\sigma^2 = \frac{(b-a)^2}{12}$$

$$\sigma^2 = \frac{(1-0)^2}{12} = \frac{1}{12}$$

$$\frac{\frac{1}{\sqrt{12}}}{\sqrt{12}} = \frac{1}{12}$$

z-score formula

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_x}$$

$$z_1 = \frac{\frac{1}{2} - \frac{1}{2}}{\frac{1}{12}} = 0$$

$$z_2 = \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{12}} = 2$$

$$P(\frac{1}{2} \leq \bar{x} \leq \frac{2}{3}) = P(z_1 < 2) - P(z_2 < 0)$$

```
0.9772 - 0.5000
```

```
## [1] 0.4772
```

$$\boxed{0.4772}$$

5.6-8

a.

The central limit theorem states that the sampling distribution of the sample mean is approximately normal with mean μ_x and standard deviation σ_x

$\mu_{\bar{x}} = \mu$

Thus $\boxed{\mu_{\bar{x}} = 24.43}$

b.

The formula for variance of $\bar{x}\ is\ \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

```
2.20/30
```

```
## [1] 0.07333333
```

$\boxed{\sigma_{\bar{x}}^2 = 0.07333333}$

c.

z-score formula

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_x}$$

z_1

```
(24.17 - 24.43) / (sqrt(0.07333333))
```

```
## [1] -0.9601137
```

z_2

```
(24.82 - 24.43) / (sqrt(0.07333333))
```

```
## [1] 1.44017
```

$$P(24.17 \leq \bar{x} \leq 24.82)$$

```
0.9251 - 0.1685
```

```
## [1] 0.7566
```

$\boxed{0.7566}$

5.6-14

Formula for mean of X is:

$$\mu_X = n * \mu$$

```
20 * 10
```

```
## [1] 200
```

Formula for standard deviation is

$$\sigma_X = \sqrt{n} * \sigma$$

```
sqrt(20) * 2
```

```
## [1] 8.944272
```

```
1- 0.2
```

```
## [1] 0.8
```

The value of z-score from the table is 0.84 In order to get X X = $\mu + z * \sigma$

```
200 + 0.84 * 8.944272
```

```
## [1] 207.5132
```

$\boxed{207.5132}$

Section 5.7

5.7-4

$$P(35 \leq X \leq 40) = P(34.5 < X < 40.5)$$

$$= P(\frac{34.5-36}{\sqrt{9}} < \frac{X-36}{\sqrt{9}} < \frac{40.5-36}{\sqrt{9}})$$

$$= P(-0.5 < W < 1.50)$$

$$= P(W < 1.50) - P(W < -0.50)$$

```
0.9332 - 0.3085
```

```
## [1] 0.6247
```

$\boxed{0.6247}$

5.7-9

a.

$$P(15 < \sum_{i=1}^{30} X \leq 22) = P(15 < \sum_{i=1}^{30} X < 22.5)$$

$$= P(\frac{15 - 30 * \frac{2}{3}}{\sqrt{30 * \frac{2}{3}}}) < \frac{\sum_{i=1}^{30} X - 30 * \frac{2}{3}}{\sqrt{30 * \frac{2}{3}}} < \frac{22.5 - 30 * \frac{2}{3}}{\sqrt{30 * \frac{2}{3}}}$$

$$= P(\frac{-5}{\sqrt{20}} < Y < \frac{2.5}{\sqrt{20}})$$

$$= P(Y < 0.56) - P(Y < -1.01)$$

```
0.7123-0.1562
```

```
## [1] 0.5561
```

$\boxed{0.5561}$

b.

$$= P(\frac{20.5 - 30 * \frac{2}{3}}{\sqrt{30 * \frac{2}{3}}}) < \frac{\sum_{i=1}^{30} X - 30 * \frac{2}{3}}{\sqrt{30 * \frac{2}{3}}} < \frac{26.5 - 30 * \frac{2}{3}}{\sqrt{30 * \frac{2}{3}}}$$

$$= P(0.11 < Y < 1.45)$$

$\boxed{0.3823}$

5.7-16

p=0.1

$$P(-1.5 < Y - 10 < 1.5) \approx \phi(\frac{1.5}{3}) - \phi(-\frac{1.5}{3})$$

```
0.6915 - 0.3085
```

```
## [1] 0.383
```

$\boxed{0.383}$

p=0.5

$$P(-1.5 < Y - 10 < 1.5) \approx \phi(\frac{1.5}{5}) - \phi(-\frac{1.5}{5})$$

```
0.6179 - 0.3821
```

```
## [1] 0.2358
```

$\boxed{0.2358}$

p=0.8

$$P(-1.5 < Y - 10 < 1.5) \approx \phi(\frac{1.5}{4}) - \phi(-\frac{1.5}{4})$$

```
0.6462 - 0.3538
```

```
## [1] 0.2924
```

$\boxed{0.2924}$

Section 5.8

5.8-1

a.

Chebyshev's inequality

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Given:

$$\mu = 33$$

$$\sigma^2 = 16$$

$$P(23 < X < 43) = P(|X - 33| < 10)$$

$$k = \frac{10}{\sigma} = \frac{10}{4} = 2.5$$

$$P(|X - \mu| < 10) \geq 1 - \frac{1}{(2.5)^2}$$

$$P(|X - \mu| < 10) \geq 1 - \frac{1}{6.25}$$

$\boxed{0.84}$

b.

$$k = \frac{14}{\sigma} = \frac{14}{4} = 3.5$$

$$P(|X - 33| \geq 14) \leq \frac{1}{(3.5)^2}$$

$$P(|X - 33| \geq 14) \leq \frac{1}{(12.25)}$$

$$P(|X - 33| \geq 14) \leq 0.0816$$

$\boxed{0.0816}$

5.8-6

$$P(75 < \bar{X} < 85)$$

$$E(\bar{X}) = \mu = 80$$

$$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{60}{15} = 4$$

$$k = \frac{5}{\sigma_{\bar{X}}} = \frac{5}{2} = 2.5$$

$$P(|\bar{X} - \mu| < k\sigma_{\bar{X}}) \geq 1 - \frac{1}{(k)^2}$$

$$P(|\bar{X} - 80| < 5) \geq 1 - \frac{1}{(2.5)^2}$$

$$P(|\bar{X} - 80| < 5) \geq 1 - \frac{1}{6.25}$$

$\boxed{0.84}$

Section 5.9

5.9-3

$$M_{S^2}(t) = M_{\frac{\sigma^2}{n-1}\chi_{n-1}^2}(t)$$

$$= M_{\chi_{n-1}^2}(\frac{\sigma^2}{n-1} * t)$$

$$= (1 - 2 * \frac{\sigma^2}{n-1} * t)^{-(\frac{n-1}{2})}$$

$$= (1 + \frac{\frac{\sigma^2}{-(n-1)}}{2} * t)^{-(\frac{n-1}{2})}$$

$$lim_{n->\infty} M_{S^2}(t) = lim_{n->\infty}(1 + \frac{\frac{\sigma^2}{-(n-1)}}{2} * t)^{-\frac{(n-1)}{2}}$$

$$= (1 - \frac{2t\sigma^2}{n-1})^{\frac{-(n-1)}{2}}$$

$$= e^{\sigma^2 t}$$

$$Therefore,\ \boxed{lim_{n->\infty}(1 + \frac{x}{n})^n = e^x}$$

5.9-4

$$Y = X_1 + X_2 + ... + X_n$$

$$X_i \approx \chi_1^2$$

$$E(X_i) = 1$$

$$V(X_i) = 2$$

$$E(Y) = E(\sum_{i=1}^{n} X_i)$$

$$= \sum_{i=1}^{n} E(X_i)$$

$$= \sum_{i=1}^{n} 1 = n$$

$$V(Y) = V(\sum_{i=1}^{n} X_i)$$

$$= \sum_{i=1}^{n} V(X_i)$$

$$= \sum_{i=1}^{n} 2 = 2n$$

$$W = \frac{Y - E(Y)}{\sqrt{V(Y)}}$$

$$= \frac{Y - n}{\sqrt{2n}}$$

$$= \boxed{\frac{Y - n}{\sqrt{2n}} \sim N(0, 1)}$$