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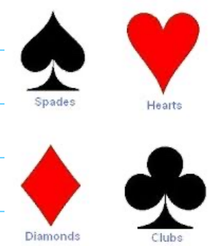
STAT 620-02
Dr. Kerr
TTH: 8-9:40pm
HW #1

1.1-3) $A = \{X : X \text{ is a jack, queen or king}\}$
$B = \{X : X \text{ is a 9, 10, jack and } x \text{ is red}\}$
$C = \{X : X \text{ is a club}\}$
$D = \{X : X \text{ is a diamond, heart, or a spade}\}$



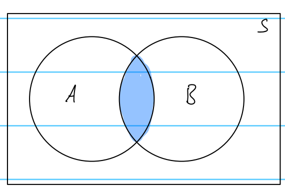
Find (a) $P(A)$

Sample Space S is the set of $m = 52$ different cards.
If A is the set of outcomes that are kings, then $P(A) = \frac{4}{52}$
If A is the set of outcomes that are jack, then $P(A) = \frac{4}{52}$
If A is the set of outcomes that are queen, then $P(A) = \frac{4}{52}$

$$P(A) = \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{156} = \boxed{\frac{1}{13}}$$

$$\boxed{\therefore P(A) = \frac{3}{13}}$$

(b) $P(A \cap B)$ Only elements that are in both A and B



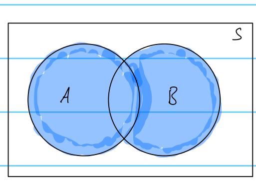
$A \cap B$

$E = \{\text{red jack}\}$

If B is the set of all outcomes that are jacks that are red then $P(A \cap B) = P(B) = \frac{2}{52} = \frac{1}{26}$

$$\boxed{\therefore P(A \cap B) = \frac{1}{26}}$$

(c) $P(A \cup B)$ Whatever is in A has to be in B.



$A \cup B$

Theorem 1.1-5 states
if A and B are any two events,
then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$B = \{x : x \text{ is a } 9, 10, \text{ jack and } x \text{ is red}\}$

If B is the set of outcomes that are 9 and red then $P(A) = \frac{2}{52}$
If B is the set of outcomes that are 10 and red then $P(A) = \frac{2}{52}$
If B is the set of outcomes that are jacks and red then $P(A) = \frac{2}{52}$

$P(B) = \frac{6}{52} = 3/26$

$P(A) + P(B) - P(A \cap B)$

$\frac{3}{13} + \frac{3}{26} - \frac{1}{26} = \frac{6}{26} + \frac{3}{26} - \frac{1}{26} = \frac{8}{26} = \frac{4}{13}$

$\boxed{\therefore P(A \cup B) \text{ is } 4/13}$

d) $P(C \cup D)$

$C = \{x : x \text{ is a club}\}$ $\qquad P(C) = 13/52$

$D = \{x : x \text{ is a diamond, heart, or a spade}\}$

If D is the set of outcomes that are a diamond $= 13/52$
If D is the set of outcomes that are a heart $= 13/52$
If D is the set of outcomes that are a spade $= 13/52$

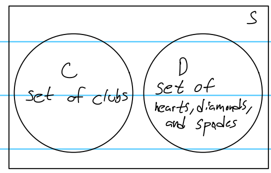
$P(D) = 39/52$

Theorem 1.1-5 states
if C and D are any two events,
then $P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$P(C) + P(D) - P(C \cap D)$ Since there are no cards in both C and D we have the empty set

$13/52 + 39/52 - 0 = \frac{52}{52} = \boxed{1}$

e) $P(C \cap D)$

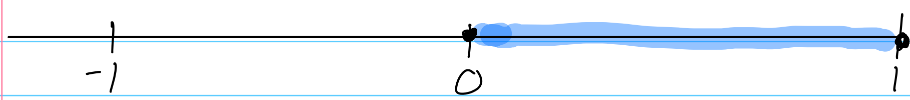


$C \cap D = \emptyset$

$$\boxed{\therefore P(C \cap D) = \emptyset}$$

1.1-12

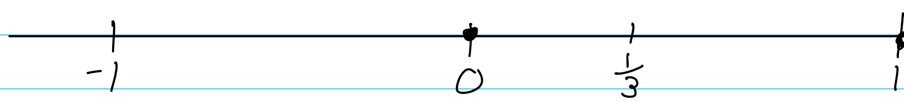
(a) $P(\{X : 0 \leq X \leq 1/3\})$ $\quad E = \{[0, \tfrac{1}{3}]\}$



$P(0 \leq X \leq 1/3) = \frac{1}{3} - 0 = \frac{1}{3}$

$$\boxed{\therefore P(0 \leq X \leq 1/3) = 1/3}$$

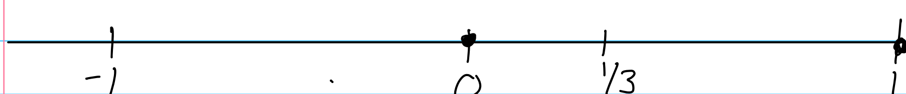
(b) $P(\{X : 1/3 \leq X \leq 1\})$ $\quad E = \{[\tfrac{1}{3}, 1]\}$



$P(1/3 \leq X \leq 1) = 1 - 1/3 = \frac{3}{3} - \frac{1}{3} = 2/3$

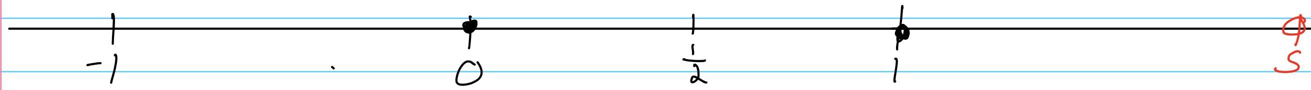
$$\boxed{\therefore P(1/3 \leq X \leq 1) = 2/3}$$

(c) $P(\{X : X = 1/3\})$



$P(X = \frac{1}{3}) = 0$ This is because there are infinite points in the interval $[0, 1]$. But the probability of landing on any specific point is approximately $\boxed{0}$

(d) $P(\{x: 1/2 < x < 5\})$



$P(\frac{1}{2} < X < 5) = 1 - 1/2 = 1/2$

$\boxed{\therefore P(1/2 < X < 5) = 1/2}$

1.1-15

(a) If $P(A_1) = P(A_2) = \ldots = P(A_m)$, show that $P(A_i) = 1/m$, $i = 1, 2, \ldots m$

mutually exclusive $A_i \cap A_j = \emptyset$, $i \neq j$

$A_1, A_2, \ldots, A_K$ are disjoint

mutually exclusive exhaustive events

we know $A_1 \cup A_2 \cup \ldots \cup A_K = S$

We know from Axiom (b) $P(S) = 1$

Given $\Rightarrow$ $A_1 \cup A_2 \cup \ldots \cup A_m = S$

Proof

$P(A_1 \cup A_2 \cup \ldots \cup A_m) = P(S) = 1$

We also know $P(A_1 \cup A_2 \cup \ldots \cup A_m) = P(A_1) + P(A_2) + \ldots + P(A_m) = 1$

Because $P(A_1) = P(A_2) = \ldots P(A_m)$

Let $P(A_i)$ for $i = 1, 2, \ldots, m$

$P(A_1) + P(A_2) + \ldots P(A_m) = p + p + p + \ldots p$ which happens m times

so $1 = m \cdot p$

$\Rightarrow \frac{1}{m} = p$

we can say $P(A_i) = \frac{1}{m}$ for $i = 1, 2, \ldots, m$ ∎

(b) If $A = A_1 \cup A_2 \cup \ldots \cup A_h$, where $h < m$, and (a) holds prove that $P(A) = h/m$

$$P(A) = P(A_1 \cup A_2 \cup \ldots \cup A_h)$$

mutually exclusive $P(A) = P(A_1) + P(A_2) + \ldots + P(A_h)$

from part (a) $P(A_i) = \frac{1}{m}$

$$\therefore \; P(A) = \underbrace{\frac{1}{m} + \frac{1}{m} + \ldots + \frac{1}{m}}_{h \text{ times}}$$

$$P(A) = h \cdot \left(\frac{1}{m} + \frac{1}{m} + \ldots + \frac{1}{m}\right)$$

$$P(A) = \frac{h}{m} \quad ∎$$

1.3-1

(a) $P(B_1)$

	B_1: Has HIV Virus	B_2: Does NOT Have HIV Virus	Total
A_1: Test Pos	4885	73,630	78,515
A_2: Test Neg	115	921,370	921,485
Totals	5000	995,000	1,000,000

$E = \{5000 \text{ ppl w/HIV virus}\}$

$$P(B_1) = 5{,}000/1{,}000{,}000 = 0.005 \text{ or } \frac{1}{200}$$

$$\boxed{\therefore P(B_1) = \frac{1}{200}}$$

$E = \{78515 \text{ positive tests}\}$

(b) $P(A_1) = 78{,}515/1{,}000{,}000 = 0.078515$

$$\boxed{\therefore P(A_1) = 0.078515}$$

(c) $P(A_1 | B_2)$ $E = \{73630 \text{ False Positive tests}\}$

$P(A_1 | B_2) = 73{,}630/995{,}000 = 0.074$ or $\boxed{\frac{37}{500}}$

(d) $P(B_1 | A_1) = 4885/78{,}515 = \boxed{0.0622174107}$

(e) In part c what $P(A_1|B_2)$ is telling us is that given that you do not have the HIV virus you have a 7.4% chance of testing positive. In medical terms this is known as a false positive

In Part D what $P(B_1|A_1)$ is telling us is that given a positive test result and having the HIV virus is about 6.2%.

1.3-3	B_1	B_2	Totals
A_1	5	7	12
A_2	14	9	23
Totals	19	16	35

(a) $P(A_1 \cap B_1)$ multiplication rule $S = \{35 \text{ students}\}$

$E = \{5 \text{ LE dom, LH dom}\}$

$P(A \cap B) = P(A)P(B|A)$

$P(A_1) = 12/35$ $P(B_1 | A_1) = 5/12$

$P(A_1 \cap B_1) = \frac{12}{35} \cdot \frac{5}{12} = \boxed{1/7}$

(b) $P(A_1 \cup B_1)$ $E = \{(12 \angle E \text{ dom}), (19 \angle H \text{ dom})\}$

Theorem 1.1-5 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A_1) + P(B_1) - P(A_1 \cap B_1)$

$= 12/35 + 19/35 - 1/7$

$\boxed{= 26/35}$

(c) $P(A_1 | B_1)$

Definition of conditional Probability

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} \rightarrow P(A_1 \cap B_1) = P(A_1) \cdot P(B_1|A_1)$

$\dfrac{12}{35} \cdot \dfrac{5}{12}$

$\dfrac{5/35}{19/35} = \dfrac{5}{\cancel{35}_1} \cdot \dfrac{\cancel{35}^1}{19} = \boxed{\dfrac{5}{19}}$

(d) $P(B_2 | A_2)$ $E = \{9 RH \text{ dom}, RE \text{ dom}\}$

$P(B_2 | A_2)$ Conditional Probability

$\dfrac{P(B_2 \cap A_2)}{P(A_2)} \rightarrow P(B_2 \cap A_2) = P(B_2) \cdot P(A_2 | B_2)$

$\dfrac{\cancel{16}^1}{35} \cdot \dfrac{9}{\cancel{16}_1}$

$\dfrac{9}{35}$

$\dfrac{9/35}{\frac{23}{35}} = \dfrac{9}{\cancel{35}_1} \cdot \dfrac{\cancel{35}^1}{23}$

$= \boxed{9/23}$

(e) Right-eye dominant (A_2)

If we hoped to select a right-eye dominant student we need to calculate left thumb on top B_1 and right thumb on top B_2.

$$P(A_2 | B_1)$$

Definition of conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A_2 \cap B_1) = P(A_2) \cdot P(B_1 | A_2)$$

$$= \frac{\cancel{23}}{35} \cdot \frac{14}{\cancel{23}}$$

$$\frac{14/35}{\frac{19}{35}} = \frac{14}{35} \cdot \frac{35}{19} = \frac{14}{19} \approx 0.73684$$

$$P(A_2 | B_2)$$

Definition of conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A_2 \cap B_2) = P(A_2) \cdot P(B_2 | A_2)$$

$$\frac{23}{35} \cdot \frac{9}{23} = \frac{9}{35}$$

$$\frac{\frac{9}{35}}{\frac{16}{35}} = \frac{9}{35} \cdot \frac{35}{16} = \frac{9}{16} \approx 0.5265$$

After calculating the probability of a right eye dominant student you would be more likely to select one given left thumb on top with $14/19 \approx 0.737$.

You would select left thumb on top.

1.3-10 D1 D2 D3 D4 D5 D6

Picking one card from each of six decks

$\frac{52}{52}$ $\frac{51}{52}$ $\frac{50}{52}$ $\frac{49}{52}$ $\frac{48}{52}$ $\frac{47}{52}$

unique possibilities

$$P(A) = \frac{52}{52} \cdot \frac{51}{52} \cdot \frac{50}{52} \cdot \frac{49}{52} \cdot \frac{48}{52} \cdot \frac{47}{52} = \frac{1.46581344E10}{1.977060966E10}$$

$$\approx \boxed{0.7414103385}$$

(b) Probability at least 2 cards match

From example 1.1-3 it might be easier to calculate $P(A^c)$

We know the $P(A) \approx 0.7141$

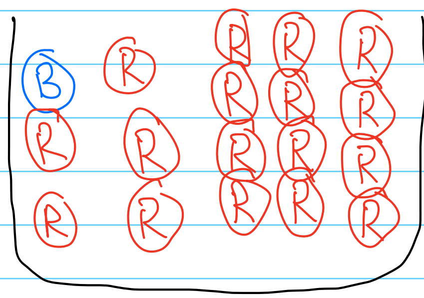
Thm 1.1-1 says $P(A) = 1 - P(A^c)$
$+P(A^c)$ $+P(A^c$

We wanna solve for $P(A^c)$ $P(A^c) + P(A) = 1$ From properties (b) and (c)

$$P(A^c) = 1 - P(A)$$

$$1 - P(A)$$
$$= 1 - 0.7141$$
$$= \boxed{0.2585896617}$$

1.3-12



$S = \{r_{17}, r_{16}, r_{15}, \dots, r_1, b_1\}$

$E = \{\text{blue chip}\}$

(a) choosing to go 1st, 5th, or Last

$P(b_1) = \boxed{\frac{1}{18}}$

$P(b_5) = P(b_1^c \cap b_2^c \cap b_3^c \cap b_4^c \cap b_5)$

$= \frac{17}{18} \cdot \frac{16}{17} \cdot \frac{15}{16} \cdot \frac{14}{15} \cdot \frac{1}{14} = \boxed{\frac{1}{18}}$

$P(b_{18}) = P(b_{18}^c \cap b_{17}^c \cap \dots \cap b_1)$

$= \frac{17}{18} \cdot \frac{16}{17} \cdot \frac{15}{16} \cdot \frac{14}{15} \cdot \frac{13}{14} \cdot \frac{12}{13} \cdot \frac{11}{12} \cdot \frac{10}{11} \cdot \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{7}{8} \cdot \frac{6}{7} \cdot \frac{5}{6} \cdot \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1}$

$= \boxed{1/18}$

The probability of drawing the blue chip is $\frac{1}{18}$ no matter the order. Therefore, 1st, 5th or last are the same!

(b) $P(b_1) = \frac{2}{18} = \boxed{\frac{1}{9}}$

$P(b_2) = P(b_1 \cap b_1) + P(r_1 \cap b_1) = \left(\frac{2}{18} \cdot \frac{1}{17}\right) + \left(\frac{16}{18} \cdot \frac{2}{17}\right) = 1/9$

$P(b_3) = P(b_1 \cap r_1 \cap b_2) + P(r_1 \cap b_1 \cap b_2) + P(r_1 \cap r_2 \cap b_1)$

$= \left(\frac{2}{18} \cdot \frac{16}{17} \cdot \frac{1}{16}\right) + \left(\frac{16}{18} \cdot \frac{2}{17} \cdot \frac{1}{16}\right) + \left(\frac{16}{18} \cdot \frac{15}{17} \cdot \frac{2}{16}\right) = 1/9$

$$P(b_5) = P(b_1 \cap r_1 \cap r_2 \cap r_3 \cap b_2) + P(r_1 \cap r_2 \cap r_3 \cap r_4 \cap b_1)$$

$$4\left(\frac{2}{18} \cdot \frac{16}{17} \cdot \frac{15}{16} \cdot \frac{14}{15} \cdot \frac{1}{14}\right) + \left(\frac{16}{18} \cdot \frac{15}{17} \cdot \frac{14}{16} \cdot \frac{13}{15} \cdot \frac{2}{14}\right) = \boxed{1/9}$$

$$P(blue_{18}) = P(r_1 \cap r_2 \cap r_3 \cap r_4 \cap r_5 \cap r_6 \cap r_7 \cap r_8 \cap r_9 \cap r_{10} \cap r_{11} \cap r_{12} \cap r_{13} \cap r_{14} \cap r_{15} \cap r_{16} \cap b_1 \cap b_2)$$

The position of the first drawn blue ball doesn't matter However, the pattern reveals that there are 17 other possible enumerations. So rather than doing this 17 more times we can just calculate one probability and multiply it by 17.

$$\frac{16}{18} \cdot \frac{15}{17} \cdot \frac{14}{16} \cdot \frac{13}{15} \cdot \frac{12}{14} \frac{11}{13} \frac{10}{12} \frac{9}{11} \frac{8}{10} \frac{7}{9} \frac{6}{8} \frac{5}{7} \frac{4}{6} \frac{3}{5} \frac{2}{4} \frac{1}{3}$$

$$= \frac{2.092278989E13}{3.201186853E15}$$

$$= 0.0065359477 \cdot 17 = \boxed{\frac{1}{9}}$$

The probability of drawing a blue chip doesn't 1/9 and the position of drawing a ball doesn't matter. 1st, 5th or last are all the same.