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STAT 620
Dr. Kerr
Midterm 2

1)

(a) $\alpha = 3$ and scale parameter $\theta = \frac{1}{2}$

$$X \sim Gamma(\alpha = 3, \theta = \tfrac{1}{2})$$

$$= \frac{x^{3-1} e^{-x/\theta}}{\Gamma(3)\theta^3}$$

$$\boxed{= \frac{x^2 e^{-2x}}{\Gamma(3)\theta^3}}$$

(b) $P(W \le 60) = 1 - \sum_{k=0}^{\alpha-1} \frac{(\lambda w)^k e^{-\lambda w}}{k!}$

$$1 - \sum_{k=0}^{2} \frac{30^k e^{-k}}{k!}$$

k <- c(0,1,2)

1- sum(30^k * exp(-30) / factorial(k))

$$\boxed{= 1}$$

2. (a) $f(x) = \frac{1}{32}(x-2), \quad 2 \le x \le 10$

$$= \int_2^{10} \frac{1}{32}(x-2)\,dx = 1$$

$$= \frac{1}{32}\int_2^{10}(x-2)\,dx = 1$$

$$= \frac{1}{32}\left[\frac{x^2}{2} - 2x\right]_2^{10} = 1$$

$$= \frac{1}{32}\left[(50-20) - (2-4)\right] = 1$$

$$= \frac{1}{32}(30) - (-2) = 1$$

$$= \frac{1}{32}(32) = 1$$

$\boxed{1 = 1 \quad \text{This is a valid pdf}}$

(b) $E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

$$= \int_2^{10} x \cdot \frac{1}{32}(x-2)\,dx$$

$$= \frac{1}{32}\int_2^{10} x \cdot (x-2)\,dx$$

$$= \frac{1}{32}\int_2^{10} x^2 - 2x\,dx$$

$$= \frac{1}{32}\left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_2^{10}$$

$$= \frac{1}{32}\left[\left(\frac{10^3}{3} - (10)^2\right) - \left(\frac{2^3}{3} - (2)^2\right]\right.$$

$$= \frac{1}{32}\left[\left(\frac{1000}{3} - (100) - \frac{8}{3} + 4\right)\right]$$

$$= \frac{22}{3}$$

$$\boxed{E[X] = \frac{22}{3}}$$

$$Var(X) = E[X^2] - [E[X]]^2$$

$$\int_2^{10} x^2 \cdot \frac{1}{32}(x-2)\,dx$$

$$= \frac{1}{32}\int_2^{10} x^3 - 2x^2\,dx$$

$$= \frac{1}{32}\left[\frac{x^4}{4} - \frac{2x^3}{3}\right]_2^{10}$$

$$= \frac{1}{32}\left[\left(\frac{10^4}{4} - \frac{2(10)^3}{3}\right) - \left(\frac{2^4}{4} - \frac{2(2)^3}{3}\right)\right]$$

$$= \frac{1}{32}\left[\left(2500 - \frac{2000}{3}\right) - \left(4 - \frac{16}{3}\right)\right]$$

$$= \frac{1}{32}\left(\frac{5500}{3}\right) - 4 + \frac{16}{3}$$

$$= \frac{1}{32}\left(\frac{5500}{3}\right) - \frac{4}{3}$$

$$= \frac{172}{3}$$

$$\frac{172}{3} - \left(\frac{22}{3}\right)^2 = \boxed{\frac{32}{9}}$$

3.

(a)
```
U <- runif(30, 2, 10)
x_sim <- sqrt(U)

x_sim
```

(b) Find $P(\bar{X} \leq 7)$

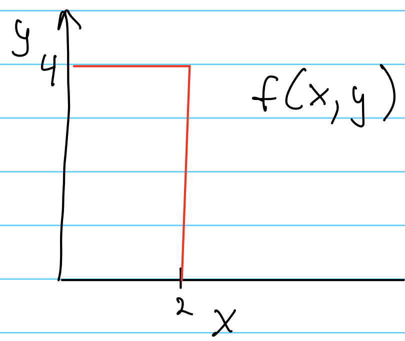
```
B <- 10000

xbar <- numeric(B)
n <- 30

for (i in 1:B) {xbar[i] <- mean(rchisq(n, 7))}

par(mfrow = c(2,1))
hist(xbar, freq= FALSE)
curve (dnorm (x, mean (xbar), sd(xbar)), min(xbar), max(xbar),
col = 'red', add= TRUE
qqnorm (xbar)
qqline (xbar, col= 'red')
par (mfrow = c(1,1))
```

4. (a) $f(x,y) = \begin{cases} \frac{x+y}{24}, & 0 \le x \le 2,\ 0 \le y \le 4 \\ 0, & \text{o.w} \end{cases}$



$f(x,y) = \frac{x+y}{24}$

$f_Y(y) = \int_0^2 \frac{x+y}{24}\,dx$

$= \frac{1}{24}\int_0^2 x+y\,dx$

$= \frac{1}{24}\left[\frac{x^2}{2} + y\right]_0^2$

$= \frac{1}{24}\left[2 + y\right]$

$= \frac{y+1}{12}$

Find $P(X > 1, Y < 2)$

$\int_1^\infty \frac{x+2}{6}\,dx$

$= \frac{1}{6}\int_1^\infty x+2\,dx$

$P(Y<2)$

$\int_0^2 \frac{y+1}{12}\,dy$

$= \frac{1}{12}\int_0^2 y+1\,dy$

$= \frac{1}{12}\left[\frac{y^2}{2}+y\right]_0^2$

$= \frac{1}{12}\left[2+2\right]$

$\boxed{= 1/3}$

b) Conditional Y, given that $X=x$

$$h(y|x)=\frac{f(x,y)}{f_X(x)}$$

$$f_X(x)=\int_0^4 \frac{x+y}{24}\,dy$$

$$=\frac{1}{24}\int_0^4 x+y\,dy$$

$$=\frac{1}{24}\left[xy+\frac{y^2}{2}\right]_0^4$$

$$=\frac{1}{24}\left[4x+\frac{4^2}{2}\right]$$

$$=\frac{1}{24}\left[4x+8\right]=\frac{4x+8}{24}=\frac{x+2}{6}$$

$$h(y|x)=\frac{\frac{x+y}{24}}{\frac{x+2}{6}}$$

$$=\frac{x+y}{\cancel{24}_{4}}\cdot\frac{\cancel{6}^{1}}{x+2}=\frac{x+y}{x+2}$$

$$\boxed{h(y|x)=\frac{x+y}{x+2}}$$