STAT 620 Midterm 1

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Problem 1

a) What is the probability that all three draws are the same suit?

Since we are drawing without replacement that means we are decreasing the amount of total cards in the deck with each draw. e.g. 52,51,50

There are 13 cards in each suit which decreases as well when we draw.

```
#Probability all same suit =
(13/52) * (12/51) * (11/50)
```

```
## [1] 0.01294118
```

The probability that all three draws are the same suit is $\boxed{0.01294118}$

b) What is the probability that the third draw is an Ace?

The probability that the third draw is an Ace. There are 4 Aces in a standard deck of cards out of a total of 52 cards. The first two draws do not matter what they are since we are only interested in the probability that the third draw is an Ace. But we have drawn twice which reduces our total card amount from 52 to 50.

```
4/50
```

```
## [1] 0.08
```

The probability that the third draw is an Ace is $\boxed{0.08}$

c) If the second and third draws are Aces, what is the probability that the first draw was an Ace?

Here we are calculating a conditional probability. P(A1|A2 and A3) = P(A1inersectA2intersectA3) / P(A2 and A3)

d) Are the events “Draw a King on the first draw” and “Draw a Diamond on the second draw” independent?

Problem 2

a) Verify that this is a valid probability mass function.

To verify that this is a valid probability mass function or p.m.f it must equal 1

```
0.1+0.3+0.4+0.2
```

```
## [1] 1
```

Therefore, this is a valid probability mass function.

b) Calculate the expected value ($E[X]$) and variance ($V[X]$) of the random variable X using the definitions.

The E(X) = The summation of x * P(x) In this case our x values are 1,2,3,4 and our P(x) are the values 0.1,0.3,0.4,0.2

```
1 * (0.1) + 2 * (0.3) + 3 * (0.4) + 4 * (0.2)
```

```
## [1] 2.7
```

The E(X) = $\boxed{2.7}$

To calculate the Var(X) =E(X^2) - [E(X)]^2

```
(1^2) * (0.1) + (2^2) * (0.3) + (3^2) * (0.4) + (4^2) * (0.2)
```

```
## [1] 8.1
```

```
# E(X^2) - [E(X)]^2
8.1 - (2.7)^2
```

```
## [1] 0.81
```

Therefore, the Var(X) = $\boxed{0.81}$

c) Find the moment generating function (MGF) of X, $MX(t)$.

To find the moment generating function (MGF) of X we can use the formula m(t)=e^(tx) * P(x) Where our x values are from the table provided and are as follows; x=1,2,3,4. and our P(X) values are 0.1,0.3,0.4,0.2

$\boxed{MX(t) = e^1 * t * (0.1) + e^2 * t * (0.3) + e^3 * t(0.4) + e^4 * t * (0.2)}$

d) Use the moment generating to find the variance of X and compare this to what you calculated in part (b).

To find the variance of x using the mgf we must find the first and second derivative.

M’X(t) = e^1*t* (0.1) + e^2*t* (0.3) * 2 + e^3*t (0.4)* 3 + e^4*t* (0.2) * 4

Then we must evaluate when t = 0

```
(0.1) * exp(0) + (0.6) * exp(2*0) + (1.2) * exp(3*0) + (0.8) * exp(4*0)
```

```
## [1] 2.7
```

M’X(t) = 2.7

M”X(t) = e^1*t* (0.1) + e^2*t* (0.3) * 4 + e^3*t (0.4)* 9 + e^4*t* (0.2) * 8

```
(0.1) * exp(0) + (1.2) * exp(2*0) + (3.6) * exp(3*0) + (6.4) * exp(4*0)
```

```
## [1] 11.3
```

Var(X) = E(X)-[E(X)]^2

```
11.3 - (2.7)^2
```

```
## [1] 4.01
```

Therefore, the Var(X) using the MGF is $\boxed{4.01}$

This is different than what we got in Part (b)

Problem 3

a) What is the probability that the player makes exactly 7 free throws out of 10?

X ~ Binom(7, 0.7)

```
choose(10,7) * (0.7)^7 * (0.3)^3
```

```
## [1] 0.2668279
```

The probability that the player makes exactly 7 free throws out of 10 is $\boxed{0.2668279}$

b) What is the probability that the player makes fewer than 5 free throws out of 10?

P(X < 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)

```
sum(dbinom(0:4,10, 0.7))
```

```
## [1] 0.04734899
```

The probability that the player makes fewer than 5 free throws out of 10 is $\boxed{0.04734899}$

c) What is the probability that the player makes at least 1 but no more than 4 successful free throws?

P(1=<X=<4)

```
sum(dbinom(1:4, 10, 0.7))
```

```
## [1] 0.04734308
```

The probability that the player makes at least 1 but no more than 4 successful free throws is $\boxed{0.04734308}$

d) Find the expected number and standard deviation of successful free throws the player will make out of 10.

Since this is a binomial distribution we calculate the E(X) as n*P where n =10 and P= 0.7

```
10 * 0.7
```

```
## [1] 7
```

The E(X) = $\boxed{7}$

Need to do Var(X) to get STD

Problem 4

a) What is the probability that more than 5 customers enter the bakery in the next 2 hours?

Let X = the number of customers entering the bakery

Since the bakery receives an average of 4 customers per hour lambda is = 2 * 4 = 8

We are interested in finding P(X>5) = 1 - P(X<5)

X ~ Poisson(8)

```
1 - ppois(5, lambda = 8)
```

```
## [1] 0.8087639
```

The probability that more than 5 customers enter the bakery in the next two hours is $\boxed{0.8087639}$

b) What is the probability that no customers enter the bakery in the next 30 minutes?

Here the P(X=0) lambda = 2

```
ppois(0, lambda = 2)
```

```
## [1] 0.1353353
```

c) If the bakery is open for 8 hours, what is the expected value and standard deviation for the number of customers for the entire day?

d) Fully justify why the distribution you used for this problem is appropriate.

The use of the Poisson distribution is used because we are modeling the number of events which is customers entering that occurs during a fixed amount of time.