

1. BK

2. BK

## Midterm #1

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STAT 631  
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1.

(a) This was an observational study. The reason this is an observational study is because there was no randomization in order to reduce bias. Perhaps only really good professor attend this workshop and so we don't get a good representation of the population of interest. Furthermore, there is no manipulation of the variables of interest.

(b) For the "Not Attend" group the dot plot appears to be right skewed. The mean is centered around 4.8. For the "Attend" group the dot plot seems more normally distributed with its mean centered around 5.5.

(c) No I don't believe that a causal relationship would be justified in this case. There was no randomization to reduce bias. There is also no repeatability in order to improve reliability.

2.

(a)  $H_0: \mu_1 = \mu_2$  vs.  $H_A: \mu_1 \neq \mu_2$

where  $\mu_1$  and  $\mu_2$  are the average sales amount for package 1 and package 2 respectively.

(b) The appropriate boxplot for checking variance is the side-by-side boxplot. From observing the boxplots of both packaging equal variance assumption seems ok. The boxplot of package 1 appears to be left skewed with the median about 16. While the boxplot of Package 2 appears to be normally distributed with a median of about 23. There are two independent random samples, i.e. Package 1 and Package 2 are independent (between) and twenty stores (within).

2(c) The assumptions that must be checked are independence, equal variance, and normality. Independence is stated in the problem. There are two independent random samples, i.e. Package 1 and Package 2 are independent (between) and twenty stores (within). To check for normality I would use a Q-Q Plot and I would also test for normality using the `shapiro.test()` function. This would confirm that normalities for both samples are satisfied. Equal variance checked via boxplots.

2(d) The proper R code to perform hypothesis testing is #2. This is because we are performing a two sample t-test. If our assumptions were not satisfied R code #1 would be best. And if we were measuring the effects of the same object Pre and Post experiment then R code #3 would be best. However with our assumptions satisfied R code #2 is our choice.

2(e) with a p-value of  $9.463e-05$  which is less than our significant level of 0.05 we reject the null

2(f) The p-value is  $9.463e-05$ . we reject the null hypothesis and can conclude that there is the means difference in mean sales between Package 1 and Package 2.

2(g) We are 95% confident that the mean sales of Package 1 is about 4.52 to 11.08 smaller than the true mean sales for Package 2

2(h) Yes the conclusion in the hypothesis test in part f agrees with the confidence interval interpretation. If we go back and re-examine the boxplot we can already see that the median sales of Package 1 is much smaller than

package 2. It's not a surprise from the conclusion to the confidence interval.

2(i) The power of design is 0.8554406. It means that this designed  $t$ -test would have 85.54% chance of correctly rejecting the null hypothesis given that the true difference in means is 5.

2(j) In order to result in a 90% chance of correctly rejecting the null hypothesis given that the true difference in means is 5, we would need 12 samples.

2(k) If the assumption in part c is not satisfied, meaning that Normality does not hold I would use a permutation test.

2(l) If the client was still concerned about variability between the stores, I would explain that we performed a variance test at the beginning of our hypothesis testing. With a p-value of 0.2585 this told us that the variances between Package 1 and Package 2 were equal. This assures us that we performed the correct analysis and came to the correct decision and conclusion.

A cornfield is divided into a certain # of subfields. Each part is treated with a different fertilizer (4 types), and varying amounts of water (standard, or enhanced) to determine which produces the most corn. To ensure robust results, five subfields are treated with one of these treatments, and 10 corn plants will be selected from each subfield to measure productivity.

a) factors: fertilizer, water

b) # of levels in each factor: 4 levels - fertilizer  
2 levels - water

c) treatment: combination of fertilizer and water

d) total # treatments: 4 fertilizer  $\times$  2 water = 8 treatment

e) experimental units: subfields

f) total # experimental units: 4 fertilizer  $\times$  2 water  $\times$  5 subfield  
= 40

g) measurement units: 10 corn plants from each subfield

h) total # measurement units:

4 fertilizer  $\times$  2 water  $\times$  5 subfield  $\times$  10 corn plants  
= 400

i) # of replication: 5 (per treatment)

Report a 95% confidence interval  
Provide a practical interpretation. (0.4995743, 4.5404257)

we are 95% confident that the true mean photoresist thickness for wafers baked at (95°C)

is about 0.5 to 4.54 larger than the true mean photoresist thickness for wafers baked at (100°C)

$H_0: \mu_1 = \mu_2$  group<sub>1</sub> = 95°C group<sub>2</sub> = 100°C

$H_A: \mu_1 \neq \mu_2$

Find the power of this test for detecting an actual difference in means of 2.5KA.

Two-sample t-test power calc

$n=8$

$\delta = 2.5$

$s_d = 1.884634$

sig. level = 0.05

Power = 0.6945829

Thus, the power of the design is 0.6946. It means that this designed t-test would have 69.46% chance of correctly rejecting the null hypothesis given the true difference in means is 2.5

Type I error ( $\alpha$ ): Rejecting  $H_0$  when it's true (false positive)

Type II error ( $\beta$ ): Failing to reject  $H_0$  when it's true (false negative)

Power of a test:  $1 - \beta$ , the probability of correctly rejecting  $H_0$

P-value = 0.03. If  $\alpha = 0.05$  what type of error could occur if the drug is actually ineffective?

• Type I error (false positive) incorrectly concluding drug works when it doesn't

## Experimental Design

- Treatment: conditions applied to experimental units
- Control Group: baseline comparison group
- Randomization: Reduces bias
- Replication: Repeating the experiment improves reliability

P-value is 0.018. we reject the null hypothesis with  $\alpha = 0.05$ , we can conclude that there is a mean difference of photoresist thickness between the two baking temperature results in wafers.

Confidence Interval of (-161.88, 411.88)

Since 0 is in the CI, we can conclude that there is no mean contribution difference in dollar amounts between group 1 and group 2.

If 0 is in CI  $\rightarrow$  No significant difference  
If 0 is NOT in CI  $\rightarrow$  Significant difference  
negative CI values  $\rightarrow$  Group 1 is lower than G

Positive CI values  $\rightarrow$  Grp 1 is higher than G

Shapiro-Wilk's test Normality Test

• p-value = 0.3457  $> 0.05$ , we fail to reject  $H_0$ , no evidence against normality.

• p-value = 0.02  $< 0.05$ , we reject  $H_0$   
meaning data deviates from normality

Test	Type of Data	Key Assumptions	Example	when US
Pooled Two-sample t-test	Two independent samples	1. Normality 2. Equal Variance	Modified vs Unmodified	Independent variances
welch's Two-sample t-test	Two independent samples	1. Normality 2. Unequal Variance	Exam Scores under diff lighting	Variance Unequal
Paired t-test (Pre/Post test)	Same subjects measured twice	1. Normality of differences 2. Observations paired	Reaction before and after caffeine	Data is before-after
Permutation Test (Randomization test)	Two independent or paired samples	No normality assumption (nonparametric) when normality is violated	Productivity before and after training	when normality assumption violated or sample size are small

### Checking Assumptions

- Independence: Ensure observations are not related
- Normality: Use QQ-plots  
Shapiro-Wilk test is supplementary
- Equal Variance: Check using box plots

### Nonparametric (Permutation Test)

- If Normality assumption do NOT hold, use permutation test
- The p-value from permutation test should match t-test p-value

### Equal Variances

- If p-value > 0.05, variances are equal  $\Rightarrow$  use Pooled t-test
- If p-value < 0.05, variances are NOT equal  $\Rightarrow$  use Welch's t-test

Decision	H <sub>0</sub> True	H <sub>0</sub> False
Reject H <sub>0</sub>	Type I error (False Positive)	Correct Decision
Fail to Reject H <sub>0</sub>	Correct Decision	Type II error (False Negative)

### Example

In a court trial

Type I: Convicting an innocent person

Type II: Acquitting a guilty person

### Perform a HI steps 1-6

$$H_0: \mu_1 = \mu_2 \text{ vs } H_A: \mu_1 \neq \mu_2$$

where  $\mu_1$  and  $\mu_2$  are the average contribution dollar amount for approach 1 and approach 2 respectively. There are two independent random samples, i.e.

Approach 1 and approach 2 are independent (between) and 16 individual sponsors (within). Also the normality assumption is satisfied since both Q-Q plots show no severe deviation

from the Q-Q line. `shapiro.test()` also confirms that normality for both samples are satisfied.

using `t.test()` output, the p-value is 0.3659, we fail to reject the  $H_0$  with  $\alpha = 0.05$ , and can conclude that there is no mean contribution difference in dollar amounts between approach 1 and

t-statistic measures how extreme a mean difference is, relative to random variation

Modified vs Unmodified

$$t = -2.1869, df = 18, p\text{-value} = 0.0$$

↓  
difference in mortar strength is 2.19 standard errors away from 0