Keck_Brandon_STAT620_HW6

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Section 4.1

Exercise 4.1-3   (a) Find fX (x), the marginal pmf of X.

$$f(x,y) = \frac{x+y}{32}, x = 1,2 \ and \ y = 1,2,3,4.$$

$$f_x(x) = \sum_{y=1}^{4}$$

$$= \frac{x+1}{32} + \frac{x+2}{32} + \frac{x+3}{32} + \frac{x+4}{32}$$

$$= \boxed{f_x(x) = \frac{4x+10}{32}, x = 1,2}$$

(b)Find fY (y), the marginal pmf of Y.

$$f(x,y) = \frac{x+y}{32}, \ x = 1,2 \ and \ y = 1,2,3,4.$$

$$f_y(y) = \sum_{x=1}^{2}$$

$$= \frac{1+y}{32} + \frac{2+y}{32}$$

$$= \boxed{f_y(y) = \frac{3+2y}{32}, \ y = 1,2,3,4}$$

(c) Find P(X > Y).

The possible paris for x,y are (1,1), (1,2), (1,3), (1,4),(2,1), (2,2), (2,3) and (2,4)

The only pair that satisfies these conditions wher x > y is (2,1)

$$P(X > Y) = \frac{2+1}{32}$$

$$= \boxed{\dfrac{3}{32}}$$

(d) Find P(Y = 2X).

The only pair that satisfies these conditions where Y = 2x is (1,2) and (2,4)

$$\frac{1+2}{32} + \frac{2+4}{32}$$

$$\boxed{\dfrac{9}{32}}$$

e Find P(X + Y = 3).

The possible paris for x,y are (1,1), (1,2), (1,3), (1,4),(2,1), (2,2), (2,3) and (2,4)

(1,2), (2,1) satisfies this condition.

$$\frac{1+2}{32} + \frac{2+1}{32}$$

$$\boxed{\dfrac{6}{32}}$$

(f) Find P(X $\leq$ 3 − Y).

The possible paris for x,y are (1,1), (1,2), (1,3), (1,4),(2,1), (2,2), (2,3) and (2,4)

$$P(X \leq 3 - Y)$$

$$P(X + Y \leq 3)$$

The only pairs that satisfies these conditions is (1,1), (1,2) and (2,1)

$$\frac{1+1}{32} + \frac{1+2}{32} + \frac{2+1}{32}$$

$$= \boxed{\dfrac{1}{4}}$$

(g) Are X and Y independent or dependent? Why or why not?

Two discrete variables are said to be independent if P(X=x, Y=y) = P(X=x)P(Y=y) for all (x,y).

$$f_x(x) = \frac{4x + 10}{32}, \ x = 1, 2$$

$$f_y(y) = \frac{3 + 2y}{32}, \ y = 1, 2, 3, 4$$

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$$f_x(x) * f_y(y) = \frac{4x+10}{32} * \frac{3+2y}{32}$$

$$= \frac{(4x+10)*(3+2y)}{1024}$$

$$f(x,y) \neq f_x(x) * f_y(y)$$

Therefore, the random variables X and Y are dependent

(h) Find the means and the variances of X and Y.

```
x <- c(1, 2)
px <- c(14/32, 18/32)

sum(x * px)
```

```
## [1] 1.5625
```

```
sum(x^2 * px) - (1.5625)^2
```

```
## [1] 0.2460938
```

$$\boxed{E(X) = 1.5625, \ V(X) = 0.2460938}$$

```
y <- c(1, 2, 3, 4)
py <- c(5/32, 7/32, 9/32, 11/32)

sum(y * py)
```

```
## [1] 2.8125
```

```
sum(y^2 * py) - (2.8125)^2
```

```
## [1] 1.152344
```

$$\boxed{E(Y) = 2.8125, \ V(Y) = 1.152344}$$

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Section 4.2

Exercise 4.2-9 f(x, y) = 1/4, (x, y)   S = {(0, 0),(1, 1),(1, −1),(2, 0)}. (a) Are X and Y independent?

To determine if X and Y are independet we need to check if each probability join event $P(X=x, Y=y) = P(X=x)P(Y=y)$

Marginal pmf of X

X	Probability
0	$\frac{1}{4}$
1	$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
2	$\frac{1}{4}$

$$\frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$

$$= 1$$

Therefore, this is a valid probability mass function because all values of probabilities are positive and add to 1

Marginal pmf of Y

Y	Probability
-1	$\frac{1}{4}$
0	$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
1	$\frac{1}{4}$

$$\frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$

$$= 1$$

Therefore, this is a valid probability mass function because all values of probabilities are positive and add to 1.

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

$$For\ (0,0)\ P(X = 0, Y = 0) = \frac{1}{4}$$

$$P(X = 0)P(Y = 0) = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$$

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$$Since \ \frac{1}{4} \neq \frac{1}{8}$$

$$\boxed{X \ and \ Y \ are \ dependent}$$

(b) Calculate Cov(X, Y) and .

$$\mu_x = E(X) = \sum x * P(X = x)$$

$$\mu_x = (0) * \left(\frac{1}{4}\right) + (1) * \frac{1}{2} + (2) * \frac{1}{4}$$

$$\mu_x = 1$$

$$\mu_y = E(Y) = \sum y * P(Y = y)$$

$$\mu_y = (-1) * \left(\frac{1}{4}\right) + (0) * \frac{1}{2} + (1) * \frac{1}{4}$$

$$\mu_y = 0$$

$$Cov(X, Y) = E(X, Y) - \mu_x \mu_y$$

$$(0)(0)\left(\frac{1}{4}\right) + (1)(1)\left(\frac{1}{4}\right) + (1)(-1)\left(\frac{1}{4}\right) + (2)(0)\left(\frac{1}{4}\right) - (1)(0)$$

$$\boxed{COV(X, Y) = 0}$$

$$\rho = \frac{Cov(X, Y)}{\sigma_X \sigma_Y} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$$E(X^2) = (0)^2 * \left(\frac{1}{4}\right) + (1)^2 * \frac{1}{2} + (2)^2 * \frac{1}{4}$$

$$E(X^2) = 1.5$$

$$\sigma_X^2 = E(X^2) - [E(X)]^2$$

$$1.5 - (1^2)$$

$$= 0.5$$