Introduction to Finance

by George W. Blazenko

Chapter 7 *Fixed Income and Debt*

"Gentlemen prefer bonds." – Andrew Mellon, U.S. financier and philanthropist, (1855-1937)

"Men are like government bonds, they mature slowly." − Kabir Bedi (actor)

In Chapter Seven You Will Learn:

1. **What is the difference between "debt" and "equity?"**
2. **What is the term structure of bond yields? What is a "flat" term structure of bond yields?**
3. **How do you value bonds?**
4. **How do current yield, yield to maturity, and coupon rate differ for premium and discount bonds?**
5. **If yield to maturity is your expected rate of return upon buying a bond, under what conditions is it also your realized holding period rate of return?**
6. **What is accrued interest in bond trading?**
7. **What is the difference between quoted price and invoice price of a bond?**
8. **How do you calculate accrued interest? How do you calculate the invoice price of a bond? How do you calculate the quoted price of a bond?**
9. **What risks do you bear as a bond investor? Can you measure these risks?**

Chapter Seven Contents

(7.1) Introduction Title Page

The fundamental difference between debt and equity as financial assets is that payments on debt are *contractually* promised. The implication of this observation is that debtholders can sue the issuer for default of principal or interest payments. On the other hand, based on their best business judgment, the board of directors of a corporation can cut or eliminate the dividends on a common share or a preferred share and the holders of these equity securities cannot (typically) sue the firm. Also, because interest is contractually promised, it is an expense of a corporate issuer,[1] and therefore, it is tax deductible. Because dividends are not contractually *guaranteed*, they are not an expense of a firm, and therefore, they are not tax deductible.

Debt can be either private or public. The following section of this book describes and discusses a number of commercial debt arrangements used by firms. Private borrowing contracts are called loans. Public debt, which trades in an organized financial market, is called a bond. Bonds have many different characteristics and there are many categories of issuers. Characteristics and features of bonds are described and discussed in sections 7.3 and 7.4 of this chapter.

(7.2) Debt Markets Title Page

Equity rather than debt is the first financial asset of any corporation. Common shares and possibly preferred shares are created in the incorporation process that also creates the corporation as a legal entity. Thereafter, the corporation enters into legal agreements that are necessary for the sale of financial assets. One of these agreements is debt financing.

The largest part of financing by small and medium sized firms is through retained earnings, which does not require any explicit financial contracting. If retained earnings are insufficient,

[1]Interest is also deductible for other issuers. For example, if you borrow to invest in a common share, the interest you pay is tax deductible because the purpose of the investment is to earn (possibly) an income in the form of dividends. On the other hand, funds you borrow for your *home* mortgage in Canada are not tax deductible because the purpose of the borrowing is not to earn income.

external financing is often obtained primarily from loans with financial institutions. In most neighborhoods, towns, and cities across Canada, firms have access to the commercial services offered by banks and trust companies. This feature of branch banking has made the private debt market very successful in Canada.

One of the characteristics of small and medium size firms is that they tend to operate in local and regional rather than national and international product and service markets. This feature of their operations limits the access of these firms to national and international investors. National and international investors have a disadvantage compared to the banks trust companies in assessing the opportunities available in local and regional markets. Because banks and trust companies have a presence in the neighborhoods of Canada through their retail network of branches, they are better able to assess the business prospects of local and regional businesses. The branches of banks and trust companies provide a wide ranging distribution network for the commercial services and products which they offer. In additional, there are economies of scale in external financial analysis. Large institutional portfolio managers can more easily invest, for example, a $100 million dollar portfolio in the financial assets of 20 or 30 larger issuers rather than 1000 firms. A large number of small holdings of financial assets is too difficult to monitor and manage for institutional investors who typically have a relatively small number of employees. Banks and trusts can more easily undertake this monitoring through their branches. If a firm prospers and grows in national and/or international product and service markets, it might then be able to access *public* debt and equity markets which are accessible by general investors.

7.2.1 Commercial Loans

Title Page

The primary credit arrangements between firms and financial institutions are short-term operating loans, term loans, and mortgages.

An *operating loan* is a specific amount that a firm borrows to cover daily operating expenses. Repayment is anticipated based on sales activity, inventory on hand, and the collection of

accounts receivable. Cash budgeting as discussed in chapter 4 of this electronic book is invaluable for assuring your commercial loans officer that plans are in place for orderly repayment. In an operating loan, there is no predefined schedule of principal repayments. These payments are typically negotiated between the firm and the commercial loans officer responsible for the account and depend upon the anticipated and realized operating results of the firm. Irrespective of these formal or informal agreements, an operating loan is typically reviewed by a lender at least once a year. The maximum amount that a firm can borrow in an operating loan is determined by the collateral it has available. As a rule of thumb, in an operating loan, banks require security in the form of 75% of "good" receivables and 50% of inventory.

A flexible way for a firm to borrow if it does know exactly the amount or the timing of financing needs is a *line of credit*. A line of credit establishes the maximum amount that can be borrowed but the timing of this borrowing is decided upon by the firm. A line of credit is usually for a specific one-time purpose. On the other hand, in a *revolving credit* arrangement, a firm can continually pay down or increase its borrowing as long as the net amount advanced is below a specified limit.

Firms typically use short-term borrowing to finance working capital and operating requirements. However, in addition, in large complex capital projects it is difficult to estimate required investment before the development phase of the project is complete. Many projects require a series of small investments, the sum total of which is difficult to estimate. Short-term borrowing is a flexible way to finance these expenditures before more long-term and permanent sources of financing can be arranged and financing needs become apparent. Also, firms change the mix of their short-term and long-term debt as a way to alter the interest rate risk exposure that their shareholders face.

Firms use term loans for longer term financing needs. Repayment is formally scheduled beyond one year. Typically as we discussed in chapter 6 of this book a portion of every scheduled payment is interest and a portion is principal repayment. Term loans are generally used to purchase fixed assets such as machinery, land, or buildings or to renovate existing premises. As

a rule of thumb, banks will lend up to 75% of land and building replacement value and 50% of equipment.

Mortgages are long-term borrowing arrangements which are secured with specific assets of a firm. In conventional mortgages real estate is used as collateral. In chattel mortgages, moveable equipment is the collateral.

(7.3) Public Debt Markets and Bond Valuation Title Page

A bond is a public financial asset that makes contractually promised payments for a finite term. The remaining term over which payments are promised is called the bond's maturity. Bond payments are often fixed and do not vary over the term of the investment. On the other hand, interest on a floating rate bond varies in a predefined way with the general level of interest in the economy.

Income payments on a bond are called *coupons*. In North America, bonds typically pay coupons semi-annually. In Europe, bonds often make annual coupon payments. At maturity of a bond, a final payment called the par value payment is made. The par value is not the value of a bond, nor is it necessarily the amount originally borrowed. However, par value influences both these bond attributes. The primary function of par value is to establish the coupons and the maturity payment. Typical par values for bonds, are $1,000, $5,000, and $10,000. These amounts are called the *denomination* of the bond. Because of economies of scale in satisfying government regulation, bonds are sold in sets of bonds, which are called bond issues. Most public bond issues are between approximately 10 million and 100 million dollars. Numerous investors typically purchase the bonds in a new public offering. A new issue of bonds is a primary market sale of a financial asset (see the discussion of primary market trades of financial assets in chapter 1 of this book).

Unlike a term loan where each payment is composed of both interest and principal reduction, in a bond, interest and principal payments are separated. Loosely speaking, par value is principal repayment and coupons are interest payments. Because principal repayment is at maturity of the bond, the entire amount of a coupon is interest.

Coupons on a bond are calculated as a per annum *coupon rate of interest* times par divided by two (if the bond makes semi-annual payments). In combination with the par value, the coupon rate of interest establishes the coupons on a bond.

 **Intro to the Bond Market: 32 Minutes**	 **A Premium Bond: 9 Minutes**

7.3.1 Yield to Maturity  Title Page

Yield to maturity is the internal rate of return on an investment in a bond on the promised payments to maturity. Recall from chapter 6 of this electronic book that the internal rate of return is that hypothetical discount rate which makes net present value of a set of predicted future payments equal zero. In the case of a bond, the promised payments are remaining coupons and par value. Relative to the required expenditure, the yield to maturity is the discount rate which makes the NPV of the investment equal zero. The term "yield to maturity" is often shortened to the *yield* of a bond.

Unlike the coupon rate, which is fixed, the yield on a bond changes continuously over time as conditions in bond markets change. When bond market traders pay greater prices for a bond,

yields fall because coupons and par are fixed. Because of this relationship between traded bond prices and yields, the yield of a bond is a market-determined rate. When rates of interest in the economy rise, yields on bonds also tend to rise (and bond prices tend to fall).

The dollar amount required to purchase a bond, which is determined in trading between bond market buyers and sellers, is called the invoice price of a bond. Presuming that the upcoming coupon on a bond is in six months, the relation between the invoice price of a bond, its yield, and promised payments to maturity is:

$$Invoice\ Price = \frac{C}{y/2}\left[1 - \frac{1}{(1+y/2)^n}\right] + \frac{M}{(1+y/2)^n},$$

where

C is the semi-annual coupon,
y is the yield (per annum) compounded semi-annually,
n is the number of six-month coupon periods to maturity,
M is the par value of the bond.

The bond matures in $n/2$ years. The invoice price of a bond is the present value of the annuity of coupons plus the present value of the lump sum par value repayment. Because the bond value is established in trading between bond market participants, and the coupons and par value repayment are fixed, the yield is a market determined rate.

Coupons on a bond can be calculated using the formula,

$$Coupons = \frac{r_c \times M}{2}$$

where r_c is the coupon rate of interest per annum, compounded semi-annually.

As an example of coupon determination, consider a bond that offers a 10% coupon rate, paid semi-annually, and that has a par value of $10,000. If the next coupon payment is due in six months, and if the bond matures in 15 years, what are promised payments on the bond?

The fact that the bond has a remaining maturity of 15 years implies that the par value is receivable in 15 years. Regardless of the fact that the bond might have been, for example,

originally issued as a 30-year bond, it is now referred to as a 15 year bond. Coupon payments are calculated as the coupon rate times par divided by two: $(0.10)\times\$10{,}000/2 = \$500$. The coupon rate is a nominal rate of interest compounded semi-annually.

Both the yield to maturity and the coupon rate of interest are annual rates compounded semi-annually. This observation means that the yield divided by 2 is the effective rate of return on your investment over one coupon period, which is six months long. Recall from chapter 6 of this book that effective rates of interest are always used for present value and future value calculations. For bonds that pay coupons annually, division by 2 in the calculation of coupons is not required and both the coupon rate of interest and the yield are annual rates compounded once per year.

7.3.2 Calculate Yield to Maturity

Title Page

If the invoice price of the bond described above is \$11,000, or \$10,000, or \$9,000, how can you determine its yield? Given the required investment (one of these three amounts), the yield to maturity is the discount rate that makes the present value of promised future receipts equal the investment. These calculations can be done by trial and error, with a dedicated financial calculator, or with a spreadsheet program.

If the bond invoice price is \$11,000, the yield to maturity must satisfy this equation:

$$\$11{,}000 = \frac{\$500}{y/2}\left[1-\frac{1}{(1+y/2)^{30}}\right] + \frac{\$10{,}000}{(1+y/2)^{30}}$$

The spreadsheet tool called *Goal Seek* is very useful when you need to find the discount rate that makes the present value of future payments equal to a particular value. The embedded worksheet shows the results of this method for each invoice price of the bond in our example.

Solution

You should be able to determine (or after a review of the above embedded worksheet you should find) that when the invoice price on the above described bond is $11,000, $10,000, or $9,000, the yield to maturity is 8.787%, 10.0%, or 11.407% respectively (per annum compounded semi-annually). Notice that the yield to maturity is inversely related to the invoice price. If you pay a greater price for a bond, your expected rate of return is lesser.

7.3.3 Yield as an Expected Return

Title Page

If you buy a bond today and hold it for any length of time less than or equal to maturity, the yield to maturity is your annualized holding period rate of return (appropriately compounded) on your investment if a number of conditions are satisfied. First, over the holding period, the yield on your bond should not change simply because the maturity is lesser (in other words, the *term structure of interest rates* is "flat"). Second, interest rates in the economy should not change over the holding period. Contrary to this condition, if interest rates have changed, and if you sell your bond before maturity, you will receive either an unexpected capital gain or loss on your bond which increases or decreases (respectively) your holding period rate of return relative to the yield to maturity. Third, you must reinvest received coupons over the holding period at a rate of interest equal to the yield to maturity. Contrary to this condition, and other things equal, if you reinvest coupons at a rate less than the yield, your annualized holding period rate of return (HPRR) will be lesser than the yield.

There are two opposing effects on your annualized HPRR on a bond investment of a fall in interest rates (opposite effects arise for an increase in interest rates). Depending upon the duration of your holding period relative to the maturity of the bond, either of these effects can dominate, and therefore, a fall in interest rates might increase or decrease your annualized HPRR. First, if interest rates fall, you will likely reinvest coupons at a lesser rate of interest. This effect tends to reduce your annualized HPRR. On the other hand, because interest rates have fallen and if you sell your bond before maturity, you will likely sell your bond for a capital gain. This effect increases your annualized HPRR. If your holding period is long relative to the maturity of the bond and, correspondingly, you reinvest coupons at a relatively low rate for a relative long period of time, the first effect tends to dominate and your annualized HPRR will decrease with the fall in interest rates. On the other hand, if your holding period is short relative to the maturity of the bond and, correspondingly, you reinvest coupons at a relatively low rate for a only a short period of time, the capital gains effect will tend to dominate and your annualized HPRR will increase.

To illustrate that the yield to maturity is your annualized holding period rate of return if these three conditions are satisfied, let us consider a numerical example. Suppose, continuing the above example, that the invoice price of a bond is $11,000. The upcoming coupon is in six months. The coupon rate of interest is 10% per annum compounded semi-annually. The maturity of the bond is 15 years. We know from calculations above, that the yield to maturity on this bond is 8.787% per annum compounded semi-annually. Suppose you buy the bond today, and sell it exactly 5 years from today immediately after a coupon payment. In the interim, you reinvest coupons at 8.787% per annum compounded semi-annually. The yield on the bond does not change over the five-year period. In other words, when you sell the bond in five years, the yield to maturity for the purchaser is 8.787% per annum compounded semi-annually. Demonstrate that the annualized holding period rate of return on your investment is, in fact, equal to 8.787% per annum compounded semi-annually.

The holding period rate of return for the five year period requires three principal values: the invoice price of the bond today, the invoice price of the bond in 5 years, and the future value of reinvested coupons. These amounts are described below:[2]

Invoice Price Today = \$11,000

$$\textbf{Invoice Price in Five Years} = \frac{\$500}{0.08787/2}\left[1-\frac{1}{(1+0.08787/2)^{20}}\right] + \frac{\$10{,}000}{(1+0.08787/2)^{20}}$$

$$= \$10{,}796.27$$

Future Value of Reinvested Coupons =

$$= \$6{,}113.75$$

Combining these three values, the five-year holding period return is:

$$\textbf{Five Year Holding Period Rate of Return} = \frac{10{,}796.27 + 6{,}113.75 - 11{,}000}{11{,}000} = 53.727\%$$

The annualized holding period return compounded semi-annually is the effective rate of return on this investment over a six-month period times two.

Annualized holding period return compounded semi-annually

$$= 2\times\left[(1.53727)^{1/10} - 1\right] = 8.767\% \text{ per annum}$$

This result confirms our above discussion. Under the three described conditions, the annualized holding period return (compounded semi-annually) is the same as the yield to maturity.

[2] These formulas are explained in Chapter 6, where we offer discussions of the future value of an annuity and holding period returns.

7.3.4 Opportunity Cost for a Bond

Title Page

If you are interested in buying a bond, a very important question is what is the bond's value? For example, how do we know that the value of the bond in the above subsection is worth its invoice price of $11,000? We can answer this question using the techniques of discounted cash flow analysis.

An important feature of any discounted cash flow analysis is the opportunity cost rate of return. One way to value a particular bond, is to use the yield on *similar risk* bonds as the opportunity cost rate of return. In the bond market there are a number of dimensions of the notion of equivalent risk. The comparison bonds you use to determine an opportunity cost rate of return should have at least the following general features:

(a) similar class of issuer (i.e., corporate, federal, provincial, or municipal governments),
(b) similar maturities,
(c) similar bond specific features (i.e., callable versus non-callable etc.)
(d) similar default risk as reflected by the rating of an independent rating agency.

Information on bond yields can be obtained from investment dealers or summaries of trading that appear in the daily financial press. We retrieve the below Canadian bond market quotes February 20, 2015 from http://www.pfin.ca .

Live Markets 20Feb15
Wholesale Pricing 02:35PM EST

Cda T-Bills	Price	Yield
1 Month 26Mar	99.96	.46
2 Month 23Apr	99.93	.46
3 Month 21May	99.89	.47
6 Month 13Aug	99.80	.42
1 Year 11Feb	99.63	.38

Cda Benchmarks	Price	Yield
2 Year	102.14	.39
5 Year	104.68	.70
10 Year	108.00	1.41
30 Year	132.76	2.06

Provincials	Price	Yield
Ontario Prov	101.39	.53

3.15/Sep15		
Manitoba 2.05/Dec16	102.54	.60
Alberta 1.6/Jun18	102.79	.74
Ontario Prov 2.1/Sep19	104.48	1.08
BC Prov 3.25/Dec21	111.94	1.41
NewBrunswick 2.85/Jun23	107.29	1.89
Ontario Prov 2.85/Jun23	107.82	1.83
Quebec 3.0/Sep23	108.78	1.88
Saskatchewan 3.2/Jun24	111.29	1.87
Quebec 5.35/Jun25	128.66	2.21
Alberta 2.9/Sep29	106.31	2.38
NewBrunswick 5.5/Jan34	138.97	2.83
Nova Scotia 4.4/Jun42	126.82	2.96
BC Prov 4.3/Jun42	129.31	2.76
Ontario Prov 3.5/Jun43	111.74	2.89
Corporates	Price	Yield
Manulife Fin 4.079/Aug15	101.43	1.13
Cominar REIT 4.274/Jun17	105.00	2.04
Sherrit Intl 8.0/Nov17	99.49	8.20
GraniteREIT 4.613/Oct18	108.40	2.17
Hydro One 2.78/Oct18	105.93	1.10
CallowayReEs 5.0/Feb19	110.62	2.17
CIBC Cap Tr 9.976/Jun19	130.99	2.42
TD Cap Tr 9.523/Jun19	129.46	2.35
Fairfax FinH 7.5/Aug19	117.79	3.21
Cameco Corp 5.67/Sep19	114.67	2.24
Sherrit Intl 7.5/Sep19	94.84	8.90
RusselMetals 6.0/Apr22	102.36	5.59
Bombardier 7.35/Dec26	100.25	7.32
CIBC Cap Tr 10.25/Jun39	144.98	6.53
TD Cap Tr 10.0/Jun39	142.83	6.48

Exhibit 7-1: Bond Market Reporting

Exhibit 7-2 gives yields for US Treasury Bonds. Exhibit 7-3 gives a plot of these yields versus term to maturity (the term structure of yields). We retrieve Exhibit 7-2 and 7-3 from www.yahoofinance.com on February 20, 2015.

US Treasury Bonds Rates

Maturity	**Yield**	**Yesterday**	**Last Week**	**Last Month**
3 Month	0.01	0.01	0.01	0.00
6 Month	0.05	0.04	0.06	0.06
2 Year	0.65	0.62	0.63	0.50
3 Year	1.07	1.04	1.04	0.88
5 Year	1.58	1.58	1.53	1.34
10 Year	2.10	2.12	2.04	1.87
30 Year	2.69	2.73	2.64	2.46

Exhibit 7-2: Bond Market Reporting

U.S. Treasury Yield Curve

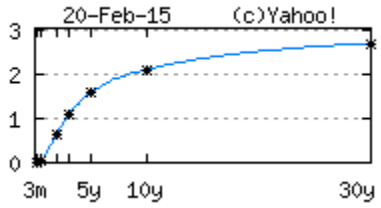


Exhibit 7-3: Bond Market Reporting

From these quotes, you should observe that for similar maturity bonds, yields are greatest for corporate bonds, next highest for provincials bonds, and lowest for Government of Canada bonds. This pattern reflects decreasing default risk. You should also be able to identify an increase in yields with respect to the maturity of bonds. This increase reflects what is called an upward sloping *term structure of interest rates*. An upward sloping term structure means that the expected per annum return for longer maturity investments is greater. One cannot identify from the above quotes special features of these bonds, like callability, that might affect yields. This type of information is available from the prospectus for the bond issue. The prospectus is the

document, required by regulation, which accompanies the issue of a bond and which describes the characteristics of both the bond and its issuer. Information on the features of a corporate bond is also available in the bond indenture agreement which is the contract between the issuer and bondholders. Because bondholders are often widely dispersed, a trustee is hired to monitor compliance with the bond indenture agreement.

Default risk of an issuer can be accessed with the aid of a rating from an independent rating agency. In order to improve the liquidity of an issue, corporations and governments often pay a rating agency to have their bonds *rated*. A rating is an independent appraisal of the credit worthiness of a bond and its issuer. The below exhibit gives ratings definitions for Fitch Corporation. We retrieve these ratings definitions from https://www.fitchratings.com

Investment grade

- **AAA** : the best quality companies, reliable and stable
- **AA** : quality companies, a bit higher risk than AAA
- **A** : economic situation can affect finance
- **BBB** : medium class companies, which are satisfactory at the moment

Non-investment grade

- **BB** : more prone to changes in the economy
- **B** : financial situation varies noticeably
- **CCC** : currently vulnerable and dependent on favorable economic conditions to meet its commitments
- **CC** : highly vulnerable, very speculative bonds
- **C** : highly vulnerable, perhaps in bankruptcy or in arrears but still continuing to pay out on obligations
- **D** : has defaulted on obligations and Fitch believes that it will generally default on most or all obligations
- **NR** : not publicly rated

- **Exhibit 7-4: Fitch Bond Ratings**

In 2015 we are in an exceptionally low interest environment and, thus, the yields in the above bond market quotes might strike you as rather modest. If you want a higher return, you will have to take on more default risk. Yahoofinance.com has a bond screener that will search for bonds with investment characteristics you desire http://screener.finance.yahoo.com/bonds.html The below table is the first page of bonds this screener identifies on February 20, 2015 when we search for corporate bonds with yields of at least 6 percent and maturity of at least 5 years. Investment grade bonds (often required by portfolio managers of pension funds and other institutional investors) have bond rating of BBB or higher. Most of these bonds are rated by Fitch Corporation (the original bond rating company in the US) as below investment grade. Charitably, these bonds are called high-yield. Less charitably, these bonds are sometimes called speculative or "junk" bonds.

BOND SCREENER RESULTS

Type	Issue	Price	Coupon(%)	Maturity	YTM(%)	Current Yield(%)	Fitch Ratings	Callable
Corp	ATWOOD OCEANICS INC	94.50	6.500	1-Feb-2020	7.813	6.878	BB	Yes
Corp	JBS USA LLC / JBS USA FINANCE	108.50	8.250	1-Feb-2020	6.292	7.604	BB	Yes
Corp	MORGAN STANLEY D W DISC SRMTNS	103.25	7.500	8-Feb-2020	6.750	7.264	BBB	No
Corp	CAESARS ENTMT OPER CO INC	81.38	8.500	15-Feb-2020	13.605	10.445	CCC	Yes
Corp	CAESARS OPER ESCROW LLC	82.75	9.000	15-Feb-2020	13.743	10.876	CCC	Yes
Corp	CAESARS OPER ESCROW LLC	81.88	9.000	15-Feb-2020	14.016	10.992	CCC	Yes
Corp	PBF HLDG CO LLC / PBF FIN CORP	106.25	8.250	15-Feb-2020	6.799	7.765	BB	Yes
Corp	SAMSON INVT CO	64.00	9.750	15-Feb-2020	21.588	15.234	B	Yes
Corp	SERVICEMASTER CO	107.88	8.000	15-Feb-2020	6.201	7.416	B	Yes
Corp	AVIS BUDGET CAR RENT	111.50	9.750	15-Mar-2020	7.096	8.744	B	Yes

Corp	CLEAR CHANNEL WW HLDGS INC	106.00	7.625	15-Mar-2020	6.269	7.193	B	Yes
Corp	CLEAR CHANNEL WW HLDGS INC	106.25	7.625	15-Mar-2020	6.215	7.176	B	Yes
Corp	CLIFFS NAT RES INC	69.50	5.900	15-Mar-2020	14.321	8.489	BB	No
Corp	LEAR CORP	106.88	8.125	15-Mar-2020	6.560	7.602	BB	Yes
Corp	RITE AID CORP	113.00	9.250	15-Mar-2020	6.313	8.186	CCC	Yes

7.3.5 Bond Valuation

Title Page

To illustrate how to value a bond, let us consider a numerical example. You want to value a bond which has the following general characteristics. The upcoming coupon is in six months. The coupon rate of interest is 10% per annum compounded semi-annually. The maturity of the bond is 15 years. The par value of the bond is $10,000. You have done some research on the bond market and have determined that yields on bonds in the same issuer class, with the same maturity, with the same features, and with the same rating have yields about 9% per annum compounded semi-annually. Using this as your opportunity cost for the particular bond under consideration, what is your assessment of its value? The value of the bond is:

$$\text{Bond Value} = \frac{\$500}{0.09/2}\left[1-\frac{1}{(1+0.09/2)^{30}}\right]+\frac{\$10{,}000}{(1+0.09/2)^{30}} = \$10{,}814.44.$$

In bond market trading, you should make sure that this is your total investment when purchasing this bond (other than commissions that you might have to pay).

7.3.6 Expected Rate of Return on a Premium Bond Title Page

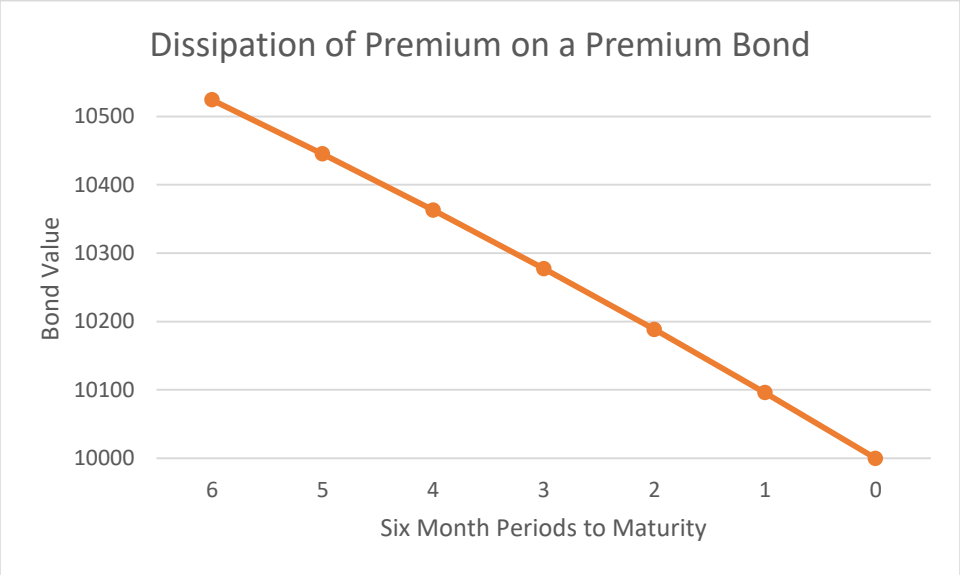
You might have noticed from the numerical example in the above subsection that a bond can have a value that is greater, lesser, or equal to par value. A bond with a value above par is called a premium bond. A bond with a value below par is called a discount bond. A bond with a value equal to par is said to trade at par.

A bond trades at a premium (at a discount) when its coupon rate is greater than (is less than) its yield to maturity. A premium bond offers an "attractive" coupon rate of interest relative to what is available in the bond market from similar risk new issue bonds. Because of this relatively higher coupon rate, bond purchasers are willing to pay a greater price. A discount bond offers a lower coupon rate than other opportunities in the market.

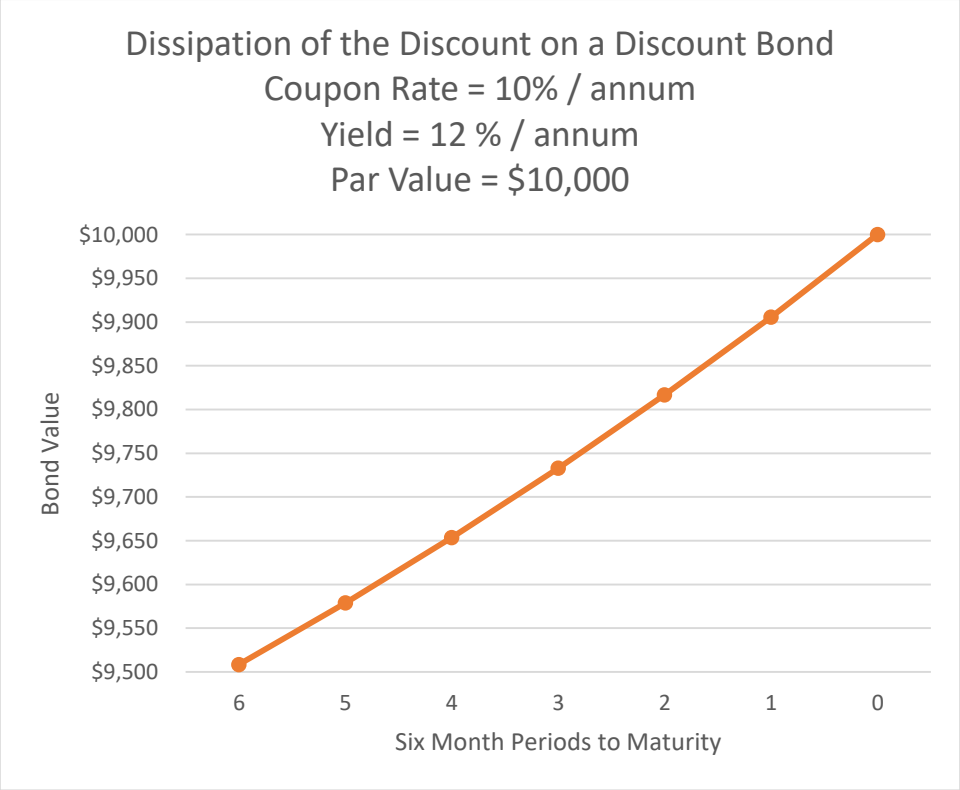
The "premium" on a premium bond depends upon its maturity. In other words, the extent to which the value of the bond exceeds par depend upon the length of time bondholders expect to receive a coupon rate which is greater than yields available on similar risk bonds. For example, consider a bond that has a 12% coupon rate and a 10 percent yield. Because this high coupon rate is attractive to bondholders, this bond will trade at a premium. It is even more attractive to receive a 12% coupon rate when similar risk bonds offer 10% for a longer period of time. The longer is the maturity of the bond, the greater will be the premium. For example, it is better to get a 12% coupon rate on a bond when similar risk bonds offer only 10% for 20 years rather than for only 10 years. The premium on the 20-year bond will be greater than the premium on the 10 year bond. This observation implies that as a premium-bond approaches maturity, its premium dissipates. The return you can expect to receive on a premium bond is, therefore, composed of a relatively high coupon rate plus a dissipation of the premium that reduces the bond's price.

The table below illustrates the value of bond with a coupon rate of 10% per annum compounded and paid semi-annually, with a par-value of $10,000, and with the next coupon in six month. The yield on the bond is 8% per annum compound semi-annually, which does not change as the bond approaches maturity. Notice that the value of the bond decreases so that the premium dissipates

as maturity of the bond approaches. At maturity the value of the bond immediately after its coupon at that time is simply the par-value of $10,000.



The opposite is true for a discount bond. The return you can expect to receive on a discount bond is composed of a relatively low coupon rate plus a dissipation of the discount that increases the bond's price. The table below illustrates the value of bond with a coupon rate of 10% per annum compounded and paid semi-annually, with a par-value of $10,000, and with the next coupon in six month. The yield on the bond is 12% per annum compound semi-annually, which does not change as the bond approaches maturity. Notice that the value of the bond decreases so that the premium dissipates as maturity of the bond approaches. At maturity the value of the bond immediately after its coupon at that time is simply the par-value of $10,000.



7.3.7 Current Yield

Title Page

The income component of the return on a bond is measured by its *current yield,* which is the coupon divided by the price of the bond. The expected rate of return on a bond is composed of its current yield plus either an expected increase or decrease in price which accompanies a discount or a premium bond. Let us refer to the decrease or increase in price that accompanies a premium or discount bond respectively as a "natural" capital gain or loss respectively. A expected capital gain or loss arises solely because of the passage of time even if interest rates do not change. If interest rates do change, then there will be an additional capital gain or loss.

The yield on a premium bond is:

$$\textit{Yield on a Premium Bond} = \textit{Current Yield} + \textit{Expected capital loss / Price}$$

The yield on a discount bond is:

Yield on a Discount Bond = Current Yield + Expected capital gain / Price

3 Rates in the Bond Market: 30 Minutes

7.3.8 Numerical Example

Title Page

To illustrate these relationships, let us consider a numerical example. You want to value a bond that has the following general characteristics. The upcoming coupon is in one year. The coupon rate of interest is 10% per annum compounded annually. The maturity of the bond is 15 years. The par value of the bond is $10,000. You have done some research on the bond market and have determined that yields on bonds in the same issuer class, with the same maturity, with the same features, and with the same rating have yields about 9% per annum compounded annually. Over the upcoming year, the expected rate of return on your bond purchase is 9%. Decompose this return into an income component and a expected capital loss component.

To determine the value of the bond today, we can do the following calculations:

Present Value of Coupons = $8,060.69,
Present Value of Par Value = $2,745.38,
Value of the Bond = $10,806.07.

Solution

The current yield on the bond is 1000/10,806.07 = 9.254%. Notice that the current coupon rate is greater than the current yield which is greater than the yield to maturity. For a premium bond:

Yield to Maturity ≤ *Current Yield* ≤ Coupon *Rate*

To find the expected capital loss on the bond, we first find the value of the bond in one year immediately after the coupon payment under the hypothetical scenario that the yield on the bond does not change.

Present Value of Coupons = $7,786.15
Present Value of Par Value = $2,992.46
Value of the Bond = $10,778.62

Solution

Notice that simply because of the passage of time, even though we have assumed that the yield on the bond has not change, the value of the premium bond falls. The amount of the fall is $10,778.62 – $10,806.07 = – $27.45. The expected capital loss relative to the original price equals – $27.45 ÷ $10,806.07 = – 0.254%. With these values we can now verify that the sum of the current yield and the expected capital loss relative to price equals the yield to maturity: 9.254%. + –0.254% = 9.0%.

When you buy a premium bond, you can expect a relatively high-income component in your return. This fact is reflected in a current yield that exceeds the yield to maturity. This relatively great income component of return is offset and lowered by a expected capital loss.

You should recognize that our hypothetical scenario in which interest rates remain unchanged is unlikely to be fulfilled. When interest rates do change over the holding period you will have an additional capital gain or loss beyond the expected capital loss.

7.3.9 Expected Rate of Return on a Discount Bond Title Page

In the case of a discount bond:

$$\textit{Coupon Rate} \leq \textit{Current Yield} \leq \textit{Yield to Maturity}$$

In the following embedded document the above analysis and numerical example for the components of return for a premium bond is replicated for a discount bond.

Discount Bond

7.3.10 Quoted Price, Invoice Price, Accrued Interest Title Page

Often as a bond buyer or seller, you will be considering a bond that pays a coupon in less than a full coupon period. For example, a bond has paid a coupon 4 months ago and will pay another coupon in 2 months. As an institutional feature of bond market trading, if you buy a bond, you must pay the seller of the bond *accrued interest*. There is no necessary reason that a bond market should require this payment (i.e., it is an institutional feature of the market); but because it exists, you must recognize this payment in your trading (i.e., buying and selling). Accrued interest is calculated (ignoring some details) as the fraction of a six month period since the last coupon was paid times the amount of the upcoming coupon. For example, if the upcoming coupon is $500 and the last coupon payment was 4 months ago, accrued interest is (4/6)*500 = $333.33. In buying the bond, this amount must be paid to the seller of the bond. Accrued interest is a linearization of the amount by which bond value has increased since the last coupon payment under the hypothetical that interest rates have not changed.

When you buy a bond, two payments are required. Expenditure on a bond is the sum of the price you agree upon through the trading process and accrued interest. The price you agree upon in trading is called the *quoted price*. This amount is "quoted" (often by an investment dealer) for sale out of an inventory of financial assets (the ask price) or into an inventory (the bid price). As

you might expect, the quoted bid price is lesser than the quoted ask price.

In trading, you want to ensure that the invoice price, which is the total amount you pay to receive the bond is equal to the value of the bond. The value of the bond is the discounted value of promised future payments.

$$\textit{Expenditure} = \textit{Invoice Price} = \textit{Bond Value} = \textit{Quoted Price} + \textit{Accrued Interest}$$

To determine the amount you should "quote" for a bond or accept in the purchase or sale of a bond, two steps are necessary. First, you must determine the bond's value (recognizing that the upcoming coupon is in less than six months) and second, you must subtract accrued interest. If you follow these two steps, the sum of the quoted price and accrued interest which is the invoice price, is equal to the value of the bond.

As an example, suppose you want to value a bond that has the following general characteristics. The upcoming coupon is in two months. The coupon rate of interest is 10% per annum compounded semi-annually. The bond has 30 remaining coupons which are paid semi-annually. The par value of the bond is $10,000. You have done some research on the bond market and determined that yields on bonds in the same issuer class, with the same maturity, with the same features, and with the same rating have yields about 9% per annum compounded semi-annually. What price should you quoted for this bond in trading?

First, what is the value of this bond? We looked at a very similar bond in the above subsection. That bond was identical except that the upcoming coupon was in exactly six months rather than two months. We determined that the value of the bond with six months to the upcoming coupon is $10,814.44. At the current time (i.e., four months later), this calculation discounts each and every coupon and the par value payment by four months "too much." The value of the bond should be higher because now we are four months closer to each and every coupon and the par value payment. The amount by which the value of the bond is higher is the future value of a

lump sum for a four-month period. The interest rate in this future value calculation is the yield to maturity divided by two. We can rectify discounting each and every coupon and the par value by too much with a future value calculation (recall that future value and present value are inverse operations).

$$\text{Bond Value} = (1+0.09/2)^{2/3}\ 10{,}814.44 = \$11{,}136.49$$

The quoted price of the bond is $11,136.49 less the accrued interest payment of $333.33 which is $10,803.16. If we agree in trading to buy the bond at $10,803.16, then the invoice price is $10,803.16 plus $333.33 which is equal to the bond value of $11,136.49.

Under the hypothetical scenario that interest rates have not changed since the last coupon payment, the value of the bond has increased by $11,136.49 – $10,814.44 = $322.05. Accrued interest of $333.33 is a linear approximation of this increase.

In exhibit 7.1, the prices reported are quoted prices. If you buy a bond at these prices, you pay accrued interest in addition.

 **Quoted Price, Invoice Price, Accrued Interest: 19 Minutes**	 **Quoted Price, Invoice Price, Accrued Interest with TI BA II Plus**

(7.4) PAR-VALUE BONDS Title Page

If a bond's coupon rate exceeds its yield, it is a premium bond. If a bond's coupon rate is less than its yield, it is a discount bond. If the coupon rate on a bond is exactly equal to its yield, then it is a *par-value bond.*

The expected rate of return on any bond is its yield. In the case of a par-value bond, this expected rate of return also equals current yield. Unlike a discount bond or a premium bond, there is no expected capital gain or loss for a par-value bond. Thus, for a par-value bond, your expected rate of return is entirely composed of coupon income.

Valuing a par-value bond is especially easy. One could calculate the value of a par-value bond as the present value of coupons plus the present value of par-value repayment at maturity. However, this calculation reduces to the following. *Regardless of bond-maturity,* if the coupon rate on a bond equals its yield to maturity, then the invoice-price (IP) of a bond is the bond's par-value with a "*k*" adjustment, where "*k*" is the fraction of a six month period (presuming the bond pays coupons semi-annually), until the next coupon on the bond,

$$IP = \frac{Par - Value}{(1 + y/2)^{k-1}}$$

If the bond has just paid a coupon so that accrued interest is zero, then the invoice-price of the bond is simply its par-value regardless of the bond's maturity.

Let's work on a former final exam question that illustrates bond valuation for a par-value bond.

Consider a bond that offers an 8.0% coupon rate, paid semi-annually, and has $10,000 par-value. There are 30 remaining coupon payments on the bond. The next and upcoming coupon is in exactly 5 months. The yield to maturity on the bond is 8 percent per annum compounded semi-annually. You buy the bond today and sell it in exactly three years and seven months. The yield

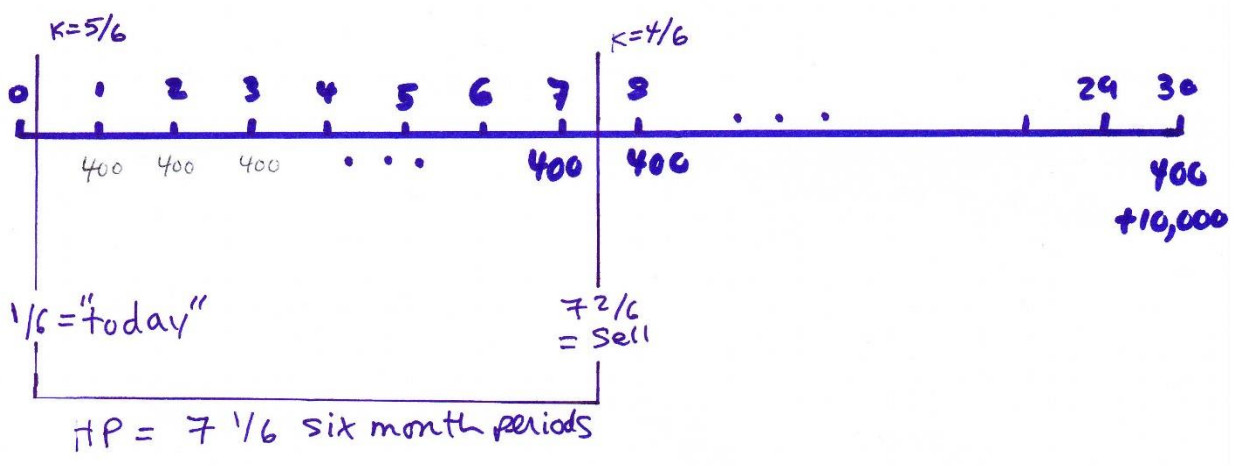
on the bond at that time is 8% per annum compounded semi-annually. Over the interim, you receive and reinvest coupons at a rate of 7.5 per cent per annum compounded monthly.

Required: What is the *annualized* holding period rate of return on your three year-seven month investment, compounded semi-annually?

First, when you buy the bond, it is a par-value bond because the coupon rate (8%) equals the yield. So, the purchase price (PP) of the bond is:

$$PP = \frac{Par - Value}{(1+y/2)^{k-1}} = \frac{10,000}{1.04^{5/6-1}} = \$10,065.58$$

The below "time-horizon" diagram represents your bond investment. Intervals represent a six-month coupon payment period. Coupons are 0.08*10,000/2 = $400. If "today" is 1/6 so that you wait for 5 months for a coupon, then k=5/6. In six-month periods, your investment holding period of 3 years and 7 months is 3*2 +1 + 1/6 = 7 1/6 six-month periods.



On the diagram, you sell the bond at 1/6 (that is, today) + 7 1/6 = 7 2/6. At that time, the buyer of the bond waits for four months to receive a coupon. Thus, at the sale, k=4/6. Also at the sale, the yield remains equal to the coupon rate so it remains a par-value bond. The sale price (SP) of the bond is:

$$SP = \frac{Par - Value}{(1+y/2)^{k-1}} = \frac{10,000}{1.04^{4/6-1}} = \$10,131.59$$

Between when you buy and sell the bond, you receive 7 $400 coupons. The reinvestment rate over a six month period is: $(1+0.075/12)^6-1 = 3.8090\%$

The future-value of reinvested coupons at the time of the last coupon is $\frac{400}{0.03809}\left[1.038090^7 - 1\right]$ = $3141.06. The future-value (FVRP) 2 months after the last coupon when you sell the bond is: $(1.03809)^{2/6}*3141.07 = 3,180.58$.

The holding period rate of return (HPRR) is:

$$HPRR = \frac{FVRP + SP - PP}{PP} = \frac{3,180.58 + \$10,131.59 - \$10,065.58}{\$10,065.58} = 32.254\%.$$

The rate of return on your investment per six-months is

$$(1.32254)^{1/7.1666} - 1 = 0.039778$$

So, finally, your annualized rate of return on you bond investment compounded semi-annually is:

$$2*0.039778 = 7.956\%$$

(7.5) Realized Versus Expected Bond Returns Title Page

When you buy a bond, your expected rate of return is the yield, which is the IRR (stated per annum compounded semi-annually for bonds paying semi-annual coupons) on the purchase of a bond. For the purpose of this calculation, remember that the total expenditure to buy a bond is the sum of quoted price (QP) and accrued interest (AI). The below video illustrates with a numerical example how to calculate the yield on a bond with a financial calculator.

Yield as an IRR with the
TI BA II PLUS Financial Calculator

Your realized rate of return on your bond investment at the end of your investment period (holding period) is exactly what you expected when you buy the bond for any holding period less than or equal bond-maturity under a number of conditions listed below. Any violation of these conditions means that your realized rate of return will be greater or less than you expected when you bought the bond in a direction you can identify from the nature of the violation. The first three conditions together mean that the yield when you sell the bond equals the yield when you buy the bond.

The conditions are:

1. **The credit-worthiness of the borrower neither improves nor deteriorates between when you buy the bond and when you sell the bond.**

2. **Interest rates in the economy neither increase nor decrease between when you buy the bond and when you sell the bond (yields in the bond market are one of the "interest rates in the economy").**

3. **Yields in the bond market neither increase nor decrease because the maturity on the bond is lesser when you sell it compared to when you buy it (a "flat" term-structure of yields).**

4. **Between when you buy the bond and when you sell the bond, you reinvest coupons at the yield that existed when you buy the bond.**

Let's work on a former final exam question on the realized rate of return for a bond investment when the above conditions are satisfied.

Consider a bond that offers a fixed per annum coupon rate of 7.2% compounded and paid semi-annually with a par value of $10,000. When you buy the bond, the yield to maturity is 6.6% per annum compounded semi-annually. Accrued interest when you buy the bond is $120. You sell the bond four years and nine months from today. At that time, the quoted price is $10,334.35. Also, when you sell the bond, the yield to maturity is 6.6% per annum compounded semi-annually. Between when you buy the bond and when you sell the bond, you reinvest coupons at 6.6% per annum compounded semi-annually.

Required: What is the realized per annum rate of return on your bond investment with reinvested coupons and interest on reinvested coupons between when you buy the bond and when you sell the bond per-annum compounded semi-annually.

I refer to this as a vicious question because it leads you in the direction of presuming that a lot of bond calculations are required to solve the problem. However, the yield when you buy the bond equals the yield when you sell the bond and, thus, the first three of the above four conditions are satisfied. In addition, you reinvest coupons between when you buy the bond and when you sell the bond at the yield that existed when you buy the bond. So, the fourth condition is also satisfied. Because all four conditions are satisfied, your realized rate of return on your bond investment is 6.6% per annum compounded semi-annually, which is the yield that existed when you buy the bond. You do not need to do any calculations to solve this problem. In fact, because the problem does not give use the maturity of the bond, we *cannot* do the detailed calculations to verify that the realized annualized rate of return on your bond investment compounded semi-annually is the yield when you buy the bond. I am a very bad person.

Yield as Both an Expected and Realized Rate of Return: 20 Minutes

(7.6) Risk and Fixed Income Securities Title Page

It is important to recognize a number of risks associated with fixed income financial assets. These risks include:

(a) Price Risk,
(b) Reinvestment Risk,
(b) Inflation Risk,
(c) Risk Induced by Contract Features,
(e) Default Risk.

7.6.1 Price Risk Title Page

The risk of a bond depends in large part upon the length of your investment horizon. If you have a short-term investment horizon but you buy a longer term bond, your investment is subject to price risk. If interest rates in the economy increase, the value of your bond decreases and vice versa. At the end of your investment horizon you sell your bond at a capital loss or gain respectively. Price risk arises only for debt securities for which there is an active secondary market. Because you have a short-term investment horizon you liquidate your bond shortly after this change and face the associated capital gain or loss on your bond. Price risk is more pronounced for bonds that have longer maturities.

To illustrate that price risk of a bond increases with its maturity (other things equal), we consider a numerical example which uses zero coupon bonds. A zero coupon bond is a bond with a coupon rate of interest equal to zero. Because a zero coupon bond offers a coupon rate which is lesser than its yield, zero coupon bonds must sell at a discount. The return you expect on a zero-

coupon bond is composed entirely of a expected capital gain. The current yield on a zero coupon bond is zero. Consider a five-year zero coupon bond and a ten-year zero coupon bond. Both bonds have par values of $10,000. Suppose that the yield on both bonds is 9% per annum compounded annually. The invoice prices of these two bonds is calculated below:

	5 Year Bond	10 Year Bond
Invoice Price	$6,499.31	$4,224.11

Solution

Now suppose the yield on these two bonds increases to 10% per annum compounded annually. The invoice prices on these bonds is now:

	5 Year Bond	10 Year Bond
Invoice Price	$6,209.21	$3,855.43

Solution

The percentage change in price from the original is:

5 Year Bond	10 Year Bond
–4.46%	–8.73%

As expected, the price sensitivity (in terms of percentage changes) for the 10 year bond is greater than for the 5 year bond. The intuition for this result is related to reinvestment. When interest rates increase, bondholders would prefer to get their par values back earlier so they can reinvest at a higher market rate of interest. Because one must wait for a longer time in the case of a 10 year bond, its price falls by more in percentage terms.

For the same maturity, bonds with higher coupon rates have lesser price sensitivity to changes in interest rates. The intuition for this sensitivity is also reinvestment. When interest rates increase, bondholders would really like to have payments on the bond as soon as possible so they can reinvest at the higher market rate of interest. For a higher coupon rate bond, you get a relatively greater component of your expected return in the form of coupons which can be reinvested, and therefore, price does not fall by as much.

Price Risk: 20 Minutes

7.6.2 Reinvestment Risk

Title Page

If your investment horizon is long-term, an important component of the risk you face as a bondholder (or debt-holder) is reinvestment risk. Moreover, reinvestment risk is opposite in direction to price risk which was described in the above subsection. This contrast illustrates that the nature of risk in debt securities depends upon your investment perspective.

Suppose your investment horizon[3] is the maturity of a bond which you own. As you receive coupons, they must be reinvested. The future value of your investment plan, which is the amount you will have in your brokerage account at the end of your investment horizon, depends upon the interest rate at which you reinvest coupons. If this rate falls over the holding period, the future value of your investment is less and vice versa.

As an example, consider a bond with an invoice price of $11,000 which offers a 10% coupon rate, paid semi-annually, and has a par value of $10,000. The upcoming coupon is in 6 months,

[3] If your investment horizon is lesser than the maturity of the bond you are considering, bondholders face both coupon reinvestment risk and price risk.

and maturity of the bond is 15 years. From calculations we have done earlier in this section, we know that the yield on this bond is 8.787% per annum compounded semi-annually. Let us demonstrate that if your investment horizon is 15 years, and if you reinvest coupons at 7 percent per annum compounded semi-annually, your annualized holding period rate of return is less than the yield on the bond.

The holding period rate of return for the fifteen year period requires three fundamental values: the invoice price of the bond today, the par value, and the future value of reinvested coupons. These amounts are described below:[4]

Invoice Price Today = \$11,000
Par Value = \$10,000

$$\textbf{Future Value of Reinvested Coupons} = \frac{\$500}{0.07/2}\left[(1+0.07/2)^{30}-1\right] = \$25{,}811.34$$

$$\textbf{Fifteen Year Holding Period Return} = \frac{\$10{,}000.00 + \$25{,}811.34 - \$11{,}000}{\$11{,}000}$$

$$= 225.56\ \%$$

The annualized holding period return compounded semi-annually, is the effective rate of return on this investment over a six-month period times two.

Annualized holding period return compounded semi-annually

$$= 2\times\left((3.2555764)^{1/30}-1\right) = 8.026\% \text{ per annum}$$

This result confirms our above discussion. The annualized holding period return (compounded semi-annually) is lesser than the yield to maturity because the rate of return received on reinvested coupons is lesser.

[4] These formulas are explained in Chapter 6, where we offer discussions of the future value of an annuity and holding period returns.

Reinvestment Risk: 16 Minutes

7.6.3 Inflation Risk

Title Page

Because most bonds offer fixed payments which are not adjust over time, inflation is an important source of risk. If you buy a bond and the rate of inflation increases, your purchasing power is diminished. To compensate for this adverse effect, bondholders require greater coupon rates when expected future inflation is greater.

The nominal rate of return per annum on a financial asset compounded annually describes the rate at which your wealth grows as the result of your investment. However, your purchasing power (i.e., your ability to consume) does not grow at this rate because of the effect of inflation. Let r* be the real rate of growth of your purchasing power associated with an investment in a particular financial asset. This rate is referred to as the *real rate of interest*. Real rates of interest are unobservable, but we can impute a value based on observed nominal rates of interest (compounded annually[5]) and our understanding of the operation of financial markets.

The nominal rate of interest which is "named" in a financial asset is used for contracting purposes between the buyer and the seller of the financial asset. Because inflation erodes the purchasing power of dollars promised in the future, if expected inflation is high, investors require

[5]The nominal rate of interest per annum compounded annually is also the effective rate of interest per annum. In the current context, it is useful to focus on nominal rates of interest to highlight the fact that observed financial market rates of interest adjust to changes in inflation in the economy.

compensation in the form of higher nominal rates of interest. The financial economist, Irving Fisher, proposed the following relationship between nominal rates of interest, real rates of interest, and expected inflation:

$$1+r = (1+r^*)\times(1+\pi)$$

where

r is the nominal rate of interest compounded once per year,
r* is the real rate of interest per annum,
π is the expected rate of inflation per annum.

As an example of this relation, suppose that expected inflation is 2% per annum and the nominal rate of interest on a financial asset is 10%. What is the real rate of interest on this investment? If you substitute these values into Irving Fisher's relation above, you will find that the real rate of interest is 7.843% per annum.

Now suppose that expected inflation increases to 5% per annum. For new issues of this financial asset, what will the offered rate of interest be? Using Fisher's relation above, $(1+r) = (1+0.07843)\times(1.05) = 1.13235$. New issues of this financial asset will be offered at about 13.235% per annum.

Inflation Risk: 2 Minutes

7.6.4 Call Risk

Title Page

Long term corporate bonds often include a *call* feature. After a period of non-callability from original issue of about 3 to 5 years, the issuer has the right to buy back a bond issue from

bondholders at par plus a small premium[6]. The issuer is said to *redeem* the issue. The premium is a partial compensation for early retirement of the bond.

After the period of non-callability, an issuer will use the call feature of their issue when market rates of interest in the economy fall. They do this because they can reissue bonds at approximately par at a lesser coupon rate of interest and reduce their interest expense. Of course, because the issuer uses the call feature to their advantage, it is to the disadvantage of bondholders. Bondholders will tend to get par value repayments from the firm when interest rates are low and not otherwise. Redeemed par is, therefore, typically reinvested at a low interest rate to the disadvantage of bondholders. Because call features are unattractive to bondholders, coupon rates tend to be higher on callable bonds compared to non-callable bonds.

7.6.5 Default Risk and Credit Assessment

Title Page

For corporate issuers, an assessment of credit quality is based in part on a number of quantitative measures. These measures include the current ratio, the quick ratio, times interest earned, fixed payment coverage, and financial leverage ratios like debt to invested capital. These ratios were introduced and discussed in chapter 2 of this book. In one way or the other, all of these ratios are meant to give an indication of the ability of a corporation to meet contractual payments. You can find a more extensive discussion of credit analysis in chapter 14 of *The Analysis and Use of Financial Statements* by Gerald I. White, Ashwinpayl Sondhi, and Dov Fried.

Alternatively, for public debt securities, as a bondholder, you might rely upon the independent credit assessment of a rating agency. This rating is based in large part upon the quantitative measures listed above. In Canada, Dominion Bond Rating Service and Canadian Bond Rating Service both offer independent ratings for debt securities for an annual fee which is paid by the issuer. The ratings of Dominion Bond Rating Service are listed below:

[6]The premium typically diminishes as maturity approaches. The premium schedule is set out in the bond indenture agreement, which is monitored by a trustee.

AAA	Highest Quality	Protection of Principal/Interest is of the highest order
AA	Superior Quality	Protection of Principal and Interest is High
A	Medium Quality	Protection of Principal and Interest is substantial
BBB	Medium Quality	Protection of Principal and Interest is adequate but with potential weakness
BB	Lower Medium Quality	Protection of Principal and Interest is uncertain. Mildly speculative.
B	Medium Speculative	Protection of Principal and Interest on a continuing basis is uncertain. Speculative.
CCC	Highly Speculative	In danger of default
CC or C	In Default	

Exhibit 7-5: Bond Ratings and their Interpretation

Credit Risk: 6 Minutes

7.6.6 Sinking Fund Bonds

Title Page

One of the characteristics of a bond is that repayment of principal (loosely speaking) is entirely at maturity rather than over the term of the bond. The ability of a firm to accumulate funds required for a large lump sum payment at maturity is uncertain for many issuers. To alleviate the fears of bondholders on an issuer's ability to repay par, issuers often include a sinking fund provision in their bond issues. In a sinking fund, a portion of par is "retired" each year to the maturity of the bond. A sinking fund transforms a bond into a financial asset that resembles a term loan where principal is repaid over the term of the contract. Smaller firms, younger firms,

and firms coming the public bond market for the first time often include sinking funds in their issues. As an example, a newly issued 20 year, $40 million par value bond might have a sinking fund that requires the firm retire 5% of the issue each year to maturity.

For a publicly traded bond, the issuer can satisfy the sinking fund obligation either by an open market purchase of their bonds in the secondary market or by a partial redemption of the issue administered by the trustee who oversees the bond issue. In the above example, the firm must retire $2 million dollars par of its bonds at the end of each year to maturity of the issue. The firm can satisfy this requirement by buying in the secondary market this amount of its bonds in par value terms. Alternatively, the trustee can *randomly* redeem $2 million par value worth of bonds. As a bondholder, if your bond is redeemed, you receive a check from the trustee in the amount of the par value of your bond. In this partial redemption, it is typically the case that no premium for early retirement is offered.

Firms use an open market purchase or a partial redemption when it is to their advantage. When interest rates in the economy have increased since original issue, the price of bonds in the secondary market is lesser than par, and therefore, firms will tend to use open market purchases to satisfy sinking fund requirements. On the other hand, if interest rates have fallen, bond prices tend to be high, and therefore, a partial redemption at par is the cheapest way to satisfy a sinking fund requirement. As a bondholder, you are most likely to receive par on your bond in a partial redemption when reinvestment rates are low. The sinking fund, therefore, adds an accentuated element of reinvestment risk to your investment. Because a sinking fund aims to reduce default risk but adds accentuated reinvestment risk the effect of a sinking fund on required coupon rates in new issue bonds is unclear. If reduced default risk has the dominant effect, a sinking fund will tend to reduce required coupon rates.

7.6.7 Floating Rate Bonds

Title Page

In a floating rate bond, the coupon rate is not fixed over the life of the bond, but it is adjusted periodically to the general level of interest rates in the economy. For example, the coupon rate on a bond might be adjust every 6 months for the upcoming coupon payment to the rate of interest on a short-term Government of Canada Treasury bill.

What is the value of a floating rate bond? Because the coupon rate on a floating rate bond is adjusted to the level of interest in the economy, the coupon rate of interest is more or less the same as the discount rate of interest with respect to present value calculation[7]. This observation means that a floating rate bond will always trade around its par value. Because price does not change greatly on a floating rate bond, regardless of changes in interest rates in the economy, it has little or no price risk. On the other hand, a floating rate bond has accentuated reinvestment risk. When general rates of interest are high, the coupon rate on the bond is high and you reinvest this relatively greater coupon at a high market rate of interest. On the other hand, when general rates of interest are low, the coupon rate on the bond is low and you reinvest this relatively low coupon at a low market rate of interest.

[7]There are some caveats to this comment. First the coupon is not adjusted continuously but only every six months (typically). Also, some of the coupons on the bond can be receivable over the long term but the expected coupon rate is based on a short-term interest in the economy. If the term structure of interest rates is not flat, the discount rate for coupons (or par) to be received in the distant future may be different from the expected coupon rate.

(7.7) Summary Title Page

Debt describes a class of financial assets which make contractually promised payments. Private debt which is generally provided by financial institutions in Canada is typically the first external financial asset which a firm sells. Because banks and trust companies have a presence in the neighborhoods of Canada through their retail network of branches, they are able to assess the business prospects of local and regional businesses. Branch offices provide banks and trust companies vast distribution networks for their products and services.

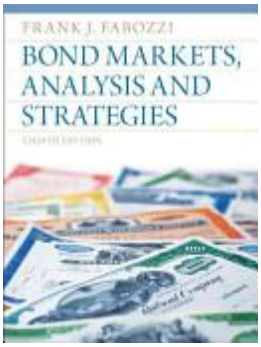
An *operating loan* is a short-term borrowing arrangement which firms use to cover daily operating expenses. Short-term borrowing can be made more flexible with a *line of credit*. A line of credit establishes the maximum amount a that can be borrowed but the timing of this borrowing is decided upon by the firm. A line of credit is usually for a specific one-time purpose. In a *revolving credit* arrangement, a firm can continually pay down or increase its borrowing as long as the net amount advanced is below a specified limit. Firms use term loans for longer term financing needs and repayment is scheduled beyond one year. Term loans are generally used to purchase plant, property, and equipment. Mortgages are long-term borrowing arrangements which are secured with specific assets of a firm. In conventional mortgages real estate is used as collateral. In chattel mortgages, moveable equipment is the collateral.

If a firm prospers and grows in national and international product and service markets, it might be able to access public debt and equity markets. A bond is a public financial asset which makes fixed contractually promised payments for a finite term. Bond payments are typically fixed and do not vary over the term of the investment. There are a number of risks associated with fixed income financial assets. These risks include price risk, reinvestment risk, inflation risk, risk induced by contract features, default risk.

(7.8) Suggested Reading

Title Page

Bond Markets, Analysis, and Strategies



Frank J. Fabozzi
Pearson, 2013

(7.9) Problems Title Page

1. ***Bond Valuation*** Title Page

Consider an 18-year bond that promises annual coupon payments. The bond has just paid a coupon payment. The coupon rate on the bond is 10% per annum (compounded once per year). The yield on the bond is 11% per annum. The par value is $10,000. Find the current yield. Explain why the current yield is greater or lesser than the yield to maturity.

Solution

2. ***Bonds and Retirement Planning*** Title Page

In planning for your retirement – in twenty years – you are considering which *one* of the two bonds shown below makes the best investment today. The price of each bond is the same, $1,171.59, and each bond yields 10% per annum, compounded semi-annually. Assume that if you buy the first bond, you will reinvest coupons at 10 percent per annum, compounded semiannually, until your retirement.

i) a twenty-year 12% per annum coupon rate bond (paid semi-annually). Forty coupons remain to be paid and the first is due in exactly six months. Par value is $1000.

ii) a bond that pays no coupons but only a lump sum amount (the face amount) in exactly 20 years (i.e., a zero-coupon bond).

Required: If your objective is to have the most money possible for your retirement, which bond is the better investment (all else equal)?

Solution

3. ***Bond Valuation and Term to Maturity*** Title Page

Consider two bonds selling in the secondary bond market. Both make equal annual coupon payments. One bond has five years to maturity; the other has ten years to maturity. The maturity values of both bonds are $1000. Both bonds have just made an interest payment. You may assume a flat term structure of interest rates (i.e., the per annum rate of interest for an investment of any term is the same). Notice that the ten-year bond promises more cash flows than the five-year bond. Nonetheless, demonstrate with a numerical example of your choosing that when these bonds trade at a discount, the value of the five-year bond exceeds the value of the ten-year bond.

Solution

4. ***Pricing Zeroes*** Title Page

A *zero-coupon bond* is a bond that pays no coupons but simply promises to make a lump sum payment at a specific time in the future. Zero-coupon bonds are originally sold at a deep discount to par value. Current interest rates on such bonds are 10% per annum (compounded once per year). If the expected capital gain on a particular zero-coupon bond (the capital gain that arises solely because of the passage of time) is currently 1% of the bond's face value (i.e., par value), what is the current maturity of the bond?
NOTE: The expected capital gain is measured over a one-year holding period starting now.

Solution

5. ***Bond Valuation*** Title Page

You have just purchased a newly issued $1000 five-year ABC Company bond at par. The fixed coupon rate is 12% paid semi-annually (the next receivable coupon is due in exactly six m nths from today). You are considering buying another ABC Company bond (another issue with possibly different features and terms). This second bond has been outstanding for some time now and it trades in the secondary market. The second bond has 5.5 years remaining to maturity, has a fixed coupon rate of 9% paid *annually* (the next coupon to be received in six months), and has a par value of $1000.

a) What is the yield to maturity on the five-year bond?

b) Employ the rate you calculated in (a) (in an appropriate manner) to value the 5.5 year bond.

Solution

6. ***Pricing Zeroes*** Title Page

A *zero-coupon bond* is a bond that pays no coupons, but simply promises to make a lump-sum payment in the future. Zero coupon bonds are sold at a discount at issue.

a) Is it possible for a zero coupon bond to sell at a premium? Explain.

b) All else equal, including default risk, are the prices of zero-coupon bonds more sensitive or less sensitive to interest rate changes than a bond with identical par value and identical maturity, but with positive coupons? Explain.

c) Which is more sensitive to interest rate changes, the price of a short-term zero-coupon bond or the price of a long-term zero-coupon bond? Explain.

d) If the price of a zero-coupon bond is the same next year as it is today, what will have happened to interest rates? Explain.

Solution

7. ***Bond Valuation*** Title Page

Five years ago, Jean Screetien purchased a fifteen-year bond at par for $10,000 that promises a 14-percent coupon rate. The bonds pay coupons once per year, the first coupon was one year after purchase. Five years after purchase, immediately after receiving a coupon, Jean sold the bond for $11,000. In the interim, Jean invested the received coupons at an annual interest rate of 12% per annum.

a) What was the yield to maturity on the bond when Jean purchased it?

b) What is the total five-year realized rate of return on Jean's overall investment of $10,000.

c) What is the annualized rate of return?

Solution

8. What characteristics of an "equity" financial asset distinguish it from debt?

Solution

9. Discuss the following assertion: "A call feature on a bond is attractive to bondholders because there is a possibility they get the par value back sooner."

Solution

10. Which of the following cases characterizes a bond selling at a premium? Explain.

	Current Yield Greater than Coupon Rate	Yield to Maturity Greater than Current Yield
a.	Yes	Yes
b.	Yes	No
c.	No	Yes
d.	No	No

Solution

11. ***Bond Valuation.***
Comment on the following assertion: "A ten year bond must always have a value greater than an otherwise identical 5 year bond (i.e., same coupon rate, same par, etc.) because it has more coupons. Use no numerical examples in your response.

Solution

12. ***Bond Valuation.***
Consider a five-year bond with a coupon rate of 9% per annum and a current yield of 8% per annum (both rates are compounded semi-annually). One year from now (immediately after a coupon payment) this bond will have a value that is:

a) higher
b) lower
c) the same
d) possibly higher or lower

Explain your reasoning.

Solution

13. ***Reinvestment Risk.***
Which has greater reinvestment risk, a premium bond or a discount bond? Explain.

Solution

14. ***Reinvested Coupons.***
A bond pays coupons December 31, and June 30 of every year. Today is March 31, 1995. The coupon rate on the bond is 11.25% per annum compounded semi-annually. The yield on the

bond is 10% per annum compounded semi-annually and the par is $10,000. The bond matures June 30, 2010.

a) What is the quoted price of the bond?

b) You buy the bond today, hold it for exactly 5 years and reinvest coupons at 9% per annum compounded semi-annually. When you sell the bond (in 5 years) you expect the yield to be 9 percent per annum compounded semiannually. What is your annualized holding period rate of return on your investment (compounded once per annum)?

Solution

15. ***Calculating the Yield to Maturity*.**

Consider a bond that offers a 10% coupon rate, paid semi-annually, and has a par value of $10,000. There are 20 remaining coupon payments on the bond. The next and upcoming coupon is in exactly 4 months.

a) Use "goal seek" in EXCEL to find the yield to maturity on this bond if it has a *value* (i.e., invoice price) of $9,000, $10,000, or $11,000. Recall that the yield to maturity is the IRR on the bond purchase stated as a per annum nominal rate compounded semi-annually.

b) What is accrued interest on this bond?

c) What is the quoted price of the bond if it has an invoice price of $9,000, $10,000, or $11,000.

Solution

16. ***Sinking Fund.***

ABC Ltd. has just sold a bond at par that makes annual coupon payments. Par value on this issue of bonds is $10,000,000 in total. Maturity of the bond is in 20 years. The coupon rate is 10% per annum compounded once per year. As part of the bond indenture agreement, there is a sinking fund for the bond. The sinking fund requires that $500,000 of *par* be "retired" at the end of each year to the maturity of the bond. The sinking fund requirement can be satisfied with open market repurchase or a partial (and random) redemption administered by the trustee of the bond issue. In the first year the bond is outstanding, if ABC uses either an open market repurchase or a partial redemption to satisfy its sinking fund requirement, it will "repurchase" bonds immediately after the first coupon.

a) What is the cost to ABC of satisfying its sinking fund requirement (ignoring the coupon payment) for the first year if, during the first year the bond is outstanding, yields in the bond market increase to 12% per annum compounded once per year.

b) What is the cost to ABC of satisfying its sinking fund requirement (ignoring the coupon payment) for the first year if, during the first year the bond is outstanding, yields in the bond market decrease to 8% per annum compounded once per year.

c) In each of the two cases described above, find the holding period rate of return for a year for a bondholder who either (as appropriate for the cases above) sells to the firm in the open market repurchase or has his/her bond redeemed in the partial redemption.

Solution

17. ***Reinvestment of Coupons.***
Consider a bond that offers a 10% coupon rate, paid semi-annually, and has a par value of $10,000. There are 20 remaining coupon payments on the bond. The next and upcoming coupon is in exactly 4 months. The yield to maturity on the bond is 11 percent per annum compounded semi-annually. You buy the bond today and sell it in exactly three years. The yield on the bond at that time is 10% per annum compounded semi-annually. Over the interim, you receive and reinvest coupons at a rate of 10.5 per cent per annum compounded daily (360 days in a year). For the purpose of calculations with this rate, you can presume that there are 30 days in a month.

a) What is your annualized holding period rate of return on your three-year investment?
b) What is the quoted price of the bond and accrued interest when you purchase it?
c) What is the quoted price of the bond and accrued interest when you sell it?

Solution

18. ***Reinvestment of Coupons.***
Consider a bond that offers an 8.0% coupon rate, paid semi-annually, and has a par value of $10,000. There are 30 remaining coupon payments on the bond. The next and upcoming coupon is in exactly 5 months. The yield to maturity on the bond is 8 percent per annum compounded semi-annually. You buy the bond today and sell it in exactly three years and seven months. The yield on the bond at that time is 8% per annum compounded semi-annually. Over the interim, you receive and reinvest coupons at a rate of 7.5 per cent per annum compounded monthly.

What is the *annualized* holding period rate of return on your three year, seven-month investment, compounded quarterly?

Solution

19. ***The current-yield versus the yield to maturity.***
Consider a bond that makes annual coupon payments. The bond has just made a coupon payment. The next and upcoming coupon on the bond is in one year. The coupon rate on the bond is 9% per annum (compounded once per year). Par value is $10,000. The *current yield* on the bond is 9.90368% (using the annual coupon). If you sell the bond in one year, after the coupon payment at that time, and if the yield to maturity on the bond does not change, you expect an invoice price of $9,232.59655 (accrued interest is zero).

a) What is the value of the bond today? (Quoted price and invoice price are the same because the bond has just made a coupon payment. Accrued interest is zero).

b) What is the "expected capital gain" on the bond between today and one year from today, relative to the price which you pay today (i.e., measured as a percentage of today's price)?

c) Using information in the statement of the problem and (b) above, find the yield to maturity on the bond (compounded once per year)?

Solution

20. ***Bond Valuation.***
One year ago you bought a bond for a price of $1,250 immediately after it had paid a coupon. The bond makes annual coupon payments. Today, you received a coupon of $100 and you sold the bond for $1,200. List and discuss the reasons why the price of the bond might have fallen over the one-year holding period.

Solution

21. ***Reinvested Coupons, Quoted Price, Invoice Price, Accrued Interest.***
Consider a fixed rate bond that makes annual coupon payments. The bond has just made a coupon payment and the next and upcoming coupon is in exactly one year. The remaining maturity on the bond is 26 years. The par value is $10,000. The current yield on the bond is 7% per annum and the coupon rate is 8% per annum (compounded once per annum). You buy the bond today and hold it for exactly 45 months. You reinvest the coupons you receive over this period at the rate of 6.75% per annum compounded quarterly. When you sell the bond, the quoted price is $11,298.00.
a) What is your annualized holding period rate of return compounded monthly on your 45-month investment?
b) Is the yield to maturity on the bond (compounded once per annum) greater than or lesser than the current yield when you purchase it? Explain.

Solution

22. ***Reinvested Coupons and the Holding Period Rate of Return***
Consider a fixed rate bond that makes semi-annual coupon payments. The coupon rate is 6.75% per annum compounded semi-annually. The yield is 6.75% per annum compounded semi-annually. The par value is $10,000. There are 25 remaining coupons on the bond and the next and upcoming coupon is in one month. You buy this bond, hold it for four years and two months, and then sell it. At this time, the yield on the bond is 6.75% per annum compounded semi-annually. In the interim (between purchase and sale of the bond), you reinvest coupons at 6.75% per annum compounded semi-annually.
Required: What is your annualized holding period rate of return on your investment, compounded monthly?

Solution

23. ***Reinvested Coupons and the Holding Period Rate of Return***
Consider a fixed rate bond that offers a 10.0% coupon rate, paid semi-annually, and has a par value of $10,000. There are 40 remaining coupons on the bond. The next and upcoming coupon is in exactly 4 months. You buy the bond today and sell it in exactly 7 years and 3 months. When you sell the bond, its yield to maturity is 6 percent per annum compounded semi-annually. The yield to maturity when you buy the bond is not necessarily the same as when you sell the bond. Between the purchase and sale of the bond, you receive and reinvest coupons at a rate of 6 per cent per annum compounded semi-annually. Your annualized holding period rate of return compounded semi-annually on your 7 year and 3 month investment is 6.44%.
Required: What was the invoice price of the bond when you purchased it?

Solution

24. ***Reinvested Coupons and the Holding Period Rate of Return***
Consider a bond that offers a fixed per annum coupon rate, paid semi-annually, and has a par value of $10,000. The bond matures in twelve years and eleven months. The coupon rate and the yield on this bond equal one another. You buy the bond today and sell it in 5 years and 3 months. When you sell the bond, the yield still equals the coupon rate. At this time, the invoice price of the bond is $10,297.79. Between your purchase and sale of the bond, you reinvest coupons at 8.5% per annum compounded monthly.

Required: What is your annualized holding period rate of return compounded semi-annually on your investment between bond purchase and sale including received coupons with interest.

Solution

25. ***Reinvested Coupons and the Holding Period Rate of Return***

Consider a bond that offers a fixed per annum coupon rate, paid semi-annually, and has a par value of $10,000. The coupon rate and the yield both equal 9% per annum compounded semi-annually. Today, the invoice price of the bond is $10,373.62 and the quoted price is $9,998.62. You buy the bond today and sell it in 12 years and 8 months. When you sell the bond, the yield still equals the coupon rate. Between your purchase of the bond and your sale of the bond, you reinvest coupons at the 8.75 percent per annum compounded semi-annually.

Required: What is the annualized holding period rate of return on your bond investment between purchase and sale including received coupons with interest?

Solution