TRAFFIC SIGNAL CONTROL ALGORITHMS OF A TRAFFIC NETWORK

E. J. Davison*, H. Shimizu and H. Naraki****

**Department of Electrical Engineering, University of Toronto, Toronto, Ontario M5S IA4, Canada*

***Department of Information Processing Engineering, Fukuyama University, Fukuyama, Hiroshima, 729-02, Japan*

Abstract. Traffic signal control algorithms of a traffic network are presented using a hierarchical control concept. The traffic congestion mechanism is described quantitatively based on the traffic volume balance at each signalized intersection. The traffic signal control system of the traffic congestion length is described by a linear time-varying discrete dynamic system in the traffic network. The hierarchical traffic signal control algorithm using the priority control algorithm is presented and simulated at twelve signalized intersections of the traffic network.

Key Words. Traffic control; feedback control; hierarchical control; control system design; control algorithms; queuing network models; adaptation; simulation.

1. INTRODUCTION

The traffic congestion has steadily increased in urban traffic networks in Japan. The traffic congestion mechanism can be described quantitatively based on the traffic volume balance at each signalized intersection(Shimizu *et al.*, 1994). The traffic signal control is effective to control the traffic congestion in traffic networks. On-line traffic signal control methods such as SCOOT(Hunt *et al.*, 1981) , decentralized control(Davison and Özgüner, 1983) and congestion pattern selection(Shibata and Yamamoto, 1985) were presented to control the traffic congestion in traffic networks. Since the incoming traffic volume at each signalized intersection increases rapidly in rush hours, the three traffic signal control parameters consisting of the cycle length, green split and offset have to be controlled adaptively and sequentially.

This paper constructs the traffic signal control system and presents the traffic signal control algorithms in a traffic network. The traffic volume balance is held at each signalized intersection of the traffic network for a certain sampling period. Based on the traffic volume balance at each signalized intersection, the traffic congestion mechanism can be described quantitatively. Based on the traffic volume balance at each signalized intersection, and regarding the excess incoming traffic volume as the state variable, the time-dependent characteristics of the traffic congestion length are described by a linear time-varying discrete dynamic system. The capacity at each signalized intersection is evaluated summing up each lane capacity. The traffic signal control system at each intersection is presented in the traffic network using a hierarchical control concept. In this control system, the reference input, control input and output are given by the permitted queue length, three traffic signal control parameters and traffic congestion length respectively. The controllability of the traffic signal control system is also considered.

Two traffic signal control algorithms of the traffic congestion length in the traffic network consisting of a rectangular grid of intersecting streets are presented; one is a "priority control algorithm", the other is a "balance control algorithm". In the priority control algorithm, the traffic congestion lengths of the arterial directions are controlled prior to the other ones, and the three traffic signal control parameters are adaptively and sequentially controlled according to the variation of incoming traffic volumes so as to minimize the sum of the traffic congestion lengths of the arterial directions. In the balance control algorithm, the traffic congestion lengths which cross each other on a road are controlled so as to become equal and minimize the sum of the traffic congestion lengths of the traffic network. The main difference between the priority control algorithm and the balance control algorithm is as follows: The green splits and cycle length are corrected for the arterial directions or the corresponding cross directions depending on control index values. Hierarchical traffic signal control algorithms are presented for the traffic network using the two above control algorithms.

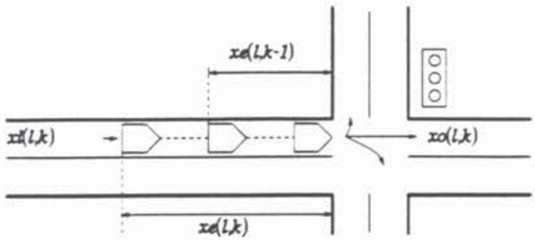

Fig. 1. Traffic volume balance at each signalized intersection.

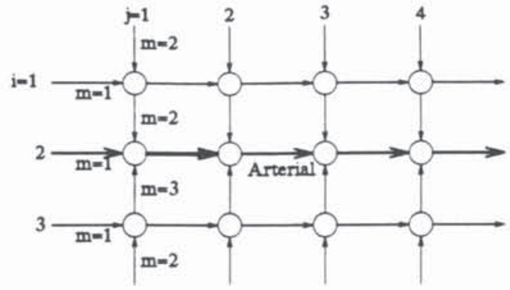

Fig. 2. Traffic network consisting of a rectangular grid of intersecting streets.

The priority and balance control algorithms of the traffic congestion length are simulated using data obtained in Fukuyama city, Japan. The simulation confirms that the cycle lengths, green time and offsets are adaptively and sequentially controlled according to the variation of incoming traffic volumes, and the two traffic signal control algorithms work well to control the traffic congestion lengths on the arterial according to each performance criterion. The presented hierarchical traffic signal control algorithm is also simulated at twelve signalized intersections in Fukuyama city. The simulation results are compared with the measurement data.

2. TRAFFIC SIGNAL CONTROL SYSTEM

Traffic signal control systems of the traffic congestion length are constructed on an arterial and in a traffic network respectively.

2.1. *Traffic congestion mechanism*

The traffic volume balance is held at each signalized intersection of the traffic network for a certain sampling period(see Fig. 1), and described by the following difference equation.

$$x_e(\ell, k) = x_e(\ell, k-1) + x_i(\ell, k) - x_o(\ell, k) \quad (1)$$

$$\begin{cases} x_o(\ell, k) < c_z(\ell, k) \\ x_e(\ell, k) \geq 0 \end{cases} \quad (2)$$

where ℓ and k denote a day of the week and time, also $x_e(\ell, k)$, $x_i(\ell, k)$, $x_o(\ell, k)$, and $c_z(\ell, k)$ denote the excess incoming traffic volume, the incoming traffic volume, the outgoing traffic volume, and the capacity at each signalized intersection respectively.

The traffic congestion mechanism can be described quantitatively based on the traffic volume balance at each signalized intersection of (1).

2.2. *Evaluation of capacity*

The capacity at each signalized intersection is evaluated summing up each lane capacity as follows.

$$\begin{aligned} c_z(\ell, k) &= r_{g\ell}(\ell, k)c_{z\ell}(\ell, k) + r_{gs}(\ell, k)c_{zs}(\ell, k) \\ &\quad + r_{gr}(\ell, k)c_{zr}(\ell, k) \end{aligned} \quad (3)$$

$$\begin{cases} c_{z\ell}(\ell, k) = s_\ell n_\ell r_\ell(\ell, k) r_t(\ell, k) r_b(\ell, k) \\ c_{zs}(\ell, k) = s_s n_s r_\ell(\ell, k) r_t(\ell, k) r_b(\ell, k) \\ c_{zr}(\ell, k) = s_r n_r r_t(\ell, k) \end{cases} \quad (4)$$

where $r_{g\ell}(\ell, k)$, $r_{gs}(\ell, k)$, $r_{gr}(\ell, k)$ are the green splits and $c_{z\ell}(\ell, k)$, $c_{zs}(\ell, k)$, $c_{zr}(\ell, k)$ are the capacities, of left-turn-, straightforward- and right-turn-traffic lanes respectively. The correction factors $r_\ell(\ell, k)$, $r_t(\ell, k)$ and $r_b(\ell, k)$ denote for left turns, trucks and local buses stopping respectively. The constant factors s_ℓ, s_s, s_r denote the saturation flows and n_ℓ, n_s, n_r denote the lane numbers, of each traffic lane.

2.3. *Traffic signal control system*

The traffic volume balance at each signalized intersection of the traffic network is rewritten as follows.

$$\begin{aligned} x_e(i, j, m, \ell, k) &= x_e(i, j, m, \ell, k-1) \\ &\quad + x_i(i, j, m, \ell, k) - x_o(i, j, m, \ell, k) \end{aligned} \quad (5)$$

$$\begin{cases} x_o(i, j, m, \ell, k) < c_z(i, j, m, \ell, k) \\ x_e(i, j, m, \ell, k) \geq 0 \end{cases} \quad (6)$$

where i, j and m denote the location of each signalized intersection and the moving direction of motor-cars respectively(see Fig. 2).

In the traffic volume balance at each signalized intersection of (5), the incoming traffic volume $x_i(i, j, m, \ell, k)$ is controlled by the three traffic signal control parameters at the upstream signalized intersection, and the outgoing traffic volume

$x_o(i,j,m,\ell,k)$ is controlled by those at the signalized intersection concerned. Therefore

$$\begin{aligned} x_i(i,j,m,\ell,k) - x_o(i,j,m,\ell,k) = \\ f[c_y(i,j,m,\ell,k), r_g(i,j,m,\ell,k), \\ t_{off}(i,j,m,\ell,k)] \end{aligned} \tag{7}$$

where
$c_y(i,j,m,\ell,k)$, $r_g(i,j,m,\ell,k)$ and $t_{off}(i,j,m,\ell,k)$ denote the cycle length, green split and offset respectively. The control input $u(i,j,m,\ell,k)$ is defined by

$$\begin{aligned} u(i,j,m,\ell,k) \triangleq f[c_y(i,j,m,\ell,k), \\ r_g(i,j,m,\ell,k), t_{off}(i,j,m,\ell,k)] \end{aligned} \tag{8}$$

The traffic signal control system is then written as follows.

$$\begin{cases} x_e(i,j,m,\ell,k) \\ \quad = x_e(i,j,m,\ell,k-1) + u(i,j,m,\ell,k) \\ y_\ell(i,j,m,\ell,k) \\ \quad = \ell_m(i,j,m,\ell,k) x_e(i,j,m,\ell,k) \end{cases} \tag{9}$$

The observation equation of the traffic congestion length $y_\ell(i,j,m,\ell,k)$ is described in such a way that the state variable is multiplied by a "transformation factor" $\ell_m(i,j,m,\ell,k)$. The condition $x_e(i,j,m,\ell,k) \leq 0$ is derived from (5) as

$$\begin{aligned} x_e(i,j,m,\ell,k-1) + x_i(i,j,m,\ell,k) \\ \leq x_o(i,j,m,\ell,k) \end{aligned} \tag{10}$$

If the equation (10) is satisfied, the traffic signal control system (9) is controllable.

In this way, the traffic signal control system of the traffic congestion length is constructed at each signalized intersection. The purpose of the system is to find such the control inputs that they make the following performance criteria minimize.

$$J_i(i,j,\ell,k) = \sum_m |e(i,j,m,\ell,k)| \tag{11}$$

for each signalized intersection

$$J_a(i,\ell,k) = \sum_j \sum_m |e(i,j,m,\ell,k)| \tag{12}$$

for each arterial

$$J_n(\ell,k) = \sum_i \sum_j \sum_m |e(i,j,m,\ell,k)| \tag{13}$$

for the traffic network

The control error $e(i,j,m,\ell,k)$ is defined by

$$e(i,j,m,\ell,k) \triangleq$$

$$\ell_r(i,j,m,\ell,k) - y_\ell(i,j,m,\ell,k) \tag{14}$$

where $\ell_r(i,j,m,\ell,k)$ denotes the permitted queue length.

3. TRAFFIC SIGNAL CONTROL ALGORITHMS

Two traffic signal control algorithms of the traffic congestion length are presented on the arterial; one is a "priority control algorithm", the other is a "balance control algorithm". The hierarchical traffic signal control algorithm is presented in the traffic network using the two above control algorithms.

3.1. *Priority control algorithm*

The priority control of the traffic congestion length means that the traffic congestion lengths of the arterial directions are controlled prior to other ones, and the three traffic signal control parameters are adaptively and sequentially controlled so as to minimize the performance criterion of the arterial directions on the arterial.

Step 1 : The parameters, control indexes and initial conditions are set.

Step 2 : The incoming traffic volume $x_i(i,j,m,\ell,k)$ is inputted, and the sampling time ΔT is equally set to the cycle length $c_y^{(n-1)}(i,j,m,\ell,k)$. The symbol $(n-1)$ denotes the repetition times of computation.

Step 3 : The incoming traffic volume is recalculated.

$$\begin{aligned} x_i'^{(n)}(i,j,m,\ell,k) = x_i^{(n-1)}(i,j,m,\ell,k) \\ + x_e^{(n-1)}(i,j,m,\ell,k-1) \end{aligned} \tag{15}$$

Step 4 : The capacity $c_x(i,j,m,\ell,k)$ is evaluated summing up each lane capacity.

$$\begin{cases} c_{x\ell}^{(n)}(i,j,m,\ell,k) = \\ \quad r_{g\ell}^{(n-1)}(i,j,m,\ell,k) c_{x\ell}^{(n-1)}(i,j,m,\ell,k) \\ c_{xs}^{(n)}(i,j,m,\ell,k) = \\ \quad r_{gs}^{(n-1)}(i,j,m,\ell,k) c_{xs}^{(n-1)}(i,j,m,\ell,k) \\ c_{xr}^{(n)}(i,j,m,\ell,k) = \\ \quad r_{gr}^{(n-1)}(i,j,m,\ell,k) c_{xr}^{(n-1)}(i,j,m,\ell,k) \end{cases} \tag{16}$$

$$\begin{aligned} c_x^{(n)}(i,j,m,\ell,k) = c_{x\ell}^{(n)}(i,j,m,\ell,k) \\ + c_{xs}^{(n)}(i,j,m,\ell,k) + c_{xr}^{(n)}(i,j,m,\ell,k) \end{aligned} \tag{17}$$

Step 5 : The green time of straightforward traffic lane $g_s(i,j,m,\ell,k)$ to ensure the outgoing traffic volume is evaluated.

$$g_s^{(n)}(i,j,m,\ell,k) = d_s + \frac{c_y^{(n-1)}(i,j,m,\ell,k) r_{gs}^{(n-1)}(i,j,m,\ell,k) x_i'^{(n)}(i,j,m,\ell,k)}{c_{xs}^{(n)}(i,j,m,\ell,k) + \frac{r_{gr}^{(n-1)}(i,j,m,\ell,k)}{r_{gs}^{(n-1)}(i,j,m,\ell,k)} c_{xr}^{(n)}(i,j,m,\ell,k)} \quad (18)$$

where d_s denotes the starting delay.

Step 6 : The cycle length is evaluated by dividing the green time by the assumed green split.

$$c_y^{(n)}(i,j,m,\ell,k) = \frac{g_s^{(n)}(i,j,m,\ell,k)}{r_{gs}^{(n-1)}(i,j,m,\ell,k)} \quad (19)$$

Step 7 : The excess incoming traffic volume $x_e(i,j,m,\ell,k)$ is evaluated based on the traffic volume balance.

$$x_e^{(n)}(i,j,m,\ell,k) = x_i'^{(n)}(i,j,m,\ell,k) - x_o^{(n)}(i,j,m,\ell,k) \quad (20)$$

$$\begin{cases} x_o^{(n)}(i,j,m,\ell,k) = \\ \quad \xi^{(n)}(i,j,m,\ell,k) c_x^{(n)}(i,j,m,\ell,k) \\ x_e^{(n)}(i,j,m,\ell,k) \geq 0 \end{cases} \quad (21)$$

$$\xi^{(n)}(i,j,m,\ell,k) = \frac{g_s^{(n)}(i,j,m,\ell,k) - d_s}{g_s^{(n)}(i,j,m,\ell,k)} \quad (22)$$

Step 8 : The traffic congestion length $y_\ell(i,j,m,\ell,k)$ is evaluated.

$$y_\ell^{(n)}(i,j,m,\ell,k) = \ell_m(i,j,m,\ell,k) x_e^{(n)}(i,j,m,\ell,k) \quad (23)$$

Proceed to Step 9 for $m = 1$(arterial direction), or return to Step 2 for $m = 2, 3$(other directions).

Step 9 : If the following control index $e^{(n)}(i,j,m,\ell,k) \geq 0$ is satisfied, we apply the green splits and the cycle length at optimum time values and proceed to Step 11.

$$e^{(n)}(i,j,m,\ell,k) = \ell_r(i,j,m,\ell,k) - y_\ell^{(n)}(i,j,m,\ell,k) \quad (24)$$

Step 10 : If $e^{(n)}(i,j,m,\ell,k) < 0$, the green splits are corrected.

$$\begin{cases} r_{gs}^{(n)}(i,j,m,\ell,k) = \\ \quad r_{gs}^{(n-1)}(i,j,m,\ell,k) + \Delta r_{gs}(i,j,m) \\ r_{g\ell}^{(n)}(i,j,m,\ell,k) = \\ \quad r_{g\ell}^{(n-1)}(i,j,m,\ell,k) + \Delta r_{g\ell}(i,j,m) \\ r_{gr}^{(n)}(i,j,m,\ell,k) = \\ \quad r_{gr}^{(n-1)}(i,j,m,\ell,k) + \Delta r_{gr}(i,j,m) \end{cases} \quad (25)$$

Return to Step 2.

Step 11 : The green time, green splits and cycle length are evaluated for the cross directions to the arterial direction based on the relationships among the traffic signal control parameters at each signalized intersection.

for $i = 1,2,3 \;\; j = 1,2,3,4 \;\; m = 2$
After the evaluations proceed to Step 12.

for $i = 2 \;\; j = 1,2,3,4 \;\; m = 3$
The traffic signal control parameters are equally set to the direction of $m = 2$. Return to Step 2.

Step 12 : The optimum relative offset $t_{off}(i,j,m,\ell,k)$ is evaluated.

$$t_{off}(i,j,m,\ell,k) = \frac{d(i,j,m)}{v(i,j,m,\ell,k)} - \frac{q^{(n)}(i,j,m,\ell,k)}{\varphi^{(n)}(i,j,m,\ell,k)} \quad (26)$$

where $d(i,j,m)$, $v(i,j,m,\ell,k)$, $q^{(n)}(i,j,m,\ell,k)$ and $\varphi^{(n)}(i,j,m,\ell,k)$ denote the road length, average speed, queue number of motor-cars while the signal at the downstream intersection has been red, and the saturation flow on the approach at the downstream intersection.

This control algorithm is executed from $k = 1$ to $k = k_f$, $i = 1,3,2 \;\; j = 1,2,3,4 \;\; m = 1,2,3$.

3.2. *Balance control algorithm*

The balance control of the traffic congestion length means that two traffic congestion lengths which cross each other on a road are controlled so as to become equal. In order to accomplish this balance control, the three traffic signal control parameters are adaptively controlled so as to minimize the performance criterion of the arterial.

Step 1 to *Step 7* : The same as the priority control algorithm.

Step 8 : The traffic congestion length is evaluated from (23).

Step 9 : The green time, green splits and cycle length are evaluated for the cross directions to the arterial direction based on the relationships among the traffic signal control parameters at each signalized intersection.

Step 10 : If the following control index

$$\max\{|e^{(\kappa)}(i,j,1,l,k)|, |e^{(\omega)}(i,j,2,l,k)|, |e^{(\lambda)}(i,j,3,l,k)|\} \leq \epsilon, \epsilon > 0$$

is satisfied, we apply the green splits and the cycle length at optimum time values and proceed to Step 12.

Step 11 : If

$$\max\{|e^{(\kappa)}(i,j,1,l,k)|, |e^{(\omega)}(i,j,2,l,k)|, |e^{(\lambda)}(i,j,3,l,k)|\} > \epsilon$$

then the green splits are corrected using (25). Return to Step 2.

Step 12 : The optimum relative offset is evaluated from (26).

This control algorithm is executed from $k = 1$ to $k = k_f$, $i = 1, 3, 2$ $j = 1, 2, 3, 4$.

The main difference between the priority control algorithm and the balance control algorithm is as follows : The green splits and cycle length are corrected for the arterial direction or the corresponding cross directions depending on the control index values.

3.3. *Hierarchical control algorithm*

The hierarchical control of the traffic congestion length means that the three traffic signal control parameters are adaptively and hierarchically controlled so as to minimize the performance criterion in the traffic network.

Step 1 : Using the two above control algorithms, the three traffic signal control parameters are evaluated so as to minimize the sum of the absolute control errors on each arterial described by (12).

Step 2 : The maximum value of the cycle lengths evaluated at Step 1 is set to all signalized intersections in the traffic network, and two other traffic signal control parameters are evaluated again so as to minimize the same performance criterion as Step 1.

Step 3 : The offsets between two arterials are evaluated so as to minimize the total sum of the absolute control errors for the traffic network described by (13).

The Hierarchical control algorithm is drawn by Fig. 3.

4. SIMULATION RESULTS AND DISCUSSIONS

4.1. *Arterial*

The priority and balance control algorithms of the traffic congestion length are simulated at two-adjacent signalized intersections, using data from 7:00 a.m. to 7:00 p.m. obtained in Fukuyama city, Japan. The parameters and the reference input are set as follows.

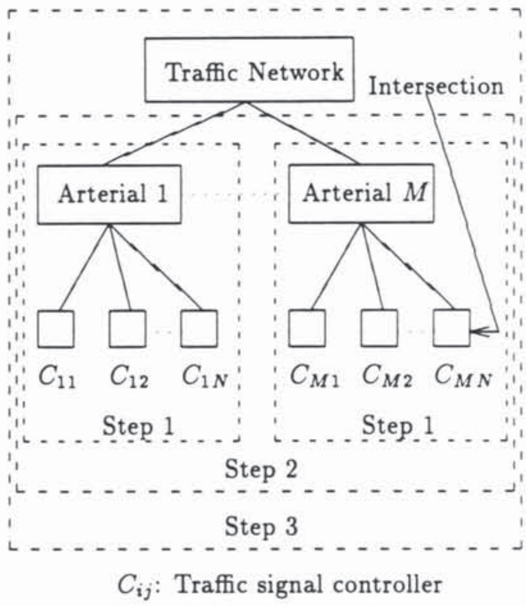

Fig. 3. Hierarchical control algorithm.

$$\begin{cases} \Delta T = c_y(i,j,m,\ell,k) \\ \ell_m(i,j,m,\ell,k) = 6.46(m) \\ \ell_r(i,j,1,\ell,k) = 0(m) \text{ for the priority control} \\ \ell_r(i,j,m,\ell,k) = 0(m) \text{ for the balance control} \end{cases}$$

From the simulation results, it is confirmed that the incoming traffic volumes at the downstream signalized intersection vary widely at rush hour in the morning and evening. The cycle lengths and the green time are adaptively controlled according to the variation of incoming traffic volumes, and the values of balance control are greater than the ones of priority control. The optimum relative offset between the upstream and the downstream signalized intersection is also adaptively controlled according to the variation of queue motor-cars at the downstream signalized intersection, and the values of balance control are less than the ones of priority control. As the results, in the case of the priority control, the traffic congestion disappears in the arterial direction at the upstream and downstream signalized intersections(see Fig. 4); the following result

$$J_a(i,\ell,k) = \sum_j |e(i,j,1,\ell,k)| = 0 \qquad (27)$$

is obtained. In the case of the balance control, the traffic congestion lengths of the two orthogonal directions at the upstream and downstream signalized intersections are controlled so as to become equal and zero (see Fig. 4); the following result

$$J_a(i,\ell,k) = \sum_j \sum_m |e(i,j,m,\ell,k)| = 0 \qquad (28)$$

is obtained. From the comparison of the simulation results for the two proposed control algorithms, it is confirmed that the balance control algorithm works more effectively to minimize the sum of the traffic congestion lengths on the arterial.

4.2. *Traffic network*

The hierarchical control algorithm using the priority control algorithm of the traffic congestion length is simulated at twelve signalized intersections in Fukuyama city(see Fig. 2). The parameters and the reference input are set similarly as the arterial case. The incoming traffic volumes, capacities and average speeds etc. are arranged for the traffic network. From the simulation results, it is confirmed that the cycle lengths and green time are adaptively controlled according to the variation of the incoming traffic volumes at all signalized intersections in the traffic network. A typical simulation result is shown in Fig. 5. As the results, the traffic congestion disappears at all signalized intersections in the traffic network; the following result

$$J_n(\ell,k) = \sum_i \sum_j \sum_m |e(i,j,m,\ell,k)| = 0 \qquad (29)$$

is obtained.

5. CONCLUSIONS

Traffic signal control algorithms of a traffic network are considered from the control theoretic viewpoint in this paper. Traffic signal control systems of the traffic congestion length are constructed for an arterial and a traffic network using the feedback adaptation concept. Two traffic signal control algorithms on the arterial and the hierarchical control algorithm in the traffic network are presented and simulated at signalized intersections. From the simulation results, it is confirmed that the presented traffic signal control algorithms work effectively to control the traffic congestion lengths on the arterial and in the traffic network respectively.

REFERENCES

Davison, E. J. and Ü. Özgüner (1983). Decentralized control of traffic networks. *IEEE Trans.*, **AC–28**, pp. 677–688.

Hunt, P. B., D. I. Robertson, R. D. Bretherton and R. I. Winton (1981). Scoot—a traffic responsive method of coordinating signals. Trrl laboratory report, 1014.

Shibata, J. and T. Yamamoto (1985). Congestion control in urban road networks. *Systems and Control* **29**, pp. 123–131.

Shimizu, H., H. Naraki and E. Watanabe (1994). Comparison of two feedback adaptation algorithms for traffic congestion length. In *Proc. 3rd International Workshop on Advanced Motion Control*. Berkeley, pp. 1019–1028.

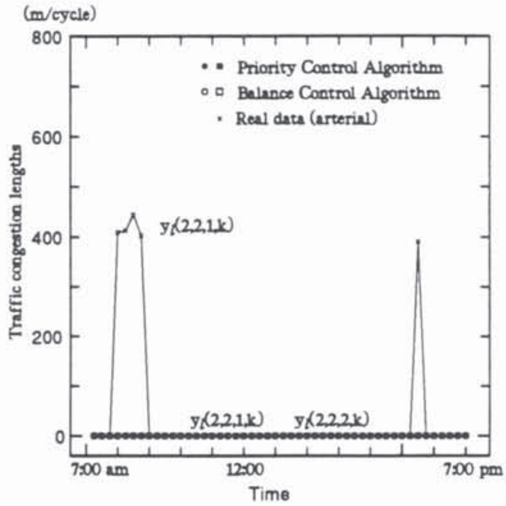

Fig. 4. Traffic congestion lengths at the downstream signalized intersection.

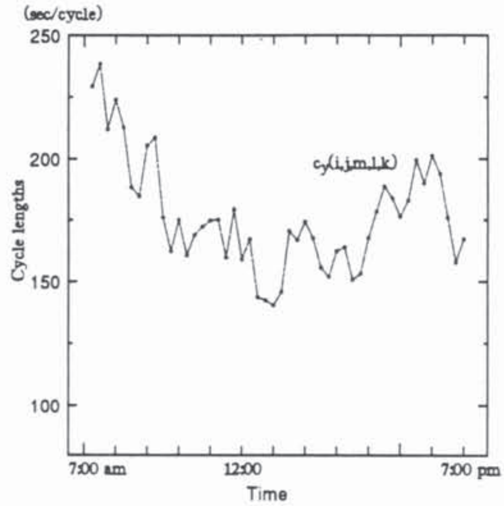

Fig. 5. Common cycle lengths of the traffic network.