Maximizing variance of projections:

$$\text{Max} \sum_{i=1}^{n} (V^T X_i)^2 \rightarrow \underset{V}{\text{Max}}\ V^T X X^T V$$

Where the variance is $V^T X X^T V$ [1]

Minimizing the mean square error:

$$\underset{V}{\text{Min}} \frac{1}{n} \sum_{i=1}^{n} \| X_i - VV^T X_i \|^2 \rightarrow \underset{V}{\text{Min}} \| X - XVV^T \|^2$$

Where the mean square error is:

$$\| X - XVV^T \|^2$$

Where X is the datapoints and XVV^T is the projections

$$\| X - XVV^T \|^2 = tr\left((X - XVV^T)(X - XVV^T)^2 \right)$$

$$= tr\left(XX^T - 2XVV^TX^T + X(VV^T)^2 X^T \right)$$

$$= \overset{\to C_1}{tr\left(\cancel{XX^T + X(VV^T)X^T} \right)} - \overset{\to C_2}{\cancel{2}}\, tr(XVV^TX^T)$$

$$= C_1 - C_2\, tr(XVV^TX^T) = C_1 - C_2\, tr(V^TX^TXV)$$

$= C_1 - C_2 V^T X^T X V$ [2] The negative sign on [2] means that minimizing this function is the same as maximizing [1]