

$$P(X|Y=i) = N(X|u_i, \Sigma)$$

$$N(X|u_i, \Sigma) = \frac{1}{(2\pi)^{D/2}} e^{-\frac{(X-u_i)^T \Sigma^{-1} (X-u_i)}{2}}$$

$$\text{if } a_i = \ln[P(X|Y=i)P(Y=i)]$$

$$\text{then } a_i = -\frac{1}{2}X^T X + (w^T)_i X + b_i$$

$$a_i = -\frac{1}{2}X^T X + u_i^T X - \frac{1}{2}u_i^T u_i + \ln[P(Y=i)] + \ln\left[\frac{1}{(2\pi)^{D/2}}\right]$$

In this case,  $a_i$  is not linear as it contains a logarithm

If the covariance is different the function does not change:

$$a_i = (\dots) + \ln\left[\frac{1}{(2\pi)^{D/2}}\right] \Bigg|_{\Sigma \neq \Sigma}$$

For all covariance  $\Sigma$  the function is still not linear.

$$\tilde{L}(\alpha) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j x_i^T x_j$$

if  $\alpha_i \geq 0 (\forall i)$  and  $\sum_{i=1}^n \alpha_i y_i = 0$

and  $\alpha^* = (\alpha_1^*, \dots, \alpha_n^*)$

Then:

$$\frac{1}{\gamma^2} = \sum_{i=1}^n \alpha_i^*$$

So:

$$\min_{w, b} \frac{1}{2} \|w\|^2 \rightarrow \mathcal{L}(w, b, \alpha) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i=1}^n \alpha_i [(z) - 1]$$

if:  $\sum_i \alpha_i \alpha_j y_i y_j \boxed{x_i^T x_j}$  and  $\sum \alpha_i y_i = 0$   
 $\sum \alpha_j y_j = 0$

Then:

$$\gamma = \left( \frac{1}{\sum \alpha_i} \right)^{1/2} \rightarrow \gamma = \frac{1}{\|w\|^2}$$