

$$P(h=1, v) = \frac{P(v, 1)}{\sum_h P(v, h)}$$

$$h \in \{0, 1\} \rightarrow \sum_h P(v, h) = P(v, 0) + P(v, 1)$$

$$\rightarrow P(h=1, v) = \frac{P(v, 1)}{P(v, 0) + P(v, 1)} = \frac{1}{1 + \frac{P(v, 0)}{P(v, 1)}} \quad (1)$$

$$\frac{P(v, 0)}{P(v, 1)} = \frac{\exp(-E(v, 0))}{\exp(-E(v, 1))} = \frac{\exp(-E(v, 0))}{\exp(-E(v, 1))}$$

$$= e^{-E(v, 0)} e^{E(v, 1)} = e^{E(v, 1) - E(v, 0)} \quad (2)$$

$$E(v, 1) - E(v, 0) = b^T v - v^T w - b^T v + c^T h$$

$$= -v^T w - c^T h \rightarrow \text{let } x = v^T w + c^T h$$

$$\rightarrow -v^T w - c^T h = -x \rightarrow E(v, 1) - E(v, 0) = -x \quad (3)$$

$$\text{Sub (3) into (2): } e^{E(v, 1) - E(v, 0)} = e^{-x}$$

Sub (2) into (1):

$$P(h=1, v) = \frac{1}{1 + \frac{P(v, 0)}{P(v, 1)}} = \frac{1}{1 + e^{-x}} = \boxed{\frac{1}{1 + \exp(-x)}}$$