Astronomy 400B Homework #2

Please Show Your Work for Full Credit

Due Feb 26, 2015 by 9:35am

1 Sparke & Gallagher Problem 2.20

Consider the spherical density distribution $\rho_H(r)$ with

$$4\pi G \rho_H(r) = \frac{V_H^2}{r^2 + a_H^2}, \tag{1}$$

where V_H and a_H are constants; what is the mass $M(< r)$ contained within a radius r? Use the equation

$$M(< R) = RV^2/G \tag{2}$$

to show that the speed $V(r)$ of a circular orbit at radius r is given by

$$V^2(r) = V_H^2[1 - (a_H/r)\arctan(r/a_H)], \tag{3}$$

and sketch $V(r)$ as a function of radius. This density law is sometimes used to represent the mass of a galaxy's dark halo – why?

2 Sparke & Gallagher Problem 2.24

We can estimate the size of an HII region around a massive star that radiates $S_\star$ photons with enrgy above 13.6eV each second. Assume that the gas within radius $r_\star$ absorbs all these photons, becoming almost completely ionized so that $n_e \approx n_H$, the density of H nuclei. In a steady state atoms recombine as fast as they are ionized, so the star ionizes a mass of gas M_g, where

$$S_\star = (4r_\star^3/3)n_H^2\alpha(T_e) = (M_g/m_p)n_H\alpha(T_e). \tag{4}$$

Use the equation

$$-\frac{dn_e}{dt} = n_e^2\alpha(T_e) \ \text{ with } \ \alpha(T_e) \approx 2 \times 10^{-13} \left(\frac{T_e}{10^4 \text{ K}}\right)^{-3/4} \text{ cm}^3 \text{ s}^{-1} \tag{5}$$

to show that a mid-O star radiating $S_\star = 10^{49}$ s^{-1} into gas of density 10^3 cm^{-3} creates an HII region of radius 0.67 pc, containing $\sim 30M_\odot$ of gas (assume that $T_e = 10^4$K). What is $r_\star$ if the density is ten times larger? Show that only a tenth as much gas is ionized. How large is the HII region around a B1 star with $n_H = 10^3$cm^{-3} but only $S_\star = 3 \times 10^{47}$ s^{-1}?

3 Sparke & Gallagher Problem 3.7

The *Navarro-Frenk-White* (NFW) model describes the halos of cold dark matter that form in cosmological simulations. Show that the potential corresponding to the density

$$\rho_{NFW}(r) = \frac{\rho_N}{(r/a_N)(1+r/a_N)^2} \text{ is } \Phi_{NFW}(r) = -\sigma_N^2 \frac{\ln(1+r/a_N)}{(r/a_N)}, \tag{6}$$

where $\sigma_N^2 = 4\pi G \rho_N a_N^2$. The density rises steeply at the center, but less so than in the singular isothermal sphere; at large radii $\rho(r) \propto r^{-3}$. Show that the speed V of a circular orbit at radius r is given by

$$V^2(r) = \sigma_N^2 \left[\frac{\ln(1+r/a_N)}{(r/a_N)} - \frac{1}{(1+r/a_N)} \right]. \tag{7}$$

4 Sparke & Gallagher Problem 3.12

Show that for the Plummer sphere model with density profile

$$\rho_P(r) = \frac{3a_P^2}{4\pi} \frac{M}{(r^2+a_P^2)^{5/2}} \tag{8}$$

and potential

$$\Phi_P(r) = -\frac{GM}{\sqrt{r^2+a_P^2}} \tag{9}$$

the potential energy is

$$PE = -\frac{3\pi}{32} \frac{GM^2}{a_P}. \tag{10}$$