Astronomy 400B Lecture 1: Review of Stellar Astrophysics

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1 Distance Measures

Astronomy is a science based on relative measurements, especially for angular separations on the sky and apparent brightness. The *angular separation* α of objects on the sky is often measured using the *small angle* formula, since the relation between the physical line-of-sight distance d and the physical separation Δx in the plane of the sky in terms of *radians* (rad) is

$$\frac{\Delta x}{d} = \sin\alpha \approx \alpha(\ll 1\text{rad}). \tag{1}$$

Angular separations can be measured in degrees (deg; 360 deg around the sky = 2π rad), arcmin ('; = 1/60 deg), and arcseconds ("; =1/60 arcmin).

Distances on the scale of solar systems are often given in terms of the *Astronomical Unit* (AU), which is the mean orbital radius of Earth about the Sun (in *centimeters*; cm):

$$1\text{AU} = 1.49597871 \times 10^{13}\text{cm}. \tag{2}$$

The *parsec* (pc) unit is defined to be the line-of-sight distance $d = 1$pc at which two objects separated by a physical distance $\Delta x = 1$AU in the plane of the sky have an apparent angular separation on the sky of $\alpha = 1''$. In terms of other units,

$$1\text{pc} = 2.06264806 \times 10^5\text{AU} = 3.08567758 \times 10^{18}\text{cm} = 3.2616334 \text{ ly} \tag{3}$$

where a *light year* (ly) is

$$1\text{ly} = 9.4605284 \times 10^{17}\text{cm} \tag{4}$$

and is defined relative to the *speed of light* (c)

$$c = 2.9979245800 \times 10^{10}\text{cm s}^{-1}. \tag{5}$$

Note that astronomers typically prefer pc to ly. We also use the units *kiloparsecs* (kpc = $10^3 pc$), *megaparsecs* (Mpc = 10^6pc), and *gigaparsecs* (Gpc = 10^9pc).

2 Stellar Mass, Luminosity, Flux, Radius, and Temperature

Stellar masses are often expressed in terms of a *solar mass* ($M_\odot$; mass of the Sun)

$$1M_\odot = 1.9891 \times 10^{33}\text{g} \tag{6}$$

in grams (g). The most massive stars are $\approx 100M_\odot$ while the least massive stars are $\approx 0.075M_\odot$.

Stellar luminosities (ergs of energy emitted per second; 1erg = 1 g cm^2 s^{-2}) are also often expressed in terms of the total (bolometric) *solar luminosity* ($L_\odot$; luminosity of the Sun)

$$1L_\odot = 3.846 \times 10^{33}\text{erg s}^{-1} \tag{7}$$

Stars range in luminosity from $10^6 L_\odot$ to less than $10^{-4}L_\odot$.

The *flux* F is the energy per unit second per unit area (erg s^{-1} cm^{-2}) that is received from an object a distance d away, and is given by the *inverse square law*

$$F = \frac{L}{4\pi d^2}. \tag{8}$$

The *flux density* ($F_\nu \equiv dF/d\nu$) of an object is best measured in *janskys*

$$1\text{Jy} = 10^{-23}\text{erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}. \tag{9}$$

Note that the *hertz* (1Hz = $1s^{-1}$) is the unit of frequency ν. Astronomers will also use a flux density ($F_\lambda = dF/d\lambda$) defined relative to wavelength. Typically, the units of F_λ are in erg cm^{-2} s^{-1} Å^{-1}, where Å is the *angstrom* (1Å = 10^{-8}cm). The wavelength and frequency of light are related by $c = \nu\lambda$, and the two flux densities are therefore related by $F_\lambda = (c/\lambda^2)F_\nu$. The total flux is related to the flux density by

$$F = \int F_\nu d\nu = \int F_\lambda d\lambda. \tag{10}$$

If the distance of an object is known, astronomers will also use the term *luminosity density* (e.g., $L_\nu = dL/d\nu$).

The luminosity, radius, and temperature of an object are related. For stellar objects, we often use the *solar radius* ($R_\odot$) to express sizes

$$1R_\odot = 6.995 \times 10^{10}\text{cm} \tag{11}$$

If the luminosity L and radius R of an object are known, we can define the effective temperature T in *Kelvin* (K) of an object through the *Stefan-Boltzmann Law*

$$L = 4\pi R^2 \sigma_{\text{SB}} T^4 \tag{12}$$

where the *Stefan-Boltzmann constant* is

$$\sigma_{\text{SB}} = 5.670373 \times^{-5}\text{ erg cm}^{-2}\text{ s}^{-1}\text{ K}^{-4}. \tag{13}$$

The Stefan-Boltzmann constant can be computed theoretically in terms of the speed of light, the *Boltzmann constant*

$$k_{\rm B} = 1.380658 \times 10^{-16} {\rm erg\ K}^{-1} \tag{14}$$

and the *Planck constant*

$$h = 6.6260755 \times 10^{-27} {\rm erg\ s}, \tag{15}$$

which gives the definition

$$\sigma_{\rm SB} \equiv \frac{2\pi^5 k_{\rm B}^4}{15 h^3 c^2}. \tag{16}$$

The effective temperature is typically a measure of the temperature of the star at its *photosphere*, or the radius where the optical depth of the star's atmosphere is $\tau \approx 1$ and it becomes opaque. For the Sun, $T \approx 5780$K. The temperature is also related to the wavelength at the peak of the black body curve via *Wien's Displacement Law*

$$\lambda_{\rm max} = \frac{2.897756 \times 10^7 {\rm \AA\ K}}{T}, \tag{17}$$

which gives $\lambda_{\rm max} \approx 5000$Å (yellow) for the Sun.

2.1 The Main Sequence

Based on their temperatures, stars are classified into *spectral types*. The spectral types are OBAFGKMLTY, and are usually subclassified into *early* (OBA) and *late* (FGKM) type stars. O type stars are hot ($T > 3 \times 10^4$K) while M type stars are relatively cool ($M \approx 3000$K), and each type is subsubclassified by decreasing temperature from $0 - 9$ (e.g., the Sun is a G2 star). Deviations from a black body are determined by the atomic and molecular species absorption in the stellar atmospheres, with strong Hydrogen absorption in A star spectra and strong molecular bands appearing in M star spectra.

See Figure 1.1 of Sparke and Gallagher.

See Table 1.1 of Sparke and Gallagher.

The *main sequence* of stars traces a band in the diagram of luminosity and temperature (the *Hertzsprung-Russell or HR diagram*), and consists of stars that shine by burning hydrogen through nuclear fusion. Main sequence stars range in radius from about $R \approx 0.1 R_\odot$ to $R \approx 25 R_\odot$, with a scaling of

$$R \sim R_\odot \left(\frac{M}{M_\odot}\right)^{0.7}. \tag{18}$$

The corresponding scaling of stellar luminosity near the main sequence is

$$L \sim L_\odot \left(\frac{M}{M_\odot}\right)^{\beta} \tag{19}$$

with $\beta \approx 5$ for $M \lesssim M_\odot$ and $\beta \approx 3.9$ for $M_\odot \lesssim M \lesssim 10 M_\odot$. For stars with $M \gtrsim 10 M_\odot$, $L \sim 50 L_\odot (M/M_\odot)^{2.2}$.

Giant and *supergiant* stars have evolved off the main sequence, and have radii and luminosities that are typically much larger than main sequence stars of the same effective temperature. Supergiants can have radii of $1000 R_\odot$ and luminosities of $10^6 R_\odot$.

White dwarf stars are not on the main sequence, but instead are the remains of the core of an evolved star that is supported through electron degeneracy pressure. White dwarfs have typical sizes of $0.01R_\odot$.

Neutron stars are the remnants of certain supernovae that leave behind a core supported by neutron degeneracy pressure. Neutron stars have radii of only $R \sim 10$km, about the size of a large city.

See Figure 1.4 of Sparke and Gallagher.

The main sequence lifetime of a star is predominately determined by its mass-to-light ratio, since hydrogen mass is being burned into helium at a rate that must fuel the stellar luminosity. For $\langle\beta\rangle \approx 3.5$ in Equation 19, we have that

$$\tau_{\rm MS} = \tau_{\rm MS,\odot}\frac{M/L}{M_\odot/L_\odot} \sim 10{\rm Gyr}\left(\frac{M}{M_\odot}\right)^{-2.5} \sim 10{\rm Gyr}\left(\frac{L}{L_\odot}\right)^{-5/7}. \quad (20)$$

Sparke and Gallagher give a more accurate approximation as

$$\log(\tau_{\rm MS}/1{\rm Gyr}) = 1.015 - 3.49\log(M/M_\odot) + 0.83(\log[M/M_\odot])^2 \quad (21)$$

(note the factor of 10 error in Sparke and Gallagher Equation 1.9).

2.2 Chemical Abundance

The presence of atomic and molecular features in the spectra of stars enable us to measure the amount of metals (elements heavier than Helium) in their atmospheres. We typically measure the metallicity Z on a log scale relative to the solar abundance, with the definition

$$[A/B] \equiv \log\left[\frac{(\text{number of A atoms/number of B atoms})_\star}{(\text{number of A atoms/number of B atoms})_\odot}\right]. \quad (22)$$

For the Sun, the total metallicity is $[Z/\mathrm{H}] = 0$ by definition, while for a star with $10\times$ the solar metallicity we have $[Z/\mathrm{H}] = 1$. Often, astronomers abbreviate metallicity with [Fe/H] even though O is typically the most abundant metal by number. The relative abundance of metals in stars is set by a combination of *Big Bang Nucleosynthesis* that determined the initial abundance of H, He, and Li (primarily) and *Stellar Nucleosynthesis* that determined the abundance of heavier elements through a variety of stellar fusion and supernovae-related processes. The presence of metals in a star can effect its evolution by increasing the atmospheric opacity. Low metallicity stars have lower opacity in their atmospheres, leading to a denser, hotter, and bluer star at fixed mass.

See Figure 1.3 of Sparke and Gallagher.

2.3 Typical life of a star

The typical star goes through several phases during its life:

1. *Pre-main sequence.* The initial formation of stars is not well-understood, but the general picture involves a collapsing gas cloud with some initial angular momentum.

As the cloud cools, the angular momentum causes the gas cloud to form a disk. The disk eventually forms a central bound object that continues to collapse roughly spherically. This cloud does shine (in the infrared) by converting gravitational potential energy to thermal energy via a virialization process. The pre-main sequence phase ends once the star is dense and hot enough in its core to undergo nuclear fusion.

2. *Main sequence.* The hydrogen-burning phase of stellar life. All stars with masses below about $0.6M_{\odot}$ have main sequence lifetimes longer than the age of the universe and remain on the main sequence. The fate of heavier stars depends on their mass.

3. *Post-main sequence for low-mass stars* ($0.6M_{\odot} \lesssim M \lesssim 2M_{\odot}$). **See Figure 1.4 of Sparke and Gallagher.** The cores of these stars contracts as hydrogen is burned, and the outer layers puff up and become cooler as the star becomes a *subgiant*. The temperature around the core becomes hot enough to burn hydrogen in a shell around the exterior of the core, leading to the *red giant* phase. The helium by-product of the hydrogen shell burning falls on the core, which causes the core to contract and get hotter. The shell burns even brighter, causing the star to become very luminous and ascend the *red giant branch* of the HR diagram. The (now) helium core eventually contracts until He can burn to C through the *triple α process*, which happens quickly in what's called the *helium flash*. The helium cores of stars with $M \lesssim 2M_{\odot}$ have about the same helium core mass and therefore the same luminosity at the tip of the giant branch. Once the core helium is mostly burned the core contracts again until both He and H are burned in shells, corresponding to the *asymptotic giant branch* (AGB) phase. In the AGB phase, the star looses its outer envelope in a *planetary nebula* phase where the expelled gas is ionized by UV light from the hot central core. This core is a degenerate white dwarf, which gradually emits as a black body (typically) modified by H and He absorption. The white dwarf cooling time (to $T \sim 0$) is longer than the age of the universe, and most white dwarfs are around ten thousand K at their surface.

4. *Post-main sequence for intermediate-mass stars* ($2M_{\odot} \lesssim M \lesssim 8M_{\odot}$). The post-main sequence lives of these stars are similar to low-mass stars, but their helium cores can become more massive and they can therefore reach higher luminosities (and are bluer) than the red clump where low-mass stars congregate in their giant phases. Some of these stars regularly pulsate as *Cepheid* variables that can be used as distance indicators. The white dwarfs of these stars may contain some Ne in addition to C and O.

5. *Post-main sequence for high-mass stars* ($M \gtrsim 8M_{\odot}$). The post-main sequence lives of high-mass stars differs substantially from lower mass populations. The cores of stars above $\sim 10M_{\odot}$ can burn through the α process C$\rightarrow$O, O$\rightarrow$Ne, all the way up to Fe where the binding energy per nucleon peaks. During this heavy element burning, the star is a blue or yellow supergiant. But after the core reaches Fe, nuclear fusion stops since burining iron to heavier elements through fusion is endothermic and requires (rather than provides) energy. The core then collapses, forms neutrons and hardens, and then the outer layers bounce and are pushed via neutrinos to form a *Supernova Type II*. The remnant is left as a neutron star. For stars $8M_{\odot} \lesssim M \lesssim 10M_{\odot}$, the

core probably exceeds the *Chandrasekar limit* of $1.4M_\odot$ where degeneracy pressure can no longer resist gravity and the star also explodes as a Type II supernova. For stars larger than $\sim 40M_\odot$ less is known, but some shed their outer layers and are seen as *Wolf-Rayet* stars with strong winds. Some heavier stars may have cores that collapse to form black holes.