JWST TECHNICAL REPORT

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Table of Contents

1 Abstract 1
  1.1 Revision A 2
2 Introduction 2
3 JWST Science Instrument Aperture File 3
  3.1 Coordinate Frames 5
    3.1.1 Detector (Det) 5
    3.1.2 Science (Sci) 5
    3.1.3 Ideal (Idl) 6
    3.1.4 V-frame 7
  3.2 Elements of the SIAF 7
    3.2.1 Recently Introduced Elements 10
      3.2.1.1 DDCName 11
      3.2.1.2 AperType 11
4 Coordinate System Transformations: Mathematical Derivations & Descriptions 18
  4.1 Detector versus Science 18
  4.2 Science versus Ideal 19
  4.3 Ideal versus Telescope (V2,V3) 20
5 Absolute Transformations 22
  5.1 Absolute coordinates 22
  5.2 Roll Angle 27
6 Example Applications 29
  6.1 Guide star selection and placement 29
  6.2 Target Acquisition 30
7 Acknowledgments 32

1 Abstract

This Technical Report was originally released as a follow-on document to *Operational Implementation and Calibration of Field-of-View Coordinate Systems for the James*

Webb Space Telescope[1]. That document examined the various coordinate frames that the JWST Science and Operations Center at STScI (S&OC) planned to utilize for JWST science operations, and outlined the considerations that drove the definition of a basic set of geometry-related parameters. The set of these parameters, referred to by the S&OC as the Science Instrument Aperture File (SIAF), was broadly addressed by that document within the context of HST lessons learned, and the JWST operational scenarios expected at that time. The goal was to inform an economical but sufficient set of SIAF parameters to support the S&OC´s subsystems used in JWST science operations.

Here, we build upon this groundwork by describing the SIAF content, processes, and products, then present some representative calculations utilizing these data..I.e. "Description and Use" The primary audience for this document are the various S&OC systems and SI teams requiring *or* producing SIAF data. The primary goals are to (a) present the set of elements comprising the SIAF, and (b) demonstrate the SIAF support of science operations by providing mathematical examples of the types of computations employed.

1.1 Revision A

Revision A of this report clarifies and updates content in a number of sections. It also reflects modifications to the SIAF specification between the document's initial release in 2009, and this version (late 2016), based on experience with the operational implementation and utilization by the stakeholding S&OC systems as these systems were matured. The XML specification for the SIAF files can be found in *PRDS to Users IRCD Vol.III: S&OC Subsystems, Rev. E*[2].

At the time of this Revision A's release, the process of delivering successively more mature SIAF files to the S&OC Project Reference Database (PRD) for test-use in advance of flight continues to be worked jointly among the SI teams (both S&OC and Instrument Definition Teams (IDT)), the S&OC systems representatives, and the S&OC's Telescopes Team via the SIAF "Working Group" [3].

This document is intended to provide an overview that is complementary and prerequisite to the series of STScI Technical Reports, *The Pre-Flight SI Aperture File, Parts 1–5* [4]

2 Introduction

The SIAF contains the geometric characteristics of the Fields of View (FOVs) pertaining to JWST Instruments' slits, apertures and Sensor Chip Assemblies (SCAs), along with sub-regions and various fiducial points of operational interest, described in terms of the basic relevant coordinate systems, along with the parameters used in the transformation between these systems. The specific pre-flight values are not finalized as of this release (Rev. A), but draw from ground testing of the Integrated SI Module (ISIM), SI modeling, and planned SI science modes. Once flight operations begin, the specific values will be updated and maintained through on-orbit calibrations, (e.g. commissioning proposal COM/OTE 1144[5]).

Section 3 presents and discusses the SIAF parameters and the coordinate frames associated with the telescope and its focal plane. Included in their own subsections are the new parameters specified since the first release of this document.

Section 4 gives the transformation equations between the different coordinate frames, and the use of the SIAF parameters in these equations, while Section 5 presents equations that allow transformation between the V-frame associated with the telescope and the inertial coordinate frame of the sky. Section 6 gives example applications to S&OC operations such as guide star selection/placement, and target acquisition. Areas for future work are discussed in appendices.

The SIAF format specification is given in the aforementioned IRCD[2]. The primary SIAF products, feeding other systems, are five XML files, one for each instrument, residing as part of the S&OC PRD. These SIAF files are read and utilized by a variety of S&OC subsystems spanning three main areas of S&OC operations:

- Pre-observation: science proposal planning (PPS)
  - Astronomer Proposal Tool (APT), including Aladin visualization of fields of view on the sky. Uses embedded copies of the SIAF xml files, and reads the derived *exposures table*.[6]
  - Guidestar selection. Utilizes SIAF content in the *exposures table* and *fine_guidance_sensor table*[7].
  - Visit planning and scheduling: SIAF feeds data in the Visit Scheduling Subsystem (VSS) *sts_pointings table*[8], which contains pointing information used by the Short Term Schedule (STS) system.
- Execution: target placement and pointing
  - Observation Plan Generation System (OPGS): for target placement and observation-specific calculations. SIAF supplies the data for the OPGS *aperture table*[9], providing the transformations among the major frames for all entities.
  - Observation Support Subsystem (OSS): for target acquisitions calculations, the handling of the Differential Distortion Compensation (DDC), and for providing the Attitude Control Subsystem (ACS) with FGS-frame inputs. Utilizes the SIAF-populated *aperture_coeff table*[10].
- Post-observation: science pipeline
  - Data Management Subsystem (DMS). Used for rotation of raw images into the "science" orientation, in support of world coordinate mapping. Also populates certain level-1b FITS header keywords. SIAF feeds a DMS-specific database table, which is read by the science data processing (SDP) code for these purposes.[11] NOTE: While DMS draws from certain SIAF data, we point out here that the SIAF is not the vehicle for capturing the best SI geometric characterizations that may be of use in pipeline processing of science data. Files that support science data processing are contained in the Calibration Reference Data Subsystem (CRDS).

3 JWST Science Instrument Aperture File

The SIAF XML specification, provided in the above-mentioned IRCD[2], describes a <SiafEntry> and its elements. Each unique SiafEntry is commonly referred to as an *aperture* and the elements contained within as *fields*. See Section 3.2 for a description of this content.

The SIAF captures certain basic geometric characteristics (in various frames) of the usable modes, subarrays, coronagraphs, physical apertures, etc., or stated generally, any unique targetable fiducial point and its associated region, used in support of JWST science operations. So in the context of the SIAF the term *aperture* is used in this broad sense.

The XML file, one for each SI plus FGS, is the PRD product utilized by the S&OC subsystems. Most of the source information, however, is calculated, maintained, and updated via an Excel spreadsheet. These spreadsheets (one for each instrument) are officially delivered to the PRD, together with the corresponding XML file, thus providing an official record and mapping of the working "source" product and calculations from which new XML files derive. These spreadsheets contain logic for calculating aperture information based on the SI teams' or ISIM group's ground or flight characterizations, and for relating the various apertures as appropriate.

For some SIAF content, calculations are performed external to the spreadsheet, (e.g. generation in some cases of the coefficients for the higher order distortion terms.) Such data are usually computed via standalone python code and then copied, as values, into the appropriate cells of the Excel spreadsheet. During earlier SIAF file updates and delivery to the Development PRD, these programs have resided on the shared volume /itar/jwst/tel/share/SIAF_WG/PythonCode. However, to support future flight-ready and in-flight SIAF work, we have instantiated a gitlab repository, https://grit.stsci.edu/ins-tel/jwst_siaf to contain, manage, and document these codes used to create SIAF content.

The XML & Excel files, and the supporting python code, are currently maintained by the Telescopes Team within the Instruments division at STScI. The group is also currently the owners of the updates and calculations. In some cases, SIAF content may be delivered directly from SI teams for insertion into the Excel file (and flowed to XML) "as-is".

The spreadsheet contains multiple sheets ("tabs"), most of which exist to perform and capture intermediate or supporting calculations which feed the special sheet named "SIAF". The number of sheets within an Excel file can vary depending on the particular SI. The "SIAF" sheet contains the parameters that map 1-to-1 to the XML files content. In fact the XML file is generated via a java executable operating *only* on the "SIAF" sheet of the Excel file. This java executable (controlled and provided by the S&OC PRD Group) creates from this sheet an XML file compliant with the above-mentioned IRCD[2] (We note here that the precision captured in the XML file depends on the display format being used in Excel at the time of XML file generation).

A SIAF tab's row maps to an XML SiafEntry, while its columns map to that SiafEntry's "elements" as shown in the example snippets in Figure 1.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Aperture Basic Info					Detector Frame				Science Frame						V-Frame		
2	InstrName	AperName	DDCName	AperType	AperShape	XDetSize	YDetSize	XDetRef	YDetRef	XSciSize	YSciSize	XSciRef	YSciRef	XSciScale	YSciScale	V2Ref	V3Ref	V3Id
21	NIRCAM	NRCB1_FULL	NRCB_CNTR	FULLSCA	QUAD	2048	2048	1024.5	1024.5	2048	2048	1024.5	1024.5	0.030689	0.030813	-120.9682	-457.7527	0.3
22	NIRCAM	NRCB2_FULL	NRCB_CNTR	FULLSCA	QUAD	2048	2048	1024.5	1024.5	2048	2048	1024.5	1024.5	0.031108	0.031303	-121.1443	-525.4582	0.
23	NIRCAM	NRCB3_FULL	NRCB_CNTR	FULLSCA	QUAD	2048	2048	1024.5	1024.5	2048	2048	1024.5	1024.5	0.030824	0.030913	-53.1238	-457.7804	-0.48
24	NIRCAM	NRCB4_FULL	NRCB_CNTR	FULLSCA	QUAD	2048	2048	1024.5	1024.5	2048	2048	1024.5	1024.5	0.031254	0.031446	-52.8182	-525.7273	-0
25	NIRCAM	NRCB5_FULL	NRCB_CNTR	FULLSCA	QUAD	2048	2048	1024.5	1024.5	2048	2048	1024.5	1024.5	0.062906	0.063219	-89.3892	-491.4440	-0.0
[illegible]	[illegible]	NRCA1_SUB160	NRCA_CNTR	SUBARRAY	QUAD	2048	2048	80.5	1968.5	160	160	80.5	80.5	0.030989	0.031131	91.1720	-498.0725	-0.

```
<SiafEntry>
	<InstrName>NIRCAM</InstrName>
	<AperName>NRCB3_FULL</AperName>
	<DDCName>NRCB_CNTR</DDCName>
	<AperType>FULLSCA</AperType>
	<AperShape>QUAD</AperShape>
	<XDetSize>2048</XDetSize>
	<YDetSize>2048</YDetSize>
	<XDetRef>1024.5</XDetRef>
	<YDetRef>1024.5</YDetRef>
	<XSciSize>2048</XSciSize>
	<YSciSize>2048</YSciSize>
	<XSciRef>1024.5</XSciRef>
	<YSciRef>1024.5</YSciRef>
	<XSciScale>0.030824</XSciScale>
	<YSciScale>0.030913</YSciScale>
	<V2Ref>-53.1238</V2Ref>
	<V3Ref>-457.7804</V3Ref>
	[...]
```


Figure 1: SIAF source spreadsheet and XML file

3.1 Coordinate Frames

The JWST SIAF aperture information deals with four coordinate systems:

- Detector pixels, (Det)
- Science/DMS pixels, (Sci)
- Ideal frame, (Idl)
- OTE-based frame (V)

The basic characteristics of these frames in the JWST context are as follows.

Note that for the particular relationships and representations of these frames specific to each instrument, refer to the aforementioned series of STScI Technical Reports, *The Pre-Flight SI Aperture File, Parts 1–5, or the SIAF itself*.

3.1.1 Detector (Det)

The Detector frame is related to each SCA's slow and fast readout directions, but this connection of slow/fast to the SIAF's Xdet,Ydet is not consistent across instruments.

There exist differing conventions in the assignment of row/column to raw readout, and some of the SCAs have 90° different rotations in the telescope focal plane. This has been a source of past confusion on occasion.

The Xdet,Ydet system used by the SIAF is oriented such that *for the NIRCam and MIRI SCAs, fast read direction is X (row), slow read direction is Y (column). For FGS, NIRISS, and NIRSpec this assignment to X & Y is reversed.*

We note that the SIAF Detector frame is equivalent to the "SI Pixel Field Coordinates" used throughout ISIM CV testing.[12]

3.1.2 Science (Sci)

The Science image frame (also referred to as the "DMS" frame) is the representation normally displayed by science analysis software. This frame also has units of pixels but can be a trimmed and/or reoriented portion of the Detector frame.

Non-illuminated or reference pixels may be excluded in the science image and this would be captured in the SIAF as differing pixel dimensions between Sci and Det frames for a given SCA. As well, SIAF apertures defining subarrays will have Sci dimensions mapping to the extents of the subarray.

There may also be differences in axes orientation (in multiples of 90 degrees) between Det and Sci frames to ensure common sky orientation among SCAs or consistent spectral representations in the science products. See the DMS Requirements documentation.[13]

Section 4.1 gives the general transformations between the Science and Detector frames.

3.1.3 Ideal (Idl)

The Ideal frame is a distortion-removed frame, given in the SIAF in units of arc-seconds. It is the output of a polynomial of up to 5th order, that is a function of Science pixel XY. Ideal frame coordinates can be treated as a location in a tangent plane projection when applied over scales of an SCA dimension. The tangent point (i.e. XY Idl origin) is defined at the given aperture's fiducial/reference point. See Sections 4.2 and 4.3 forelating the Ideal frame to the Science and V frames respectively. These sections includes the rigorous treatment of Idl as an angular frame relevant when transforming between SIs.

We note that the notion of Ideal frames associated with each SIAF aperture is an extension of the JWST Mission-level ICS (Idealized Coordinate System) frame definition, which specifies a distortion-removed angular frame associated with the Guiders. The definitions of the FGS ICS frames are given in the *Coordinate Systems Definition Document Plan, SE-20*[14]. The specific definition of the FGS ICS frames are fixed and common between OSS and ACS since pointing and slewing information is passed across the OSS-ACS interface[15] via this frame. The Ideal frames for the other instruments, however, are used only by the S&OC for target acquisitions, and science pointing calculations, so need only be defined and utilized in an internally consistent manner.

Figure 2 provides a representative example of the Det, Sci, and Idl frames' general locations for the case of the SIAF Apertures specifying the NIRCam short-wave SCAs.

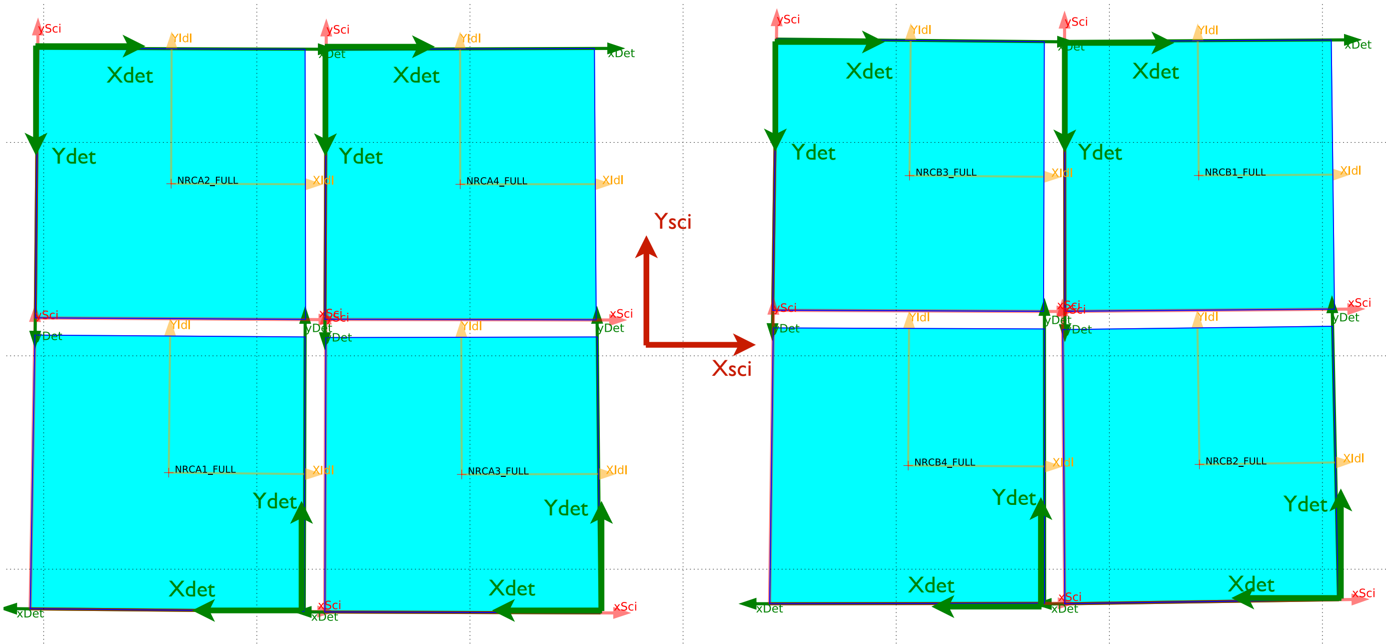


Figure 2: Illustration shows: A) the SCA-specific Detector frame, B) the Science (DMS) frame, which provides continuity of the imaged sky across the SCAs, and C.) The Ideal frame, corrected for distortion, and centered on the SCA reference location.

3.1.4 V-frame

The V2,V3 coordinates, also expressed in arc-seconds, serve as field angles that locate points over the OTE whole field of view, encompassing all instruments. The actual 3-dimensional V-frame (V1,V2,V3) is tied to the JWST OTE physical axes. Once in flight however, the V-frame's alignment becomes arbitrary and will be defined with respect to FGS1's location, which will be assumed fixed in that frame from the time of the first in-flight SIAF update (COM/OTE 1144)

Celestial coordinates (α, δ) and a given position angle are mapped to the V-frame via Euler rotations using an attitude matrix or quaternions, (described in Section 6) while instrument Ideal frame locations are related by the S&OC to FGS (or other SI) Ideal locations via calculations described in Section 4.3.

3.2 Elements of the SIAF

The contents for the JWST SIAF are summarized in Table 1 giving the parameter name and description.

The SIAF elements, as represented in the SIAF Excel file as columns, are grouped as follows:

- Columns 1 – 5: Science instrument, Aperture name, Differential Distortion Compensation point name, Aperture type. and Aperture shape.
- Columns 6–9: Characteristics of an Aperture in terms of detector pixels.
- Columns 10–15: Characteristics of an aperture´s corresponding science image.
- Columns 16-19: Relation of Ideal and V frames via a zero point, angle and parity.

Note: this utilizes a tangent plane approximation, treating the V frame as a cartesian system. See section 5.3 for treatment of the spherical solution and planar. approximation.

- Columns 20-21: Detector to Science angle and parity.
- Columns 22-23: Orientations of the Science axes with respect to the V frame.
- Columns 24–31: Aperture corners in the Ideal frame (Four X values followed by the four Y values)
- Columns 32–33: Date and Comment
- Columns 34–118: The degree and the coefficients of the distortion polynomial relating the Science to Ideal frame. The coefficients represented by these SIAF elements are applied as described in Section 4.2

Table 1: The contents of the Science Instrument Aperture File

Excel Column	Element Label	Description
1	InstrName	Science instrument to which the aperture belongs
2	AperName	A label for the aperture
3	DDCName	Name of aperture to be used for differential distortion correction
4	AperType	Identifies the type of SIAF Aperture. This determines which SIAF columns/elements will be populated and how this aperture will be used
5	AperShape	Quadrilateral or circle
6	XDetSize	Number of pixels in the detector x direction
7	YDetSize	Number of pixels in the detector y direction
8	XDetRef	x pixel position of an aperture reference point on the detector
9	YDetRef	y pixel position of an aperture reference point on the detector
10	XSciSize	Number of pixels in the science image in the x direction
11	YSciSize	Number of pixels in the science image in the y direction
12	XSciRef	X pixel position of reference point on science image frame.
13	YSciRef	Y pixel position of reference point on science image frame.
14	XSciScale*	Scale in arcsec/pixel along the Sci x direction at the reference point.
15	YSciScale*	Scale in arcsec/pixel along the Sci y direction at the reference point.
16	V2Ref	V2 position of the reference point (arcsec).
17	V3Ref	V3 position of the reference point (arcsec).
18	V3IdlYAngle	Angle from the V3 axis** to the Y axis of the ideal frame measured in the V3 to V2 direction at the ideal frame origin.
19	VIdlParity	Relative sense of rotation between Idl x to y & V2 to V3. (+/–1,

Excel Column	Element Label	Description
		where +1 indicates same XY and V2V3 have same sense)
20	DetSciYAngle	Angle in degrees from YDet to YSci in direction XDet to YDet
21	DetSciParity	Relative sense of x to y rotation between Det and Sci (+/- 1).
22	V3SciXAngle*	Angle from the V3 axis** to the Sci x axis (degrees) at the reference point measured from V3 towards V2.
23	V3SciYAngle*	Angle from the V3 axis** to the Sci y axis (degrees) at the reference point measured from V3 towards V2.
24..27	XIdlVert …	Array of x ordinates of aperture corners in Idl frame.
28..31	YIdlVert …	Array of y ordinates of aperture corners in Idl frame.
32	UseAfterDate	Date before which this data does not apply.
33	Comment	Optional comment about this entry.
34	Sci2IdlDegree	Degree of distortion polynomial.
35..55	Sci2IdlXij…	Coefficients in Idl x polynomial.
56..76	Sci2IdlYij…	Coefficients in Idl y polynomial.
77..97	Idl2SciXij…	Coefficients in Sci x polynomial.
98..118	Idl2SciYij…	Coefficients in Sci y polynomial.

* These values are derivable from the Sci2Idl coefficients and V3IdlYang and V3IdlParity.

** Tangent plane approximation defined at V2Ref,V3Ref

3.2.1 Recently Introduced Elements

The elements DDCName & AperType (Excel columns 3 & 4) were added to the SIAF after the first release of this report, and the XML specification was updated accordingly in 2016[2]. In the case of DDCName this was to accommodate new requirements flowed to the S&OC. For AperType, this field was introduced by the SIAF Working Group to better support the multiple systems' utilization of SIAF content..

3.2.1.1 DDCName

See the report *Field Points for Differential Distortion Compensation*[16] for a description of the DDC concept and a full documenting of the SIAF implementation that is briefly summarized below.

Differential Distortion Compensation (DDC), as implemented, relies on a limited number of field points stored by ACS as the allowable points at which to stabilize a target. The S&OC ground system must map each science pointing to the most proximal one of these points. The SIAF DDCName is populated for every SIAF aperture, and provides OPGS with the string value required in the command to initiate an Absolute Vehicle Slew Maneuver.

The SIAF Excel spreadsheets contain the formulae to populate the DDCName value with one of the allowable field point names, by identifying the closest such point to the given aperture's V2Ref, V3Ref.

The aforementioned document[14] is the reference for this SIAF DDC implementation and includes plots, science rationale, and error assessments.

3.2.1.2 AperType

AperType allows a more explicit categorization of the SIAF apertures. Each type can have a somewhat different purpose and utilization, and each type will have only the applicable elements (columns) populated. AperType can be assigned values described below. There is currently no need or plan to extend the types, though doing so is allowed by the SIAF specification.[2]

"FULLSCA"

Denotes an entry that describes the detectors' full extent.

This type of entry characterizes the detector in each of the four frames described in Section 3.1. All its XML elements will contain real values. (Excepting that 5th order polynomial coefficients may not be populated if not applicable. This case will be indicated by the value of "Sci2IdlDeg" which will be 4 instead of 5).

Multiple SIAF apertures of AperType=FULLSCA can exist for a given SCA. There will always exist at least one, containing a fiducial/reference point that is centrally located on the SCA (See Figure 3) There may also be others to describe the same SCA but defining a different reference point.

These entries will be of interest to APT, PPS, OPGS, as well as DMS, which will relate the SIAF entry used in various observing modes & templates to a set of one or more SCAs.

"OSS"

This AperType maps 1:1 with the set of sixteen FULLSCA entries with centered reference points. They are very similar to their FULLSCA counterpart, containing the same populated elements, but with certain frames defined differently for on-board efficiency and (in the case of FGS) for compatibility with the ISIM-Spacecraft interface (e.g. OSS-->ACS).

Specifically, OSS apertures have a Science frame (Xsci,Ysci) that is defined to be identical to the Detector frame (Xdet,Ydet). This redefinition, together with altered polynomial coefficients, means that the input to the polynomial is XY pixel as directly determined by OSS (e.g. during a Target Aquisition) and the output is an ideal frame with a differing parity than the normal non-OSS counterpart. (parity +1 for OSS and -1 for all others. See table 1 row 19 for parity definition.)

See Figure 4, and compare with Figure 3.

To reduce the chance of misapplication, SIAF apertures of AperType=OSS will also have an AperName ending with the string "_OSS". This AperType is only for use by OSS[17]. It need not be read by any other system.

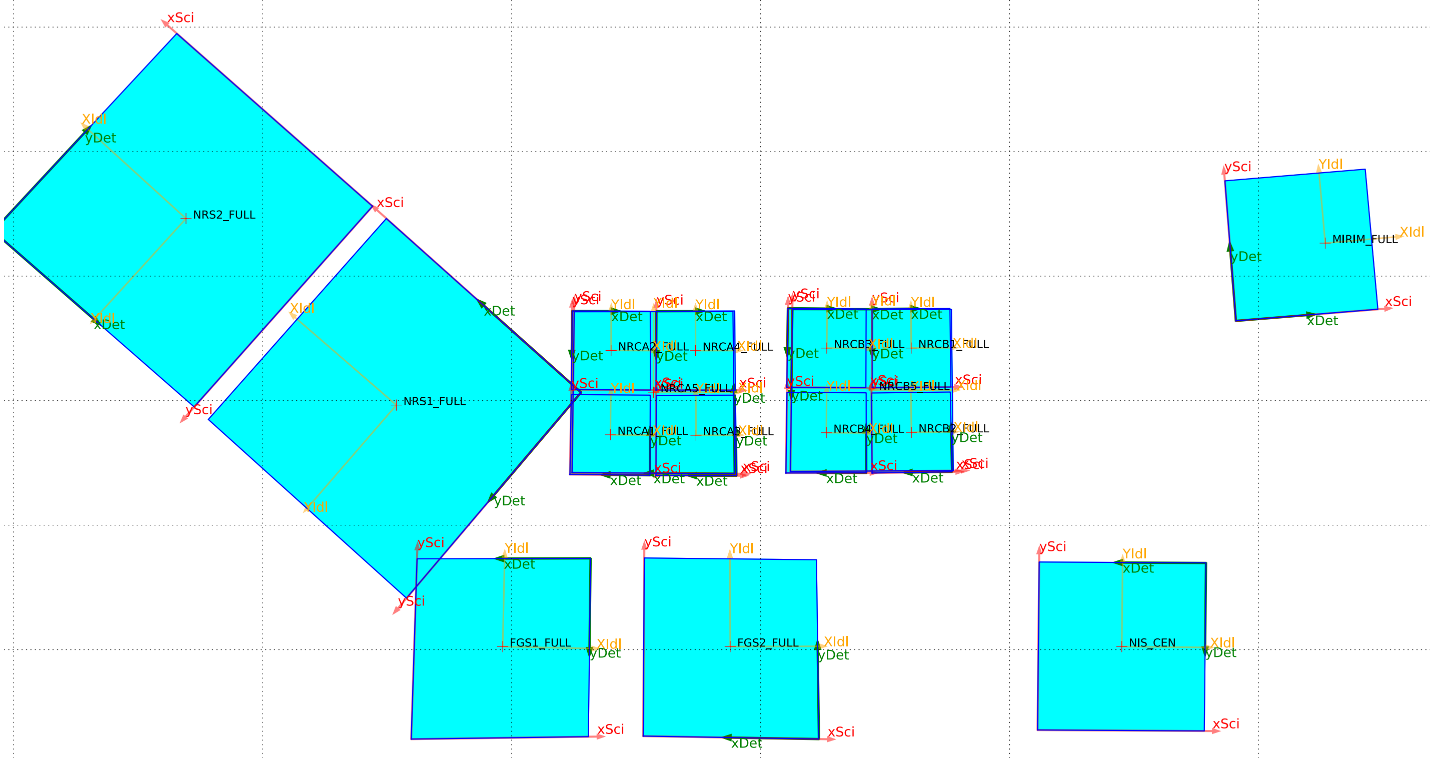


Figure 3: SIAF Apertures with AperType=FULLSCA and centrally located reference/fiducial points shown in the V2V3 frame with V2 to the left and V3 up.

Fi

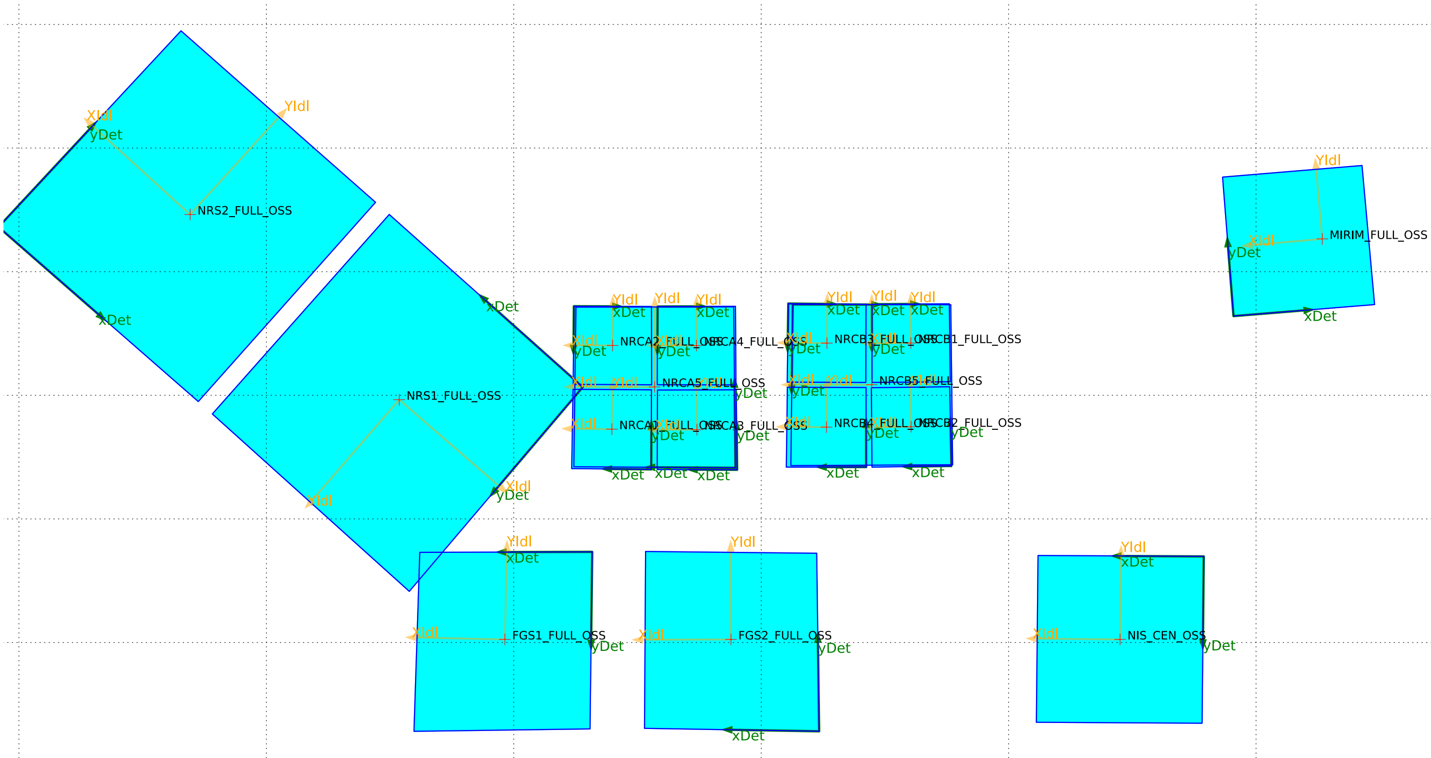


Figure 4: SIAF Apertures with AperType=OSS. Note the differing Ideal (Idl) frame shown in the V2V3 frame with V2 to the left and V3 up.

"COMPOUND"

This AperType is used for a SIAF aperture that encompasses several others.

e.g. detectors, or IFU "slices".

This type of entry has values populated only for elements relating the region to V2V3 (i.e. V2ref, V3ref, vertices in Ideal frame, orientation with respect to V3, and parity). With no connection to a detector, pixel-related values are inapplicable and are not populated.

The construct is used primarily to provide only a targetable reference/fiducial point (V2ref,V3ref) for proposal planning.

Example 1: NIRCam Imaging using Module=ALL would call upon this entry to provide the inter-module location at which to center an extended target spanning both modules.

Example 2: MIRI MRS observations would call upon a compound aperture encompassing the many slice FOVs in order to provide the location at which to center a target among them. See example in Figure 5.

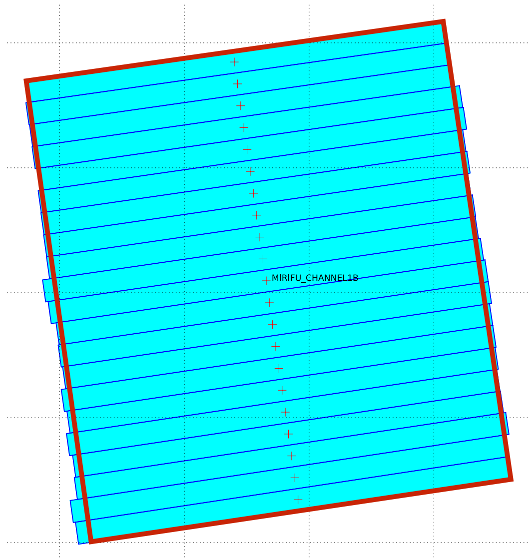


Figure 5: Example of a SIAF Aperture with AperType=COMPOUND. MIRIFU_CHANNEL1B (shown in red) is a compound aperture encompassing the individual slice fields of view. The central reference point of this compound aperture determines the target placement in the currently implemented PPS, while the vertices provide a convenient approximation of the extent of the multiple individual slices. The slices are also represented in the SIAF for future use as appropriate and are of AperType=SLIT, discussed below.

"SLIT"

This AperType denotes a SIAF entry that describes a physical aperture, slit, IFU slice, or any other region that is not tied to a particular SCA's pixel frame, but whose definition is required for target placement and/or for understanding its footprint on the sky. See Figure 6 for examples from NIRSpec.

Usually this type applies to entrance apertures for spectral modes. The relationship of spectrum to detector is handled outside of the SIAF (via the science pipeline and the calibration reference database system) so these entries contain only the elements sufficient to describe a slit/aperture/slice FOV in V2,V3 for use in science planning or by the archive. The limited set of elements populated by AperType=SLIT is identical to that for COMPOUND.

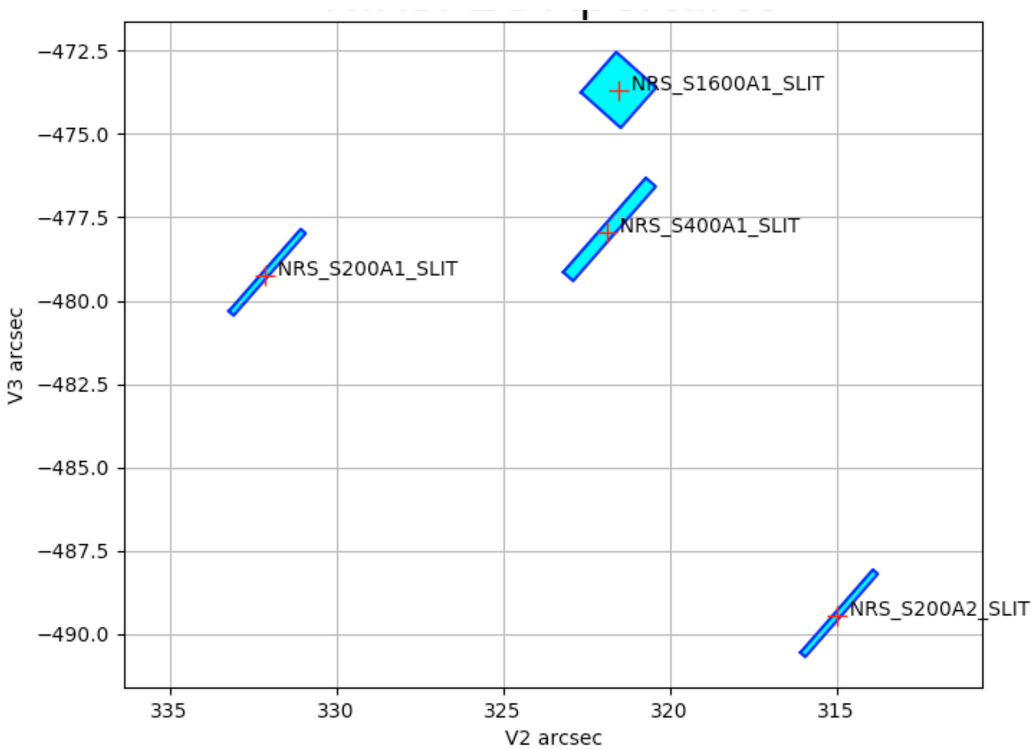


Figure 6: Some NIRSpec SIAF Apertures of AperType=SLIT. These entries define the V2,V3 location of a region's center for target placement, and the vertices for understanding projection onto the sky. AperType=SLIT contains no pixel frame information.

We point out the SIAF handles the NIRSpec MSA by defining the four quadrants as apertures of AperType=SLIT, in order to supply only the reference location of each quadrant to the MSA Planning Tool (MPT). For more information on the intersection of the SIAF and MPT, please refer to the aforementioned NIRSpec SIAF document, *The Pre-Flight SI Aperture File, Part 4: NIRSpec,* JWST-STScI-005921.

"SUBARRAY"

AperType=SUBARRAY denotes a fixed region of an SCA that is read out by OSS.

These subarrays have been specified by the SI Operations Working Groups and reside in OSS "definition tables" as specified in the OSS level 5 requirements.

While not the defining source for this information, the SIAF will contain such subarrays if they are needed for pointing and proposal planning (i.e. typically if they are used in imaging or slitless spectroscopy where FOV on the sky is relevant for proposal planning). Subarrays used in other spectral modes are not contained in the SIAF because the pointing in these cases is determined by the slit or physical aperture specified in PPS through which the source light passes.

As captured in the SIAF this AperType will have its characteristics fundamentally defined in pixel space, and will have all fields populated. Its Det frame characteristics locate its reference pixel and dimensions on the SCA (which is indicated in the AperName) while its Sci frame is referenced entirely to the subarray region, (See as Figure 10 in Section 4.1).

Figure 7 illustrates some examples of SIAF subarrays for the case of MIRI.

Figure 7: Example of MIRI SIAF Apertures of AperType=SUBARRAY

Figure 8: All MIRI SIAF Apertures of AperType=ROI.

"ROI"

AperType ROI indicates a *Region of Interest* or "search box" within a full SCA if it is fixed with respect to the detector. The SIAF would not capture a dynamically defined ROI. This type of entry will have all fields populated and is otherwise very similar to type SUBARRAY in its use in pointing and visualization.

The ROI differs from SUBARRAY in that it is not defined by OSS as a sub-region of the detector to be read out. In 2015 the SIAF Working Group created this AperType to distinguish it from the OSS-defined SUBARRAY. Figure 8 shows MIRI ROI entries.

"TRANSFORM"

Denotes a SIAF entry that describes a particular transformation polynomial that differs from the normal SIAF convention of XY(pixel)-->XY(ideal).

Entries of this type will have the polynomial coefficients populated, as well as a limited number of fundamental zero-point values, whose interpretation also differs from the normal SIAF spec. Other fields will be unpopulated. Examples currently are only found in NIRSpec's SIAF, representing a special two-step pixel to field-angle mapping ("_OTE") and MSA-related transformations. For NIRSpec the entries of AperType FULLSCA supply polynomials which transform between pixels and the Grating Wheel Assembly (GWA) plane rather than the OTE field angle. The TRANSFORM type rows with AperNames ending in "_OTE" contain the transformations between this GWA plane and field angle plus constants related to the reflection and rotation that occurs at the GWA plane. Other TRANSFORM rows with names including "OTEIP" transform between the MSA assembly and field angle. A detailed description of this NIRSpec-unique implementation and AperType is provided in the aforementioned NIRSpec SIAF document, *The Pre-Flight SI Aperture File, Part 4: NIRSpec,* JWST-STScI-005921.

4 Coordinate System Transformations: Mathematical Derivations & Descriptions

This section describes in detail how the quantities contained in the SIAF described above are used in coordinate transformations, and how those transformations are derived. It shows that the SIAF parameters that were defined are necessary and sufficient to support the various operations relying on the database which will be produced from the set of instrument SIAFs.

4.1 Detector versus Science

The transformations between Detector and Science only involve simple changes in the axis directions in multiples of 90 degrees with possible shifts of origin and changes of axis parity. It turns out for JWST that all rotations are multiples of 180 degrees, i.e changes of sign.

$$XSci - XSciRef = DetSciParity\left[\left(XDet - XDetRef\right)\cos(DetSciYAngle) + \left(YDet - YDetRef\right)\sin\left(DetSciYAngle\right)\right]$$
$$YSci - YSciRef = -\left(XDet - XDetRef\right)\sin(DetSciYAngle) + (YDet - YDetRef)\cos(DetSciYAngle)$$

In practice this formula will be simpler than it looks above, since the angles between Det and Sci axes will be multiples of 90 degrees and the sine and cosine terms can only have values -1, 0, or +1. Only one term to the right of the equals sign in each row will be non-zero. The (XSciRef, YSciRef) position will be at or near the center of the science aperture which may be a subarray on the detector. The matching (XDetRef, YDetRef) can be anywhere on the detector. If the detector indices are zero based while the science image starts from 1, this can easily be accommodated at this point. Although we had allowed for the possibility of a zero based detector system this has not arisen. All pixel counting for JWST starts with (1,1) being the central point within the first pixel.

The inverse transformation is

$$XDet - XDetRef = DetSciParity\left(XSci - XSciRef\right)\cos(DetSciYAngle) - (YSci - YSciRef)\sin(DetSciYAngle)$$
$$YDet - YDetRef = DetSciParity\left(XSci - XSciRef\right)\sin(DetSciYAngle) + (YSci - YSciRef)\cos(DetSciYAngle)$$

Figure 10 gives an example illustrating most of the complications that may arise between detector and science frames. In the direction from XDet to YDet the YSci axis is at 90 degrees from YDet. So sin(DetSciYangle) = 1 and the cosine is zero. From x to y is anti-clockwise in both cases so the parity is positive. The transformation equations are

XSci-XSciRef = YDet-YDetRef

YSci-YSciRef = -(XDet-XDetRef)

XDet-XDetRef = -(YSci-YSciRef)

YDet-YDetRef = (XSci-XSciRef)

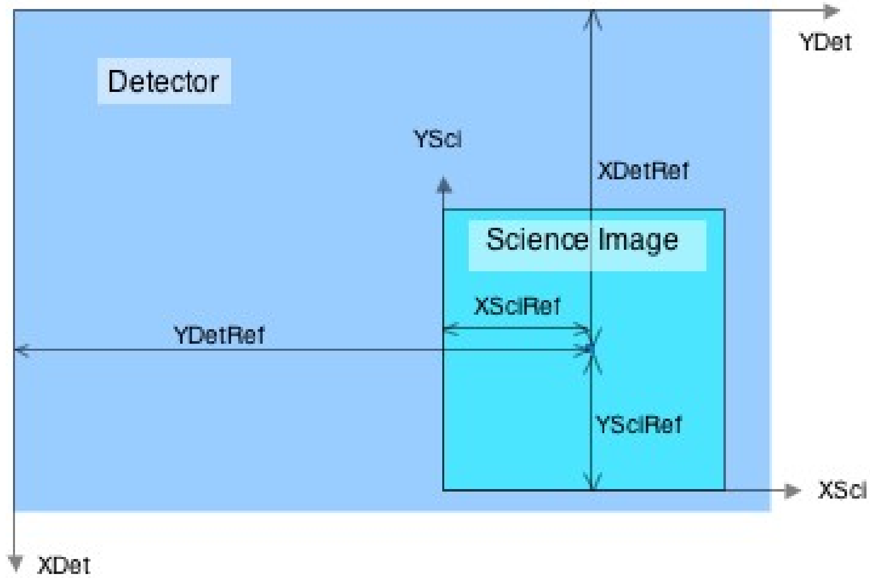


Figure 10 Detector and Science coordinate frames

4.2 Science versus Ideal

The transformation from science to ideal coordinates follows the HST method using polynomials which express the distortion correction including varying scales and non-orthogonality of the axes.

<u>Science to Ideal</u>

$$XIdl = \sum_{i=1}^{degree} \sum_{j=0}^{i} Sci2IdlCoefX_{i,j} (XSci - XSciRef)^{i-j} (YSci - YSciRef)^{j}$$

$$YIdl = \sum_{i=1}^{degree} \sum_{j=0}^{i} Sci2IdlCoefY_{i,j} (XSci - XSciRef)^{i-j} (YSci - YSciRef)^{j}$$

in which *degree* is identical to *Sci2IdlDegree*.

<u>Ideal to Science</u>

$$XSci - XSciRef = \sum_{i=1}^{degree} \sum_{j=0}^{i} Idl2SciCoefX_{i,j} XIidl^{i-j} YIdl^{j}$$

$$YSci - YSciRef = \sum_{i=1}^{degree} \sum_{j=0}^{i} Idl2SciCoefY_{i,j} XIdl^{i-j} YIdl^{j}$$

The equations do not appear to be quite symmetrical because the origin of the Idl system is chosen to be at the reference point which is not at the Sci origin. Within a program one would probably use intermediate variables equal to *XSci-XSciRef* and *YSci-YSciRef* and then the symmetry is more apparent.

The coding of the coefficients is such that they refer to powers of x and y in the order

1, x, y, x^2, xy, y^2 … (This is a more natural order than is used on HST which results in 1, y, x, y^2, xy, x^2… The change should be noted in case software is reused.) The number of terms in each of the four groups is (*degree*+1)(*degree*+2)/2. For instance, a third order polynomial will require 10 terms in each polynomial.

The coefficients define scales in arcsec/pixel in all directions at any position in the image. Axis directions and hence skewness can be calculated and so the values XSciScale, YSciScale, V3SciAngleX, V3SciAngleY are not independent entries.

In practice, problems like the 34th row effect in HST/WFPC2 and other systematic deviations from the polynomial solution as found in HST/ACS may arise. However, if the effects are too small to impact telescope on-board pointing calculations during Target Acquisitions then the SIAF polynomial is sufficient, with more accurate pixel-sky mappings being relevant for post-observation astrometric processing, which draws from calibration products containing the most accurate mappings for science.

4.3 Ideal versus Telescope (V2,V3)

The transformation from Ideal to Telescope (V2,V3) is given in its most general form by spherical trigonometry formulae. However, in the tangent plane approximation, the equations reduce to a simpler two-dimensional linear transformation. The distinction between tangent plane and spherical trigonometry calculations may not be meaningful for displacements across a single aperture. At ten arc-minutes from the tangent point the difference is 1.7 mas. The difference varies in its lowest Taylor approximation order as the cube of the displacement. Even for a use case that defines a large mosaic using Ideal coordinate offsets, the errors should be acceptable since mosaic tile positioning does not generally need milli-arcsecond precision (nor can such accurate relative pointing be achieved for offsets larger than a few arc-seconds).

In the tangent plane approximation, the coordinate transformation is a simple shift and rotation operation, both sets of scales being in arc-seconds. The parity is almost always -1 because the on-sky view in terms of *V2* and *V3* has the *V2* to *V3* rotation as clockwise while an x-y frame conventionally has x to y in an anti-clockwise direction. Different conventions might be preferred in some cases especially for spectrographic apertures.

The coordinate transformation formulae in the tangent plane approximation are:

$$V2-V2Ref = VIdlParity \cdot XIdl \cdot \cos(V3IdlYang) + YIdl \cdot \sin(V3IdlYang)$$

$$V3-V3Ref = -VIdlParity \cdot XIdl \cdot \sin(V3IdlYang) + YIdl \cdot \cos(V3IdlYang)$$

$$XIdl = VIdlParity \cdot \{(V2-V2Ref) \cdot \cos(V3IdlYang) - (V3-V3Ref)\,) \cdot \sin(V3IdlYang)\}$$

$$YIdl = (V2-V2Ref) \cdot \sin(V3IdlYang) + (V3-V3Ref) \cdot \cos(V3IdlYang)$$

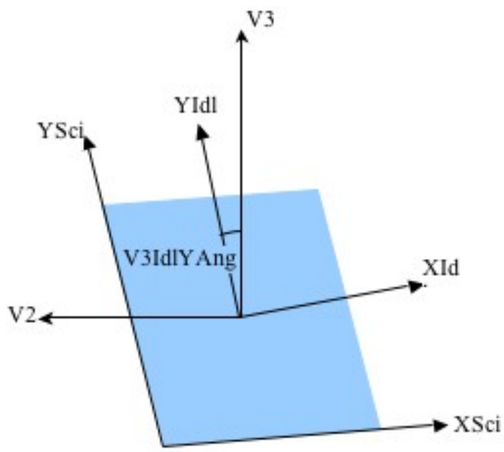


Figure 11: Ideal and Telescope (V2,V3) coordinate frames.

The angle *V3IdlYang* can now have any value and will be a result of measuring the orientation of each detector within the telescope. The orientation of the ideal frame with respect to the science frame is arbitrary, but we normally choose it to make YIdl align with YSci. Because of distortion, this does not imply XIdl aligns exactly with XSci. In Figure 11, the V2,V3 and Idl axes are normal whereas the Sci axes are not when displayed in the distortion corrected view. The sense of *V3IdlYang* has been chosen to be the same as that of the position angle on the sky. So by adding the angle from North to V3 to this angle, we get the position angle of both y axes on the sky.

In some cases, depending on the application, it may be necessary to use the more general spherical trigonometry formulae. This might be the case if there is a need for both a very large Ideal coordinate offset and very high precision. To derive the formulae, we first consider converting an *XIdl, YIdl* position into exact *V2* and *V3* values. We begin by using the *V2-V2Ref* and *V3-V3Ref* values calculated above for the linear approximation. Let us call them *t* and *u* respectively. Then the total linear displacement on the plane tangent at (*V2Ref ,V3Ref*) will be $s = \sqrt{t^2 + u^2}$ and the corresponding angle subtended on the unit sphere will be ρ, where $\tan\rho = s$ Then, calling the spherical coordinate positions, *V2s* and *V3s* we have

$$V2s = V2Ref + \arcsin\left(t\cos\rho/\cos V3Ref\right)$$

and

$$V3s = \arcsin\left(\cos\rho\left(u\cos V3Ref + \sin V3Ref\right)\right)$$

Admittedly, the derivation of these equations is not obvious but they are based on formulae found in a spherical astronomy text book (Smart 1960, Section 161).

For the inverse calculation *V2s* and *V3s* are known and we wish to find the accurately calculated values of *XIdl* and *YIdl* on the tangent plane. We start by calculating

$$\cos\rho = \cos(V2s - V2Ref)\cos V3s \cdot \cos V3Ref + \sin V3s \cdot \sin V3Ref$$

Then the displacements along the V2 and V3 axes in the tangent plane are

$$V2 - V2Ref = \sin(V2s - V2Ref)\cos V3s/\cos\rho$$

and

$$V3 - V3Ref = (\sin V3s/\cos\rho - \sin V3Ref)/\cos V3Ref$$

These values can be used directly in the second pair of linear transformations given above at the beginning of this section to obtain *XIdl* and *YIdl*.

Note that for all the spherical trigonometry formulae referenced in this section, angles must be expressed in radians whereas in the tangent plane case it is often convenient to work in arc-seconds.

5 Absolute Transformations

In practice one needs to be able to calculate for a given pixel not only where it is located in the focal plane, but also which RA, Dec on the sky it corresponds to and where the direction of North is. The data and calculations given in the previous sections support all necessary transformations between detector, science, ideal, and (V2,V3) coordinates. Therefore, we need to address only the relation between (V2,V3) and absolute coordinates. These relations do not depend on any of the parameters in the SIAF file, and are therefore strictly outside the scope of this document. Nonetheless, the issue is of sufficient practical interest that we discuss it here.

5.1 Absolute coordinates

We define an inertial coordinate system through a right-handed orthogonal set of three axes. The first axis, A, points at the First Point of Aries, the origin of the RA measurement. The second axis which we call B is in the equatorial plane at right angles to A and points to a sky position with a Right Ascension of 6 hours and a Declination of zero. The third axis, N, points North and is normal to A and B (see Figure 12).

We also define a similar coordinate system that is fixed within the telescope. V1 corresponds to the optical axis in the viewing direction. The V2-V3 plane is then normal to the optical axis. The V3-axis has been chosen to point towards the single secondary mirror support structure strut. The exact details of this definition are not relevant beyond understanding that the axes are defined and fixed with respect to the telescope. The axes V1, V2 and V3 are aligned relatively in the same way as A, B and N, both describing right-handed orthogonal coordinate systems. The (V1,V2,V3) axes are rotated with respect to the inertial frame, and we therefore refer to it as the rotated frame.

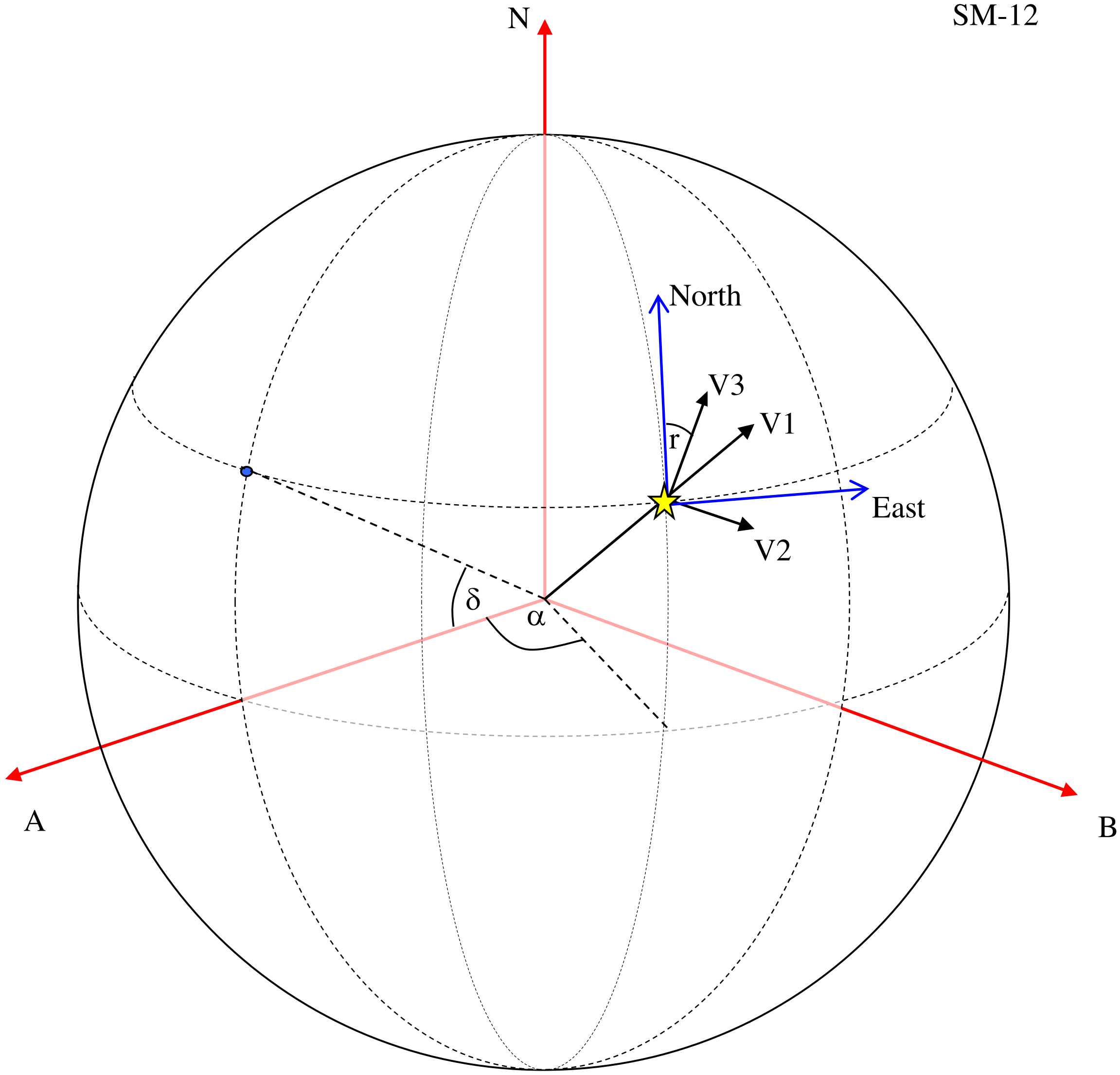


Figure 12. The inertial and rotated coordinate frames. In this example, the V1 axis points at a star with absolute coordinates (α,δ) and the V3 axis is at a position angle r, measured from North towards East. The vectors shown as North, East, V2 and V3 all lie in then tangent plane at (α,δ)

A unit vector in either system can be described by two Euler angles (i.e., the polar angles of a spherical polar coordinate system). In the inertial frame the commonly used angles are α and δ (i.e., RA and Dec). In the rotated frame the angles are *v2* and *v3*. These are the same angles that were used in the previous discussion of the SIAF file. There they were denoted with capital letters. Here we use lower-case italics to differentiate the angles from the coordinate axes. Let ***w*** = (x,y,z) be a unit vector in the inertial frame, and ***w*´** = (x´,y´,z´) a unit vector in the rotated frame. Then the relations to the Euler angles are

$$\begin{aligned} x' &= \cos(v2)\cos(v3) \\ y' &= \sin(v2)\cos(v3) \\ z' &= \sin(v3) \end{aligned} \quad \text{and} \quad \begin{aligned} x &= \cos(\alpha)\cos(\delta) \\ y &= \sin(\alpha)\cos(\delta) \\ z &= \sin(\delta) \end{aligned}$$

The inverses are

$$v2 = \arctan(y'/x') \qquad \text{and} \qquad \alpha = \arctan(y/x)$$
$$v3 = \arcsin(z') \qquad\qquad \delta = \arcsin(z)$$

In calculating the arctan to derive to derive α (and similarly for $v2$), the values of x and y should be retained separately and used in an ATAN2(y,x) function as is supplied in the Fortran, Python and Java programming languages. This places the angle in the correct quadrant. Evaluating y/x and using the one-parameter ATAN function would lead to an angle which may differ by π from the correct value. The angles $v3$ or δ should always be considered as lying between $-\pi/2$ and $+\pi/2$ which will be the default supplied by an arcsine function.

Let $\boldsymbol{u}_j$ for $j=1,\ldots,3$ be the three unit vectors along the axes of the inertial system, and

let $\boldsymbol{u}_j'$ for $j=1,\ldots,3$ be the three unit vectors along the axes of the rotated system. Consider now a known vector in the three-dimensional space (irrespective of coordinate frame). Let this vector have carthesian coordinates $\boldsymbol{w} = (w_1,w_2,w_3)$ in the inertial frame and $\boldsymbol{w}' = (w_1',w_2',w_3')$ in rotated frame. Since we are considering one and the same vector, one obtains the identity $\Sigma\, w_j\, \boldsymbol{u}_j = \Sigma\, w_j'\, \boldsymbol{u}_j'$. Upon taking the inner product with $\boldsymbol{u}_i$ on either side of the equation, for $i=1,\ldots,3,$ one obtains the matrix equation

$$\boldsymbol{w} = \boldsymbol{M}\, \boldsymbol{w}',$$

where the matrix $\boldsymbol{M}$ has components $M_{ij} = (\boldsymbol{u}_i \cdot \boldsymbol{u}_j')$, with i numbering the rows and j the columns. Note that the three columns of $\boldsymbol{M}$ correspond to the unit vectors in the inertial frame pointing in the directions of the V1, V2, and V3 axes, respectively. The matrix $\boldsymbol{M}$ determines the coordinate transformation between the two frames. The goal is to express the matrix $\boldsymbol{M}$ in terms of known quantities. This then allows us to calculate α and δ for any given $v2$ and $v3$, or vice versa, using $\boldsymbol{w} = \boldsymbol{M}\, \boldsymbol{w}'$ with the equations for $\boldsymbol{w} = (x,y,z)$ and $\boldsymbol{w}' = (x',y',z')$ defined above.

To understand the contents of the matrix $\boldsymbol{M}$, it is useful to consider a mapping T in the rotated frame that moves any vector to a different direction. Let the mapping be such that it moves each unit vector of the inertial frame to the corresponding unit vector of the rotated frame, i.e., $T(\boldsymbol{u}_j) = \boldsymbol{u}_j'$ for $j=1,\ldots,3$. The unit vector $\boldsymbol{u}'_j$ can be written as

$\boldsymbol{u}'_j = \Sigma\, (\boldsymbol{u}_i \cdot \boldsymbol{u}_j')\, \boldsymbol{u}_i = \Sigma\, M_{ij}\, \boldsymbol{u}_i$ where the summation is over $i = 1,\ldots,3$. An arbitrary vector $\boldsymbol{w}' = \Sigma\, w_j'\, \boldsymbol{u}'_j$ in the rotated frame will be mapped to

$\mathbf{w}'' = T(\boldsymbol{w}') = \Sigma\, w_j'\, T(\boldsymbol{u}_j') = \Sigma\, w_j'\, \Sigma\, M_{ij}\, T(\boldsymbol{u}_i) = \Sigma\, \boldsymbol{u}_i'\, (\Sigma\, M_{ij}\, w_j')$. For the vector $\mathbf{w}''$, the

coordinates w_i'' in the rotated frame satisfy by definition the equation $\mathbf{w}'' = \Sigma\, w_i''\, \boldsymbol{u}_i'$. Hence, the coordinates satisfy $w_i'' = \Sigma\, M_{ij}\, w_j'$, for $i = 1,\ldots,3$. This corresponds to the matrix equation ***w*´´ = *M w*´** in the rotated frame. Thus, the matrix ***M*** corresponds to the mapping in the rotated frame that maps each unit vector of the inertial frame to the corresponding unit vector of the rotated frame.

To obtain a convenient expression for ***M***, it is useful to resort to the use of rotation matrices. A rotation matrix describes the rotation of a vector in the given system, around a given axis by some angle. The set of rotation matrices in the rotated frame about V1 and V2 and V3 by the angles λ, μ, and ν, respectively, are:

$$R1(\lambda) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & -\sin\lambda \\ 0 & \sin\lambda & \cos\lambda \end{pmatrix} \quad R2(\mu) = \begin{pmatrix} \cos\mu & 0 & \sin\mu \\ 0 & 1 & 0 \\ -\sin\mu & 0 & \cos\mu \end{pmatrix} \quad R3(\nu) = \begin{pmatrix} \cos\nu & -\sin\nu & 0 \\ \sin\nu & \cos\nu & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

All rotations are defined as positive when they are right handed around the axis in question. This means that if you imagine the thumb of your right hand pointing in the positive direction of the rotation axis, the rotation is in the direction your fingers curve to close. For rotation matrices the inverse is equal to the transpose and in turn is the same as changing the sign of the angle. Overall, this simply states that the inverse of a rotation in one direction is a rotation in the other direction. Note that if one starts with the rotated and inertial frames in alignment, then RA corresponds to a positive rotation around V3, Dec corresponds to a negative rotation around V2, and the position angle measured from North through East corresponds to a negative rotation about V1.

In a practical situation, we will know from information provided by the ACS what the telescope roll angle r is, defined here as the position angle (measured from North over East) of the V3 axis, at the V1-axis pointing. We will also know for at least one $(v2,v3)$ point what the corresponding absolute position (α,δ) is (this may be the $(v2,v3)$=(0,0) position corresponding to the V1 axis, but for the purpose of this discussion it could be any other point). The unit vectors of the inertial frame can then be aligned with the unit vectors of the rotated frame using a sequence of 5 rotation matrices in the rotated frame.

First, we align the position (α,δ) with the V1 unit vector. This is achieved by the successive rotations R3($-v2$) and R2($v3$). Second, we align the local tangent plane direction of North with the V3 unit vector. This is achieved by the rotation R1($-r$). Third, we align the A axis with the V1 unit vector. This is achieved by the rotations R2($-\delta$) and R3(α). The multiplication of the corresponding rotation matrices yields the matrix

$$\boldsymbol{M} = \mathrm{R3}(\alpha)\ \mathrm{R2}(-\delta)\ \mathrm{R1}(-r)\ \mathrm{R2}(v3)\ \mathrm{R3}(-v2)$$

where operations are executed right to left. Note that in the first step we change the $(v2,v3)$ position of a known inertial pointing, while in the third step we change the inertial pointing of a known $(v2,v3)$ position. These operations are mathematical inverses, and hence the order of the rotations R3 and R2 and the signs of their arguments are opposite in the two steps. It is convenient to break up ***M*** into two successive matrices,

$$\boldsymbol{M} = \boldsymbol{M}2\,\boldsymbol{M}1,$$

where

$$M1 = R2(v3)R3(-v2) = \begin{pmatrix} \cos v3 & 0 & \sin v3 \\ 0 & 1 & 0 \\ -\sin v3 & 0 & \cos v3 \end{pmatrix} \begin{pmatrix} \cos v2 & \sin v2 & 0 \\ -\sin v2 & \cos v2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos v2 \cos v3 & \sin v2 \cos v3 & \sin v3 \\ -\sin v2 & \cos v2 & 0 \\ -\cos v2 \sin v3 & -\sin v2 \sin v3 & \cos v3 \end{pmatrix}$$

$$M2 = R3(\alpha)R2(-\delta)R1(-r) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos r & \sin r \\ 0 & -\sin r & \cos r \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\delta & -\sin\alpha\cos r + \cos\alpha\sin\delta\sin r & -\sin\alpha\sin r - \cos\alpha\sin\delta\cos r \\ \sin\alpha\cos\delta & \cos\alpha\cos r + \sin\alpha\sin\delta\sin r & \cos\alpha\sin r - \sin\alpha\sin\delta\cos r \\ \sin\delta & -\cos\delta\sin r & \cos\delta\cos r \end{pmatrix}$$

An alternative way of thinking about the transformation M involves rotating the telescope V1,V2,V3 frame within the A,B,N frame. The rotations R1,R2 and R3 are now about A, B and N respectively. We start with V1,V2,V3 aligned with A,B,N, which is equivalent to having the telescope optical axis pointing at the first point of Aries with a roll angle of zero. An arbitrary aperture reference point is at a position (*v2*,*v3*) and is represented by a vector fixed in the V coordinate system. The first step is to align this vector with the A direction by the rotations R3(-*v2*) followed by R2(*v3*), the same mathematical steps as described above as the M1 matrix. At this point we apply a designated roll R1(-*r*) about A which will not move the reference vector. The final operation is to place this vector at the sky position (α,δ) with the sequence R2(-δ) followed by R3(α). The last three steps correspond to the matrix M2, and the full sequence is, as above, R3(α)R2(-δ)R1(-*r*)R2(*v3*)R3(-*v2*)

In the following we will often refer to ***M*** with its arguments as the attitude matrix $\boldsymbol{M}(v2_0, v3_0, \alpha_0, \delta_0, r_0)$. We add here a subscript 0 to the arguments, to indicate that they are the values that define the attitude of the telescope. To calculate the values (α,δ) for other values of (*v*2,*v*3) we need to evaluate the equation $\boldsymbol{w} = \boldsymbol{M}(v2_0, v3_0, \alpha_0, \delta_0, r_0)\ \boldsymbol{w}'$ with $\boldsymbol{w}'$ =(cos *v*2 cos *v*3 , sin *v*2 cos *v*3, sin *v*3). The result equals ***w***=(x,y,x)=(cos α cos δ, sinα

cos δ, sinδ), from which $\alpha = \arctan(y/x)$ and $\delta = \arcsin(z)$ can be calculated (using the ATAN2 function to get the correct quadrant for α).

As a simple verification of these equations, one may set ($v2$, $v3$)= ($v2_0$, $v3_0$). Then $\boldsymbol{M}1\ \boldsymbol{w}'$ can be evaluated to be (1,0,0), and hence $\boldsymbol{M}\ \boldsymbol{w}$ equals the first column of $\boldsymbol{M}2$, which is (cos α_0 cos δ_0, sinα_0 cos δ_0, sinδ_0), as it should. As another verification, consider the case in which the inertial and rotated systems are aligned. Then r_0=0 and at any ($v2_0$,$v3_0$) the absolute coordinates are (α_0,δ_0)=($v2_0$,$v3_0$). Hence, $\boldsymbol{M}(\alpha_0,\delta_0,\alpha_0,\delta_0,r_0)$ should equal the identity matrix $\boldsymbol{I}$. That this is indeed the case follows from $\boldsymbol{M}(\alpha_0,\delta_0,\alpha_0,\delta_0,r_0)$ =

$$R3(\alpha)\ R2(-\delta)\ R1(-r)\ R2(\delta)\ R3(-\alpha) = R3(\alpha)\ R2(-\delta+\delta)\ R3(-\alpha) = R3(\alpha-\alpha) = \boldsymbol{I}.$$

5.2 Roll Angle

The local roll r, defined as the position angle (measured from North over East) of the V3 axis (defined as in the footnote to Table 1), is not constant in the focal plane. At arbitrary ($v2$,$v3$), its value will not be the same as the roll r_0 at ($v2$,$v3$)=(0,0) (i.e., the at the V1-axis). However, the value or r can be calculated using the elements M_{ij} of the matrix $\boldsymbol{M}$.

A small displacement along the V3 axis causes shifts in α and δ. We calculate $\frac{d\alpha}{dv3}$ and $\frac{d\delta}{dv3}$ and then the local roll angle is given by $\tan(r) = \cos\delta \frac{d\alpha}{dv3} \Big/ \frac{d\delta}{dv3}$, as illustrated in Figure 13 This turns out not to be a very difficult calculation. For arbitrary ($v2,v3$), the coordinates (w_1,w_2,w_3) in the inertial frame are obtained from the equation $\boldsymbol{w} = \boldsymbol{M}\ \boldsymbol{w}'$ as

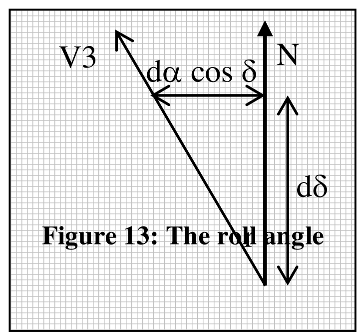


Figure 13: The roll angle

$$w_j = \left(M_{j,1}\cos v2 + M_{j,2}\sin v2\right)\cos v3 + M_{j,3}\sin v3$$

Also, (w_1,w_2,w_3) =(cos α cos δ, sinα cos δ, sinδ), and therefore tan α = w_2/w_1. Taking the derivative with respect to $v3$ yields

$$\sec^2\alpha \frac{d\alpha}{dv3} = \frac{w_1 \frac{dw_2}{dv3} - w_2 \frac{dw_1}{dv3}}{w_1^{\ 2}},$$

which yields

$$\frac{d\alpha}{dv3} = \frac{w_1 \frac{dw_2}{dv3} - w_2 \frac{dw_1}{dv3}}{w_1^{\ 2} + w_2^{\ 2}}.$$

Also,

$$\frac{d\delta}{dv3} = \frac{1}{\cos\delta}\frac{dw_3}{dv3}$$

$w_1^2+w_2^2 = \cos^2\delta$. Hence,

$$\tan(r) = \frac{w_1\dfrac{dw_2}{dv3} - w_2\dfrac{dw_1}{dv3}}{\dfrac{dw_3}{dv3}}$$

The differentials are

$$\frac{dw_j}{dv3} = -(M_{j,1}\cos v2 + M_{j,2}\sin v2)\sin v3 + M_{j,3}\cos v3$$

In evaluating the difference of the products in the numerator many terms cancel leaving a surprisingly simple final formula, tan(r)=Y/X in which

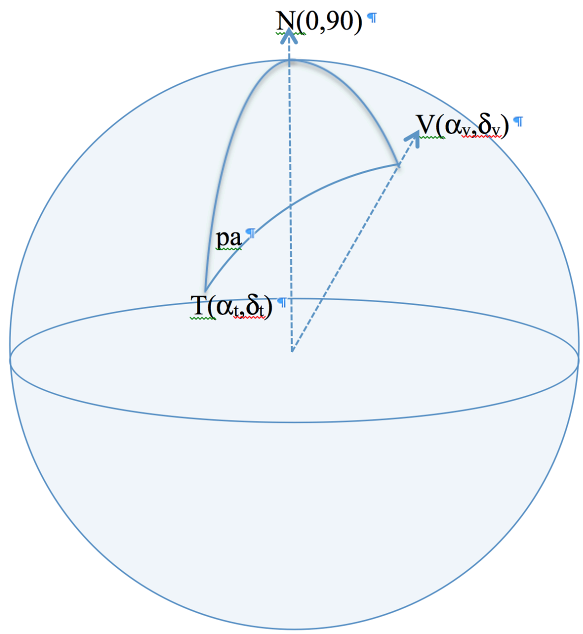
$$X = -\left(M_{31}\cos v2 + M_{32}\sin v2\right)\sin v3 + M_{33}\cos v3$$
$$Y = (M_{11}M_{23} - M_{21}M_{13})\cos v2 + \left(M_{12}M_{23} - M_{22}M_{13}\right)\sin v2$$

As mentioned above, in evaluating arc tangents the π ambiguity must be kept in mind. Again, if the values X and Y are kept separate and used in the functions ATAN2(Y,X), the angle will be placed in the correct quadrant. In deriving the formula, the only common divisor is $\cos^2\delta$, which is non-negative. The special case where $\cos\delta=0$ refers to positions at the poles where the roll angle would require a specific definition. Here, the ATAN2 function does not cause a divide-by-zero error but returns a value of zero with no special warning.

As a simple verification of these equations, consider the case in which the attitude matrix $\boldsymbol{M}(v2_0,v3_0,\alpha_0,\delta_0,r_0)$ is defined at the V1 axis, so that $(v2_0,v3_0) = (0,0)$. Then M1 is the identity matrix, and $\boldsymbol{M}$ = M2. Then for arbitrary $(v2,v3)$ close to the origin,

$$X \approx M_{33} = \cos\delta_0\cos r_0$$
$$Y \approx M_{11}M_{23} - M_{21}M_{13} = \cos\delta_0\sin r_0$$

and therefore $r \approx r_0$, as expected.



The position angle may also be calculated using the inertial coordinates, RA and Dec of the target. In the diagram we show a spherical triangle NTV with the target at vertex T with RA and Dec of (α_t, δ_t), the North pole at position N (0, 90) and the V3 axis intersecting the unit sphere at V. The position of the V3 axis can be easily calculated from the same attitude matrix *M*; the elements of the third column of *M* are the vector components of V3 in the inertial system.

Hence $\alpha_v = \text{atan}(M_{32}/ M_{31})$ and

$\delta_v = \text{asin}(M_{33})$ The angular arc length NT is 90-δ_t and the arc NV is of length 90-δ_v. The angle at N is α_v-α_t and so we have the spherical triangle fully defined by two sides and the included angle. The position angle is the angle labeled *pa* at T. A standard spherical trigonometric formula for tan(*pa*) gives

$$\tan(pa) = \cos(\delta_v)\sin(\alpha_v - \alpha_t) / (\sin(\delta_v)\cos(\delta_t) - \cos(\delta_v)\sin(\delta_t)\cos(\alpha_v - \alpha_t))$$

6 Example Applications

We discuss here two cases of operational interest that can be addressed with the equations that we have presented, namely the selection and placement of a guide star, and the process of target acquisition. The methods have been coded in IDL and tested. Some have been ported to a Java environment (see Section 1) and are being tested in the APT system.

6.1 Guide star selection and placement

Suppose we wish to place a target onto a given pixel in a science aperture, using a predefined telescope roll r_0. First we calculate the $(v2,v3)$ position of the pixel using the SIAF file information and its corresponding transformations. Combining this with the target coordinates and the desired roll defines the attitude matrix ***M***. For each corner pixel of a chosen FGS aperture we can then also calculate the $(v2,v3)$ position using the SIAF file information and its corresponding transformations. The known matrix ***M*** can be applied to these $(v2,v3)$ positions to determine the absolute sky coordinates of the corners. The Guide Star Catalog 2 can then be used to identify an appropriate guide star in the region enclosed by the corner positions. The inverse of the matrix ***M*** can be applied to the guide star coordinates to obtain its $(v2,v3)$ position. The corresponding pixel position in the FGS aperture in which we need to place the guide star can be found using the SIAF file information and its corresponding transformations. The roll angle r at this position can be calculated using the equations in Section 5.2. This parameter is needed as input to the FGS commanding. The fundamental reference information is going to be the guide star position (in FGS ideal coordinates) and the roll about that point (rather than about the V1 axis or the target).

6.2 Target Acquisition

Figure 6 illustrates a target T, being found within a science instrument aperture near to, but not exactly on, the desired acquisition point, A. At the same pointing a guide star is found within one of the FGS apertures at position G. The question is, what are the pixel coordinates of the position H to which we need to move the guide star so that the target ends up exactly at A?

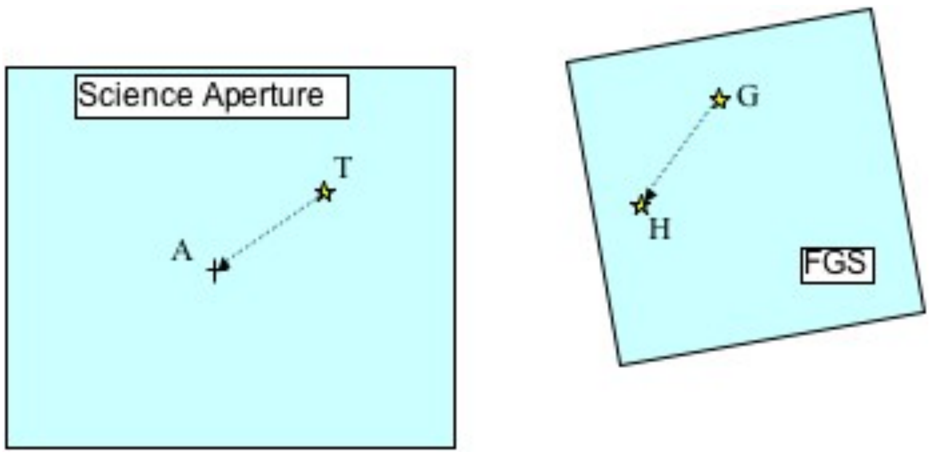


Figure 6 Target acquisition movements

Let the target coordinates RA and Dec be (α_T,δ_T), which need not be known for the purposes of these calculations. The (x,y) position of the target in the science aperture is measured. The corresponding $(v2_T,v3_T)$ values can be calculated using the SIAF file information and its corresponding transformations. Let the telescope roll be r_0. Then the attitude matrix is $\boldsymbol{M}(v2_T,v3_T,\alpha_T,\delta_T,r_0)$.

We can define a second matrix $\boldsymbol{N}(v2_A,v3_A,\alpha_T,\delta_T,r_0)$ which would place the same target on the acquisition point, using a telescope roll r_1 (which need not be the same as r_0). This combines the target RA and Dec with the $v2,v3$ of the acquisition position A.

The guide star is found at a measured (x,y) position in the FGS aperture. The corresponding $(v2_G,v3_G)$ values can be calculated using the SIAF file information and its corresponding transformations. These define a unit vector $\boldsymbol{w}_G{}'=(\cos v2_G \cos v3_G , \sin v2_G \cos v3_G, \sin v3_G)$ in the rotated frame. The corresponding vector in the inertial frame is given by $\boldsymbol{w} = \boldsymbol{M}\, \boldsymbol{w}_G{}'$. When the telescope is moved to the desired attitude, corresponding to the matrix $\boldsymbol{N}$, the guide star coordinates in the rotated frame will become $\boldsymbol{w}_H{}' = \boldsymbol{N}^{-1}\, \boldsymbol{w} = \boldsymbol{N}^{-1} \boldsymbol{M}\, \boldsymbol{w}_G{}'$, where $\boldsymbol{N}^{-1}$ is the inverse of the matrix $\boldsymbol{N}$. The coordinates $(v2_H,v3_H)$ can be calculated from the vector $\boldsymbol{w}_H{}'$. With help of the SIAF file information and its corresponding transformations, one obtains the pixel position (x,y) in the FGS aperture. This is the pixel to which the guide star should be moved, so as to move the target from T to A in the science aperture. To support script writing, this position has to be supplied in FGS ideal coordinates. The local telescope roll at the new position of the guide star can be calculated as in Section 5.2.

The matrix $\boldsymbol{N^{-1} M}$ is given by:

$$\boldsymbol{N^{-1}M} = \mathrm{R3}(v2_A)\ \mathrm{R2}(-v3_A)\ \mathrm{R1}(\mathrm{r}_1)\ \mathrm{R2}(\delta_T)\ \mathrm{R3}(-\alpha_T)\ \mathrm{R3}(\alpha_T)\ \mathrm{R2}(-\delta_T)\ \mathrm{R1}(-r_0)\ \mathrm{R2}(v3_T)\ \mathrm{R3}(-v2_T)$$
$$= \mathrm{R3}(v2_A)\ \mathrm{R2}(-v3_A)\ \mathrm{R1}(\mathrm{r}_1 - r_0)\ \mathrm{R2}(v3_T)\ \mathrm{R3}(-v2_T)$$

The absolute coordinates (α_T,δ_T) of the target drop out of the matrix, and therefore need not be known. The absolute coordinates of the guide star need not be known either. However, to calculate the local telescope roll at the position of the guide star one does need to know the absolute pointing and roll of the telescope which will normally be defined at a guide star position. As written, the matrix $\boldsymbol{N^{-1}M}$ corresponds to a combination of a slew from T to A with a roll over an angle r_1-r_0. Such a "delta-roll" may be necessary for certain target acquisitions. If not needed, i.e., $\mathrm{r}_1 - r_0 = 0$, then $\boldsymbol{N^{-1}M}$ reduces to $\mathrm{R3}(v2_A)\ \mathrm{R2}(v3_T - v3_A)\ \mathrm{R3}(-v2_T)$, which is purely a slew from T to A.

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