Implement a New Measure The BRAPH 2 Developers August 14, 2024

This is the developer tutorial for implementing a new measure. In this tutorial, you will learn how to create the generator file *.gen.m for a new measure, which can the be compiled by braph2genesis. All measures are (direct or indirect) extensions of the element Measure. Here, you will use as examples the measures Degree, DegreeAv, Distance, and Triangles.

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Implementation of unilayer measures

Nodal unilayer measure (Degree)

You will start by implementing in detail the measure Degree, which applies to most graphs and is a direct extension of the element Measure.

Code 1: Degree element header. The header section of the generator code in _Degree.gen.m provides the general information about the Degree element.

```
1 %% iheader!
_{2} Degree < Measure (m, degree) is the graph degree. (1)
4 %% idescription!
_{\rm 5} The degree of a node is the number of edges connected to the node within a
       laver.
6 Connection weights are ignored in calculations.
8 %% ibuild!
9 1
```

Code 2: Degree element prop update. The props_update section of the generator code in _Degree.gen.m updates the properties of the Measure element. This defines the core properties of the measure.

```
%% iprops_update!
3 %% iprop!
4 NAME (constant, string) is the name of the degree.
5 %%% idefault!
6 'Degree'
8 %% iprop!
9 DESCRIPTION (constant, string) is the description of the degree.
10 %%% idefault!
'The degree of a node is the number of edges connected to the node within a
       layer. Connection weights are ignored in calculations.'
13 %% iprop!
14 TEMPLATE (parameter, item) is the template of the degree.
15 %%% isettings!
16 'Degree'
18 %% iprop!
19 ID (data, string) is a few-letter code of the degree.
20 %%% idefault!
<sup>21</sup> 'Degree ID'
23 %% iprop!
24 LABEL (metadata, string) is an extended label of the degree.
25 %%% idefault!
26 'Degree label'
28 %% iprop!
29 NOTES (metadata, string) are some specific notes about the degree.
30 %%% idefault!
```

(1) The element Degree is defined as a subclass of Measure. The moniker will be m.

```
<sup>31</sup> 'Degree notes'
33 %% iprop! (1)
_{\rm 34} SHAPE (constant, scalar) is the measure shape \_\_{\rm Measure.NODAL}\_\_ .
  %%% idefault!
36 Measure.NODAL
38 %% iprop! (2)
  SCOPE (constant, scalar) is the measure scope __Measure.UNILAYER__.
  %%% idefault!
41 Measure.UNILAYER
43 %% iprop! (3)
44 PARAMETRICITY (constant, scalar) is the parametricity of the measure
       __Measure.NONPARAMETRIC__.
  %%% idefault!
46 Measure.NONPARAMETRIC
48 %% iprop! (4)
  COMPATIBLE_GRAPHS (constant, classlist) is the list of compatible graphs.
  %%% idefault!
51 {'GraphWU' 'GraphBU' 'MultigraphBUD' 'MultigraphBUT' 'MultiplexWU' '
       MultiplexBU' 'MultiplexBUD' 'MultiplexBUT' 'OrdMxWU' 'OrdMxBU'}
53 %% iprop! (5)
54 M (result, cell) is the degree.
  %%% icalculate!
  g = m.get('G'); (6)
  A = g.get('A'); (7)
59 degree = cell(g.get('LAYERNUMBER'), 1); (8)
  parfor li = 1:1:g.get('LAYERNUMBER')
      Aii = A\{li, li\};
      Aii = binarize(Aii); (9)
      degree(li) = {sum(Aii, 2)}; (10)
65
  end
  value = degree; (11
```

Code 3: **Degree element tests.** The tests section from the element generator _Degree.gen.m. A test should be prepared for each graph with which the measure is compatible. The test should at least verify in some simple cases that the value of the measure is correct.

```
%% itests!
3 %% iexcluded_props! (1)
4 [Degree.PFM]
6 %% itest! (2)
7 %%% iname!
8 GraphWU
9 %%% iprobability! (3)
10 .01
11 %%% icode!
```

- (1) Measures have a *shape*: Measure.GLOBAL (a value for the whole brain graph, e.g., average degree), Measure. NODAL (a value for each brain region, e.g., degree), or Measure.BINODAL (a value for each couple of brain regions, e.g., distance between couples of nodes).
- (2) Measures have a *scope*: Measure.SUPERGLOBAL (a result for the whole multi-layer graph, e.g., overlapping strength), Measure.UNILAYER (a result for each layer, e.g., average strength), or Measure.BILAYER (a result for each couple of layers).
- (3) Measures are either Measure.NONPARAMETRIC (the usual case) or Measure. PARAMETRIC (depending on some parameter).
- (4) Each measure has a list of compatible graphs for which the measure can be used.
- (5) The property M contains the code to be executed to calculate the measure. Here is where most of the action happens.
- (6) retrieves the graph from the property G of the measure m.
- (7) retrieves the cell with the adjacency matrix (for graph) or 2D-cell array (for multigraph, multiplex, etc.).
- (8) preallocates the variable to contain the result of the measure calculation.
- (9) binarizes the adjacency matrix (removing diagonal).
- (10) calculates the degree of the node for layer li.
- (11) returns the calculated value of the measure degree assigning it to the output variable value.
- List of properties that are excluded from testing.
- (2) Test for GraphWU. Similar tests should be implemented for each graph compatible with the measure.
- (3) assigns a low test execution probability.

```
<sub>12</sub> B = [
      0
         .6 1
13
      .6 0 0
      1 0 0
15
      ];
18 known_degree = {[2 1 1]'}; 4
g = GraphWU('B', B); (5)
m_outside_g = Degree('G', g); (6)
assert(isequal(m_outside_g.get('M'), known_degree), ... (7)
      [BRAPH2.STR ':Degree:' BRAPH2.FAIL_TEST], ...
      [class(m_outside_g) ' is not being calculated correctly for ' class(g) '
27 m_inside_g = g.get('MEASURE', 'Degree'); (8)
  assert(isequal(m_inside_g.get('M'), known_degree), ... 8
      [BRAPH2.STR ':Degree:' BRAPH2.FAIL_TEST], ...
      [class(m_inside_g) ' is not being calculated correctly for ' class(g) '.
       '])
```

- (4) is the expected value of the measure calculated by external means.
- (5) creates the graph.
- (6) creates the measure.
- $\overline{7}$ tests that the value of the measure coicides with its expected value.
- (8) extracts the measure from the graph.
- (8) tests that the value of the measure extracted from the graph coicides with its expected value.

Global unilayer measure (DegreeAv)

You can now use Degree as the basis to implement the global measure DegreeAv. The parts of the code that are modified are highlighted.

Code 4: DegreeAv element header. The header section of the generator code in _DegreeAv.gen.m provides the general information about the DegreeAv element. ← Code 1

```
1 %% iheader!
2 DegreeAv < Degree (m, average degree) is the graph average degree. (1)
4 %%% idescription!
5 The average degree of a graph is the average of all number of edges
      connected to a node within a layer.
6 Connection weights are ignored in calculations.
8 %%% ibuild!
9 1
```

Code 5: DegreeAv element prop update. The props_update section of the generator code in _DegreeAv.gen.m updates the properties of the Degree element. ← Code 2

```
1 %% iprops_update!
3 %% iprop!
4 NAME (constant, string) is the name of the average degree
5 %%%% idefault!
6 'DegreeAv'
8 %% iprop!
9 DESCRIPTION (constant, string) is the description of the average degree.
10 %%% idefault!
'The average degree of a graph is the average of all number of edges
       connected to a node within a layer. Connection weights are ignored in
       calculations.'
13 %% iprop!
14 TEMPLATE (parameter, item) is the template of the average degree
15 %%% isettings!
16 'DegreeAv'
18 %% iprop!
19 ID (data, string) is a few-letter code of the average degree
20 %%% idefault!
<sup>21</sup> 'DegreeAv ID'
23 %% iprop!
24 LABEL (metadata, string) is an extended label of the average degree.
25 %%% idefault!
26 'DegreeAv label'
28 %% iprop!
29 NOTES (metadata, string) are some specific notes about the average degree
30 %%% idefault!
31 'DegreeAv notes'
```

1 DegreeAv is a child of Degree.

```
33 %% iprop!
34 SHAPE (constant, scalar) is the measure shape __Measure.GLOBAL__.
35 %%% idefault!
36 Measure.GLOBAL
38 %% iprop!
39 SCOPE (constant, scalar) is the measure scope __Measure.UNILAYER__.
40 %%% idefault!
41 Measure.UNILAYER
43 %% iprop!
44 PARAMETRICITY (constant, scalar) is the parametricity of the measure
       __Measure.NONPARAMETRIC__ .
45 %%% idefault!
46 Measure.NONPARAMETRIC
48 %% iprop!
49 COMPATIBLE_GRAPHS (constant, classlist) is the list of compatible graphs.
50 %%% idefault!
51 {'GraphWU' 'GraphBU' 'MultigraphBUD' 'MultigraphBUT' 'MultiplexWU' '
       MultiplexBU' 'MultiplexBUD' 'MultiplexBUT' 'OrdMxWU' 'OrdMxBU'}
53 %% iprop!
54 M (result, cell) is the average degree.
55 %%% icalculate!
56 degree = calculateValue@Degree(m, prop); (1)
58 g = m.get('G');
60 degree_av = cell(g.get('LAYERNUMBER'), 1);
61 parfor li = 1:1:g.get('LAYERNUMBER')
      degree_av(li) = {mean(degree{li}));
62
_{63} end
65 value = degree_av;
```

(1) calculates the value of the degree calling its parent Degree.

Code 6: DegreeAv element tests. The tests section from the element generator $_DegreeAv.gen.m. \leftarrow Code 3$

```
1 %% itests!
3 %% iexcluded_props!
4 [DegreeAv.PFM]
6 %% itest!
7 %%% iname!
8 GraphWU
9 %%% iprobability!
10 .01
11 %%% icode!
12 B = [
   0 .6 1
13
    .6 0 0
14
15
    1 0 0
18 known_degree_av = {mean([2 1 1])};
g = GraphWU('B', B);
```

```
22 m_outside_g = DegreeAv('G', g);
23 assert(isequal(m_outside_g.get('M'), known_degree_av), ...
         [BRAPH2.STR ':DegreeAv:' BRAPH2.FAIL_TEST], ...
         [class(m_outside_g) ' is not being calculated correctly for ' class(g)
27 m_inside_g = g.get('MEASURE', 'DegreeAv');
28 assert(isequal(m_inside_g.get('M'), known_degree_av), ...
29 [BRAPH2.STR ':DegreeAv:' BRAPH2.FAIL_TEST], ...
30 [class(m_inside_g) ' is not being calculated correctly for ' class(g)
```

Binodal unilayer measure (Distance)

Now you will implement the binodal measure Distance. Again the differences in the code will be highlighted.

Code 7: Distance element header. The header section of the generator code in _Distance.gen.m provides the general information about the Distance element. \leftarrow Code 1

```
1 %% iheader!
Distance < Measure (m, distance) is the distance.</pre>
4 %% idescription!
5 The distance of a graph is the shortest path between all pairs of nodes
      within a layer of the graph.
6 For weighted graphs, the distance is calculated with the Dijkstra algorithm
      using the inverse weight as the distance associated to the edge.
8 %% ibuild!
9 1
```

Code 8: Distance element prop update. The props_update section of the generator code in _Distance.gen.m updates the properties of the Measure element. ← Code 2

```
%% iprops_update!
3 %% iprop!
_{\rm 4} NAME (constant, string) is the name of the {\tt distance}
5 %%% idefault!
6 'Distance'
8 %% iprop!
9 DESCRIPTION (constant, string) is the description of the distance
10 %%% idefault!
_{	ext{11}} 'The distance of a graph is the shortest path between all pairs of nodes
       within a layer of the graph. For weighted graphs, the distance is
       calculated with the Dijkstra algorithm using the inverse weight as the
       distance associated to the edge.'
13 %% iprop!
14 TEMPLATE (parameter, item) is the template of the distance
15 %%% isettings!
16 'Distance'
18 %% iprop!
19 ID (data, string) is a few-letter code of the distance
20 %%% idefault!
<sup>21</sup> 'Distance ID'
23 %% iprop!
24 LABEL (metadata, string) is an extended label of the distance
25 %%% idefault!
26 'Distance label'
28 %% iprop!
29 NOTES (metadata, string) are some specific notes about the distance.
30 %%% idefault!
31 'Distance notes'
```

```
34 SHAPE (constant, scalar) is the measure shape __Measure.BINODAL__.
35 %%% idefault!
36 Measure.BINODAL
38 %% iprop!
39 SCOPE (constant, scalar) is the measure scope __Measure.UNILAYER___
40 %%% idefault!
41 Measure.UNILAYER
43 %% iprop!
44 PARAMETRICITY (constant, scalar) is the parametricity of the measure
       __Measure.NONPARAMETRIC__
45 %%% idefault!
46 Measure.NONPARAMETRIC
48 %% iprop!
49 COMPATIBLE_GRAPHS (constant, classlist) is the list of compatible graphs.
50 %%% idefault!
51 {'GraphBD' 'GraphBU' 'GraphWD' 'GraphWU' 'MultigraphBUD' 'MultigraphBUT' '
       MultiplexBD' 'MultiplexBU' 'MultiplexWD' 'MultiplexWU' 'MultiplexBUD' '
       MultiplexBUT' 'OrdMxBD' 'OrdMxBU' 'OrdMxWD' 'OrdMxWU'}
<sub>53</sub> %% iprop!
54 M (result, cell) is the distance
55 %%% icalculate!
56 g = m.get('G');
57 A = g.get('A');
59 distance = cell(g.get('LAYERNUMBER'), 1);
60 connectivity_type = g.get('CONNECTIVITY_TYPE', g.get('LAYERNUMBER'));
61 connectivity_type = diag(connectivity_type);
62 Aii_tmp = {};
63 for li = 1:1:g.get('LAYERNUMBER')
      Aii_tmp{li} = A{li, li};
64
65 end
66 for li = 1:1:g.get('LAYERNUMBER')
67
      Aii = Aii_tmp{li};
      connectivity_layer = connectivity_type(li);
68
      if connectivity_layer == Graph.WEIGHTED % weighted graphs
           distance(li) = {getWeightedCalculation(Aii)}; (1)
                                                                                        (1) and (2) call some callback functions
       else % binary (i.e., non-weighted) graphs
                                                                                        that are provided below in (3)
           distance(li) = {getBinaryCalculation(Aii)}; (2)
73
74
<sub>75</sub> end
77 value = distance;
78 % icalculate_callbacks! (3)
                                                                                        (3) This section contains the callback
79 function weighted_distance = getWeightedCalculation(A)
                                                                                        functions for the calculation of the
80
                                                                                        measure.
81 end
82 function binary_distance = getBinaryCalculation(A)
83
84 end
```

Code 9: Distance element tests. The tests section from the element generator _Distance.gen.m. ← Code 3

```
1 %% itests!
3 %% iexcluded_props!
4 [Distance.PFM]
6 %% itest!
7 %%%% iname!
8 GraphWU
9 %%% iprobability!
10 .01
11 %%% icode!
<sub>12</sub> B = [
      0
                 .2 .25 0
           . 1
13
14
      .125 0
                 0
                     0
                 0 .25 0
15
      .2 .5
     .125 10
                 0 0
                          0
16
           0
17
      ];
18
known_distance = {[
     0 5 5 4 Inf
      5 0 2 1 Inf
5 2 0 3 Inf
22
23
     4 1 3 0 Inf
     Inf Inf Inf Inf 0
25
     ]};
_{28} g = GraphWU('B', B);
30 m_outside_g = Distance('G', g);
31 assert(isequal(m_outside_g.get('M'), known_distance), ...
      [BRAPH2.STR ':Distance: BRAPH2.FAIL_TEST], ...
      [class(m_outside_g) ' is not being calculated correctly for ' class(g)
35 m_inside_g = g.get('MEASURE', 'Distance');
36 assert(isequal(m_inside_g.get('M'), known_distance), ...
      [BRAPH2.STR ':Distance: BRAPH2.FAIL_TEST], ...
      [class(m_inside_g) ' is not being calculated correctly for ' class(g)
38
39
```

Implementation of measure parameters (Triangles)

Now, you will implement the (nodal unilayer) measure Triangles, which depends on a parameter RULE, which we add as a property of category parameter. The parts of the code that are modified are highlighted.

Code 10: **Triangles element header.** The header section of the generator code in _Triangles.gen.m provides the general information about the Triangles element. ← Code 10

```
1 %% iheader!
2 Triangles < Measure (m, triangles) is the triangles.</pre>
4 %% idescription!
_{\scriptsize 5} The triangles are calculated as the number of neighbors of a node that are
      also neighbors of each other within a layer.
6 In weighted graphs, the triangles are calculated as geometric mean of the
      weights of the edges forming the triangle.
8 %% ibuild!
9 1
```

Code 11: Triangles element calculate. The calculate section of _Triangles.gen.m utilizes the rule property to select which algorithm it will use to calculate the Triangles element. ← Code 11

```
2 %% iprops_update!
4 %% iprop!
5 NAME (constant, string) is the name of the triangles
6 %%% idefault!
7 'Triangles'
10 DESCRIPTION (constant, string) is the description of the triangles
11 %%% idefault!
12 'The triangles are calculated as the number of neighbors of a node that are
      also neighbors of each other within a layer. In weighted graphs, the
       triangles are calculated as geometric mean of the weights of the edges
       forming the triangle.'
13
14 . . . .
15
16 %% iprop!
17 M (result, cell) is the triangles
18 %% icalculate!
19 g = m.get('G'); % graph from measure class
20 A = g.get('A'); % cell with adjacency matrix (for graph) or 2D-cell array (
      for multigraph, multiplex, etc.)
L = g.get('LAYERNUMBER');
23 triangles = cell(L, 1);
25 directionality_type = g.get('DIRECTIONALITY\_TYPE', L);
26 for li = 1:1:L
27 Aii = A{li, li};
```

```
if directionality_type == Graph.UNDIRECTED % undirected graphs
29
      triangles_layer = diag((Aii.^(1/3))^3) / 2;
30
      triangles_layer(isnan(triangles_layer)) = 0; % Should return zeros, not
31
      triangles(li) = {triangles_layer};
32
33
    else % directed graphs
34
      directed_triangles_rule = m.get('RULE'); (1)
      switch lower(directed_triangles_rule)
        case 'all' % all rule
37
          triangles\_layer = \frac{diag((Aii.^(1/3) + transpose(Aii).^(1/3))^3)}{2};
        case 'middleman' % middleman rule
          triangles_layer = diag(Aii.^(1/3) * ...
40
                transpose(Aii).^(1/3) * Aii.^(1/3));
41
        case 'in' % in rule
42
          triangles_layer = diag(transpose(Aii).^(1/3) * (Aii.^(1/3))^2);
43
        case 'out' % out rule
44
          triangles_layer = diag((Aii.^(1/3))^2 * transpose(Aii).^(1/3));
45
        otherwise % {'cycle'} % cycle rule
          triangles_layer = diag((Aii.^(1/3))^3);
47
48
      triangles_layer(isnan(triangles_layer)) = 0; % Should return zeros, not
      triangles(li) = {triangles_layer};
    end
52 end
54 value = triangles;
56 % iprops!
57
58 %% iprop!
_{59} RULE (parameter, option) is the rule to determine what is a triangle in a
       directed graph. (2)
60 %%% isettings!
61 {'all' 'middleman' 'in' 'out' 'cycle'} (3)
62 %%% idefault!
63 'cycle' (4)
```

1) gets the rule to calculate the measure Triangle. See below how this rule is defined in (2), (3), and (4)

- (2) defines a parameter to determine how the measure is calculated. This property must be of category parameter.
- (3) defines the available triangle rule options, which are case sensitive.
- (4) defines the default option, in this case 'cycle'.