CSCI 3104 Spring 2023
Instructors: Prof. Layer and Chandra Kanth Nagesh

Midterm 2 Standard 13 - Analyzing Code I: Independent nested loops

Due Date ..... April 28th
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Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **mssEc**

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1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 13 - Analyzing Code I: Independent nested loops

2.1 Problem 1

Problem 1. Analyze the runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

```
for i = 1, i <= n
    i = 3 * i
    for j = 1, j <= n
        j = 2 * j
        print()
```

Answer. Here is the runtime:

- At line 1, we have our outer loop. This line takes 1 step for initialization and 1 step every loop for comparison.
- At line 2, we have arithmetic and assignment which takes 2 steps total every loop.
- At line 3, we have our inner loop which also takes 1 step for initialization and 1 step every loop for comparison.
- At line 4 we also have arithmetic and assignment which takes 2 steps every loop.
- Finally at line 5, we have a print statements that takes 1 step every loop.

Now we sum the non-looping runtime with the inner and outer loop runtimes.

This gives us the following:

$$1 + \sum_{i=1}^{(\log_3(n-1))+1} (1 + 2 + 1 + \sum_{j=1}^{(\log_2(n-1))+1} (1 + 2 + 1))$$

Which equals:

$$1 + \sum_{i=1}^{(\log_3(n-1))+1} (4 + \frac{4\log(2(n-1))}{\log(2)})$$

Which equals:

$$\frac{4(\log(2(n-1))+\log(2))\log(3(n-1))}{\log(2)\log(3)}$$

So, $T(n) \in \Theta(n \log n)$.

□