Midterm 2 Standard 22 - Dynamic Programming: Write down recurrences

Due Date .... April 28th
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Quiz Code (enter in Canvas to get access to the LaTeX template) .... **hNBDh**

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1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 22 - Dynamic Programming: Write down recurrences

2.1 Problem 1

Problem 1. Suppose we have an m-letter alphabet $\Sigma = \{0, 1, \ldots, m-1\}$. Let W_n be the set of strings $\omega \in \Sigma^n$ such that ω does not have 101 as a substring. Let $f_n := |W_n|$. Write down an explicit recurrence for f_n, including the base cases. Clearly justify each recursive term.

Answer.

$$f_n = \begin{cases} 1 & : n = 0, \\ m & : n = 1, \\ m^2 & : n = 2, \\ (m-1)f_{n-1} + (m-1)f_{n-2} + (m-1)f_{n-3} & : \text{else.} \end{cases}$$

For base case of 1, when we have $n = 0$, there is only one string to consider, and it is not 101, so we have size 1. When $n = 1$, we only have one instance of $\Sigma = \{0, 1, \ldots, m-1\}$ to consider, which does not have 101, so we have m. When $n = 2$, we only have 2 instances of $\Sigma = \{0, 1, \ldots, m-1\}$ to consider, none of which has 101, so we have m^2. Finally, when we have $n > 2$, we have n of $\Sigma = \{0, 1, \ldots, m-1\}$ to consider, so we subtract the 101 out of however many strings we are considering, so we get $(m-1)f_{n-1} + (m-1)f_{n-2} + (m-1)f_{n-3}$.

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