CSCI 3104 Spring 2023
Instructors: Prof. Layer and Chandra Kanth Nagesh

Midterm 1 Standard 10 - Ford Fulkerson

Due Date .... March 10
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Quiz Code (enter in Canvas to get access to the LaTeX template) .... **JGHSS**

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1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 10 - FordFulkerson

2.1 Problem 1

Problem 1. Consider a flow F on the network $G = (V, E, c)$, where V is the set of vertices, E is the set of edges, and c is the capacity function for each edge. Let $s \in V$ be the source and $t \in V$ be the sink. If there is no $s - t$ path in G_F, why must there be an $s - t$ cut in G whose capacity equals the value of F?

Answer. If there is no $s-t$ path in G_F there must be an $s-t$ cut in G whose capacity equals the value of F because with no $s \to t$ path, we can assume that all source nodes are in one cut and all sink nodes in the other. With this information, we know there are no more vertices in which to push positive flow. The sum of the capacities of the cut are those of the edges which we cannot push any more flow through from the cut that contains s into the cut that contains t. Since these edges are full and at capacity, that means they hold the flow amount F as their sum. Therefore, there must be an $s \to t$ cut that's capacity equals the value of F.

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