CSCI 3104 Spring 2023
Instructor: Prof. Ryan Layer and Chandra Kanth Nagesh

Quiz 1 Standard 1

Due Date ..... TODO
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Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **PLQJR**

Contents

Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
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Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
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- I have neither copied nor provided others solutions they can copy.

Agreed (I agree to the above, Blake Raphael). □

1 Standard 1: Proof by Induction

Problem 1. Consider the sequence defined by:

$$T_n = \begin{cases} 0 & : n = 0 \\ 1 & : n = 1 \\ 2T_{n-1} + 1 & : n \geq 2 \end{cases}$$

Prove by induction that for every $n \geq 0$, we have:

$$\sum_{i=0}^{n} T_i = T_{n+1} - (n+1).$$

Proof. **Base Case:**

$$\begin{cases} 0 & : n = 0 \\ 1 & : n = 1 \\ 3 & : n = 2 \end{cases}$$

Inductive Hypothesis:
Assume that for all $k \geq 2$, $\sum_{i=0}^{n} T_k = T_{k+1} - (k+1)$ holds true.

Inductive Step:
Show that $\sum_{i=0}^{n} T_{k+1} = T_{k+2} - ((k+1)+1)$.

$$\begin{aligned} T_{k+1} &= T_{k+1} - ((k+1)+1) \\ 2T_k + 1 &= T_{k+2} - ((k+1)+1) \\ 2T_k &= T_{k+2} - (k+1) \\ 2T_k &= 2T_{k+1} + 1 - (k+1) \\ T_k &= T_{k+1} - (k+1) \end{aligned}$$

So $\sum_{i=0}^{n} T_{k+1} = T_{k+2} - ((k+1)+1)$ by our Inductive Hypothesis.

□