CSCI 3104 Spring 2023
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Quiz 19 Standard 19 – (Divide & Conquer) MergeSort

Due Date .... TODO
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Quiz Code (enter in Canvas to get access to the LaTeX template) .... **IKJOP**

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Instructions

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Honor Code (Make Sure to Virtually Sign)

Problem HC.
- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (I agree to the above, Blake Raphael). □

19 Standard 19 – (Divide & Conquer) MergeSort (4 points)

Problem 19. Consider a modified version of MergeSort, which divides the array into 6 (as equally-sized as possible) partitions (**Recall:** Standard MergeSort partitions the array into 2-equal sized sets). This variation of MergeSort is called the 'k-way' MergeSort, where $k = 6$. Instead of dividing the input array into two subarrays, '6-way' MergeSort divides the array into **six** subarrays, sorts them recursively, and then call a '6-way' Merge which combines six sorted arrays into one sorted array. (4 points)

Do the following **three** parts of the question.

1. **Write down** the number of comparisons that occur in the '6-way' Merge function, in the worst case. Assume we have six 'M-element' arrays. (1 point)
   (**Note:** In the worst case, a '2-way' Merge function performs $2M - 1$ comparisons, where we assume to have two 'M-element' arrays. **Donot** give a Big-O notation.)

   Answer. The 6-way Merge function would perform $6M - 1$ comparisons at the worst case. □

2. Now, with the the number of comparisons from part (1) **write down** the recurrence relation for the runtime of this modified version of '6-way' MergeSort. **Justify your recurrence relation in 1–2 sentences.** (1 point)

   Answer.

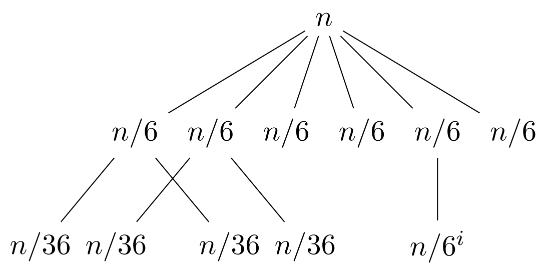
$$T(n) = \begin{cases} \Theta(1) & : n \leq 1, \\ 6T(n/6) + \Theta(n) & : n > 1. \end{cases}$$

   This is the recurrence relation because with each recursion, we break the array down 6 more times, which is $6 * n/6$.

   □

3. Solve your recurrence relation from part (1), by any method you choose. **Show your work.** Find a function $f(n)$ such that the runtime $T(n)$ is $T(n) = O(f(n))$. (2 points)
   (**Further:** You need not show this work, but would a '6-way' MergeSort sort elements faster than '2-way' MergeSort.)

   Answer. Here we will use the tree method to solve the recurrence relation.


n
$n/6$ $n/6$ $n/6$ $n/6$ $n/6$ $n/6$
$n/36$ $n/36$ $n/36$ $n/36$ $n/6^i$

So we see that the non-recursive work at each level is $(n/6^i) * 6^i$.

We also hit our base case at $n/6^k \leq 1$ or $n \leq 6^k$. Solving for k, we get $k \leq \log_6(n)$.

So the total work performed by our 6-way mergesort is $\displaystyle\sum_{i=0}^{\log_6(n)} n = n \cdot \log_6(n)$.

So we see that $T(n) \in \Theta(n \log(n))$.

A 6-way mergesort would not solve any faster than a 2-way mergesort. They have the same Θ runtimes. $\square$