CSCI 3104 Spring 2023
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Quiz 23 Standard 23 – DP – Solve using recurrence

Due Date ..... TODO
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Quiz Code (enter in Canvas to get access to the LaTeX template) ..... **JCVBA**

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Instructions

- You may either type your work using this template, or you may handwrite your work and embed it as an image in this template. **If you choose to handwrite your work, the image must be legible, and oriented so that we do not have to rotate our screens to grade your work.** We have included some helpful LaTeX commands for including and rotating images commented out near the end of the LaTeX template.
- You should submit your work through the **class Gradescope page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section). Failure to do so will result in your assignment not being graded.

Honor Code (Make Sure to Virtually Sign)

Problem HC.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (I agree to the above, Blake Raphael). □

23 Standard 23: Dynamic Programming: Solve using recurrence

Problem 23. The Subset-Sum problem is defined as follows.

- Input: We are given n items with positive weights $w_1, \ldots, w_n > 0$, as well as a target threshold $W > 0$.
- Decide: Is there a subsequence $w_{i_1}, \ldots, w_{i_k}$ such that:

$$\sum_{j=1}^{k} w_{i_j} = W.$$

That is, can we select a subsequence of items whose combined weights add to W?

For example, consider the input array $A = [4, 15, 8, 16, 23, 42]$. If $w = 31$ then the answer is "TRUE" since there is a subsequence $[15, 16]$ where the sum is equal to $15 + 16 = 31$. However, if $w = 13$ then the answer is "FALSE" since no subsequence of A has sum equal 13.

The Subset-Sum problem satisfies the following recurrence. For $0 \leq i \leq n$, and $0 \leq w \leq W$, define $T[i, w]$ be to TRUE if and only if there exists a subsequence of the first i elements $[w_1, \ldots, w_i]$ that sum to w. Then we have:

$$T[i,w] = \begin{cases} TRUE & w = 0 \\ FALSE & i = 0 \text{ and } w > 0 \\ T[i-1, w] & i > 0 \text{ and } w_i > w \\ T[i-1, w] \text{ OR } T[i-1, w-w_i] & i > 0 \text{ and } w_i \leq w \end{cases}$$

(Note that in the final case, "OR" is the Boolean OR operation.)

Consider $A = [3, 4, 2, 1]$ and $W = 5$. Design and fill in a lookup table for this problem. Clearly indicate whether a solution exists, and which cell of the table tells you this.

Answer. **Here is some LaTeX code for the start of a table that you may find useful:**

$w \rightarrow$	0	1	2	3	4	5
A[0]	T	F	F	T	F	F
A[1]	T	F	F	T	T	F
A[2]	T	F	T	T	T	T
A[3]	T	T	T	T	T	T

A solution does exist, and the bottom right cell of our table tells us this. It shows us that for the weights within A, there is a sub-sequence of A whose combined weights w add to W

□