

Quiz 13 Standard 13 - Analyzing Code I

Due Date TODO
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Quiz Code (enter in Canvas to get access to the LaTeX template) **VGHBN**

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Instructions

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Honor Code (Make Sure to Virtually Sign)

Problem HC. • My submission is in my own words and reflects my understanding of the material.

- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (I agree to the above, Blake Raphael).

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13 Standard 13: Analyzing Code I – Independent Nested Loops (4 points)

Problem 13. Analyze the *worst-case* runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) = \Theta(f(n))$). Specifically, we are looking for these **two** objectives: (a) number of operations and run time of the inner loop (b) number of operations and run time of the outer loop.

Algorithm 1 Nested Algorithm 1

```
1: procedure FOO2(Integer  $n$ )
2:   if  $n \leq 10$  then
3:      $n \leftarrow n * 100$ 
4:   for  $i \leftarrow 1; i \leq n^2; i \leftarrow i + 2$  do
5:     print "from outer"
6:     for  $j \leftarrow 1; j \leq n; j \leftarrow j + 2$  do
7:       print "from inner"
```

Answer. • At lines 2 and 3 we have an if-then statement. This takes 3 steps total: 1 for comparison and 2 for arithmetic and assignment.

- At line 4 we have a for-loop which takes 1 step for initialization and every iteration takes 1 step for comparison, 2 steps for arithmetic and assignment, 1 step for printing, 1 step for the inner for-loop initialization, and the sum of the whole inner loop.
- At line 6 we have the inner for-loop which takes 1 step for initialization (covered in the outer loop), 1 step for comparison, 2 steps for arithmetic and assignment, and finally 1 step for print.

Now we need to sum the non-looping part of the code + the outer-loop runtime which sums with the inner-loop runtime.

So we get the following:

$$3 + 1 + \sum_{i=1}^{((n^2-1)/2)+1} (1 + 2 + 1 + 1 + \sum_{j=1}^{((n-1)/2)+1} (1 + 2 + 1))$$

Which equals:

$$4 + \sum_{i=1}^{((n^2-1)/2)+1} (5 + 2(n + 1))$$

Which equals:

$$4 + \frac{1}{2}(2n + 7)(n^2 + 1)$$

$$\text{So, } T(n) \in \Theta(n^3)$$

□