

Homework 16

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1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to \LaTeX .
- You should submit your work through the **class Gradescope page** only (linked from Canvas). Please submit one PDF file, compiled using this \LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (I agree to the above, Blake Raphael).

□

3 Standard 16 - Solving Recurrences I: Unrolling

Problem 1. For each of the following recurrences, solve them using the unrolling method (i.e. find a suitable function $f(n)$ such that $T(n) \in \Theta(f(n))$). **Note:** Show all the work.

3.1 Problem 1a (2 points)

a.

$$T(n) = \begin{cases} 2 & : n < 2, \\ 2T(n/2) + 2 & : n \geq 2. \end{cases}$$

Answer. We have a base case when $\frac{n}{2^k} < 2$

This gives us $\frac{n}{2^k} = 1$

So $n = 2^k$ and $k = \log_2(n)$

Unroll $T(n)$

$$- 2T\left(\frac{n}{2}\right) + 2$$

$$- 2(2T\left(\frac{n}{4}\right) + 2) + 2$$

$$- 2^2T\left(\frac{n}{2^i}\right) + 2^1 \cdot 2 + 2^0 \cdot 2 = 2^i \cdot 2$$

So

$$T(n) = 1 \cdot 2^{\log_2(n)} + \sum_{i=0}^{\log_2(n)} 2^i \cdot 2 \quad (1)$$

$$= 2^{\log_2(n)} + 4n - 2 \quad (2)$$

$$= 5n - 2 \quad (3)$$

$$(4)$$

So $T(n) \in \Theta(n)$

□

3.2 Problem 1b (2 points)

b.

$$T(n) = \begin{cases} 4 & : n < 2, \\ 7T(n-4) + 4n & : n \geq 2. \end{cases}$$

Answer. Base case when $n - 4k < 2 \rightarrow n - 4k = 1$

Unroll $T(n)$:

$$- 7T(n-4) + 4n$$

$$- 7(7T(n-8) + 4n) + 4n$$

$$- 7^2T(n-4k) + 7^1 \cdot 4n + 7^0 \cdot 4n \rightarrow 7^i \cdot 4n$$

$$\text{So } T(n) = \text{cost of base case} \cdot \text{number of base case} + \sum_{i=0}^{\frac{n-1}{4}} 7^i \cdot 4n$$

$$T(n) = 1 \cdot 7^{\frac{n-1}{4}} + \sum_{i=0}^{\frac{n-1}{4}} 7^i \cdot 4n \quad (5)$$

$$= 7^{\frac{n-1}{4}} + \frac{2}{3}(7^{(n+3)/4} - 1)n \quad (6)$$

$$(7)$$

$$\text{So } T(n) \in \Theta(n7^{\frac{n}{4}})$$

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