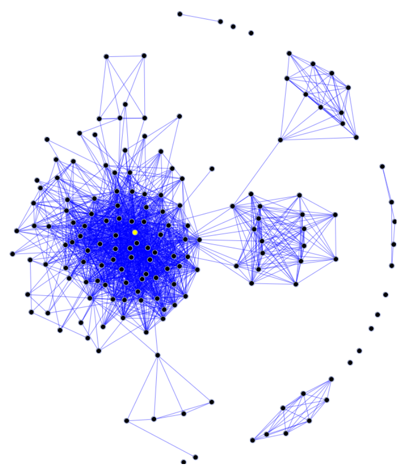
CSCI3104
Spring 2023

Recitation 2: Graph traversal: BFS and DFS

This week we will discuss "graph search" with BFS and DFS. Question: Why would we be interested in exploring graphs? Before proceeding with the formal definitions, and then some problems, here is an illustration of a graph...



...in which each person is represented by a dot called a node and the friendship relationship is represented by a line called an edge (Source: **Wikipedia**, Social Graph). Graphs encode relationships between objects, they are especially handy when we want to study the global properties of a large set of objects. For example, a social network is conceptualized as a graph, and the properties of this graph informs us about the properties of an entire social network which we could not easily understand by looking at individual social agents / nodes. (This becomes even more interesting if we include, e.g., weights on the edges, signifying the strength of the social bonds between agents.)

Other applications: Decision engineering and social systems [1], economics wiki, neuroscience [2], web crawling on Google...

Now, we proceed to recall some concepts which will be useful in exploring graphs.

A **(simple, undirected) graph** is defined as a pair of sets $G = (V, E)$ where V is any set (commonly the set $[n] = \{1, 2, \dots, n\}$) and $E \subseteq \binom{V}{2}$. V is called the set of vertices, and E is called the set of edges. The edges of an undirected graph can be viewed as symmetric relationships between pairs of vertices (accounts being friends on Facebook, sports teams playing each other in a tournament, etc).

A **directed graph (digraph)** is defined as a pair of sets $G = (V, E)$ where V is any set (commonly the set $[n] = \{1, 2, \dots, n\}$) and $E \subseteq (V \times V)$. The edges of a undirected graph can be viewed as asymmetric relationships between pairs of vertices (an account following another on Twitter, but is not followed in return).

Let $G = (V, E)$ be a graph. For $v, w \in V$ we say that $v \sim w$ if v and w are connected by an edge, i.e. $(v, w) \in E$. For an edge $e \in E$ and a vertex $v \in V$, we say that e is **incident** on v if v is one of the endpoints of e. The **degree** $\deg(v)$ of a vertex $v \in V$ is the number of edges incident on v.

The **undirected path** of length n, P_n, is a graph whose vertex set is

$$V(P_n) = [n + 1] = \{1, 2, \dots, n, (n + 1)\}$$

and whose edges are

$$E(P_n) = \{(i, i + 1) \mid 1 \leq i \leq n\}$$

The **directed path** of length $n, \vec{P}_n$, is a graph whose vertex set is

$$V\left(\vec{P}_n\right) = [n+1] = \{1, 2, \ldots, n, (n+1)\}$$

and whose edges are

$$E\left(\overrightarrow{P_n}\right) = \{i \to i+1 \mid 1 \leq i \leq n\}$$

The **directed cycle** of length n, C_n, is a graph whose vertex set is

$$V\left(C_n\right) = [n] = \{1, 2, \ldots, n\}$$

and whose edges are

$$E\left(C_n\right) = \{(i, i+1 \mod n) \mid 1 \leq i \leq n\}$$

The **undirected cycle** of length $n, \vec{C}_n$, is a graph whose vertex set is

$$V\left(\vec{C}_n\right) = [n] = \{1, 2, \ldots, n\}$$

and whose edges are

$$E\left(\vec{C}_n\right) = \{i \to i+1 \mod n \mid 1 \leq i \leq n\}$$

A graph is **connected** if every node is reachable from every other node. A graph is **complete** if every node has a direct edge to every other node. A graph is **acyclic** if it does not contain any cycles. A **tree** is a connected acyclic graph. A **spanning tree** of a simple graph G is a subgraph of G that is a tree containing every vertex of G.

Problems

Theorem: A simple graph with n vertices is connected if and only if it has a spanning tree.

Question 0, 1. We will solve two problems regarding implementations of BFS and DFS.

Question 2. We will solve a problem regarding cycle detection via BFS / DFS (related to HW 2).

Question 3. We will concentrate on a theoretical result regarding the equivalency between BFS and DFS trees for a given input type.

References

[1] Hisashi Ohtsuki, Christoph Hauert, Erez Lieberman, and Martin A. Nowak. A simple rule for the evolution of cooperation on graphs and social networks. *Nature*, 441(7092):502–505, May 2006.

[2] Farzad V. Farahani, Waldemar Karwowski, and Nichole R. Lighthall. Application of graph theory for identifying connectivity patterns in human brain networks: A systematic review. *Frontiers in Neuroscience*, 13, June 2019.