# Kalman Filter for PMU

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# Introduction

- This document is to understand the basics of KF and how it is applied to the BBM PMU encoder data
- It will not enter into deep details of KF theory or scripts, just provide an insight on how the KF is used in this context
- Basically, a linear KF such as in this case is very similar to a recursive least-squares filter.

# KF Summary

- The KF is a model based algorithm used to filter the noise out from some data
- We require a model of the real world described by:

$$\dot{\boldsymbol{x}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{w}$$

- And a set of state dependent measurements given by:

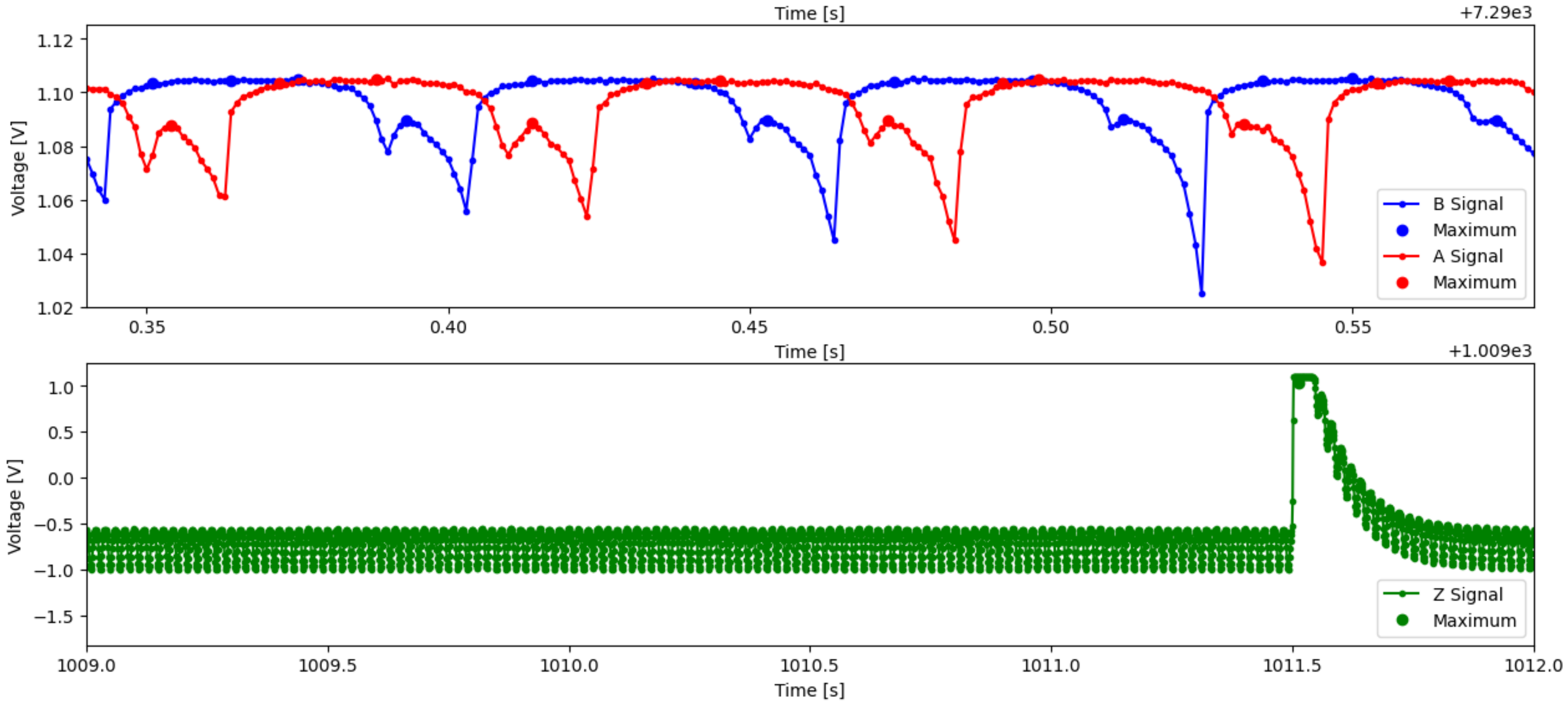
$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$$

- Where x is the state vector, F and H the dynamics and measurements matrices, w and v some white noises.

# PMU Encoder Data

This is how data looks like in the stable rotation.

We have to figure out a good observable that we can use as a state and another to be used as measurements.



# State and Measurements Vectors

- We are interested in the position angle and the frequency of the rotor, so a suitable choice for the state vector is the angle itself.

$$\boldsymbol{x} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix}, \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dddot{\theta} \end{bmatrix}$$

- The voltage is not a good measurement since it is not a linear function of the state and also we would need a very accurate model of the encoder data, which is not great and it is computationally very expensive.

# State and Measurements Vectors (cont.)

- We can think about using the angle as a measurement too: if we are able to determine when the voltage signal reaches its maximum we know the angular position of the rotor (using a counter that is synchronized with encoder Z)
- In this way we would have

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$$

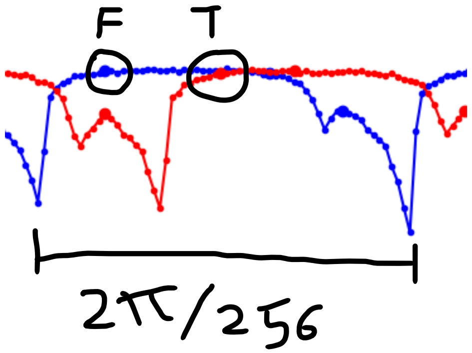
$$\boldsymbol{z} = [\mathbf{1} \quad \mathbf{0} \quad \mathbf{0}] \begin{bmatrix} \theta \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} + \boldsymbol{v} = \theta + v$$

- Now we have to define the noises

# Noise Vectors

$\dot{\boldsymbol{x}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{w}$ $\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$

- We have to define a process noise w and a measurements noise v
- w can be considered as a parameter and «tuned» appropriately
- For v we can proceed in this way:
  - We have 256 holes, so every hole covers approximately $2\pi\ /256$ radians.
  - The maximum angular error we commit if we consider point F as maximum insted of the actual maximum T is half that value, so $2\pi\ /512$.
  - We take $2\pi/512$ as the 3σ of the noise distribution, so that we have: $v\ =\ 2\pi\ /512/3$



# Initialization Parameters

- We have to initialize the filter
- We can say that the initial error on the angular position is 2π/512 as we discussed in the previous slide, this is a kind of worst case scenario
- We initialize the angular velocity as 2 π plus 0.006*2 π, in this way we obtain the mean angular velocity retrieved from the previously analyzed encoder data.
- If the supposed initialized state is not «too far» from the true one, the KF will undergo a first transitory phase and ultimately converge to a steady state very quickly
- If we set the initial angle and acceleration to zero we have:

$$\begin{bmatrix} \theta_0 \\ \dot{\theta}_0 \\ \ddot{\theta}_0 \end{bmatrix} = \begin{bmatrix} 0 + \dfrac{2\pi}{512} \\ 2\pi + 0.006 \times 2\pi \\ 0 \end{bmatrix}$$

# KF Workflow

- The goal of the KF is to determine the best frequency to propagate the angle between two consecutive maximums.
- We wait until we have a maximum on encoder Z, then we switch on the filter and start computing.
- Between two maximums the KF predicts the state using the last stored frequency (linear angular motion)
- Everytime a maximum is recorded we have the measurement of the angular position (see next slides), the KF filters the noise, gives the correction to be applied on the angle and updates the angular frequency.

# A couple more words on the KF Workflow…

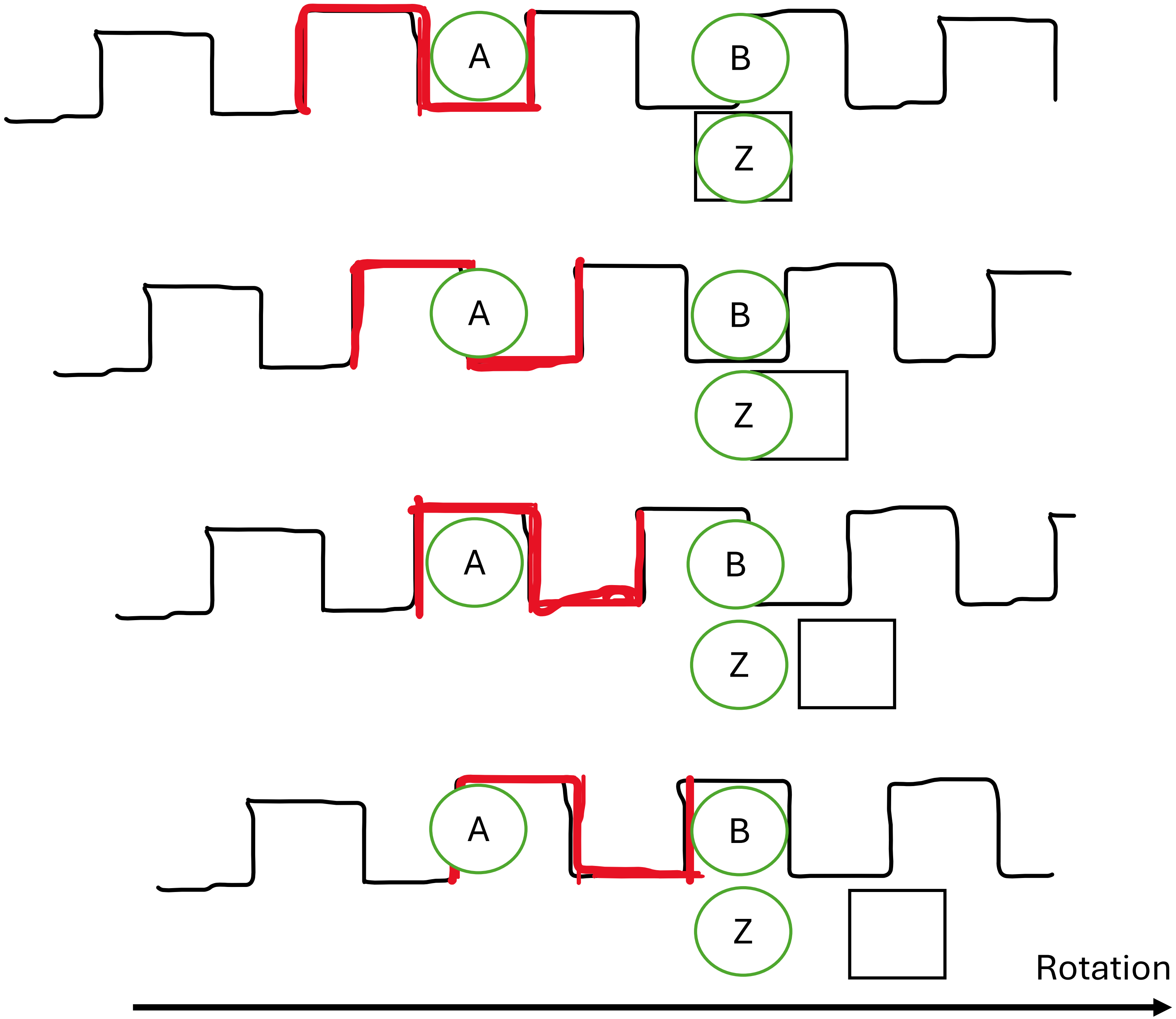
- Next slide is how encoders work and read the signal
- Then we see how we pass from voltage to angle
- Finally we see how we apply KF

t = 0 u
Theta = 0
Signal A, Z

t = 1 u
Theta = 2π/512 * 1
Signal B

t = 2 u
Theta = 2π/512 * 2
No signal

t = 3 u
Theta = 2π/512 * 3
No signal



So the signal pattern is the following:

| | T = 0 | T = 1 | T = 2 | T = 3 | T = 4 | T =5 | ... | T = 511 | T = 512 |
|---|---|---|---|---|---|---|---|---|---|
| A | ✓ | **closing** | X | **opening** | ✓ | **closing** | ... | **opening** | ✓ |
| B | **opening** | ✓ | **closing** | X | **opening** | ✓ | ... | X | **opening** |
| Z | ✓ | **closing** | X | X | X | X | ... | **opening** | ✓ |
| Theta | 0 | $\frac{2\pi}{512} \times 1$ | ? | ? | $\frac{2\pi}{512} \times 4$ | $\frac{2\pi}{512} \times 5$ | ... | ? | $2\pi$ |

We ‘’know’’ (**mind the quotes**) the angular position when we get the signal from the encoders, the goal is to find the ‘’right’’ angular velocity to propagate the position between measurements

**Also, this estimation of theta is not really correct since it depends on how good we are at estimating the ‘’maximum’’ of the signal**, that is when the angle is equal to the values in the table (see slide 7).

So the theta row in the table is our measurement, that will correspond to the true angular position plus some noise, which will be related to the amplitude of every signal peak and/or some other things

The workflow of the Kalman Filter is therefore the following:

(We process one raw signal (A, B, Z) at a time, since we are on-board)

- We initialize the state, setting the initial angle to zero, the initial angular velocity to $2\pi$ rad/s and the initial acceleration to zero.
- We look at the signal from Z, if the signal is maximum, we proceed and start the filtering, otherwise we wait. In this way we lose a bit of information (from the start to the first Z peak) but we remove the uncertainty on the initial position, since we start exactly at our reference zero.
- We go forward in time using the initial angular velocity to propagate the position until we find a peak in one of the encoder signals
- When we encounter a maximum signal, we apply the KF to retrieve the updates to the state.

In this way we can have four independent ways of retrieving the angular velocity of the rotor

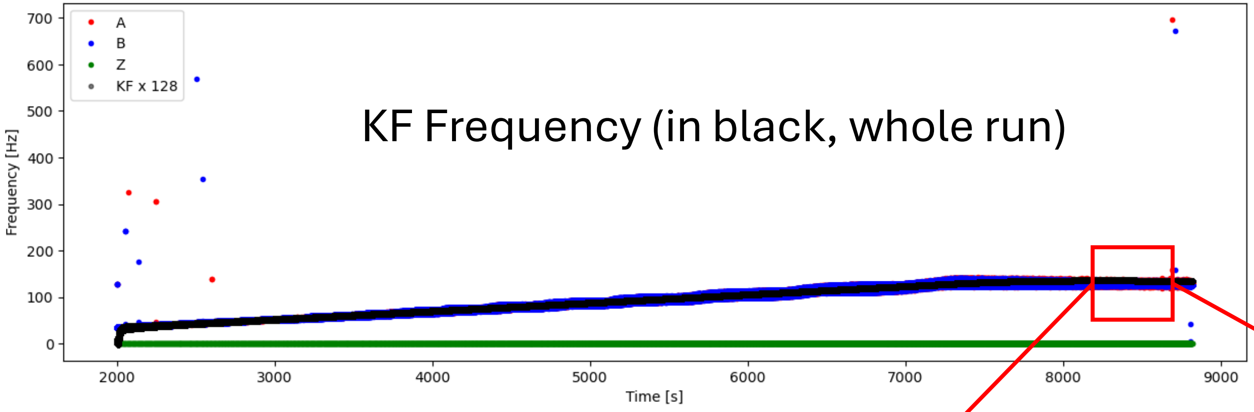
- We can measure the time interval between two Z peaks, retrieve a frequency and use that to propagate the angle until the next peak ($f_Z$)
- The same thing can be applied to A and B ($f_A, f_B$)
- Then we have the angular velocity given by the KF converted to frequency ($f_{KF}$)

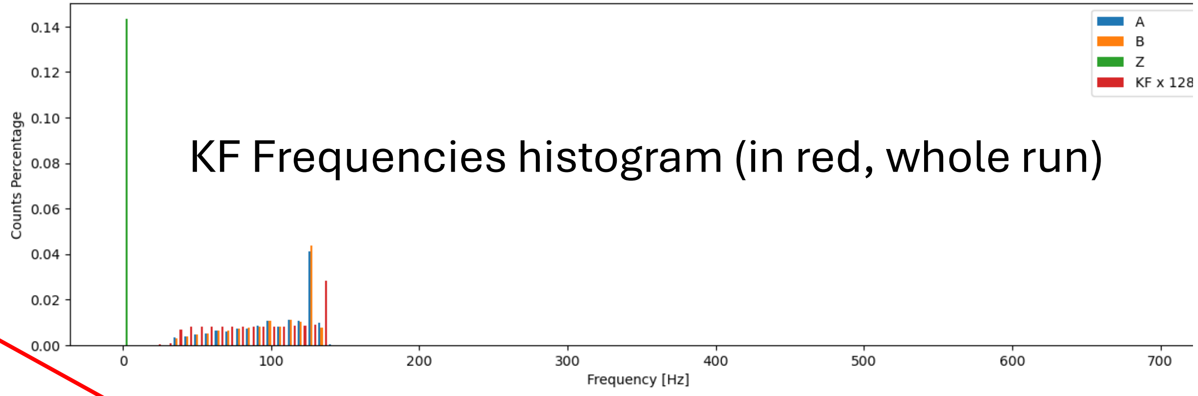
When we compare the angle retrieved from measurement to the angle obtained with the propagation using those frequencies, we might see that the angle retrieved from $f_{A,B,Z}$ is closer to the measurement than the angle obtained through $f_{KF}$, but **the measurement angle is not the true position of the rotor**!

# KF Issues

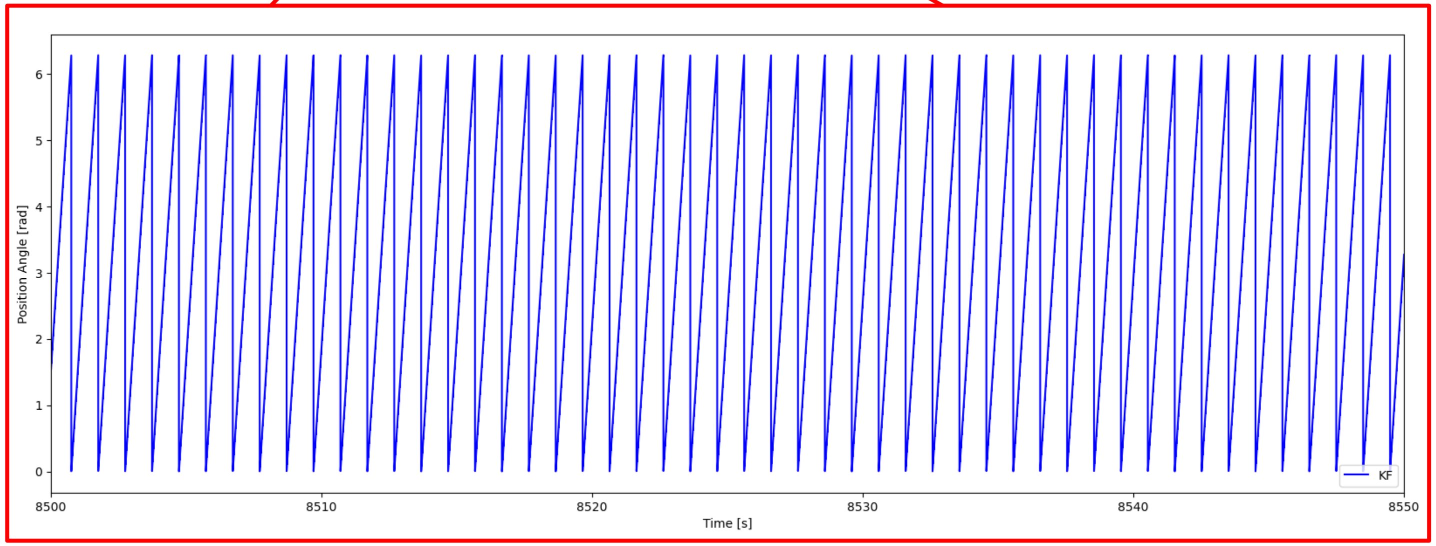
- On real data, KF runs smoothly and gives reasonable results, however the accuracy must be assessed using simulated data.
- The precision of the KF depends on the goodness of the algorithm used to retrieve the maximums in the encoder signals.
- KF can be used also for non-constant frequencies, for example in the acceleration phase of the rotor.
- In this case I managed to obtain a good estimation of the frequency but not of the angle, but that once again depends on the maximum-retrieving algorithm not on the KF.

# Some KF Images

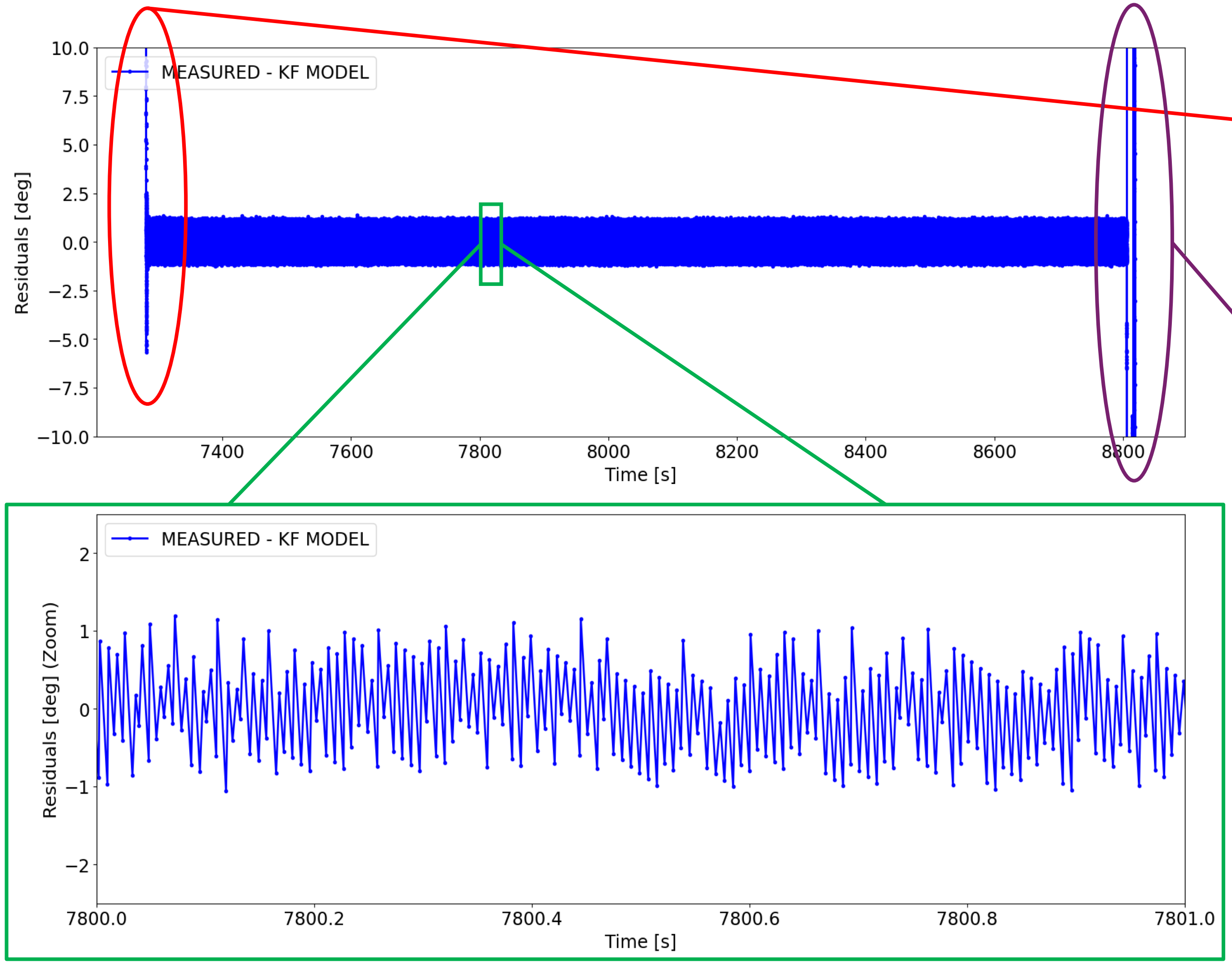



KF angular
Position
(stable rotation)



# Some KF Images (cont.)




Transitory phase of the KF due to the initialization of the frequency to zero instead of 2π.

Spin-down phase of the PMU.