1 Introduction

Clear demonstration that the proton has a finite size (unlike the electron?) had to wait for the availability of electron beams with energy up to 500 MeV. In his extensive, 1956, review Robert Hofstadter presented data for the angular distribution of the elastically scattered electrons up to 140 degrees which differ from the expectation for a point charge and magnetic distribution by a factor of 5 at the largest $q^2 = 14 \times 10^{-26} cm^{-2} = 14 fm^{-2} = (3.7 fm^{-1})^2$ or $\approx 0.73 GeV^2$. A proton radius of 0.80 fm was derived from these data. We take note that the JLab data from double polarization experiments completed in 2000 with electrons of 5 GeV differed from what was then believed to be the gold standard proton form factors by a similar factor; the standard data base at the time was entirely defined by cross section measurements, and suggested that both the electric and magnetic form factors behaved approximately like the dipole form factor $G_D = (1 + \frac{Q^2}{0.71})^{-2}$, with the momentum transfer squared Q^2 is in units of GeV^2. The series of experiments started in 1998 in Hall A at JLab showed that the electric and magnetic form factor ratio decreased linearly with Q^2, reaching a factor 1/5 at the highest Q^2 investigated so far (5.6 GeV^2). And almost 60 year after this work we are discussing whether the proton radius is $\tilde{0}.875$ or 0.76 fm.

1.1 Dirac, Pauli and Sachs Form Factors

The lowest order approximation for electron nucleon scattering is the single virtual photon exchange model, or Born term. The approximation is expected to be valid because of the weak electro-magnetic coupling of the photon with the charge and magnetic moment of the nucleon. The amplitude for the process is the product of the four-component leptonic and hadronic currents, ℓ_ν and $\mathcal{J}_\mu$, and can be written as:

$$\begin{aligned} i\mathcal{M} &= \frac{-i}{q_\mu^2} \ell_\mu \mathcal{J}^\mu \\ &= \frac{-ig_{\mu\nu}}{q_\mu^2} \left[ie\bar{u}(k')\gamma^\nu u(k)\right] \left[-ie\bar{v}(p')\Gamma^\mu(p',p)v(p)\right] \quad (1) \end{aligned}$$

where k, k', p, p' are the the four-momenta of the incidenta and scattered, electron and proton, respectively, Γ^μ contains all information of the nucleon structure, and $g_{\mu\nu}$ is the metric tensor. To insure relativistic invariance of the amplitude $\mathcal{M}$, Γ^μ can only contain p, p' and γ^μ, besides numbers, masses and Q^2.

The most general form for the hadronic current for the spin $\frac{1}{2}$-nucleon, satisfying relativistic invariance and current conservation, and including an internal structure is:

$$\mathcal{J}^\mu_{hadronic} = ie\overline{\nu}(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_p} \kappa_j F_2(Q^2)\right] \nu(p) \quad (2)$$

where j=p,n for proton and neutrons, srespectively, and $Q^2 = \boldsymbol{q}^{\,2} - \omega^2 = -q_\mu^2$, is the negative of the square of the invariant mass q_μ^2 of the virtual photon exchanged in the one-photon approximation of eN scattering. The Dirac and Pauli form factors, $F_1(Q^2)$ and $F_2(Q^2)$, the Dirac and Pauli form factors, are the only strucutre functions allowed in the Born term by relativistic invariance. $\kappa_p = \mu_p - 1$ and $\kappa_n = \mu_n$ are the anomalous magnetic moments of the proton and neutron, in units of the nuclear magneton, $\mu_N = \frac{e\hbar}{2M}$with M the nucleon mass; their values are κ_p =1.7928 and κ_n=-1.9130, . In the static limit, $Q^2 = 0$, $F_{1p} = 1$, $F_{2p} = \kappa_p$ and $F_{1n} = 0$ and $F_{2n} = \kappa_n$, for the proton and neutron, respectively.

The Lab frame differential cross section for detection of the electron in elastic ep or en scattering is then:

$$\begin{aligned} \tfrac{d\sigma}{d\Omega_e} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E_e}{E_{beam}} \times \quad (3) \\ \left\{F_1^2(Q^2) + \tau\left[F_2^2(Q^2) + 2(F_1(Q^2) + F_2(Q^2))^2 \tan^2\frac{\theta_e}{2}\right]\right\} \end{aligned}$$

with $\tau = Q^2/4M_p^2$ and $(\frac{d\sigma}{d\Omega})_{Mott}$ is the Mott cross section, including the recoil factor $\frac{E_e}{E_{beam}} = (1 + \frac{2E_{beam}}{m}\sin^2\frac{\theta_e}{2})^{-1}$, given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2}{4E_{beam}^2 \sin^4\frac{\theta}{2}} \frac{E_e}{E_{beam}} \cos^2\frac{\theta}{2} \quad (4)$$

Experimental cross section data are most easily analyzed in terms of another set of form factors, the Sachs form factors G_{Ep}, G_{En} and G_{Mp}, G_{Mn}:

$$\begin{aligned} G_{E_{p,n}} &= F_{1_{p,n}} - \tau F_{2_{p,n}} \quad (5) \\ G_{M_{p,n}} &= F_{1_{p,n}} + F_{2_{p,n}}, \quad (6) \end{aligned}$$

The scattering cross section Eq. 4 can then be written in a much simpler form, without interference term, leading to a separation method for $G_{E_p}^2$ and $G_{M_p}^2$ known as Rosenbluth (or Longitudinal-Transverse) method, as will be seen below. Now the cross section is:

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \left(\frac{\alpha}{2E\sin^2(\frac{\theta_e}{2})}\right)^2 \frac{E_e}{E_{beam}} \\ &\times \left(\frac{\cot^2(\frac{\theta_e}{2})}{1+\tau}\left[G_{E_p}^2 + \tau {G_{M_p}}^2\right] + 2\tau {G_{M_p}}^2\right) \quad (7) \end{aligned}$$

where r_e is the electron Compton wave length. G_{E_p}, G_{M_p}, G_{E_n} and G_{M_n} have the static values of the charge and magnetic moments, of the proton and neutron, respectively:

$$\begin{aligned} G_{E_p} = 1,\ G_{M_p} = \mu_p;\ F_{1_p} = 1,\ F_{2_p} = 1 \quad (8) \\ G_{E_n} = 0,\ G_{M_n} = \mu_n;\ F_{1_n} = 0,\ F_{2_n} = 1. \quad (9) \\ (10) \end{aligned}$$

Defining the polarization of the virtual photon as $\epsilon = \frac{1}{1+2(1+\tau)\tan^2\frac{\theta_e}{2}}$, equ. 7 takes the much simpler form:

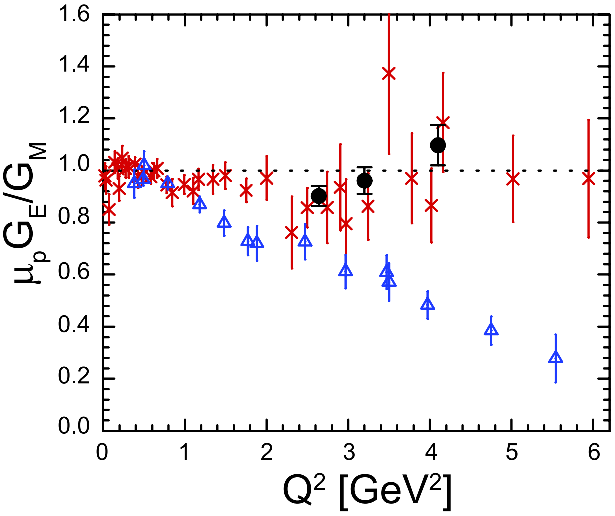

Fig. 1. whatever

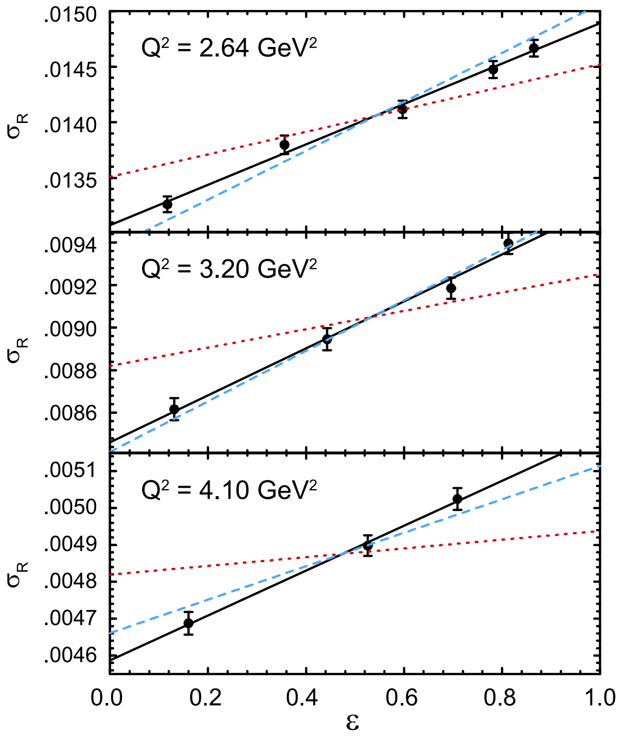

Fig. 2. whatever else

$$\frac{d\sigma}{d\Omega} = (\frac{d\sigma}{d\Omega})_{Mott} \left[G_E^2 + \frac{\tau}{\epsilon} G_M^2 \right] / (1 + \tau) \tag{11}$$

The modern version of the Rosenbluth separation technique takes advantage of the linear dependence in ϵ, of the FFs in the reduced cross section based on Eq. (11), as follows:

$$\left(\frac{d\sigma}{d\Omega}\right)_{red} = \frac{\epsilon(1+\tau)}{\tau} \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = G_M^2 + \frac{\epsilon}{\tau} G_E^2, \tag{12}$$

Testing the bibliography [1]. Testing again [2]. And again [3–5].

2 Theoretical Interpretations of Nucleon Form Factors

2.1 Models of Nucleon Form Factors

2.2 Dyson-Schwinger Equations and Diquark Models

2.3 Links between Deep-Inelastic Scattering and Nucleon Form Factors

2.4 Lattice QCD Calculations of Nucleon Form Factors

Lattice calculations of nucleon form factors are available only for isovector form factors.

Isoscalar form factors require calculations of disconnected diagrams, which are diagrams with quark loops not connected to the quark lines emanating from or ending on the lattice nucleon source or sink. The gluons that attach the quark loops to the valence quarks are not indicated in lattice diagrams, hence the phrase “disconnected.” Contributions from the disconnected loops require computer time intensive calculations, and the calculations remain undone. However, the disconnected diagrams contribute equally to proton and neutron, and the isovector case has been assayed.

Several isovector form factor lattice results have been reported in the past year or so [Alexandrou, Bhattacharya, Green]. They cover low to moderate Q^2, up to 1.4 GeV^2 for [Alexandrou, Bhattacharya, Green] together, and have pion masses in the range 373 MeV down to 149 Mev. The best appearing results are found in [Green], who have m_π = 149 MeV and perhaps more importantly have reduced the contributions from excited nucleon contamination. They show results for Q^2 up to 0.5 GeV^2, and have decent agreement of G_E^v and G_M^v with experimental data, represented by the Kelly form factor fits [Kelly], with uncertainty limits in the 20% range for Q^2 about 0.4 GeV^2.

Refs. [Alexandrou, Bhattacharya] cover a larger range of Q^2, but have larger pion masses, in the 213–373 MeV range. Their results for the isovector form factors tend to be about 50%, sometimes more, above the data for G_E^v or F_1^v in the Q^2 region above about 0.6 GeV^2, with uncertainties indicated at about 10%. The results are closer to data for G_M^v or F_2^v. The authors of these works do point out that all the lattice treatments with these pion masses are consistent with each other.

One may specifically focus on nucleon radii calculated from lattice gauge theory. In the future, it may be possible and desirable to calculate using a dedicated correlator which gives directly the slope of the form factor at zero momentum transfer. Finding such correlators by taking derivatives of known correlators is suggested and studied [de Divitiis] for lattice calculations of form factors at points where the Lorentz factors they multiply go to zero.

Applications in [de Divitiis] are to form factors for semileptonic scalar meson decay, and to hadronic vacuum polarization corrections to the muon $(g-2)$.

At present, lattice calculations of nucleon radii proceed by calculating the form factor at several non-zero Q^2, fitting to a suitable form, typically a dipole form, and finding the radius by extrapolating to zero Q^2. Results are available only for the isovector nucleon. Ref. [Syritsyn] presents a plot of Dirac radius results for lattice calculations at various pion masses. The results are typically about 50% of the experimental (electron measured) isovector Dirac radii-squared, albeit rising with decreasing pion mass, with quoted uncertainties in the 10% range. One may say there opportunity for further work. An uncertainty of 1% or less for the proton alone is needed for a lattice calculation to impact the proton radius puzzle.

References

1. R. Hofstadter, H. Fechter, J. McIntyre, Phys. Rev. **92**, 978 (1953)
2. R. Hofstadter, R.W. McAllister, Phys. Rev. **98**, 217 (1955)
3. M.R. Yearian, R. Hofstadter, Phys. Rev. **110**, 552 (1958)
4. M.K. Jones *et al.*, Jefferson Lab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
5. O. Gayou *et al.*, Jefferson Lab Hall A Collaboration, Phys. Rev. Lett. **88**, 092301 (2002)