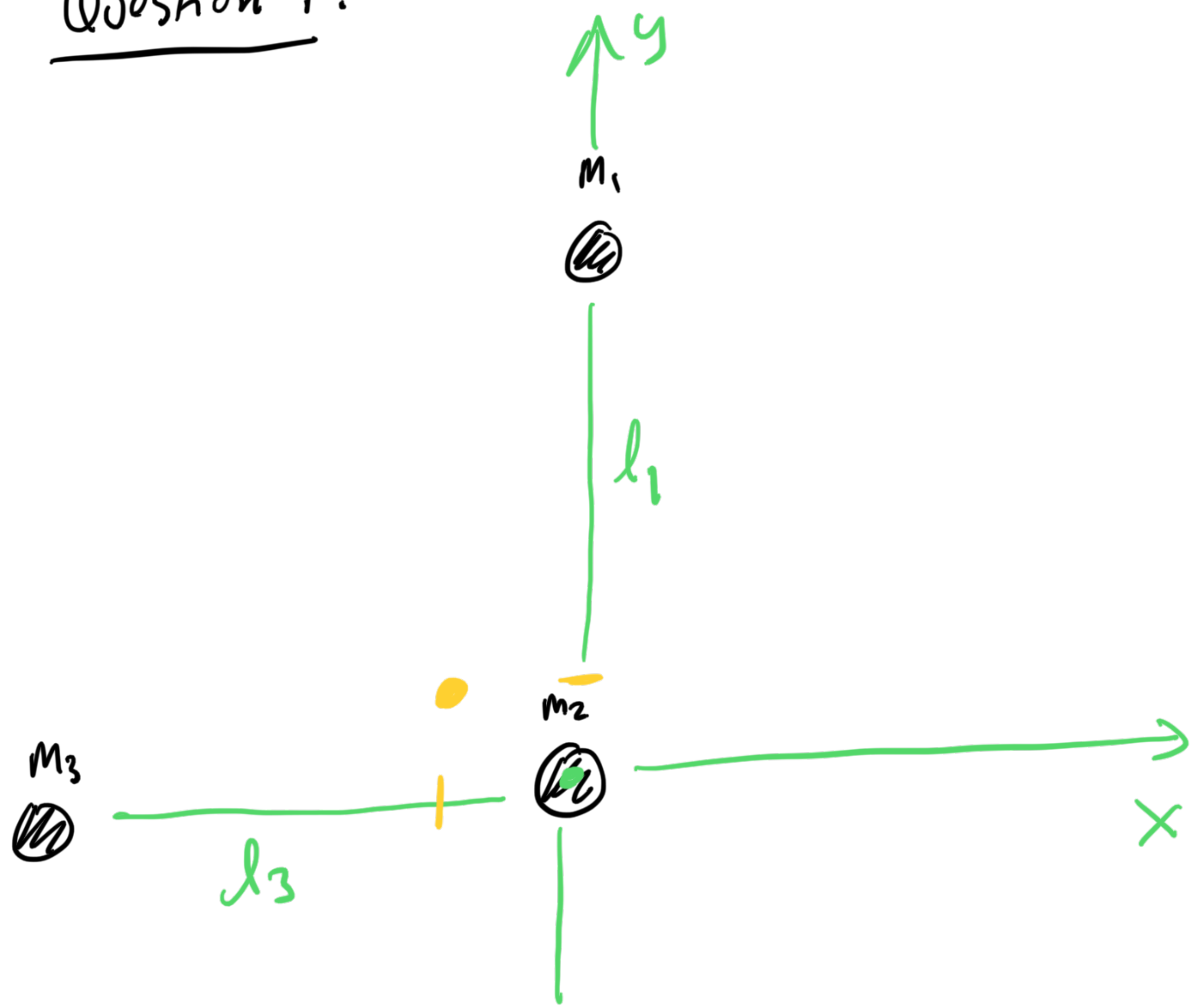
Physics 201 - Lecture 24

Question 7:



$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\, dm$$

$$x_{cm} = \frac{1}{M} \underbrace{\int x \, dm}_{= x_1 m_1 + x_2 m_2 + x_3 m_3}$$

$$= \frac{1}{0.360} \left((0)(.06) + (0)(.190) + (-0.06)(.110) \right)$$

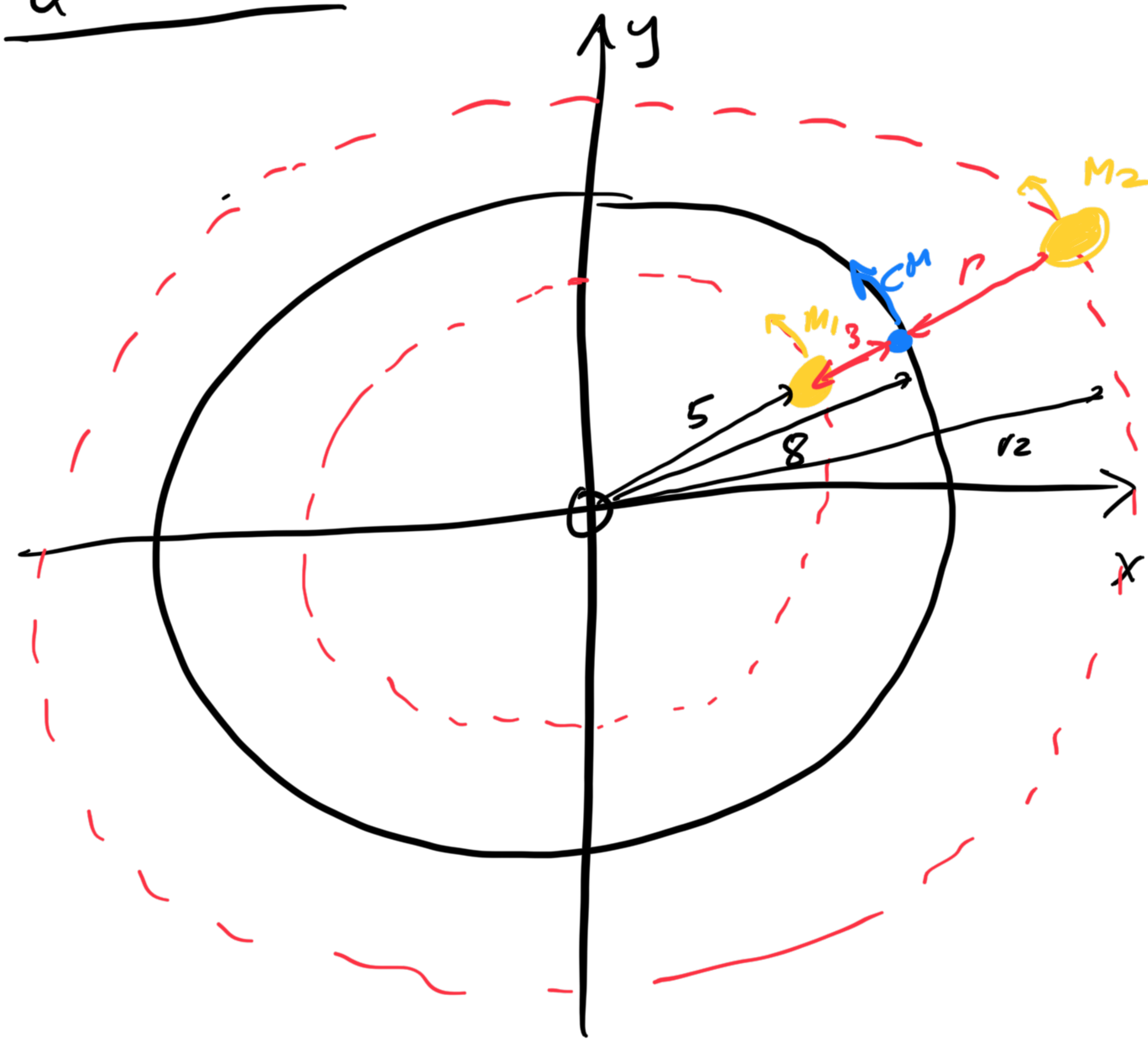
$$= -0.0183 \text{ m} = -1.83 \text{ cm}$$

$$y_{cm} = \frac{1}{M} \underbrace{\int y \, dm}_{= y_1 m_1 + \cancel{y_2 m_2} + \cancel{y_3 m_3}}$$

$$= \frac{1}{0.360} \left((0.05)(.06) \right)$$

$$= 0.00833 \text{ m} = 0.833 \text{ cm}$$

Question 8 :



$x_1 = 5\cos(4t) \qquad y_1 = 5\sin 4t$

$x_1^2 + y_1^2 = 25\left(\cos^2(4t) + \sin^2(4t\right.$

$$\underbrace{x_1^2 + y^2 = 25}_{\text{\& radius 5}}$$

Circle of radius ...

$\therefore$ (a) $\rightarrow$ 5 m

Similarly, $x_{cm}^2 + y_{cm}^2 = 8^2$

$$\boxed{5(m_1) + r_2(m_2) = 8(m_1+m_2)}$$

$$5m_1 + r_2 m_2 = 8m_1 + 8m_2$$

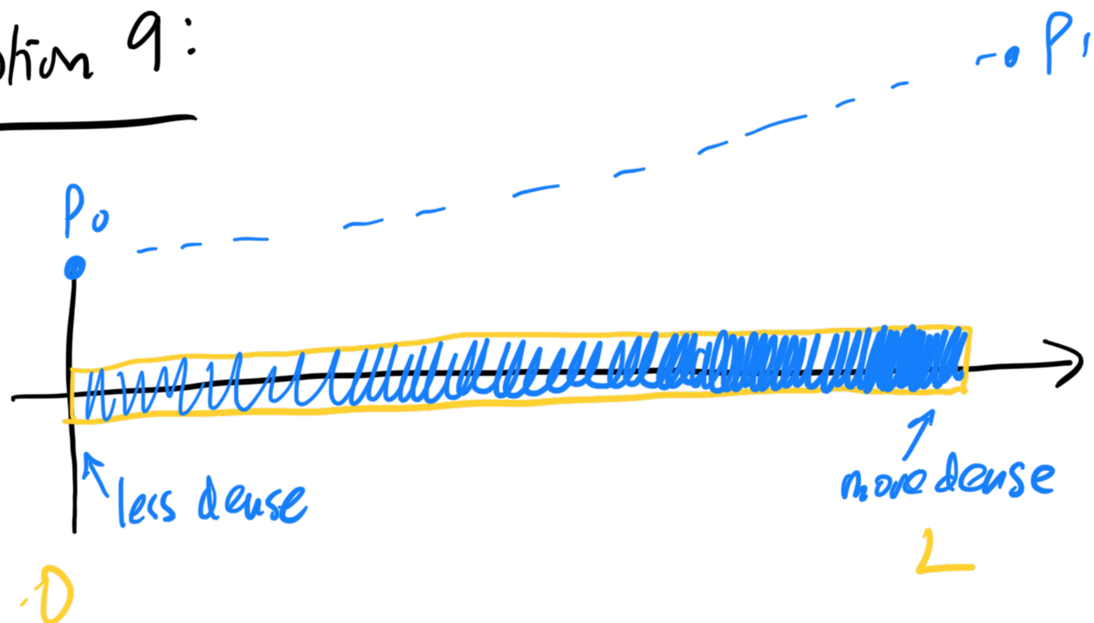
$$r_2 = \frac{3m_1 + 8m_2}{m_2}$$

$$\boxed{r_2 = 8 + \frac{3m_1}{m_2}}$$

$$\therefore x_2 = \left(8 + \frac{3m_1}{m_2}\right)\cos(4t)$$

$$y_2 = \left(8 + \frac{5m_1}{m_2}\right) \sin(4t)$$

Question 9:



$$\rho(x) = \rho_0 + (\rho_1 - \rho_0)\left(\frac{x}{L}\right)^2$$

at $x = 0$, $\rho = \rho_0$

at $x = L$, $\rho = \rho_1$

$$x_{cm} = \frac{1}{M} \int^{L} x \, \rho(x) \, dx$$

$M \quad \int_0 \quad \hat{i}$

$?$

$$\rho_0 + (\rho_1 - \rho_0)\left(\frac{x}{L}\right)^2$$

$$x_{cm} = \frac{1}{M} \underbrace{\int_0^L x\left(\rho_0 + (\rho_1 - \rho_0)\left(\frac{x}{L}\right)^2\right.}_{\text{wolfram alpha !!}}$$

$$= \frac{1}{4} L^2 (\rho_0 + \rho_1)$$

$$M = \int_0^L \rho(x)\, dx$$

$$= \underbrace{\int_0^L \left(\rho_0 + (\rho_1 - \rho_0)\left(\frac{x}{L}\right)^2\right) dx}$$

Wolfram Alpha

$$= \frac{1}{3} L (2\rho_0 + \rho_1)$$

$$x_{cm} = \frac{\frac{1}{4} \cancel{L}^{\cancel{2}\,L} (\rho_0 + \rho_1)}{\frac{1}{3} \cancel{L} (2\rho_0 + \rho_1)}$$

$$= \frac{3}{4} L \left(\frac{\rho_0 + \rho_1}{2\rho_0 + \rho_1} \right)$$

Question 10:

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$$

$$= \frac{\Delta (m \vec{v})}{}$$

$$\overline{\quad\Delta t\quad}$$

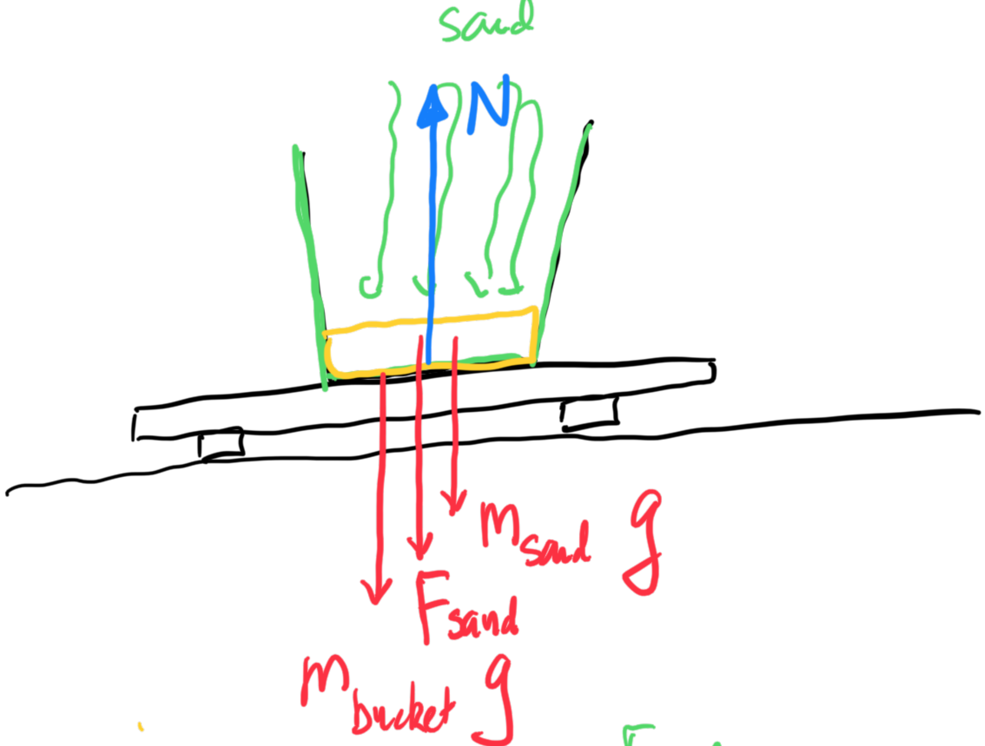
force due to sand

$$\vec{F}_{net} = m\frac{\Delta \vec{v}}{\Delta t} + \vec{v}\,\frac{\Delta m}{\Delta t}$$

{ product rule of calculus

$\to \vec{a}$

real external forces



F_{sand}

$$\vec{F}_{net} - \vec{v}\left(\frac{\Delta m}{\Delta t}\right) = m\,\vec{a}$$

$$= 0 \quad \text{(not moving)}$$

$$N - m_B g - m_s g - \left|\vec{v}\left(\frac{\Delta m}{\Delta t}\right)\right| = 0$$

$$N = m_B g + m_s g + \left|\vec{v}\left(\frac{\Delta m}{\Delta t}\right)\right.$$

$$= (0.540)(9.8) + (0.300)(9.8) + (3.70)(.0755$$

$$= 5.292 + 2.940 + 0.2794$$

$$= 8.232 + 0.2794$$

$$= 8.511 \text{ N}$$

Question 11 :

$$\vec{F} = m\vec{a} + \vec{v}\,\frac{\Delta m}{\Delta t}$$

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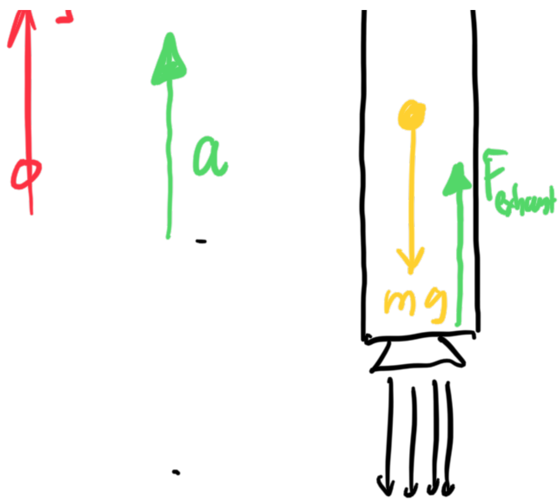

$$\vec{F}_{net} - \vec{v}\cdot\underbrace{\frac{\Delta m}{\Delta t}}_{-ve} = m\vec{a}$$

$$-mg + \left|v\frac{\Delta m}{\Delta t}\right| = ma$$

$$a = -g + \frac{1}{m}\left|v\frac{\Delta m}{\Delta t}\right|$$

$$= -1.60 + \frac{1}{6.45\times10^{3}}\left|2.20\times10^{3}\cdot 6.4\right|$$

$$a = -1.60 + 2.183$$

$$a = 0.583 \text{ m/s}^2$$