Physics 201 – Lecture 18

Work & Energy

→ Forces cause objects to accelerate!

↳ $\boxed{\text{velocity changes}}$

VECTORS ☹

SCALARS ☺ algebra, trig, calculus

Kinetic Energy → Energy of Motion.

$$K \equiv \frac{1}{2} m v^2$$

How difficult is it to stop an object that is in motion?

$$\begin{cases} v_i = \text{large} \rightarrow \text{large kinetic energy} \\ v_f = 0 \rightarrow \text{zero kinetic energy.} \end{cases}$$

$$K_i = \frac{1}{2} m v_i^2$$

$$K_f = \frac{1}{2} m v_f^2 = 0$$

$$\Delta K \equiv K_f - K_i$$

$$= 0 - \frac{1}{2} m v_i^2$$

$$\boxed{\Delta K = -\frac{1}{2} m v_i^2}$$

Stopping an object → taking kinetic energy from

energy along
the system.

$$K = \frac{1}{2} m v^2$$

$$[K] = kg \cdot \left(\frac{m}{s}\right)^2 = \frac{kg \cdot m^2}{s^2} = N$$

$$= \text{Joule}$$

$$= J$$

How do we change the kinetic energy of an object?

→ "**Work**" on the object.

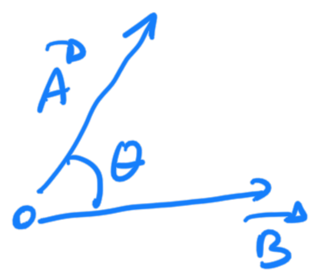
$$\boxed{K = \frac{1}{2} m v^2} \leftarrow \text{scalar}$$

$$= \frac{1}{2} m \left(\underbrace{\vec{v} \cdot \vec{v}}_{= |\vec{v}|^2} \right) = v^2$$

$\boxed{\text{Work}}$ done by a force $\vec{F}$

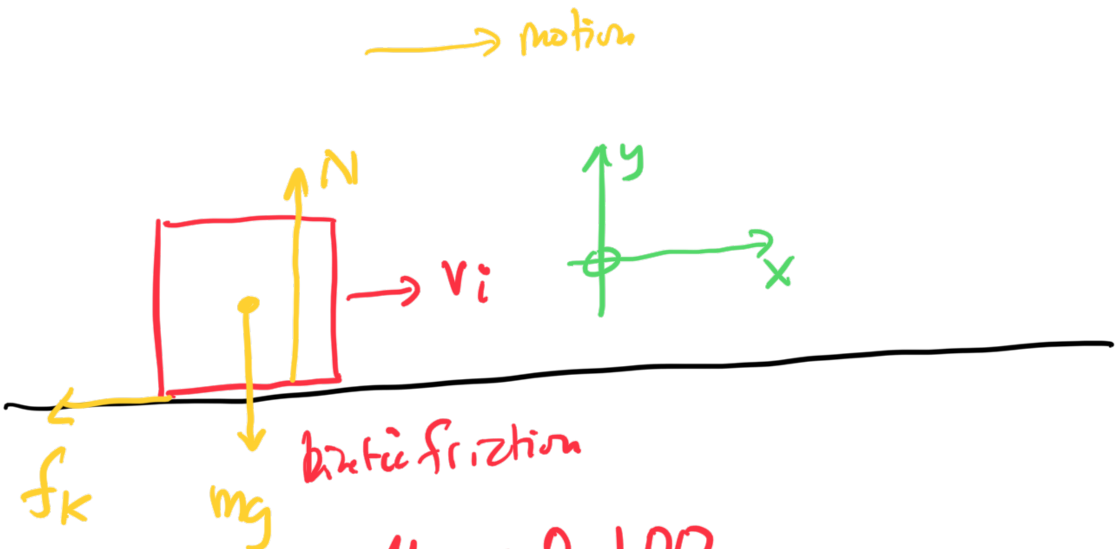
$$W_F \equiv \vec{F} \cdot \Delta\vec{x}$$

$$\left(= \int \vec{F} \cdot d\vec{x} \right)$$



$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\theta$

$$W_F = |\vec{F}| \, |\Delta\vec{x}| \cos\theta$$



$$\mu_K = \underline{0.100}$$

$$m = 10\ kg$$

$$v_i = 10 m/s \leftarrow \sim 20 mph$$

① What is the acceleration?

② How long does it take to stop?

③ How far does it travel before stopping?

X	Y

$a_x = a$

$\sum F_x = m a_x$

$-f_k = m a_x$

$a_y = 0$

$\sum F_y = m a_y = 0$

$N - mg = 0$

$N = mg$

$f_k = \mu_k N$

$= \mu_k m g$

$-\mu_k \cancel{m} g = \cancel{m} a_x$

$a_x = -\mu_k g$

$$\boxed{a = -0.98 \text{ m/s}^2}$$ a)

b) t ,

$v_i = 10$ m/s ①

$v_f = 0$ m/s ②

$a = -0.98$ m/s^2 ③

$v_f = v_i + a t$

$0 = 10 - 0.98 t$

$\frac{10}{...} = 10.2$ s

$t = \frac{\quad}{0.98}$

c) $\Delta x = ?$

$$\Delta x = \cancel{v_f}^{\,0} t - \frac{1}{2} a t^2$$

$$= -\frac{1}{2}(-0.98)(10.2)^2$$

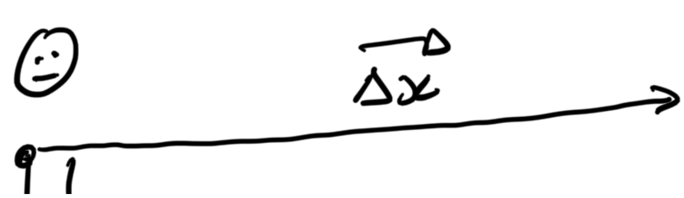
$$= 51.0 \text{ m}$$

What is the work done by **each** of the forces?

$$W_{mg} = \vec{F_g} \cdot \vec{\Delta x}$$

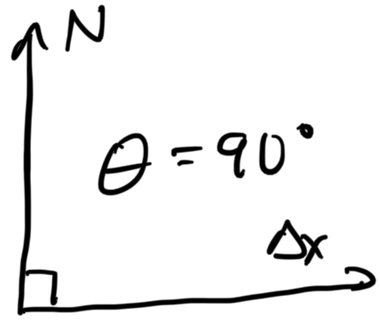
$$= |\vec{F_g}|\,|\vec{\Delta x}|\,\boxed{\cos \overset{0}{\theta}}$$

$$\boxed{W_{mg} = 0}$$

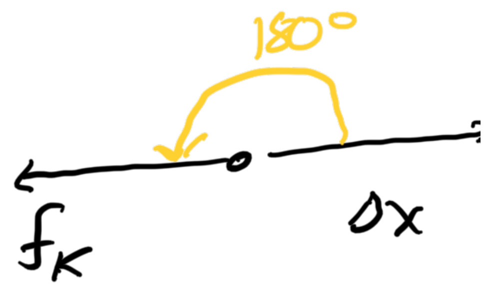


$\cos(90°) = 0$!

$$\boxed{W_N = 0}$$



$$W_f = |f_K||\Delta x|\cos\theta$$



$$\boxed{W_f = -f_K \cdot \Delta x}$$

$\cos(180°) = -1$

negative! Take energy out of the system!

Work-Energy Theorem!

$$\boxed{W_{\substack{all\\forces}} = \Delta K}$$

$$\cancel{W_{mg}}^{0} + \cancel{W_N}^{0} + W_f = \Delta K$$

$$W_f = \Delta K$$

$$-f_K \cdot \Delta x = \Delta K$$

$$= \cancel{\tfrac{1}{2} m v_f^2}^{0} - \tfrac{1}{2} m v_i^2$$

$$-f_K \cdot \Delta x = -\tfrac{1}{2} m v_i^2$$

$$\boxed{f_K \cdot \Delta x = \tfrac{1}{2} m v_i^2}$$

$$(\mu_K \cancel{m} g)(\Delta x) = \tfrac{1}{2} \cancel{m} v_i^2$$

2

$$\Delta x = \frac{v_i^2}{2\mu_k g}$$

$$= 51.0 \text{ m}$$

$$W_{\substack{\text{all} \\ \text{forces}}} = \Delta K$$

$$\vec{F_1} \cdot \vec{\Delta x} + \vec{F_2} \cdot \vec{\Delta x} + \ldots \vec{F_n} \cdot \vec{\Delta x} = \Delta K$$

$$\underbrace{(\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \ldots \vec{F_n})} \cdot \vec{\Delta x} = \Delta K$$

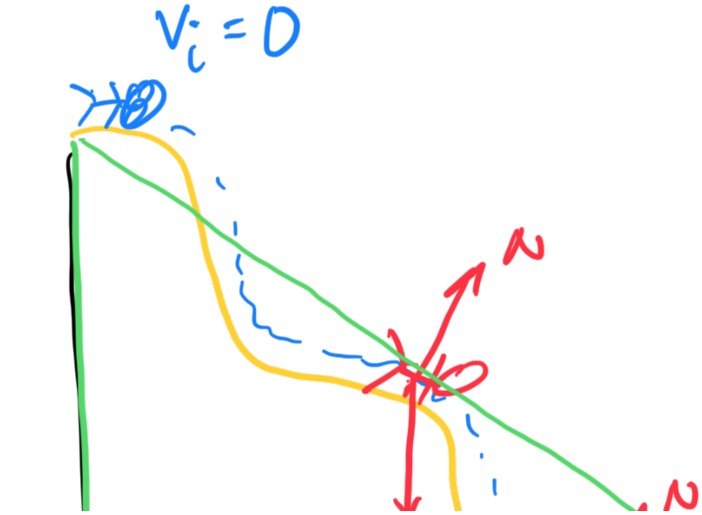
$$\vec{F}_{net} \cdot \vec{\Delta x} = \Delta K$$

$$m\vec{a}\cdot\Delta\vec{x} = \Delta K$$

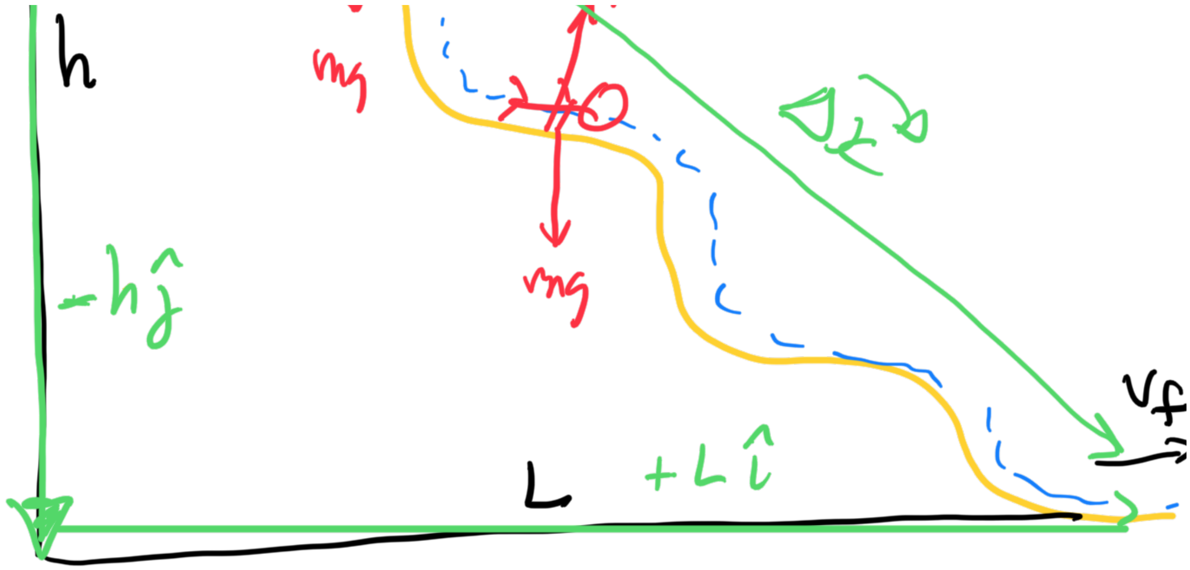
$$\cancel{m}\vec{a}\cdot\Delta\vec{x} = \frac{1}{2}\cancel{m}v_f^2 - \frac{1}{2}\cancel{m}v_i$$

$$2\vec{a}\cdot\Delta\vec{x} = v_f^2 - v_i^2$$

$$\boxed{v_f^2 = v_i^2 + 2\vec{a}\cdot\Delta\vec{x}}$$



friktionless!



→ gravity

→ normal force.

$W_{mg} = ?$

$W_N = 0$

$\vec{N} \perp \Delta\vec{x}$

$$W_{mg} = \underbrace{\vec{F}_g}_{(-mg\hat{j})} \cdot \underbrace{\Delta\vec{x}}_{(-h\hat{j} + L\hat{i})}$$

$$= mgh\,\underbrace{(\hat{j}\cdot\hat{j})}_{=1} - mgL\,\underbrace{(\hat{j}\cdot\hat{i})}_{=0}$$

$$\boxed{W_{mg} = mgh}$$

① $W_g = mgh$ (down)

② $W_g = -mgh$ (up)

$$W_{\text{all forces}} = \Delta K$$

$$\cancelto{0}{W_N} + \underbrace{W_g}_{mgh} = \Delta K$$

$$\cancel{m}gh = \Delta K = \frac{1}{2}\cancel{m}v_f^2 - \cancelto{0}{\frac{1}{2}mv_i^2}$$

$$\boxed{v_f = \sqrt{2gh}}$$

$h = 30\ m$

$v_f = 24.2\ m/s \sim \underline{\underline{45 mph}}$

$W_N = \vec{N} \cdot \Delta\vec{x}$

$dW_N = \underline{\underline{\vec{N} \cdot d\vec{x}}}$

$= 0$