Physics 201 - Lecture 27

Translational			Rotational
$\vec{x}$	position	$\rightarrow$	θ
$\vec{v}$	velocity	$\rightarrow$	ω
$\vec{a}$	acc.	$\rightarrow$	α
$\Delta\vec{x}$	disp.	$\rightarrow$	$\Delta\theta$

Newton's 2nd Law

$$\boxed{\sum_i \vec{F}_i = \vec{F}_{net} = m\vec{a}} \rightarrow \boxed{?}$$

$\sum F_x = ma_x$ $\quad$ $\sum F_y = ma_y$

$$\boxed{? = ?}$$

$$? = \boxed{?}\,\alpha$$

moment of inertia.

What is the equivalent

Question 1: What is ...
quantity for <u>mass</u> in rotational motion?

Question 2: What is the equivalent quantity for <u>force</u> in rotational motion?

1: Rotational Moment of Inertia

mass → how hard is it to make an object move?

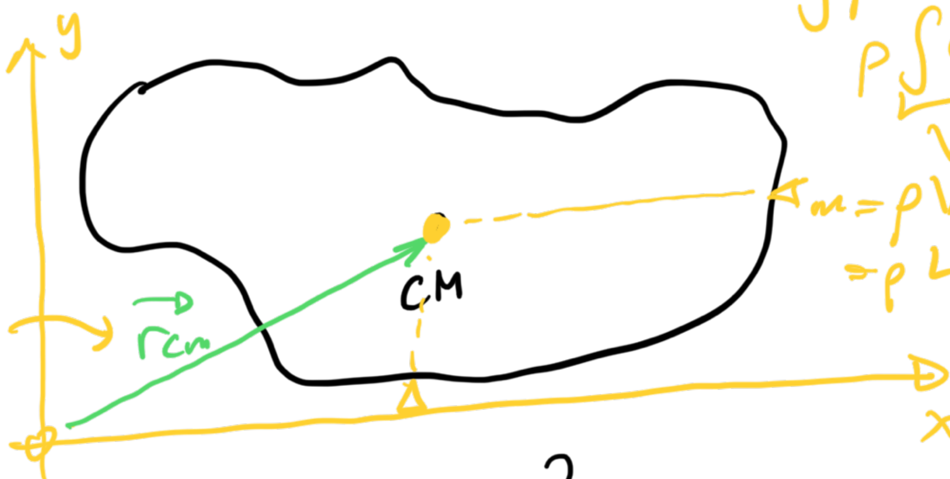
moment of inertia → how hard is it to make an object rotate?

<u>Moments</u>

"$\int \rho \, dV$"

$\rho \underbrace{\int dV}_{V}$

$m = \rho V = \rho L^3$



What is the total mass?

$$\boxed{m} = \int_{\text{volume}} \rho \, dV \qquad \text{density kg/m}^3$$

(scalar)

Where is the centre of mass?

$$m \, \vec{r}_{CM} = \int_{\text{volume}} \vec{r} \, \rho \, dV$$

$$\boxed{\vec{r}_{CM} = \frac{\int_{\text{vol}} (\vec{r}) \rho \, dV}{\int \rho \, dV}}$$

(vector)

What is the rotational moment of inertia? Well, it depends on which axis of rotation we are talking about!! Consider, for now, an axis through the center of mass.

$$\boxed{m(\vec{r}_{cm})^2} = \int_{vol} (\underline{\vec{r}})^2 \rho \, dV$$

$$I \equiv \boxed{\text{rotational moment of inertia}}$$

$$m = \int_V \rho \, dV = \boxed{\text{zeroth moment of inertia.}}$$

$$m \underline{\vec{r}_{cm}} = \int_V \vec{r} \rho \, dV = \boxed{\text{first moment of inertia}}$$

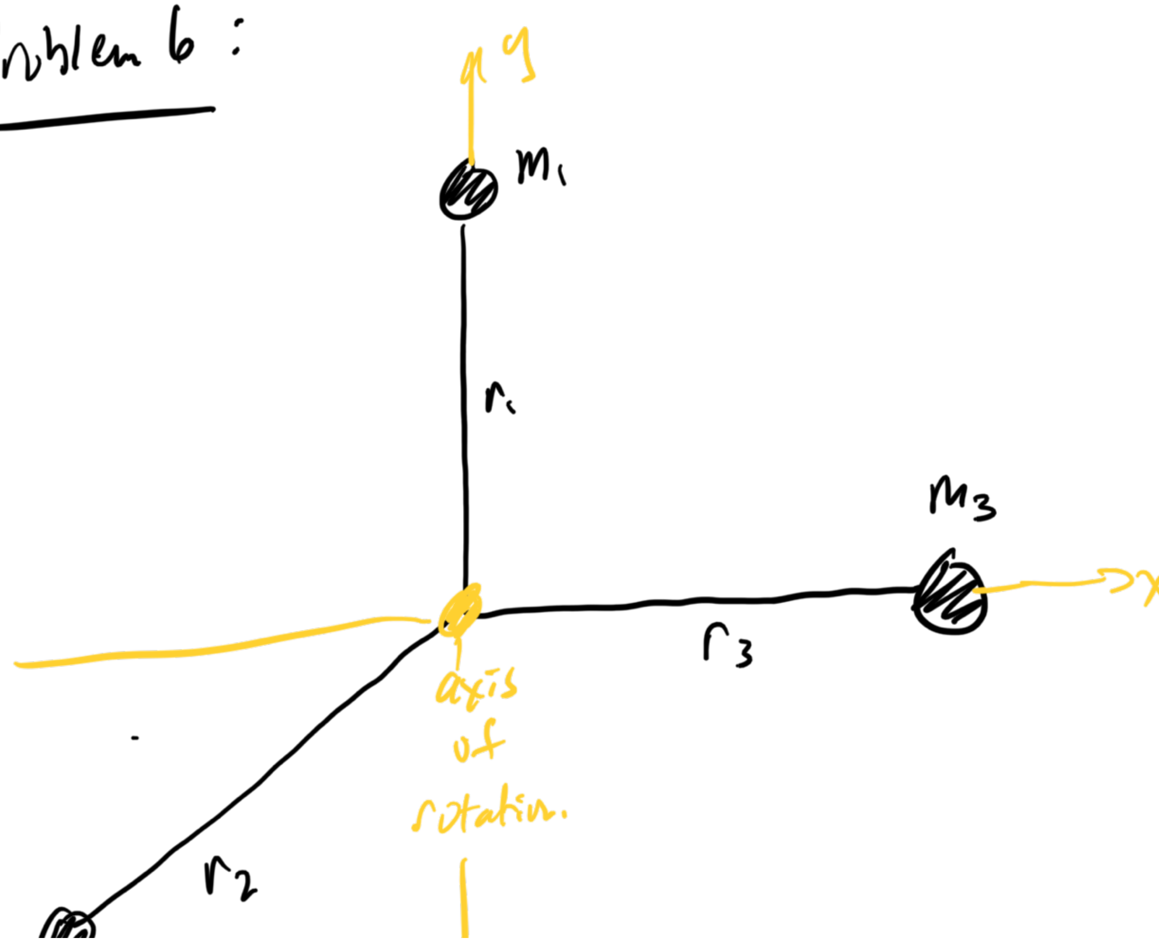
$$m(\vec{r}_{cm})^2 = \int_V (\vec{r})^2 \rho \, dV = \text{Second moment of inertia.}$$

$$m(\vec{r}_{cm})^3 = \int_V (\vec{r})^3 \rho \, dV = \text{third moment of inertia}$$

$\vdots$

etc.

Problem 6:



m_2

$$\boxed{I = \int (\vec{r})^2 \rho \, dV}$$

$$\int \vec{r}^2 dm$$

$m_1 \vec{r}_1^2 + m_2 \vec{r}_2^2 + m_3 \vec{r}_3^2 +$

$$= \int r^2 \, dm$$

"S" → sum → all op

$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= (.63)(.35)^2 + (.63)(.62)^2 + (.63)(.22)^2$$

$$= 0.35 \text{ kg} \cdot \text{m}^2$$

Kinetic Energy

$$\boxed{K = \frac{1}{2} m v^2}$$

$$\Rightarrow \boxed{K = \frac{1}{2} I \omega^2}$$

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b) $\omega = 4.1 \frac{\cancel{rev}}{s} \times \frac{2\pi \text{ rad}}{\cancel{rev}}$

$= \underline{8.2\pi} \text{ rad/s}$

$$K = \frac{1}{2}(I)(\omega^2)$$

$$= \frac{1}{2}(.35)(8.2\pi)^2$$

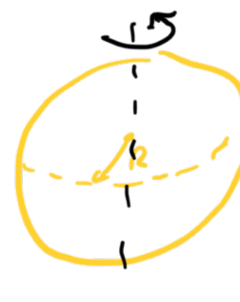
$$= 116 \text{ J}$$

Question: What is the moment of inertia of a wheel, a cylinder, a sphere, a meter stick, etc....

These are all common shapes, and someone has already done the calculus for us!

$$I_{\text{disk/wheel}} = \frac{1}{2} M R^2$$

$$I_{\text{sphere}} = \frac{2}{5} M R^2$$

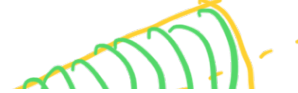


$$I_{\text{rod}} = \frac{1}{12} M L^2$$

(about center)

$$I_{\text{rod}} = \frac{1}{3} M L^2$$

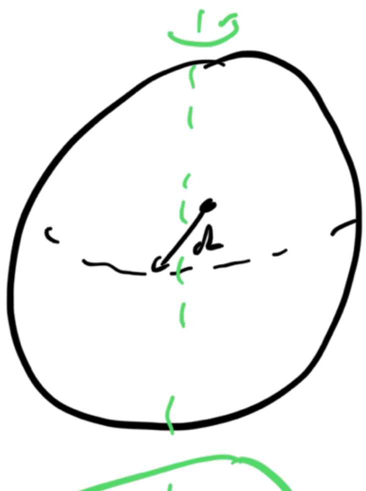
(about one



and)

$$I_{cylinder} = \frac{1}{2} M R^2$$

Question 7:



1 rev in one "day"
↑
T

$$I_{sphere} = \frac{2}{5} M R^2$$

$$\omega = \frac{2\pi}{T} \text{ rad}$$

a) $R_{KE} = \frac{1}{2} I \omega^2$

$$= \frac{1}{2}\left(\frac{2}{5} M R^2\right)\left(\frac{4\pi^2}{T^2}\right)$$

$=$

$$= \frac{4\pi^2}{5} M \frac{R^2}{T^2}$$

$D = 12100\,\text{km}$

$R = 6050\,\text{km}$

$= 6.05 \times 10^6\,\text{m}$

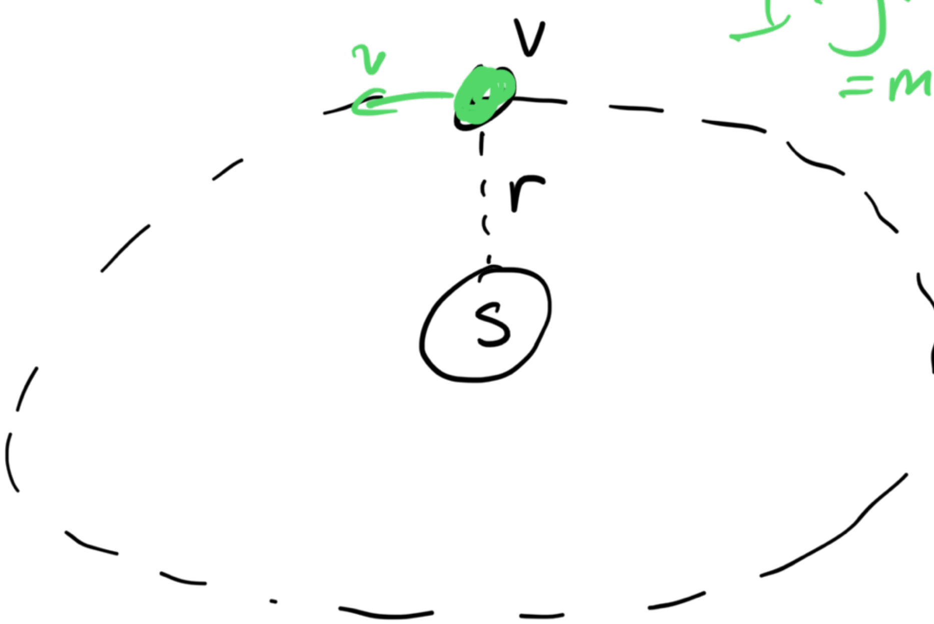
$$= \frac{4\pi^2}{5}\left(4.9 \times 10^{24}\right)\left(\frac{6.05 \times 10^6\,\text{m}}{5830 \times 3600}\right)^2$$

$$= \boxed{3.21 \times 10^{24}}\ \text{J}$$

b)

$I = \int r^2\,dm$

$= mr^2$



$$I = mr^2 \qquad \omega = \frac{2\pi}{T}$$

$$R_{KE} = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (m r^2) \left(\frac{2\pi}{T}\right)^2$$

$$= \frac{2\pi^2 m r^2}{T^2}$$

$$= 2\pi^2 (4.9 \times 10^{24}) \left(\frac{1.08 \times 10^{11}}{225 \times 24 \times 3600}\right)^2$$

$$= 2.98 \times 10^{33}\ J$$

Back to Newton's 2^{nd} Law for rotational motion ....

$$? = I\alpha$$

What is the rotational equivalent of

... linearly :

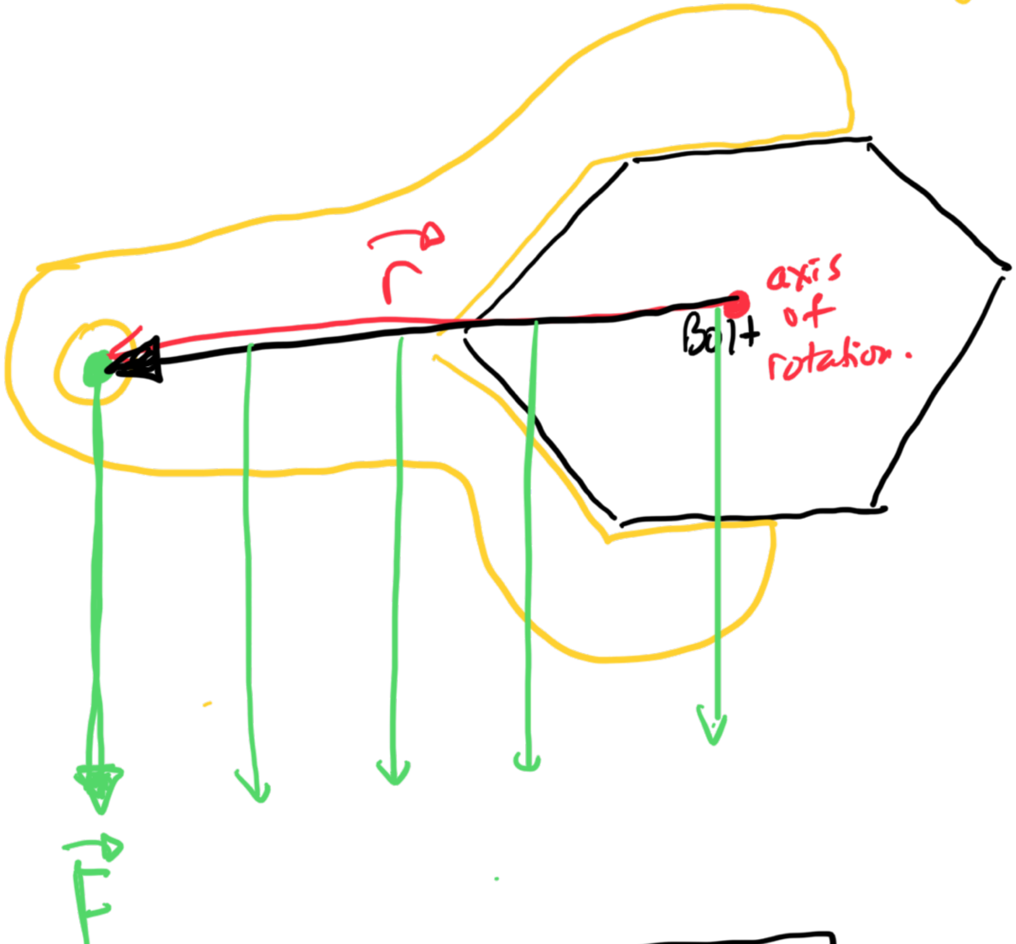
Force → accele...

force ?   Torque → accelerate rotationally.

⇒ $\boxed{\text{Torque}}$ (τ)

Stupid IKEA wrench

(Top View)



$\boxed{\vec{F} \equiv \text{applied force}}$

$\vec{r} \equiv$ vector which points from the axis of rotation to the ... of the

: point of application of

force.

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

" $\vec{r}$ cross $\vec{F}$ "

Torque is a vector!

$\vec{r} \times \vec{F}$ is the vector / cross product of $\vec{r}$ and $\vec{F}$. We need to understand how to calculate both the size and direction of this torque vector.

$W = \vec{F} \cdot \Delta\vec{x}$

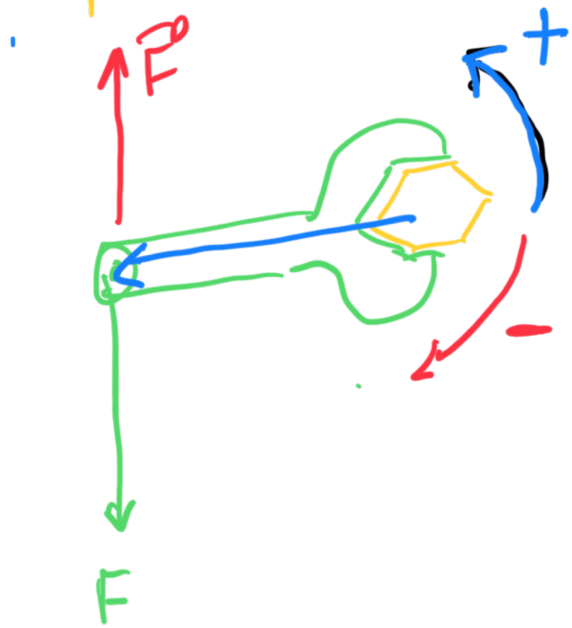
$= |\vec{F}||\Delta\vec{x}| \cos\theta$

size:

$$|\vec{\tau}| = \tau = \boxed{|\vec{F}||\vec{r}| \sin\theta}$$

direction:

- Think about which direction of rotation the torque will tend to cause.



∴ this is a positive torque.

Question 8:

$N = 14\ N$

$$f = \mu_k N$$
$$= (0.40)(14) = 5.6\ N$$

$$\tau_f = -f \cdot r \cdot \sin(90°)$$
$$= -(5.6)(.4)(1)$$

$$\boxed{\tau_f = -2.24\ N \cdot m}$$

$$\tau_N = N \cdot r \cdot \sin(180°) = 0\ !$$

$$\tau_{F_{axle}} = (F_{axle})(0) \sin(0°) = 0\ !$$

$$\therefore \tau_{NET} = \tau_f + \tau_N + \tau_{F_{axle}}$$

$$= -2.24 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{2} m r^2 \qquad \text{(disk)}$$

$$= \frac{1}{2}(91)(.4)^2$$

$$= 7.28 \text{ kg}\cdot\text{m}^2$$

$$\tau_{NET} = I \alpha$$

$$-2.24 = 7.28 \alpha$$

a) $$\alpha = -0.3077 \text{ rad/s}^2$$

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b) $$\omega_i = 65 \text{ rpm} = 65 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}}$$

$\times \dfrac{1\,min}{60\,s}$

$= 6.8068\ rad/s$

$\omega_f = 0$

$\alpha = -0.3077\ rad/s^2$

$\Delta\theta = ?$

$\cancel{\omega_f^2}^{\,0} = \omega_i^2 + 2\alpha\Delta\theta$

$0 = (6.8068)^2 - 2(.3077)\Delta\theta$

$\Delta\theta = 75.288\ rad$

$\#\ rev = \dfrac{\Delta\theta}{2\pi} = 11.98\ rev$

$\sim 12\ rev.$