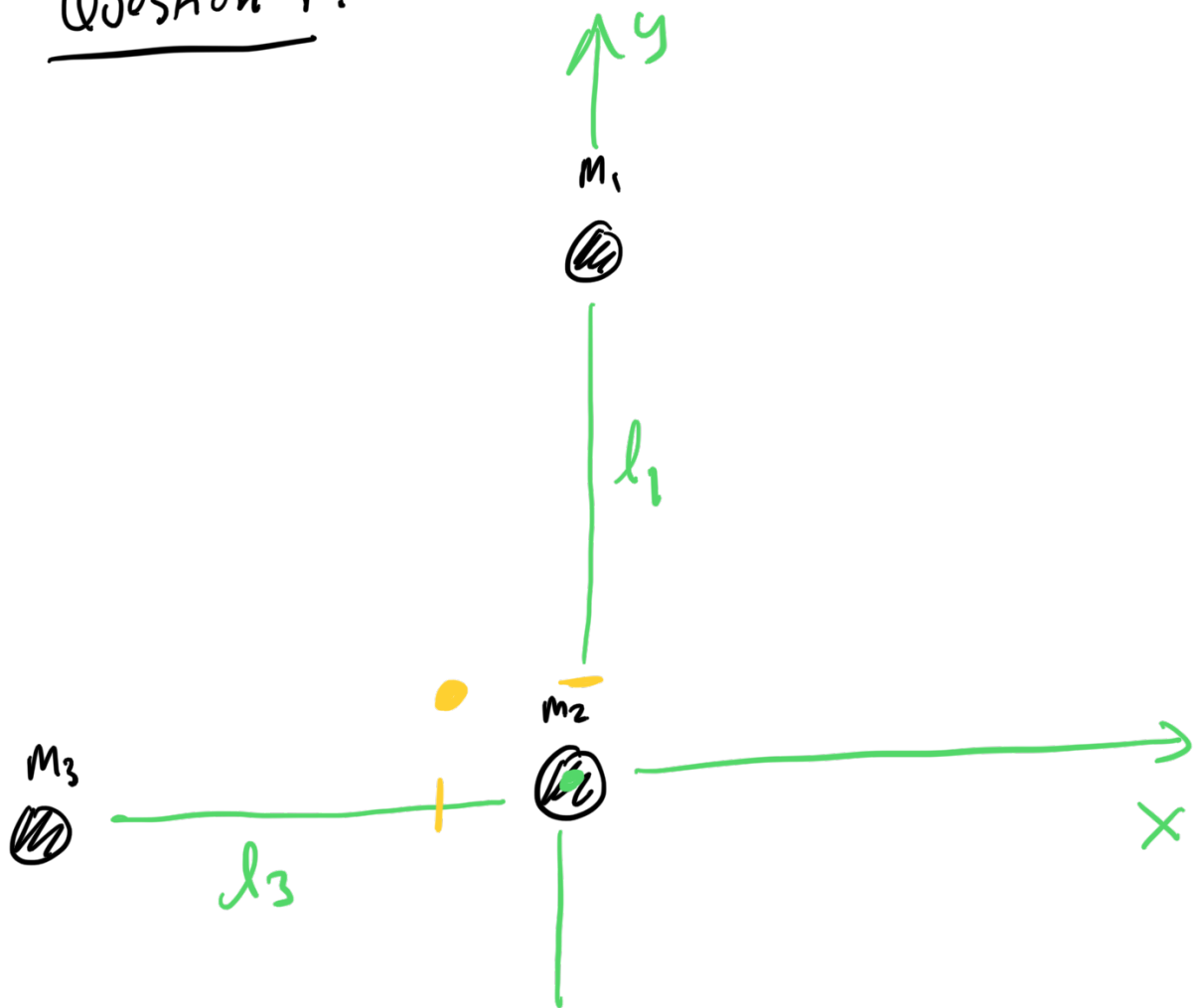


# Physics 201 - Lecture 24

Question 7:



$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r} dm$$

$$x_{cm} = \frac{1}{M} \int x dm$$

$$= x_1 m_1 + x_2 m_2 + x_3 m_3$$

$$= \frac{1}{0.360} \left( (0)(.06) + (0)(.146) + (-0.06)(.112) \right)$$

$$= -0.0183 \text{ m} = -1.83 \text{ cm}$$

$$y_{cm} = \frac{1}{M} \int y dm$$

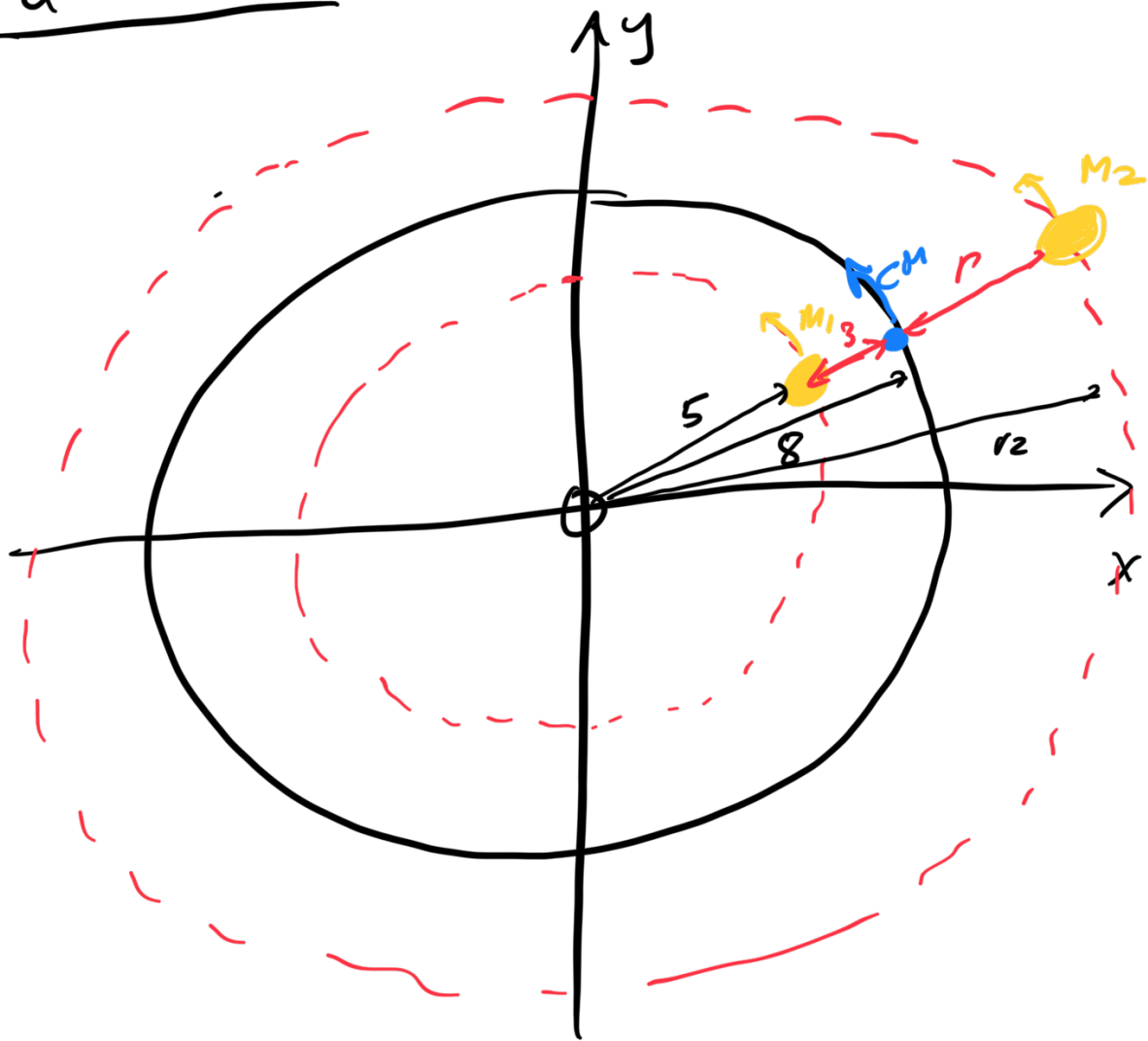
$$= y_1 m_1 + \cancel{y_2 m_2} + \cancel{y_3 m_3}$$

$$= \frac{1}{0.360} \left( (0.05)(.06) \right)$$

$$= 0.00833 \text{ m} = 0.833 \text{ cm}$$


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Question 8 :



$$x_1 = 5 \cos(4t) \quad y_1 = 5 \sin 4t$$

$$x_1^2 + y_1^2 = 25 (\cos^2(4t) + \sin^2(4t))$$

$$x_1^2 + y_1^2 = 25$$

radius 5

Circle of radius 8

$$\therefore (a) \rightarrow 5m$$

$$\text{Similarly, } x_{cm}^2 + y_{cm}^2 = 8^2$$

$$\boxed{5(m_1) + r_2(m_2) = 8(m_1 + m_2)}$$

$$5m_1 + r_2 m_2 = 8m_1 + 8m_2$$

$$r_2 = \frac{3m_1 + 8m_2}{m_2}$$

$$\boxed{r_2 = 8 + \frac{3m_1}{m_2}}$$

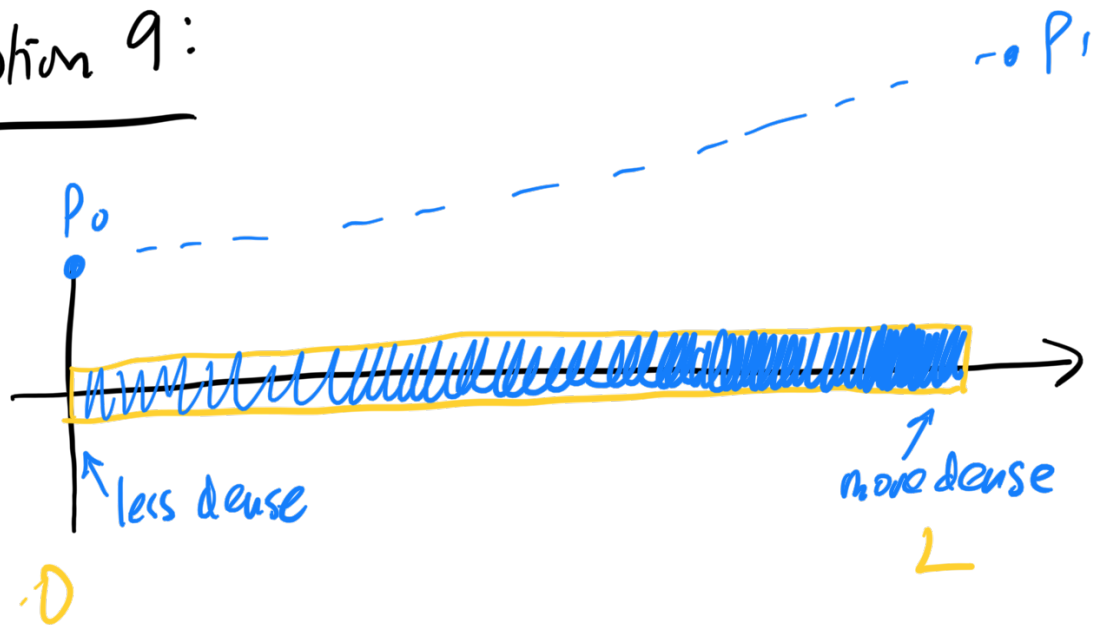
$$\therefore x_2 = \left(8 + \frac{3m_1}{m_2}\right) \cos(4t)$$

... ..

$$y_2 = \left( 8 + \frac{5m_1}{m_2} \right) \sin(4\pi)$$


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Question 9:



$$\rho(x) = \rho_0 + (\rho_1 - \rho_0) \left( \frac{x}{L} \right)^2$$

$$\text{at } x=0, \quad \rho = \rho_0$$

$$\text{at } x=L, \quad \rho = \rho_1$$

$$x_{cm} = \frac{1}{L} \int_0^L x \rho(x) dx$$

$\overset{M}{\uparrow}$   $\overset{J}{\downarrow}$   $\overset{1}{\uparrow}$   
 ~~$\rho_1$~~ ?  $\rho_0 + (\rho_1 - \rho_0) \left(\frac{x}{L}\right)^2$

$$X_{cm} = \frac{1}{\cancel{M}} \int_0^L x \left( \rho_0 + (\rho_1 - \rho_0) \left(\frac{x}{L}\right)^2 \right) dx$$

$\underbrace{\hspace{15em}}$   
 we know alpha!! (:)

$$\begin{aligned}
 M &= \int_0^L \rho(x) dx \\
 &= \int_0^L \left( \rho_0 + (\rho_1 - \rho_0) \left(\frac{x}{L}\right)^2 \right) dx
 \end{aligned}$$

Wolfram Alpha

$$= \frac{1}{3} L (2p_0 + p_1)$$

$$x_{cm} = \frac{\frac{1}{4} L (p_0 + p_1)}{\frac{1}{3} L (2p_0 + p_1)}$$

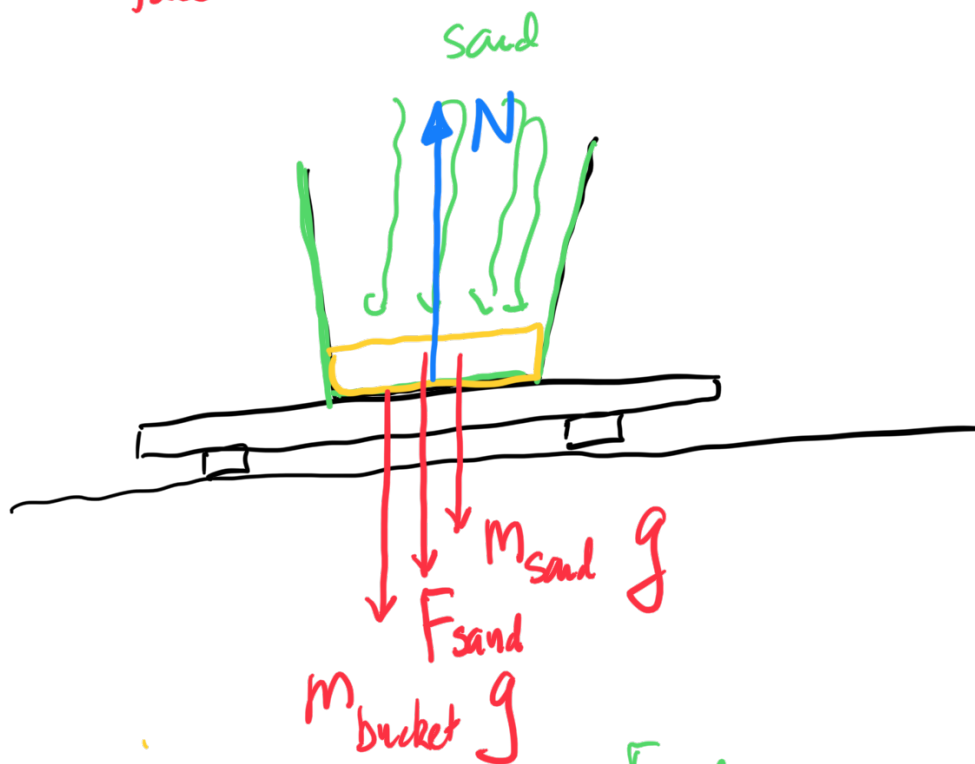
$$= \frac{3}{4} L \left( \frac{p_0 + p_1}{2p_0 + p_1} \right)$$

Question 10:

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$$
$$= \Delta (m \vec{v})$$

$$\vec{F}_{\text{net}} = m \frac{\Delta \vec{v}}{\Delta t} + \vec{v} \frac{\Delta m}{\Delta t}$$

$\vec{F}_{\text{net}}$  (red arrow) → real external forces  
 $\frac{\Delta \vec{v}}{\Delta t}$  (circled in red) → product rate of change →  $\vec{a}$   
 $\vec{v} \frac{\Delta m}{\Delta t}$  (circled in red) → force due to sand



$$\vec{F}_{\text{net}} - \vec{v} \left( \frac{\Delta m}{\Delta t} \right) = m \vec{a}$$

$\vec{F}_{\text{net}}$  (red arrow) →  $\vec{a}$  (red arrow)  
 $\vec{v} \left( \frac{\Delta m}{\Delta t} \right)$  (circled in green) →  $F_{\text{sand}}$  (green arrow)  
 $= 0$  (not mag)



$$N - m_B g - m_S g$$

$$- \left| \vec{v} \left( \frac{\Delta m}{\Delta t} \right) \right| = 0$$

$$N = m_B g + m_S g + \left| \vec{v} \left( \frac{\Delta m}{\Delta t} \right) \right|$$

$$= (0.540)(9.8) + (0.300)(9.8) + (3.70)(.0755)$$

$$= 5.292 + 2.940 + 0.2794$$

$$= 8.232 + 0.2794$$

$$= 8.511 \text{ N}$$

Question 11 :

.4



$$\vec{F} = m \vec{a} + \vec{v} \cdot \frac{\Delta m}{\Delta t}$$

↓
↑
↓

Diagram illustrating a rocket engine nozzle. A fluid element of mass  $m$  is shown inside the nozzle. The forces acting on it are the thrust force  $F_{thrust}$  (upward) and the gravitational force  $mg$  (downward). The net force is  $F_{net}$ . The equation of motion is given by:

$$F_{net} - v \cdot \frac{\Delta m}{\Delta t} = m \cdot a$$

where  $\frac{\Delta m}{\Delta t}$  is negative, indicating mass loss. The simplified equation is:

$$-mg + \left| v \frac{\Delta m}{\Delta t} \right| = ma$$

$$a = -g + \frac{1}{m} \left| v \frac{\Delta m}{\Delta t} \right|$$

$$= -1.60 + \frac{1}{6.45 \times 10^3} \left| 2.20 \times 10^3 \cdot 6.4 \right|$$

$$a = -1.60 + 2.183$$

$$a = 0.583 \text{ m/s}^2$$

