Lecture 3

The Algebra of Scalars

→ this is easy! It's just the same math you have been doing your whole life!

Addition, subtraction, multiplication, divisi work exactly the same way!

BIG CAVEAT! UNITS MATTE

✓ $m^2 = m_1^2 + m_2^2$ (m is mass

$(kg^2) \quad (kg^2) \quad (kg^2)$

✗ $m = \frac{2m_1}{} + \frac{3m_2^2}{}$

$(kg) \quad (kg) \quad (kg^2)$

Algebra of vectors

The Algebra of

→ This is much different!

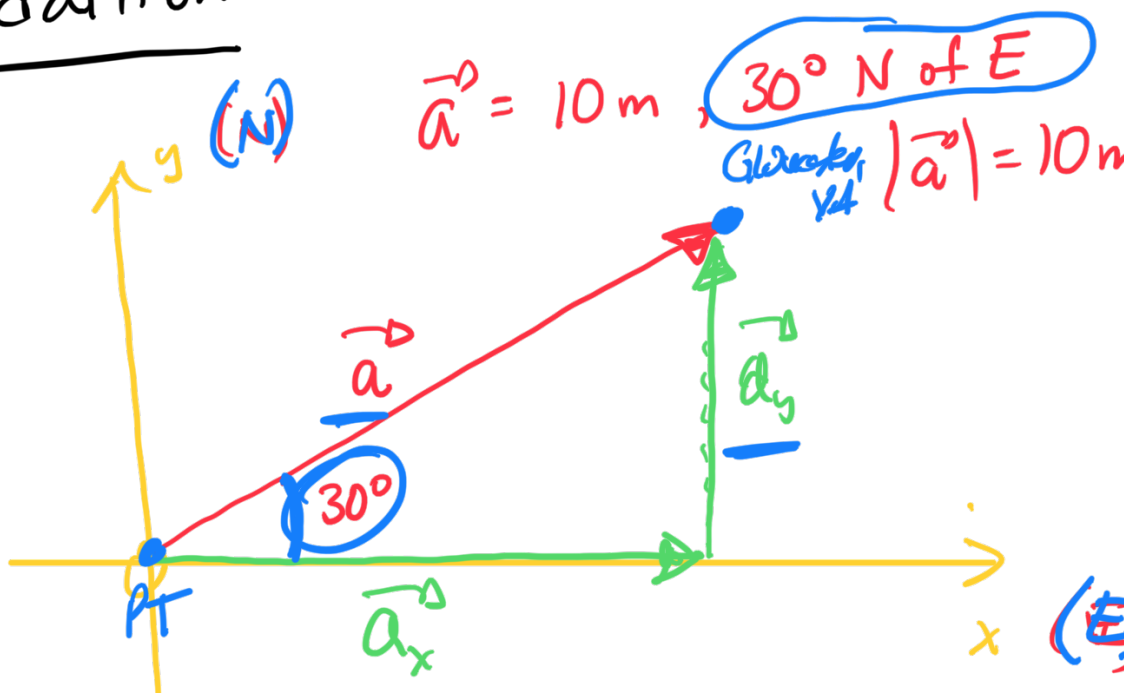
→ Direction Matters!!

$a = 2m + 3m \quad \therefore a = 5\,m$

$\vec{b} = 2\,m \text{ South} + 3m \text{ North}$

$= ?$

Addition



$$\vec{a} = \vec{a_x} + \vec{a_y}$$

i.e. We split the vector $\vec{a}$ int

an x-component and a

y-component

Step 1: Pythagorean Theorem.

$$|\vec{a}|^2 = |\vec{a_x}|^2 + |\vec{a_y}|^2$$

$$100 = (10m)^2 = |\vec{a_x}|^2 + |\vec{a_y}|^2$$

Step 2: Trigonometry.

Triangle reasons

SOHCAHTOA

$$\sin\theta = \frac{opp}{hyp}, \quad \cos\theta = \frac{adj}{hyp}$$

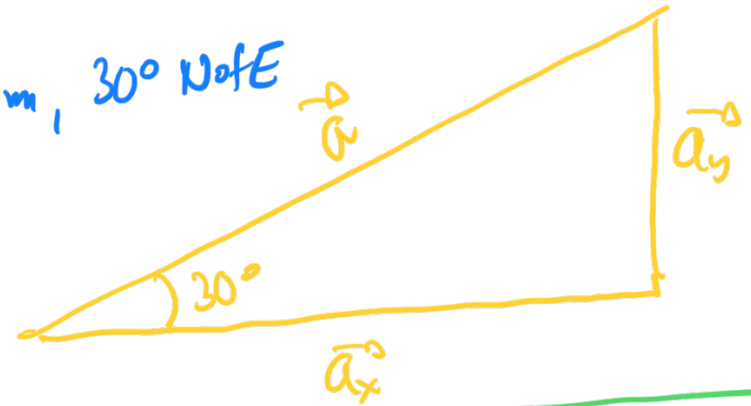
$$\tan\theta = \frac{opp}{adj}$$

$$\sin 30^\circ = \frac{|\vec{a_y}|}{|\vec{a}|} \qquad \cos 30^\circ = \frac{|\vec{a_x}|}{|\vec{a}|}$$

$$|\vec{a_y}| = |\vec{a}| \sin 30^\circ$$
$$= (10\,m)(0.500)$$
$$= 5\,m$$

$$|\vec{a_x}| = |\vec{a}| \cos$$
$$= (10\,m)(0.8$$
$$= 8.66\,m$$

$\vec{a} = 10\,m,\ 30^\circ$ North of East



$$\vec{a} = 8.66\,m \text{ East} + 5.00\,m \text{ N}$$

This [illegible] to keep writing the

It's ... units and the directions out all the time. Let's develop a "shorthand" notation.

Unit Vectors

Let $\hat{i}$ = 1 m East = 1 m North

"i hat" "j hat"

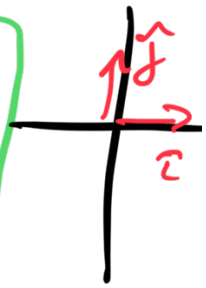
$\hat{j}$ = 1 m in +ve z-direction

(+ve) x-direct

(+ve y-direct

(Note) 1 m South = $-\hat{j}$

1 m West = $-\hat{i}$



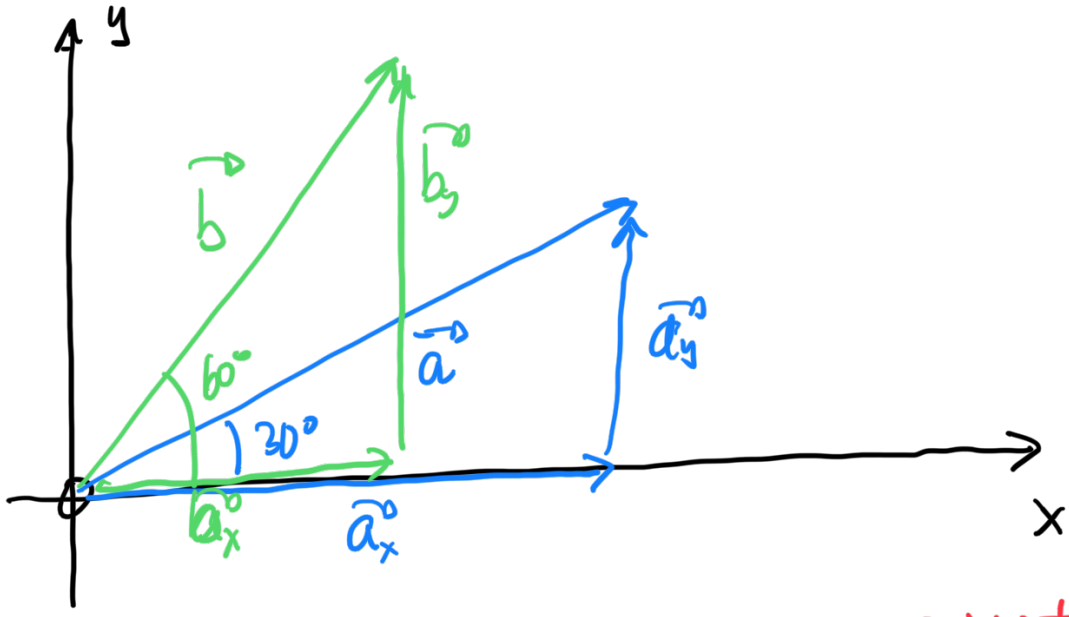
$$\vec{a} = 8.66\,\hat{i} + 5.00\,\hat{j}$$

Check: $|\vec{a}|^2 = (8.66)^2 + (5.00)^2$

$= 75.00 + 25.00$

$= 100.00$

$|\vec{a}| = 10$ ✓



Experiment : $\vec{a}$ = 100 m, 30° N of Ea

$\vec{b}$ = 90 m, 60° N of Ea

Step 1: x/y components

$$\vec{a} = \vec{a_x} + \vec{a_y}$$

$$= 100\cos 30^\circ \hat{i} + 100 \sin 30^\circ \hat{j}$$

$$= 86.60\hat{i} + 50\hat{j}$$

$$\vec{b} = \vec{b_x} + \vec{b_y}$$

$$90\cos 60^\circ \hat{i} + 90 \sin 60^\circ \hat{j}$$

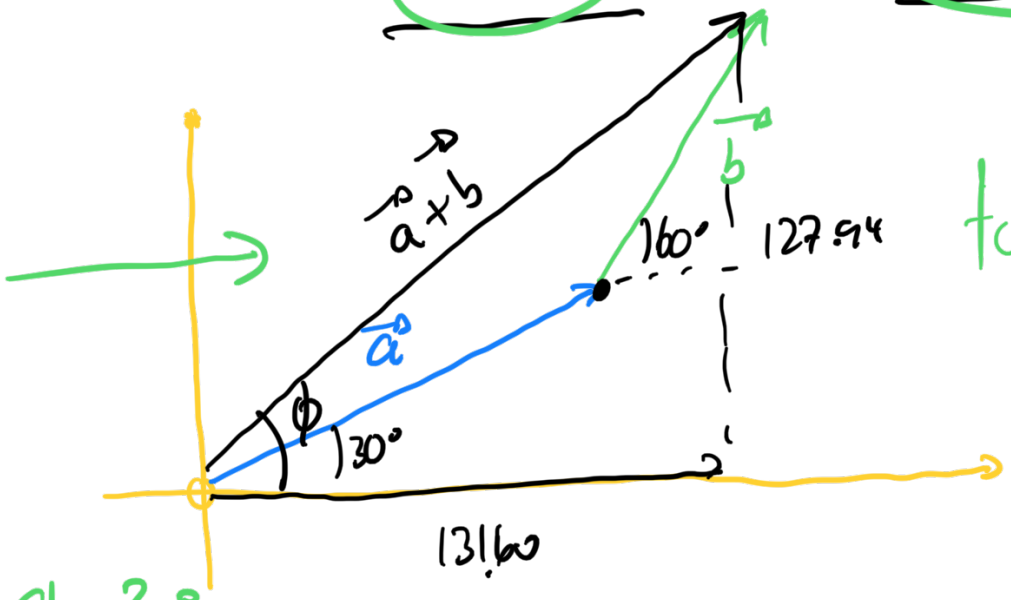
$= 45\,\hat{i} + 77.94\,\hat{j}$ ✓

Step 2:

What is $\vec{a} + \vec{b}$?

$$\vec{a} + \vec{b} = (86.60\,\hat{i} + 50\,\hat{j}) + (45\,\hat{i} + 77.94\,\hat{j})$$

$$= 131.60\,\hat{i} + 127.94\,\hat{j}$$



$\tan = \frac{opp}{adj}$

Step 3:

$$|\vec{a} + \vec{b}|^2 = 131.60^2 + 127.94^2$$

$$|\vec{a} + \vec{b}| = \sqrt{131.60^2 + 127.94^2}$$

$|a + b|$

size $= 183.54$

$\tan\phi = \dfrac{127.94}{131.00} = 0.9722$

angle. $\tan^{-1}(.9722)$

$\phi = 44.19°$

$\therefore \quad \vec{a} + \vec{b} = 183.54\ m,\ 44.19°$ North of East

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Subtraction:

This is now easy!

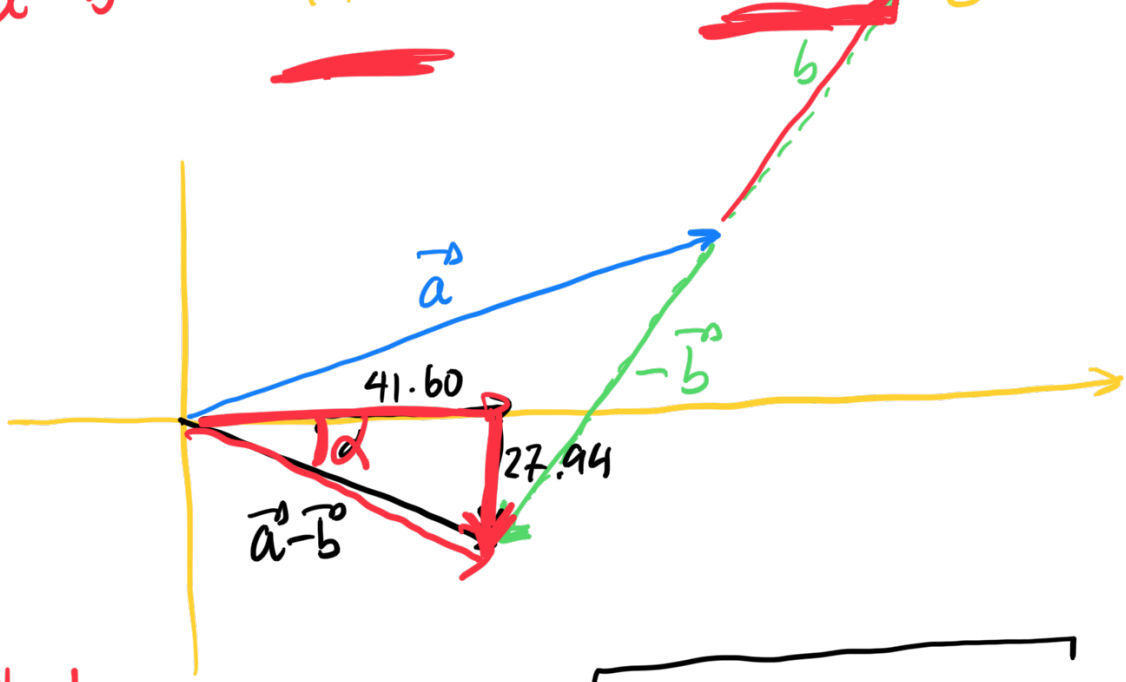
$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

vector going in a direction opposite to $\vec{b}$!

$= 86.60\,\hat{i} + 50.00\,\hat{j}$

$-(45\,\hat{i} + 77.94\,\hat{j})$

$\vec{a} - \vec{b} = 41.60\,\hat{i} - 27.94\,\hat{j}$



Step 3

$$|\vec{a} - \vec{b}| = \sqrt{41.60^2 + 27.94^2}$$

$$= 50.11 \text{ m}$$

$$\tan\alpha = \frac{|27.94|}{|41.60|} = 0.6716$$

$$\alpha = 33.89^\circ$$

$|\vec{a} - \vec{b}| = 50.11$ m, 33.89° South of

$|a - b|$ -

East

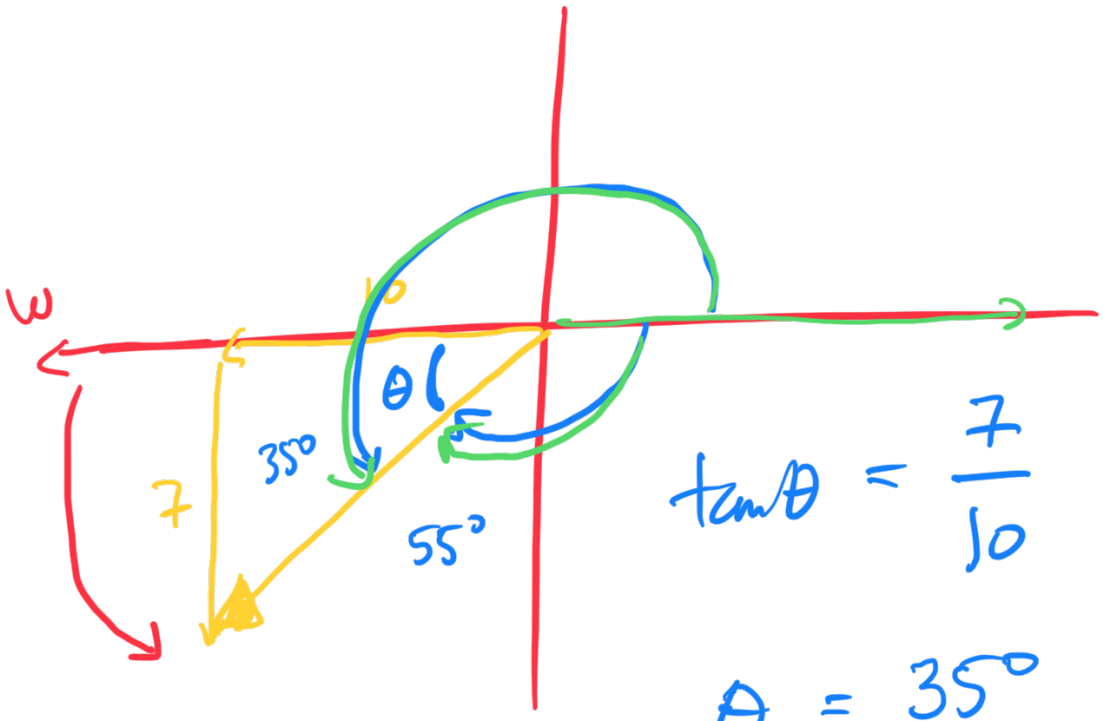
Hints for Vector Addition and Subtraction.

1. Break up all vectors in the problem into x- and y- components.

2. Remember Pythagorean Theorem and SOHCAHTOA

3. **Draw Pictures!!** This is crucial to get the signs of angles correct!!

$$\vec{C} = -10\,\hat{i} - 7\,\hat{j}$$



$$\tan\theta = \frac{7}{10}$$

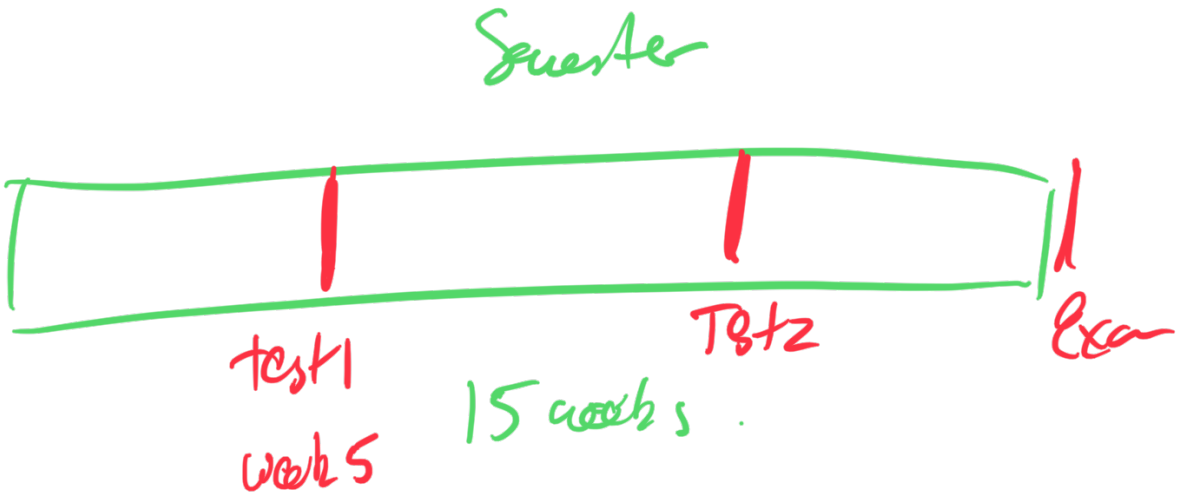
$$\theta = 35^\circ$$

✓ 35° South of West

✓ $\theta = 215^\circ$ $(180^\circ + 35^\circ)$

✓ $\theta = -145^\circ$ ↖ N of E ↖ S of E

M II & Dn. → WAIT



$$y = mt + b$$

y: length (m)

m: length (m) / time (s)

b: length (m)

$$[m][t] = L$$

$[t]$: time (s)

dimension: L

units: m

$C_{m3} = L/T = L\,l$

$v_{m/s}$ m/s