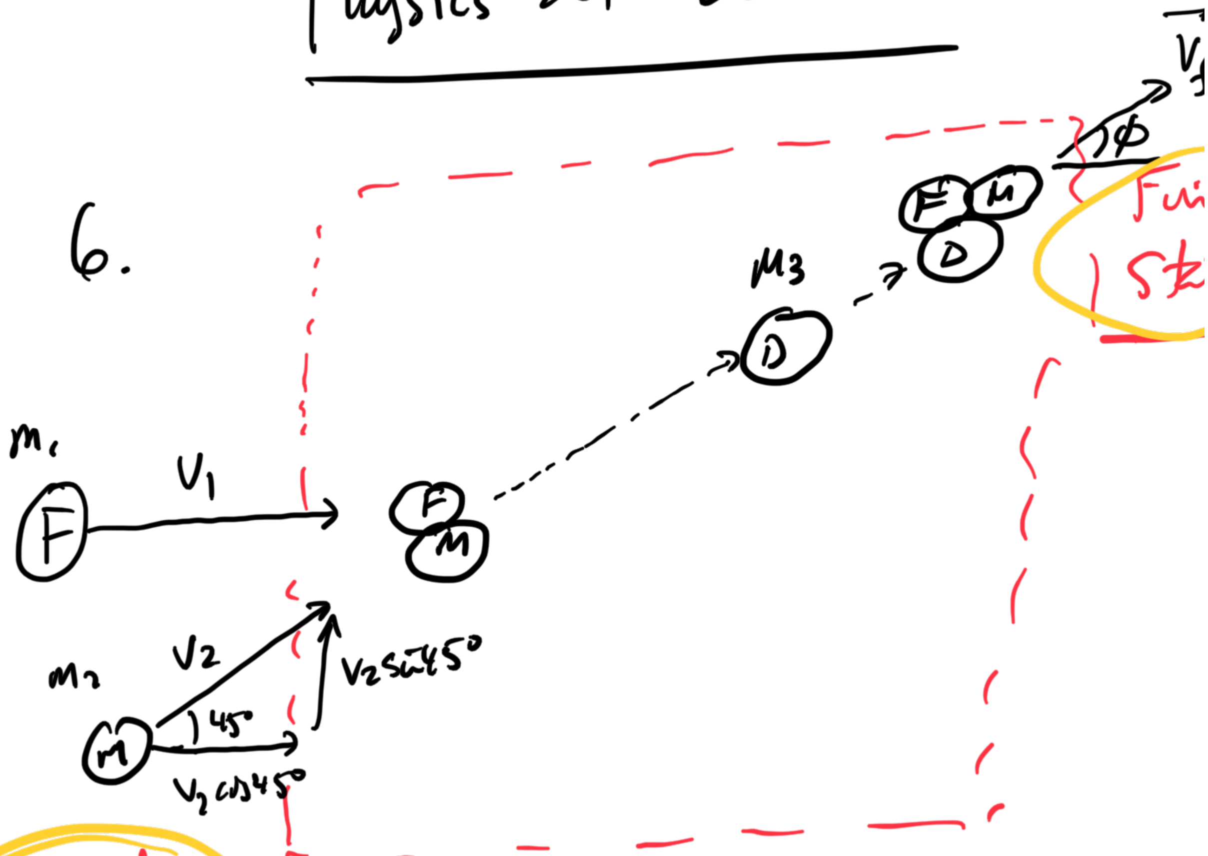
Physics 201 - Lecture 23

6.



Initial State

$$\vec{p}_i^{\,sys} = \vec{p}_i^{\,F} + \vec{p}_i^{\,M} + \cancel{\vec{p}_i^{\,D}}^{\,0}$$

$$\vec{p}_i^{\,sys} = m_1 v_1 \hat{i} + m_2 v_2 \cos 45^\circ \hat{i}$$

$$+ m_2 v_2 \sin 45^\circ \hat{j}$$

$$\vec{p}_f^{\,sys} = (\underset{m_1 + m_2 + m_3}{M}) v_f \cos\phi \, \hat{i} + (M) v_f \sin\phi \, \hat{j}$$

(1) $m_1 v_1 + m_2 v_2 \cos 45^\circ = M v_f \cos\phi$

(2) $m_2 v_2 \sin 45^\circ = M v_f \sin\phi$

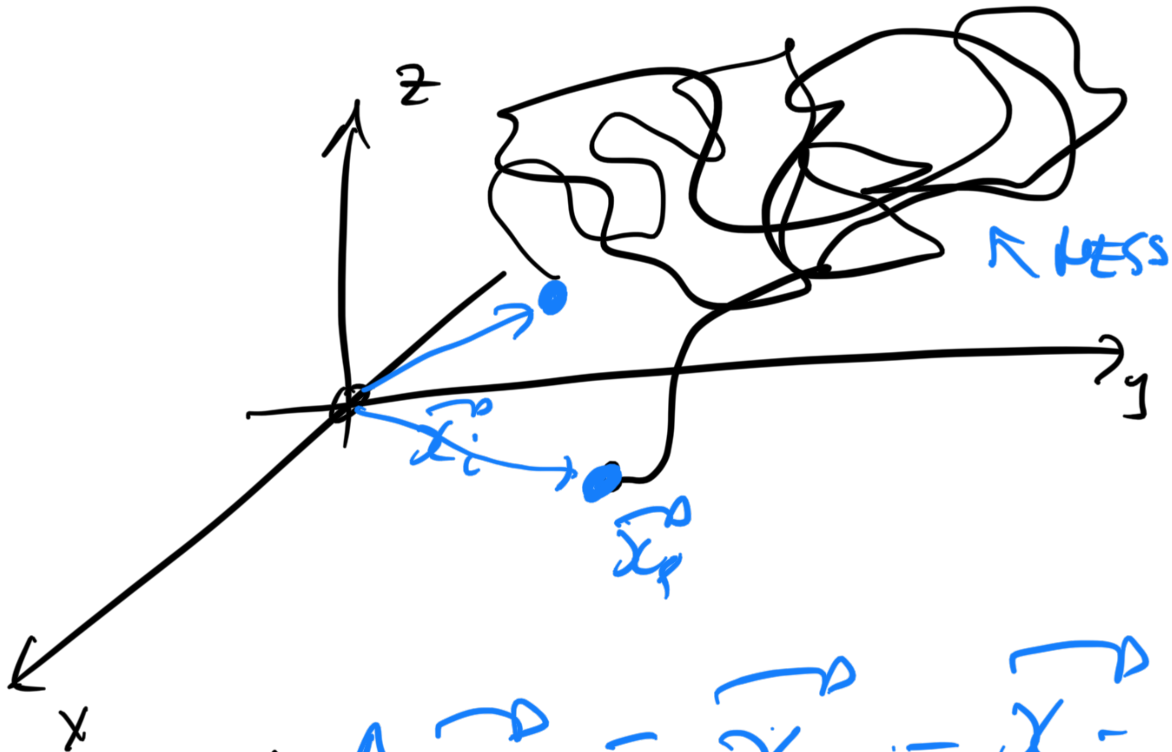
$$\frac{(2)}{(1)} \quad \left(\frac{m_2 v_2 \sin 45^\circ}{m_1 v_1 + m_2 v_2 \cos 45^\circ} \right) = \frac{\cancel{M}\cancel{v_f} \sin\phi}{\cancel{M}\cancel{v_f} \cos\phi} = \tan\phi$$

$$\phi = \tan^{-1}(\text{—})$$

$$= 14^\circ$$

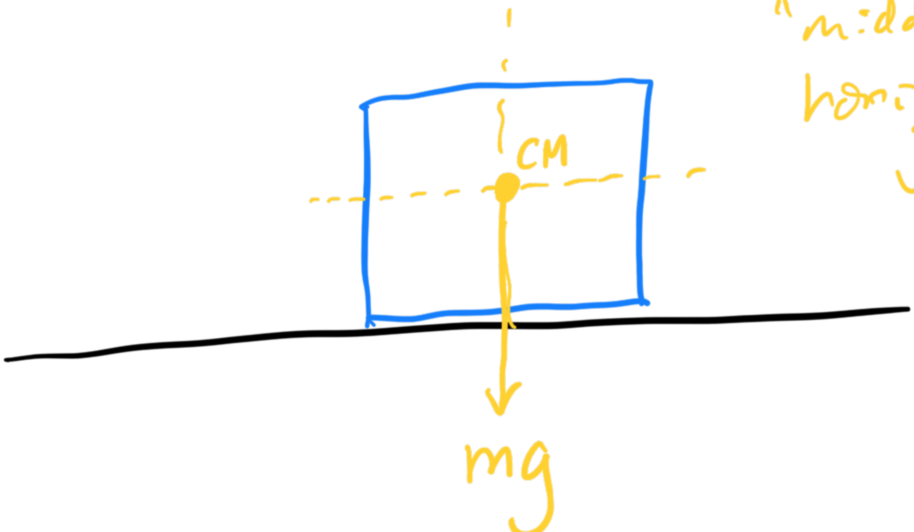
(2) $$v_f = \frac{m_2 v_2 \sin 45^\circ}{M \sin\phi}$$

$$= 4.17 \text{ m/s}$$

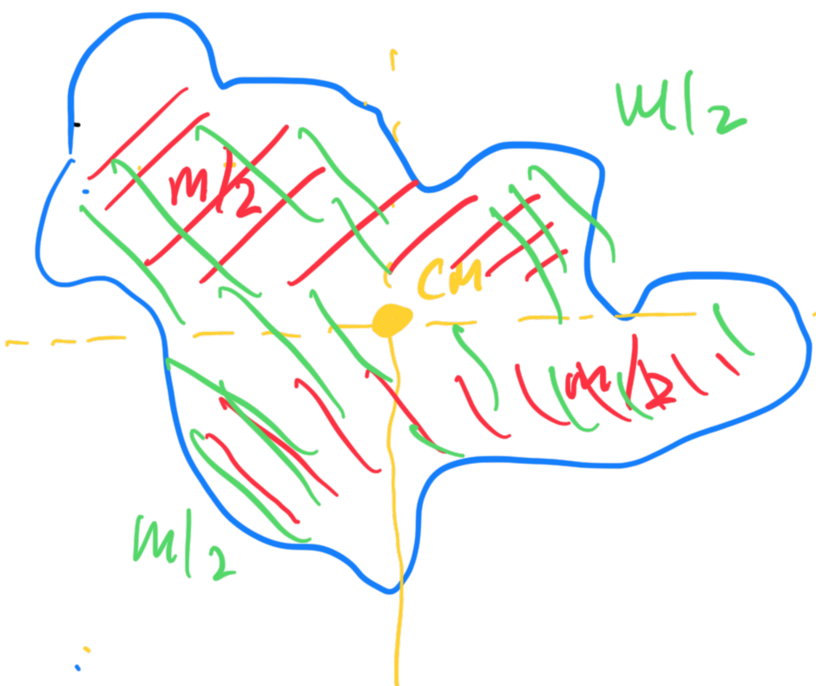


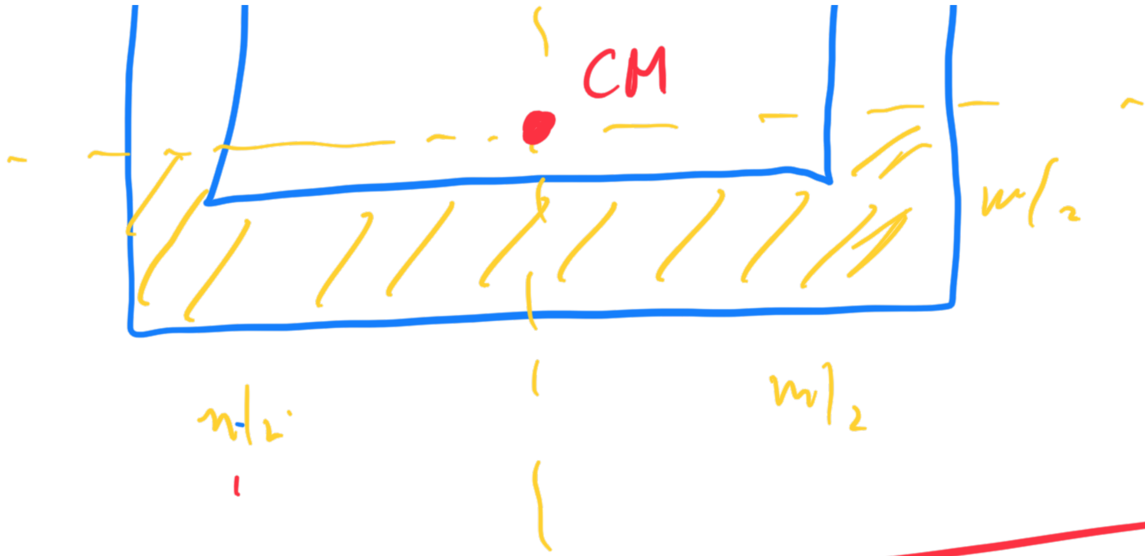
$$\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

Center of Mass → Center of Gravity.



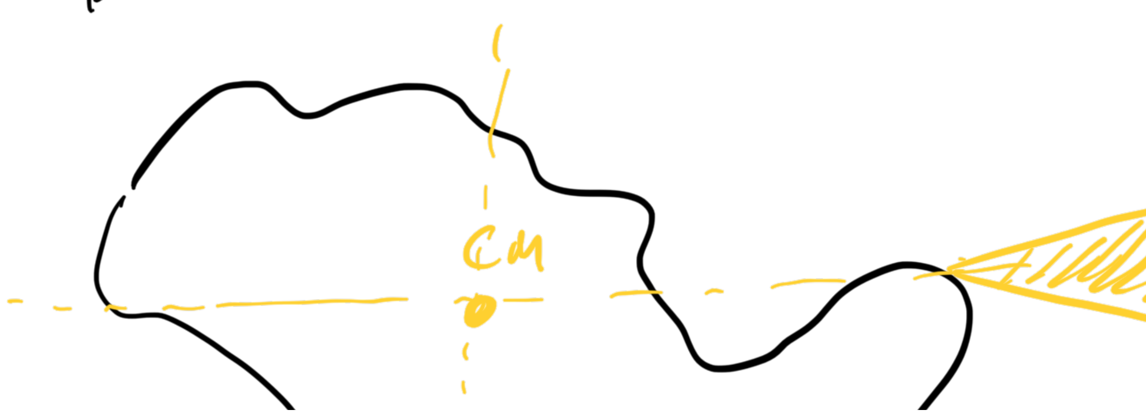
"middle of object horizontally and vertically"





Center of Mass → center of the object, based on how the mass is distributed in space.

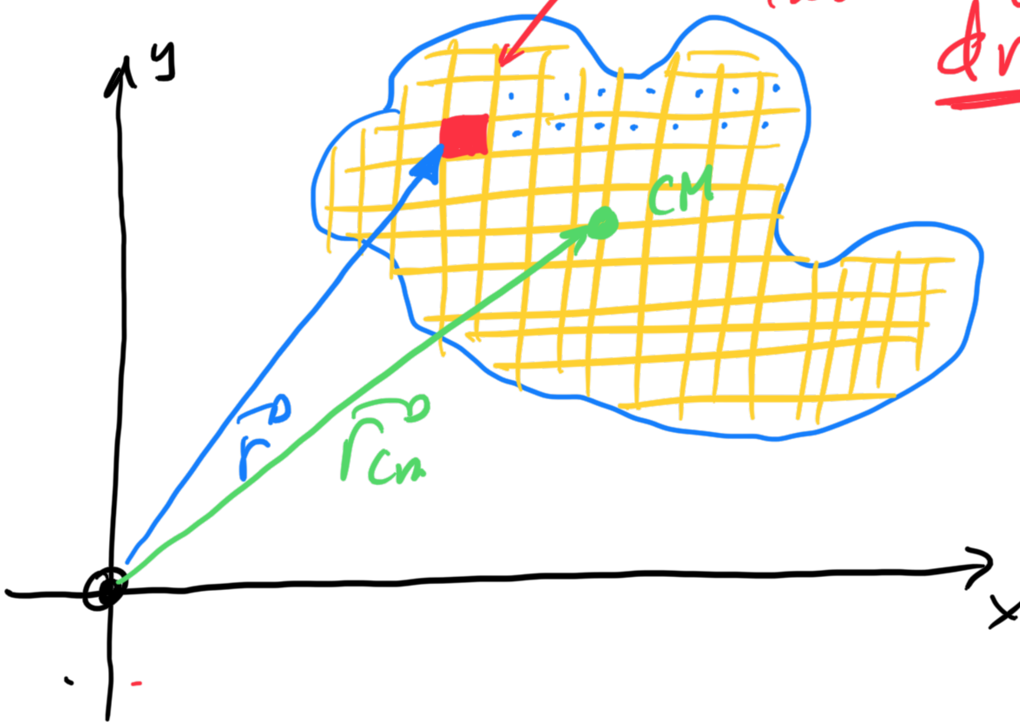
"Balance Point"



Distribution of mass

what is the mass of this rectangle?

dm



$$M\vec{r}_{cm} = \int \vec{r} \cdot dm$$

mass distribution

$$\boxed{\vec{r}_{cm} \equiv \frac{1}{M}\int \vec{r}\cdot dm}$$

continue.

$$\vec{r}_{cm} = x_{cm}\,\hat{i} + y_{cm}\,\hat{j}$$

$$\vec{r} = x\,\hat{i} + y\,\hat{j}$$

$$\therefore \quad x_{cm} = \frac{1}{M}\int x\, dm$$

$$y_{cm} = \frac{1}{M}\int y\, dm$$

What is dm? Really...

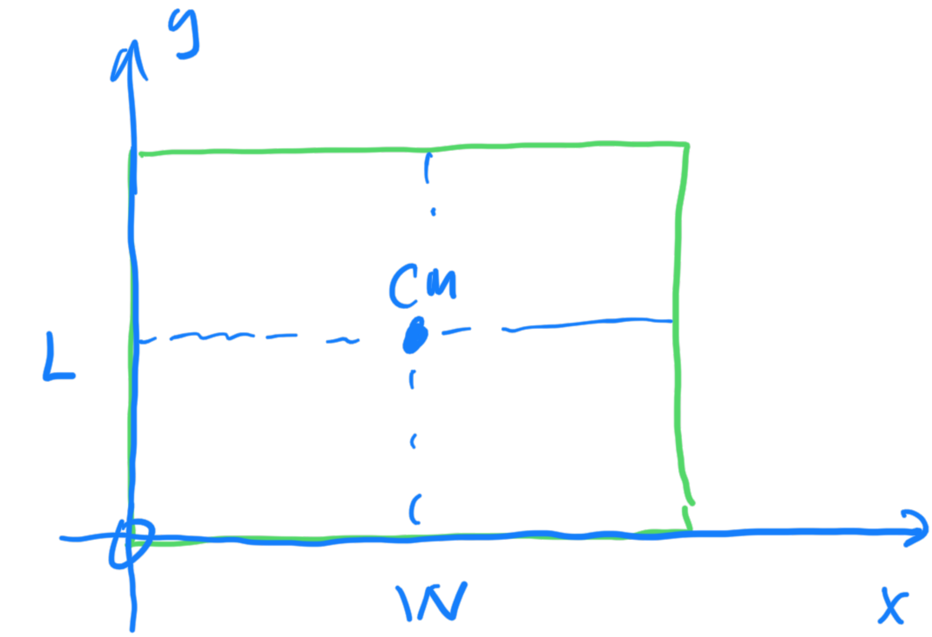
$$d \qquad \rho\, dx\, dy$$

$dm = \rho$

density (in an areal sense)

$$\therefore \quad x_{cm} = \frac{1}{M} \iint x \rho \, dx \, dy$$

$$y_{cm} = \frac{1}{M} \iint y \rho \, dx \, dy$$



$$A = LW \qquad \therefore \quad \rho = \frac{M}{LW}$$

$$x_{cm} = \frac{1}{M}\iint x\rho \, dx \, dy$$

$$= \frac{1}{M} \cdot \frac{M}{LW} \cdot \iint x \, dx \, dy$$

$$= \frac{1}{M} \cdot \frac{M}{LW} \cdot L \cdot \underbrace{\int_0^W x \, dx}_{W^2/2}$$

$$= \frac{1}{\cancel{M}} \cdot \frac{\cancel{M}}{\cancel{L}\cancel{W}} \cdot \cancel{L} \cdot \frac{W^{\cancel{2}}}{2}$$

$$\boxed{x_{cm} = W/2}$$

Similarly, $y_{cm} = \frac{L}{2}$