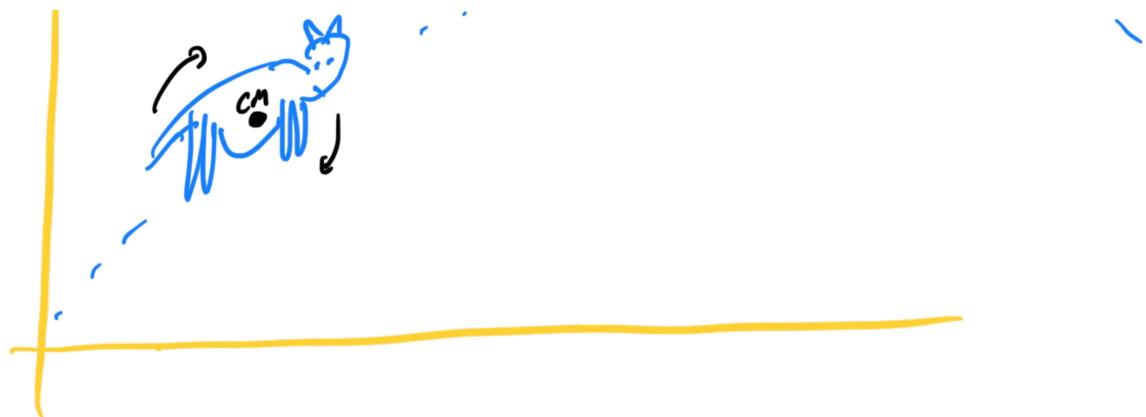
Physics 201 - Lecture 25

1. All 6 assignments are open / reopened until next Thursday!

2. Test is next Friday, April 9.
   More on the format next week, but for now → $\boxed{\text{will cover A4, A5, A6}}$

3. Today: Next Topic ....
   rotational motion.

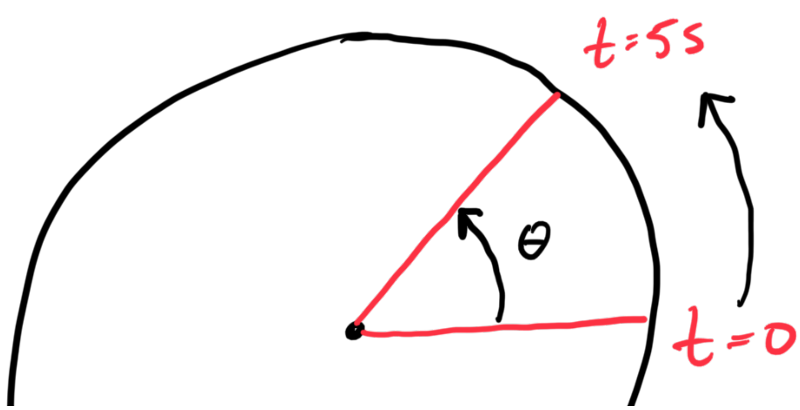
Imagine a complex shaped object traveling through the air.

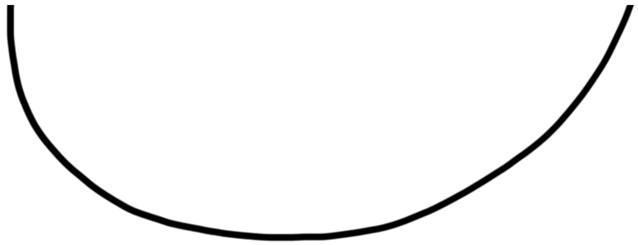


The complete motion can be described in two parts.

we know this! → (i) the translational motion of the centre of mass.

(ii) The rotational motion around the centre of mass. ← We need to know this!!





Consider a simple wheel, of radius R, that is rotating.

Questions.

1. Where is it? position $\vec{x}(t)$
2. How far has it moved from its original position? displacement $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$
3. How fast is it moving, and in what direction? velocity $\vec{v}(t)$
4. How is the velocity changing? acceleration $\vec{a}(t)$

① where is it?

↳ where is the red line?

$\theta(t)$

$\theta(t)$

$\boxed{\text{angular position}}$

(2) How far has it moved from its original position?

$$\Delta\theta = \theta_f - \theta_i$$

$\boxed{\text{angular displacement}}$

(3) How fast is it moving and in what direction?

$\boxed{\text{Angular velocity}}$

$$\omega = \frac{d\theta}{dt} \quad \left(\frac{\Delta\theta}{\Delta t}\right)$$

omega

$$\left(\vec{v} = \frac{d\vec{x}}{dt}\right)$$

rate of change of the

angular position.

④ How is the velocity changing?

$\boxed{\text{angular acceleration.}}$

$$\alpha = \frac{d\omega}{dt} \quad \left(\frac{\Delta\omega}{\Delta t}\right)$$

rate of change of the angular velocity.

alpha.

$$\left(\vec{a} = \frac{d\vec{v}}{dt}\right)$$

Translational		Rotational	
$\vec{x}$	$\rightarrow$	θ	(rad)
$\Delta\vec{x}$	$\rightarrow$	$\Delta\theta$	(rad)
$\vec{v}$	$\rightarrow$	ω	(rad/s)
$\vec{a}$	$\rightarrow$	α	(rad/s^2)

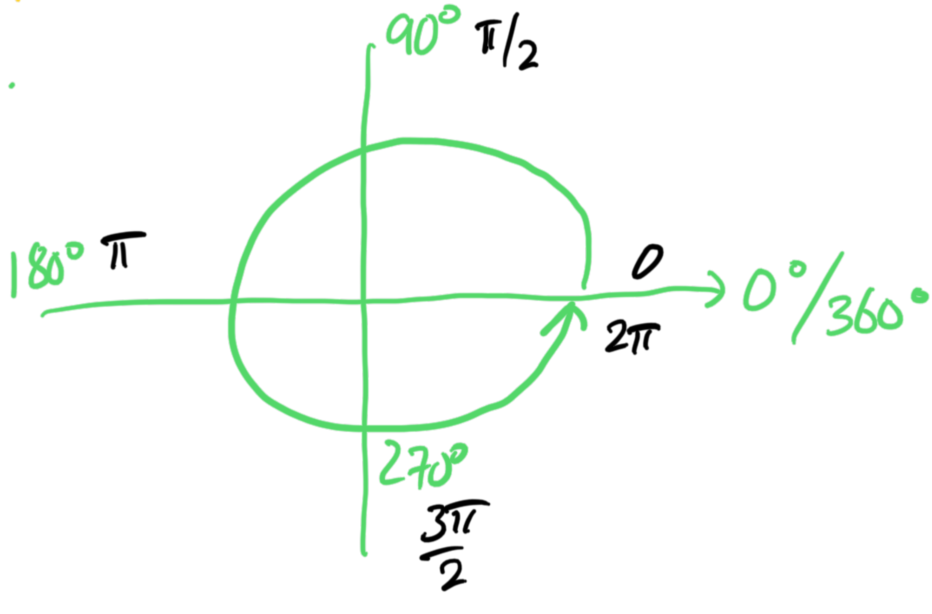
ω

vectors | ?

Problems

(1) Units

$\theta \rightarrow$ ~~degrees~~ $\boxed{\text{radians}}$



$$\boxed{\pi = 180^\circ}$$

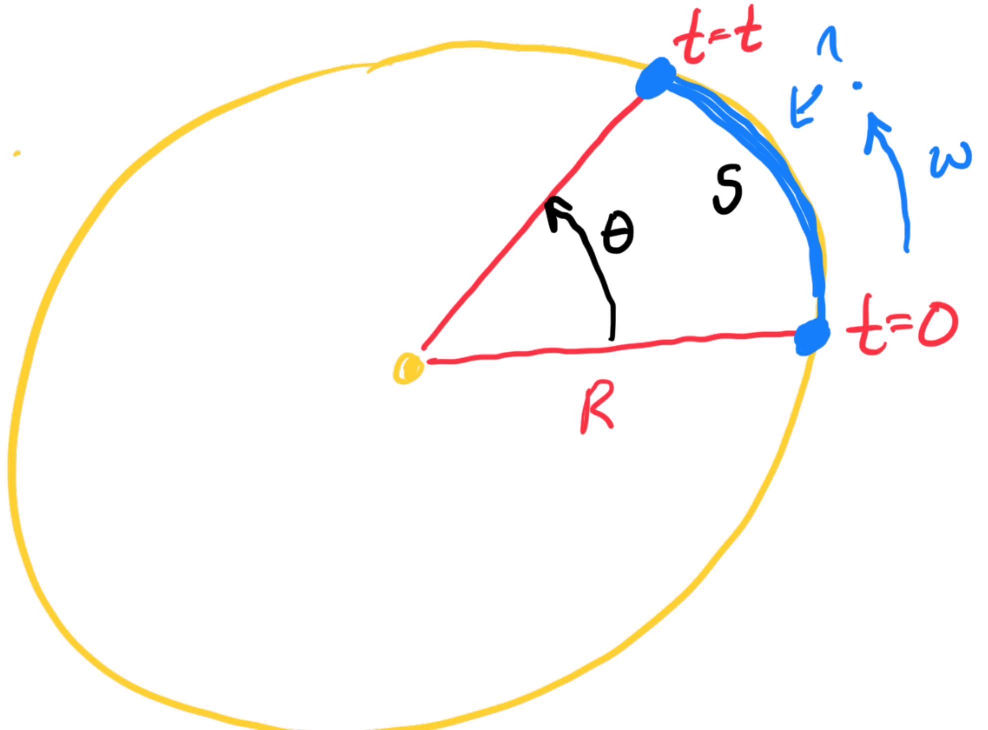
$\Delta\theta \rightarrow$ radians

$\Delta\theta$

$$\omega \rightarrow \left(\frac{d\theta}{dt}\right) \rightarrow \frac{rad}{s}$$

$$\alpha \rightarrow \left(\frac{d\omega}{dt}\right) \rightarrow rad/s^2$$

2 rotational $\longleftrightarrow$ translational.



Imagine that the wheel rotates for t seconds.

Question: How far did the blue striker move?

What is the path length?

$$\boxed{S = R \cdot \theta} \leftarrow \underbrace{\frac{ds}{dt}}_{v_T} = R \cdot \underbrace{\frac{d\theta}{dt}}_{\omega}$$

Radians!!

Question: How fast is the blue striker moving?

$$\boxed{v_T = R \cdot \omega}$$

(translational velocity)

$$\underbrace{\frac{dv_T}{dt}}_{a_T} = R \cdot \frac{d\omega}{dt} = R \cdot \alpha$$

How is the velocity of the

Question :

blue sticker changing?

$$\boxed{a_T = R \cdot \alpha}$$

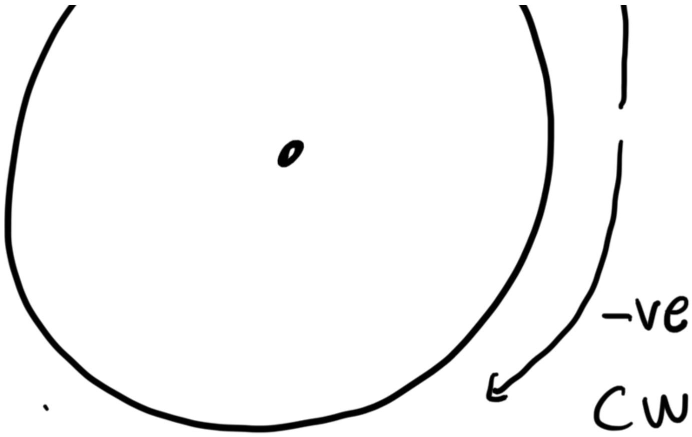
$$\boxed{\begin{aligned} S &= R \cdot \theta \\ v_T &= R \cdot \omega \\ a_T &= R \cdot \alpha \end{aligned}}$$

③ What about $\boxed{\text{vectors?}}$

It's complicated....

$\boxed{1D}$

2



Motion in 1D with constant acceleration.

$\vec{a}$ = constant

5 variables

Δx

v_i

v_f

a

t

5 equations.

$$v_f = v_i + at$$

$$\Delta x = v_i t + \frac{1}{2}at^2$$

$$\Delta x = v_f t - \frac{1}{2}at^2$$

$$\Delta x = \left(\frac{v_f + v_i}{2}\right) t$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

5 variables

$\Delta\theta$

ω_i

ω_f

α

t

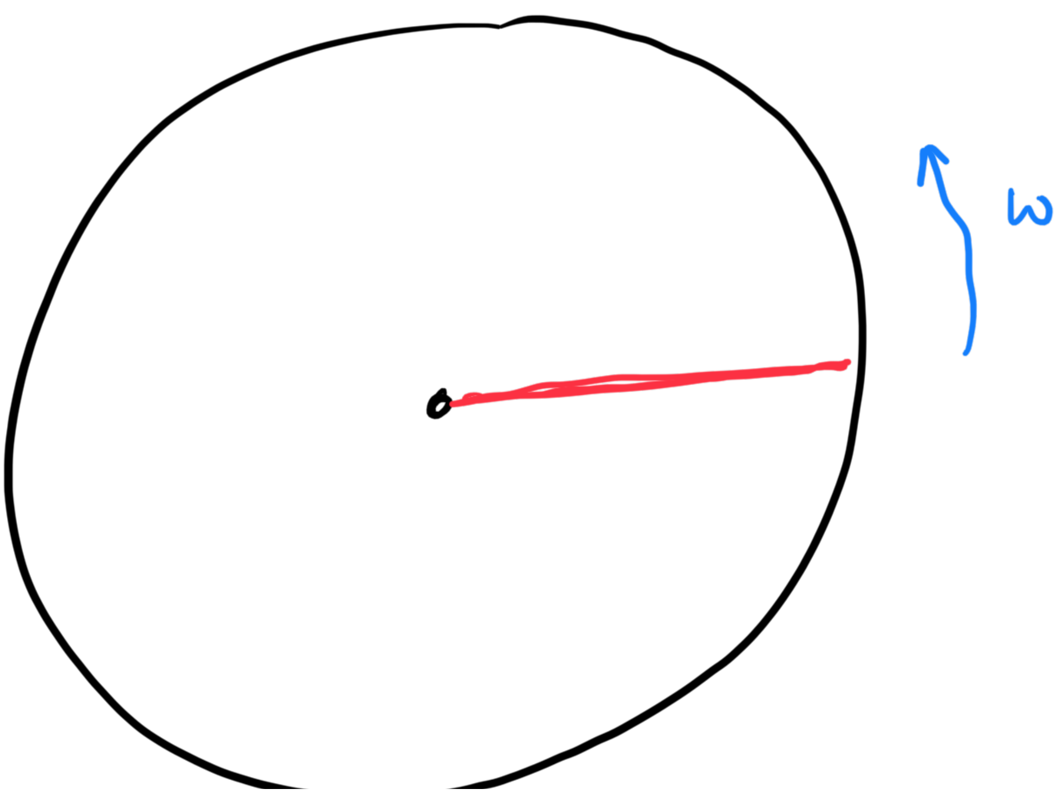
$$\omega_f = \omega_i + \alpha t$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \left(\frac{\omega_f + \omega_i}{2}\right) t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$



$$\omega = 1.6 \text{ rad/s}$$

$$\alpha = 0$$

$$\theta_i = 0$$

$$\Delta\theta = \omega_i t + \cancel{\tfrac{1}{2}\alpha t^2}^{\,0}$$

$$\Delta\theta = 1.6t$$

$$= \theta_f - \cancel{\theta_i}^{\,0}$$

$$\boxed{\theta_f = 1.6t}$$

$$t = 11.3529 \text{ s}$$

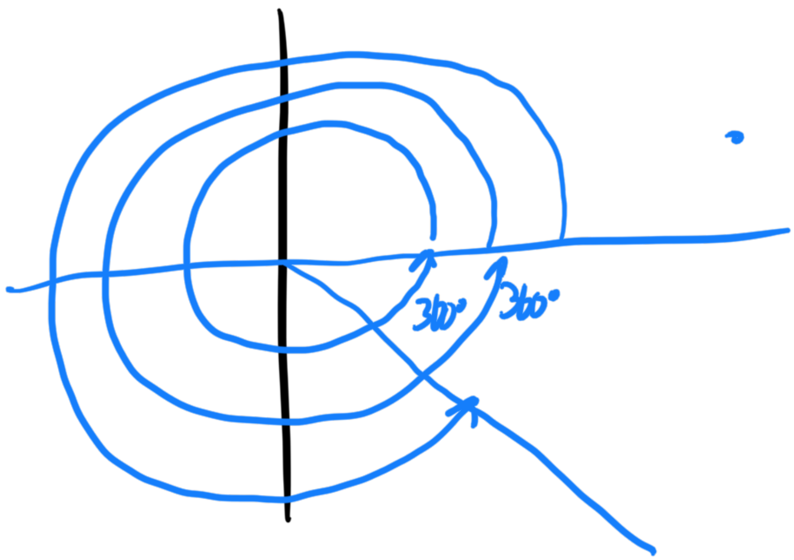
$$\theta_f = (1.6)(11.3529)$$

$$= 18.1646 \text{ radians.}$$

$= 18.1646$

$$= 18.1646 \times \frac{180^\circ}{\pi}$$

$= 1040.8$ degrees.



$$360^\circ + 360^\circ + \underline{x} = 1040.8$$

$$x = 1040.8 - 720$$

$$= 320.8^\circ$$