Physics 201 – Lecture 22

Today → Assignment 6

① $\boxed{\vec{p} = m\vec{v}}$ $\hat{i}$

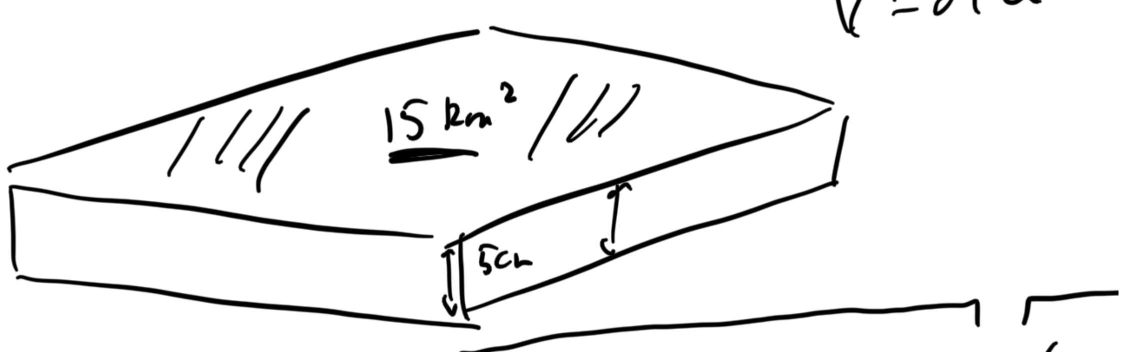
$= (2200\ \text{kg})(23\ \text{m/s East})$

$= 50600\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$ East

② "5 cm of rain over 15 km^2 in 3 hrs"

How much water (in kg) falls in 1 s?

$V = Ad$



$$\text{Volume} = 15 \times (1000\text{ m}) \times (1000\text{ m}) \times (.05\text{m}$$

$$= \underline{750{,}000}\ \text{m}^3$$

$$\text{Volume in } \underline{1\text{s}} = \frac{750{,}000\ \text{m}^3}{\underline{3 \times 3600\ \text{s}}}$$

$$= 69.44\ \text{m}^3/\text{s}$$

$\underline{1\ \text{m}^3}$ of water has a mass of $\underline{1000\ \text{kg}}$

$$\text{mass of water in } \underline{1}\text{s} = \frac{69\,444.4\ \text{kg}}{\times 6}$$

$$\text{mass of water in } 6\text{s} = \boxed{416{,}666.67\ \text{kg}}$$

$$\vec{p} = m\vec{v}$$

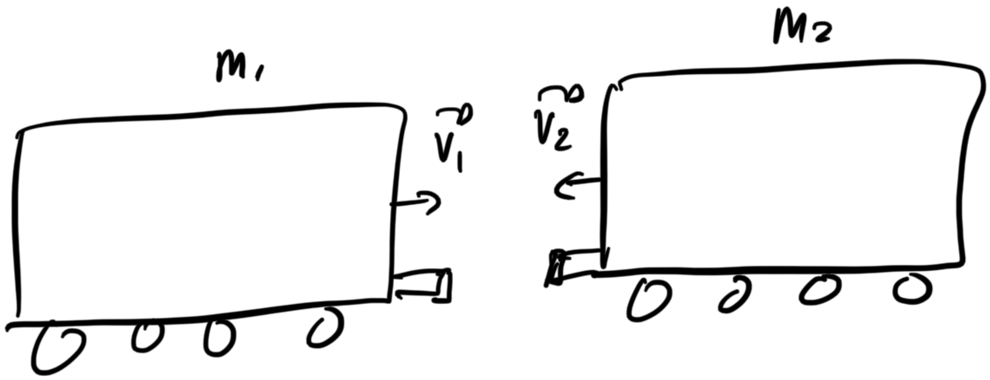
$$\vec{v} = 10\ \text{m/s} \downarrow$$

$$\vec{p} = (416{,}666.67\ \text{kg})(10\ \text{m/s} \downarrow)$$

$$= \underline{4.17 \times 10^6}\ \text{kg m/s} \downarrow$$

$$|\vec{p}| = 4.17 \times 10^{0} \ \mathrm{kg \cdot m/s}$$

3



$$\vec{P}^{(i)}_{sys} = \vec{p}_1 + \vec{P}_2$$

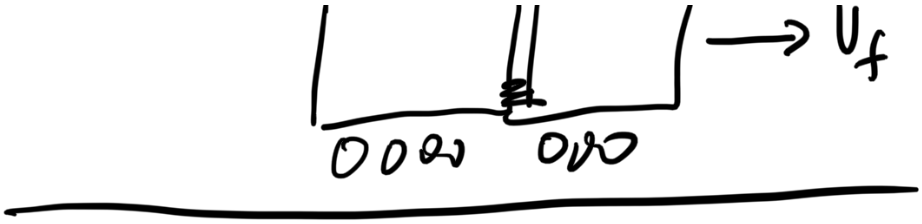
$$= m_1\vec{v}_1 + m_2\vec{v}_2$$

$$= (1.51 \times 10^5)(0.30 \mathrm{m/s}\ \hat{i})$$

$$+ (1.06 \times 10^5)(-0.12\ \hat{i}$$

$$= 4.53 \times 10^4\ \hat{i} - 1.272 \times 10^5$$

$$= \boxed{3.258 \times 10^4}\ \hat{i}$$



$$\vec{p}^{\,f}_{sys} = (m_1 + m_2)\, v_f\, \hat{i}$$

$$= \boxed{(2.56 \times 10^5)\, v_f}\ \hat{i}$$

$$\vec{p}_i = \vec{p}_f \qquad \Delta\vec{p} = 0$$

$$\vec{F}_{net} = 0$$

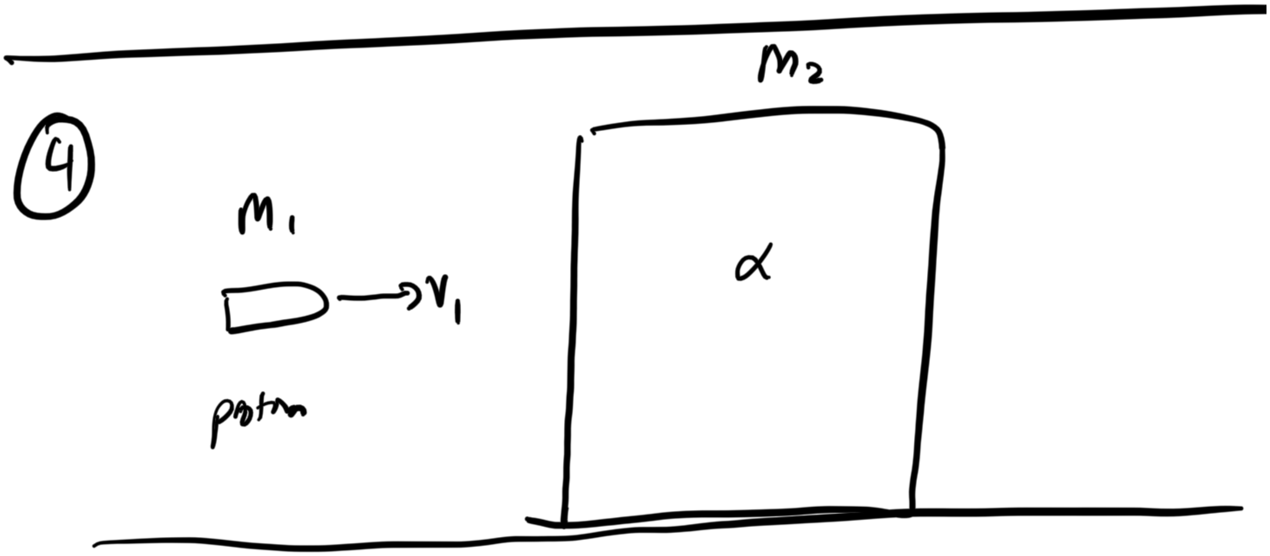
$$3.258 \times 10^4 = 2.56 \times 10^5\, v_f$$

$$\boxed{v_f = 0.127 \text{ m/s}}$$

$$\vec{v}_f = \boxed{0.127\, \hat{i}}$$

"stick together" → totally inelastic

4

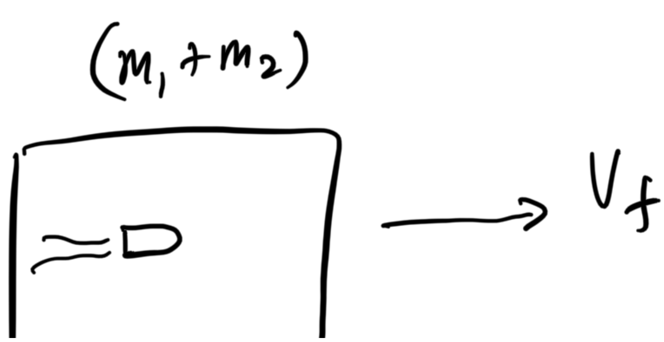


$$\vec{P}^{\,i}_{sys} = \vec{P}_1 + \vec{P}_2$$

$$= m_1 \vec{v}_1 + \cancel{m_2 \vec{v}_2}^{\,0}$$

$$= (.230\,kg)(405\,m/s\,\hat{i})$$

$$= \underline{93.15\,kg\,m/s\,\hat{i}}$$



$$\vec{p}^{\,f}_{sys} = (m_1 + m_2)\vec{v}_f$$

$$= (4.23\ \text{kg})\ \vec{v}_f$$

$$\Delta \vec{p}_{sys} = 0$$

$$4.23\ \vec{v}_f = 93.15\ \hat{i}$$

$$\boxed{\vec{v}_f = 22.0\ \text{m/s}\ \hat{i}}$$

impulse? $\Rightarrow$ $\Delta \vec{p}$

position $\rightarrow$ $\vec{x}$
displacement $\rightarrow$ $\Delta\vec{x}$

momentum $\rightarrow$ $\vec{p}$
impulse $\rightarrow$ $\Delta\vec{p}$

b) $\Delta \vec{p}_{bullet}$

$\vec{p}_i \text{ (bullet)} = m_1 \vec{v}_1 = 93.15\ \hat{i}$

$\vec{p}_f \text{ (bullet)} = m_1 \vec{v}_f = (.230)(22\ m/s\ \hat{i}$

$= 5.065\ \hat{i}$

$\Delta \vec{p}_{bullet} = \vec{p}_f - \vec{p}_i$

$= 5.065\ \hat{i} - 93.15\ \hat{i}$

$= -88.1\ \hat{i}$

$|\Delta \vec{p}| = 88.1$

direction $\rightarrow$ west

c) $\Delta \vec{P}_{block}$

$\vec{p}_i = 0$

$\vec{p}_f = (4.0\,kg)(22.0\,m/s\ \hat{i})$

$= 88.1\ kg\,m/s\ \hat{i}$

$\Delta \vec{P}_{block} = \vec{p}_f - \vec{p}_i$

$= 88.1\,\hat{i} - 0\,\hat{i}$

$= +88.1\ kg\,m/s\ \hat{i}$

East

$\boxed{\Delta \vec{P}_{system} = 0} = -88.1\,\hat{i} + 88.1\,\hat{i}$

d) $\boxed{F_{net} = \frac{\Delta p}{\Delta t}}$

$$= \frac{-88.1 \text{ kgm/s } \hat{i}}{0.003 \text{ s}}$$

$$= -29,362 \underbrace{\text{kg m/s}^2}_{N} \hat{i}$$

$$\boxed{\vec{F}_{net} = -29,362 \text{ N } \hat{i}}$$

$\xleftarrow{F_{net}}$

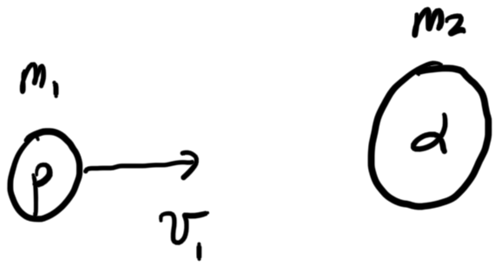
D → 405 m/s　　　　D → 22 m/s

$\vec{v}_3, \vec{v}_4$

(3)

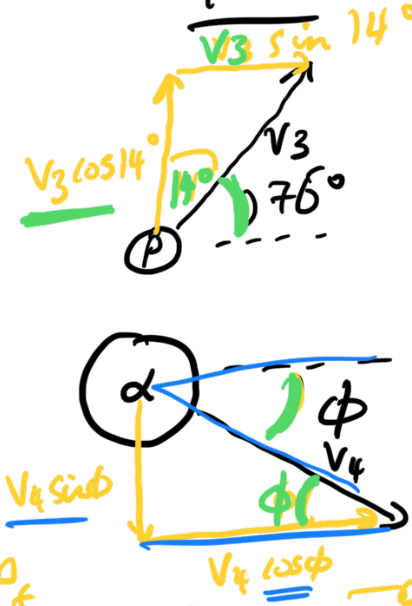
v_1, v_2

Initial



$$\boxed{\vec{p}^{\,i}_{sys} = m_1 v_1 \hat{i}}$$

Final



$$\vec{p}^{\,f}_{sys} = \vec{p}_3 + \vec{p}_4$$

$$= m_1\left(v_3 \cos 14° \hat{j} + v_3 \sin 14° \hat{i}\right) + m_2\left(-v_4 \sin\phi \hat{j} + v_4 \cos\phi \hat{i}\right)$$

x: $$\boxed{m_1 v_1 = m_1 v_3 \sin 14° + \underset{4m_1}{\cancel{m_2}} v_4 \cos}$$

y: $$\boxed{0 = m_1 v_3 \cos 14° - \underset{4m_1}{\cancel{m_2}} v_4 \cos}$$

3 unknowns

"elastically" → Kinetic energy is conserved.

$$K_i = K_f$$

$$\boxed{\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 V_3^2 + \frac{1}{2} \cancel{m_2}_{4m_1} v}$$

Wolfram Alpha ← try this.

$\boxed{m_2 = 4m_1}$

3.1×10^6 m/s

x

y

$V_1 = V_3 \sin 14° + 4 V_4 \cos\phi$ ①

$0 = V_3 \cos 14° - 4 V_4 \sin\phi$ ②

$\frac{1}{2} v_1^2 = \frac{1}{2} \boxed{V_3}^2 + 2 \boxed{V_4}^2$ ③

Oops ... WA fails !

$$\left(\cos\phi\right)^2 = \frac{V_1 - V_3 \sin(\theta)}{4 V_4}$$

TRICK

$$\left(\sin\phi\right)^2 = \frac{V_3 \cos\theta}{4 V_4}$$

SQUARE + ADD

(1) $$1 = \left(\frac{V_1 - V_3 \sin\theta}{4 V_4}\right)^2 + \left(\frac{V_3 \cos\theta}{4 V_4}\right)^2$$

(2) $$V_1^2 = V_3^2 + 4 V_4^2$$

Wolfram Alpha success !!

$$V_3 = 2.556 \times 10^6 \text{ m/s}$$

$$V_4 = 8.771 \times 10^5 \text{ m/s}$$

$$(\phi = 44.98^\circ)$$

Kinetic Energy Fraction

$$f = \frac{\frac{1}{2} m_1 v_3^2}{\frac{1}{2} m_1 v_1^2} = \frac{v_3^2}{v_1^2}$$

$$= \left(\frac{2.556}{3.1}\right)^2 = \underline{0.68}$$

$$= \underline{68\%}$$