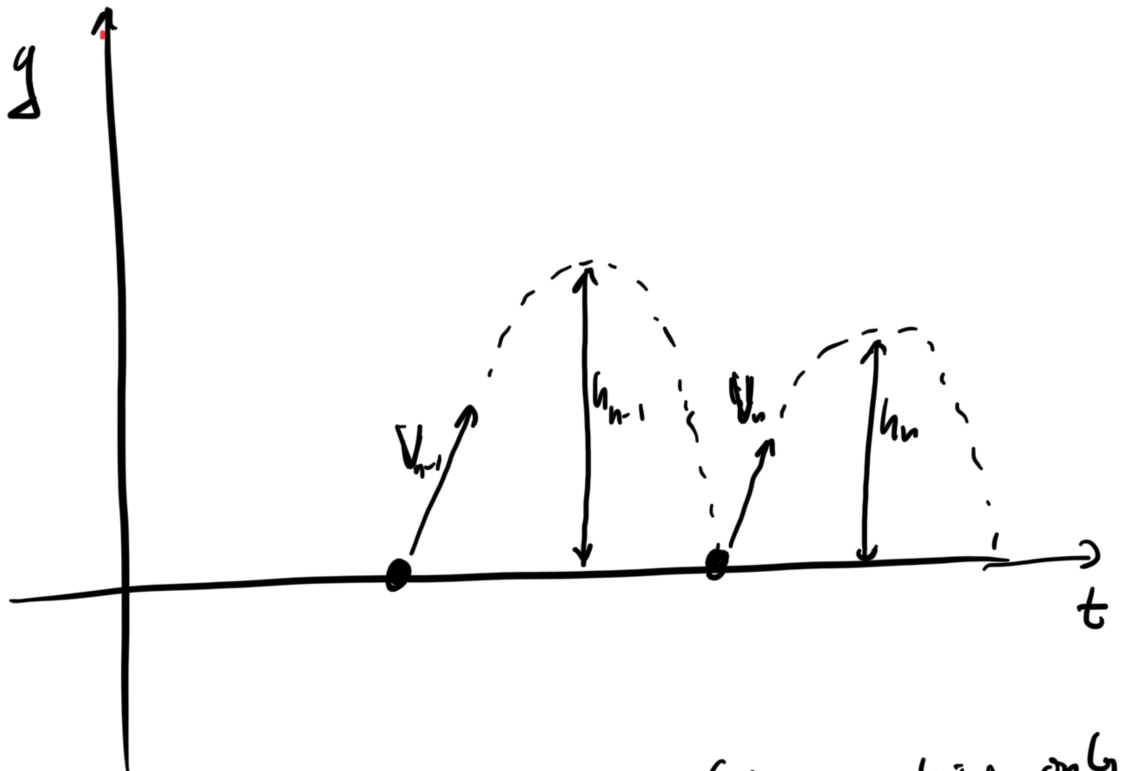


→ Assume vertical bouncing (i.e. motion only in the y direction.

$$V_{iy} = V_{n-1}$$

$$V_{fy} = 0$$

$$a_y = -g$$

$$\Delta y = h_{n-1}$$

$$V_{fy}^2 = V_{iy}^2 + 2a_y \Delta y$$

$$0 = v_{n-1}^2 - 2g\,h_{n-1}$$

$$h_{n-1} = \frac{v_{n-1}^2}{2g} \quad \therefore v_{n-1} = \sqrt{2g\,h_{n-1}}$$

Similarly, $h_n = \frac{v_n^2}{2g} \quad \therefore v_n = \sqrt{2g\,h_n}$

① Method 1

$$e \equiv \frac{v_n}{v_{n-1}} = \frac{\sqrt{2gh_n}}{\sqrt{2g\,h_{n-1}}} = \sqrt{\frac{h_n}{h_{n-1}}}$$

$$\boxed{e = \left(\frac{h_n}{h_{n-1}}\right)^{1/2}}$$

Let $v_n = e\,v_{n-1}$

then $v_1 = e\,v_0$

$v_2 = e\,v_1 = e^2 v_0$

$v_3 = e\,v_2 = e^3 v_0$

etc.

$\therefore v_n = e^n v_0$

$$\therefore \; v_n = e^n v_0$$

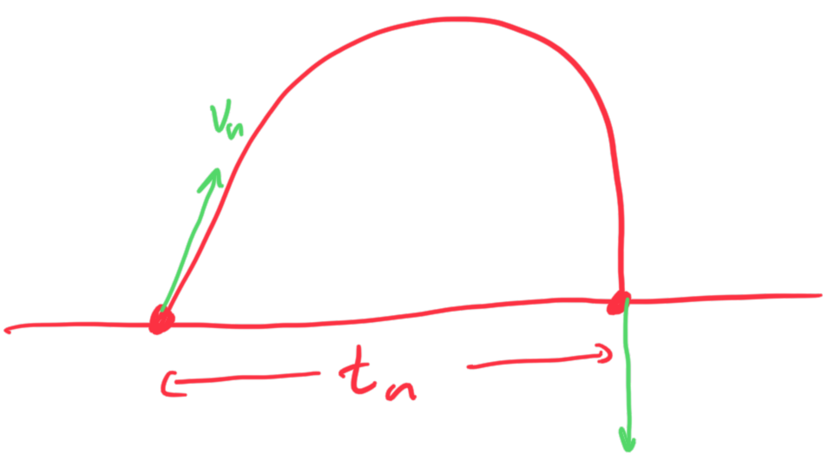
$$\therefore \; e = \left(\frac{v_n}{v_0}\right)^{1/n} = \left(\frac{\sqrt{2g h_n}}{\sqrt{2g h_0}}\right)^{1/n}$$

$$= \left(\sqrt{\frac{h_n}{h_0}}\right)^{1/n}$$

$$= \left(\left(\frac{h_n}{h_0}\right)^{1/2}\right)^{1/n}$$

$$\boxed{e = \left(\frac{h_n}{h_0}\right)^{\frac{1}{2n}}}$$

Method 2 :



v_n

$v_i = v_n$

$v_f = -v_n$

$a = -g$

$t = ?$

$$v_f = v_i + at$$

$$-v_n = v_n - g t_n$$

$$t_n = \frac{2v_n}{g} \quad \therefore v_n = \frac{g t_n}{2}$$

$$e \equiv \frac{v_n}{v_{n-1}} = \frac{g t_n/2}{g t_{n-1}/2} = \left(\frac{t_n}{t_{n-1}}\right)$$

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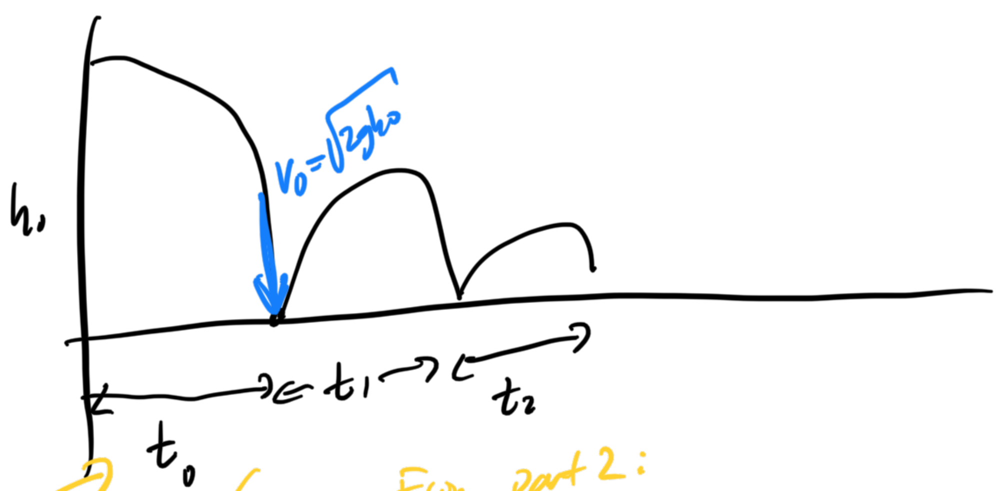
3. <u>Time to stop.</u>



$t_0 = ?$

From part 2:

$$\left(t_n = \frac{2v_n}{\ } \quad (n \geq 1) \right.$$

g

$v_i = 0$

$\Delta y = -h_0$

$a_y = -g$

$\Delta y = \cancel{v_i t} + \frac{1}{2} a_y t^2$

$-h_0 = \frac{-g}{2} t_0^2$

$$\boxed{t_0 = \sqrt{\frac{2h_0}{g}}}$$

$t_1 = \frac{2}{g}(eV_0)$

$t_2 = \frac{2}{g}(e^2 V_0)$

$t_3 = \frac{2}{g}(e^3 V_0)$

$\vdots$

$t_n = \frac{2}{g}(e^n V_0)$

$$t_s = t_0 + \sum_{n=1}^{\infty} t_n$$

$$t_s = \sqrt{\frac{2h_0}{g}} + \sum_{n=1}^{\infty} \frac{2}{g} e^n V_0$$

$= \sqrt{2gh_0}$

$$t_s = \sqrt{\frac{2h_0}{g}} + \frac{2\sqrt{2gh_0}}{g} \underbrace{\sum_{n=1}^{\infty} e^n}_{= \frac{e}{1-e}}$$

from Wolfr Al

if $e < 1$ which it is!

$$t_s = \sqrt{\frac{2h_0}{g}} + \sqrt{\frac{2h_0}{g}} \left(\frac{2e}{1-e}\right)$$

$= c$ $\qquad c$

$$t_s = c + \frac{2ce}{1-e}$$

$$t_s = \frac{c(1-e) + 2ce}{1-e}$$

$$t_s = \frac{c - ce + 2ce}{1-e} = \frac{c + ce}{1-e}$$

$$= c\left(\frac{1+e}{1-e}\right)$$

$$t_s(1-e) = c(1+e)$$

$$t_s - t_s e = c + ce$$

$$t_s - c = ce + t_s e$$

$$= e(c + t_s)$$

$$e = \frac{t_s - c}{t_s + c} = \frac{t_s - \sqrt{\frac{2h_0}{g}}}{t_s + \sqrt{\frac{2h_0}{g}}}$$ ✓