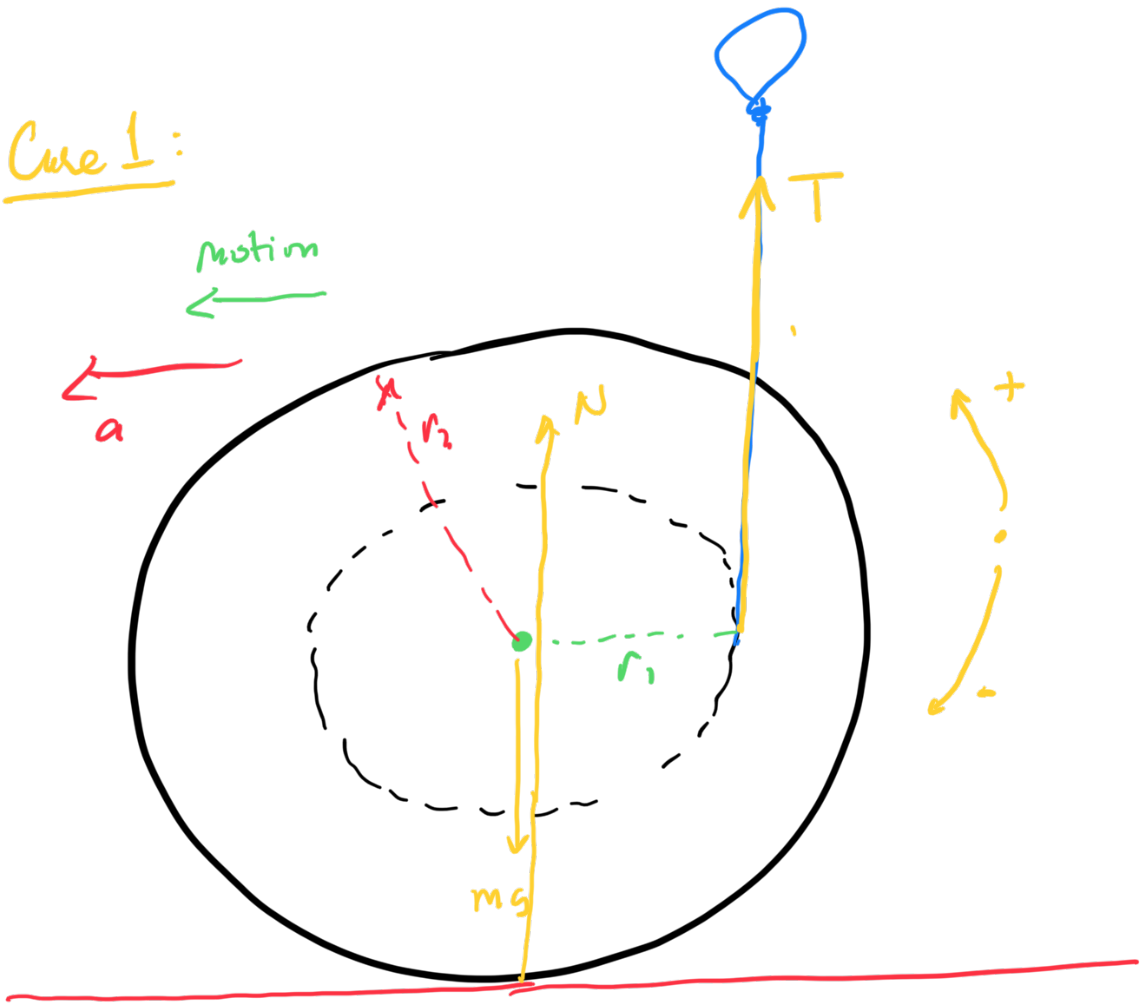
The Yo-Yo Problem.

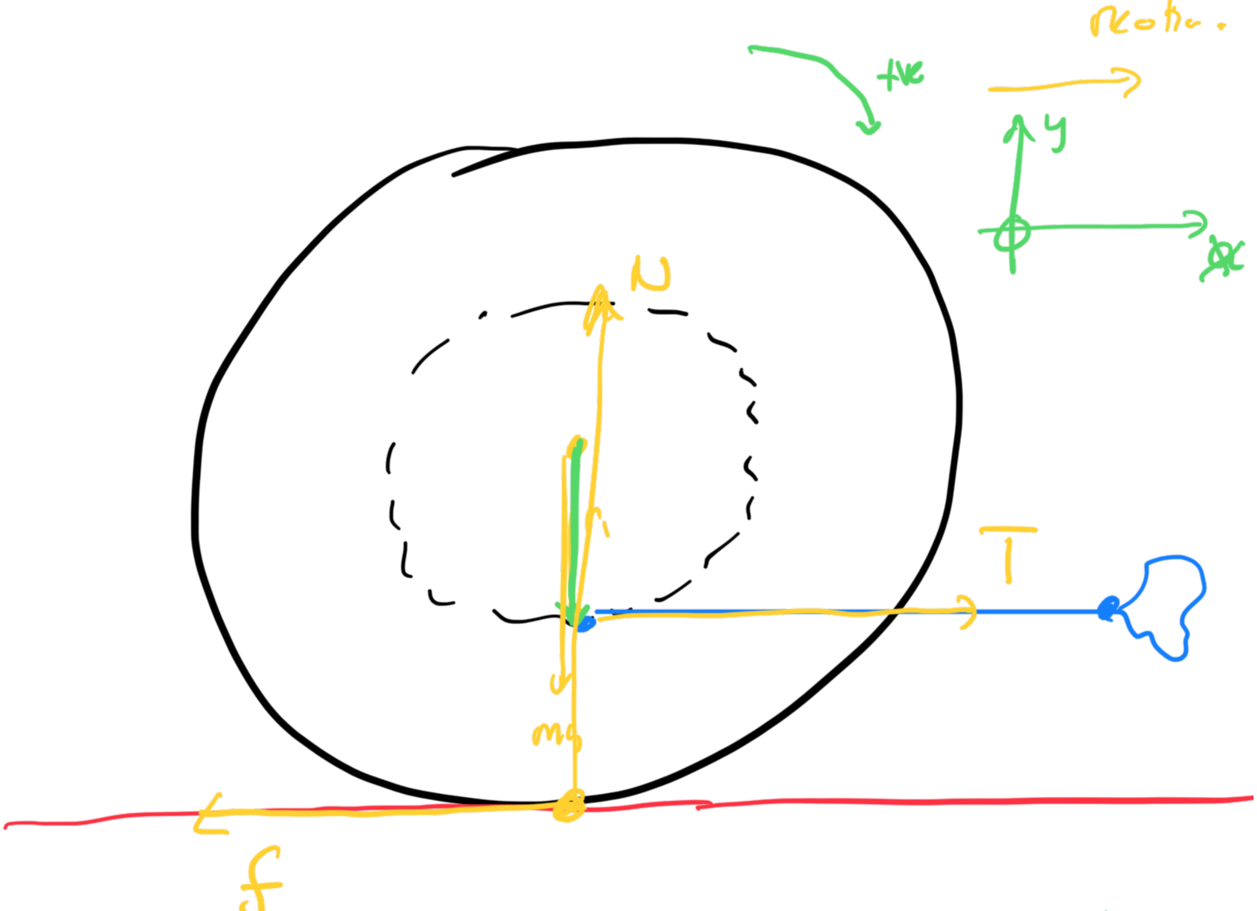
Case 1:



$$r_1 T = I\alpha = I\left(\frac{a}{r_2}\right)$$

$$\boxed{a = \frac{T r_1 r_2}{I}}$$

Case2 :



Problem : Tension causes a negative torque, but the acceleration is positive. How?

$\boxed{\text{FRICTION}}$

$T - f = ma$

x: $T - f = m a_x$

y: $N - mg = 0$

$\therefore N = mg$

$$T - \mu m g = m a_x$$

Torque

$$f r_2 - T r_1 = I\alpha$$

$$= I\left(\frac{a_x}{r_2}\right)$$

$$\boxed{\begin{gathered} \mu m g r_2 - T r_1 = I \frac{a_x}{r_2} \\ T - \mu m g = m a_x \end{gathered}}$$

Solve:

$$T = m a_x + \mu m g$$

$$\mu m g r_2 - (m a_x + \mu m g) r_1 = \frac{I a_x}{r_2}$$

$$\mu m g \left(r_2 - r_1 \right) = a_x \left(\frac{I}{r_2} + m \right.$$

true!

$$\boxed{a_x = \frac{\mu m g \left(r_2 - r_1 \right)}{\frac{I}{r_2} + m r_1}}$$

$$\boxed{a_x > 0}$$

<u>Question</u>: what is μ?

Well we don't know! It is somewhere between 0 and μ_s^{max}.

We should probably instead solve for the acceleration in terms of T!

$$② \rightarrow \quad T - \mu m g = m a_x$$

$$\mu = \frac{T - m a_x}{m g}$$

Substitute into ①

$$\mu m g r_2 - T r_1 = \frac{I a_x}{r_2}$$

$$\left(\frac{T - m a_x}{\cancel{mg}}\right)\cancel{mg}\, r_2 - T r_1 = \frac{I a_x}{r_2}$$

$$T(r_2 - r_1) - m a_x r_2 = \frac{I a_x}{r_2}$$

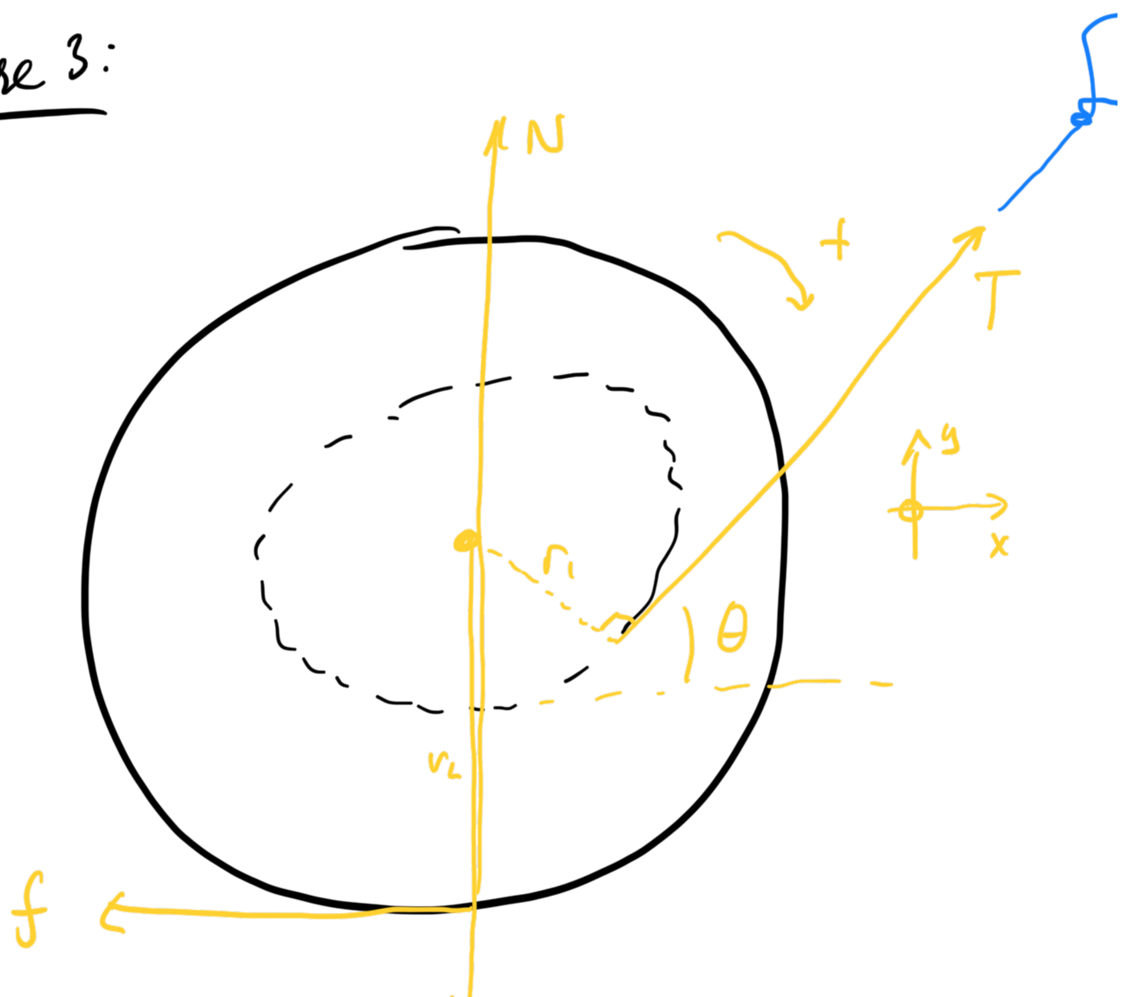
$$T(r_2 - r_1) = a_x\left(\frac{I}{r_2} + m r_2\right)$$

$$\boxed{a_x = \frac{T(r_2 - r_1)}{\quad}}$$

$$\boxed{\quad a_x \qquad \frac{I}{r_2} + m r_2 \quad}$$

Then $\mu = \dfrac{T - m a_x}{mg}$

<u>Case 3:</u>



mg

Torque: $-Tr_1 + fr_2 = I\alpha$

x: $T\cos\theta - f = ma_x$

y: $N + T\sin\theta - mg = ma_y$

No motion in the x- or y- directions!

$$\boxed{\begin{aligned} T\cos\theta - f &= 0 \\ N + T\sin\theta - mg &= 0 \end{aligned}}$$

$N = mg - T\sin\theta$

$f = \mu(mg - T\sin\theta)$

$T\cos\theta - \mu(mg - T\sin\theta) = 0$

Critical angle $\rightarrow$ $\mu = \mu_s^{max}$

Critical angle ,

$$T\cos\theta_c - \mu_s^{max}(mg - T\sin\theta_c) = 0$$

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$$T(\cos\theta_c + \mu_s^{max}\sin\theta_c) = \mu_s^{max} mg$$

$$\boxed{\cos\theta_c + \mu_s^{max}\sin\theta_c = \frac{\mu_s^{max} mg}{T}}$$

Highly non-linear function with no (obvious)

analytical solution!

Solve numerically.

(i) LHS = $\cos\theta_c + \mu_s^{max}\sin\theta_c$

RHS = $\mu_s^{max} mg$

RHS = $\dfrac{[illegible]}{T}$

(2) vary θ_2 between 0° and 90°
and see where LHS = RHS