

Physics 201 - Lecture 27

Translational

$$\begin{array}{ll}
 \vec{x} & \text{position} \rightarrow \theta \\
 \vec{v} & \text{velocity} \rightarrow \omega \\
 \vec{a} & \text{acc.} \rightarrow \alpha \\
 \Delta \vec{x} & \text{displ.} \rightarrow \Delta\theta
 \end{array}$$

Rotational

Newton's 2nd Law

$$\sum_i \vec{F}_i = \vec{F}_{net} = m \vec{a}$$

$\sum F_x = ma_x$ $\sum F_y = ma_y$

$? = ?$ $? = ?$

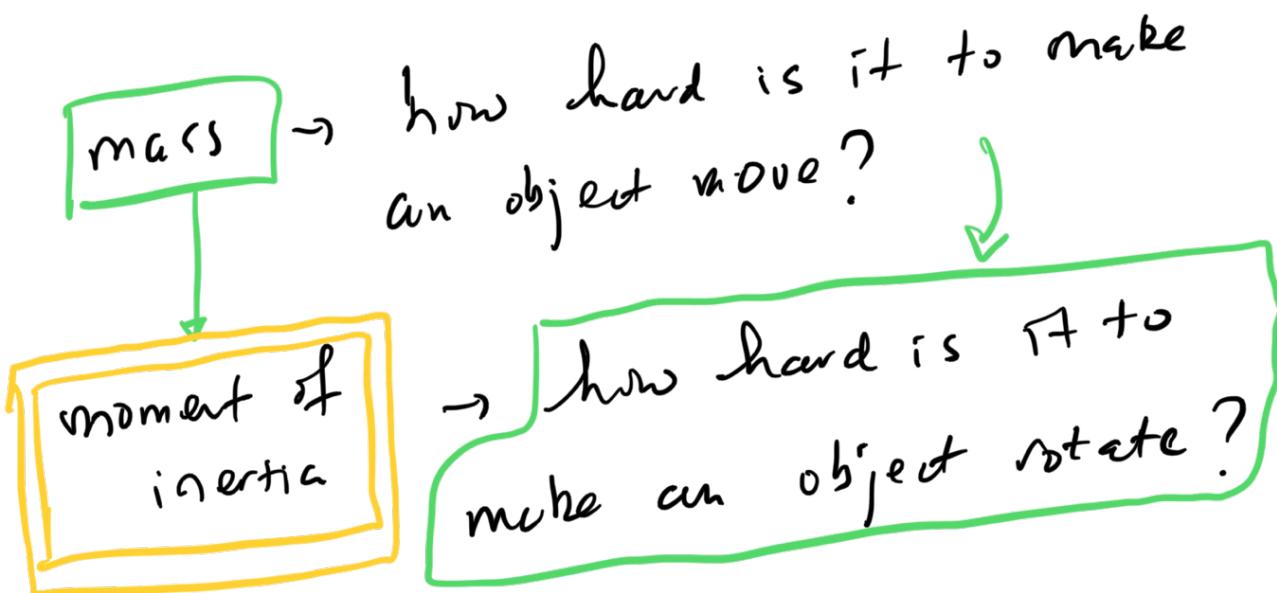
$? = ? \alpha$ moment of inertia

..... \rightarrow the equivalent

Question 1 : What is the equivalent quantity for mass in rotational motion?

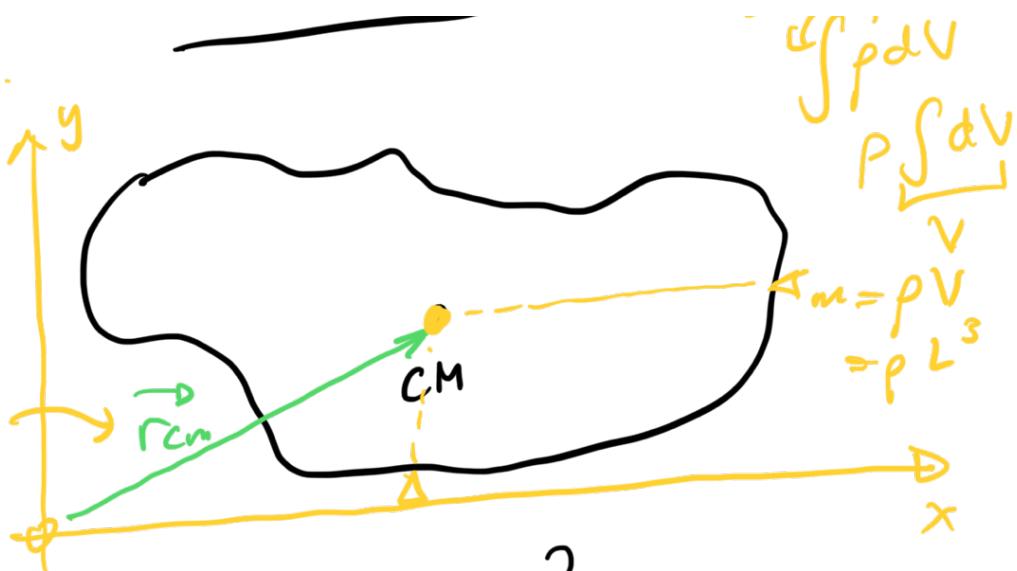
Question 2 : What is the equivalent quantity for force in rotational motion?

I : Rotational Moment of Inertia



Moments





What is the total mass?

$$m = \int_{\text{volume}} p \, dV$$

density kg/m^3 (scalar)

Where is the centre of mass?

$$m \vec{r}_{CM} = \int_{\text{volume}} \vec{r} p \, dV$$

$$\vec{r}_{CM} = \frac{\int_{\text{vol}} (\vec{r}) p \, dV}{\int p \, dV}$$

(vector)

What is the rotational moment of inertia? Well, it depends on which axis of rotation we are talking about. Consider, for now, an axis through the center of mass.

$$m(\vec{r}_{cm})^2 = \int_{\text{vol}} (\vec{r})^2 \rho dV$$

$I \curvearrowright =$ [rotational moment of inertia]

$$m = \int_V \rho dV =$$

[zeroth moment of inertia.]

$$\underline{m \vec{r}_{cm}} = \int_V \vec{r} \rho dV =$$

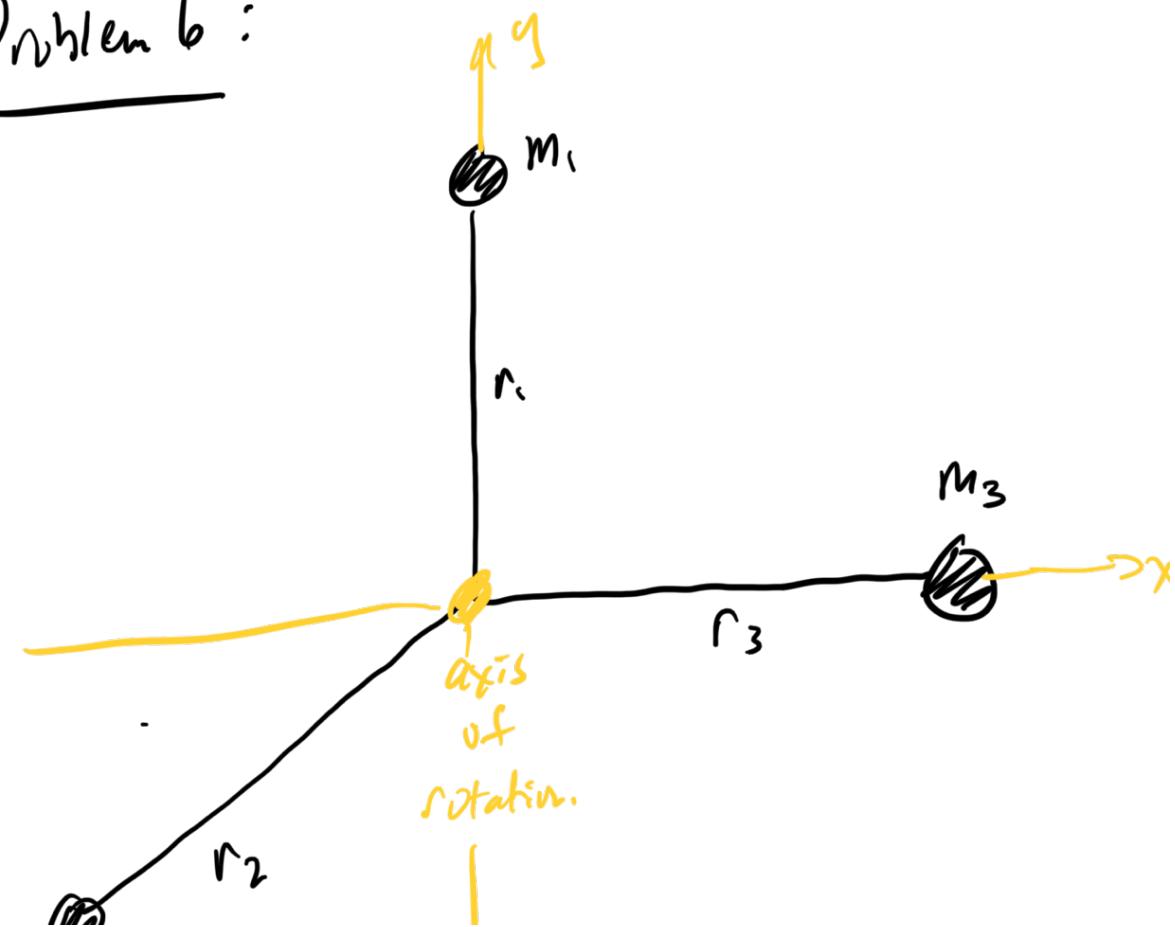
[first moment of inertia]

$$m(\vec{r}_{cm})^2 = \int_V (\underline{\vec{r}})^2 \rho dV = \boxed{\text{Second moment of inertia.}}$$

$$m(\vec{r}_{cm})^3 = \int_V (\underline{\vec{r}})^3 \rho dV = \text{third moment of inertia}$$

⋮
⋮
etc.

Problem 6 :



m_2
 $I = \int (\vec{r})^2 \rho dV$

 $= \int r^2 dm$

 $\text{, } S \rightarrow \sum \rightarrow \text{all op}$

 $= \frac{m_1 r_1^2}{\text{---}} + \frac{m_2 r_2^2}{\text{---}} + \frac{m_3 r_3^2}{\text{---}}$

 $= (.63)(.35)^2 + (.63)(.62)^2$

 $+ (.63)(.22)^2$

 $= 0.35 \text{ kg} \cdot \text{m}^2$

Kinetic Energy

$K = \frac{1}{2} m v^2$

$$\Rightarrow R = \frac{1}{2} I \omega^2$$

b) $\omega = 4.1 \frac{\text{f/s}}{\text{s}} \times \frac{2\pi \text{ rad}}{\text{rev}}$

$$= 8.2\pi \text{ rad/s}$$

$$R = \frac{1}{2} (I)(\omega^2)$$

$$< \frac{1}{2} (.35)(8.2\pi)^2$$

$$= 116 \text{ J}$$

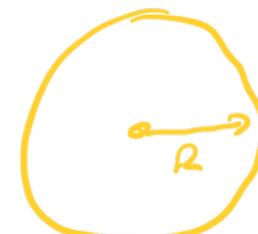
Question: What is the moment of inertia of a wheel, a cylinder,

a sphere, a meter stick, etc... -

These are all common shapes, and
someone has already done the
calculus for us!



$$I_{\text{disk/wheel}} = \frac{1}{2} M R^2$$



$$I_{\text{sphere}} = \frac{2}{5} M R^2$$



$$I_{\text{rod}} = \frac{1}{12} M L^2$$

(about center)



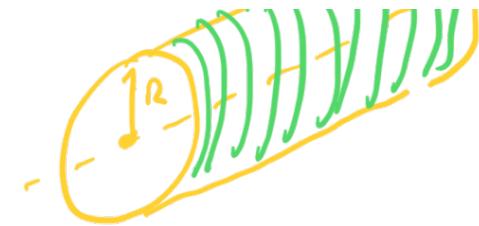
$$I_{\text{rod}} = \frac{1}{3} M L^2$$

(about one end)

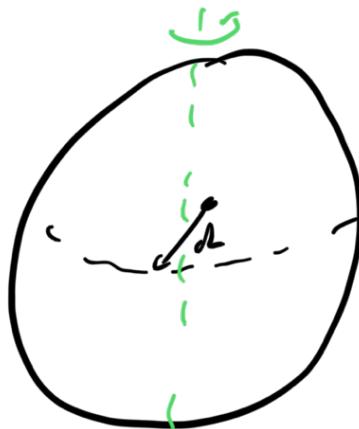


end).

$$I_{\text{Cylinder}} = \frac{1}{2} M R^2$$



Question 7:



1 rev in
one "deg"
 $\frac{1}{T}$

$$I_{\text{Sphere}} = \frac{2}{5} M R^2$$

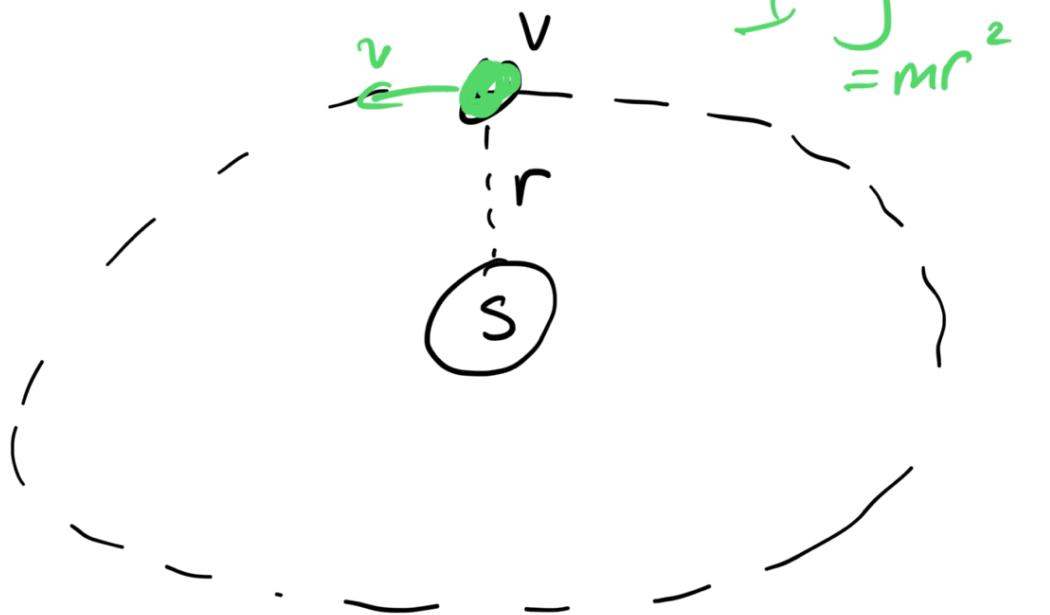
$$\omega = \frac{2\pi}{T} \text{ rad}$$

a) $R_{\text{KE}} = \frac{1}{2} I \omega^2$

$$= \frac{1}{2} \left(\frac{2}{5} M R^2 \right) \left(\frac{4\pi^2}{T^2} \right)$$

$$\begin{aligned}
 &= \frac{4\pi^2}{5} M \frac{R^2}{T^2} \\
 &= \frac{4\pi^2}{5} \left(\frac{4.9 \times 10^{24}}{5830 \times 3600} \right) \left(\frac{6.05 \times 10^6 m}{5830 \times 3600} \right) \\
 &= \boxed{3.21 \times 10^{24}} \text{ J}
 \end{aligned}$$

b)



$$I = \int r^2 dm = mr^2$$

$$I = mr^2 \quad \omega = \frac{2\pi}{T}$$

$$\begin{aligned}
 R_{\text{KE}} &= \frac{1}{2} I \omega^2 \\
 &= \frac{1}{2} (mr^2) \left(\frac{2\pi}{T}\right)^2 \\
 &= \underline{2\pi^2 m} \frac{r^2}{T^2} \\
 &= 2\pi^2 \left(\underline{4.9 \times 10^{24}} \right) \left(\frac{\underline{1.08 \times 10^{11}}}{\underline{225 \times 24 \times 3600}} \right)^2 \\
 &= \underline{2.48 \times 10^{33} \text{ J}}
 \end{aligned}$$

Back to Newton's 2nd Law for
rotational motion ...

$$? = I \alpha$$

What is the rotational equivalent of
Newton's 2nd Law for linear motion?

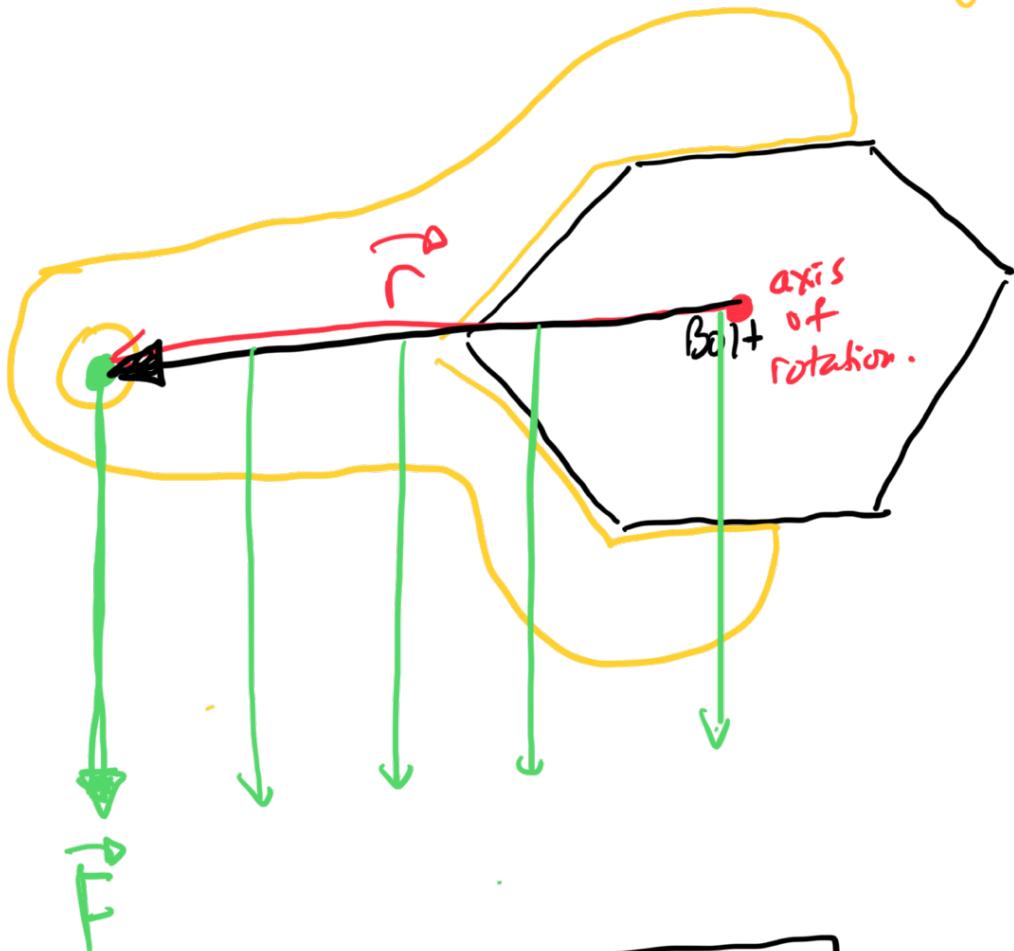
force?

Force \rightarrow accelerate
Torque \rightarrow accelerate stationary.

Torque (τ)

Stupid IKEA
wrench

(Top View)



\vec{F} = applied force

\vec{r} = vector which points from the
axis of rotation to the
location of the

- point of application
force.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

\vec{r} cross \vec{F}

Torque is a vector!

$\vec{r} \times \vec{F}$ is the vector / cross product of \vec{r} and \vec{F} . We need to understand how to calculate both the size and direction of this torque vector.

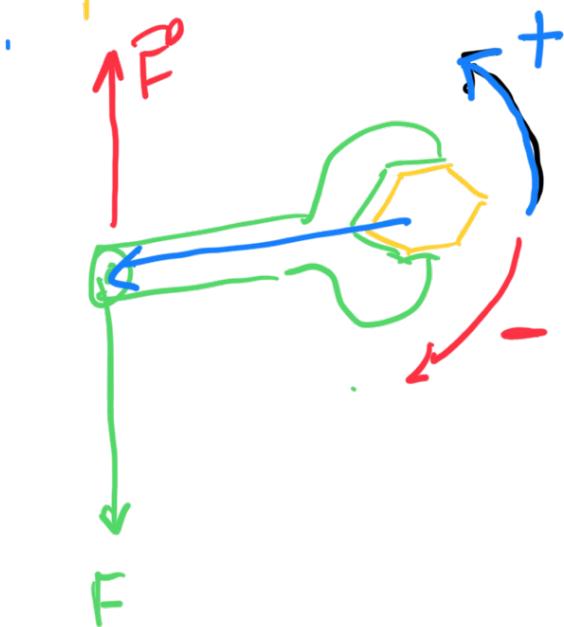
$$W = \vec{F} \cdot \vec{dr}$$
$$= |\vec{F}| |\vec{dr}| \cos \theta$$

size:

$$|\vec{\tau}| = \tau = |\vec{F}| |\vec{r}| \sin \theta$$

direction:

- think about which direction of rotation the torque will tend to cause.



∴ this
is a positive
torque.

Question 8:





$$N = 14 \text{ N}$$

$$\begin{aligned} f &= \mu_K N \\ &= (0.40)(14) = 5.6 \text{ N} \end{aligned}$$

$$\tau_f = -f \cdot r \cdot \sin(90^\circ)$$

$$= -(5.6)(.4)(1)$$

$$\boxed{\tau_f = -2.24 \text{ N} \cdot \text{m}}$$

$$\tau_N = N \cdot r \cdot \sin(180^\circ) = 0 !$$

$$\tau_{\text{F}_{\text{axle}}} = [F_{\text{axle}}](0) \sin(0^\circ) = 0 !$$

$$\text{O} \quad \tau_{NET} = \tau_f + \tau_N + \tau_{F_{axle}} \\ = -2.24 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{2} m r^2 \quad (\text{disk}) \\ = \frac{1}{2} (91) (0.4)^2 \\ = 7.28 \text{ kg}\cdot\text{m}^2$$

$$\tau_{NET} = I \alpha \\ -2.24 = 7.28 \alpha$$

a)

$$\alpha = -0.3077 \text{ rad/s}^2$$

b)

$$\omega_i = 65 \text{ rpm} = 65 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}}$$

$$\times \frac{60 \text{ min}}{60 \text{ s}}$$

$$= 6.8068 \text{ rad/s}$$

$$\omega_f = 0$$

$$\alpha = -0.3077 \text{ rad/s}^2$$

$$\Delta\theta = ?$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$0 = (6.8068)^2 - 2(0.3077)\Delta\theta$$

$$\Delta\theta = 75.288 \text{ rad}$$

$$\# \text{ rev} = \frac{\Delta\theta}{2\pi} = 11.98 \text{ rev}$$

$\sim 12 \text{ rev.}$

