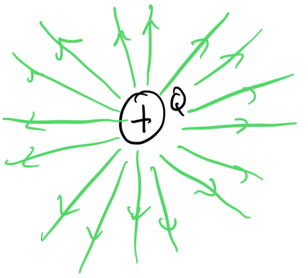
Electric Flux

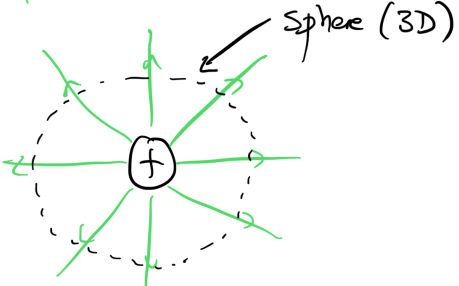
So, exactly how many electric field lines does a given charge produce?



I mean, this is just a schematic, right?

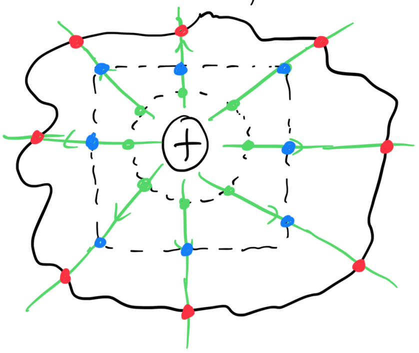
How could we count them?

Answer: surround the charge with a "surface", and then count the # of lines that pass through the surface.



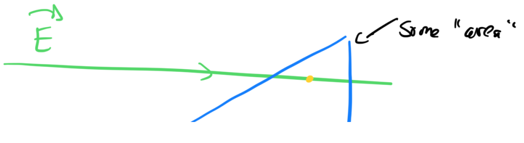
→ the surface should be "closed", to make sure we get all of them.

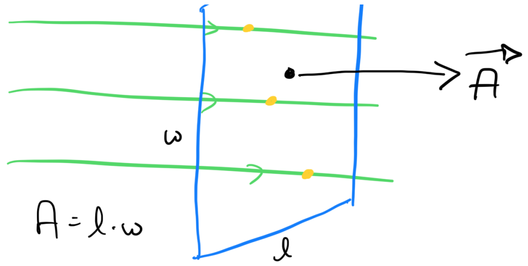
→ the surface is rather arbitrary, since as long as it is closed it will catch all of the lines.



(There are 8 lines in all three cases!)

Maths: It turns out that there is a mathematical way of doing this "counting"!





$A = l \cdot w$

We need to define the AREA VECTOR. this is a vector that points $\perp$ to the surface, and whose length is the area of the surface.

We count the lines pass through this area/surface using:

$$\Phi_E \equiv \vec{E} \cdot \vec{A}$$

$$= |\vec{E}| \cdot |\vec{A}| \cdot \cos(\theta_{EA})$$

electric flux ← Φ_E

↖ Latin: flow

↑ size of $\vec{E}$ ↑ size of $\vec{A}$ $= l \times w$ ↑ angle between $\vec{E}$ and $\vec{A}$

In the above Example,

$\vec{E} = E_0 \hat{i}$ $\quad \vec{A} = l w \hat{i}$

$$\therefore \quad \Phi_E = E_0 l w \cos(0^\circ)$$

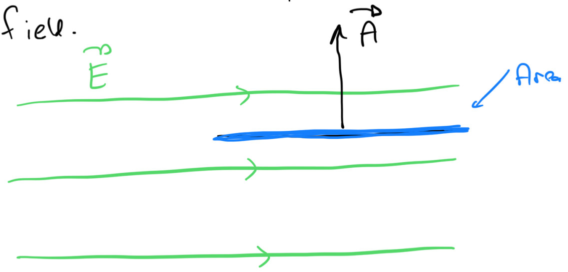
$$= E_0 l w$$

↑ N/C ↑ m ↑ m

$\rightarrow \dfrac{kg\, m}{s^2 . C} . m . m$

$\rightarrow \dfrac{kg \cdot m^3}{s^2 C} \equiv 1\, Wb$ (Weber)

What if we rotate the area so that the area itself is parallel to the field.



Now, $\Phi_E = |\vec{E}|\, |\vec{A}| \underbrace{\cos(90^\circ)}_{=0!}$

$= 0$!!

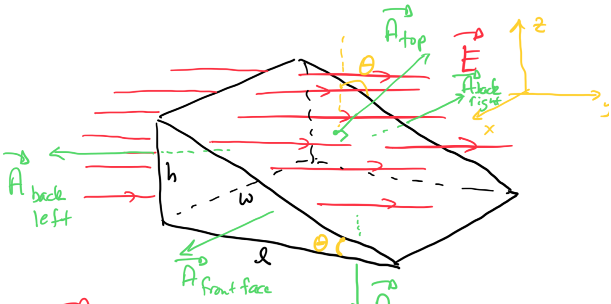
That is why this is the correct mathematical entity:

increase $|\vec{E}|$ → flux ↑

increase $|\vec{A}|$ → flux ↑

angle takes into account alignment.

A3Q3



$$\vec{E} = E_0\,\hat{j}$$

$$\vec{A}_{front\ face} = A_{front\ face}\,\hat{\imath}$$

$$\vec{A}_{back\ right} = -A_{back}\,\hat{\imath}$$

$$\vec{A}_{back\ left} = -A_{back\ left}\,\hat{j}$$

$$\vec{A}_{bottom} = -A_{bottom}\,\hat{k}$$

$$\vec{A}_{top} = A_{top}\cos\theta\,\hat{k} + A_{top}\sin\theta\,\hat{j}$$

$$A_{front\ face} = \frac{1}{2}\ell h = A_{back\ right}$$

$$A_{back\ left} = hw \qquad A_{bottom} = \ell w$$

$$A_{top} = w\sqrt{\ell^2 + h^2}$$

$$\Phi_{front\ face} = (E_0\hat{j})\cdot\left(\tfrac{1}{2}\ell h\,\hat{\imath}\right) = 0$$

$$\Phi_{back\ right} = (E_0\hat{j})\cdot\left(-\tfrac{1}{2}\ell h\,\hat{\imath}\right) = 0$$

$$\Phi_{back\ left} = (E_0\hat{j})\cdot(-hw\,\hat{j}) = -E_0 hw$$

$$\Phi_{bottom} = (E_0\hat{j})\cdot(-\ell w\,\hat{k}) = 0$$

$$\Phi_{top} = (E_0\hat{j})\cdot\left(A_{top}\cos\theta\,\hat{k} + A_{top}\sin\theta\,\hat{j}\right)$$

$$= E_0 A_{top}\sin\theta$$

$$= E_0\, w\,\cancel{\sqrt{\ell^2+h^2}}\cdot\frac{h}{\cancel{\sqrt{\ell^2+h^2}}}$$

$$= +E_0 hw$$

∴ total flux = $0 + 0 - E_0 hw + 0$

so total flux =

$+ E_0 h\omega$

$= 0$