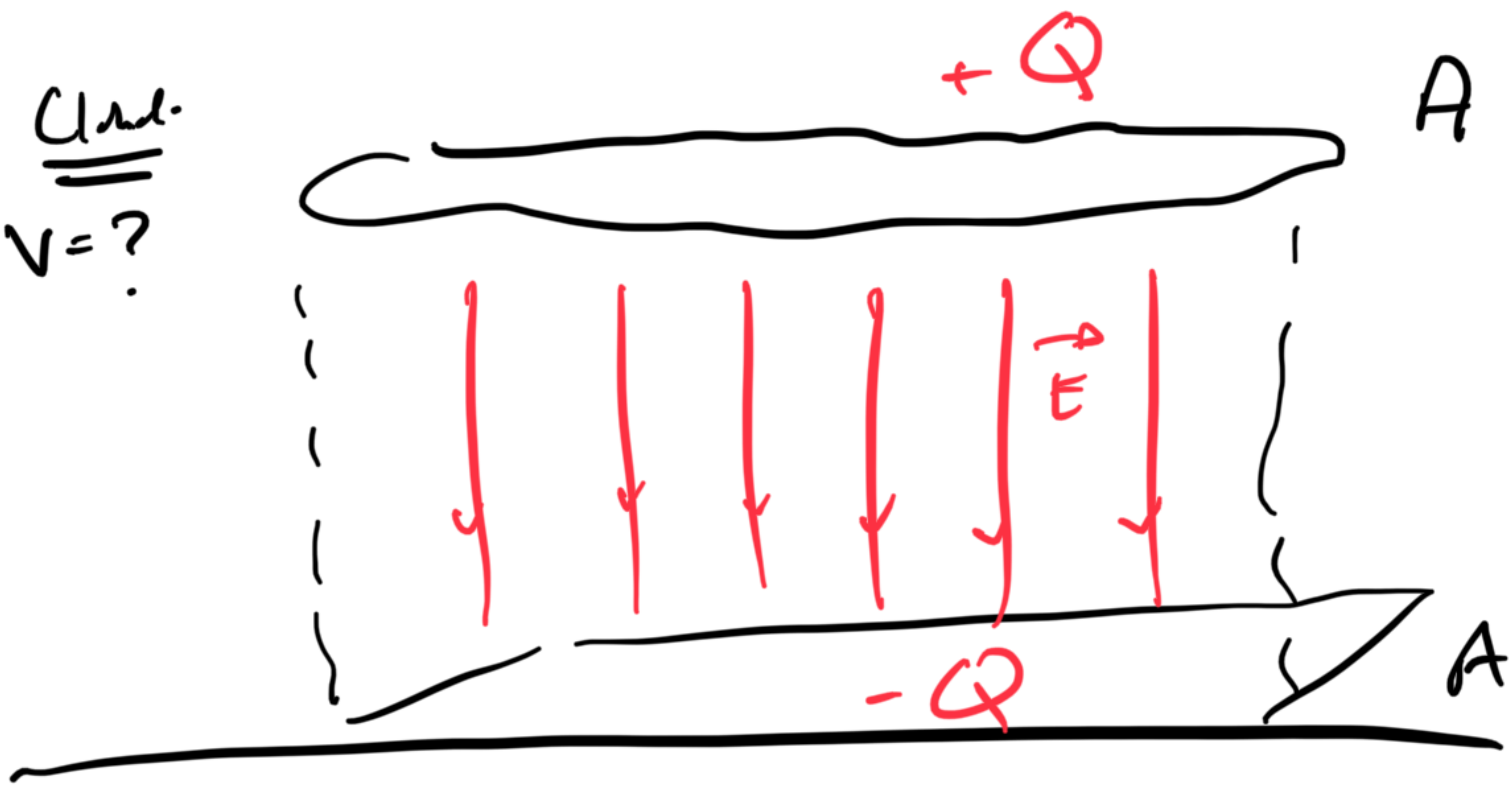
Physics 202 - A4-1-6

1.

Cloud.
V=?



Earth
V = 0

Capacitance: $$C = \frac{\varepsilon_0 A}{d}$$

where $k_e = \frac{1}{4\pi\varepsilon_0}$, or $\varepsilon_0 = \frac{1}{4\pi k_e}$

$$= 8.854 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

$$C = \frac{(8.854 \times 10^{-12})(1,000,000\,m^2)}{750\,m}$$

$$= 1.18 \times 10^{-8} \text{ Farads}$$

1.2 nF

$= 11.8$

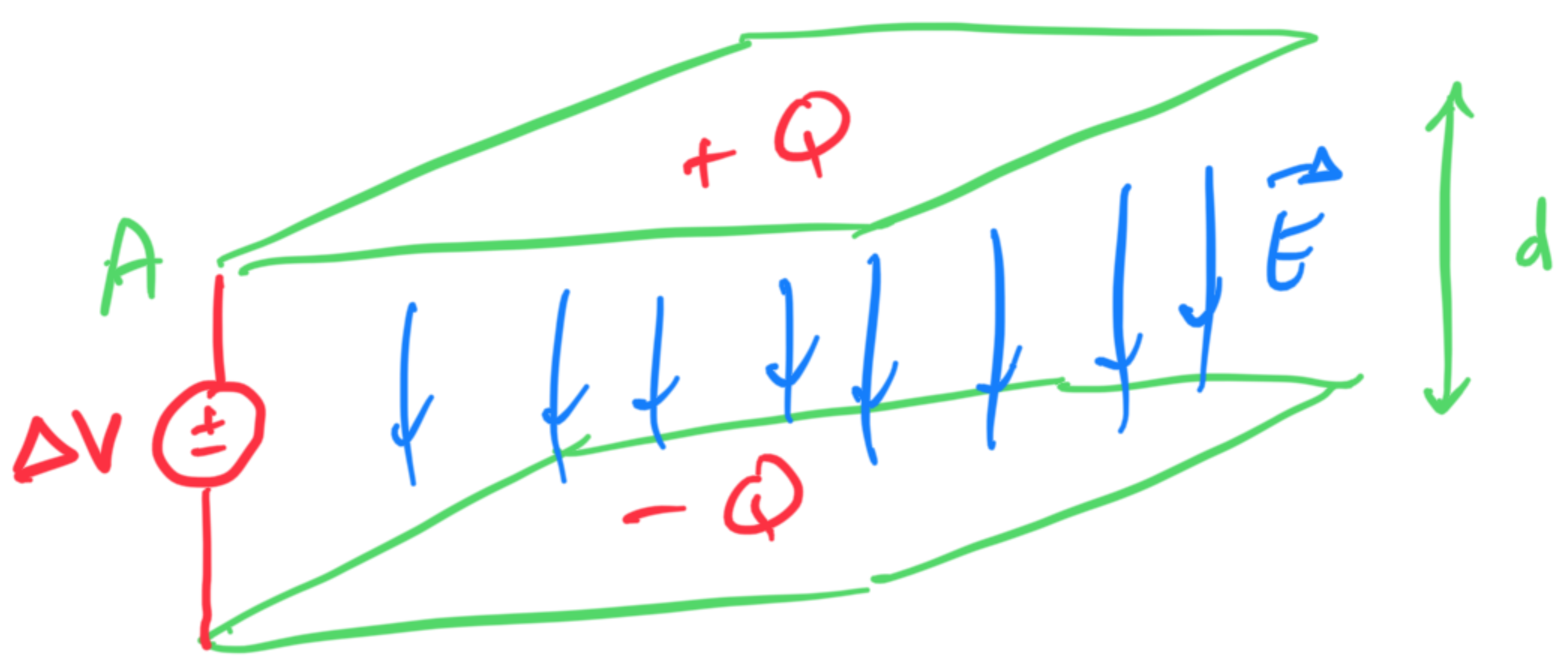
10^{-9}

Farad $\rightarrow$ $$1\,F = \frac{1\,\frac{C^2}{N\cdot \cancel{m^2}}\cdot \cancel{m^2}}{m}$$

$$= 1\ C^2/N\cdot m$$

$$1\,F = 1\ C^2/J$$

2.



What do we know?

$$C = \frac{\varepsilon_0 A}{d}$$ ← find this.

$$Q = C\,\Delta V$$

Surface Charge Density $$\sigma = \frac{Q}{A} \qquad (C/m^2)$$

Put this all together:

$$Q = C\,\Delta V = \left(\frac{\varepsilon_0 A}{d}\right)\Delta V$$

$$\therefore \quad d = \frac{\varepsilon_0 A \,\Delta V}{Q}$$

$$= \varepsilon_0 \left(\frac{A}{Q}\right)\Delta V$$

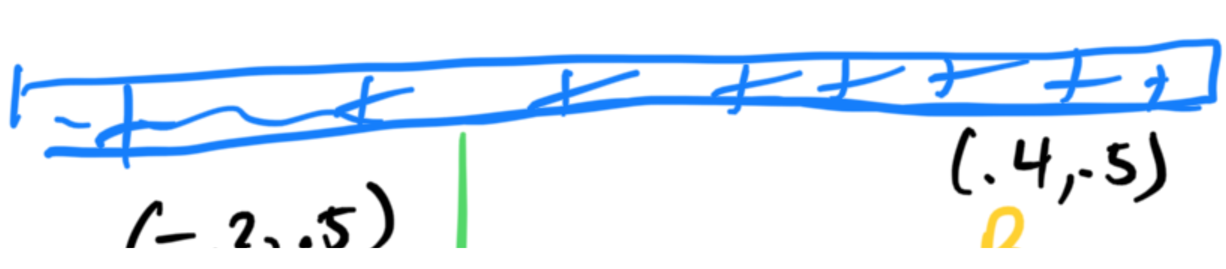
$$\boxed{d = \frac{\varepsilon_0 \,\Delta V}{\sigma}}$$

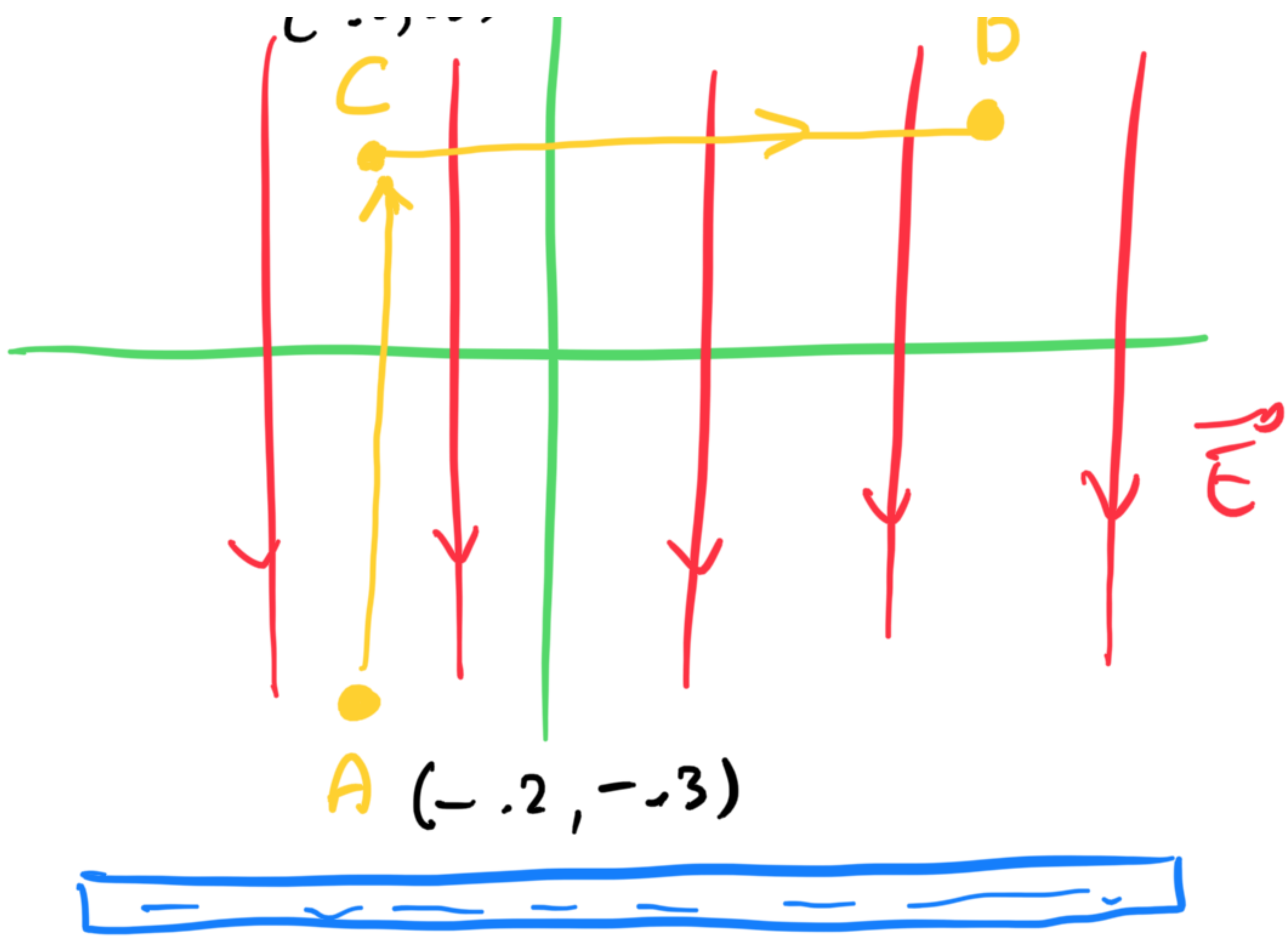
$$\frac{nC}{cm^2} = \frac{10^{-9}\,C}{10^{-4}\,m^2} = 10^{-5}\ C/m^2$$

$$d = \frac{(8.854\times10^{-12})(150)}{(30\times10^{-5})} = 4.43\times10^{-6}\,m = 4.43\ \mu m$$

We also know that $|\vec{E}| = \dfrac{\Delta V}{d}$

$$|\vec{E}| = \frac{\Delta V}{\left(\frac{\varepsilon_0 \,\Delta V}{\sigma}\right)} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

3. 



Path A → C

What is the work done?

$$W = \vec{F} \cdot \vec{\Delta y}$$

$$= -|\vec{F}||\vec{\Delta y}|$$

$$= -q|\vec{E}|\Delta y$$

definition:
$$\Delta V = -\frac{W}{q}$$

$$\therefore \quad \Delta V = |\vec{E}|\Delta y$$

$$\Delta V_{A \to C} = 300\,V/m\,(0.8)$$

$$= 240\ V$$

Path $C \to B$ $\quad W = 0$!

$\therefore \Delta V_{C \to B} = 0$

$$\therefore \boxed{\Delta V_{A \to C \to B} = 240 \text{ V}}$$

4.

$$\Delta V = |\vec{E}| \cdot |\Delta \vec{x}|$$

$$|\vec{E}| = \frac{\Delta V}{|\Delta \vec{x}|} = \frac{25000 \text{ V}}{0.015 \text{ m}}$$

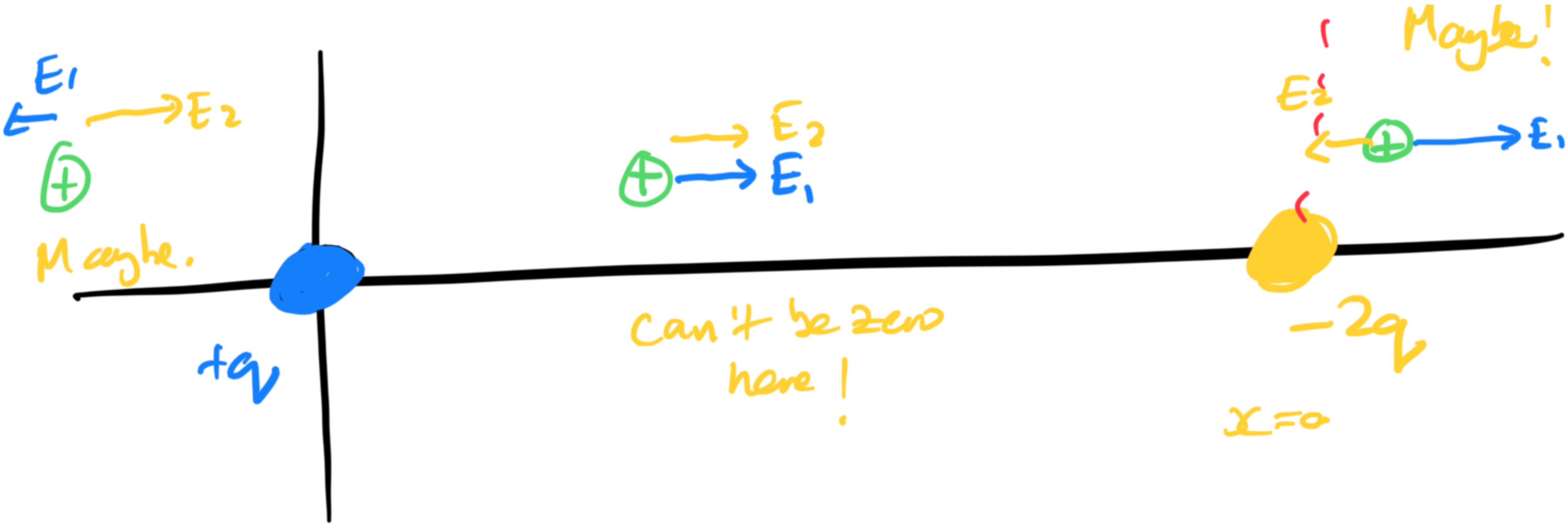
$$= 1.67 \times 10^6 \text{ V/m}$$

$$= 1.67 \times 10^6 \text{ N/C}$$

$$= 1.67 \text{ MN/C}$$

5.

I $\qquad$ II $\qquad$ III



III: $$|\vec{E_1}| = \frac{k_e q}{x^2}$$

$$|\vec{E_2}| = \frac{k_e (2q)}{(x-a)^2}$$

Let $$\frac{\cancel{k_e}\,\cancel{q}}{x^2} = \frac{2\cancel{k_e}\,\cancel{q}}{(x-a)^2}$$

$$\frac{1}{x^2} = \frac{2}{(x-a)^2}$$

$$x^2 = \frac{(x-a)^2}{2}$$

$$x = \frac{x-a}{\sqrt{2}}$$

$$\sqrt{2}x = x - a$$

$$(\sqrt{2}-1)\,x = -a$$

$$x = \frac{-a}{(\sqrt{2}-1)} < 0 \;!$$

But x is positive! ∴ No solution!

I :

$$|\vec{E_1}| = \frac{k_e q}{x^2} \qquad (x<0)$$

$$|\vec{E_2}| = \frac{2k_e q}{(x-a)^2} \qquad (x<0)$$

Similarly,

$$x = \frac{-a}{(\sqrt{2}-1)} < 0 \;✓$$

$$x = -12{,}6\ \text{m}$$

b) $V(x) = V_1 + V_2$

$$V(x) = \frac{k_e q}{|x|} - \frac{2k_e q}{|x-a|}$$

$$0 = k_e q\left(\frac{1}{|x|} - \frac{2}{|x-a|}\right)$$

$$\boxed{\frac{1}{\ } - \frac{2}{\ } = 0}$$

$|x| \qquad |x-a|$

Solution 1: $x > 0$, but less than a (i.e. region II)

$$\frac{1}{x} - \frac{2}{a-x} = 0$$

$$\frac{1}{x} = \frac{2}{a-x}$$

$$a - x = 2x$$

$$a = 3x$$

$$x = \frac{a}{3} = 1.73\ \text{m}$$ ✓

(largest x)

Solution 2: $x > 0$, and bigger than a (i.e. region III)

$$\frac{1}{x} - \frac{2}{x-a} = 0$$

$$\frac{1}{x} = \frac{2}{x-a}$$

$$x - a = 2x$$

$$-a = x$$

$\boxed{x = -a} < 0$ ∴ no solution!

Solution 3 :

$x < 0$

$$-\frac{1}{x} - \frac{2}{-x+a} = 0$$

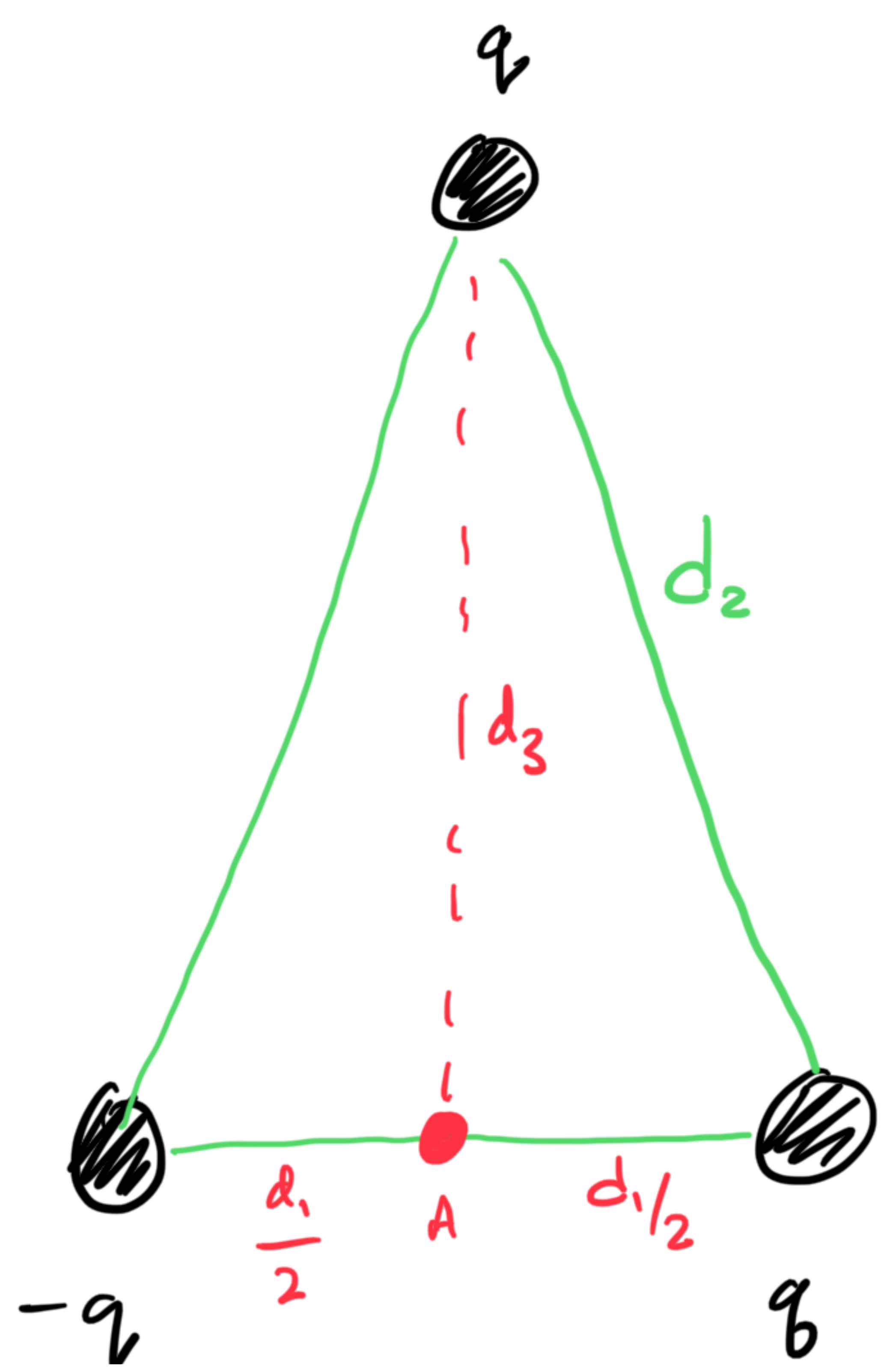
$$-\frac{1}{x} = \frac{2}{-x+a}$$

$$x - a = 2x$$

$$\boxed{x = -a} = -5.2\,m$$

(smallest x)

6.



$$V_A = \frac{k_e q}{d_3} - \frac{k_e q}{(d_1/2)} - \frac{k_e q}{(d_1/2)}$$

$$= \frac{k_e q}{d_3} - 4\frac{k_e q}{d_1}$$

What is d_3?

$$d_2^2 = \left(\frac{d_1}{2}\right)^2 + d_3^2$$

$$\therefore \quad d_3 = \sqrt{d_2^2 - \frac{d_1^2}{4}}$$

$$V_A = k_e q\left(\frac{1}{\sqrt{d_2^2 - \frac{d_1^2}{4}}} - \frac{4}{d_1}\right)$$

$$= -1.17 \times 10^7 \text{ V}$$

$$= -11.7 \text{ MV}$$