Generating Magnetic Fields

1. Electric charges <u>generate</u> electric fields.

2. We can use Gauss's Law for electrostatics to understand this.

$$\boxed{\iint_S \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}}$$

3. This is based on the concept of electric flux:

$$\Phi_E = \iint_S \vec{E} \cdot d\vec{A}$$

What are the equivalent "rules" for magnetic fields???

1. Electric Currents generate (moving charges) Magnetic fields!!

→ they just do!

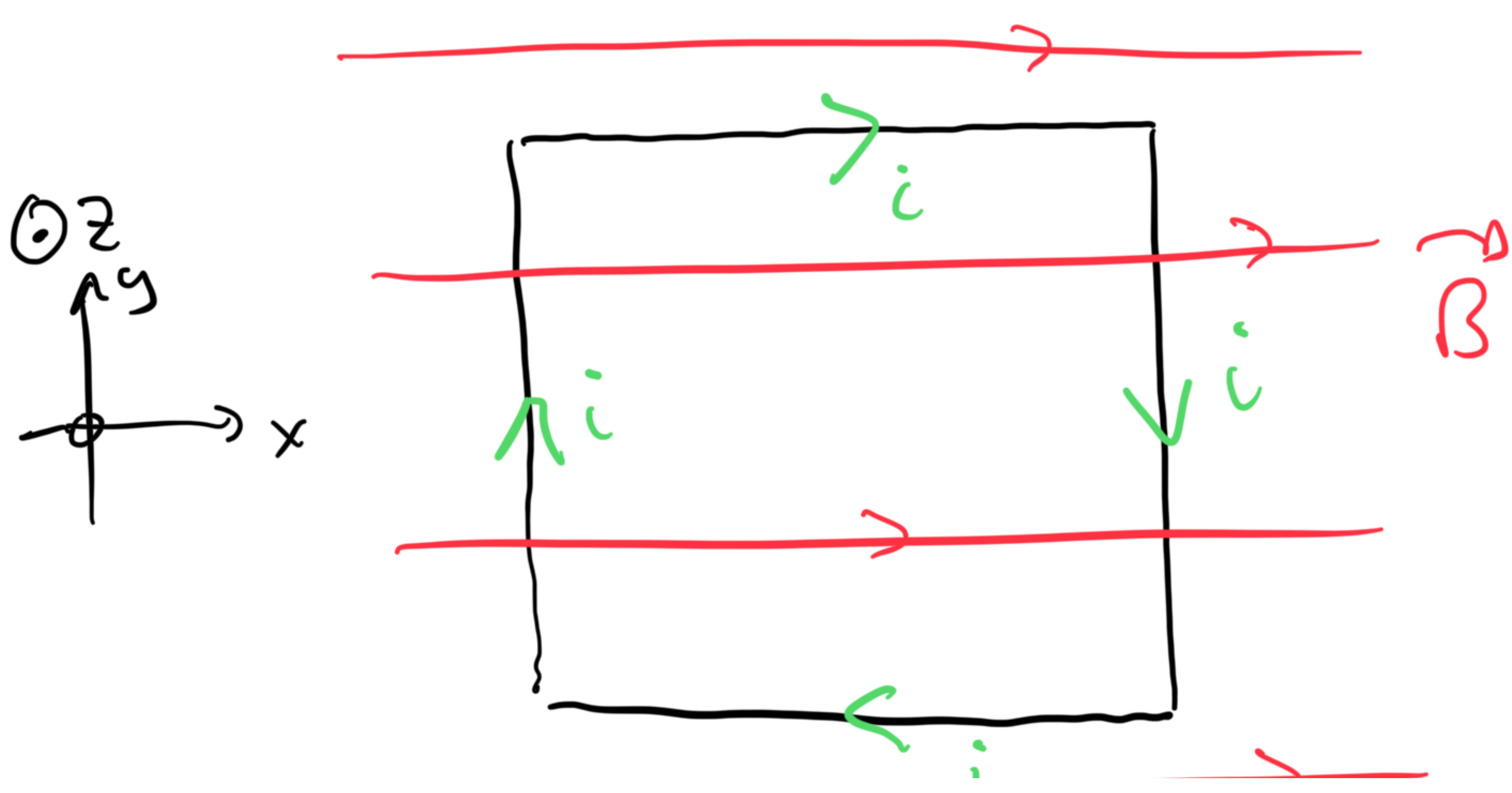
2. We can understand how this works using Ampère's Law for Magnetostatics:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \, i_{enclosed}$$

There is a lot to unpack here!!

Let's take a bit of a detour:

Imagine a piece of wire formed into a square loop, with side length l



Place this into a $\vec{B}$ field in the positive x direction:

$$\vec{B} = B_0 \hat{i}$$

Now, let's calculate the force on each of the four sections of wire.

left: $$\vec{L} = l \hat{j}$$

$$\vec{F}_M = i \vec{L} \times \vec{B}$$

$$= i l B \hat{j} \times \hat{i}$$

$$= -i l B \hat{k}$$

right : $$\vec{L} = -l \hat{j}$$

$$\vec{F}_m = i \vec{L} \times \vec{B}$$

$$= i l B (-\hat{j} \times \hat{i})$$

$$= i l B \hat{k}$$

top: $\vec{L} = l\hat{i}$

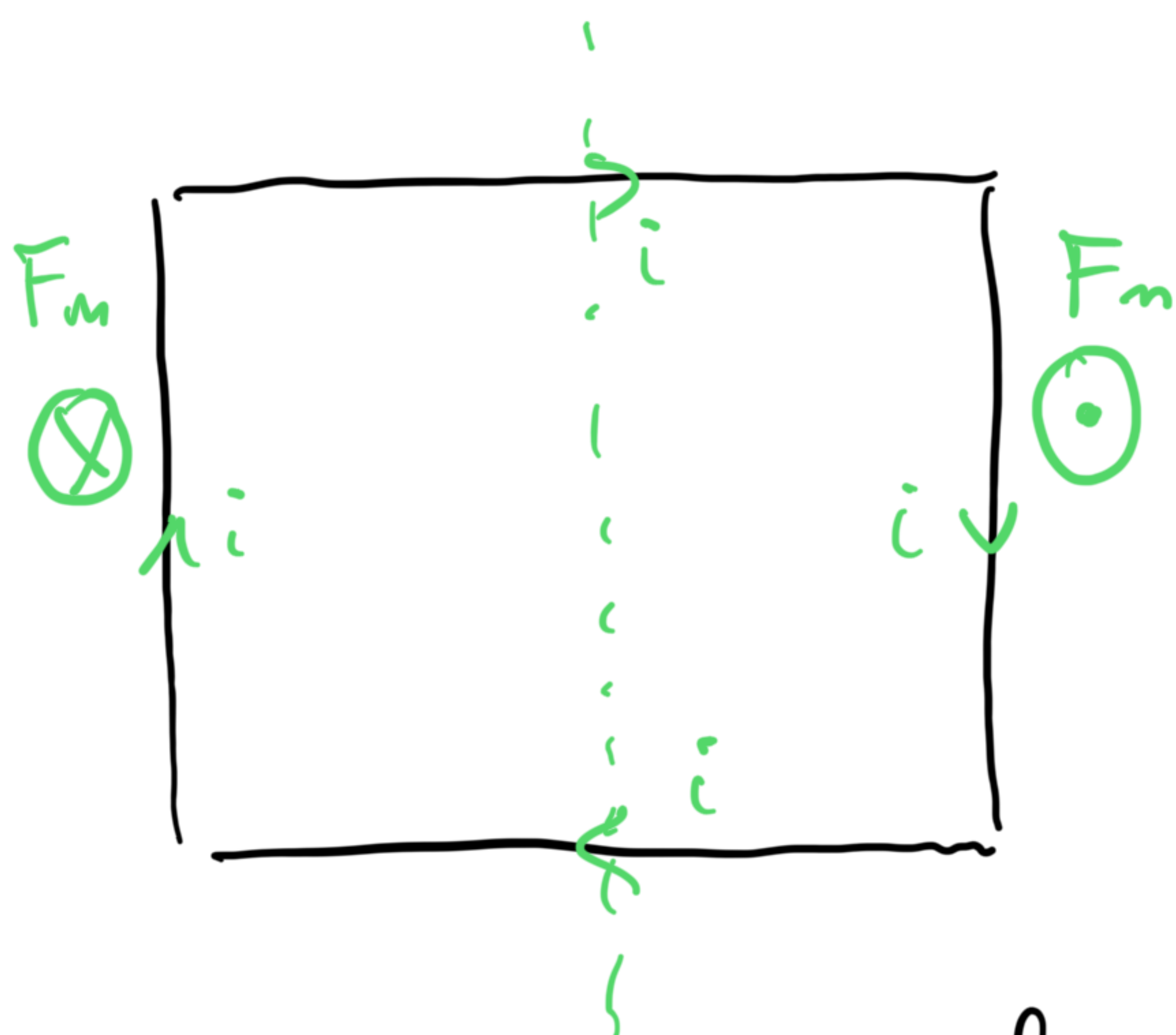
$$\vec{F}_m = i l B(\hat{i} \times \hat{i})$$

$$= 0 \,!$$

bottom: $\vec{L} = -l\hat{i}$

$$\vec{F}_m = -i L B(\hat{i} \times \hat{i})$$

$$= 0 \,!$$



These forces will cause this loop to <u>rotate</u>: there will be a <u>torque</u> on the loop!!

Phys 201:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

moment arm

$\equiv$ vector from axis of rotation to point of application of the force.

Left:

$$\vec{r} = -\frac{L}{2}\hat{i}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= -\frac{L}{2}\hat{i} \times -iLB\hat{k}$$

$$= i\frac{L^2}{2}B\underbrace{\hat{i}\times\hat{k}}_{-\hat{j}} = -\frac{iL^2B}{2}\hat{j}$$

Right:

$$\vec{r} = +\frac{L}{2}\hat{i}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = -\frac{iL^2B}{2}\hat{j}$$

$\therefore$ total torque:

$$\vec{\tau} = -iL^2B\,\hat{j}$$

$$\boxed{\vec{\tau} = -iAB\,\hat{j}}$$

In general, we can write:

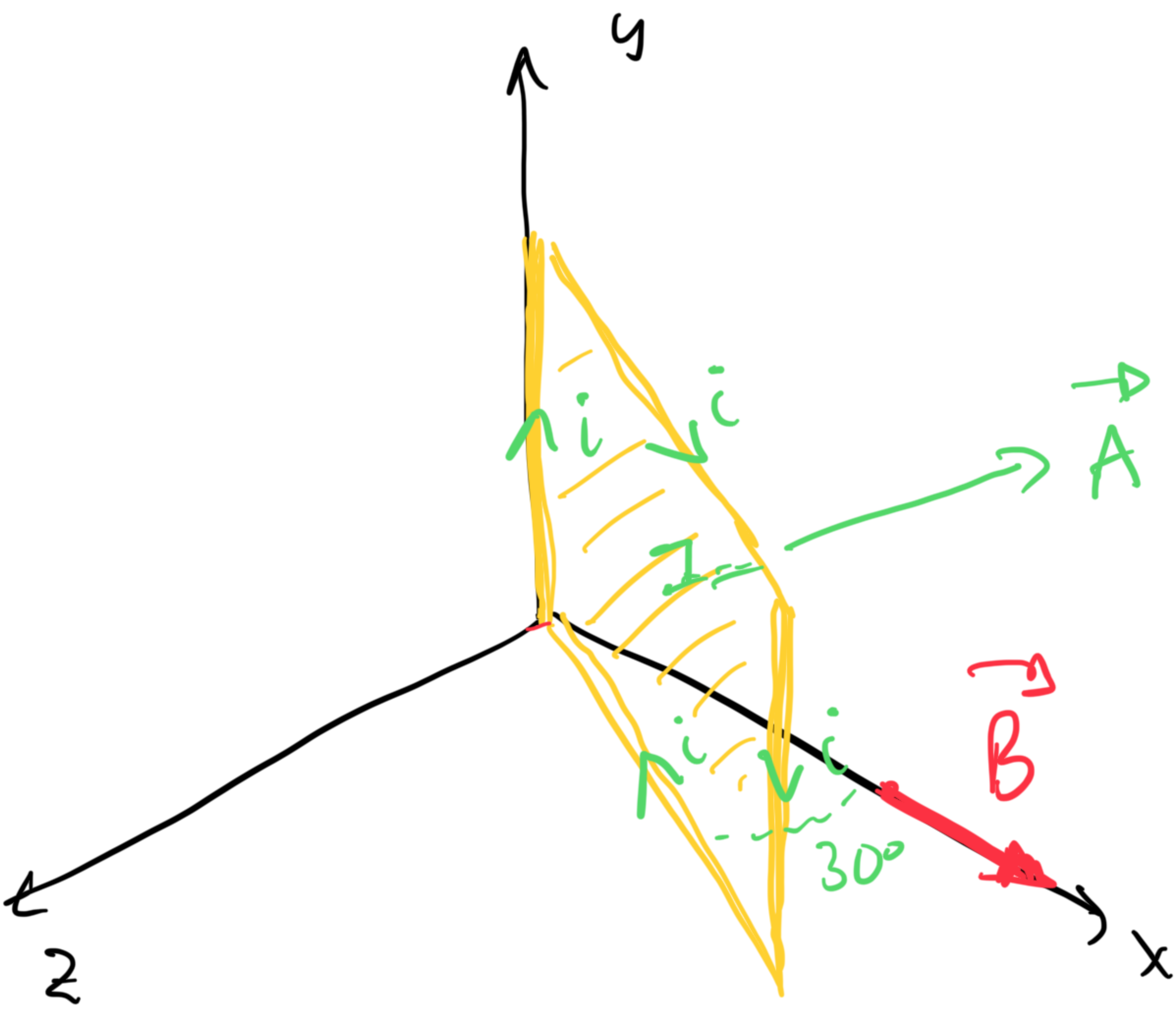
$$|\vec{\tau}| = iAB\underbrace{\sin\theta}$$

angle between $\vec{B}$ field and $\vec{A}$ vector.

If we have multiple loops of wire, i.e. a coil of wire, we can write:

$$\boxed{|\vec{\tau}| = NiAB\sin\theta}$$

WebAssign Q9 :



$\theta = 60^\circ$!!!

$|\vec{\tau}| = N i A B \sin\theta$

$= (110)(1.20)(.4 \times .3)(0.8)$

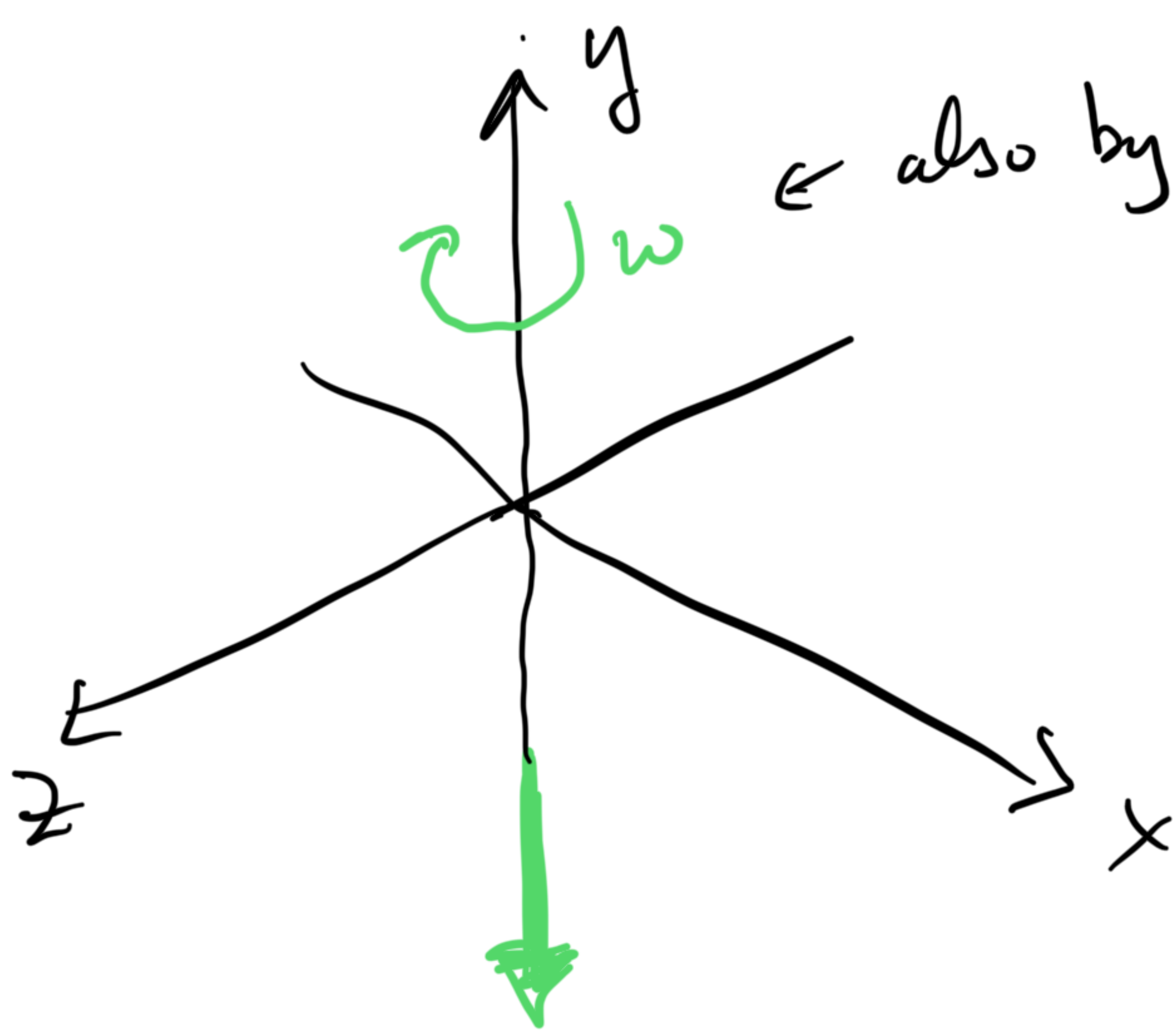
$\times \sin(60^\circ)$

$= 10.97 \ N \cdot m$

What is the direction?

$\vec{\tau} = N i \vec{A} \times \vec{B}$

$\Rightarrow -\hat{j}$ direction by RHR!



← also by RHR!

10.

$$|\vec{\tau}| = NiAB \underbrace{\sin\theta}_{\text{always } -1 \,\ldots\ldots\, 1}$$

a) $$|\vec{\tau}|_{max} = NiAB \qquad (5.68 \times 10^{-4}\ N\cdot m)$$

b) $$P_{max} = |\vec{\tau}| \cdot \omega \qquad (0.214\ W)$$

$$\omega = 3600\ \frac{rev}{min} \times \frac{2\pi\ rad}{rev} \times \frac{1\ min}{60\ s} = 120\pi\ rad/s \approx 376.8\ rad/s$$