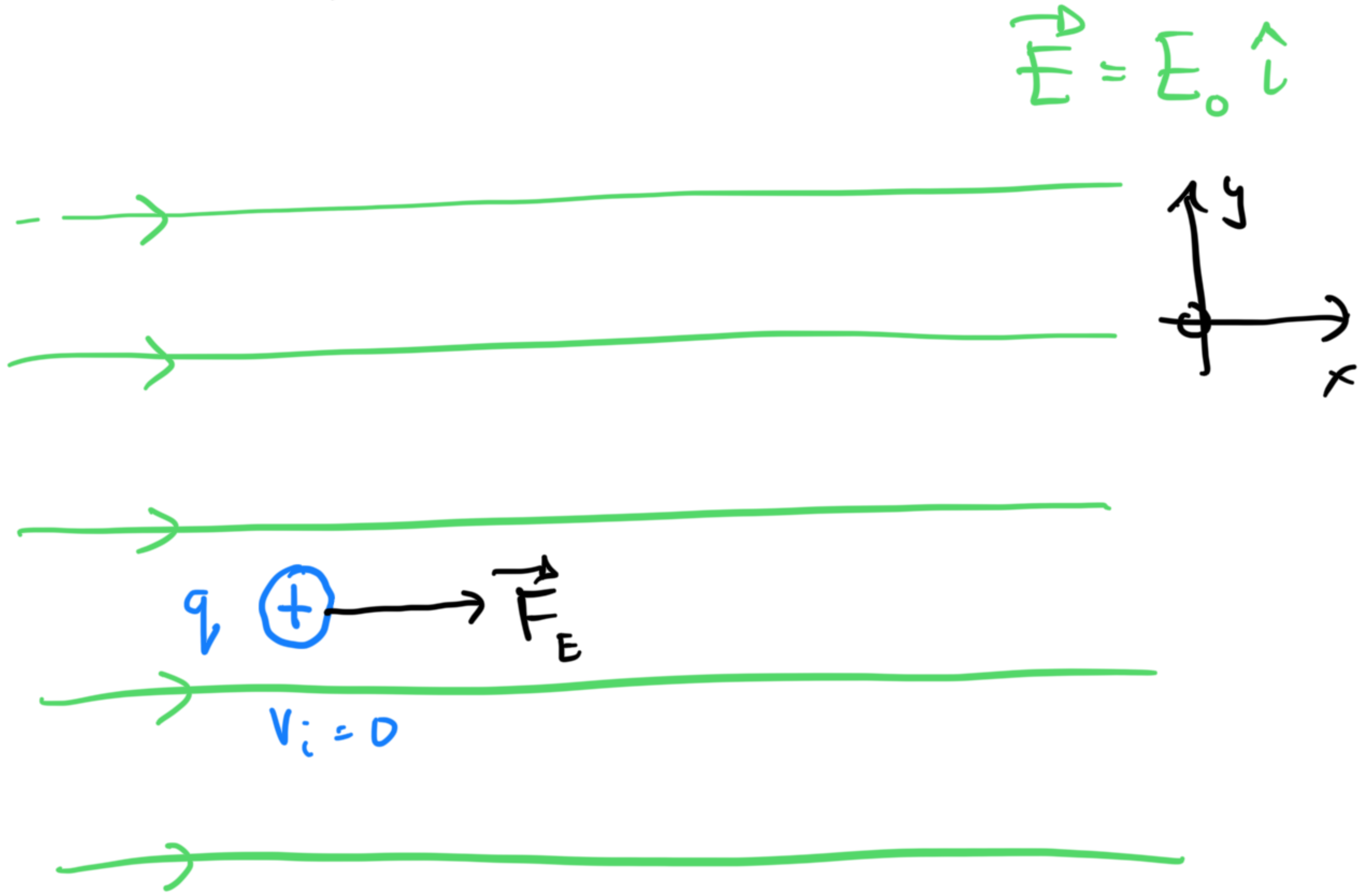
# Voltage and Electric Potential Energy.

Suppose we have some electric field that exists in space (created externally).




We place a +ve charge, q, in this field. It will experience a force,

$$\vec{F}_E = q\vec{E} = qE_0 \hat{\imath}.$$

It will <u>accelerate</u>, $\vec{a} = \dfrac{\vec{F}_E}{m} = \dfrac{qE_0}{m}\hat{\imath}$

Suppose $\vec{v}_i = 0$, and it moves to the right through some distance, $d$.

displacement: $\Delta\vec{x} = d\,\hat{\imath}$

What is the work done by the electric field?

$$W_E = \vec{F}_E \cdot \vec{\Delta x}$$
$$= q E_0 \hat{\imath} \cdot d\,\hat{\imath}$$
$$= q E_0 d$$

We define the change in electric potential energy, $\Delta U_E$ as:

$$\Delta U_E = -W_E = -q E_0 d$$

$$= U_f - U_i$$

this leads us to define the electric potential energy, IN THIS CASE, as:

as:

$$\boxed{U_E = -qE_0 x}$$

$$\therefore \quad U_i = -qE_0 x_i$$
$$U_f = -qE_0 x_f$$

$$\Delta U = -qE_0 x_f - (-qE_0 x_i)$$
$$= -qE_0 (x_f - x_i)$$
$$= -qE_0 d \quad \text{, as required} \checkmark$$

The problem with this is that it contains $\boxed{q}$, and so the answer will be different for every charge we place in the field. Keeping with our idea of "field models", we now define the VOLTAGE FIELD, as:

$$\boxed{V = \frac{U}{q}} \quad \begin{matrix} \leftarrow \text{Joules} \\ \leftarrow \text{Coulomb} \end{matrix}$$

J/ = Volt

So, now we have:

$$V = \frac{-\cancel{q} E_0 x}{\cancel{q}}$$

$$V = -E_0 x$$

or, $\Delta V = V_f - V_i$

$$= -E_0 x_f - (-E_0 x_i)$$

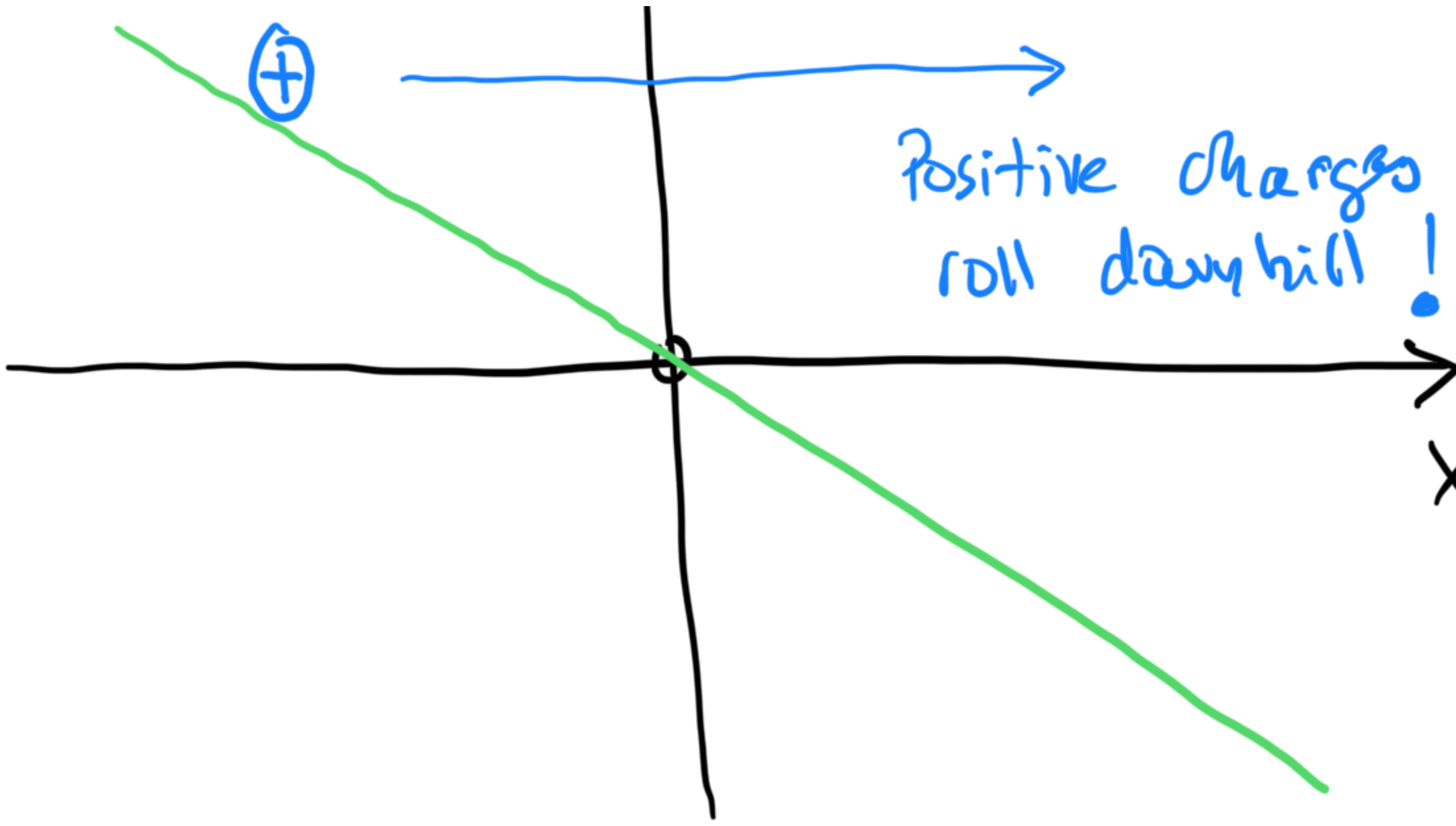
$$= -E_0 (x_f - x_i)$$

$$\boxed{\Delta V = -E_0 d}$$

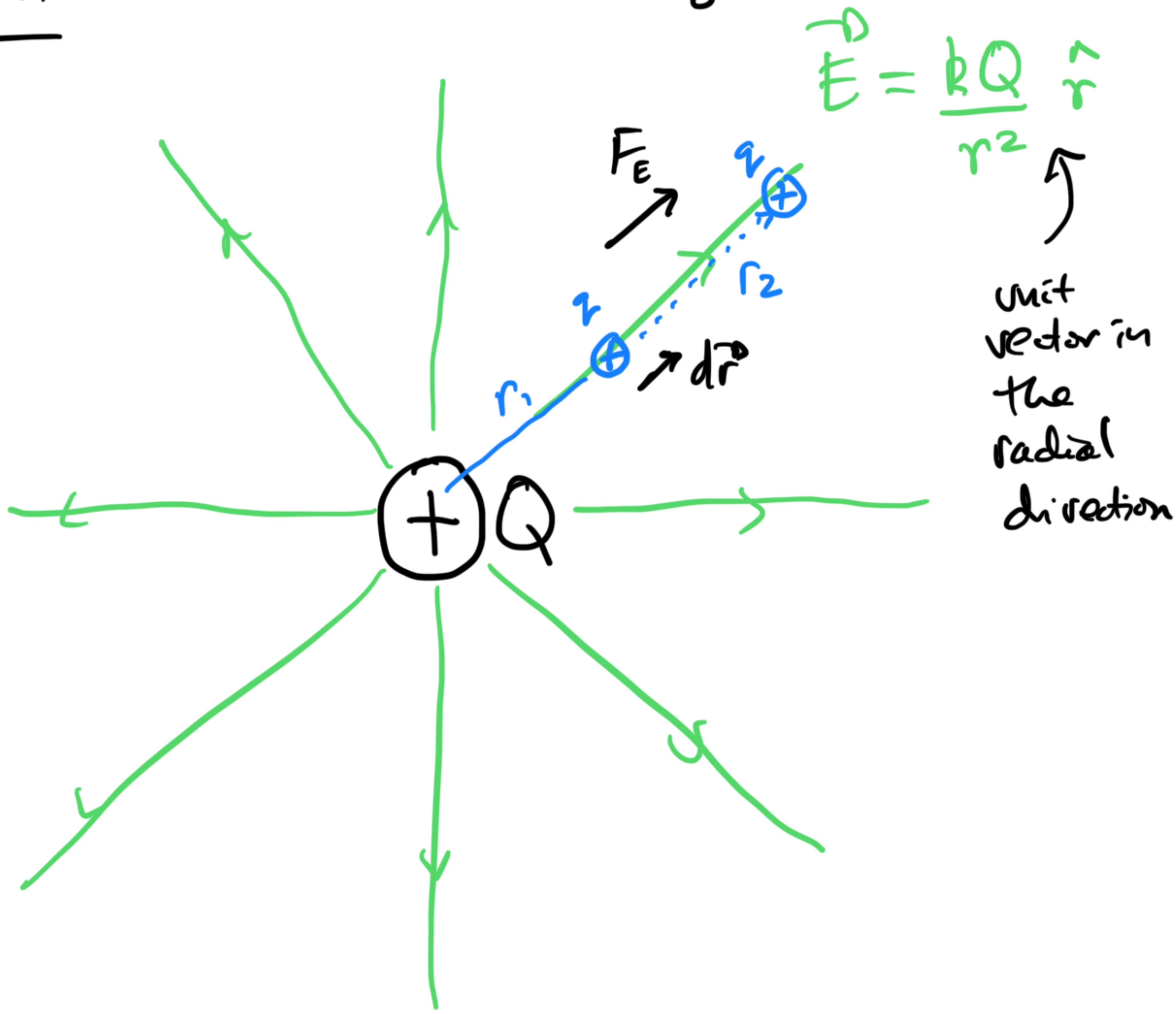
Most important concept in this course:

Understand the difference between voltage at a point ($V$) and the voltage difference between two points ($\Delta V$)

$$\uparrow V(x) = -E_0 x$$




## Example 2: Point Charge.




Imagine placing a positive charge, q, in this field.

$$F_E = \frac{kQq}{r^2}\, r$$

Notice that $r$ is <u>not</u> constant !! (as we move from $r_1$ to $r_2$)

$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$d\vec{r} = dr\,\hat{r}$$

$$\vec{F} = \frac{kQq}{r^2}\,\hat{r}$$

$$= \int_{r_1}^{r_2} \frac{kQq}{r^2}\, dr$$

$$= -\frac{kQq}{r}\Bigg|_{r_1}^{r_2}$$

$$W = -kQq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As before we define :

$$\Delta U = -W$$

$$= kQq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$= U_f - U_i$$

which leads us to the conclusions:

$$U_{\substack{\text{point}\\ \text{charge}}} = \frac{kQq}{r}$$

$$V_{\substack{\text{point}\\ \text{charge}}} = \frac{U}{q} = \frac{kQ}{r}$$

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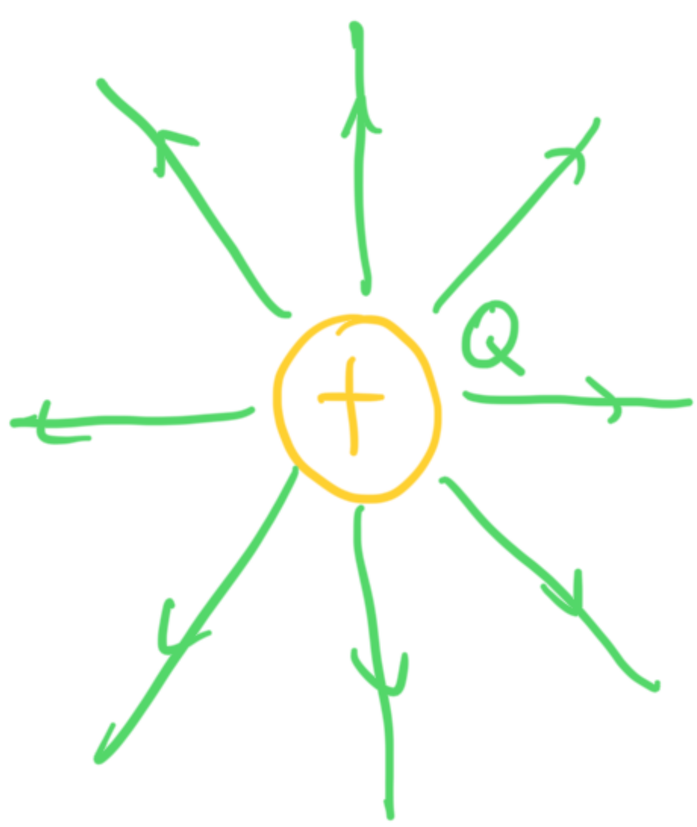
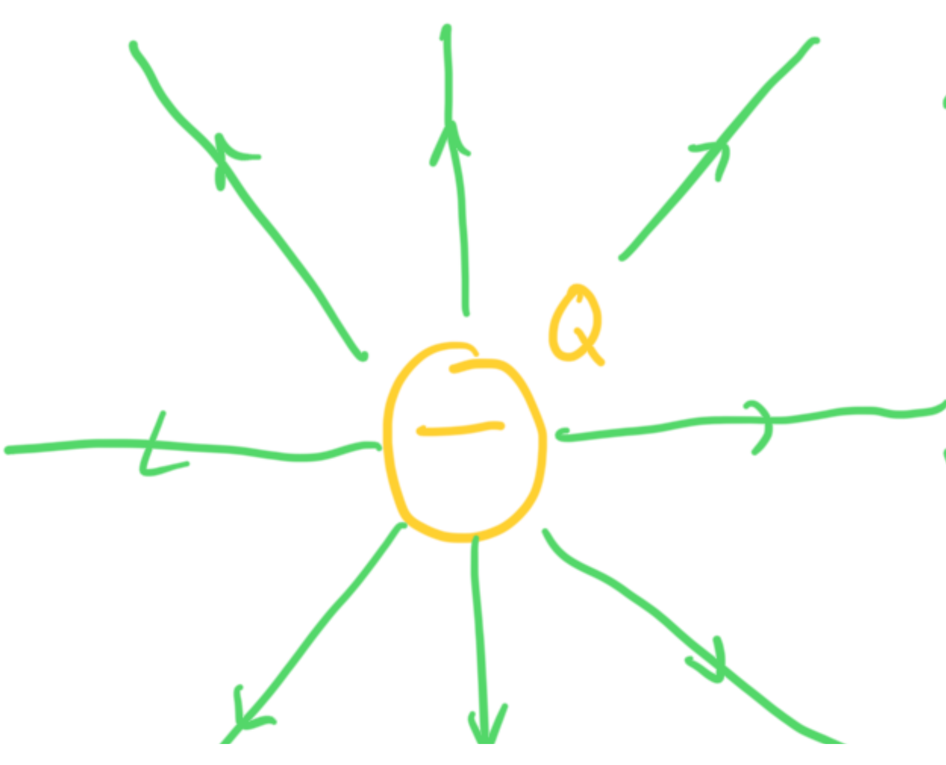
## Summary for Point Charges:




$$\left.\begin{array}{ll} \underline{\text{vector}} & \vec{E} = \dfrac{k|Q|}{r^2}\hat{r} \\ \underline{\text{scalar}} & V = \dfrac{kQ}{r} \end{array}\right\} \text{Fields}$$

$\uparrow$




$$\vec{E} = -\frac{k|Q|}{r^2}\hat{r}$$

$$V = \frac{kQ}{r}$$

voltage is positive for positive charges, and negative for negative for negative charges.

→ I hate vectors, and I ♡ scalars!

So, much prefer to work with voltage and electrical potential energy.

THIS IS WHY YOU'VE HEARD OF VOLTAGE BEFORE, BUT PROBABLY NOT Electric Field!! ☺

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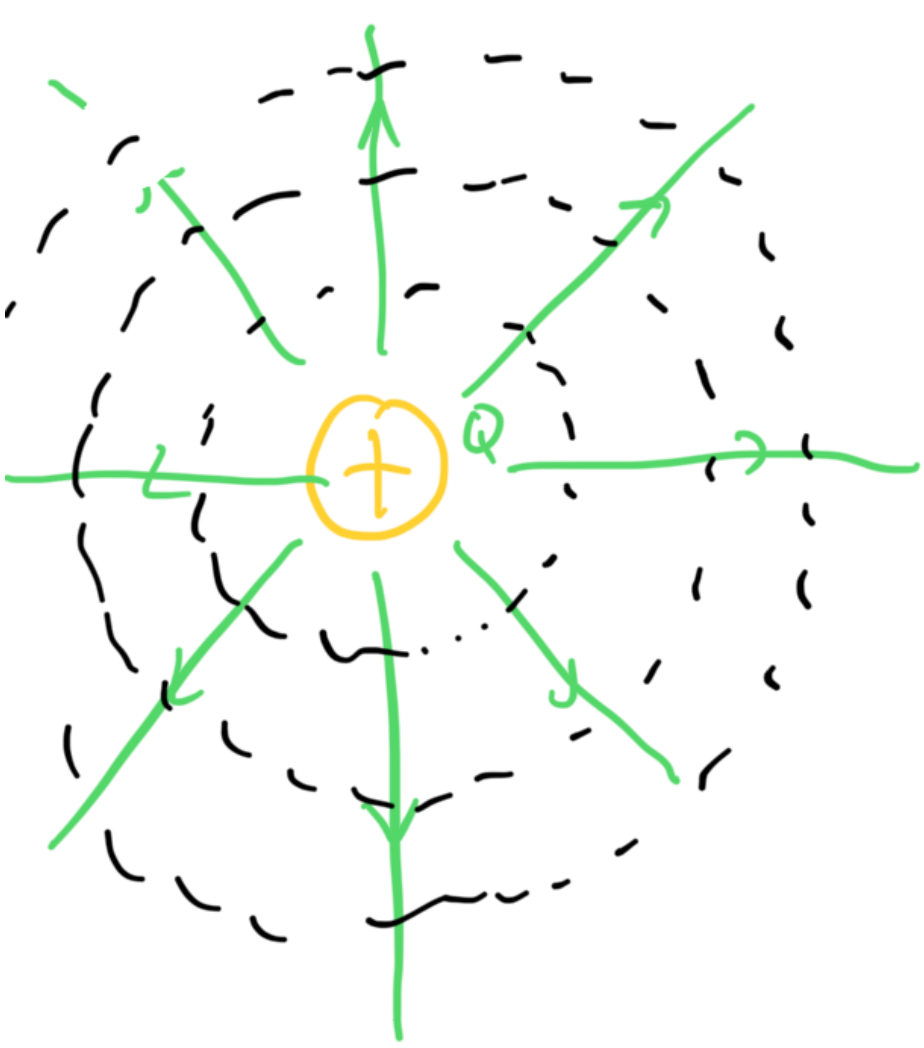
Equipotential Lines.

equal voltage




$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

$$V = \frac{kQ}{r}$$

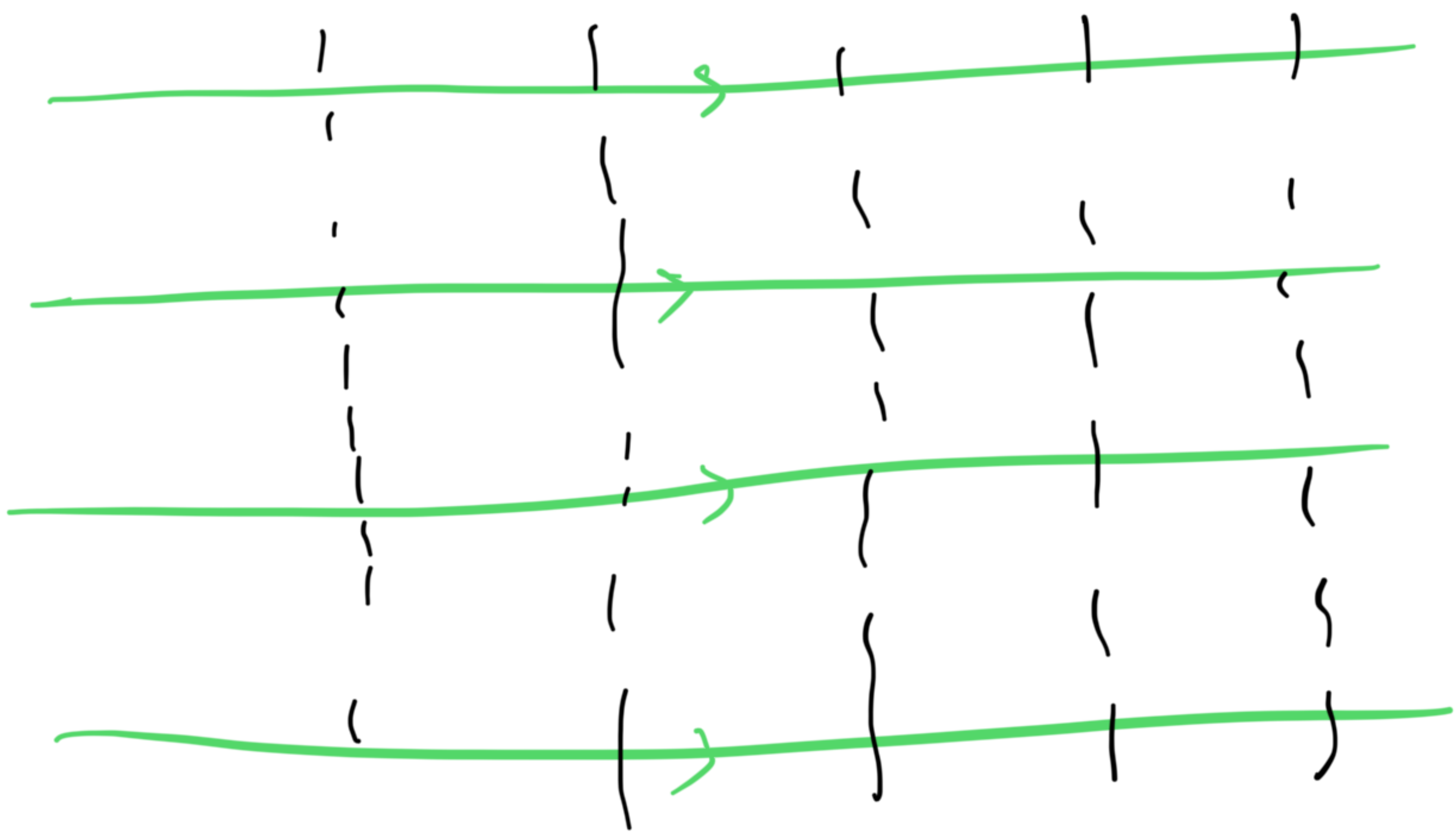
let $V$ = constant

$$\frac{kQ}{r} = C$$

$r = \frac{kQ}{1}$ , $r = \frac{kQ}{2}$ ,

$r = \frac{kQ}{3}$ , etc.

$r = \frac{kQ}{c}$ ← i.e. radius = constant



$\vec{E} = E_0 \hat{i}$ $\quad \underbrace{V = -E_0 x}_{= \text{constant}}$

$-E_0 x = c$

$x = -\frac{c}{E_0}$ , i.e. $x$ = constant.

$x = -\frac{1}{E_0}, +\frac{1}{E_0}, -2/E_0, +\frac{2}{E_0}$, etc....