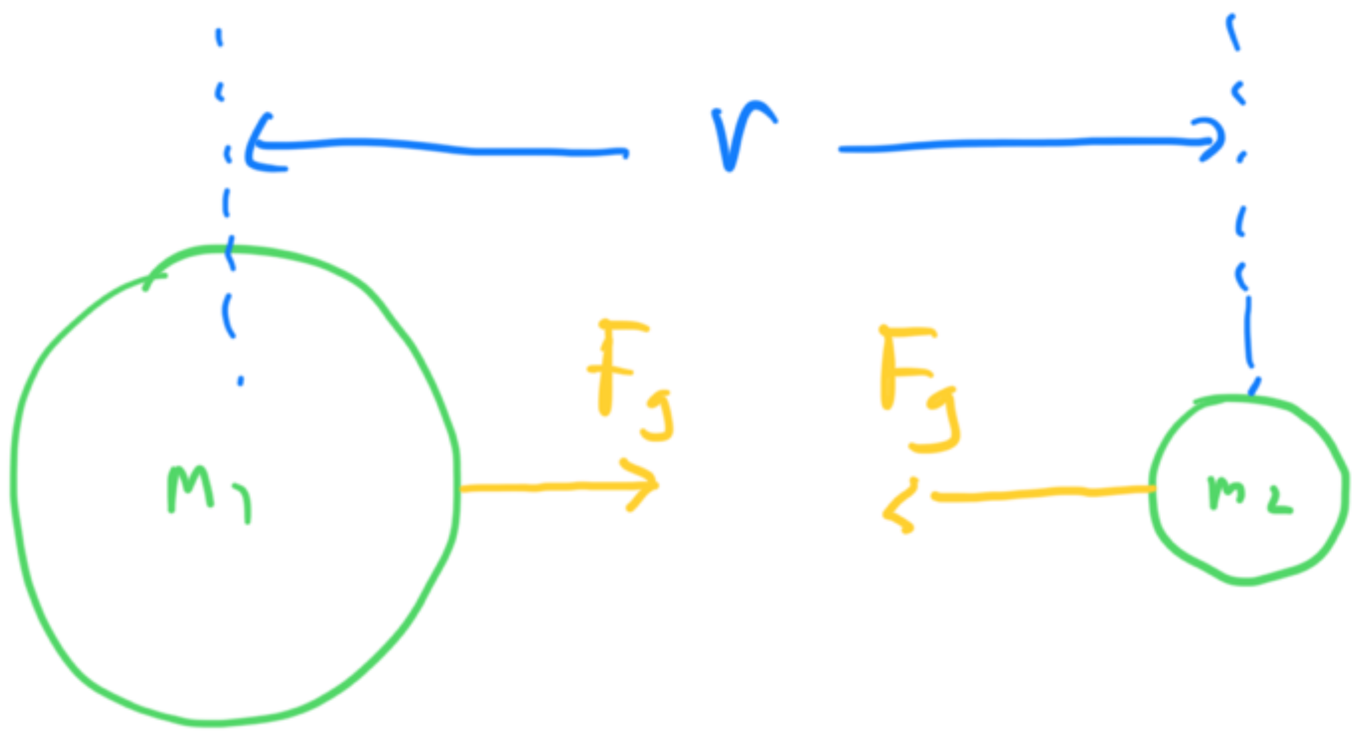
Gravity



Important Concepts:

① Forces always come in <u>pairs</u>!!!

— Newton's Third Law

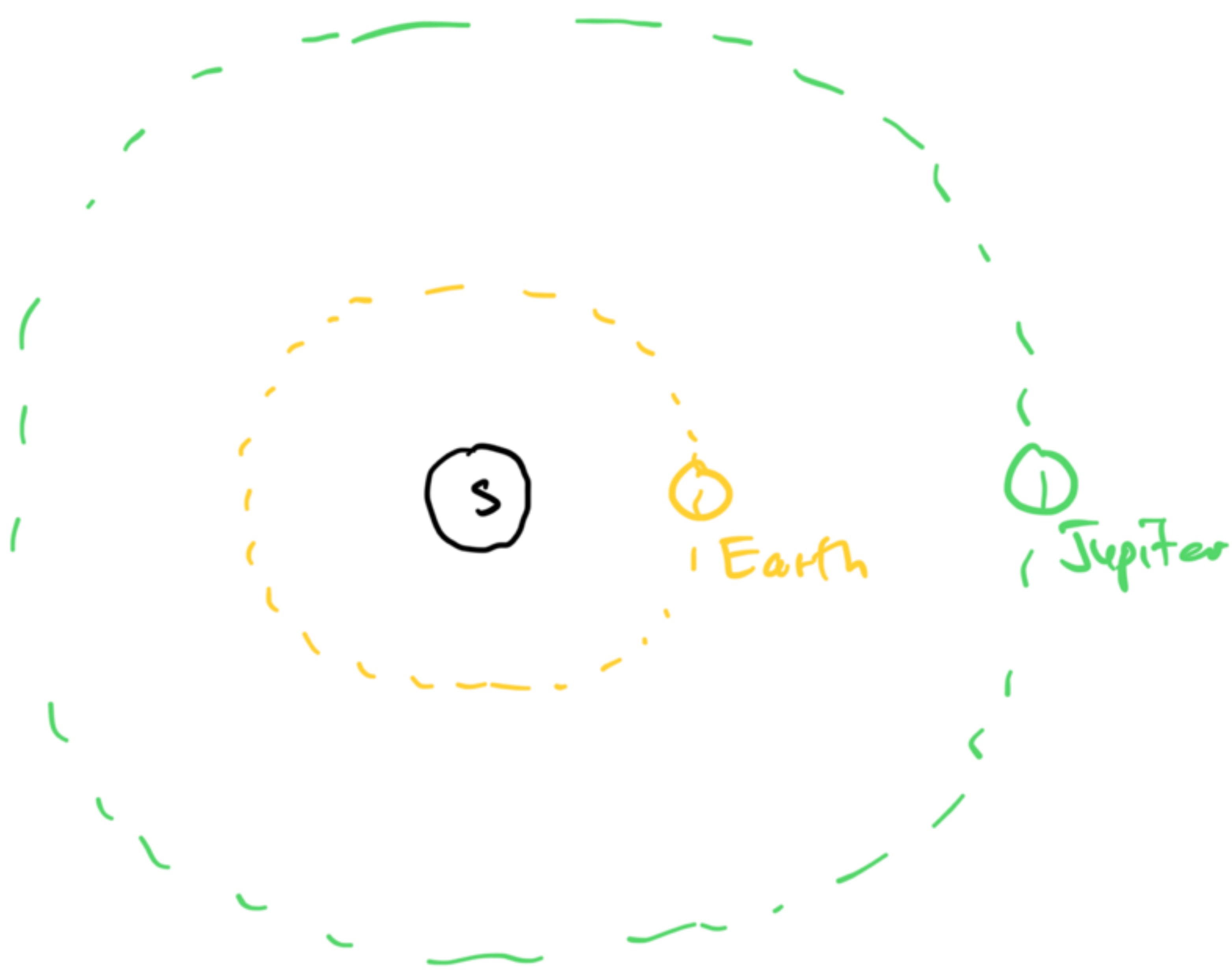
$$\vec{F}_{AB} = -\vec{F}_{AB}$$

"equal and opposite"

② $$|\vec{F}_g| = F_g = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

Example: Calculate the Force of Jupiter on a human baby, due to gravity. Assume Jupiter is at its <u>closest</u> approach to Earth.



$$R_{Earth/Sun} = 1.50 \times 10^{11}\ m$$

$$R_{Jupiter/Earth} = 7.78 \times 10^{11}\ m$$

$$\therefore\ R_{Earth/Jupiter} = (7.78 - 1.50) \times 10^{11}\ m$$

$$= 6.28 \times 10^{11}\ m$$

$m_{Jupiter} = 1.90 \times 10^{27}$ kg

$m_{Baby} = \sim 7$ lbs $= 3.2$ kg

$$|\vec{F}_g| = \frac{G \, m_{baby} \, m_{Jupiter}}{R_{E/J}^2}$$

$$= \frac{(6.67 \times 10^{-11})(3.2)(1.90 \times 10^{27})}{(6.28 \times 10^{11})^2}$$

$$|\vec{F}_g| = 1.02 \times 10^{-6} \text{ N}$$

b) Calculate the force of gravity of the doctor on the baby.

$$m_{doctor} \approx 180 \text{ lbs} = 81.7 \text{ kg}$$

$$R_{doctor/baby} \approx 0.1 \text{ m}$$

$$|\vec{F}_g| = \frac{G\, m_{baby}\, m_{doctor}}{R_{D/B}^2}$$

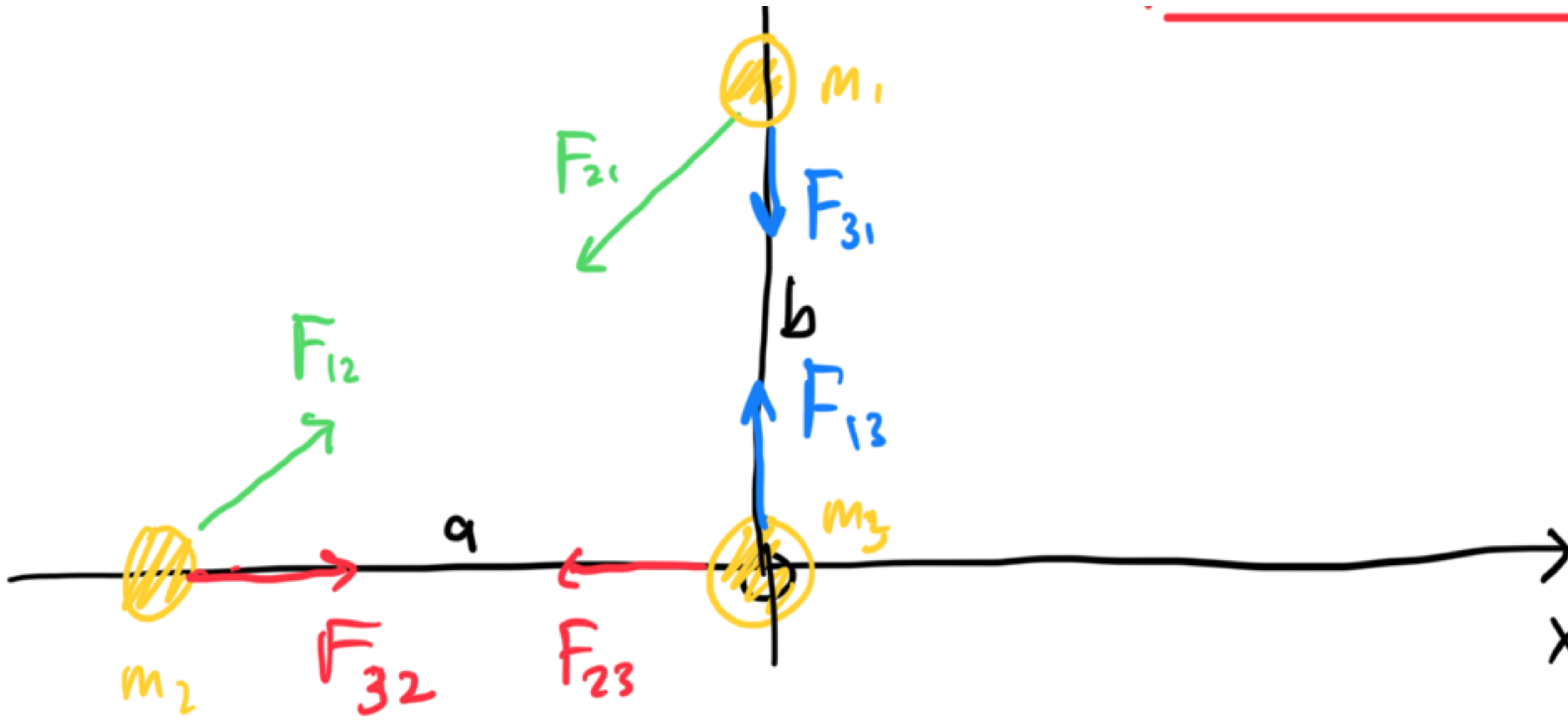
$$= \frac{(6.67 \times 10^{-11})(3.2)(81.7)}{(.1)^2}$$

$$|\vec{F}_g| = 1.74 \times 10^{-6} \text{ N}$$

Conclusion: Astrology should pay more attention to who your OB/GYN is than where the planets are.

A1 Q2:

$\boxed{\text{Forces come in pairs!}}$



Question what is the <u>net</u> force on m_3?

$$\vec{F}_{NET} = \underbrace{\vec{F}_{23}}_{\text{on } 3} + \underbrace{\vec{F}_{13}}_{\text{on } 3}$$

$$|\vec{F}_{23}| = \frac{G m_3 m_2}{a^2}$$

$$\therefore \vec{F}_{23} = -\frac{G m_3 m_2}{a^2}\hat{i}$$

unit vector in positive x direction

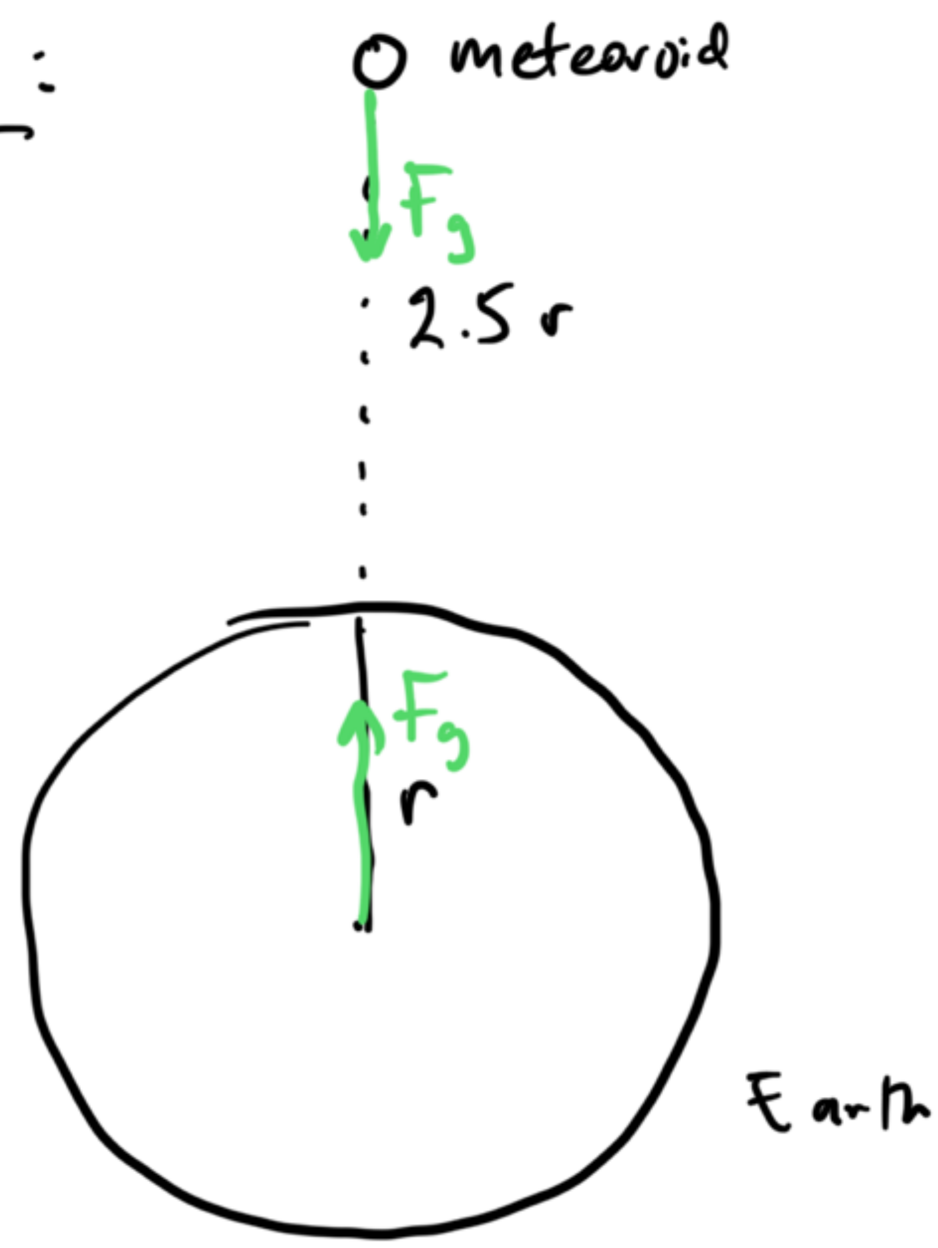
$$|\vec{F}_{13}| = \frac{G m_3 m_1}{b^2}$$

$$\therefore \vec{F}_{13} = +\frac{G m_3 m_1}{b^2}\hat{j}$$

unit vector in positive y direction.

$$\therefore \vec{F}_{net} = -\frac{G m_3 m_2}{a^2}\hat{i} + \frac{G m_3 m_1}{b^2}\hat{j}$$

A1 Q3:



What is the acceleration due to gravity of the meteroid?

$$|\vec{F}_g| = \frac{G \cancel{m_{meteroid}} m_{Earth}}{(3.5r)^2} = \cancel{m_{meteroid}} a_g$$

Note: Surface

$G = 6.67 \times 10^{-11}$

$m_{Earth} = 5.98 \times 10^{24}$ kg

$r = 6.378 \times 10^{6}$ m

$$\frac{G \cancel{m} m_E}{r^2} = \cancel{m} g$$

$$\boxed{g = \frac{G m_E}{r^2}}$$

$$a_g = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{3.5^2 (6.378 \times 10^6)^2}$$

$$= 0.800 \text{ m/s}^2$$

Easier Way:

$$a_g^{surface} = g = 9.8 \text{ m/s}^2$$

$$a_g \propto \frac{1}{(distance)^2}$$

$$\therefore a_{meteor} = a_{surface} \times \left(\frac{1}{3.5^2}\right) = 0.8 \text{ m/s}^2$$

A1 Q4: Classic Physics Problem →

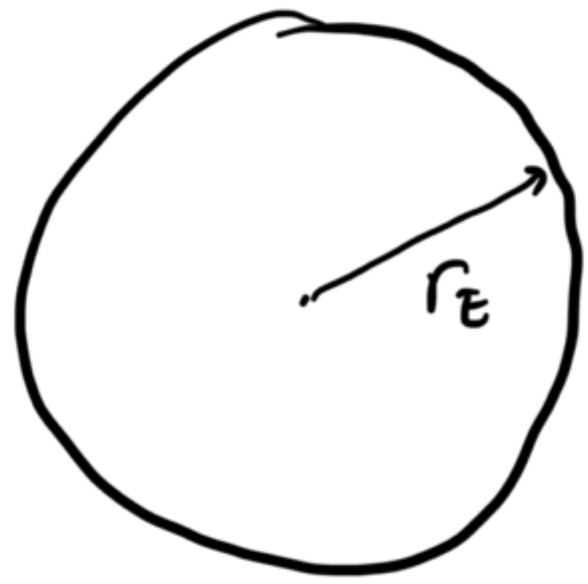
Two situations.

Venus	Earth

m_V

r_V

$$g_V = \frac{G m_V}{r_V^2}$$



m_E

r_E

$$g_E = \frac{G m_E}{r_E^2}$$

$$g_V \approx g_E$$

$$\therefore \quad \frac{\cancel{G} m_V}{r_V^2} = \frac{\cancel{G} m_E}{r_E^2}$$

$$\therefore \quad m_V = m_E \left(\frac{\overset{.65 r_E}{r_V}}{r_E} \right)^2$$

$$\boxed{m_V = (0.65)^2 \, m_E}$$

$\rho = \frac{m_V}{\;}$ $\qquad\qquad$ $\rho = \frac{m_E}{\;}$

$$\rho_v = \frac{\quad}{V_v} \qquad\qquad \rho_E = \frac{\quad}{V_E}$$

$$= \frac{m_v}{\frac{4}{3}\pi r_v^3} \qquad\qquad = \frac{m_E}{\frac{4}{3}\pi r_E^3}$$

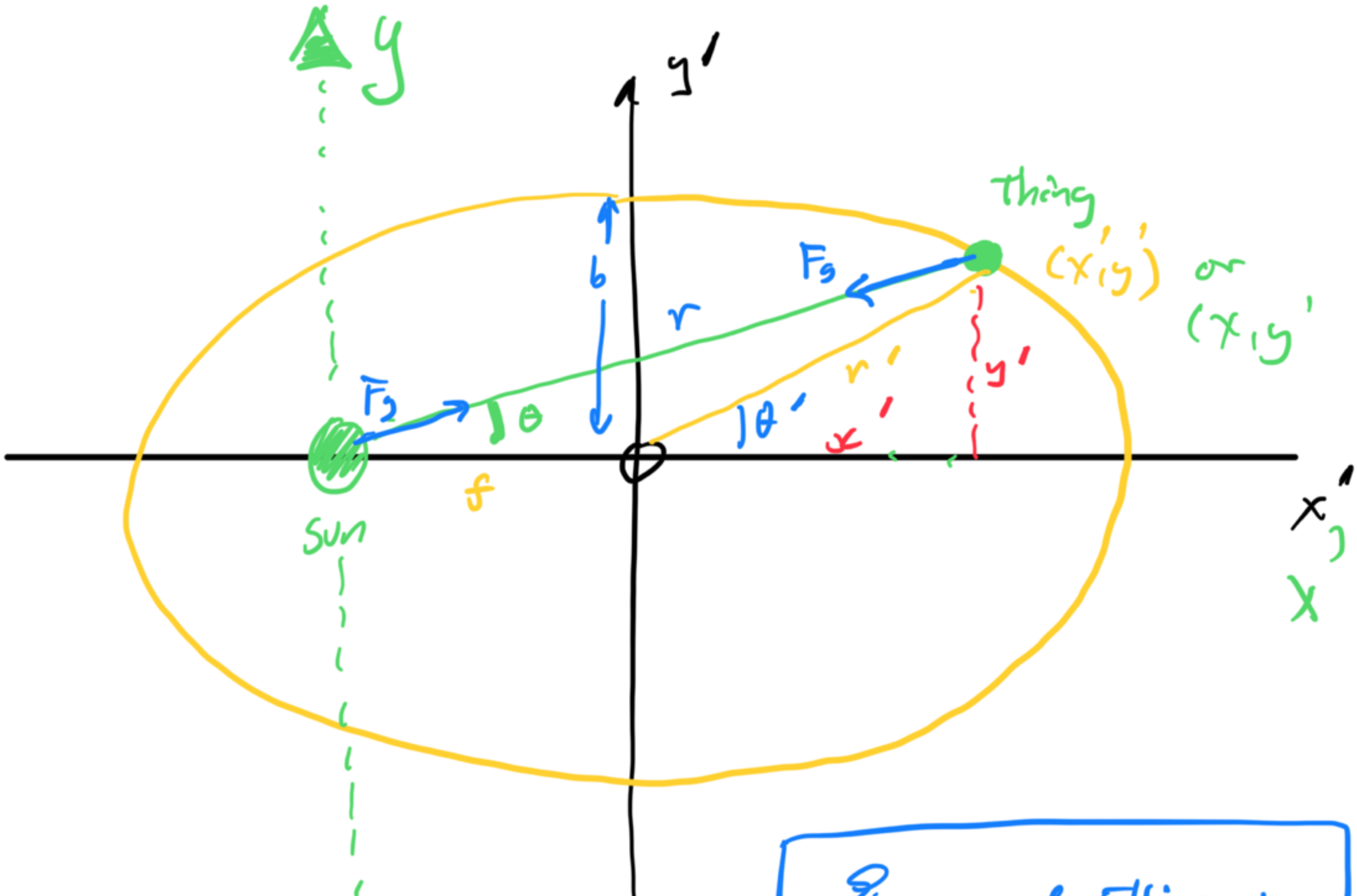
$$\therefore \frac{\rho_v}{\rho_E} = \frac{\frac{m_v}{\frac{4}{3}\pi r_v^3}}{\frac{m_E}{\frac{4}{3}\pi r_E^3}} = \frac{r_E^3}{r_v^3} \cdot \frac{m_v}{m_E}$$

$$= \left(\frac{1}{.65}\right)^3 \cdot (.65)^2$$

$$\frac{\rho_v}{\rho_E} = \frac{1}{0.65} = \underline{\underline{1.54}}$$

AIQS: Comets

- Johannes Kepler ... orbits are ellipses!! The Sun is at one focus of the ellipse.
- Hmmmm... I wonder why?



$$|\vec{F_g}| = \frac{G M_T M_s}{d^2}$$

$\boxed{\text{Equation of Ellipse: } \dfrac{(x')^2}{a^2} + \dfrac{(y')^2}{b^2} = 1}$

Definition of Ellipse:

$$d_1 + d_2 = \text{constant}$$

1 Change of coordinate systems! Let's use the x/y coordinate system to define r and θ.

$$|\vec{F}_g| = \frac{G m_T m_s}{r^2}$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

② Conservation of Energy

Kinetic Energy $= \frac{1}{2}mv^2 \equiv T$

Gravitational Potential Energy $\equiv V$

$$|\vec{F}_g| = -\frac{\partial V}{\partial r}$$

$$\therefore \boxed{V = -\frac{G m_T m_s}{r}}$$

Total Energy :

$$E = T + V = \frac{1}{2}m_T v^2 - \frac{G m_T m_s}{r}$$

$= \text{constant}$

Conservation of Angular Momentum:

$$\vec{L} = \vec{r} \times \vec{p} = \text{constant}$$

$$L = m r v \sin\theta = \text{constant}$$

Substitutions:

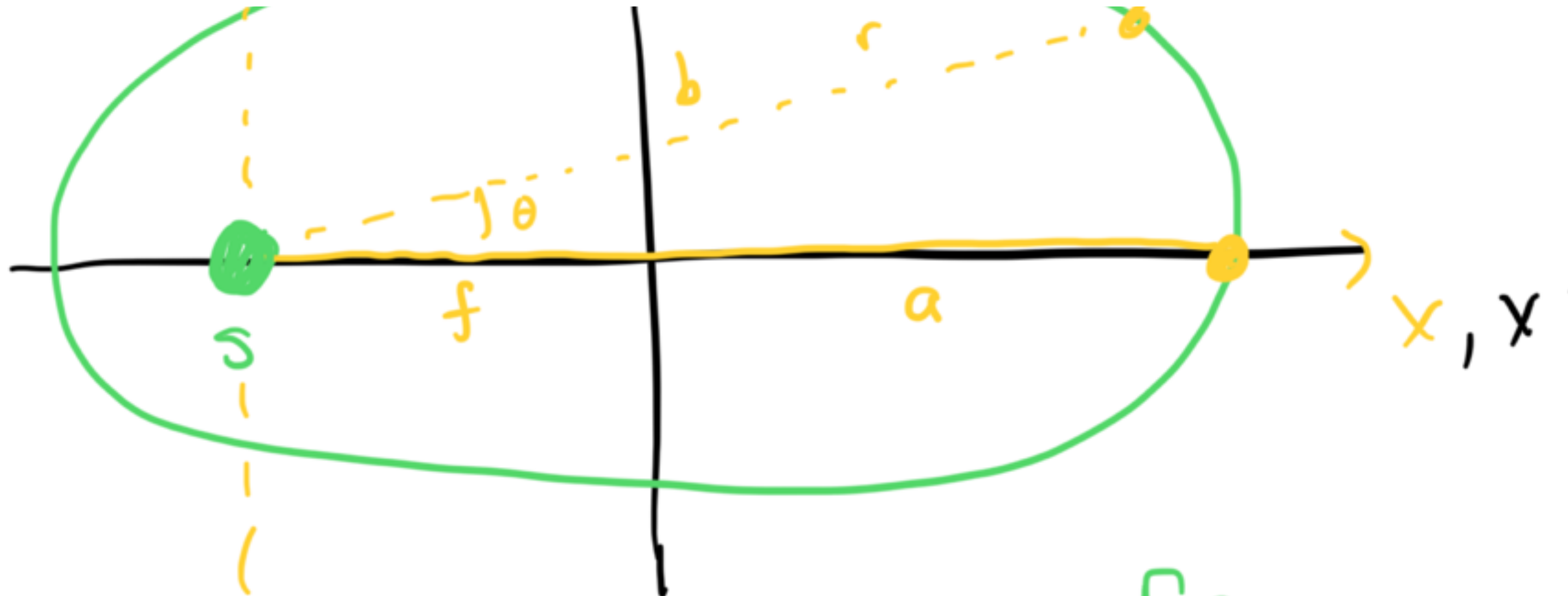
$$r_0 = \frac{L^2}{G M_S m_T^2}$$

$$e^2 = 1 + \frac{2 E r_0}{G M_S m_T}$$

$$\Big\downarrow \text{math}$$

$$\boxed{r = \frac{r_0}{1 - e\cos\theta}}$$

This is the equation of an ellipse in polar coordinates !!



At $\theta = 0$, $r = f + a = \dfrac{r_0}{1+e}$

at $\theta = 180°$, $r = a - f = \dfrac{r_0}{1-e}$

$$\boxed{r = \frac{a(1-e^2)}{1 - e\cos\theta}}$$

$$2a = r_0\left(\frac{1}{1+e} + \frac{1}{1-e}\right)$$

$$= r_0\left(\frac{1-e+1+e}{1-e^2}\right)$$

$$2a = \frac{2r_0}{1-e^2}$$

$$r_0 = a(1-e^2)$$

$e \equiv$ eccentricity $= \sqrt{1 - \dfrac{b^2}{a^2}}$

notes: ① $e = 0$ for a circle

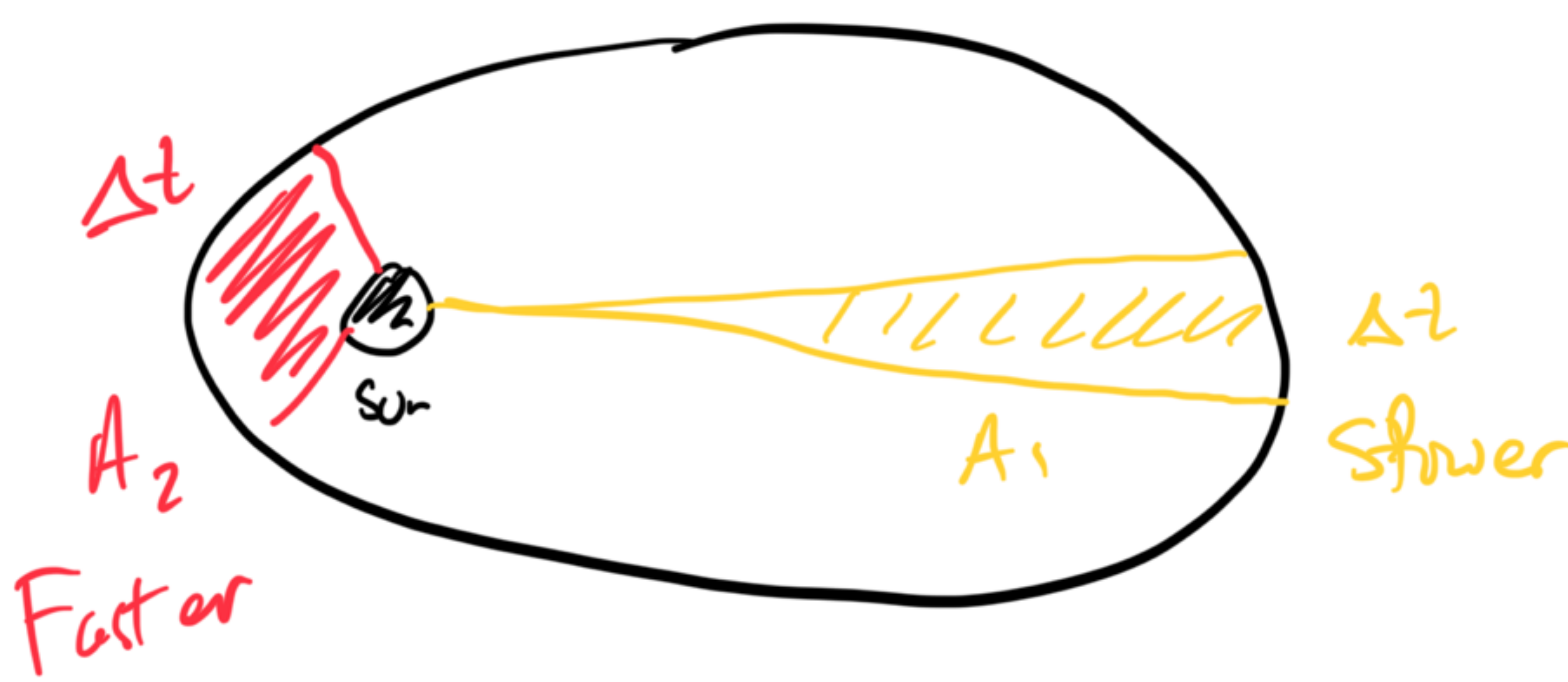
② most elliptical orbit → Mercury

$e_{Mercury} \sim 0.20$

$\rightarrow \quad b = 0.98\,a$

(still, almost circular !!)

<u>Kepler's 2^{nd} Law</u> : area swept out is equal in equal time intervals.



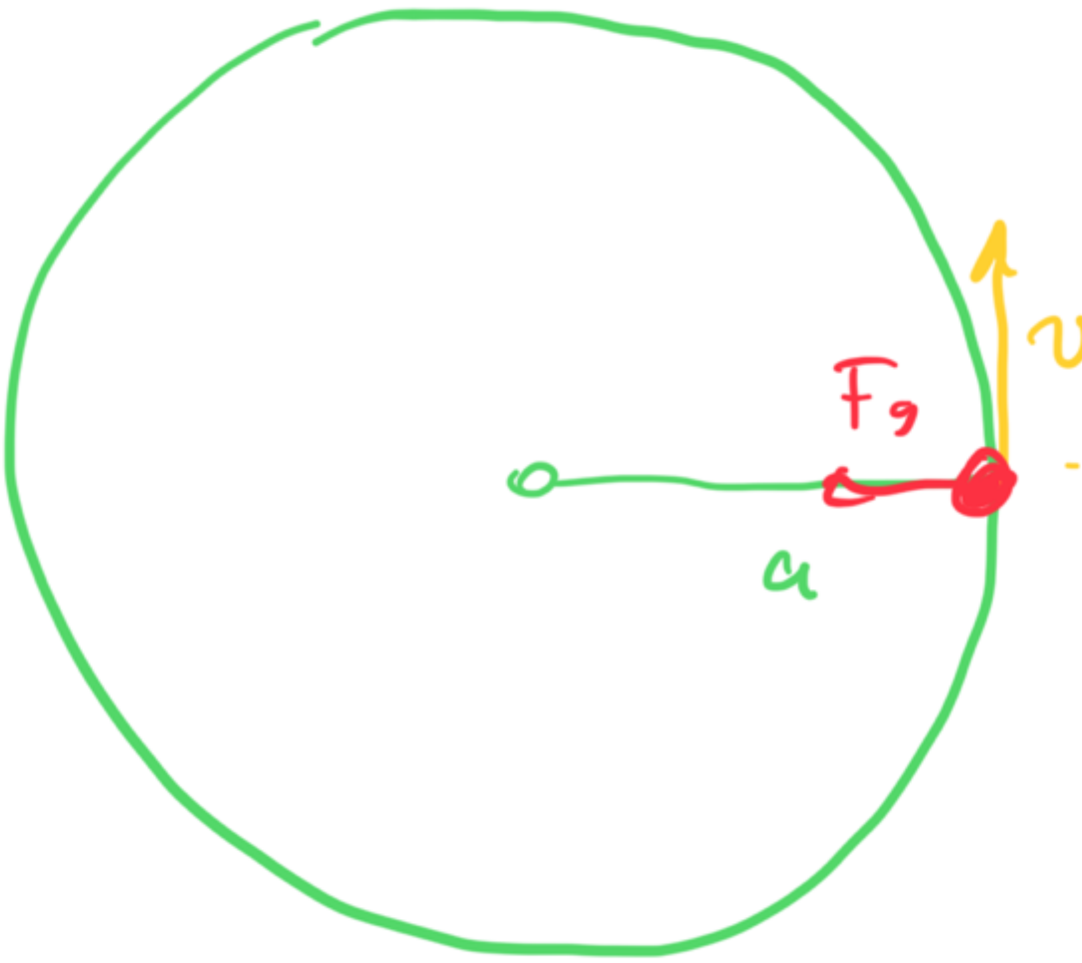
→ This can be derived easily from conservation of angular momentum.

<u>Kepler's third Law</u>

a^3

$$\frac{a^3}{T^2} = \text{constant}$$

Consider a <u>circular</u> orbit:



$$C = 2\pi a$$

$$t = T \leftarrow \text{period.}$$

$$v = \frac{C}{t} = \frac{2\pi a}{T}$$

$$F_g = \frac{G \cancel{m} m_s}{a^2} = \cancel{m} a = \frac{v^2}{a} \leftarrow \text{Centripetal acc.}$$

$$\frac{G m_s}{a^{\cancel{2}}} = \frac{v^2}{\cancel{a}}$$

$$a^2 \quad G m_s = 4\pi^2 a^2$$

$$v^2 = \frac{G m_s}{a} = \frac{4\pi^2 a^2}{T^2}$$

$$\frac{a^3}{T^2} = \boxed{\frac{G m_s}{4\pi^2}} = \text{constant!}$$

What is this # ?

We could work it out → $G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

$m_s = \ldots$

Or, we could use Earth!!

$a = 1$ A.U. $(= 1.50 \times 10^{11} m)$

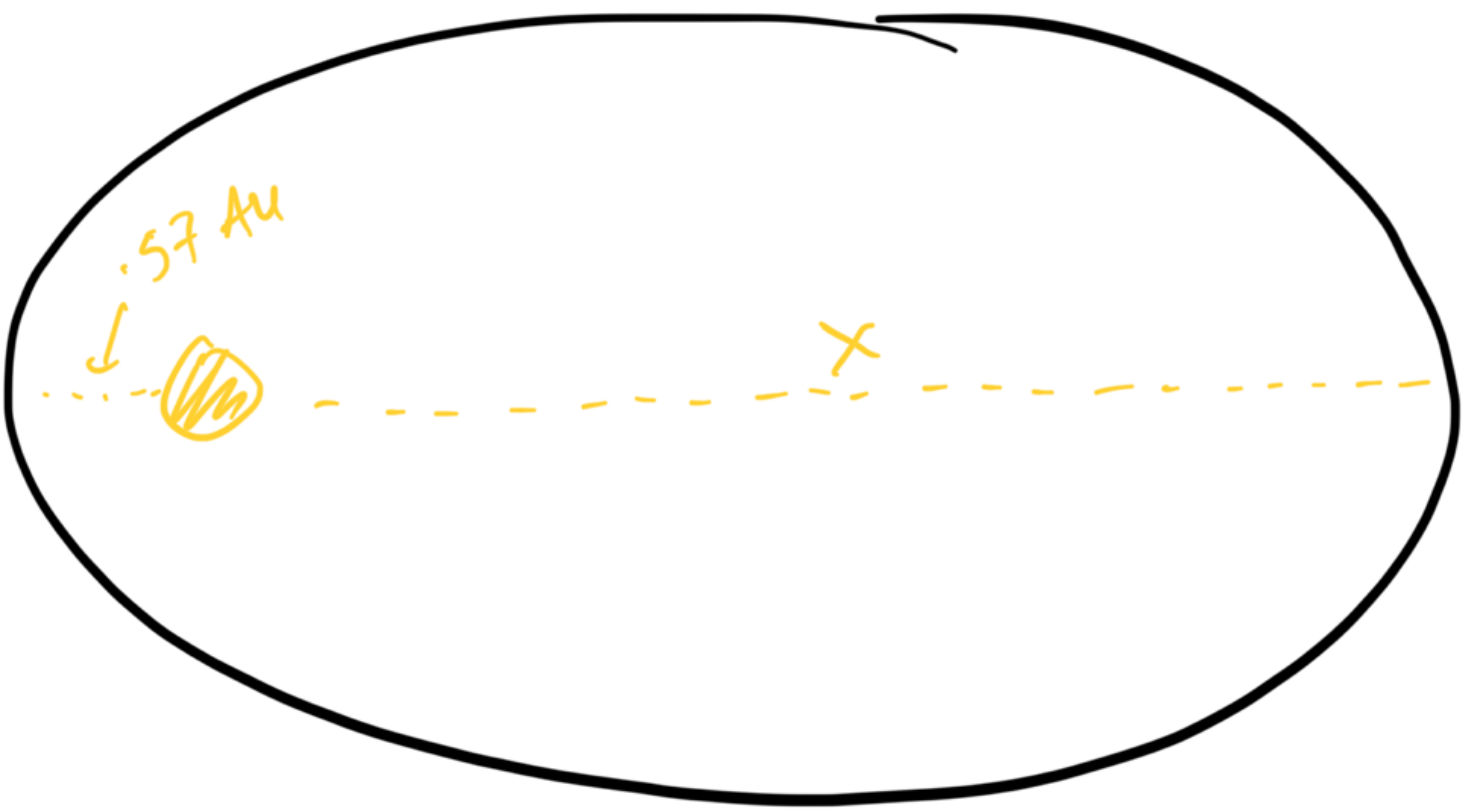
$T = 1$ year (by definition!)

$$\therefore \frac{a^3}{T^2} = \frac{(1 \, Au)^3}{(1 \, year)^2} = 1 \, \frac{Au^3}{yr^2}$$

Now, we can use this constant for

everything that orbits the Sun, incl.

comets !!



$$2a = x + 0.57$$

$$\frac{a^3}{T^2} = 1\ \frac{Au^3}{yr^2}$$

$$a^3 = 1\ \frac{Au^3}{yr^2}\ (83.6\ yr)^2$$

$$= (83.6^2)\ Au^3$$

$$a = \sqrt[3]{(83.6)^2}\ Au = 19.12\ Au$$

$2a = 38.24\ Au = x + .57\,Au$

$x = 37.7\ Au.$

A1Q6: Io, a satellite of Jupiter

$a = 4.22 \times 10^8\ m$

$T = 1.77 \times 24 \times 3600\ s$

$\therefore \dfrac{a^3}{T^2} = \dfrac{(4.22 \times 10^8)^3}{(1.77 \times 24 \times 3600)^2} = 3.21 \times 10^{11}\ \dfrac{m^3}{s^2}$

$= G m_J$

$$= \frac{\ldots}{4\pi^2}$$

$$\therefore \quad m_J = \frac{4\pi^2 \cdot 3.21 \times 10^{11}}{G}$$

$$= 1.90 \times 10^{27} \text{ kg.}$$

Fun fact: This is how we determine the masses of planets, the mass of the sun, etc.

→ Most of the time, we can only measure

$$\boxed{\begin{array}{l} \text{1. speed} \\ \text{2. time} \end{array}}$$

we then have to use those to determine other things, based on some underlying theory!!