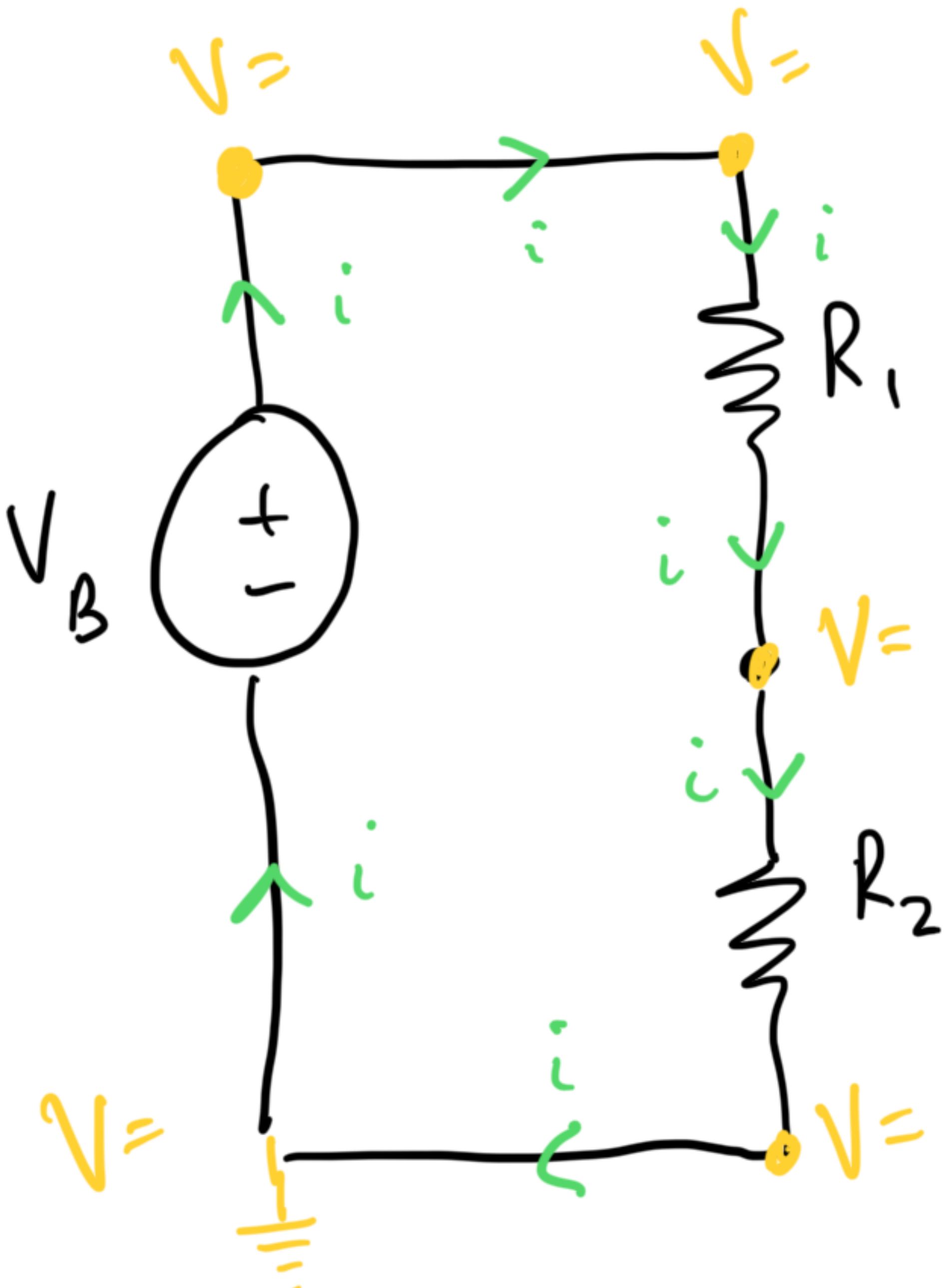
Phys 202 - Resistors in Series and parallel



Imagine connecting two devices, one after the other, in series.

What is i ?

$$\Delta V_{R1} = i R_1$$

$$V_B - V_M = i R_1$$

$$V_M = V_B - i R_1$$

$$\Delta V_{R2} = i R_2$$

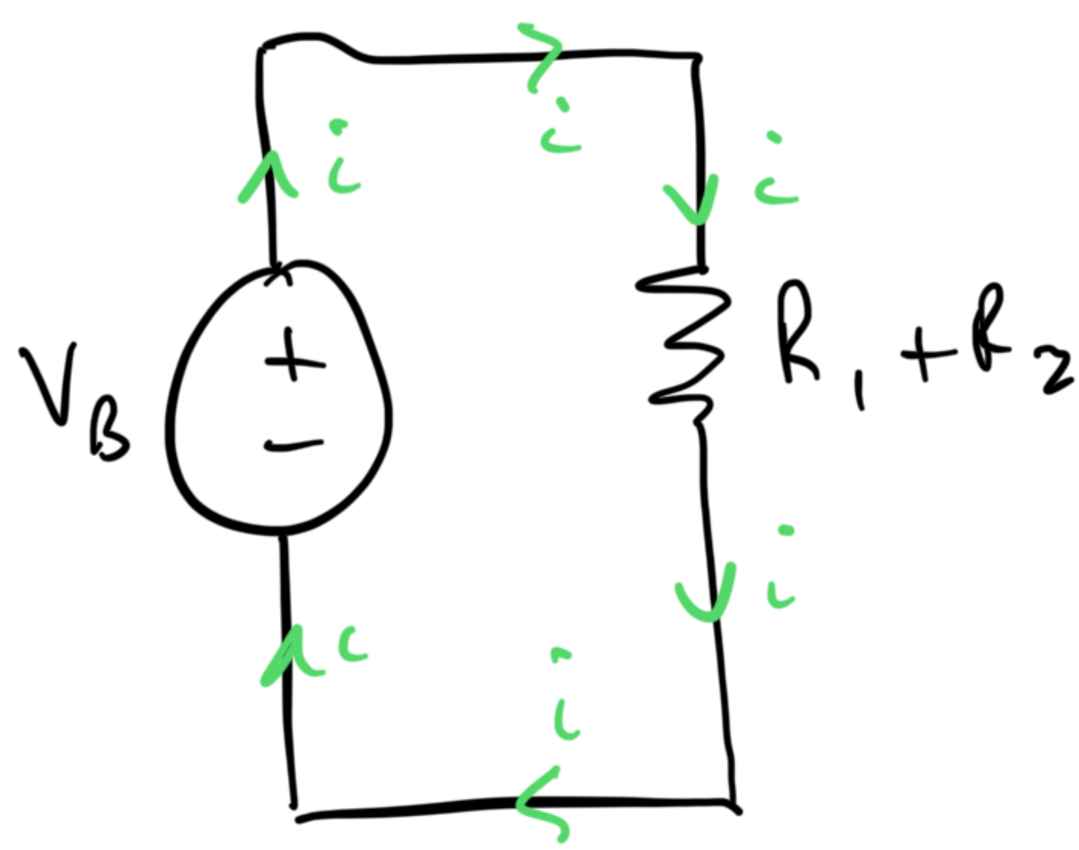
$$V_M - 0 = i R_2$$

$$V_B - iR_1 = iR_2$$

$$i(R_1 + R_2) = V_B$$

$$\boxed{i = \frac{V_B}{R_1 + R_2}}$$

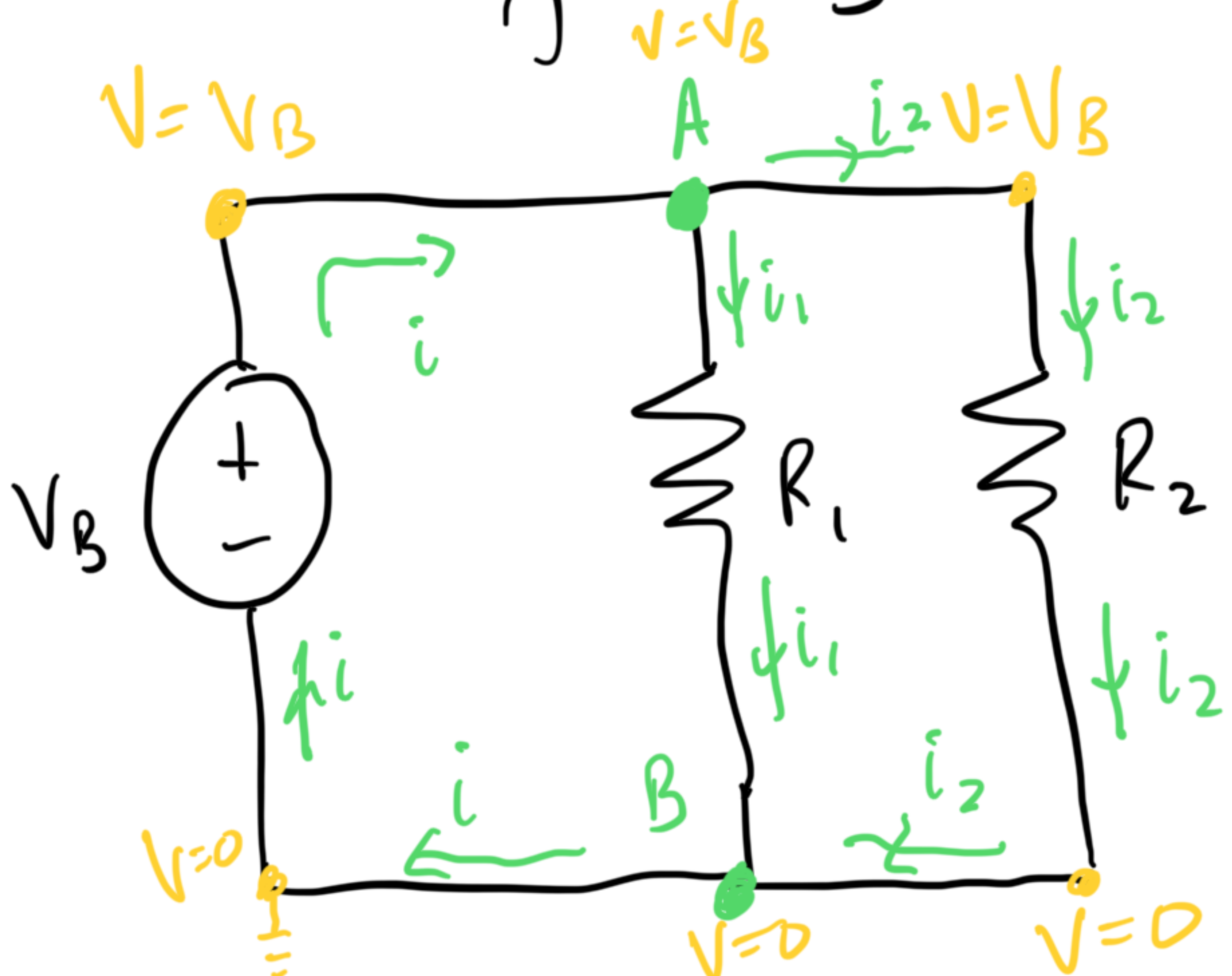
Now, notice that this is equivalent to the following:



Resistors in Series:

$$R_{\text{equivalent}} = R_1 + R_2 + \dots$$

Consider the following circuit:



Junction Rule:

current in = current out

A: $i = i_1 + i_2$

B: $i_1 + i_2 = i$

} consistent

$$\Delta V_{R_1} = i_1 R_1$$

$$V_B - 0 = i_1 R_1 \rightarrow i_1 = \frac{V_B}{R_1}$$

$$\Delta V_{R_2} = i_2 R_2$$

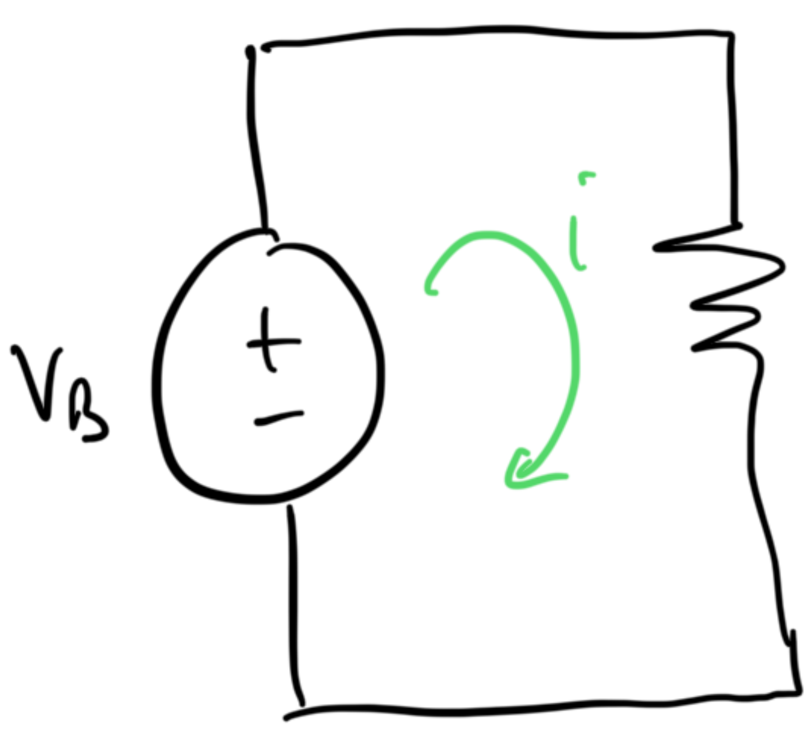
$$V_B - 0 = i_2 R_2 \rightarrow i_2 = \frac{V_B}{R_2}$$

$$i = i_1 + i_2 = \frac{V_B}{R_1} + \frac{V_B}{R_2}$$

$$i = V_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V_B = i\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}\right)$$

Now, this is the same as:

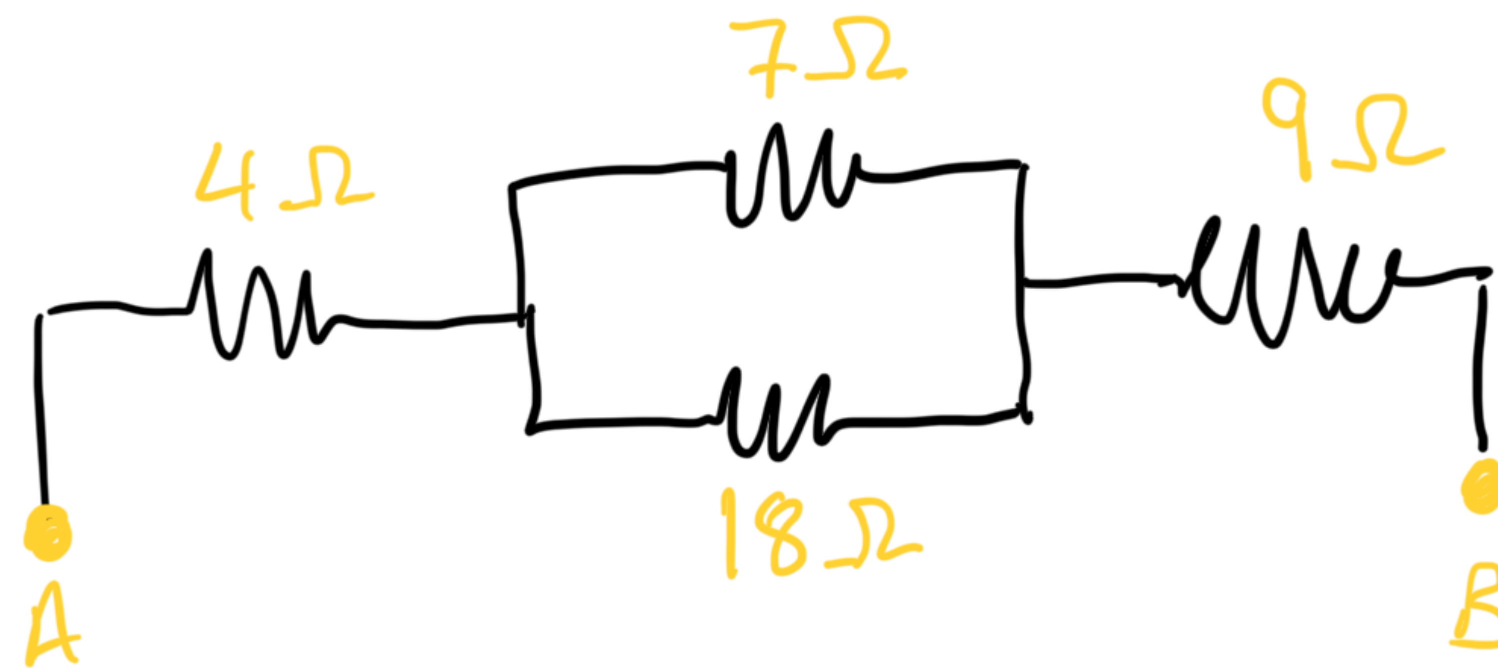


$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Resistors in Parallel:

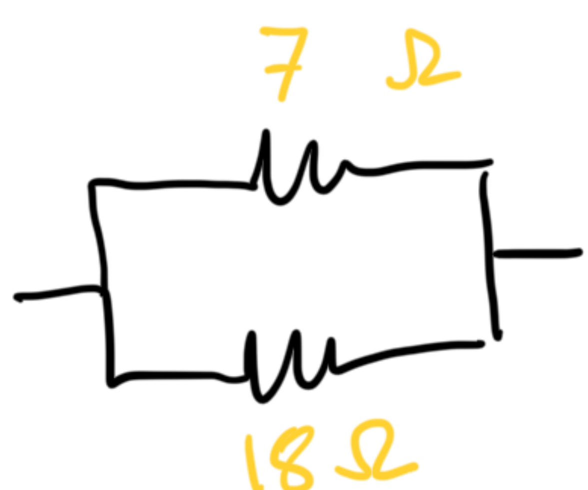
$$\boxed{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots\cdots}}$$

Example:



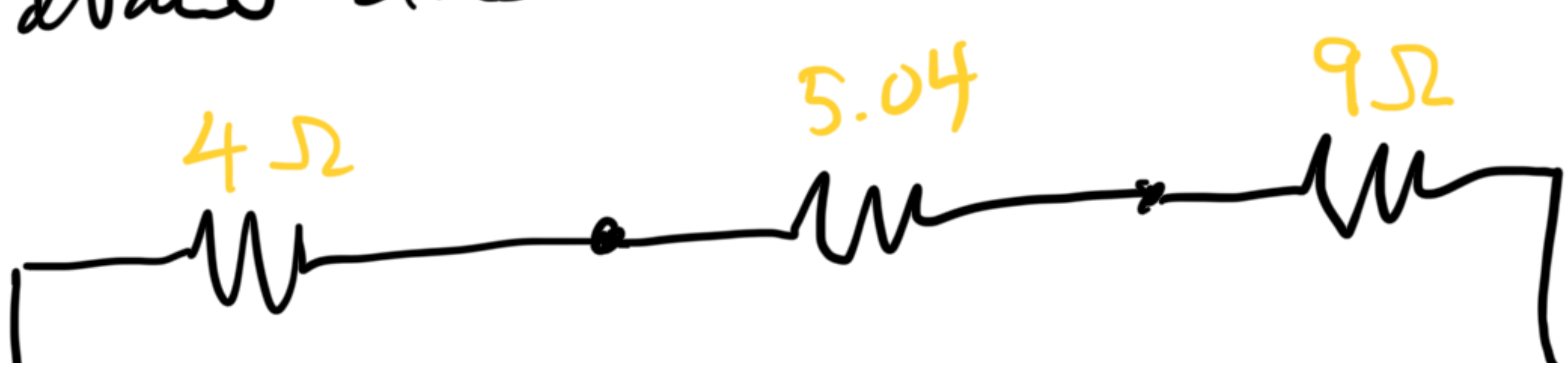
What is the equivalent resistance of this combination?

① Start with the resistors in parallel in the middle.



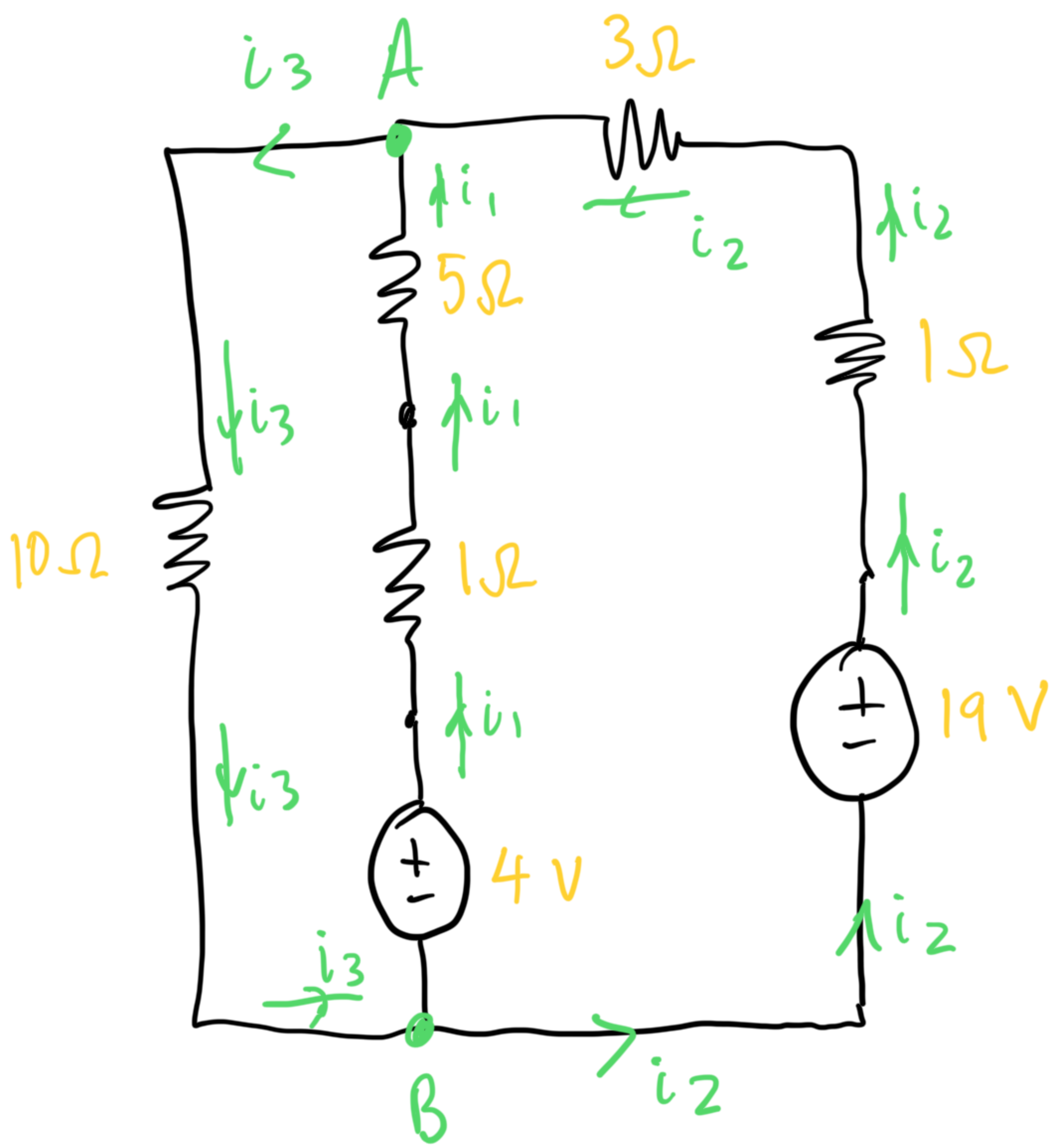
$$R_{eq} = \frac{1}{\frac{1}{7} + \frac{1}{18}} = 5.04\ \Omega$$

② Redraw the circuit...



③ Add in Series

$$R_{eq} = 4\,\Omega + 5.04\,\Omega + 9\,\Omega$$
$$= 18.04\,\Omega$$



Analyze this . . .

Step 1: Assign currents.

Step 2: Use Junction Rule

A: $i_1 + i_2 = i_3$

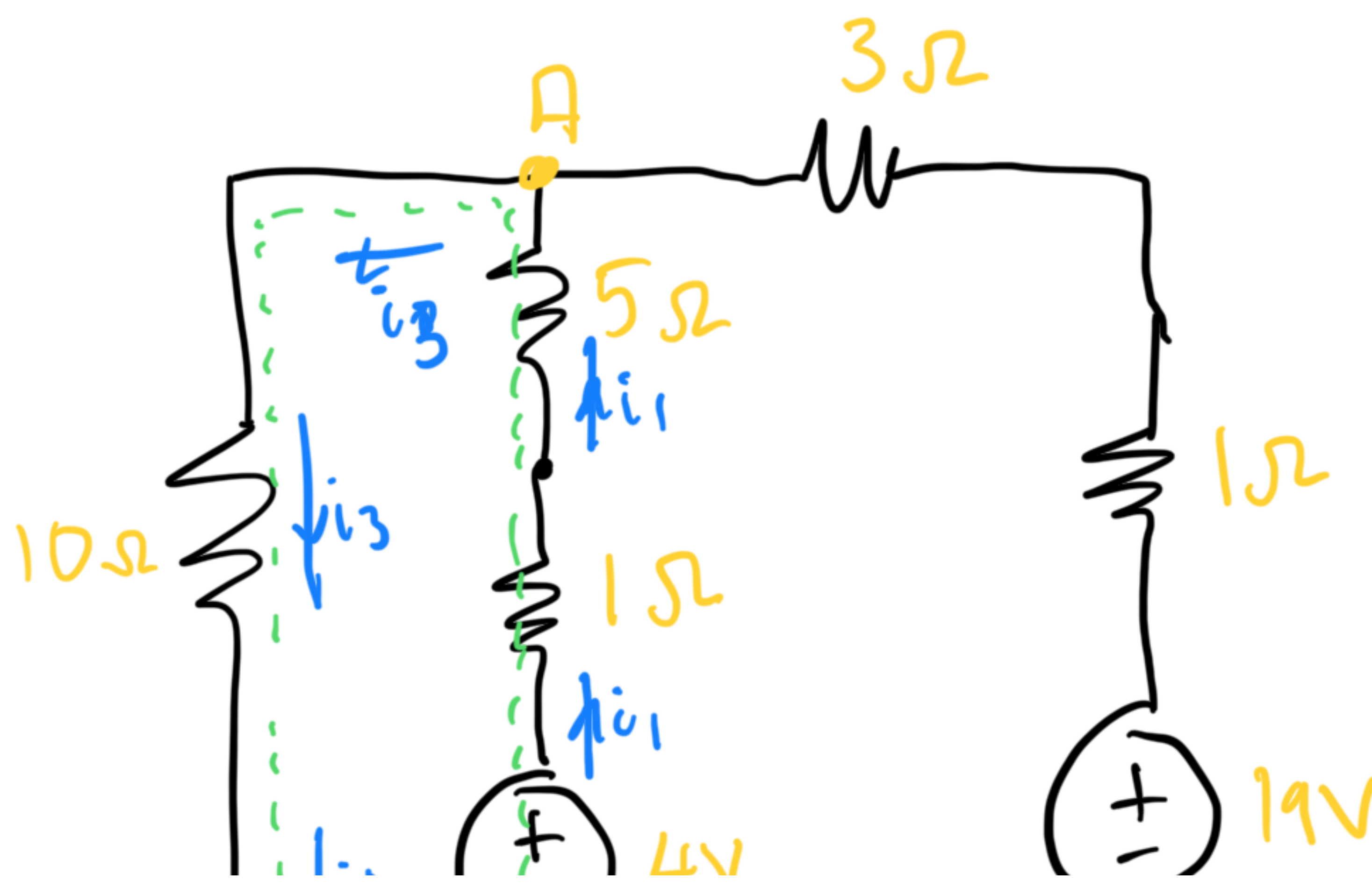
B: $i_3 = i_1 + i_2$

$\Big\}$ consistent!

Step 3: Use Loop Rule

- What's the loop Rule ???

" the sum of voltage drops and rises around any closed loop in a circuit is zero."

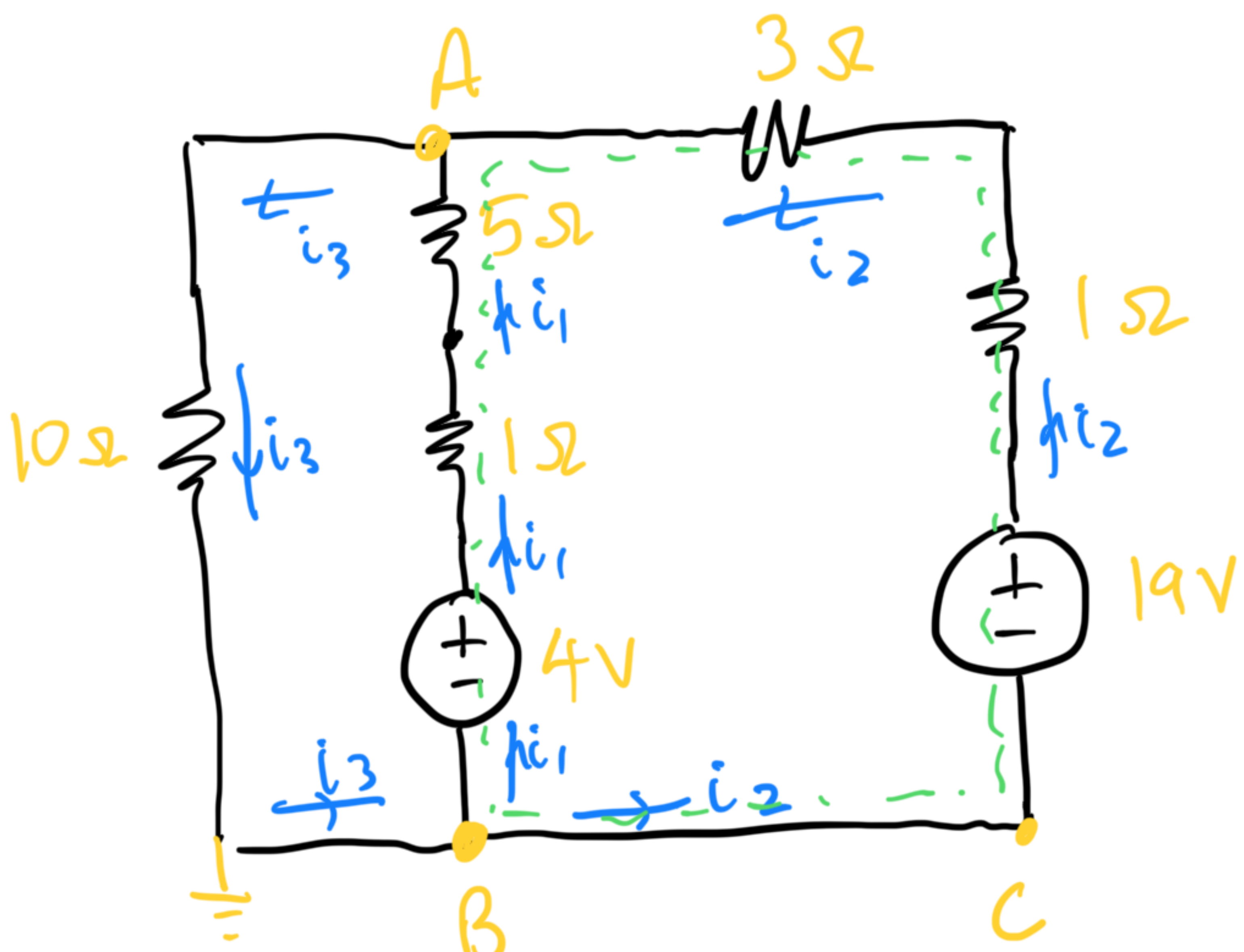


Start at B, go towards A:

$$+4 - i_1(1) - i_1(5) - i_3(10) = 0$$

Equation 1 $\boxed{4 - 6i_1 - 10i_3 = 0}$

$$-6x + 0y - 10z = -4$$



Start at C, go towards A:

$$+19 - i_2(1) - i_2(3)$$

$$+ i_1(5) + i_1(1) - 4$$

$$= 0$$

Equation 2 $\boxed{15 - 4i_2 + 6i_1 = 0}$

$+6x - 4y + 0z = -15$

Equation 3 $\boxed{i_1 + i_2 - i_3 = 0}$

$x + y - z = 0$

3 equations in 3 unknowns !!

Solve for i_1, i_2, i_3

Solution:

$i_1 = -1.08\ A$

$i_2 = +2.13\ A$

$i_3 = +1.05\ A$

What does this tell us...

Well, apparently we got the direction of i_1 wrong ☺

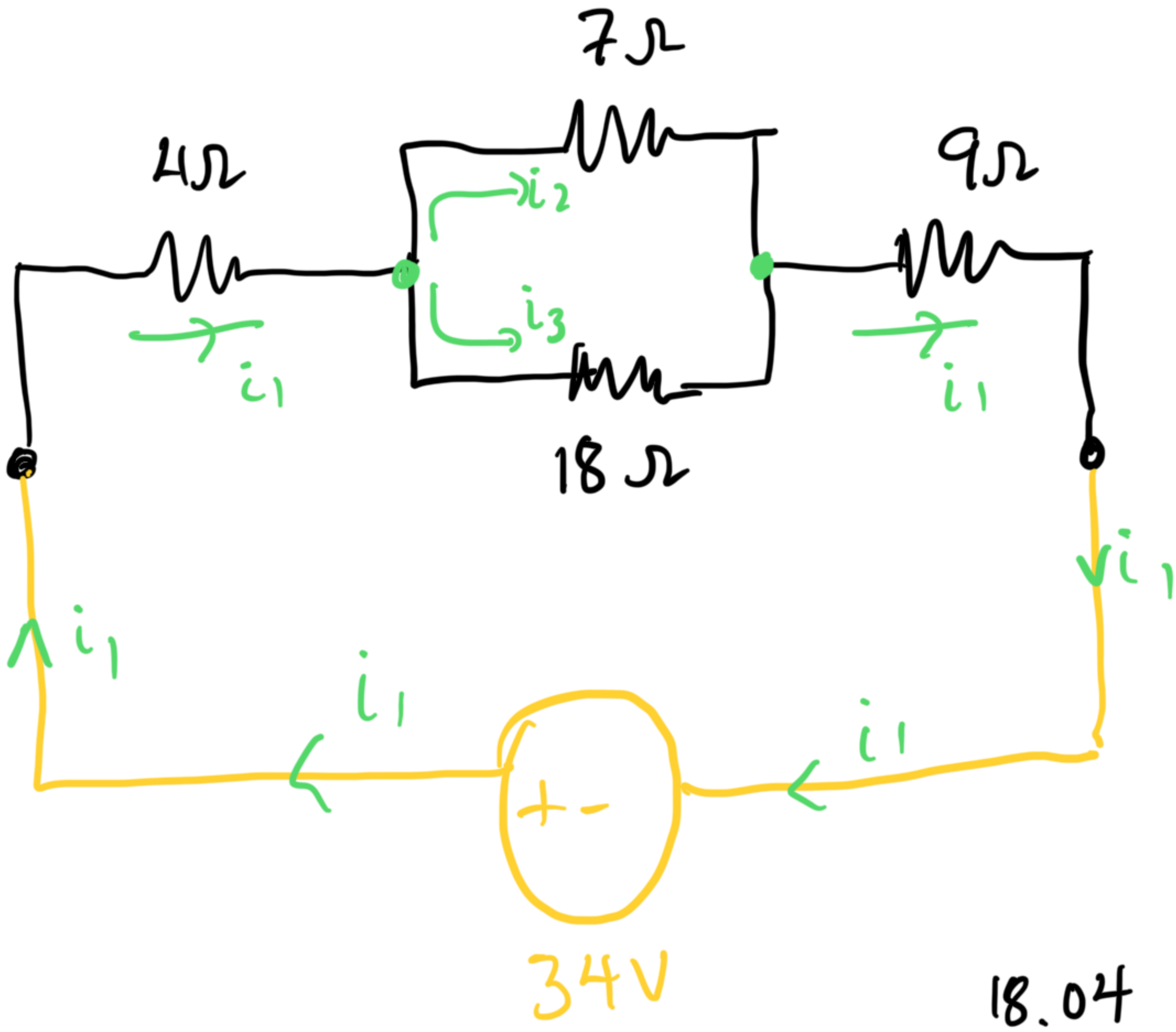
But that's cool, because math saved us... YAY!!!

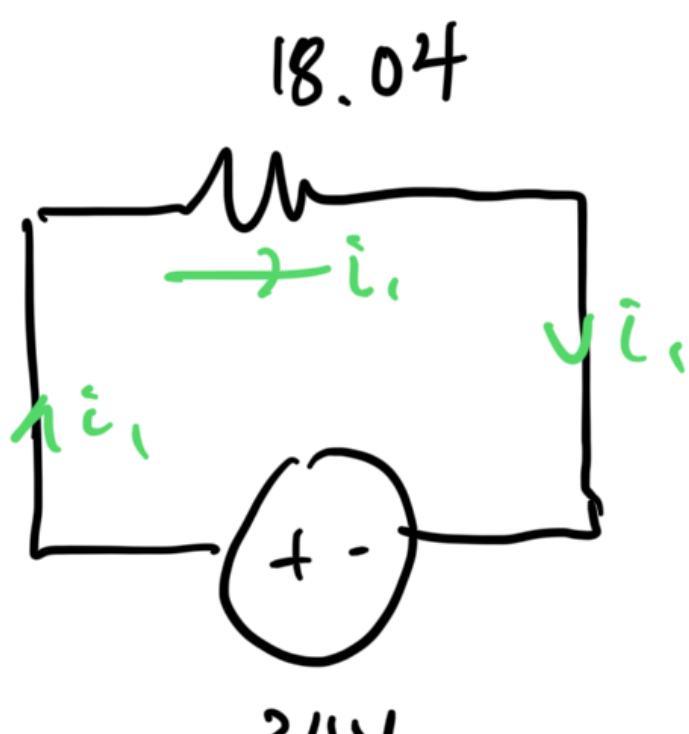
Example Websites:

[system equation calculator - Wolfram|Alpha]

[3 Unknown Calculator]

Let's go back to Question 7:



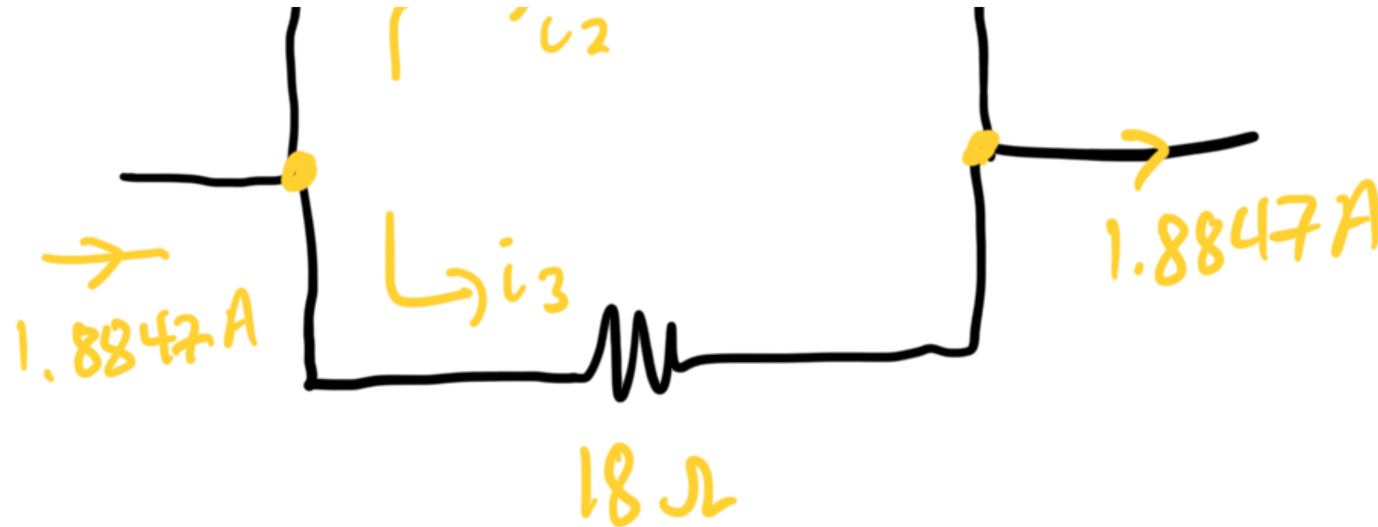


a) $R_{eq} = 18.04\ \Omega$

$i_1 = 34\ V / 18.04\ \Omega$

$= 1.8847\ A$

b)



$$\Delta V_7 = 7 i_2 = \Delta V_{18} = 18 i_3$$

$$\therefore \quad 7 i_2 = 18 i_3 \qquad \therefore i_3 = \frac{7}{18} i_2$$

$$i_2 + i_3 = 1.8847$$

$$i_2 + \frac{7}{18} i_2 = 1.8847$$

$$i_2 = 1.3570 \text{ A}$$

$$i_3 = 0.5277 \text{ A}$$