1. Two point charges of equal magnitude (size) are 0.76 m apart. At the **midpoint** of the line connecting them, their combined electric field has a magnitude of 73 N/C.

(i) Based on the information given above, which of the following statements is TRUE?

A) The charges will repel one another.

B) Both charges have positive electric charge.

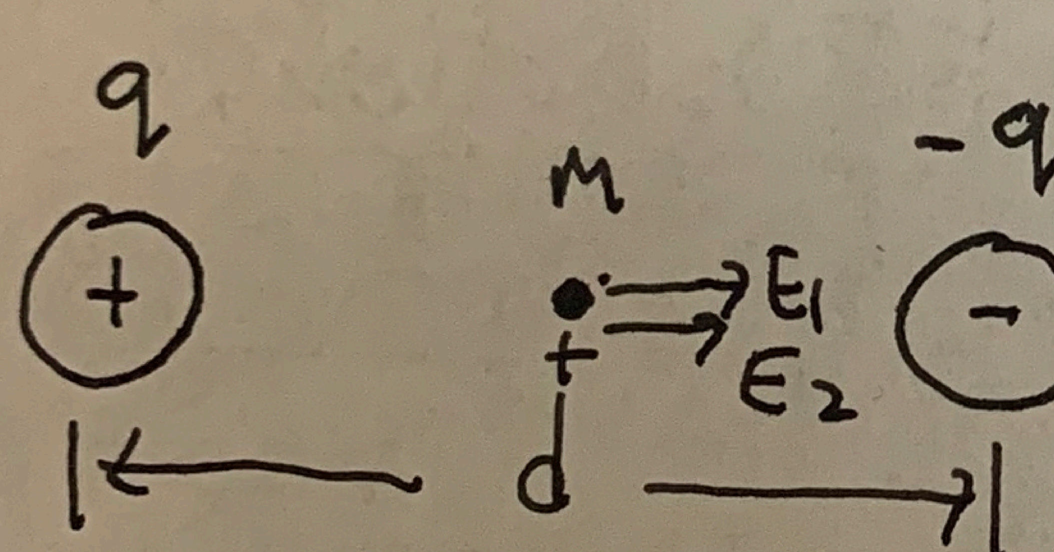
C) The charges must have opposite electric charge.

D) At least one of the charges must have zero electric charge.

E) The net electric force is zero.

If the charges are both +ve, or both -ve, then $|\vec{E}|_{midpoint} = 0$. Since it's not, they have opposite charges!!

(ii) Derive and expression for the size/magnitude of each charge, in terms of the electric field at the midpoint, the distance between the charges, and Coulomb's constant, k.



$$|\vec{E}_{\text{total at MP}}| = E_1 + E_2 = \frac{kq}{(d/2)^2} + \frac{kq}{(d/2)^2} = \frac{4kq}{d^2} + \frac{4kq}{d^2} = \frac{8kq}{d^2}$$

$$\therefore q = \frac{|\vec{E}|\cdot d^2}{8k}$$

(iii) Calculate the numerical value of the size/magnitude of each charge.

$$q = \frac{(73\,N/C)(.76m)^2}{8\left(8.99\times 10^9\,\frac{Nm^2}{C^2}\right)} = 5.7\times 10^{-10}\,C$$

2. A parallel-plate capacitor has plates separated by 0.59 x 10^{-3} m. The plates are connected to a battery which results in a uniform electric field between the plates.

(i) Which of the following statements must be TRUE?

~~A)~~ The electric field between the plates depends only on the area of the plates. No → $|\vec{E}| = \frac{Q}{\varepsilon_0 A}$ → also depends on charge.

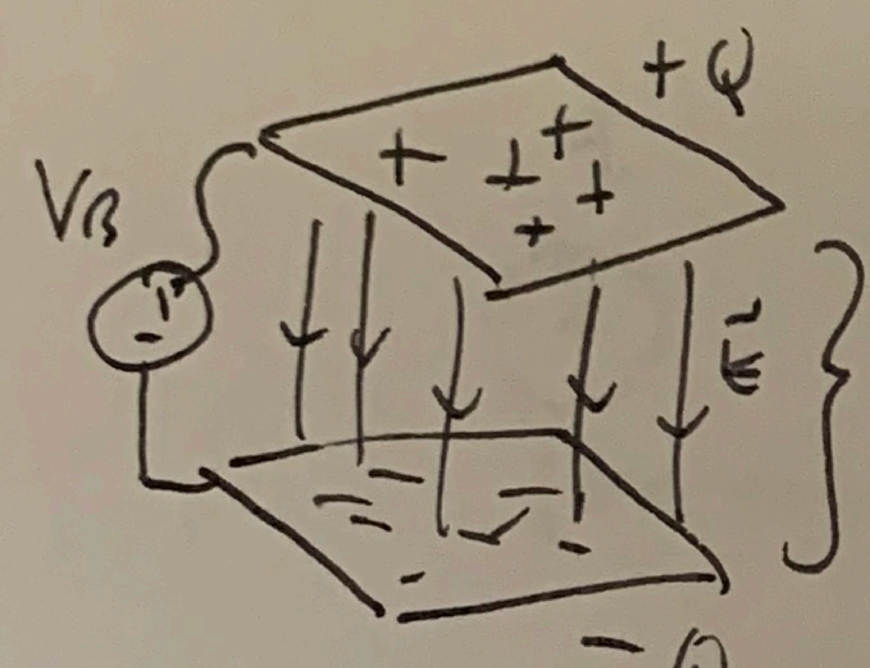
~~B)~~ Decreasing the distance between the plates will decrease the size of the electric field. No. $V_B = |\vec{E}| \cdot d$ $\therefore |\vec{E}|$ increases

~~C)~~ The electric field between the plates increases as the square of the battery voltage. No. $|\vec{E}| \propto V$, not V^2

~~D)~~ Doubling the distance between the plates will decrease the electric field by a factor of four. No, $|\vec{E}| \propto 1/d$, not $1/d^2$

E) Decreasing the distance between the plates by a factor of three will increase the electric field by a factor of three. Yes, $|\vec{E}| \propto \frac{1}{d}$

(ii) Derive an expression for the electric field between the plates, in terms of the battery voltage and the distance between the plates.



$|\vec{E}| \cdot d = V_B$

$\therefore |\vec{E}| = V_B / d$

(iii) Calculate the numerical value of the battery voltage if the electric field between the plates has a magnitude of 100,000 V/m?

$V_B = |\vec{E}| \cdot d = (100{,}000 \frac{V}{m})(.59 \times 10^{-3} m)$

$= 59$ Volts.

(iv) Assuming that the battery voltage is the same as you found in part (iii), what would the separation between the plates need to be for the electric field to reach the dielectric breakdown strength of dry air (3,000,000 V/m)?

(N.B. If you could not find the answer to part (iii), use a value of 1 Volt, and continue with this part of the question.)

$V_B = 59\ V$

$|\vec{E}| = 3 \times 10^6\ V/m$

$d = V_B / |\vec{E}| = \dfrac{59\ V}{3 \times 10^6\ V/m} = 1.9 \times 10^{-5}\ m$

3. Two points in space, A and B, have voltages of 273 V and 129 V, respectively. When an electron released from rest at point A, it arrives at a third point, C, and its kinetic energy is K_A. When the electron is released from rest at point B, however, its kinetic energy when it reaches point C is $K_B = 4K_A$.

(i) Which of the following statements must be true?

✓ A) Protons move from regions of high voltage to regions of low voltage. Yes, protons go uphill!

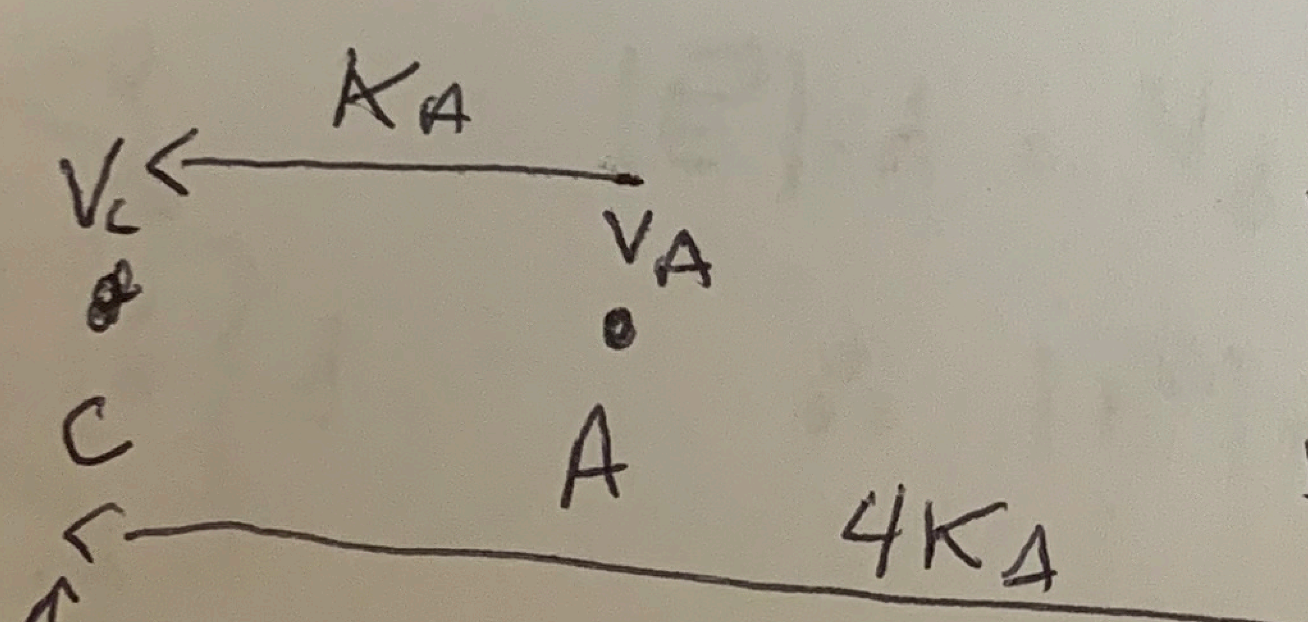
✓ B) Electrons move from regions of low voltage to regions of high voltage. Yes, electron go down hill!

✓ C) As protons move from high voltage to low voltage, their kinetic energy increases. Yes, as protons go downhill they speed up!

✓ D) As electrons move from low voltage to high voltage, their electric potential energy decreases. Yes, as electrons go uphill, they speed up, so potential energy decreases!

E) All of the above are true.

(ii) Draw a diagram of the above situation, thinking carefully about where point C must be located, relative to points A and B.



must be closer to A than to B, since $K_B = 4K_A$

(iii) Derive an expression for the voltage at point C, in terms of the voltages at points A and B.

$\Delta U \equiv q\,\Delta V$

$\Delta K \equiv -\Delta U$ $\quad\therefore\quad q\,\Delta V = -\Delta K$

Note: since $q<0$, makes sense → $q(V_C - V_A) = -K_A$ ①

$q(V_C - V_B) = -4K_A$ ②

① × 4 $\quad 4q(V_C - V_A) = -4KA$

② $\quad q(V_C - V_B) = -4KA$

$$4\cancel{q}(V_C - V_A) = \cancel{q}(V_C - V_B)$$

$$3V_C = 4V_A - V_B$$

$$V_C = \frac{4V_A - V_B}{3}$$

(iv) What is the numerical value of the voltage at point C?

$$V_C = \frac{4(273) - 129}{3} = 321 \text{ Volts.}$$

Elegant solution:

Voltage is a measure of how much energy can be extracted from the field. As the electron gets 4 times as much energy going from B to C than from A to C, we can say.

$$\frac{1}{4}(V_C - V_B) = (V_C - V_A)$$

$$V_C - V_B = 4V_C - 4V_A$$

$$3V_C = 4V_A - V_B$$

$$V_C = \frac{4V_A - V_B}{3}$$

$$= 321 \text{ Volts.}$$

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