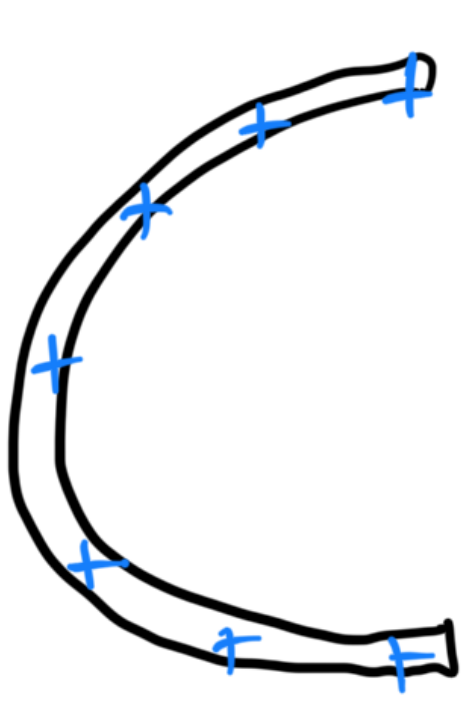
Charge Distributions

So far, we have been dealing with electric fields from individual (or a few) charges → point charges.

A more practical situation is when we have a bunch of charge that is distributed in some specific way.

Ex.

line of charge.



Semi circular arc

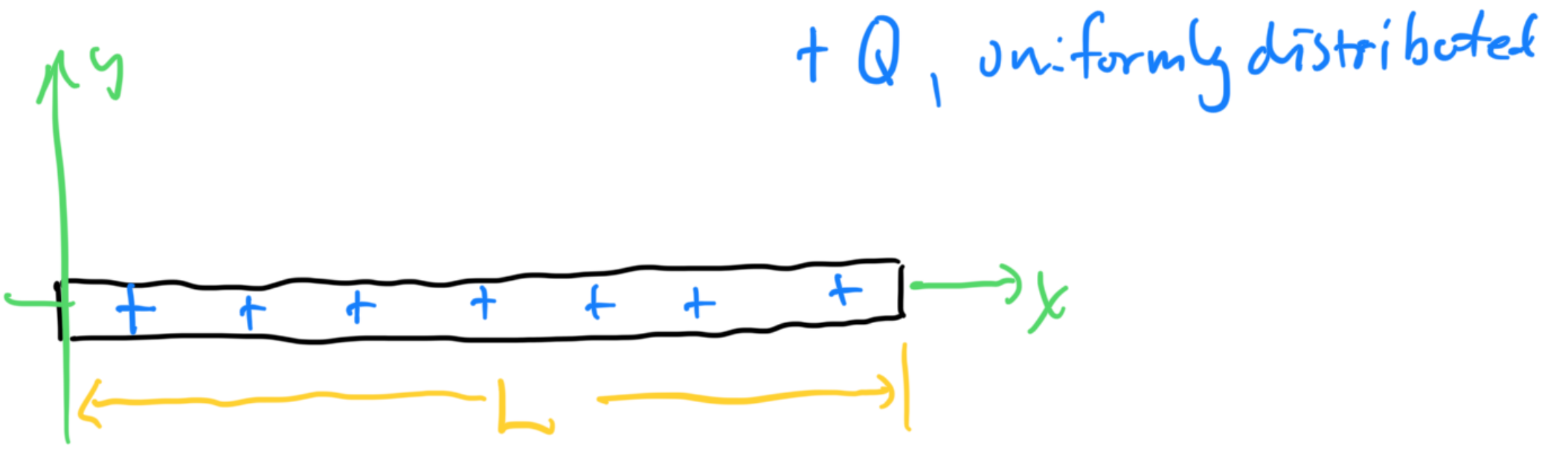
Sheet of charge

⋮

etc.

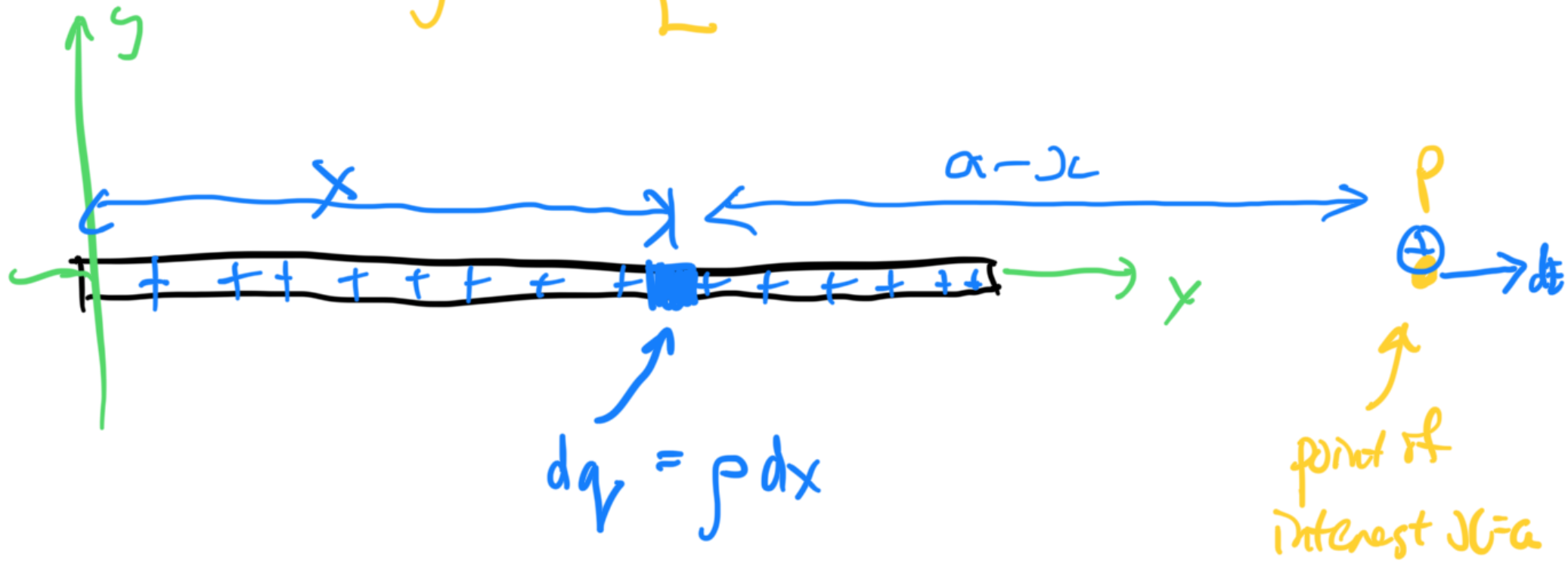
What does the electric field, voltage

field, and potential energy look like in those cases???



Question: what is the charge density?

$$\rho = \frac{Q}{L} \quad \text{(Coulombs per meter)}$$



(i) consider a small piece of charge, dq, located at x.

(ii) This will create a small electric field, $d\vec{E}$ at $x = a$:

$$d\vec{E} = \frac{k\,dq}{(\quad)^2}\,\hat{i} \qquad dq = \rho\,dx$$

$$(a-x)^2$$

(iii) Add up for all of the dq's

$$\vec{E} = \left[\int_0^L \frac{k\rho}{(a-x)^2}\,dx\right]\hat{\imath}$$

$$= k\rho\left[\frac{1}{(a-x)}\right]_0^L \hat{\imath}$$

$$\vec{E} = k\rho\left[\frac{1}{(a-L)} - \frac{1}{a}\right]\hat{\imath}$$

$$\vec{E} = k\rho\left[\frac{a-(a-L)}{(a-L)(a)}\right]\hat{\imath}$$

$$= \frac{k\rho L}{(a-L)(a)}\hat{\imath} \qquad \text{But: } \rho = \frac{Q}{L}$$

$$\boxed{\vec{E} = \frac{kQ}{(a-L)(a)}\hat{\imath}}$$

Voltage:

$$dV = \frac{k\,dq}{a-x}$$

$$V = \int_0^L \frac{k\rho\,dx}{(a-x)}$$

$$= k\rho \ln(a-x)\Big|_0^L$$

$$= k\rho\left[\ln(a-L) - \ln(a)\right]$$

$$= k\rho \ln\left(\frac{a-L}{a}\right)$$

$$\boxed{V = \frac{kQ}{L}\ln\left(1-\frac{L}{a}\right)}$$

Sommary: It's hard. The usual way is to solve these problems using numerical integration on a computer!

See, for example, efield.ipynb on

my GitHub repo !!