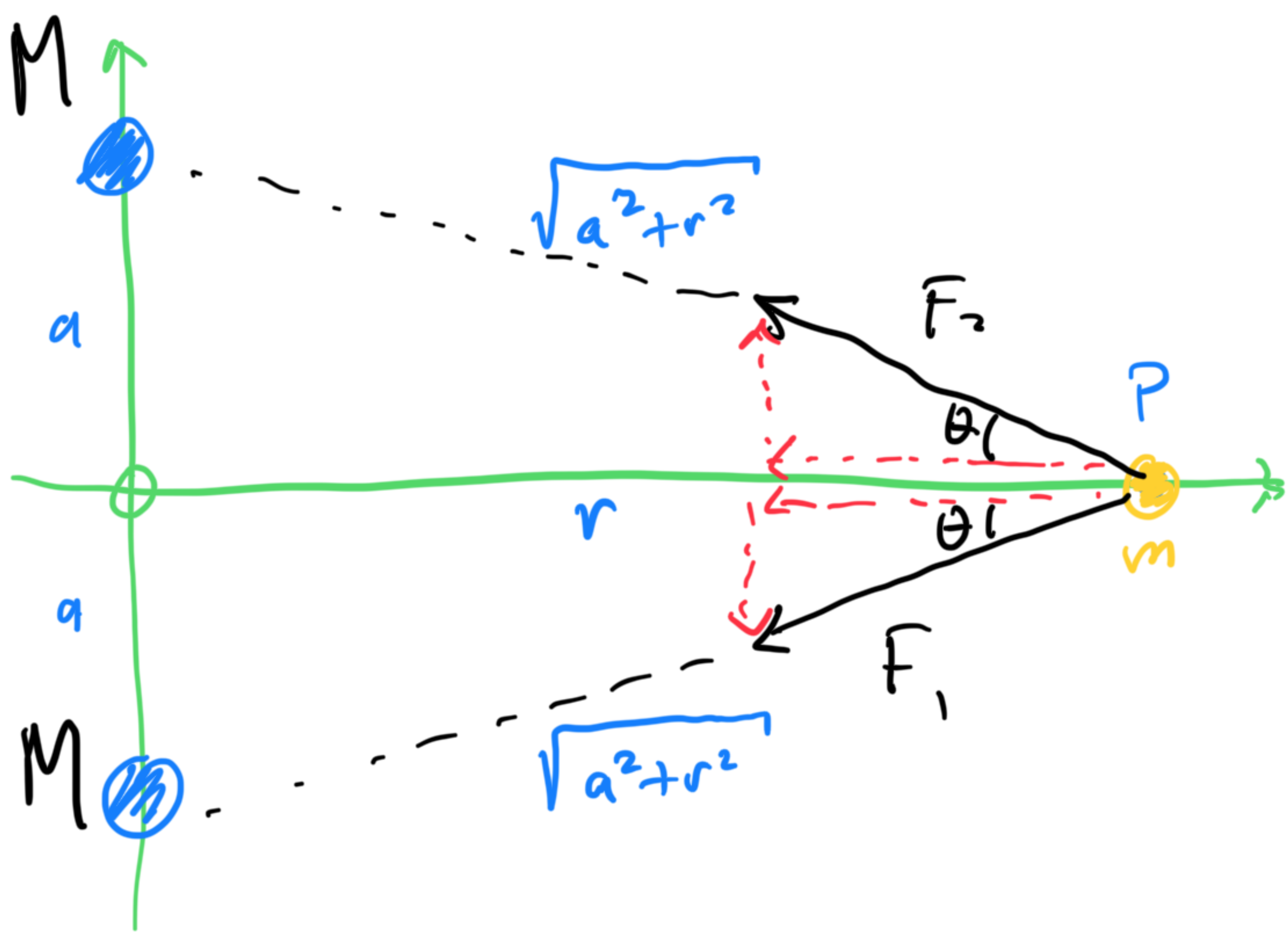
More Gravity

A1 Q7:



"gravitational field" → what does this even mean?

We will talk a lot more about this later, but what we want here is the acceleration, $\vec{a}$.

Step 1: Place a small "test mass" at the point of interest.

Step 2: Calculate the net force on this mass.

$$|\vec{F}_1| = |\vec{F}_2| = \frac{GmM}{d^2}$$

$$= \frac{GmM}{\left(\sqrt{a^2+r^2}\right)^2} = \frac{GmM}{a^2+r^2}$$

$$F_{NET}^{x} = F_1^{x} + F_2^{x}$$

$$= -\frac{GmM}{a^2+r^2}\cos\theta - \frac{GmM}{a^2+r^2}\cos\theta$$

$$= -\frac{2GmM}{a^2+r^2}\cos\theta$$

$$F_{NET}^{y} = F_1^{y} + F_2^{y}$$

$$= \frac{GmM}{a^2+r^2}\sin\theta - \frac{GmM}{a^2+r^2}\sin\theta$$

$$= 0 \, !$$

$$\cos\theta = \frac{adj}{hyp} = \frac{r}{\sqrt{a^2+r^2}}$$

$$\therefore \vec{F}_{NET} = -\frac{2GmM}{a^2+r^2} \cdot \frac{r}{\sqrt{a^2+r^2}}\,\hat{i}$$

$$= -\frac{2GmMr}{(a^2+r^2)^{3/2}}\,\hat{i}$$

Now, we have $\vec{F}_{NET} = m\vec{a}$

$$\therefore \vec{a} = -\frac{2GMr}{(a^2+r^2)^{3/2}}\,\hat{i}$$

$$g = |\vec{a}| = \frac{2GMr}{(a^2+r^2)^{3/2}}$$

<u>AIQ8</u>:

This is an <u>energy</u> problem!

<u>Potential Energy</u>: in Physics,

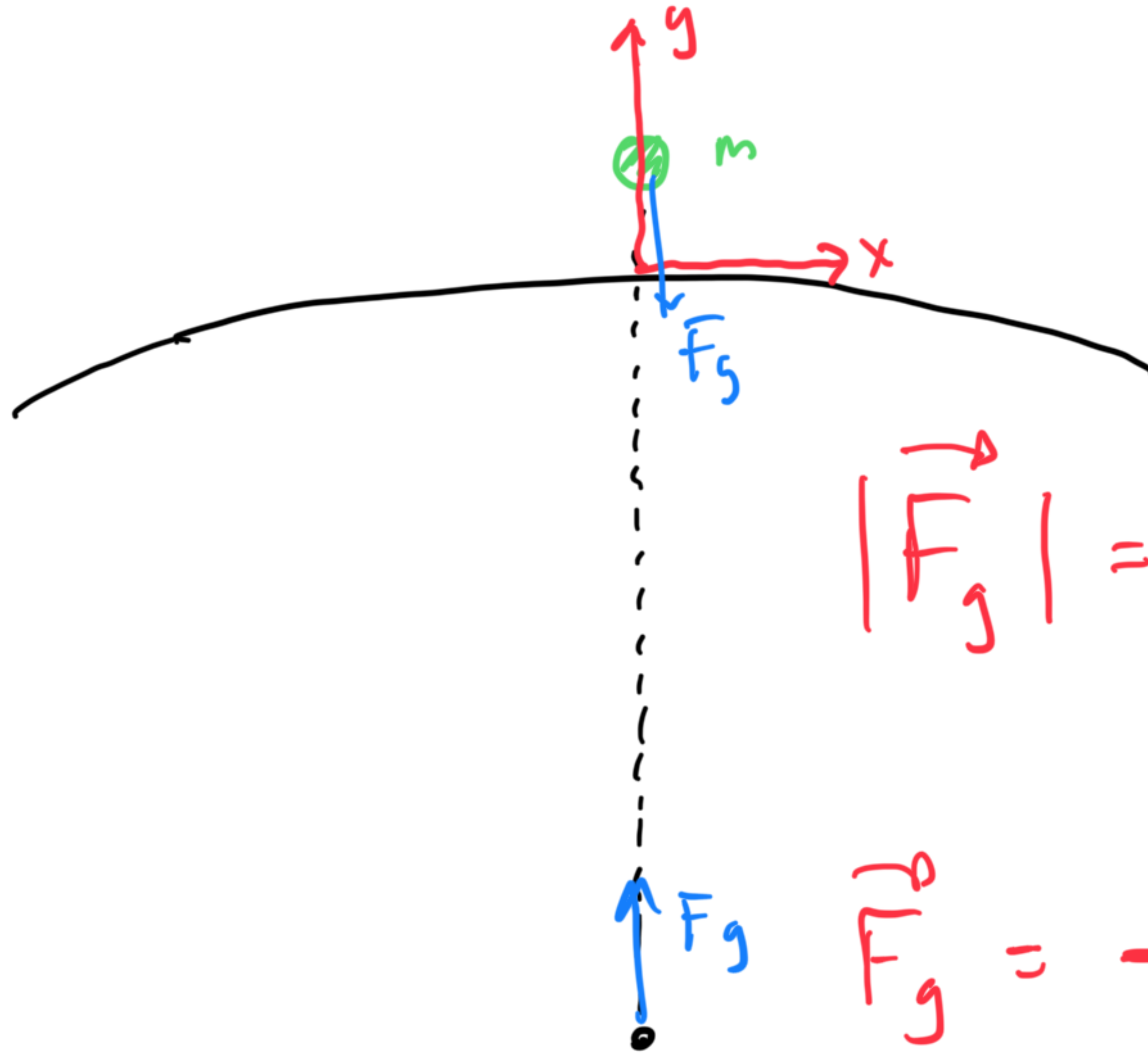
when we have a <u>conservative force</u>, like gravity, we can define a potential energy associated with that force, according to:

$$\boxed{\vec{F} = -\vec{\nabla} \cdot U}$$

where $\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\equiv$ gradient operator.

So, what is $\vec{F}$?



$$|\vec{F_g}| = \frac{G m M_E}{(r_E + y)^2}$$

$$\vec{F_g} = -\frac{G m M_E}{(r_E+y)^2}\,\hat{j}$$

$$\vec{F_g} = \cancel{\hat{i}\frac{\partial u}{\partial x}} + \hat{j}\frac{\partial u}{\partial y} + \cancel{\hat{k}\frac{\partial u}{\partial z}}$$

$$\vec{F_g} = -\frac{G m M_E}{(r_E+y)^2}\,\hat{j} = -\hat{j}\,\frac{\partial u}{\partial y}$$

$$\therefore \quad \frac{\partial u}{\partial y} = + \frac{G m M_E}{(r_E+y)^2}$$

$$\therefore \quad U = -\frac{G m M_E}{(r_E + y)} = -\frac{G m M_E}{d_{\text{center}}}$$

$$\left(\text{Check:} \quad \frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(-G m M_E (r_E+y)^{-1}\right) \right.$$

$$= G m M_E (r_E + y)^{-2}$$

$$= \frac{G m M_E}{(r_E+y)^2}$$

$$\left. \therefore \vec{F} = -\frac{G m M_E}{(r_E+y)^2}\,\hat{y} \quad \checkmark \right)$$

<u>Initial:</u>

$$T_i = \frac{1}{2} m v^2$$

$$U_i = -\frac{G m M_E}{r_E}$$

<u>Find:</u>

$$T_f = 0$$

$$U_f = -\frac{G m M_E}{(r_E + h)}$$

$$E_i = \frac{1}{2} m v^2 - \frac{G m M_E}{r_E}$$

$$E_f = -\frac{G m M_E}{(r_E + h)}$$

$$\therefore \boxed{\frac{1}{2}\cancel{m}v^2 - \frac{G\cancel{m}M_E}{r_E} = -\frac{G\cancel{m}M_E}{(r_E+h)}}$$

$$\frac{1}{2}v^2 = G M_E\left(\frac{1}{r_E} - \frac{1}{(r_E+h)}\right)$$

$$\frac{1}{r_E} - \frac{1}{(r_E+h)} = \frac{v^2}{2GM_E}$$

$$\frac{1}{(r_E+h)} = \frac{1}{r_E} - \frac{v^2}{2GM_E}$$

$$\frac{1}{r_E+h} = \frac{1}{r_E} - \frac{v^2}{2GM_E}$$

$$r_E + h = \frac{1}{\frac{1}{r_E} - \frac{v^2}{2GM_E}}$$

$$h = \frac{1}{\frac{1}{r_E} - \frac{v^2}{2GM_E}} - r_E$$

$r_E = 6.378 \times 10$ m

$v = 8400$ m/s

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2

$m_E = 5.98 \times 10^{24}$ kg

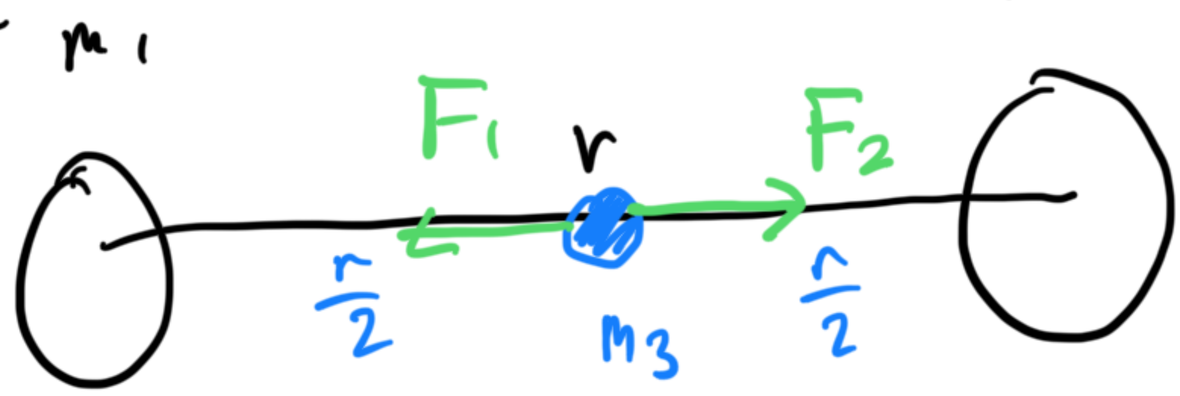
$h =$

8.255×10^6 m

<u>Main Point</u>:

$$\boxed{U = -\frac{G m m_E}{(r_E + y)}}$$

<u>A1Q9</u>:



$m_1 = 135$ kg

$m_2 = 435$ kg

$r = 0.45$ m

$$|\vec{F}_1| = \frac{G m_1 m_3}{(r/2)^2} = \frac{4 G m_1 m_3}{r^2}$$

$$\vec{F}_1 = -\frac{4 G m_1 m_3}{r^2}\hat{i}$$

$$|\vec{F_2}| = \frac{G m_2 m_3}{(r/2)^2} = \frac{4 G m_2 m_3}{r^2}$$

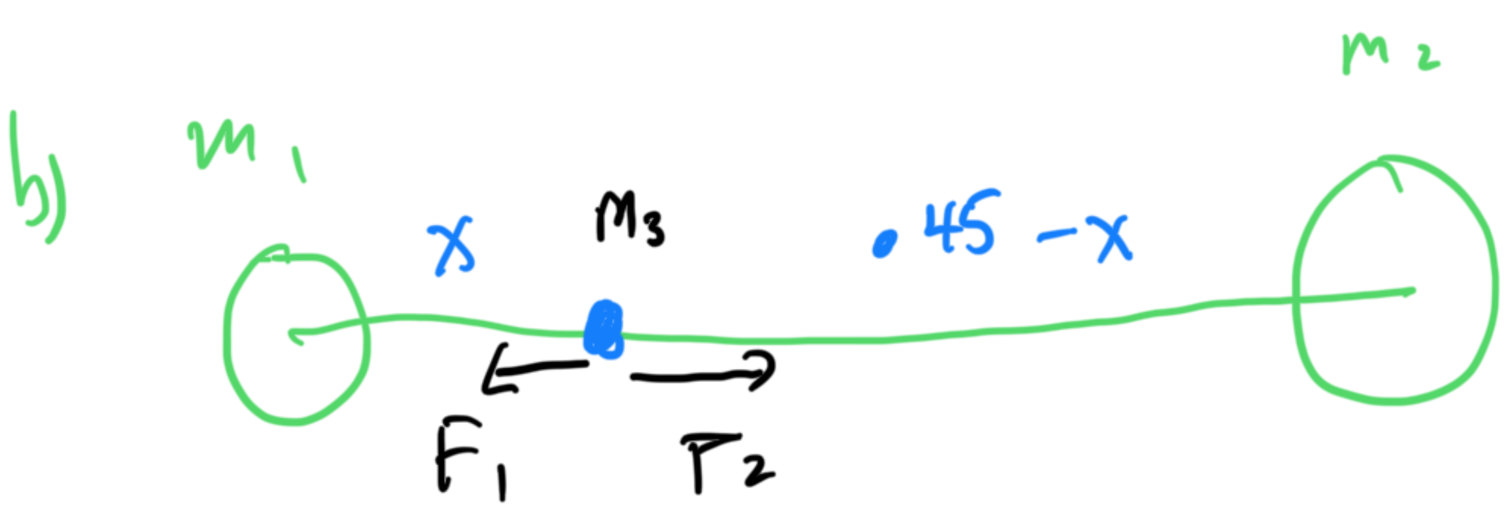
$$\vec{F_2} = + \frac{4 G m_2 m_3}{r^2}\,\hat{i}$$

$$\vec{F} = \vec{F_1} + \vec{F_2} = \frac{4 G m_3}{r^2}(m_2 - m_1)\,\hat{i}$$

$$= \frac{4(6.67\times10^{-11})(33)}{(.45)^2}(435-135)\,\hat{i}$$

$$\vec{F} = 1.30 \times 10^{-5}\,\hat{i}$$

b)



$$|\vec{F_1}| = \frac{G m_1 m_3}{x^2}$$

$$|\vec{F_2}| = \frac{G m_2 m_3}{(.45 - x)^2}$$

$$\frac{\cancel{G} m_1 \cancel{m_3}}{x^2} = \frac{\cancel{G} m_2 \cancel{m_3}}{(.45-x)^2}$$

$$\frac{m_1}{x^2} = \frac{m_2}{(.45-x)^2}$$

$$\frac{x^2}{m_1} = \frac{(.45-x)}{m_2}$$

$$\frac{x}{\sqrt{m_1}} = \frac{.45-x}{\sqrt{m_2}}$$

$$x\sqrt{m_2} = .45\sqrt{m_1} - x\sqrt{m_1}$$

$$x\left(\sqrt{m_2}+\sqrt{m_1}\right) = .45\sqrt{m_1}$$

$$x = .45\left(\frac{\sqrt{m_1}}{\sqrt{m_2}+\sqrt{m_1}}\right)$$

$$x = 0.161 \text{ m}$$

(so, 0.289 m from the larger mass)

10.

a) "escape velocity"

m $v=0$ at ∞ !!

$$T_i = \frac{1}{2} m v_{esc}^2 \qquad T_f = 0$$

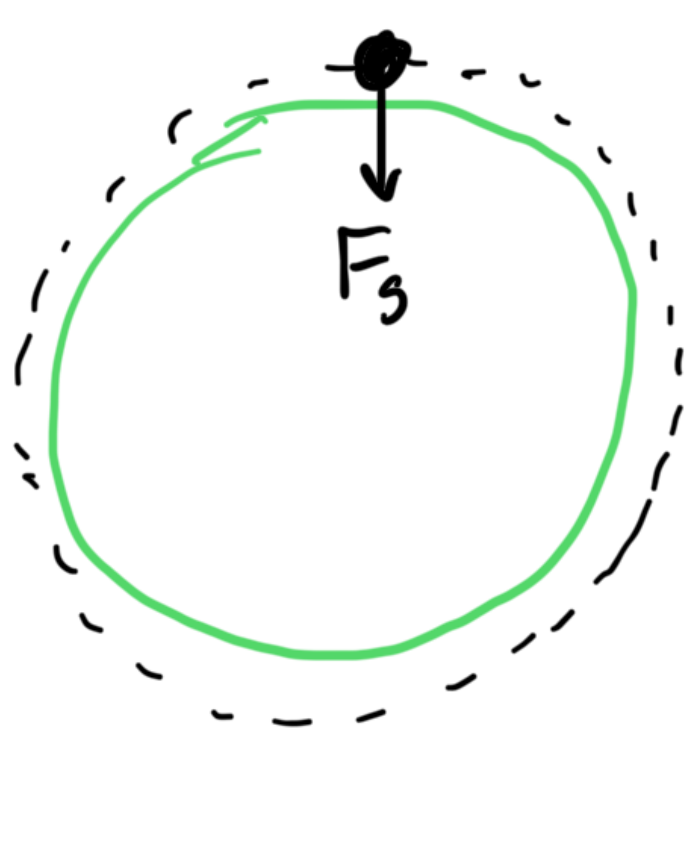
$$U_i = -\frac{G m M_E}{r_E} \qquad U_f = \frac{-G m M_E}{\infty} = 0$$

$$\frac{1}{2}\cancel{m} v_{esc}^2 - \frac{G \cancel{m} M_E}{r_E} = T_f + U_f = 0$$

$$v_{esc}^2 = \frac{2 G M_E}{r_E}$$

$$v_{esc} = \sqrt{\frac{2 G M_E}{r_E}} \qquad \left(= 11.2 \text{ km/s for earth} \right)$$

$$= \sqrt{2}\sqrt{\frac{G M_E}{r_E}}$$



$$F_g = \frac{G \cancel{m} M_E}{r_E^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r_E}}$$

$$v^2 = \frac{G M_E}{r_E}$$

$$v = \sqrt{\frac{G M_E}{r_E}}$$

$$\therefore \boxed{v_{esc} = \sqrt{2}\, v}$$