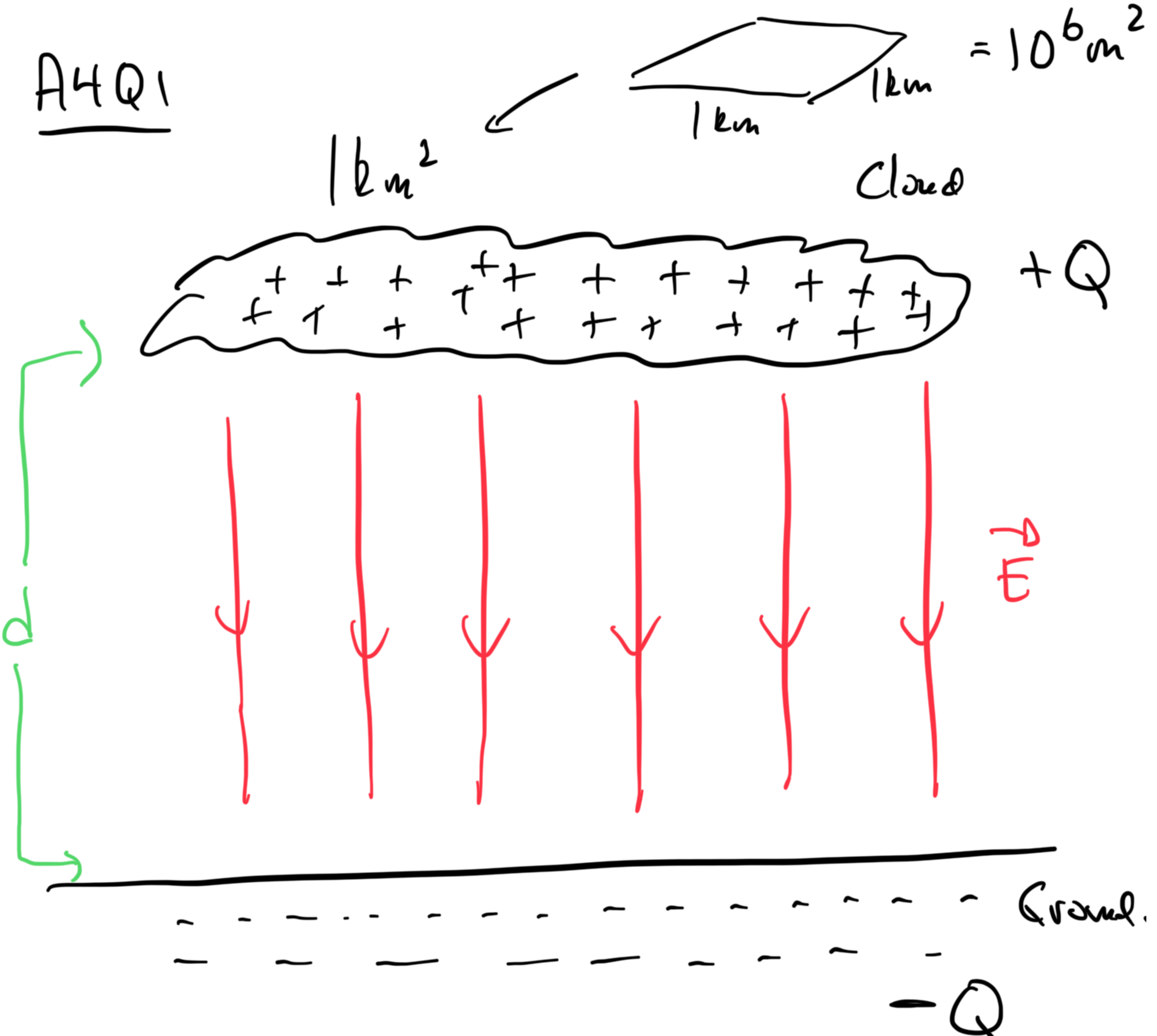
More on Capacitors

A4Q1



a) $C = \frac{\varepsilon_0 A}{d} = \frac{(8.854 \times 10^{-12})(10^6 m^2)}{850 m}$

$= 1.04 \times 10^{-8}$ F

$= 10.4$ nF

nano Farads

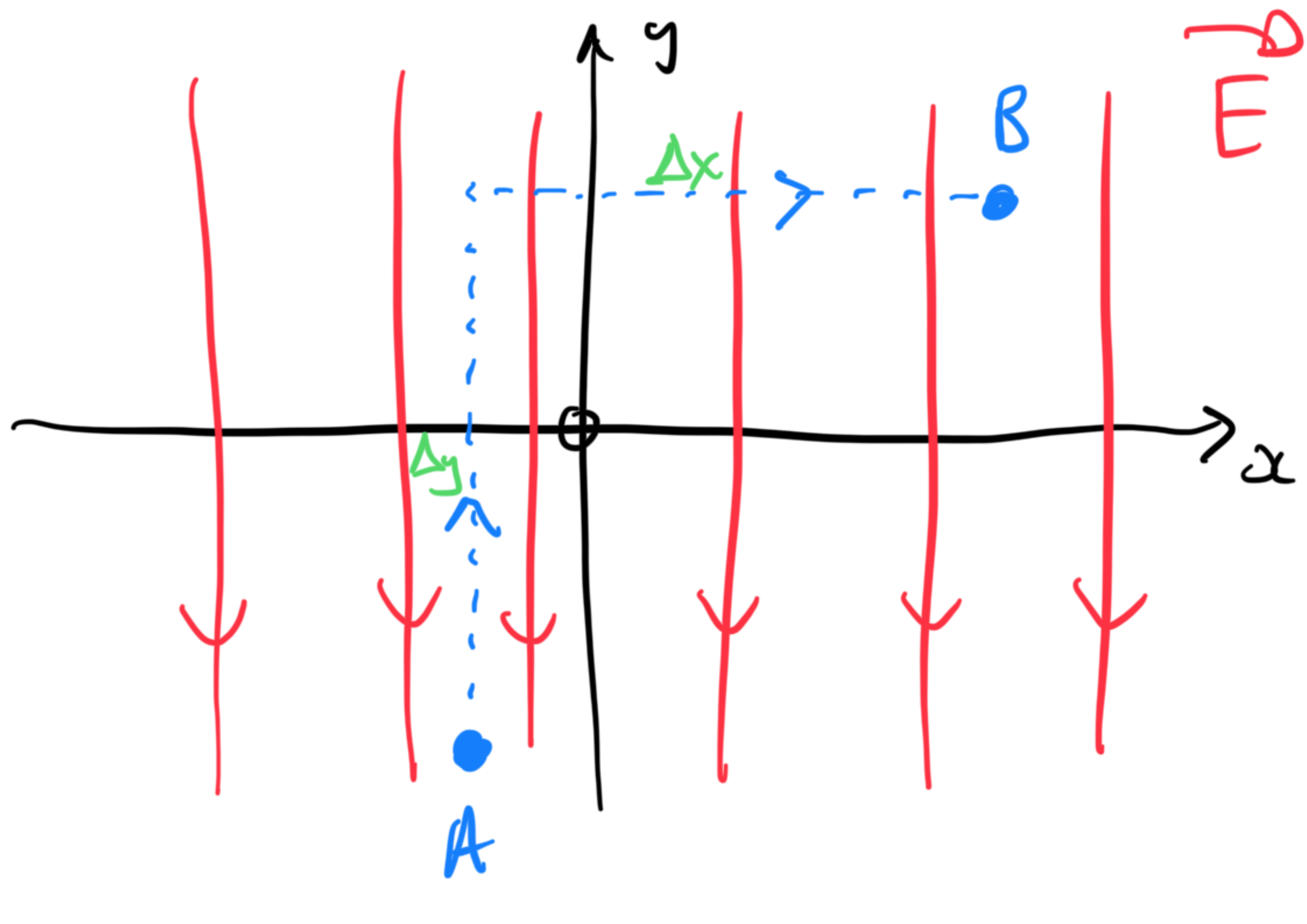
b) $|\vec{E}| = \frac{\Delta V}{d} \Rightarrow \Delta V = |\vec{E}| \cdot d$

$$Q = C \Delta V = C |\vec{E}| d$$

$$= (1.04 \times 10^{-8})(2.00 \times 10^{6})(850)$$

$$= 17.7 \text{ Coulombs}$$

3.



Step 1: Calculate to total Work Done.

$$\vec{F}_E = q\vec{E} = -qE_0\hat{j}$$

$$\vec{\Delta x}_{A\to B} = \Delta y\,\hat{j} + \Delta x\,\hat{i}$$

$$\therefore\ W_{A\to B} = \vec{F}_E \cdot \vec{\Delta x}$$

$$= -qE_0\,\hat{j} \cdot \left(\Delta y\,\hat{j} + \Delta x\,\hat{i}\right)$$

$$= -qE_0\,\Delta y$$

$$\Delta U \equiv -W$$

$$\therefore\ \Delta U_{A\to B} = qE_0\,\Delta y$$

$$\Delta V \equiv \frac{\Delta U}{q}$$

$$\therefore\ \boxed{\Delta V_{A\to B} = E_0\,\Delta y}$$

$$= \left(260\ \text{V/m}\right)\left(0.8\ \text{m}\right)$$

$$\boxed{\Delta V_{A\to B} = 208\ \text{V}} = V_B - V_A$$

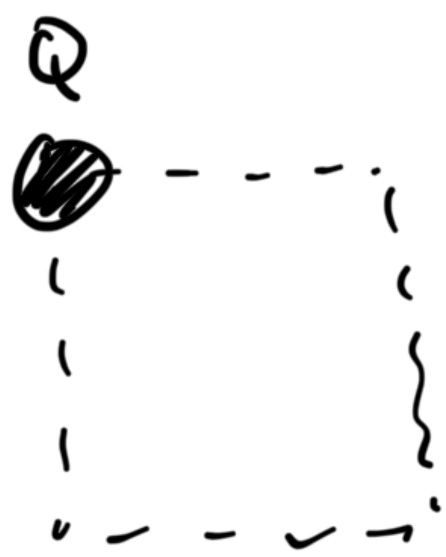
A4 Q6

Assemble four charges (Q) in a square of side s.

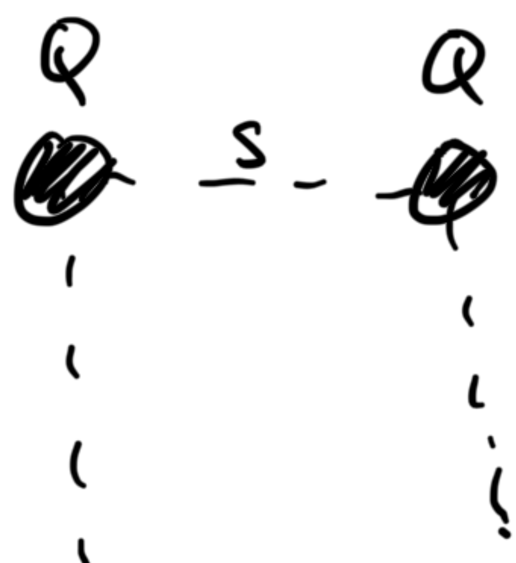
Start with an empty universe...

Step 1: Bring first charge from ∞ to one of the corners.

How much work? $\rightarrow \emptyset$



Step 2: Bring a second charge from ∞ to a different corner.



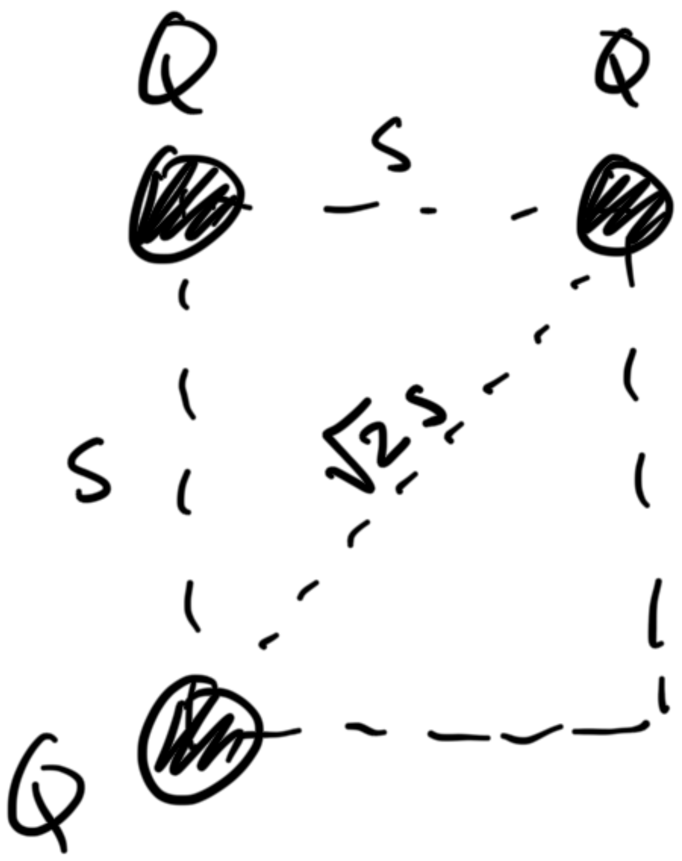
What is ΔU?

$$\Delta U = U_f - U_i$$

$$= -\frac{kQQ}{s} - \cancel{\left(-\frac{kQQ}{\infty}\right)}^{\,0}$$

$$= -\frac{kQQ}{s}$$

Step 3: Bring in a third charge ... this has to work against <u>both</u> of the existing charges.

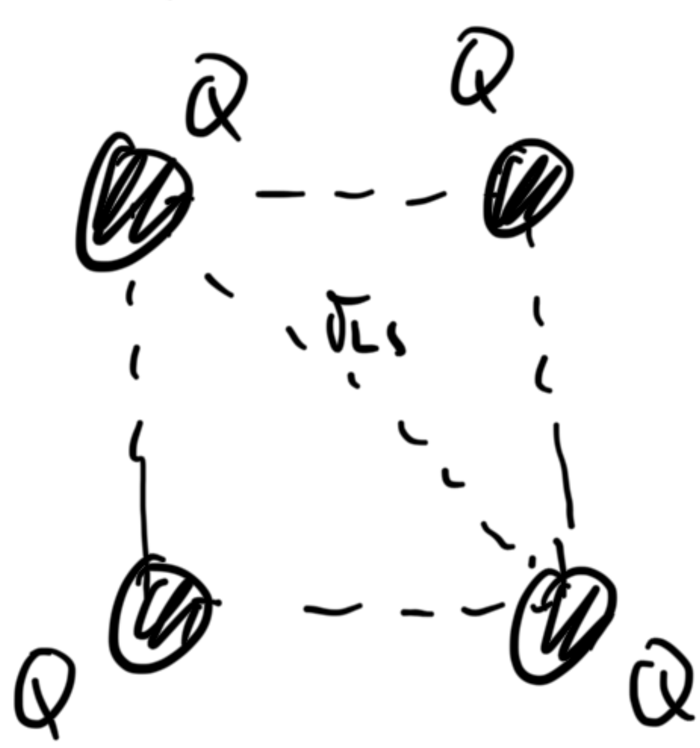


$$\Delta U = U_f - U_i$$

$$= -\frac{kQQ}{s} - \frac{kQQ}{\sqrt{2}s} - \cancelto{0}{\left(-\frac{kQQ}{\infty} - \frac{kQQ}{\infty}\right)}$$

$$= -\frac{kQQ}{s} - \frac{kQQ}{\sqrt{2}\,s}$$

Step 4: Bring in the fourth charge ...



$$\Delta U = U_f - U_i$$

$$= -\frac{kQQ}{s} - \frac{kQQ}{s} - \frac{kQQ}{\sqrt{2}s} - \cancelto{0}{\left(-\frac{kQQ}{\infty} - \frac{kQQ}{\infty} - \frac{kQQ}{\infty}\right)}$$

$$= -2kQQ - kQQ$$

$$s \qquad \sqrt{2}\, s$$

<u>Step 5</u>: Add it all up!

$$\Delta U_{TOTAL} = -\frac{kQQ}{s} - \frac{kQQ}{s} - \frac{kQQ}{\sqrt{2}\, s}$$

$$- \frac{2kQQ}{s} - \frac{kQQ}{\sqrt{2}\, s}$$

$$= -\frac{kQ^2}{s}\left[1 + 1 + \frac{1}{\sqrt{2}} + 2 + \frac{1}{\sqrt{2}}\right]$$

$$= -\frac{kQ^2}{s}\left[4 + \frac{2}{\sqrt{2}}\right]$$

$$\Delta U_{TOTAL} = -5.14121\ \frac{kQ^2}{s}$$

$$W_{TOTAL} = -\Delta U_{TOTAL} = +5.14121\ \frac{kQ^2}{s}$$

<u>Alternative method</u>:

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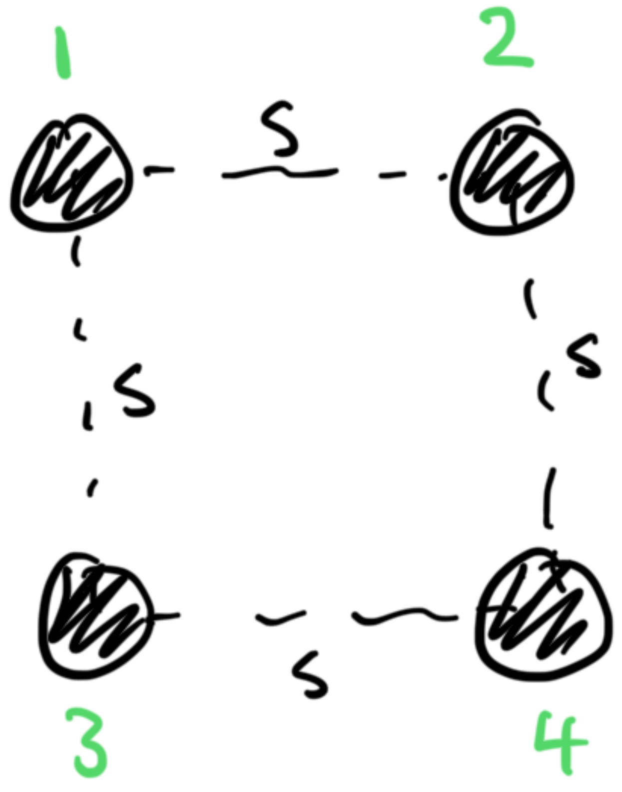
Calculate the potential energy of the final configuration between all pairs of charges.



$$\Delta U_{TOTAL} = \Delta U_{12} + \Delta U_{13} + \Delta U_{14} + \Delta U_{23} + \Delta U_{24} + \Delta U_{34}$$

$$= -\frac{kQ^2}{s} - \frac{kQ^2}{s} - \frac{kQ^2}{\sqrt{2}\,s} - \frac{kQ^2}{\sqrt{2}\,s} - \frac{kQ^2}{s} - \frac{kQ^2}{s}$$

$$= -\frac{kQ^2}{s}\left[1 + 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 + 1\right]$$

$$= -\frac{kQ^2}{s}\left[4 + \frac{2}{\sqrt{2}}\right] = -5.14121\,\frac{kQ^2}{s}$$

$$\therefore W_{TOTAL} = 5.14121\,\frac{kQ^2}{s}$$