1. Web Assign

2. Gravity ... phys 201 review

→ vectors

→ gravitation forces

→ gravitational potential energy

Vectors :

mass, temp., density } math is easy!

m T ρ

$\boxed{\text{Algebra}}$ $+, -, \times, \div, (\ )^{-}, \sqrt{\ }, \ldots$

Vector quantities → size, and direction.

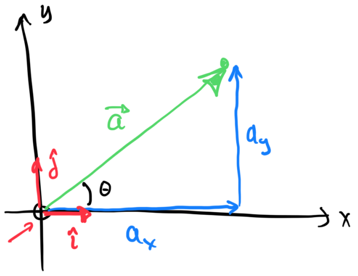
$\vec{x}$ -- 3.0 miles East

$\vec{v}$

$\vec{a}$, $\vec{F}$, $\vec{p}$, ...

Math is not the same as for scalars.

Algebra of Vectors → $+, -, \times, \div, (\ )^{-}$



① chose a coordinate system.

↳ choose origin!

② Vectors point from the origin to the poin

③ $\boxed{\text{SOH}}$CAHTOA

$$\sin\theta = \frac{O}{H} = \frac{a_y}{|\vec{a}|} \qquad \cos\theta = \frac{A}{H} = \frac{a_x}{|\vec{a}|}$$

$$a_y = |\vec{a}|\sin\theta \qquad a_x = |\vec{a}|\cos\theta$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j}$$

$$\vec{c} = (a_x \pm b_x)\hat{i} + (a_y \pm b_y)\hat{j}$$

Multiplication

$$k\vec{a} = k(a_x \hat{i} + a_y \hat{j})$$

$$= k a_x \hat{i} + k a_y \hat{j}$$

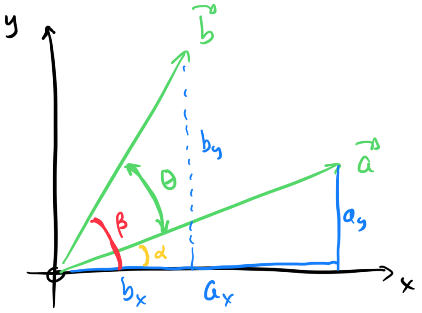
Scalar Multiplication

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = \text{Scalar}$$

Scalar Product



$a_x = a \cos\alpha \qquad a_y = a \sin\alpha$

$b_x = b \cos\beta \qquad b_y = b \sin\beta$

$$\vec{a} \cdot \vec{b} = a b \cos\alpha \cos\beta + a b \sin\alpha \sin\beta$$

$$= a b \underbrace{(\cos\alpha \cos\beta + \sin\alpha \sin\beta)}_{\cos(\beta - \alpha)}$$

$\cos(\theta)$

$$\vec{a} \cdot \vec{b} = a b \cos(\theta)$$

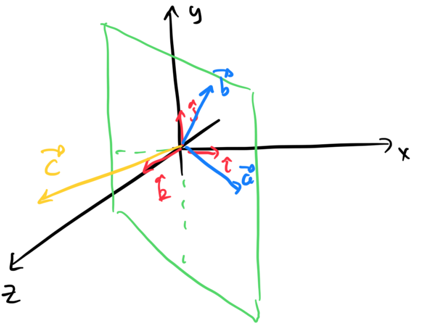
$$\boxed{\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{a b}}$$

$\theta = 90°$ (vectors are $\perp$ to one another)

$\cos(90°) = 0$!!

Work $\qquad W = \vec{F} \cdot \Delta\vec{x}$

many more !!!



Can I find a third vector, of unit length, that is $\perp$ to the plane?

① $\vec{c} \cdot \vec{a} = 0$

② $\vec{c} \cdot \vec{b} = 0$

③ $|\vec{c}| = 1$

$$\vec{c} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

a miracle occurs

Chat GPT

$$\vec{C} = \hat{i}\,(a_y b_z - a_z b_y) + \hat{j}\,(a_z b_x - a_x b_z) + \hat{k}\,(a_x b_y - b_y a_x)$$

$$\vec{C} = \vec{a} \times \vec{b}$$

Vector Product

Cross Product

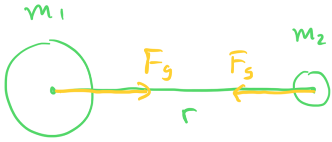
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{S} = \vec{E} \times \vec{B}$$

many more.

Gravity:

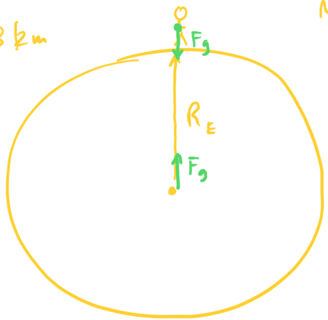
Isaac Newton → Universal Gravitation Law



$$|\vec{F_g}| = \frac{G\, m_1 m_2}{r^2}$$

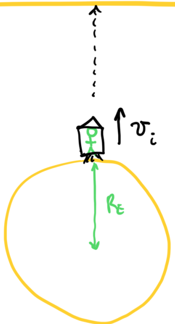
$$G = 6.67 \times 10^{-11} \ \frac{N \cdot m^2}{kg^2}$$

$R_E = 6378 \text{ km}$ $\quad M_E = 5.97 \times 10^{2}$



$$|\vec{F_g}| = \frac{G \, \cancel{m_{gym}} \, M_{Earth}}{R_E^2} = \cancel{m_{gym}} \, g$$

$$g = \frac{G \, m_E}{R_E^2}$$



Work-Energy theorem.

$$\sum \text{Work Done by All Forces} = \Delta \text{ Kinetic Energy}$$

$\checkmark \ \frac{1}{2}mv^2$

$$= \cancel{\tfrac{1}{2} m v_f^2}^{\,0} - \frac{1}{2} m v_i^2$$

$$W_{gravity} = -\frac{1}{2} m v_i^2$$

$\cancel{W_F = \vec{F} \cdot \Delta\vec{x}}$ Constant Forces

$$W = \int \vec{F} \cdot d\vec{x}$$

$$W = -\int_{R_E}^{\infty} \frac{G m M_E}{r^2} \, dr$$

$\frac{1}{x} \rightarrow -\frac{1}{x^2}$ $\quad G m M_E \Big|^{\infty}$

$$= \cancel{\frac{GmM_E}{\infty}} - \frac{GmM_E}{R_E}$$

$$W_G = -\frac{GmM_E}{R_E}$$

Gravitational Potential Energy

$$\boxed{W = -\Delta U}$$

$$\boxed{\Delta U_G = \frac{GmM_E}{R_E}}$$

$$\cancel{-}\frac{G\cancel{m}M_E}{R_E} = \cancel{-}\frac{1}{2}\cancel{m}v_i^2$$

$$v_i = \sqrt{\frac{2GM_E}{R_E}} = 11\ \text{km/s}$$