More Electricity

A2 Q1

Situation 1:

(i) Since B is repelled by A, either both charges are + or both are − ; let's assume both +

(ii) So, there is another force (pairs!) to the left on A.

$$\underset{\checkmark}{|\vec{F}_{E1}|} = \underset{\checkmark}{k_e} \frac{\overset{?}{|q_A|}\,\overset{?}{|q_B|}}{\underset{\checkmark}{r_1^2}} \Rightarrow k_E |q_A||q_B| = |\vec{F}_{E1}|\, r_1^2$$

Situation 2:

17.9 mm

$$|\vec{F}_{E2}| = \frac{k_e\, |q_A||q_B|}{r_2^{\,2}}$$

$$\Rightarrow \quad k_e |q_A||q_B| = |\vec{F}_{E2}|\, r_2^{\,2}$$

$$\therefore \quad |\vec{F}_{E1}|\, r_1^{\,2} = |\vec{F}_{E2}|\, r_2^{\,2}$$

$$|\vec{F}_{E2}| = |\vec{F}_{E1}| \cdot \frac{r_1^{\,2}}{r_2^{\,2}}$$

$$= (2.45\,\mu N)\left(\frac{13.7\text{ mm}}{17.9\text{ mm}}\right)^2$$

$\vec{F}_{E2} = 1.39\ \mu N$ to the left.

A2Q2:

Part c

$$|\vec{F}_E| = \frac{k_e\, |q_p||q_p|}{\ldots}$$

r^2

$\therefore$ can calculate $|\vec{F}_E|$ numerically

$$|\vec{F}_G| = \frac{G\, m_p\, m_p}{r^2}$$

$\therefore$ can calculate $|\vec{F}_G|$ numerically.

New universe: $|\vec{F}_E| = |\vec{F}_G|$

$$\therefore \quad \frac{k_e |q_p|^2}{\cancel{r^2}} = \frac{G\, m_p^2}{\cancel{r^2}}$$

$$\frac{q_p^2}{m_p^2} = \frac{G}{k_e}$$

$$\frac{q_p}{m_p} = \sqrt{\frac{G}{k_e}} = 8.61 \times 10^{-11} \frac{C}{kg}$$

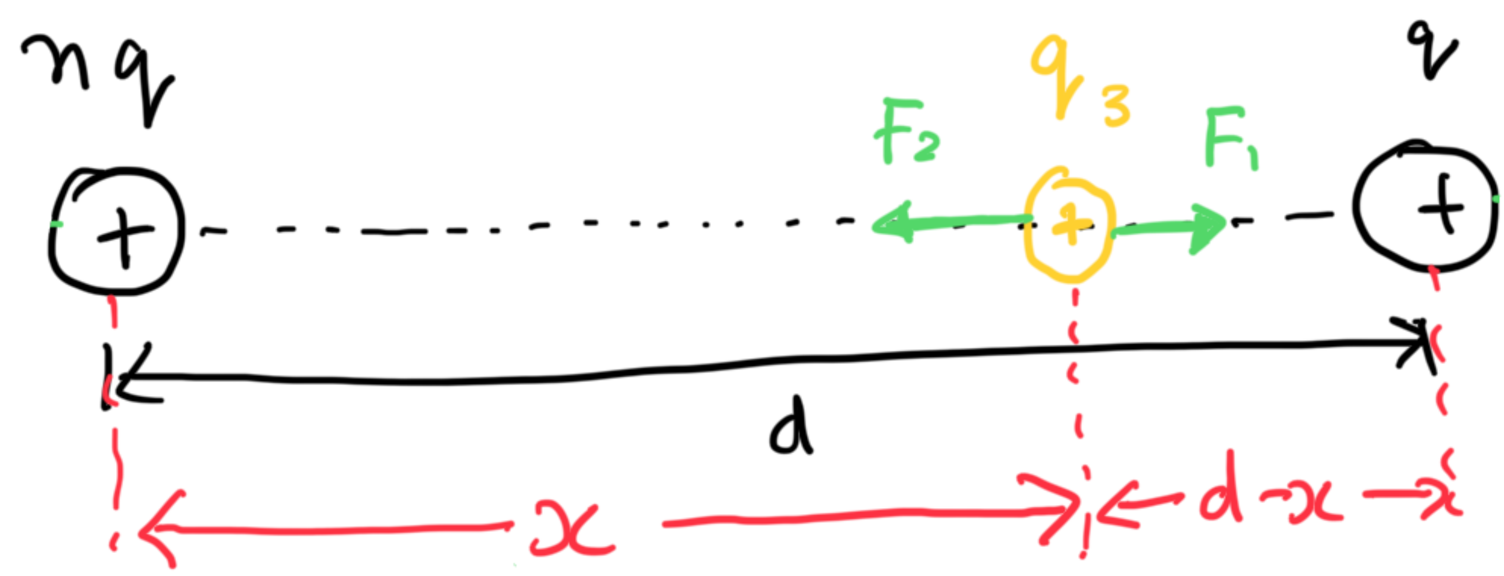
<u>Note</u>: this part of the question is stupid ☹

Why change q_p/m_p, but not G

or k_e ???

I don't like "what if" the universe were different questions.

A2 Q4



$$|\vec{F}_1| = \frac{k_e |nq||q_3|}{x^2}$$

$$|\vec{F}_2| = \frac{k_e |q||q_3|}{(d-x)^2}$$

Suppose $|\vec{F}_1| = |\vec{F}_2|$:

$$\frac{\cancel{k_e}(n\cancel{q})(\cancel{q_3})}{x^2} = \frac{\cancel{k_e}(\cancel{q})(\cancel{q_3})}{(d-x)^2}$$

$$\frac{n}{x^2} = \frac{\quad}{(d-x)^2}$$

Solve for x:

$$\frac{x^2}{n} = (d-x)^2$$

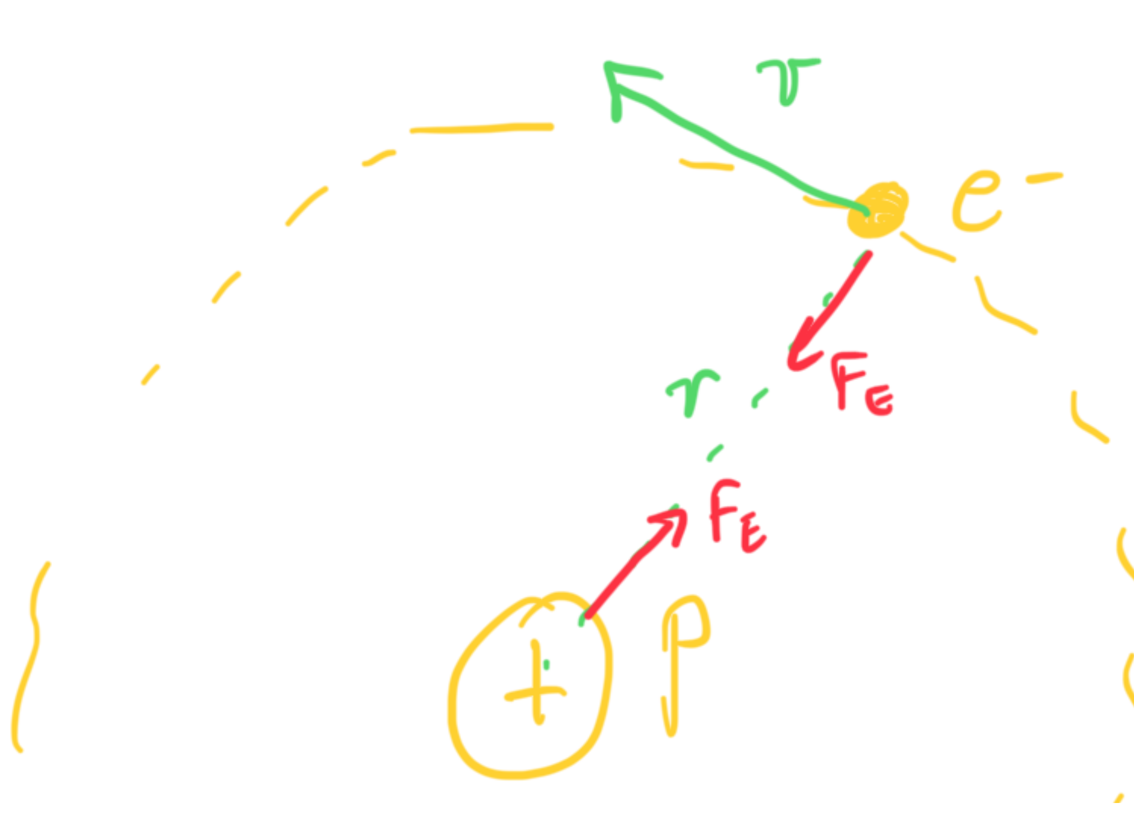
$$\frac{x}{\sqrt{n}} = d - x$$

$$x\left(\frac{1}{\sqrt{n}} + 1\right) = d$$

$$x = \left(\frac{1}{\frac{1}{\sqrt{n}} + 1}\right) d$$

$n = 12$: $$x = \left[\frac{1}{\frac{1}{\sqrt{12}} + 1}\right] d = 0.776\, d$$

A2Q5:



$$|\vec{F}_E| = \frac{k_e |q_p||q_e|}{r^2} = m_e a_c = \frac{m_e v^2}{r}$$

$$\frac{k_e |q_p||q_e|}{r^{\cancel{2}}} = \frac{m_e v^2}{\cancel{r}}$$

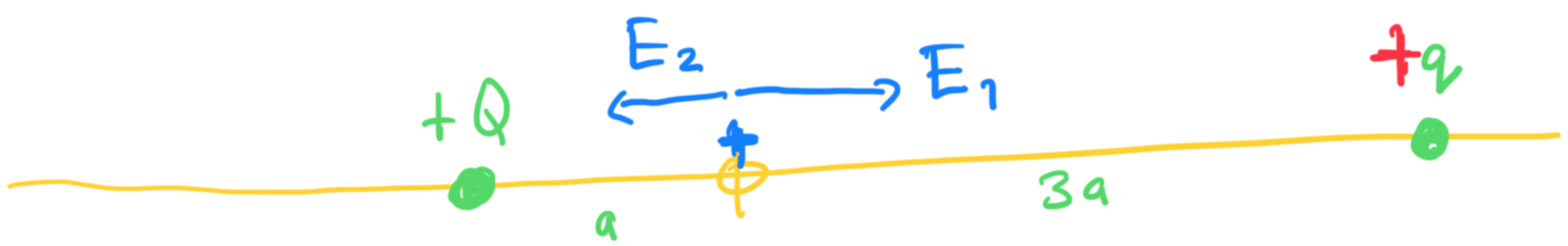
$$v^2 = \frac{k_e |q_p||q_e|}{r}$$

$$v = \sqrt{\frac{k_e |q_p||q_e|}{r}} = 2.18 \times 10^6 \text{ m/s}$$

$(0.007c \text{ !!})$

A2Q6

Possibility 1:



(i) place small test charge at the pt. of interest.

(ii) consider forces on this charge to get directions

electric fields

of associated electric fields.

$\rightarrow$ one for each charge!!

(iii) calculate size of each electric field part.

$$|\vec{E}_1| = \frac{k_e Q}{a^2}$$

$$|\vec{E}_2| = \frac{k_e q}{(3a)^2}$$

(iv) Add up these fields like vectors.

$$\vec{E}_{TOTAL} = \vec{E}_1 - \vec{E}_2$$

$$\frac{2\cancel{k}Q}{\cancel{a^2}} = \frac{\cancel{k}Q}{\cancel{a^2}} - \frac{\cancel{k_e}q}{9\cancel{a^2}}$$

$$2Q = Q - \frac{q}{9}$$

$$Q = -\frac{q}{9}$$

$$q = -9Q$$ .... contradiction!! :(

Solution:

$$-\frac{2\cancel{k}Q}{\cancel{a^2}} = \frac{\cancel{k}Q}{\cancel{a^2}} - \frac{\cancel{k}q}{9\cancel{a^2}}$$

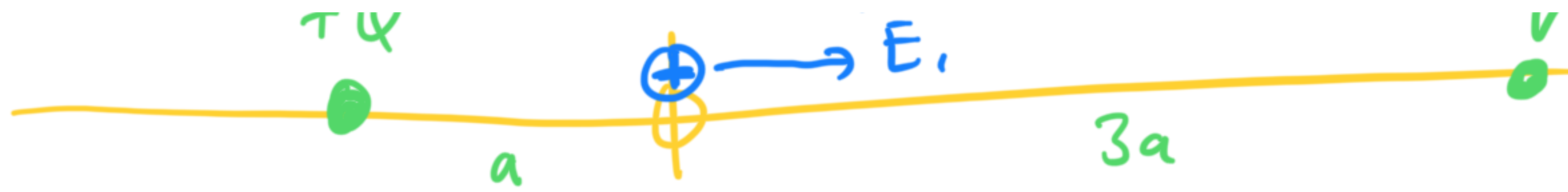
$$-2Q = Q - \frac{q}{9}$$

$$-3Q = -\frac{q}{9}$$

$$\boxed{q = 27Q}$$

Possibility 2: what if q is negative.

$+Q$ $\longrightarrow E_2$ $-q$



$$|\vec{E_1}| = \frac{kQ}{a^2} \qquad |\vec{E_2}| = \frac{kq}{(3a)^2}$$

$$|\vec{E}_{TOTAL}| = |\vec{E_1}| + |\vec{E_2}|$$

$$\frac{2k_eQ}{a^2} = \frac{k_eQ}{a^2} + \frac{k_eq}{(3a)^2}$$

$$\frac{\cancel{k_e}Q}{\cancel{a^2}} = \frac{\cancel{k_e}q}{9\cancel{a^2}}$$

$$\boxed{q = 9Q}$$

A2Q8 :

$$\vec{E}_{TOTAL} = \vec{E}_2 - \vec{E}_1$$

$$E_{TOTAL} = \frac{kq}{(x-a)^2} - \frac{kq}{(x+a)^2}$$

$$E_{TOTAL} = kq\left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right)$$

How does this behave at large x?

binomial approximation:

$$\frac{1}{(x-a)^2} = \frac{1}{x^2\left(1-\frac{a}{x}\right)^2}$$

$$= \frac{1}{x^2}\left(1-\frac{a}{x}\right)^{-2}$$

For $\frac{a}{x} \ll 1$: $\left(1-\frac{a}{x}\right)^{-2} \approx 1 + \frac{2a}{x}$

$$\frac{1}{(x-a)^2} \approx \frac{1}{x^2}\left(1+\frac{2a}{x}\right)$$

$$\frac{1}{(x+a)^2} \approx \frac{1}{x^2}\left(1-\frac{2a}{x}\right)$$

$$\therefore \frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} = \frac{1}{x^2}\left(1 + \frac{2a}{x}\right) - \frac{1}{x^2}\left(1 - \frac{2a}{x}\right)$$

$$= \frac{1}{x^2}\left(1 + \frac{2a}{x} - 1 + \frac{2a}{x}\right)$$

$$= \frac{1}{x^2}\left(\frac{4a}{x}\right)$$

$$= \frac{4a}{x^3}$$

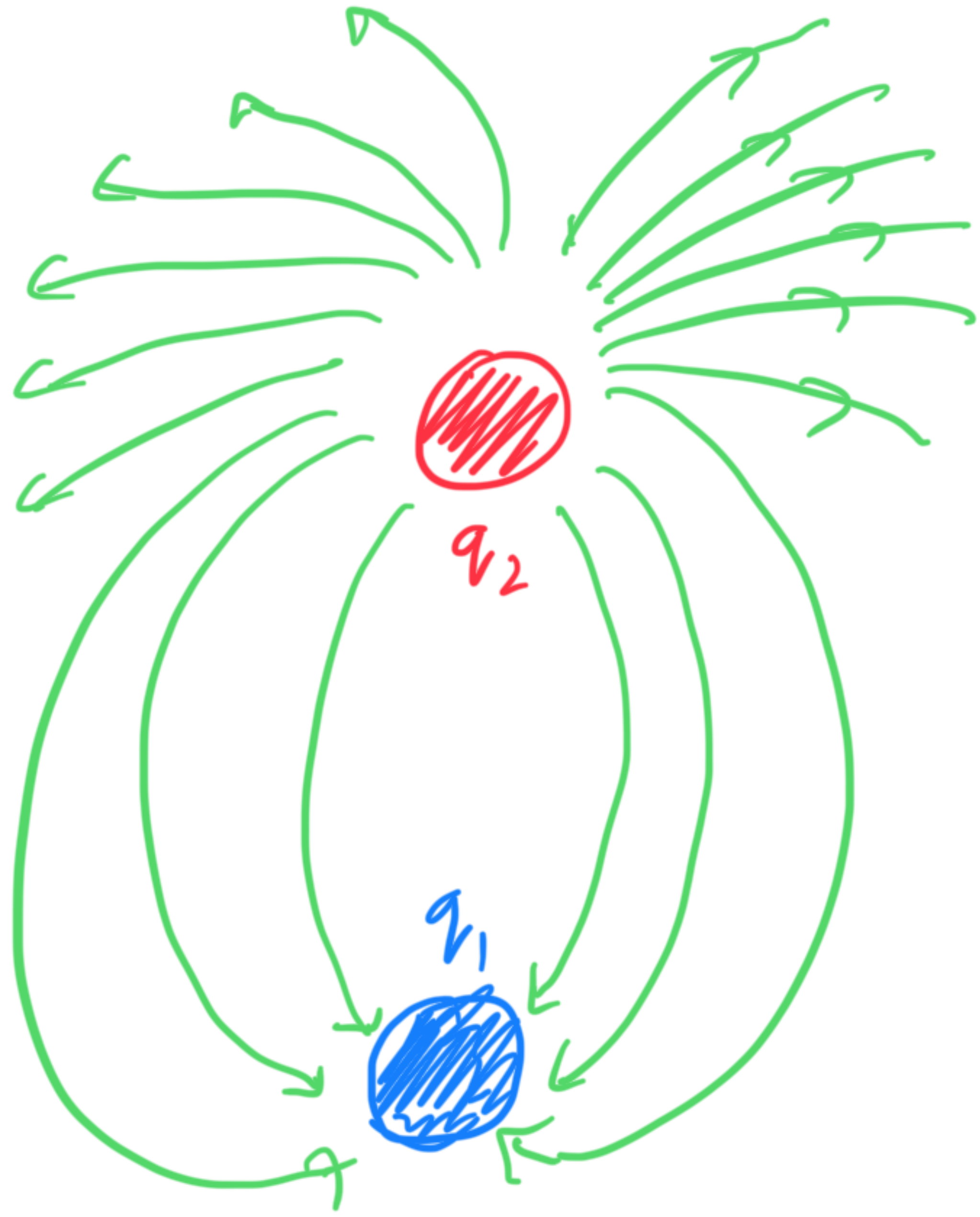
$$E_{TOTAL} = kq\left(\frac{4a}{x^3}\right) = \frac{4kqa}{x^3} \checkmark$$

<u>Note</u>: This tends to zero as $x \to \infty$

This makes sense ... the "dipole" looks like zero charge at very large distances.

<u>Example</u>: a hydrogen atom at large distances appears neutral. So do all other non-ionized atoms.

A2Q9 :



① lines go into q_1 $\therefore$ $q_1 < 0$ (negative)

② lines come out of q_2 $\therefore$ $q_2 > 0$ (positive)

③ six lines into q_1

eighteen lines out of q_2

$$\therefore \quad |q_2| = 3|q_1|$$

$$\therefore \quad \frac{|q_1|}{\ } = \frac{1}{3}$$

$|q_2|$

$$\boxed{\frac{q_1}{q_2} = -\frac{1}{3}}$$