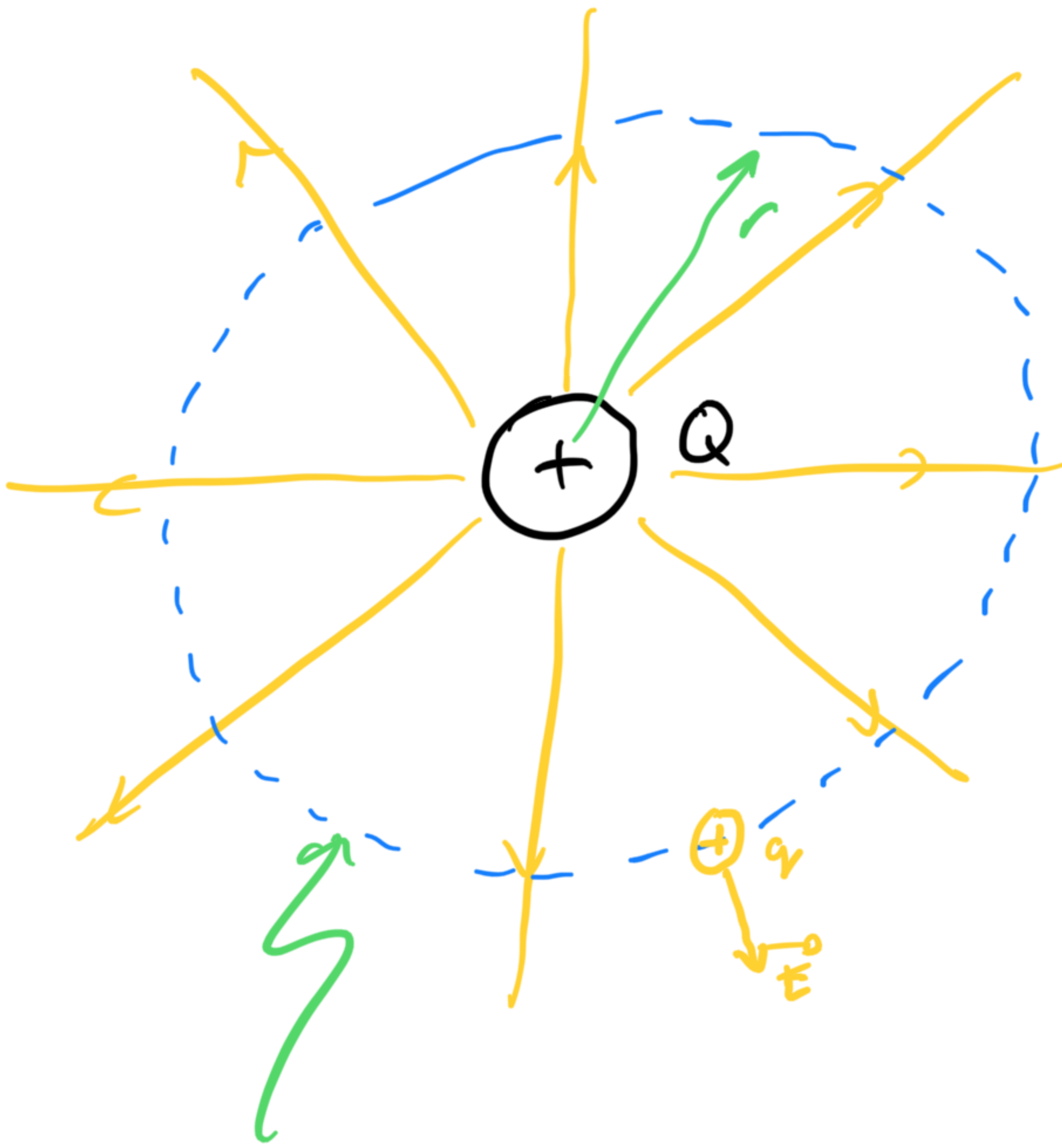
Revisiting Coulomb's Law and Electric Field



Sphere of radius r → $|\vec{E}|$ is the same everywhere on this sphere.

$$|\vec{E}| = \frac{kQ}{r^2} \qquad \underbrace{|\vec{F}_E| = \frac{kQq}{r^2}}_{\text{Coulomb's law.}} = q|\vec{E}|$$

Carl Frederich Gauss

→ how does our universe work ?

→ Mathematize genius!!

→ Because of the fact that $|\vec{E}|$ is constant on the sphere, somehow this must be related to geometry!!

$$A_{sphere} = 4\pi r^2 !!$$

<u>Idea</u> :

① $|\vec{E}| \propto Q$ (makes sense)

② $|\vec{E}| \propto \frac{1}{A_{sphere}}$

$$\therefore \ |\vec{E}| \propto \frac{Q}{4\pi r^2}$$

Define constant of proportionality → $\frac{1}{\varepsilon_0}$

$$|\vec{E}| = \frac{Q}{4\pi\varepsilon_0 r^2}$$

But also, $|\vec{E}| = \frac{k_e Q}{r^2}$

$$\therefore \quad k_e = \frac{1}{4\pi\varepsilon_0}$$

$$\varepsilon_0 = \frac{1}{4\pi k_e} \approx 8.854 \times 10^{-12} \ C^2/N\cdot m^2$$

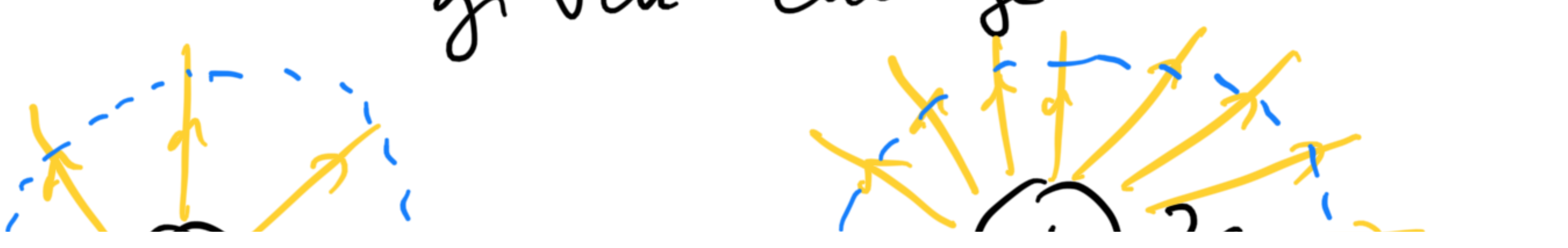
$\varepsilon_0 \equiv$ permittivity of free space

$\Rightarrow$ the fundamental strength of electricity in our universe.

$\Rightarrow$ a measure of how our universe "permits" electric fields to be generated by charges.

Gauss's Law

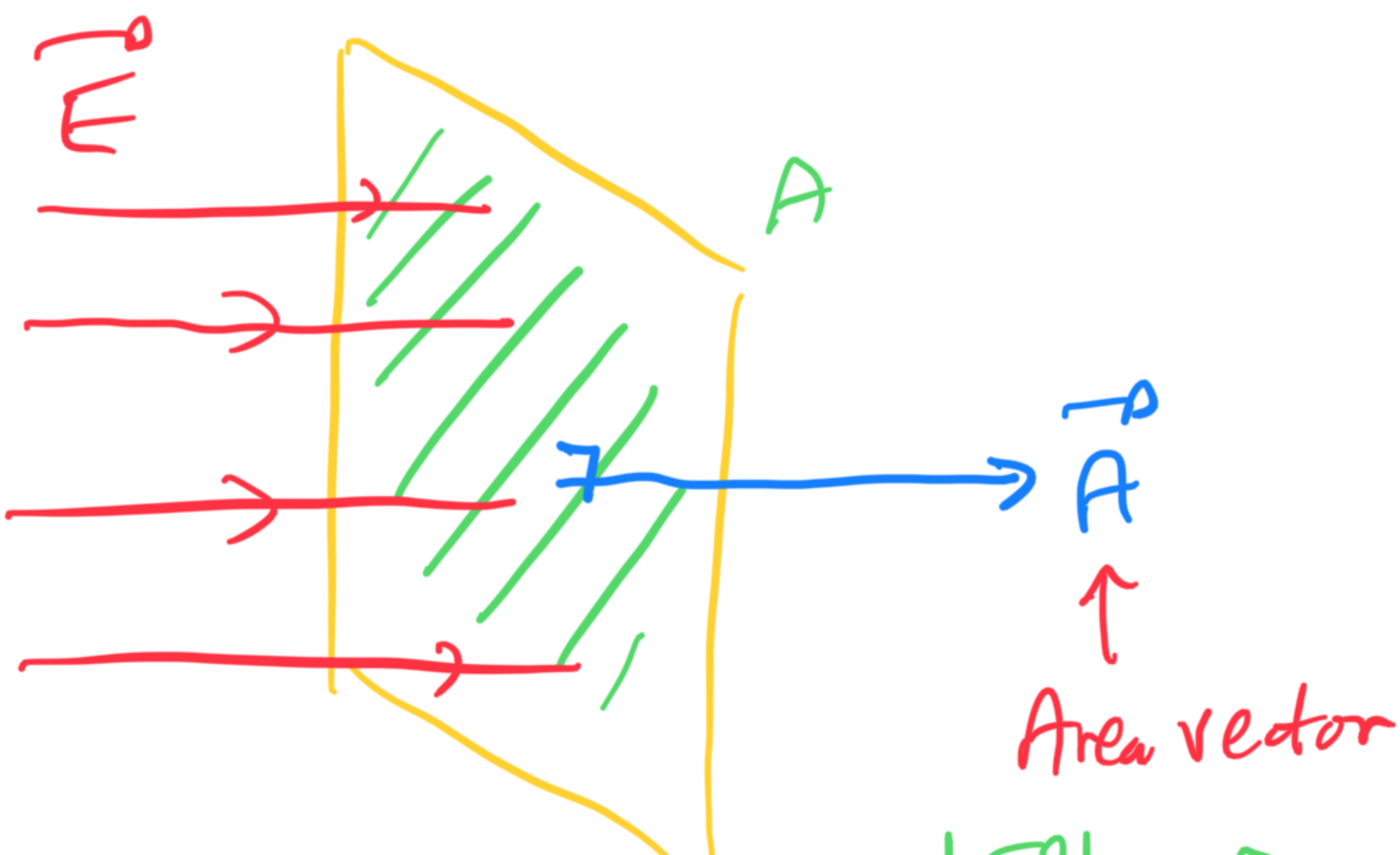
Idea 1 : How many electric field lines are generated by a given charge?



Think about surrounding the sphere with a sphere, and the counting the number of field lines that "pierce" the sphere.

Electric Flux

$$\Phi_E \equiv \vec{E} \cdot \vec{A}$$



$|\vec{A}| = A$

direction = $\perp$ to surface.

$$\Phi_E = |\vec{E}||\vec{A}| \cos\theta$$

$|\vec{E}||\vec{A}|$

In general,

$$\underbrace{\Phi_E = \oint\!\!\!\!\oint_{\text{surface}} \vec{E} \cdot d\vec{A}}_{\text{integral over the complete surface}}$$

Idea 2:

The total electric flux through a closed surface surrounding a charge is:

$$\Phi_E^{\text{TOTAL}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$\therefore \quad \boxed{\oint\!\!\!\!\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}}$$

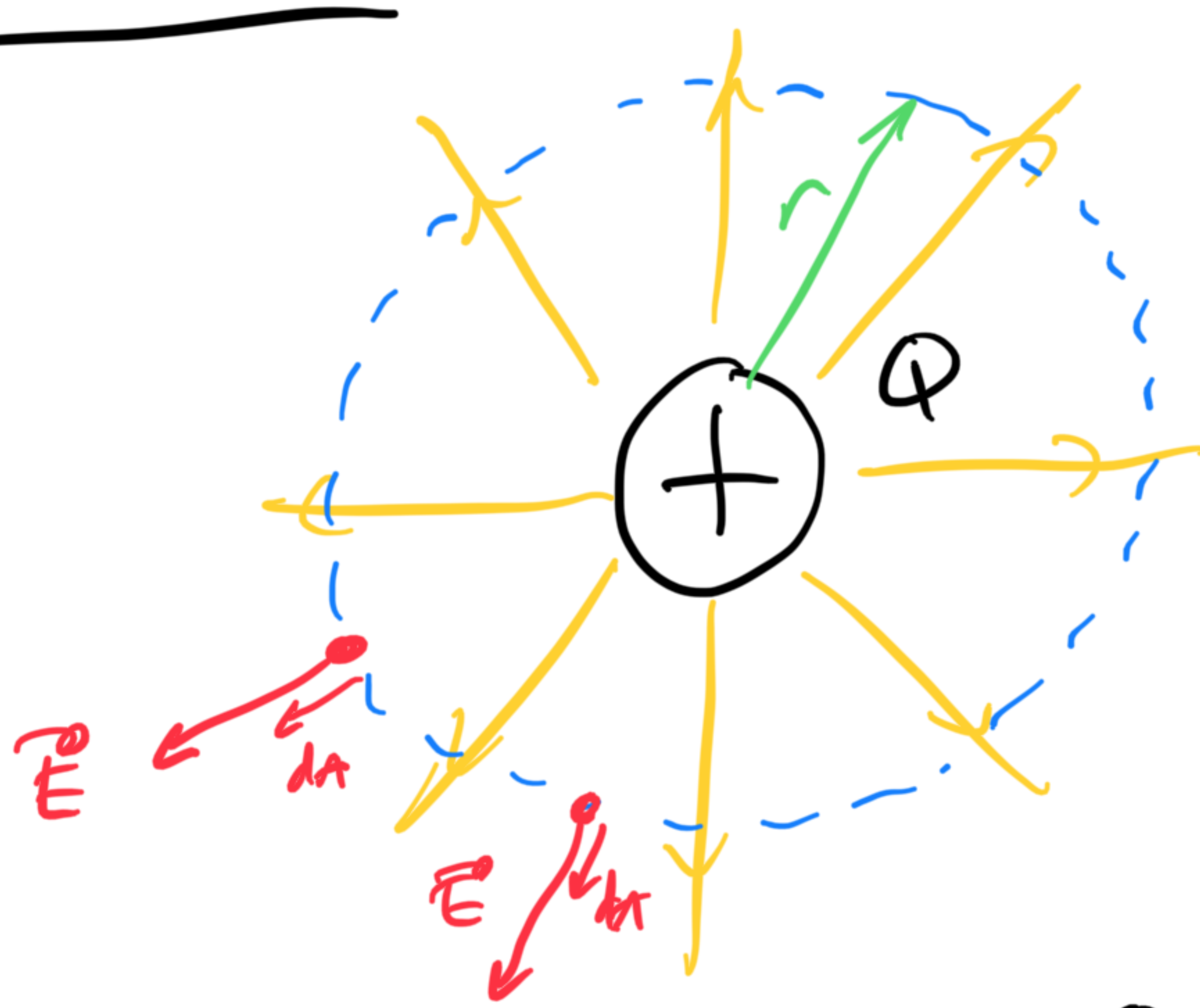
Gauss's Law

→ much more fundemental than Coulomb's Law

→ works for any distribution of charge !!!

Purpose: Find $\vec{E}(x,y,z)$ for any charge distribution !!

Example 1:



① $\Phi_E = \dfrac{Q_{enclosed}}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0}$

constant

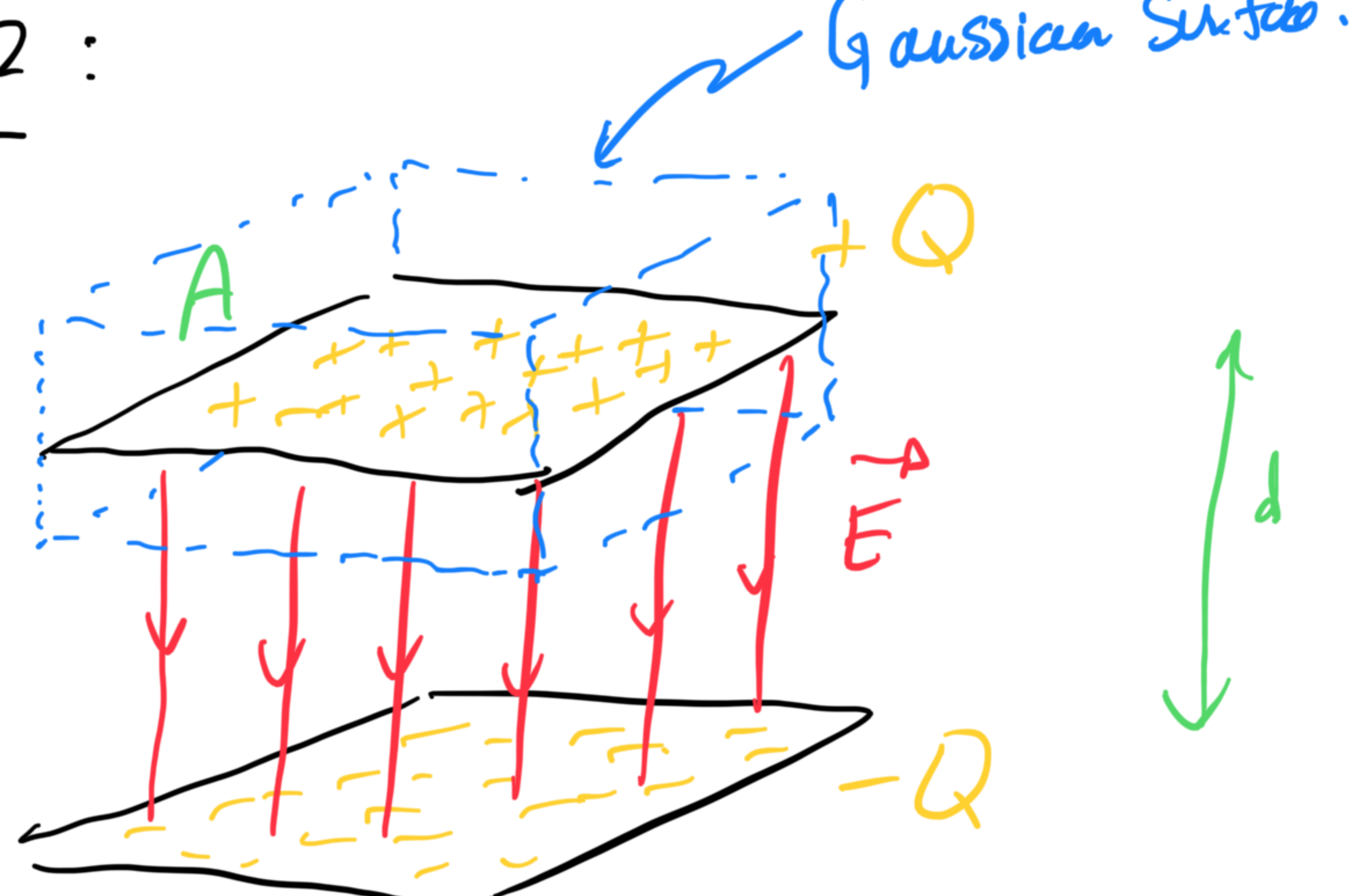
② $\Phi_E = \oint\!\!\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = |\vec{E}| \oint\!\!\oint_{\text{surface}} dA$

$= |\vec{E}| \cdot 4\pi r^2$

$\therefore \quad |\vec{E}|(4\pi r^2) = \dfrac{Q}{\varepsilon_0}$

$|\vec{E}| = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ ✓ ☺

<u>Example 2</u> :



① $\Phi_E = \dfrac{Q}{\varepsilon_0}$

② $\oint\!\!\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = |\vec{E}| \cdot A$

Q

$\therefore \quad |\vec{E}| \cdot A = \frac{\ }{\varepsilon_0}$

$$\boxed{|\vec{E}| = \frac{Q}{\varepsilon_0 A}}$$

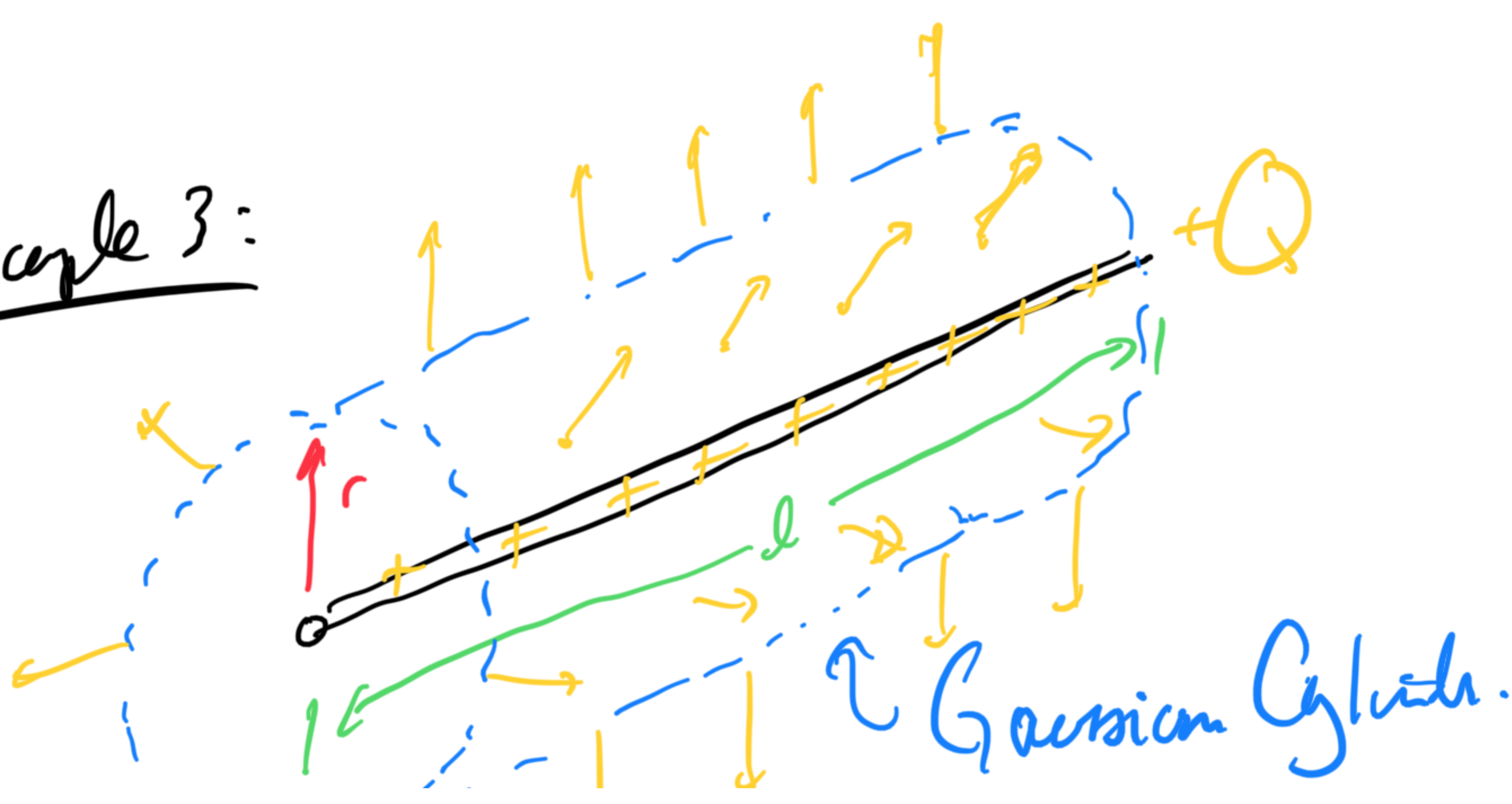
Recall: $Q = C\,\Delta V$

$$|\vec{E}| = \frac{C\,\Delta V}{\varepsilon_0 A}$$

But Also, $|\vec{E}| = \frac{\Delta V}{d}$

$$\frac{\cancel{\Delta V}}{d} = \frac{C\,\cancel{\Delta V}}{\varepsilon_0 A} \qquad \boxed{C = \frac{\varepsilon_0 A}{d}}$$

Example 3:



① $\Phi_E = \dfrac{Q}{\varepsilon_0}$

② $\oint_S \vec{E} \cdot \vec{dA} = |\vec{E}| \cdot 2\pi r l$

$$|\vec{E}|(2\pi r l) = \frac{Q}{\varepsilon_0}$$

$$|\vec{E}| = \frac{Q}{2\pi \varepsilon_0 r l} \qquad \left(= \frac{\text{charge}}{n\pi\varepsilon_0 (\text{Area})} \right)$$

But, $\dfrac{Q}{l} = \lambda$ (linear charge density)

$$\boxed{|\vec{E}| = \frac{\lambda}{2\pi \varepsilon_0 r}}$$