Phys 340 – Second Midterm Test

1 hour and 15 minutes
Professor: Dr. Edward J. Brash

Rules and Regulations:

1. Calculators, with memory cleared, are permitted.
2. You may bring as many pencils, pens, and erasers with you as you like.
3. You may use your notes, as needed.
4. The test consists of three (3) questions where you should present full solutions. The full solution questions are worth 10 points each (30 points total).
5. You should complete your solutions to the full solution questions on the exam paper itself.
6. Your solutions should contain a combination of diagrams, equations, and English word sentences explaining your reasoning.
7. If you cannot complete one part of a question, you may use the given result of that part in subsequent sections.

STUDENT NAME: ______________________________

STUDENT ID NUMBER: ______________________

SIGNATURE: ______________________________

1. Vector Algebra (10 points)

Consider the three vectors

$$\vec{A} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$$

$$\vec{B} = -\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$$

$$\vec{C} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$$

(a) Compute the scalar triple product

$$\vec{A} \cdot (\vec{B} \times \vec{C})$$

(b) Compute the quantity

$$\vec{B} \cdot (\vec{C} \times \vec{A})$$

(c) By comparing your results from parts (a) and (b), show explicitly that

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A})$$

Explain briefly why this must be true in general.

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2. Eigenvalues and Eigenvectors (10 points)

Consider the following system of coupled first-order differential equations:

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = 4x + 3y$$

(a) Write the system in matrix form

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}$$

Find the eigenvalues of the transformation matrix $\mathbf{A}$.

(b) Find the normalized eigenvectors corresponding to each eigenvalue.

(c) Given the initial conditions

$$x(0) = 1, \qquad y(0) = 0$$

write down the complete time-dependent solution for $x(t)$ and $y(t)$.

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3. Moment of Inertia (10 points)

Consider a thin uniform rod of length L and total mass M. The rod lies along the x-axis, extending from $x = 0$ to $x = L$.

(a) Write an expression for the linear mass density λ of the rod.

(b) Calculate the moment of inertia of the rod about the y-axis (which passes through the origin and is perpendicular to the rod).

That is, compute

$$I_y = \int x^2 \, dm$$

and express your answer in terms of M and L.

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