Physics 421 / PCSE 503
Lecture 17

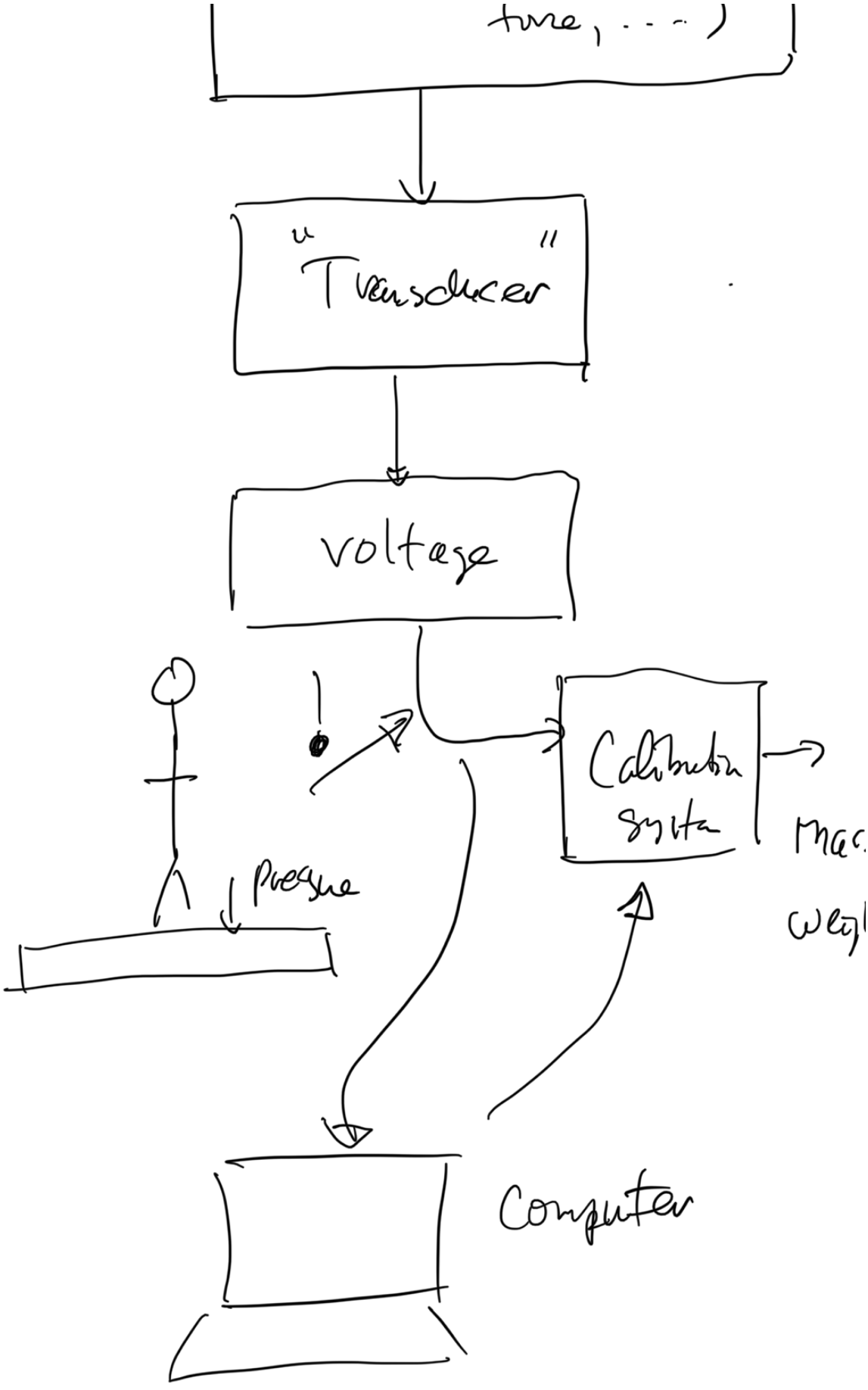
Fourier Analysis

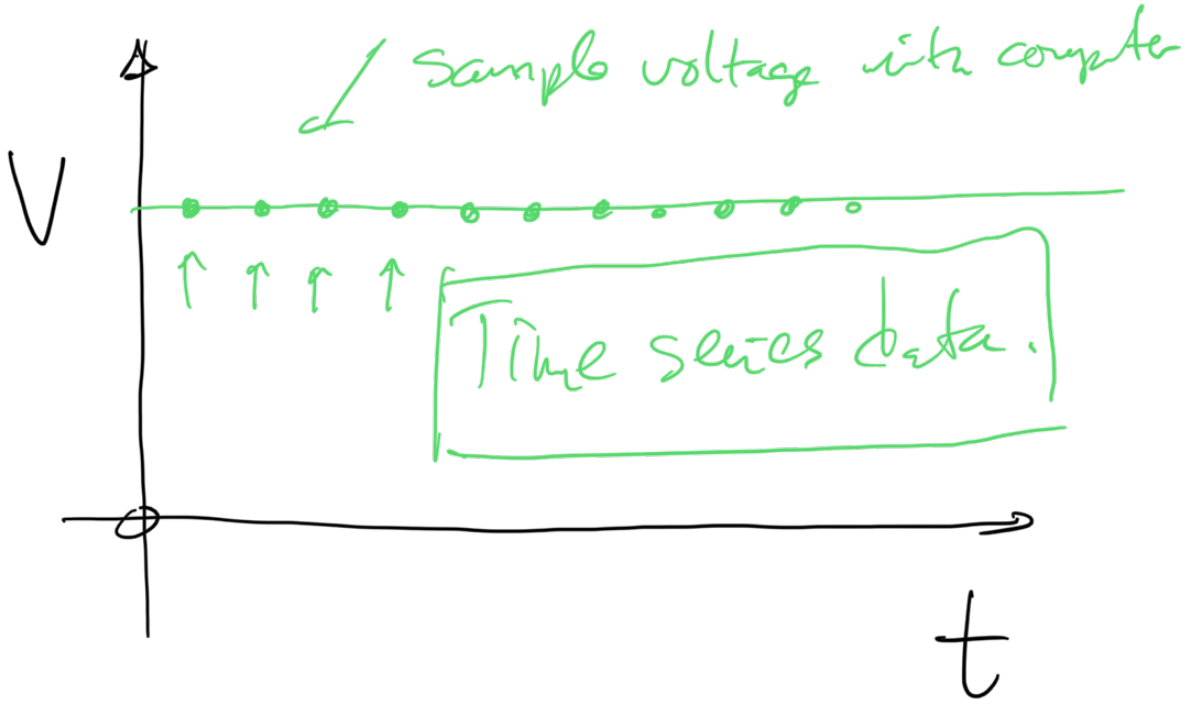
→ MATH ($\int \ldots$)

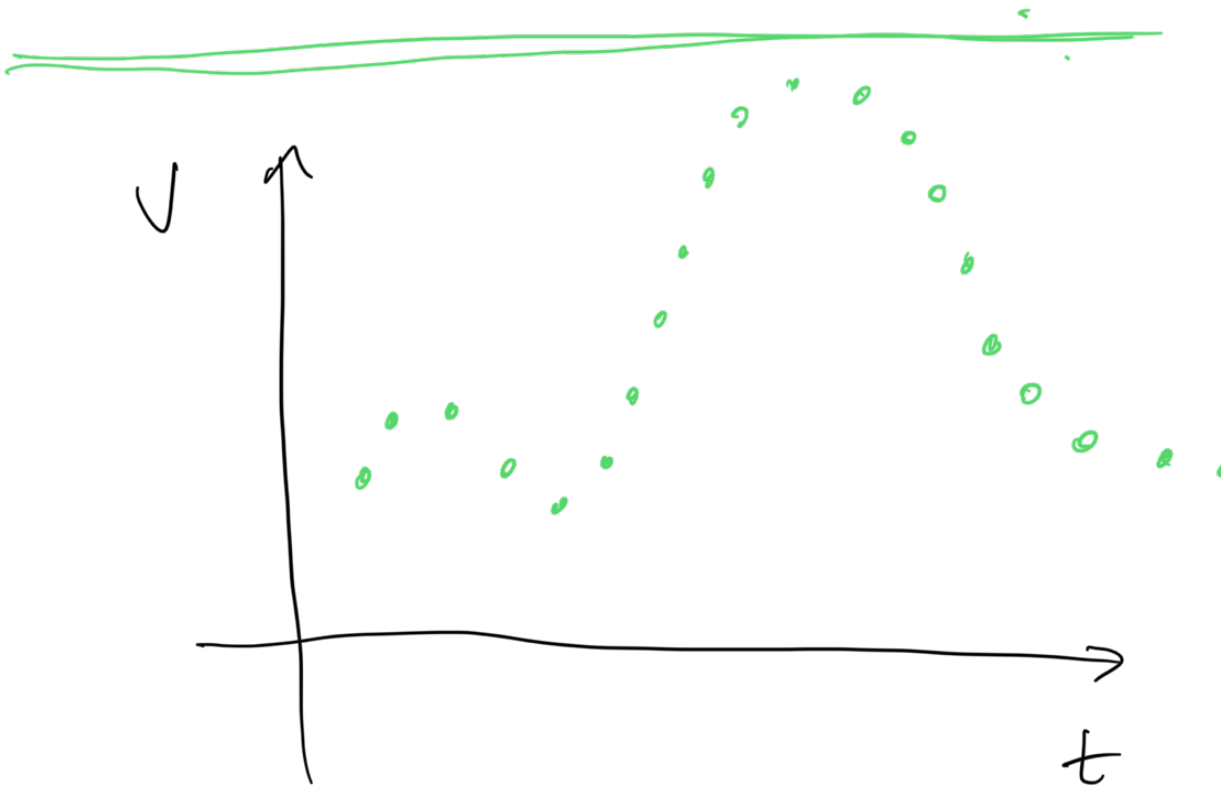
→ Math-Meth. (PHYS 340
ENGR 340
ENGR 210)

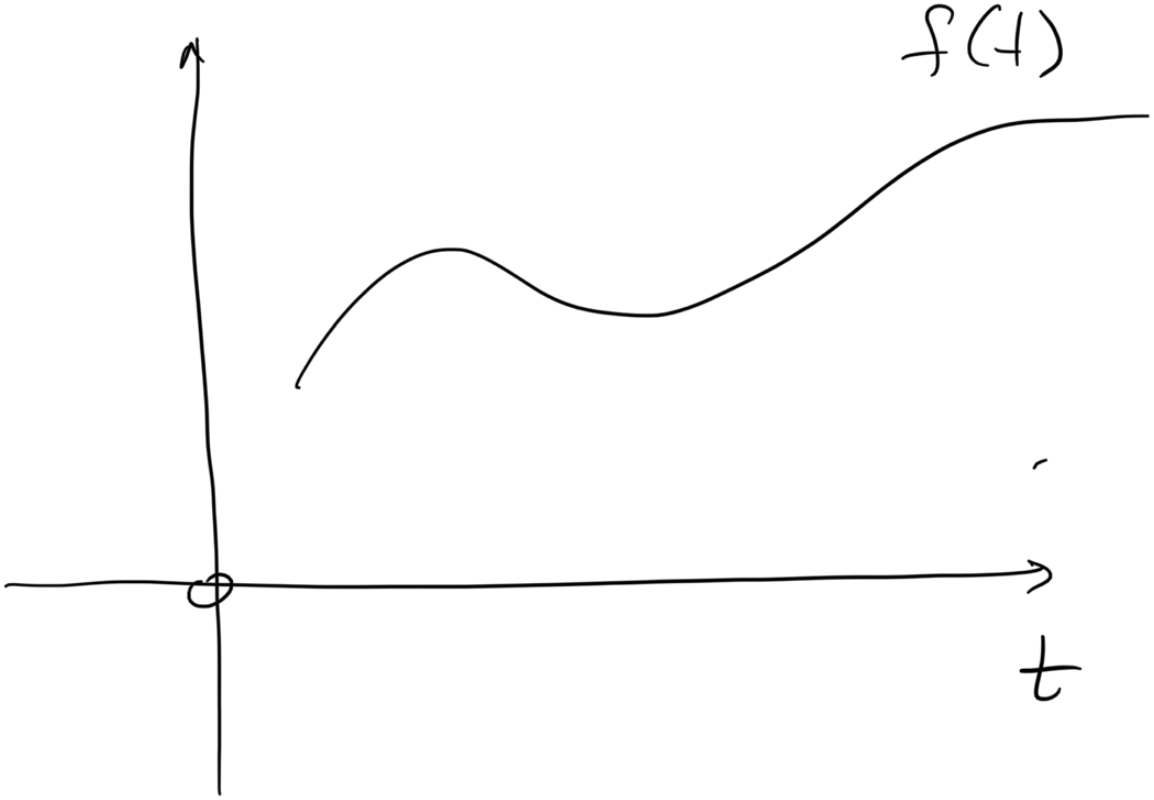
Time Series Data

Physical Quantity
(mass, voltage, density,


time, ...)
"Transducer"
voltage
Calibration system
Mac
weig
Pressure
Computer


sample voltage with computer
V
Time series data.
t


V
t



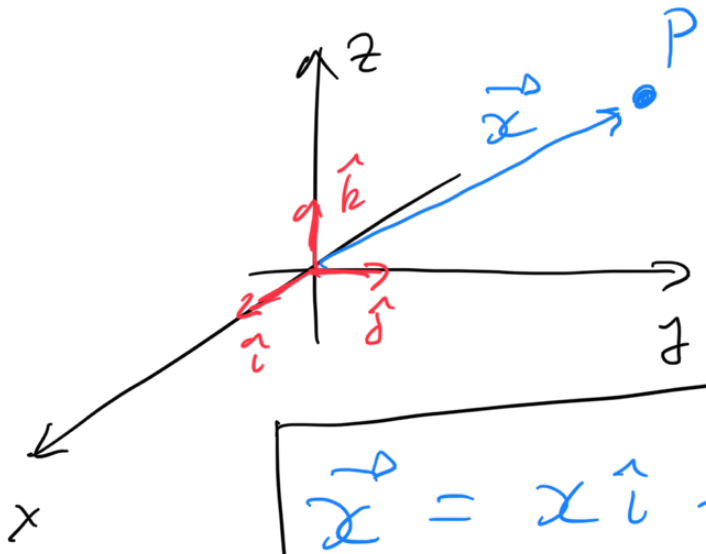
Approximation of $f(t)$

~~$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots$~~

Bad as $t \to \infty, -\infty$!!

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi t}{\cdot}\right) + b_n \sin(a\pi t) \right)$$

Fourier Approximation



$$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$$

basis vectors

"span" 3D. space.

mutually $\perp$

Hilbert:

$f(t)$ = Linear combination of basis functions.

$= a_0 + a_1 \phi_1(t)$

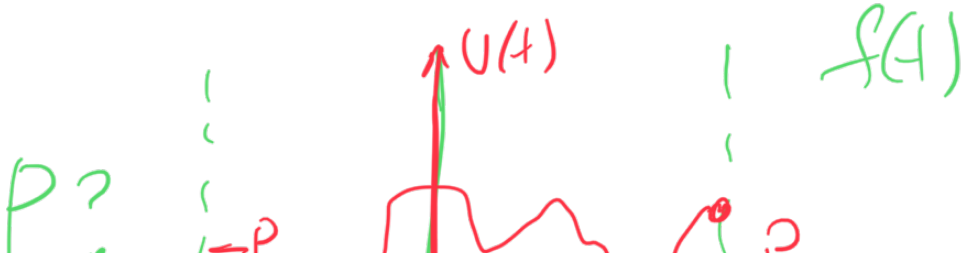
$$+ a_2 g_2(t) + a_3 g_3(t)$$

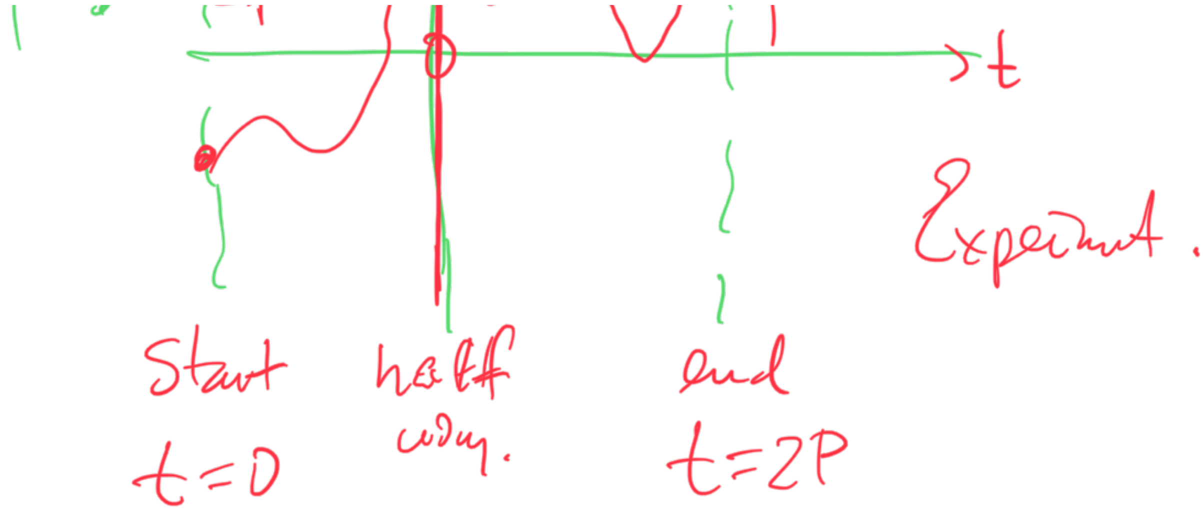
$$\int_{-\infty}^{\infty} g_1(t)\, g_2(t)\, dt = 0$$

$$a_0 = \frac{1}{P}\int_{-P}^{P} f(t)\, dt$$

$$a_n = \frac{1}{P}\int_{-P}^{P} f(t)\cos\left(\frac{n\pi t}{P}\right) dt$$

$$b_n = \frac{1}{P}\int_{-P}^{P} f(t)\sin\left(\frac{n\pi t}{P}\right) dt$$

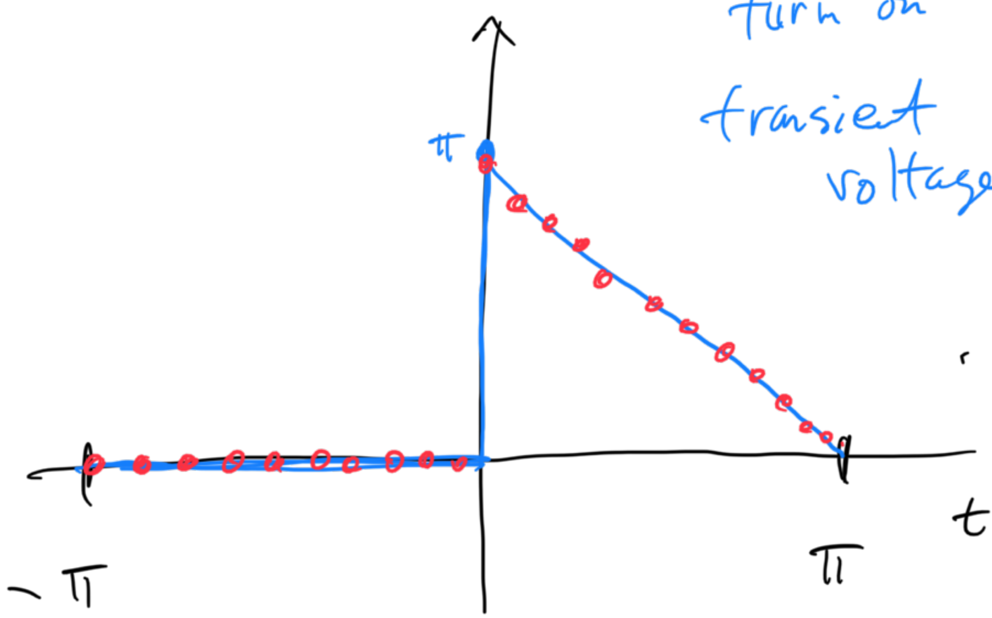




$$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ (\pi - t) & 0 \le t < \pi \end{cases}$$

turn on $t=0$

transient voltage spike



between

Sample 1000 [illegible]

$t = -\pi$ and $t = \pi$

x[ ], fx[ ]

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (0)\, dt + \underbrace{\int_{0}^{\pi} (\pi - t)\, dt}_{} \right]$$

$$\pi t - \frac{t^2}{2} \Big|_0^{\pi}$$

$$\pi^2 - \frac{\pi^2}{2}$$

$$= \pi^2/2$$

$$= \frac{1}{\pi} \left(\frac{\pi^2}{2} \right)$$

$$\boxed{a_0 = \pi/2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos\left(\frac{n\pi t}{\pi} \right) dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} (\pi - t)\cos(nt)\,dt \right]$$

Integration by Parts.

WA Technique.

Wolfram Alpha.

$$a_n = \frac{1 - \cos(\pi n)}{\pi n^2} = \frac{1-(-1)^n}{\pi n^2}$$

$\cos(\pi n) = (-1)^n$

$n=1 \rightarrow (-1)\ \cos(\pi)$

$n=2 \rightarrow (1)\ \cos(2\pi)$

$n=3 \rightarrow (-1)\ \cos(3\pi)$

$\vdots$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\,dt$$

$$= \frac{\pi n - \overbrace{\sin(\pi n)}^{0}}{\pi n^2} = \frac{1}{n}$$

$$f(t) = \frac{\pi}{a} + \sum_{n=1}^{\infty}\left[\left(\frac{1-(-1)^n}{\pi n^2}\right)\cos(nt) + \frac{1}{n}\sin(nt)\right]$$

← high frequency.

$\cos(nt)$

$\cos(\omega t)$

$\omega \uparrow 2\pi f$

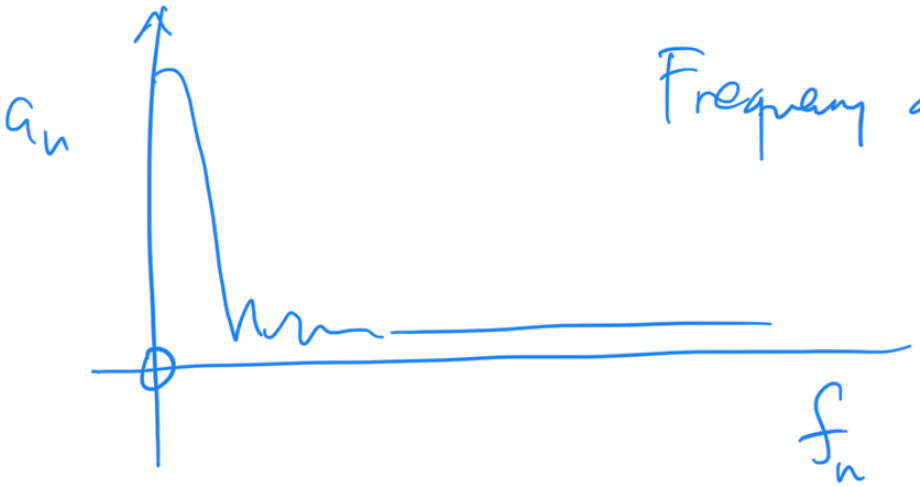
$f_n = \frac{n}{2\pi}$

$\sin(nt)$

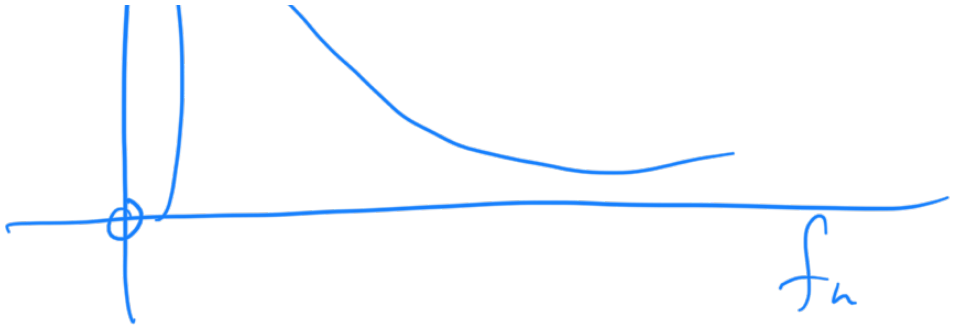
$\sin(\omega t)$

$\omega \uparrow 2\pi f$

$f_n = \frac{n}{2\pi}$



Frequency decomposition



$$a_0 = \frac{1}{P}\int_{-P}^{P} f(t)\,dt$$

data.

numerical integration!

$$a_n = \frac{1}{P}\int_{-P}^{P} f(t)\cos\left(\frac{n\pi}{P}t\right)dt$$

$$b_n = \frac{1}{P}\int_{-P}^{P} f(t)\sin\left(\frac{n\pi}{P}t\right)dt$$

$$T = \frac{5\pi}{2.5} = 2\pi$$

$$f = \frac{1}{T} = \frac{1}{2\pi}$$

$$f = \frac{1.30}{2\pi} = 0.2070 \text{ Hz}$$

$$f_s = \frac{1}{dt} = \frac{1}{10\pi/1000} = \frac{1000}{10\pi}$$

$$= \frac{100}{\pi}$$

$$f_{Nyquist} = \frac{100}{2\pi} = 15.92 \text{ Hz}$$

$P = 5\pi$ $\qquad \sin\left(\frac{n\pi t}{P}\right)$

$= \sin\left(n \frac{\pi}{5\pi} t\right)$

$\omega = \frac{n\pi}{P}$

$f = \frac{\omega}{2\pi} = \frac{n\pi}{2\pi P} = \frac{n}{2P}$

David Collar	OENB	PTR
Jessica Kelly	MATH	PTR
Leslie Rollins	PSYC	PTR
Rebecca Bromley-Trujillo	POLS	Prom to Assoc
Andrew Falk	HIST	Full

All Reds

→ John McGuire

→ Angela Spranger

→ ~~Mohamed Amdag~~

David Collar:

OENB
EVAL-4

① Teaching → UE4

② Research →
"clear pattern of student mentoring, research activities, and scientific relevance"

③ Service →
valued / highly valued
+ university committee / Core advisor

DRC statement →

① Teaching → course development / redesign
(BIOL 213/213L)
(BIOL 370)
→ teaching across the curriculum
→ CET workshops

→ three summer scholars
three grad + students
with student authors.

IDEA → scores mostly above average but
mix below, mostly early o

② Research →

→ 5 pubs, 1 under review, 1 in revisi
→ internal and external collaboration
→ internal and external funding (small $ amounts)
→ Associate journal edito

P.

③ Sonia → true

→ LLC + lots of other stuff.

Jessica Kelly - MATH

Teaching →

③ Curriculum changes and enhancement

① ~~Stop~~ IDEA scores with mostly very outstanding comments - very consistent

② broad spectrum of classes, across the curriculum. Majors & non-majors.

Research →

Ph.D. → (i) Recap. ortho. polys

NEW → (ii) Lyapunov Inequalities

(iii) Matrix-Based Tech

(3+2) Weak? '24 Actree

<u>Service</u> → LLC

→ Reviewer and referee

EVAL - 8 2018

"Work completed for this successful 4th year review is not itself adequate for the tenure and promotion review"

— Dougherty.

① Teaching → very high scores the last year / two years.

② Pubs → 2020 (PRIMUS)
2019 (Lyapunov)
2019 (Laguerre Poly)
2019 (PRIMUS)

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nothing before this related to new work

Leslie Rollins – PSYC

DE4 → teaching (UE4)
→ reseach (peer-reviewed article exp. with co-authors)
student

Teaching → IDEA scores and comments are good (grey bad, or above) mostly positive comments (virtually, era)
→ Psych and Neuro Comps. developpt
→ lots of work at CET
→ 38 UG students in char lab.

Scholarship → 11 refereed publications
(one-fraction, electrical brain

(e[illegible] activity' lists)

→ reviewer for a lot of manuscripts.

Service → Core advisor, IRB, UAC

→ lots of stuff.

Some comment from Dave?!?!?...

→ I think it comes because a lot of her earlier pubs were with her Ph.D. advisor. (all but one pub, actually)
