Physics 441/541 Assignments – Part A

Due Date: Friday, September 20, 2019

1. Comparison of First Derivative Calculations

One can calculate the first derivative of polynomials exactly, or by using a numerical approximation. We want to compare the results for the function $f(x)=x^2$ at $x=1$. We define the difference between the exact and so-called "forward difference" numerical approximations as:

$$\Delta h = \left| f'(x) - \frac{f(x+h) - f(x)}{h} \right|$$

The assignment is to create a plot of Δh vs. h for values of h between 10^{-20} and 10^{0}, first by using the forward difference approximation, and then also by using the "centered difference" approximation.

Use double precision variables in all cases.

2. Root-finding Using Newton's Method

This assignment has multiple parts:

Begin by considering the polynomial:

$$f(x) = x^3 + 0.40x^2 - 4.05x + 1.80$$

which as we discussed in class, has three real roots at x=-2.4,0.5, and 1.5

a) Write a standalone program which calculates the roots of this polynomial using the bisection algorithm. The program should fill an array of 10,000 data points between x=-4.0 and x=4.0. Then, the program should do a sparse search of this array, i.e. once every n_search elements, to look for sign changes, and subsequently using the bisection algorithm to find the root. The program should also keep track of the approximate total number of instructions taken to find the three roots (n_steps).

b) Create a plot, of the number of steps taken (n_steps) vs. the sparsification size (n_search), for n_search values between 1 and 10,000.

c) Deduce the source of the various patterns that you observe in part b). Hint: You might consider adjusting the number of data points in the original array, as well as the coefficients of the polynomial, slightly.

d) Create a single program that both finds the roots of the polynomial, as well as creates the plot from part b)

e) Modify the program created in part d) to allow for the possibility to find the roots of other polynomials or functions (by defining a general function).

3. Differential Equations Part I – Projectile Motion

I am providing to you a program that will calculate the motion of a projectile under the influence of gravity, including air resistance. It uses the "forward derivative" method for calculating derivatives. The program does the following:

(a) Takes as input the initial height above the ground, the initial speed, and the initial launch angle.
(b) Calculates the path of the projectile (i.e. the (x,y) position of the particle) for the case of zero air resistance, and including air resistance.
(c) Creates (x,y) plots of these two paths, for comparison.
(d) Creates plots of x vs. t and y vs. t for both paths, for comparison.

The assignment is to take this program and modify it to do the following:

Include the effect of the object "bouncing" of the ground. In order to do this, you will have to make some assumptions:

a. The coefficient of restitution between the projectile and the ground should be, say, 0.30 in the y-direction and 0.90 in the x-direction.
b. When a "bounce" happens, the projectile reverses its y-velocity, subject to the coefficient of restitution in the y-direction, and its x-velocity is reduced according to the coefficient of restitution in the x-direction.
c. The object has to "stop" at some point – say when the total velocity has dropped below some threshold … use 0.5 m/s.

Run the code and produce plots for the following situation: h0=2m, v0=40 m/s, theta0=45 degrees.

4. Differential Equations Part II – Simple Pendulum

I am providing you with a program that will calculate the "path" of a simple pendulum over time in the case of zero air resistance. The final output is a plot of θ vs. t for several oscillations of the pendulum. The assignment is to modify the program to include air resistance. You should be able to take advantage of the algorithm that was used in the program of part 3 above.

Create a plot of theta vs. t for several oscillations for the case of theta0=45 degrees, m=1kg, radius of mass=0.10m, length of pendulum=9.807m.