ECE 203

Circuits I

Inductors

Lecture 9-2

Inductance

- An inductor is just a coil of wire.
- Sometimes also referred to as a coil, or as a choke.
- Symbol is L (because I is already taken!)
- Has units of Henrys
- 1 Henry is 1 V-sec/A

Inductance

- When a current flows through a wire, it sets up a magnetic field.
- If the current is time varying, a voltage appears across the inductor that is proportional to the time rate of change of current

$$v(t) = L\frac{di(t)}{dt}$$

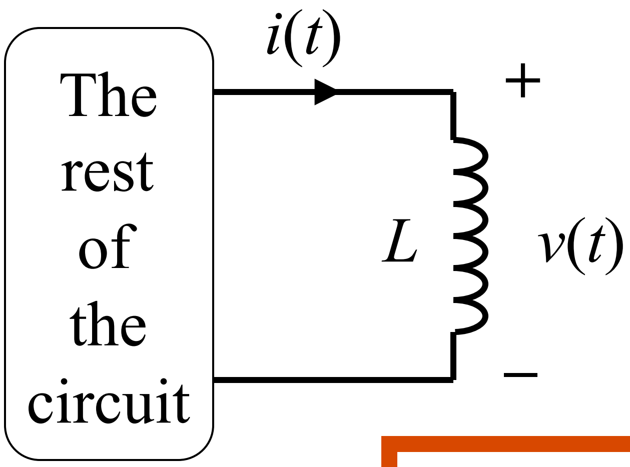
Joseph Henry

Joseph Henry

(1797-1878)

In 1846, Henry became the first secretary of the newly organized Smithsonian Institution, where he established a continuing tradition of research. Under his leadership, weather reporting stations were connected by telegraph in the United States. These weather reporting stations were organized and maintained by the U.S. Army signal corps. This organization would become, in 1891, the U.S. Weather Bureau (Service). Henry also directed the resources of the Smithsonian Institution to encourage research in the areas of astronomy, botany and Native American anthropology.
In the spring of 1863 Henry became one of the founding members of the National Academy of Science, and served as Academy president beginning in 1867. He served as both the National Academy of Science president and secretary of the Smithsonian Institution until his death in 1878. In 1893 his name was given to the standard electrical unit of inductive resistance, the henry.

Inductor



$$\boxed{v(t) = L\frac{di(t)}{dt}}$$

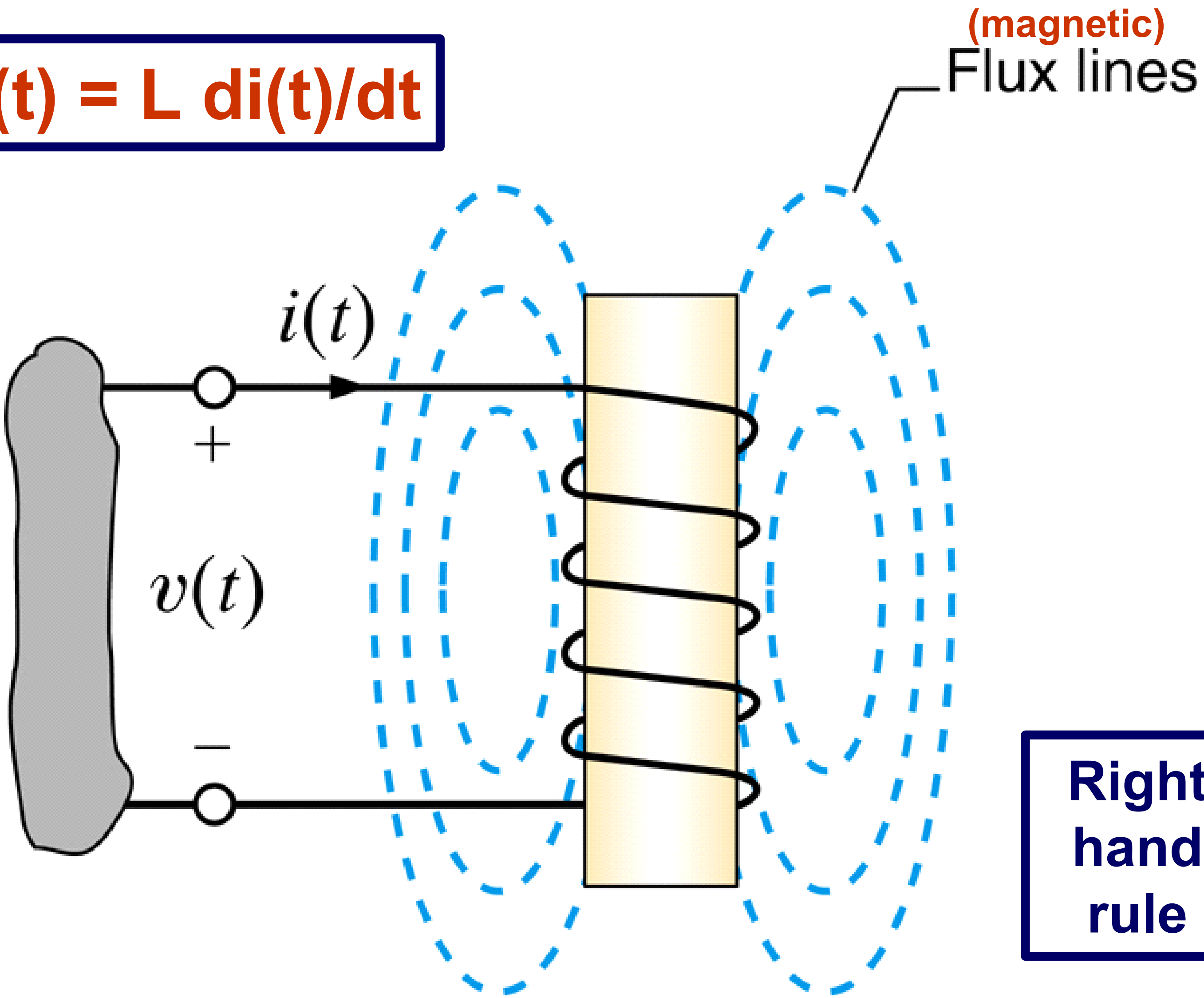
$$\boxed{i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)dx}$$

DC characteristics

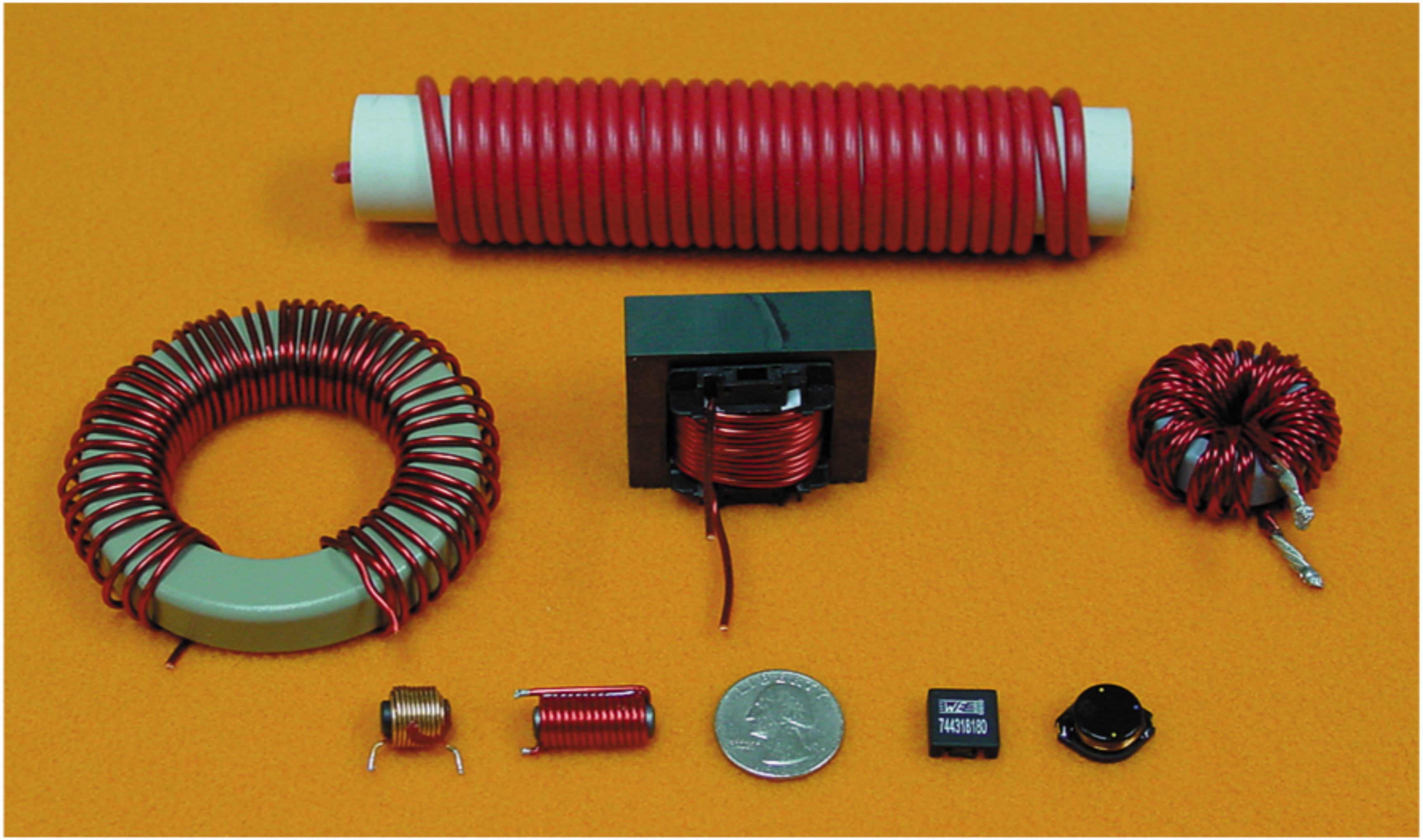
- Since voltage across an inductor is proportional to the time rate of change of current, we only have a voltage across an inductor when the current is changing with time
- So, for DC, an inductor looks like a wire, and V = 0   No voltage across an inductor for DC!

Inductor

$$v(t) = L\ di(t)/dt$$



Right hand rule



A variety of inductors

Inductors

- Inductors are coils of wire, sometimes around a ferrite or iron core.
- The ferrite core is a composite with small magnetic particles. Works at high frequencies where iron doesn't.

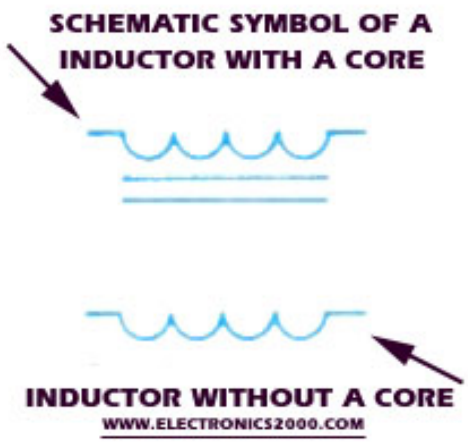
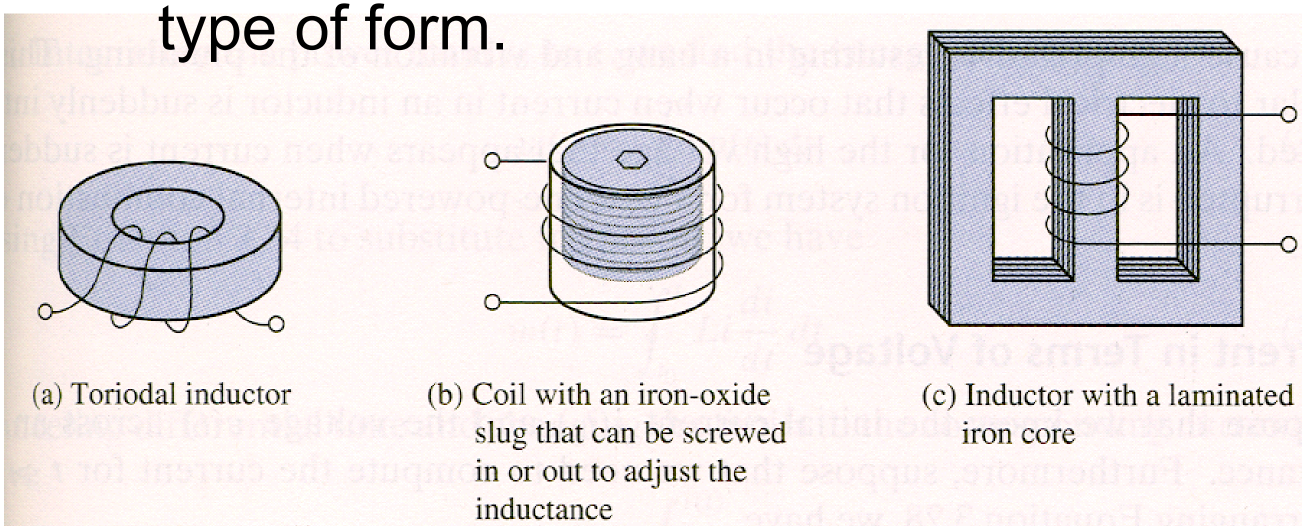
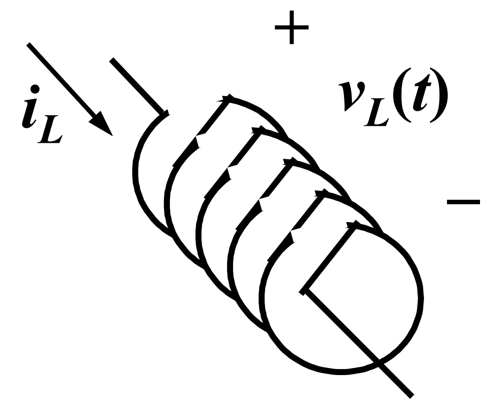


TABLE 6.2 Standard inductor values

nH	nH	nH	μH	μH	μH	mH	mH	mH
1	10	100	1.0	10	100	1.0	10	100
1.2	12	120	1.2	12	120	1.2	12	
1.5	15	150	1.5	15	150	1.5	15	
1.8	18	180	1.8	18	180	1.8	18	
2	20	200	2.0	20	200	2.0	20	
2.2	22	220	2.2	22	220	2.2	22	
2.7	27	270	2.7	27	270	2.7	27	
3	33	330	3.3	33	330	3.3	33	
4	39	390	3.9	39	390	3.9	39	
5	47	470	4.7	47	470	4.7	47	
6	51	510	5.1	51	510	5.1	51	
7	56	560	5.6	56	560	5.6	56	
8	68	680	6.8	68	680	6.8	68	
9	82	820	8.2	82	820	8.2	82	

- An inductor is constructed by coiling a wire around some type of form.





- Current flowing through the coil creates a magnetic field and a magnetic flux that links the coil: Li_L
- When the current changes, the magnetic flux changes → a voltage across the coil is induced:

$$\boxed{v_L(t) = L\frac{di_L}{dt}}$$

<u>Note</u>: In "steady state" (dc operation), time derivatives are zero → *L* is a short circuit

An Inductance Example

The current in a 2-mH inductor is

$$i(t) = 2 \sin 377t \text{ A}$$

Determine the voltage across the inductor and the energy stored in the inductor.

$$\begin{aligned} v(t) &= L \frac{di(t)}{dt} \\ &= \left(2 \times 10^{-3}\right) \frac{d}{dt} (2 \sin 377t) \\ &= 1.508 \cos 377t \text{ V} \end{aligned}$$

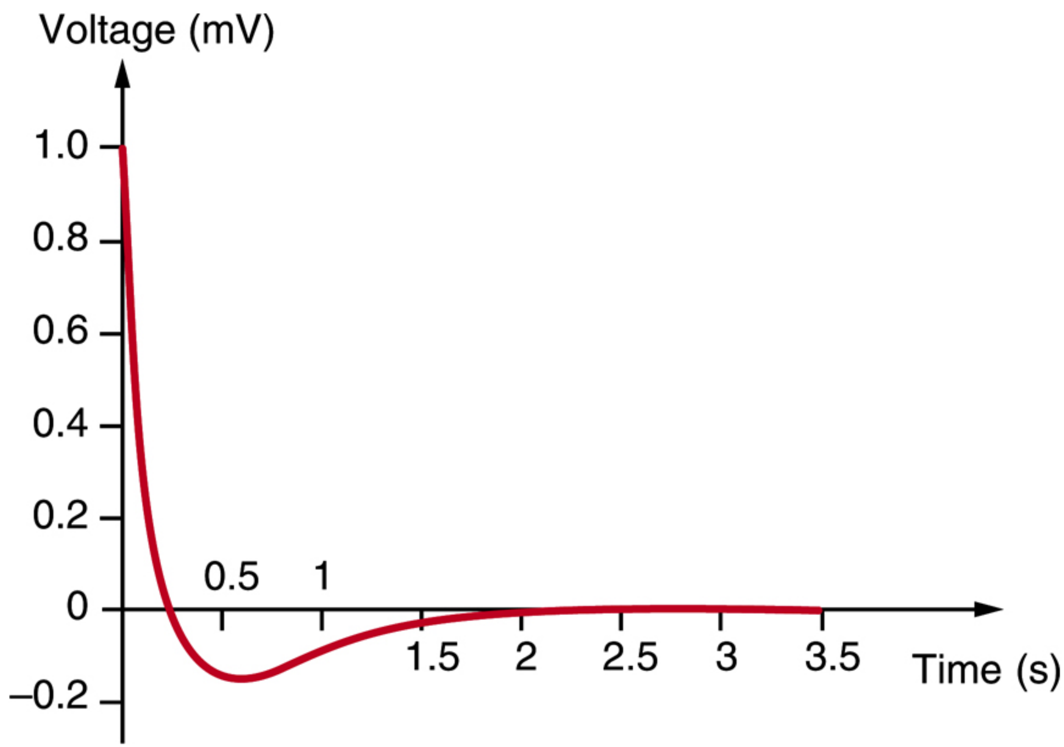
$$\begin{aligned} w_L(t) &= \frac{1}{2} L i^2(t) \\ &= \frac{1}{2} \left(2 \times 10^{-3}\right) (2 \sin 377t)^2 \\ &= 0.004 \sin^2 377t \text{ J} \end{aligned}$$

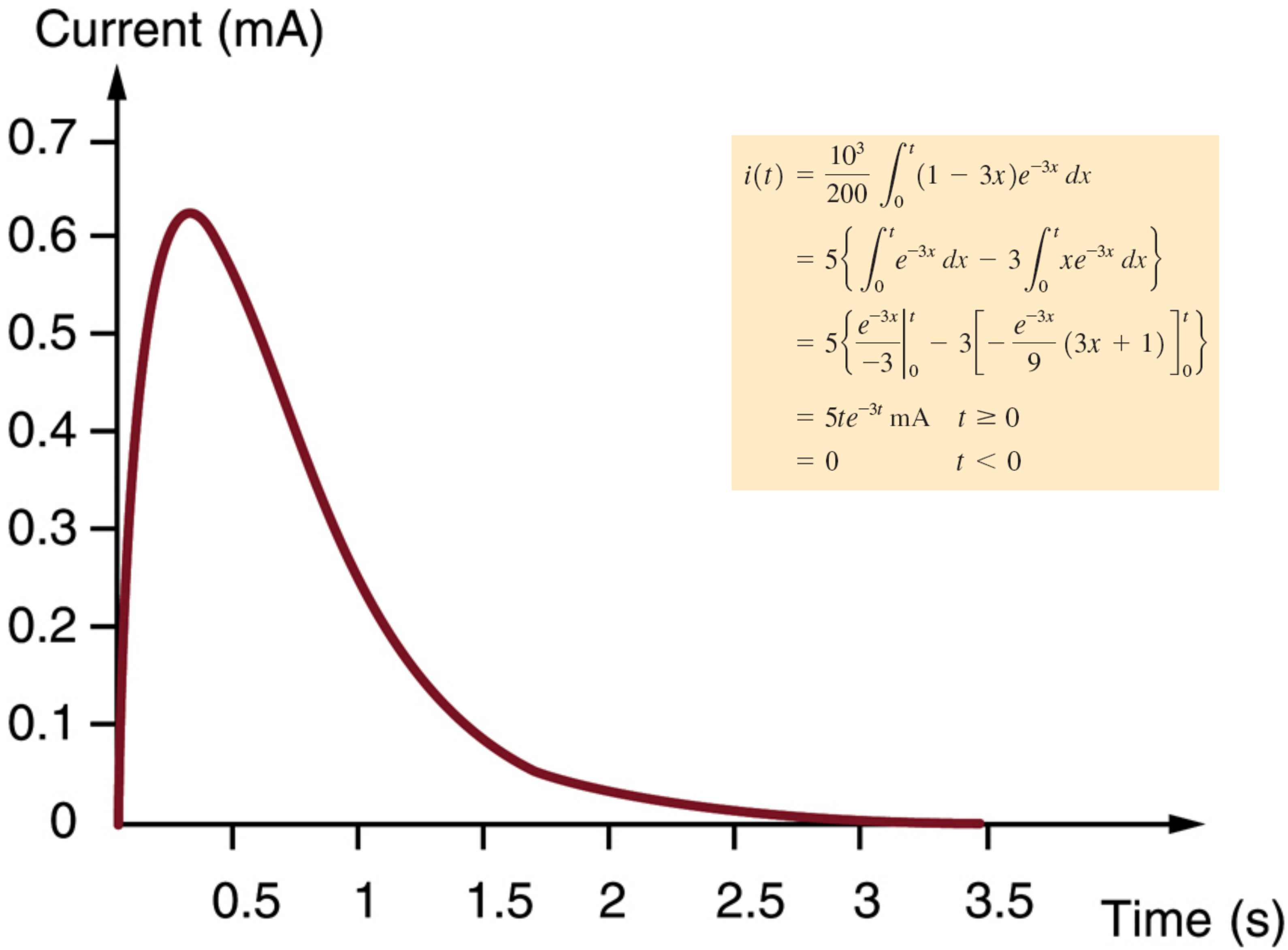
Another Inductance Example

The voltage across a 200-mH inductor is given by the expression

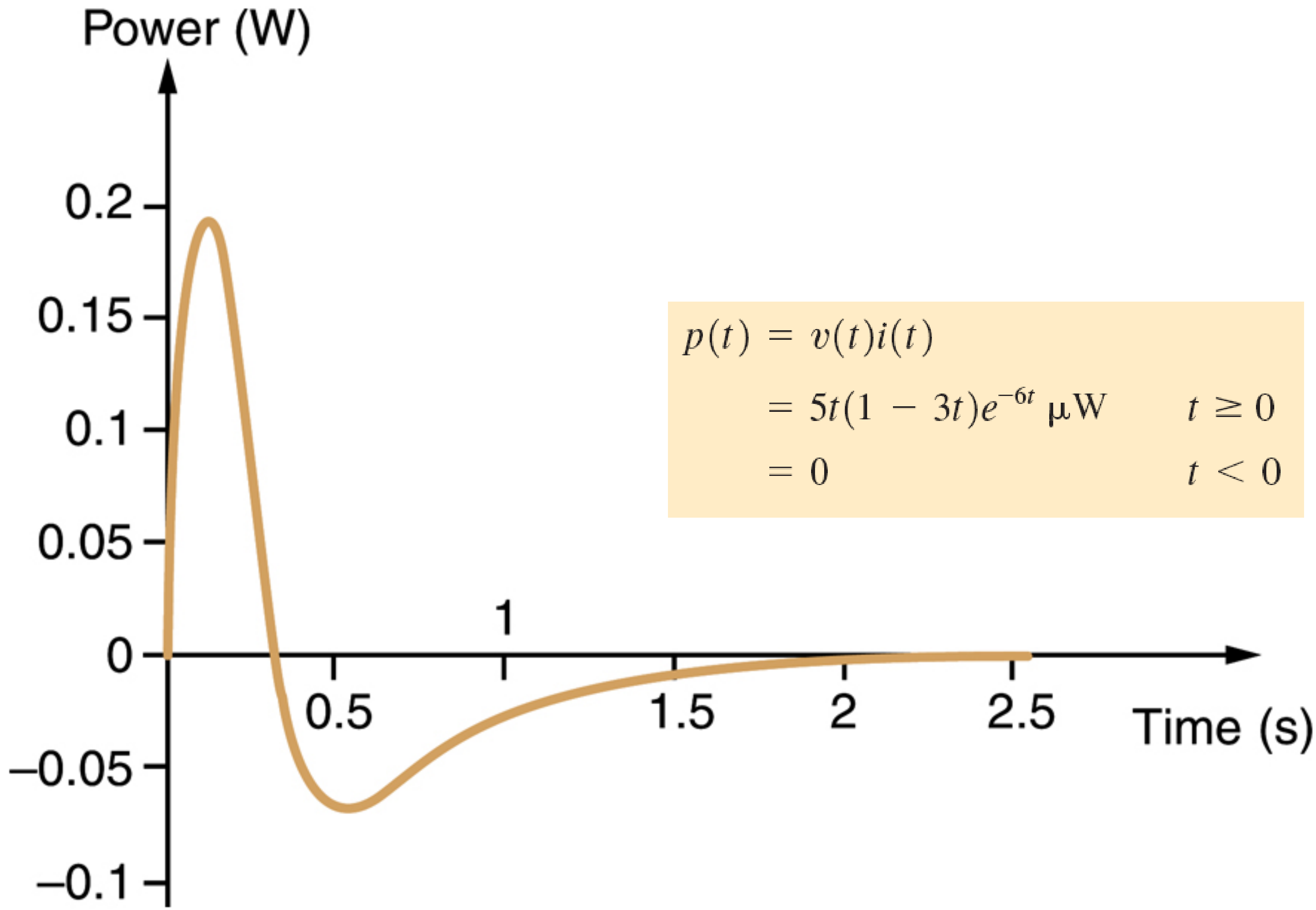
$$v(t) = (1 - 3t)e^{-3t}\text{mV} \quad t \geq 0$$
$$= 0 \quad t < 0$$

Let us derive the waveforms for the current, energy, and power.

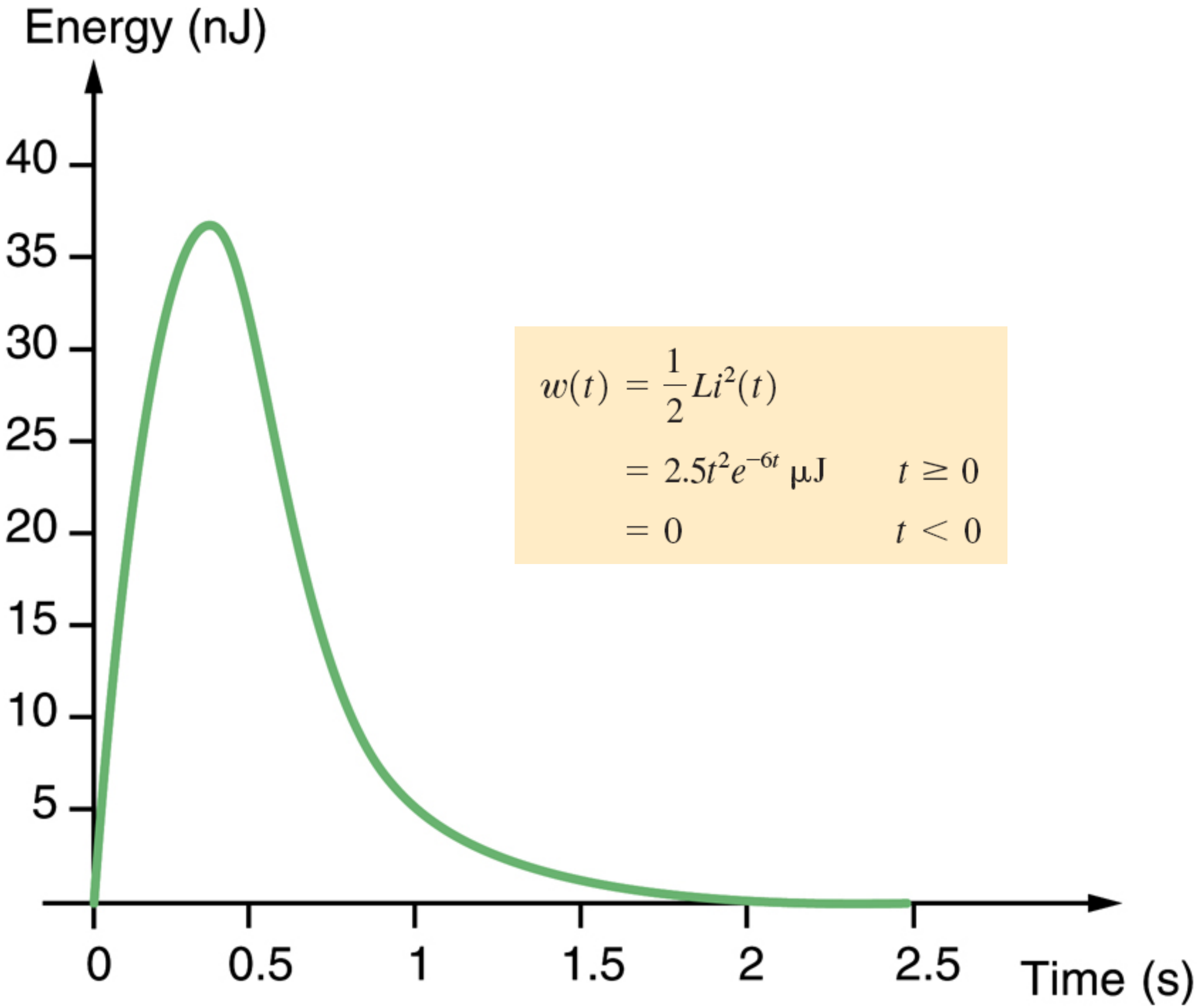




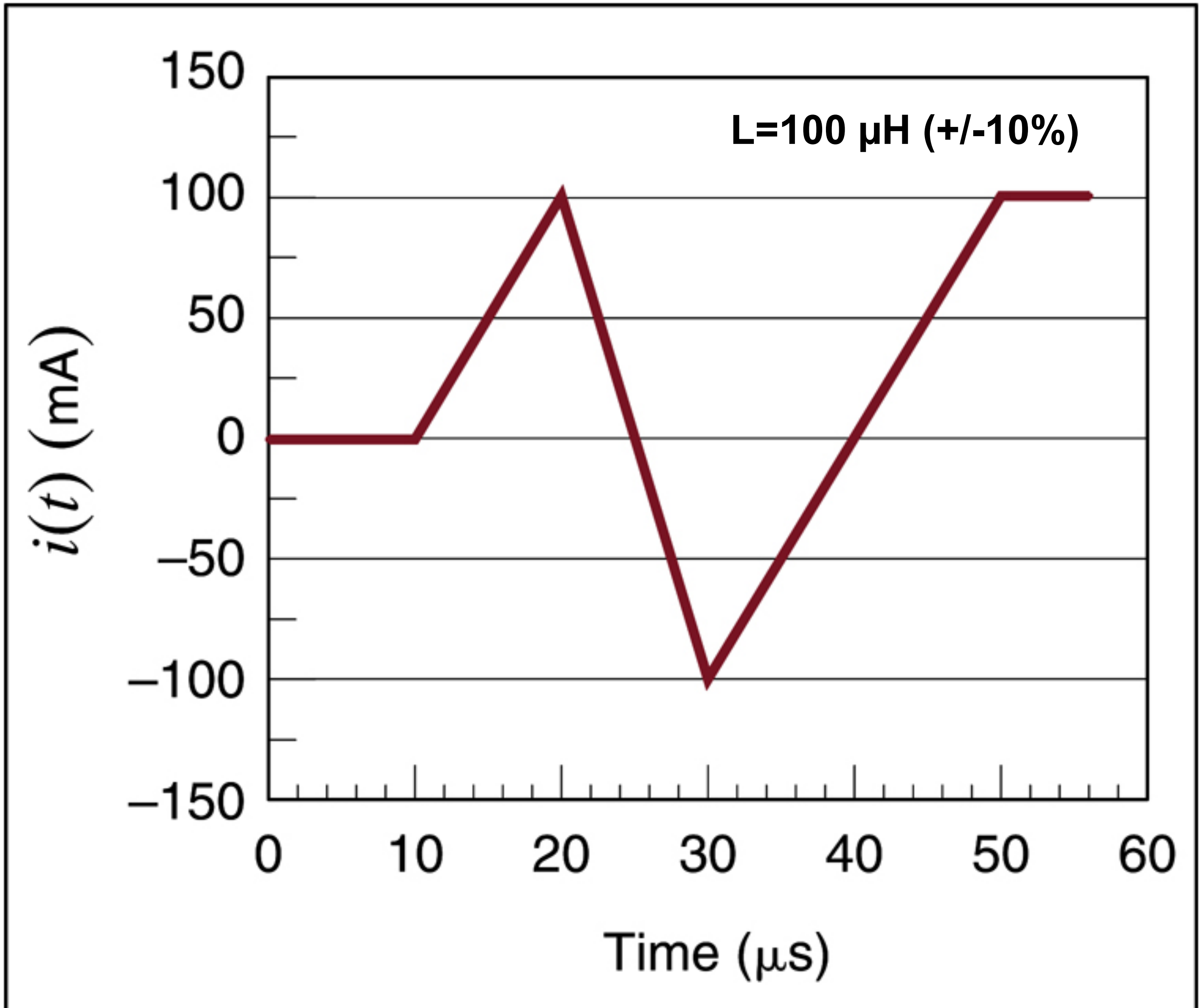
$$
\begin{aligned}
i(t) &= \frac{10^3}{200}\int_0^t (1 - 3x)e^{-3x}\,dx \\
&= 5\left\{\int_0^t e^{-3x}\,dx - 3\int_0^t xe^{-3x}\,dx\right\} \\
&= 5\left\{\frac{e^{-3x}}{-3}\bigg|_0^t - 3\left[-\frac{e^{-3x}}{9}(3x + 1)\right]_0^t\right\} \\
&= 5te^{-3t}\ \text{mA} \qquad t \geq 0 \\
&= 0 \qquad\qquad\quad\ \ t < 0
\end{aligned}
$$

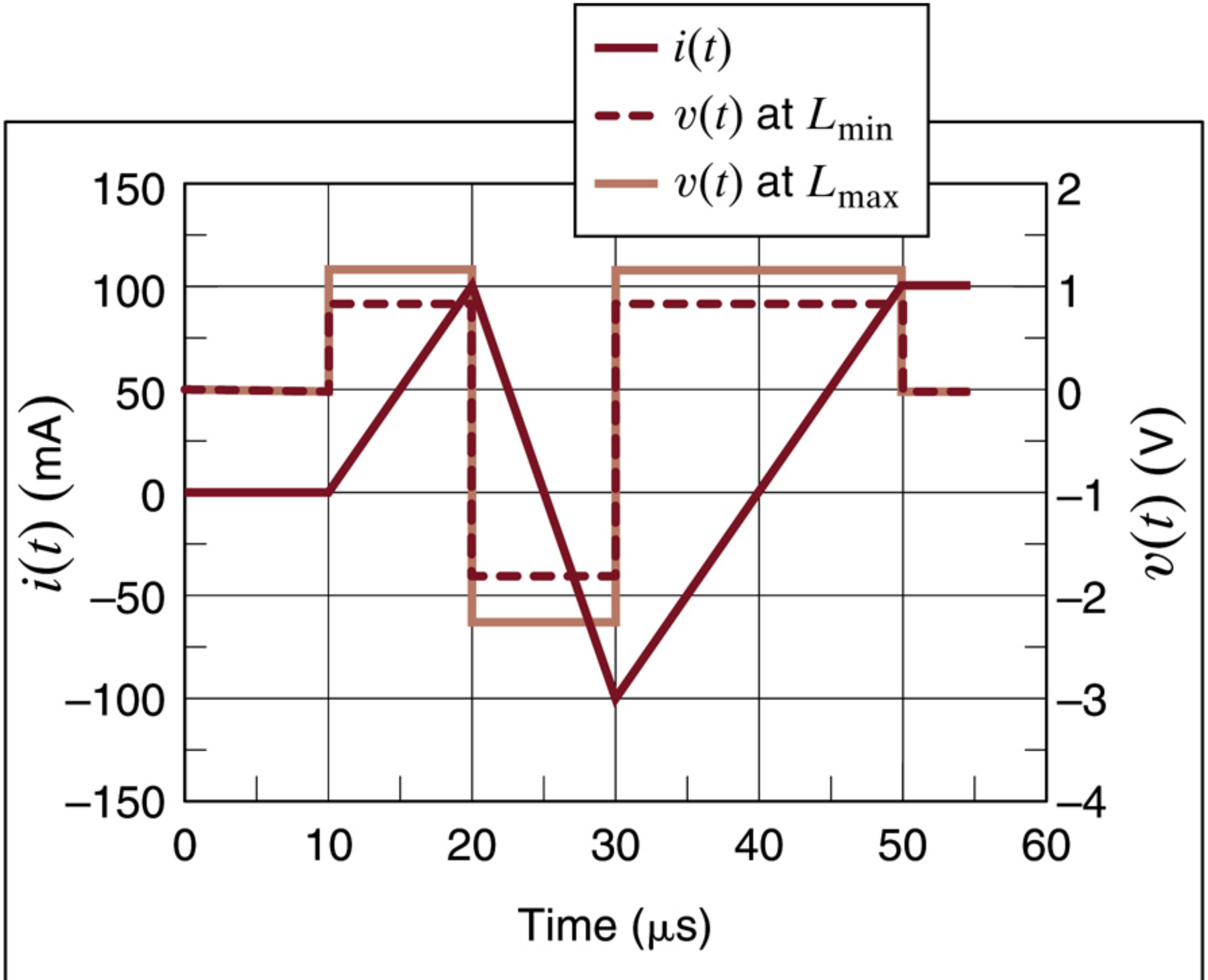


Power (W)
0.2
0.15
0.1
0.05
0
–0.05
–0.1
0.5
1
1.5
2
2.5
Time (s)
$p(t) = v(t)i(t)$
$= 5t(1 - 3t)e^{-6t}$ μW $\quad t \geq 0$
$= 0 \quad t < 0$


Energy (nJ)
40
35
30
25
20
15
10
5
0
0.5
1
1.5
2
2.5
Time (s)
$w(t) = \frac{1}{2}Li^2(t)$
$= 2.5t^2e^{-6t}$ μJ $\quad t \geq 0$
$= 0 \quad t < 0$

Another example





$i(t)$
$v(t)$ at L_{min}
$v(t)$ at L_{max}
150
100
50
0
–50
–100
–150
2
1
0
–1
–2
–3
–4
0
10
20
30
40
50
60
$i(t)$ (mA)
$v(t)$ (V)
Time (μs)

Combining Inductors

Inductors add like resistors:

In series: $L_{total} = L_1 + L_2 + \ldots$

In parallel: $1/L_{total} = 1/L_1 + 1/L_2 + \ldots$

Go to Example 9-2.1

Inductive voltage and current dividers

Work exactly like resistive voltage dividers and current dividers

Go to Example 9-2.2