HW 10 | Engr 2910 4-15 | Ansel Bejos

Question 2

$i = 40\,mA \quad t \leq 0$

$i(t) = A_1 e^{-10000t} + A_2 e^{-40000t} \quad t \geq 0$

$\lim_{t \to 0} i(t) = \lim_{t \to 0^+} i(t)$

$40 \times 10^{-3} A = A_1 e^0 + A_2 e^0$

$40 \times 10^{-3} = A_1 + A_2$ (1)

Voltage across inductor when $t \geq 0$

$V_L(t) = L \frac{di}{dt}$

$V_L(t)\Big|_{t=0} = 20 \times 10^{-3} H \left[-A\, 10^4 e^{-10^4 t} - A_2 (4 \times 10^4 e^{-4000t} \right]$

$V_L(0) = 28 = 200\,[-A_1 - 4A_2]$

$.14 = -A_1 - 4A_2$ (2)

$$\begin{bmatrix} 1 & 1 & 40 \times 10^{-3} \\ -1 & -4 & .14 \end{bmatrix}$$

$A_1 = .1 \qquad A_2 = .06$

$$\boxed{V_L(t) = 48e^{-40000t} - 20e^{-10000t} \qquad t > 0}$$

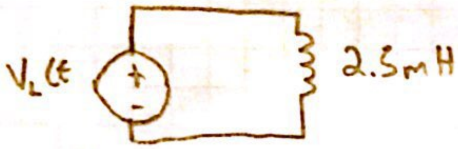
Scanned with CamScanner

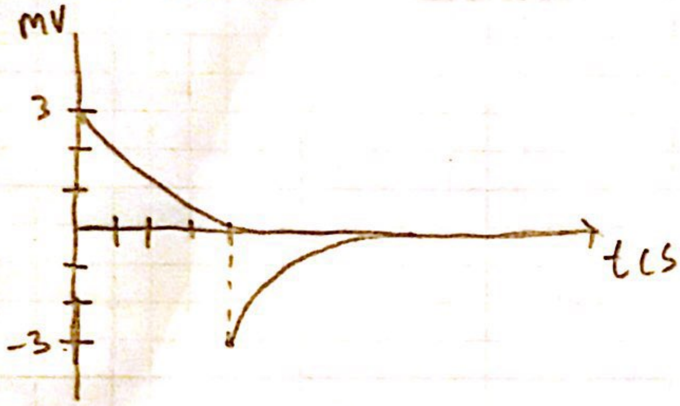
Question 2

$t < 0$ $\quad$ $i(t) = 1\,mA$

$V_L(t) = 3e^{-4t}\ mV \qquad 0^+ \le t \le 2s$

$V_L(t) = -3e^{-4t+8}\ mV \qquad 2 \le t \le \infty$





$$i(t) = \frac{10^{-3}}{L}\int_0^t 3e^{-4\tau}\,dt + i(t_0$$

$$i_1(t) = \frac{3}{20}\frac{e^{-4\tau}}{-4} + 1 \Rightarrow i(t) = -\frac{3}{30}e^{-4t} + 1$$

$i(2) = .99998$

$\boxed{i(2) \approx 1}$

$0^+ \le t \le 2s$

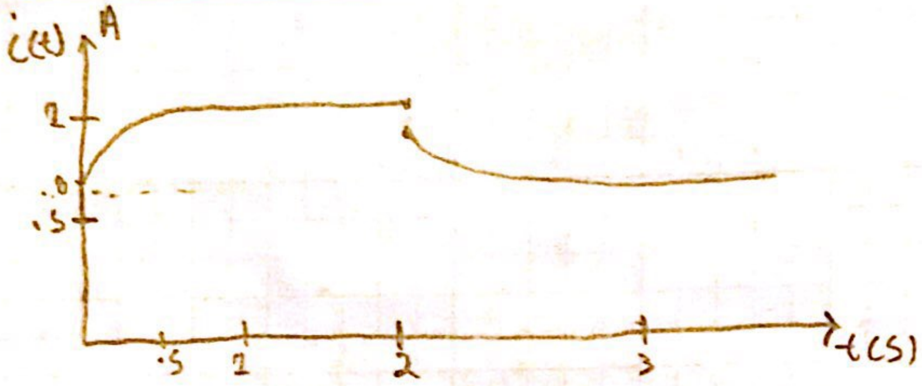
$$i_2(t) = \frac{1}{20}\int_2^t -3e^{-4(t-2)}\,dt + 1 \qquad \{2s \le t \le \infty\}$$

$$i_2(t) = -\frac{3}{20}\left.\frac{e^{-4(t-2)}}{-4}\right|_2^t + 1$$

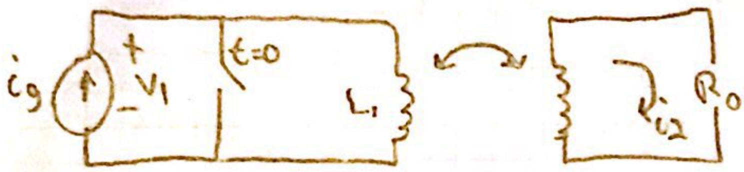
$$i_2(t) = -\frac{3}{80}\left[e^{-4(\tau-2)} - e^{-4(0)}\right] + 1$$

$$\boxed{i_2(t) = \frac{3}{80}e^{-4(t-2)} + \frac{77}{80}}$$

Contd $\rightarrow$


i(t)
A
2
.0
.5
.5
1
2
3
t(s)

Question 3



(i) $R i_2 + L_2 \frac{di_2}{dt} + M \frac{di_g}{dt} = 0$

$$\boxed{10 i_2 + .2 \frac{di_2}{dt} = -.5 \frac{di_g}{dt}}$$

(ii) $V_1 = L_g \frac{di_g}{dt} + M_{12} \frac{di_2}{dt}$

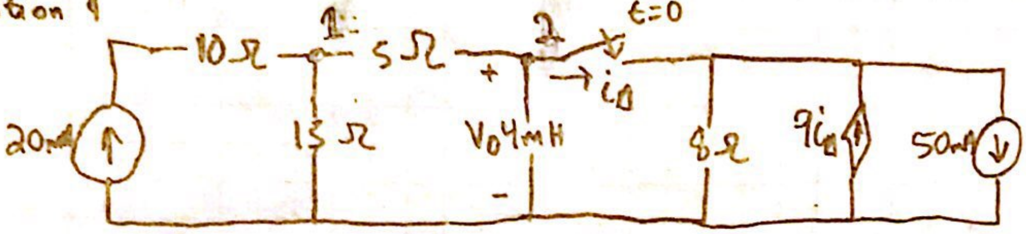
$V_1 = 5 \frac{di}{dt} + .5 \frac{di_2}{dt}$

$V_1 = 5 \frac{d}{dt}(e^{-10t} - 10) + .5 \frac{di_2}{dt}$

$V_1 = 5(-10e^{-10t}) + .5(6.25e^{-10t} + 12.5e^{-50t})$

$$\boxed{V_1(0) = -46.875\,V}$$

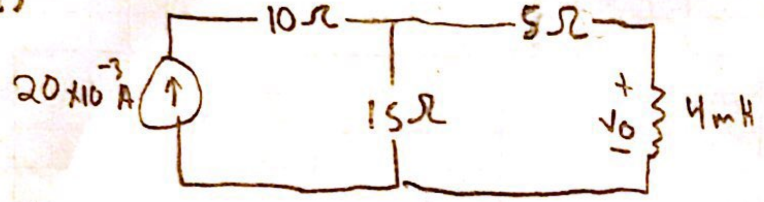
Question 1



(i) $t<0$
$i = 0\,A$

(ii)

$\underline{t<0}$



$$\boxed{i_2(0^+) = \frac{15}{20}(2.0\times10^{-2}\,A) = 0.015A}$$

(iii) Node 1

$$-2\times10^{-2}A + \frac{V_1 - V_2}{5} + \frac{V_1}{15} = 0$$

$$15\left(\frac{V_1}{15}\right) + 15\left(\frac{V_1 - V_2(0^+)}{5}\right) = 15(2.0\times10^{-2}A)$$

$$V_1 + 3V_1 - 3V_2(0^+) = 3.0\times10^{-2}A$$

① $4V_1 - 3V_2(0^+) = .3A$

② $i_\Delta = \frac{V_2(0^+)}{8} - 9i_\Delta + 0.05$

Node 2

$$\frac{V_2(0^+) - V_1}{5} + .015 + \frac{V_1(0^+)}{8} - 9i_\Delta + 0.05 = 0$$

$$V_2(0^+) - V_1 + .075 + \frac{5}{8}V_1 - 45i_\Delta + .25 = 0$$

$$8V_2(0^+) - 8V_1 + .6 + 5V_1(0^+) - 360i_\Delta + 2 = 0$$

$$13V_2(0^+) - 8V_1 - 4.5V_2(0^+) - 1.8 + 2.6 = 0$$

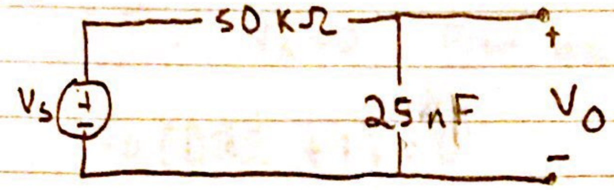
$$-8V_1 + 8.5V_2(0^+) = -0.8 \quad (3)$$

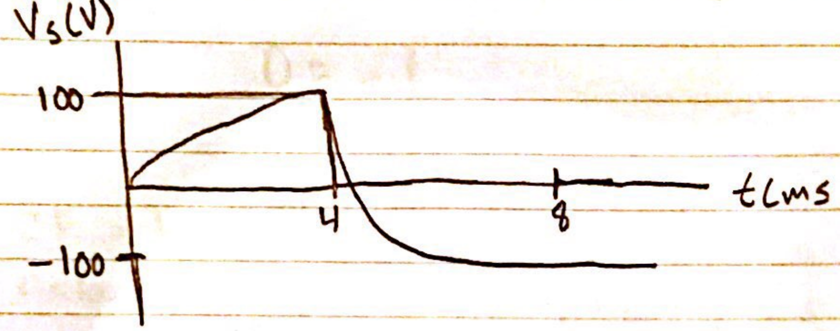
Using ① + ③

$V_b(0^+) = -0.08V$

$V_0(\infty) = 0$

Question 5





(i)

$$V_s(t) = \begin{cases} 0 & t < 0 \text{ ms} \\ 100 & 0 \le t \le 4\text{ms} \\ -100 & 4\text{ms} \le t \le 8\text{ms} \\ 0 & t \ge 8\text{ms} \end{cases}$$

Case 1: $t < 0$ source Voltage = 0 V $\therefore$ Voltage across the capacitor is also 0.

@ $t < 0$ $V_o(t) = 0 = V_0$

Case 2: $0 \le t \le 4\text{ms}$

KVL e: $-V_s + RC\dfrac{dv}{dt} + v_o = 0$

$$RC\frac{dv_o}{dt} + v_o = V_s$$

$$RC(Dv_o) + v_o = V_s$$

$$v_o(DRC + 1) = V_s \quad (1)$$

Here $D = \dfrac{dv_o}{dt}$

Solution to equation (1) is

$$V_o(t) = CF + PI \quad (2)$$

→ contd.

To get CF: $V_s = 0$ From eq 1

$$V_0(DRC+1) = 0$$

$$D = -\frac{1}{RC}$$

$CF = Ke^{-t/RC}$

calculate

$\star$ $\&$ $PI = V_s$, $Ke^{-t/RC}$ for CF

$$V_0(t) = Ke^{-t/RC} + V_s \quad (3)$$

at $t=0$ $V_0 = 0$

$$0 = Ke^0 + V_s$$

$$V_s = -K \Rightarrow K = -V_s$$

case 2 sub $-V_s$ for K in eq (3)

$$V_0(t) = -V_s e^{-t/RC} + V_s$$

$$V_0(t) = V_s - V_s e^{-t/RC} \quad (4)$$

$$\tau = RC = 50 \times 10^3 (25 \times 10^{-9}) = .00125 = \frac{1}{800} s$$

$$\boxed{V_0(t) = 100 - 100e^{-800t}}$$

case 3

$$V_0(4ms) = 100(1 - e^{-800(4 \times 10^{-3})}) \quad \{4ms \le t \le 8ms\}$$

$$\boxed{V_0(4ms) = 95.92V}$$

$$V_0(t) = V_0\infty + (V_0(4ms) - V_0\infty)e^{-800(4 \times 10^{-3})}$$

$V_0(\infty) = -100V$

→ contd

$$V_0(t) = -100 + (95.92 + 100)e^{-800t(4\times10^{-3})}$$

$$\boxed{V_0(t) = -100 + 195.92e^{-800t + 3.2}}$$

Case 4: For $t \geq 8ms$

$$V_0(t) = V_0(\infty) + (V_0(8ms) - V_0(\infty))e^{-800(t - 8\times10^{-3})}$$

$$V_0(8ms) = -100 + 195.92e^{-800(8\times10^{-3}) + 3.2}$$

$$\boxed{V_0(8ms) = -92.14\ V}$$

$V_0(\infty) = 0$ $t = \infty$, capacitor is open

$$V_0(t) = 0 + 92.14(e(\frac{\infty - 8ms}{RC}))$$

$$V_0(t) = -92.14\ V$$

$$\begin{cases} 0V & t \leq 0ms \\ 100 - 100e^{-800t} & 0 \leq t \leq 4ms \\ -100 + 195.92e^{-800t+3.2} & 4ms \leq t \leq 8ms \\ -92.14 & t \geq 8ms \end{cases}$$