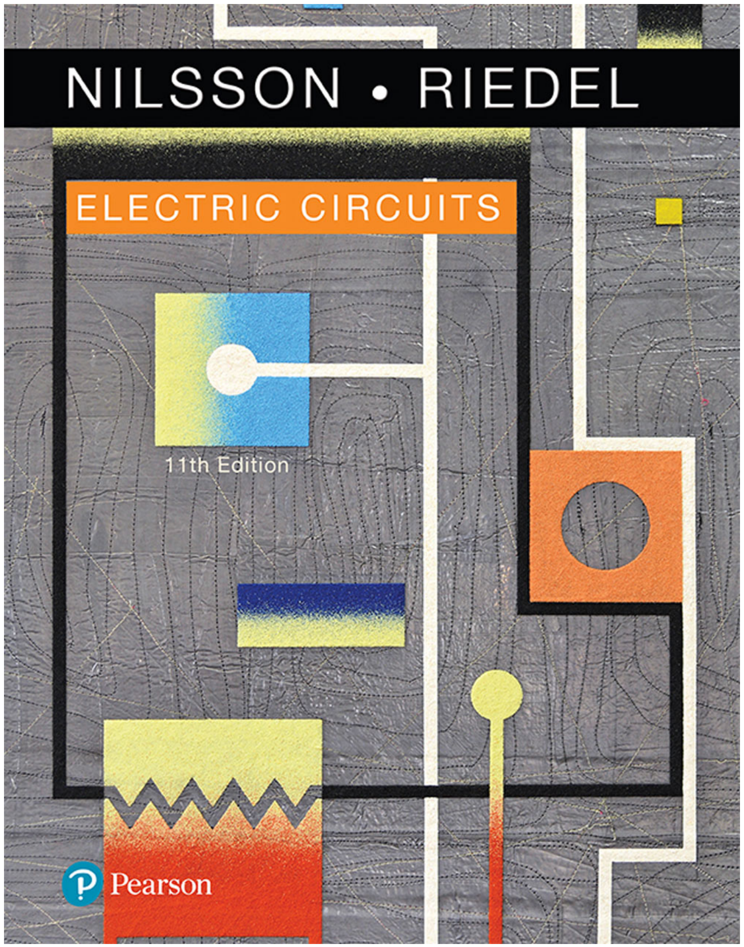
Electric Circuits

Eleventh Edition



Chapter 3

Simple Resistive Circuits

Learning Objectives

- Resistors in Series
- Resistors in Parallel
- The Voltage/Current--Divider Circuit
- Voltage/Current Division
- Measuring Voltage and Current
- Measuring Resistance--The Wheatstone Bridge
- Delta–to–Wye (Pi–to–Tee) Equivalent Circuit

Practical Perspective - Resistive Touch Screens

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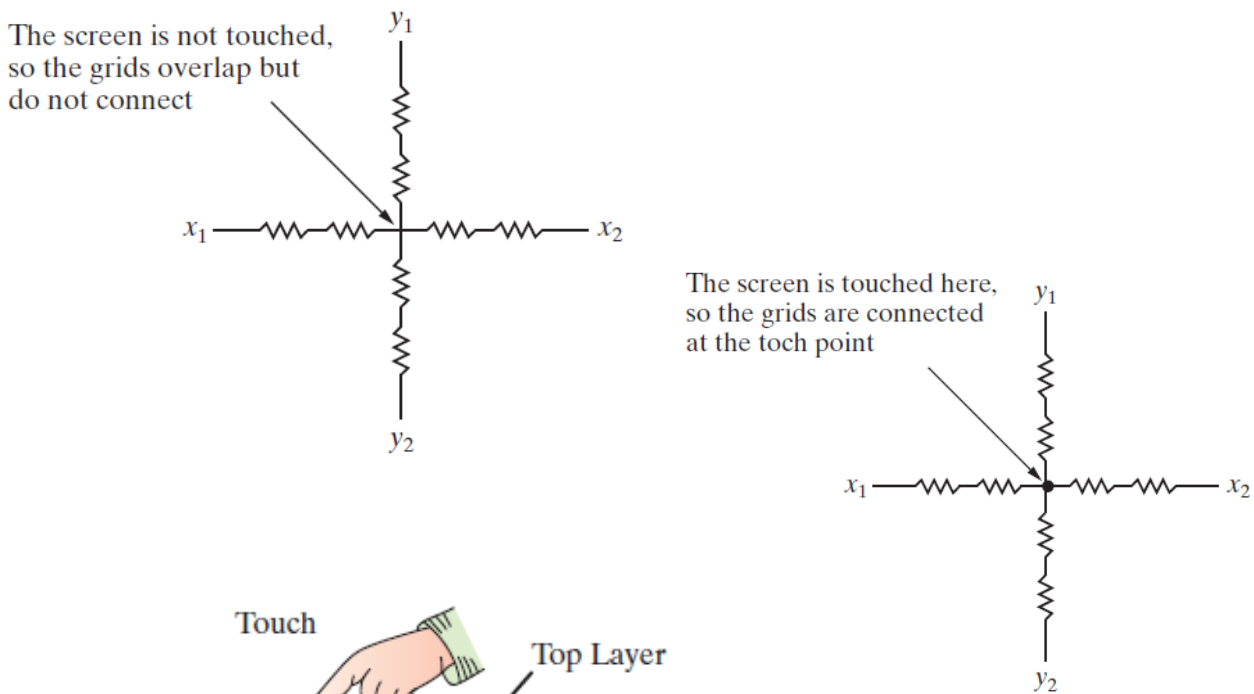
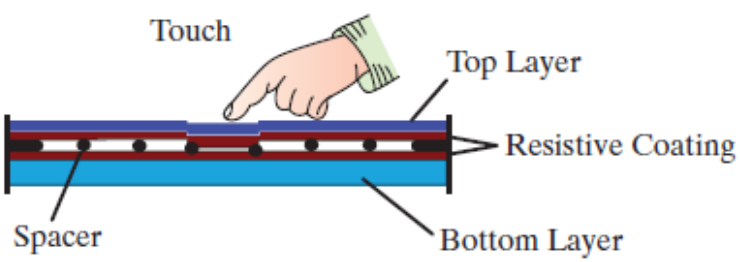
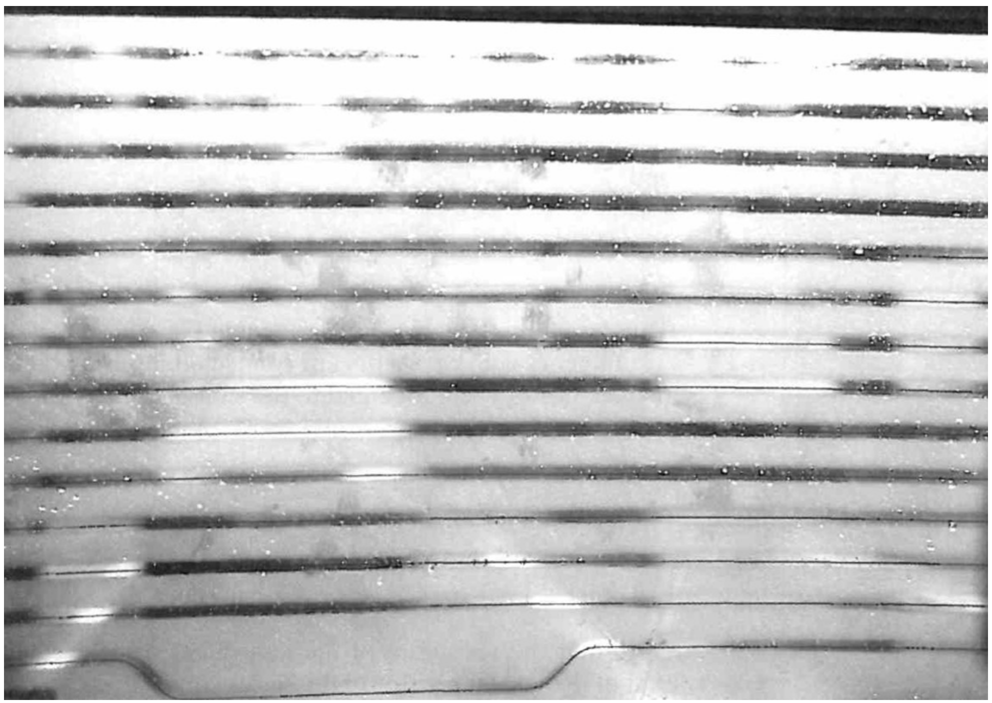
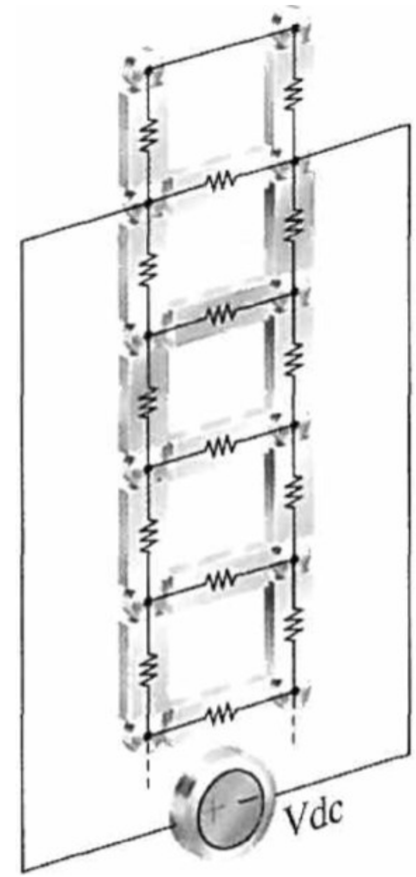




A Rear Window Defroster



A Rear Window Defroster



A resisEve circuit

3.1 Resistors in Series

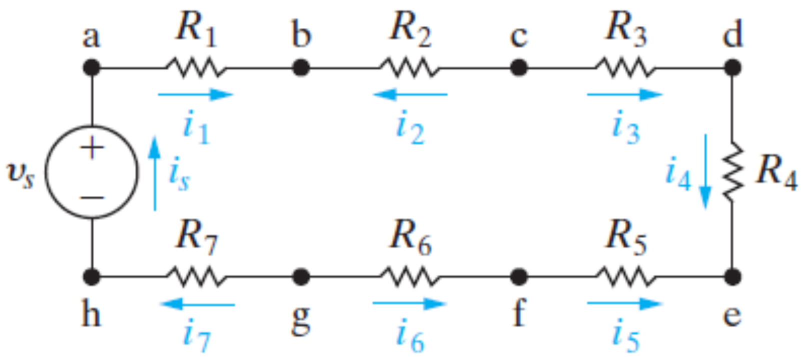


Figure 3.1: Resistors connected in series.

$$\boxed{i_s = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7}$$

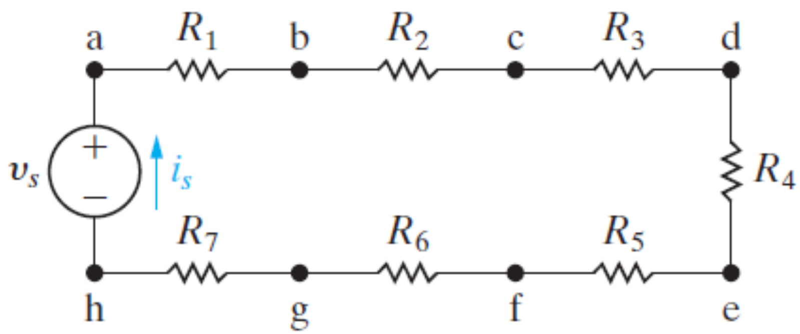


Figure 3.2: Series resistors with a single unknown current i_s.

$$\boxed{v_s = i_s(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7)}$$

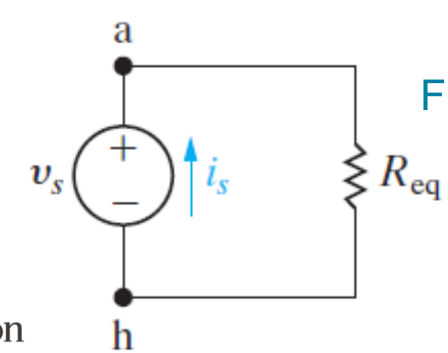


Figure 3.3: A simplified version of the circuit shown in Fig. 3.2.

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7}$$

$$\boxed{v_s = i_s R_{eq}}$$

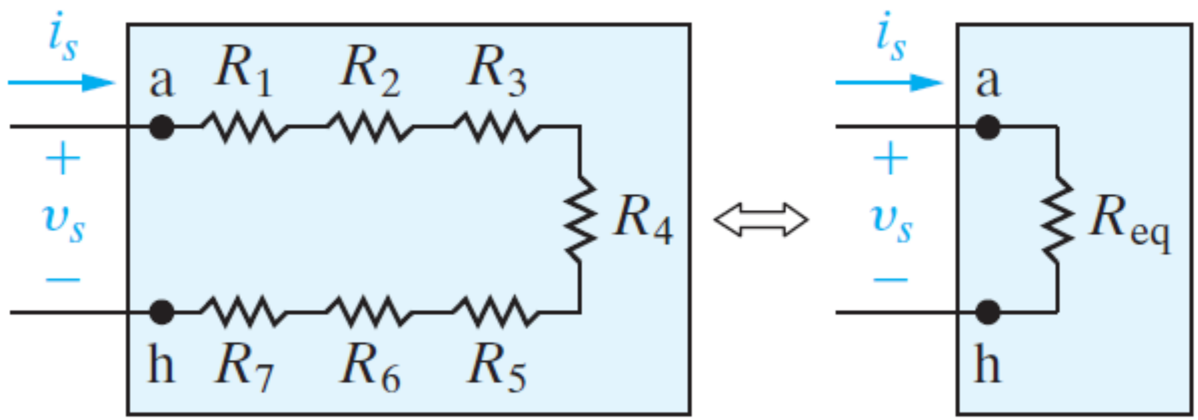


Figure 3.4: The black box equivalent of the circuit shown in Fig. 3.2.

In general, if k resistors are connected in series, the equivalent single resistor has a resistance equal to the sum of the k resistances:

$$\boxed{R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k}$$

3.2 Resistors in Parallel

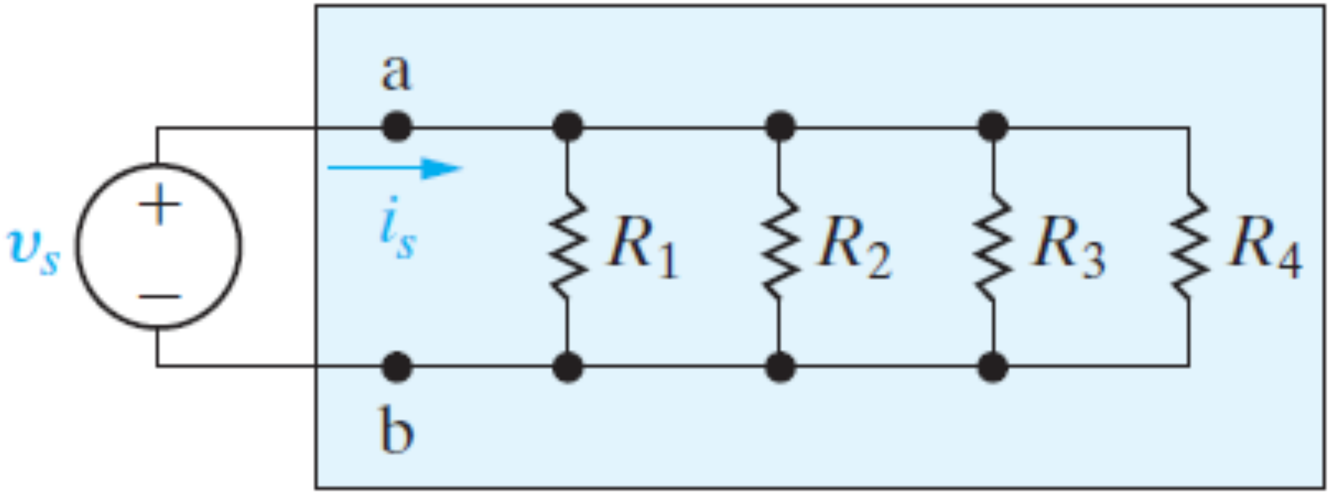


Figure 3.5: Resistors in parallel.

$$\boxed{i_s = i_1 + i_2 + i_3 + i_4}$$

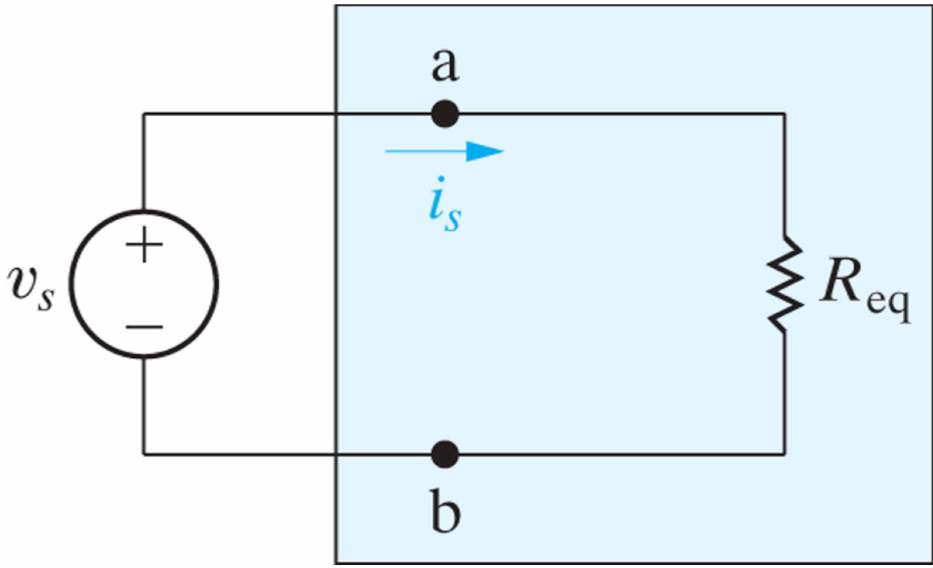
$$\boxed{i_1 R_1 = i_2 R_2 = i_3 R_3 = i_4 R_4 = v_s}$$

$$i_1 = \frac{v_s}{R_1} \qquad i_2 = \frac{v_s}{R_2} \qquad i_3 = \frac{v_s}{R_3} \qquad i_4 = \frac{v_s}{R_4}$$

$$i_s = v_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$\frac{i_s}{v_s} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Figure 3.7: Replacing the four parallel resistors shown in Fig. 3.5 with a single equivalent resistor.



$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \cdots + G_k$$

Conductance

Typical Cases

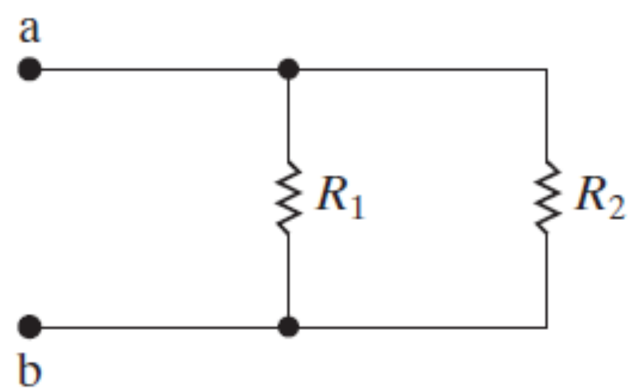


Figure 3.8: Two resistors connected in parallel.

- If k = 2, we have

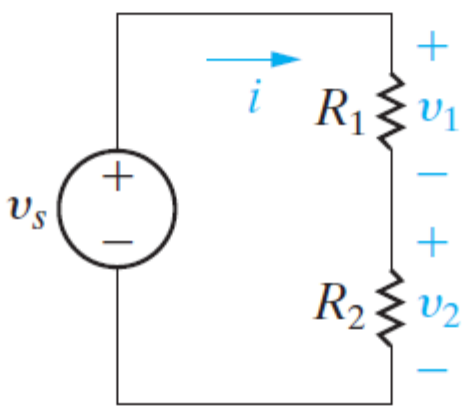
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- If $R_1 = R_2 = \ldots\ldots = R_k = R$, we have

$$1/R_{eq} = k/R \Rightarrow R_{eq} = R/k$$

3.3 Voltage-Divider & Current-Divider Circuits

The Voltage-Divider Circuit



(b)

$$i = \frac{v_s}{R_1 + R_2}$$

$$v_1 = iR_1 = v_s \frac{R_1}{R_1 + R_2}$$

$$v_2 = iR_2 = v_s \frac{R_2}{R_1 + R_2}$$

Figure 3.14: (b) the voltage-divider circuit with current i indicated.

The Current-Divider Circuit

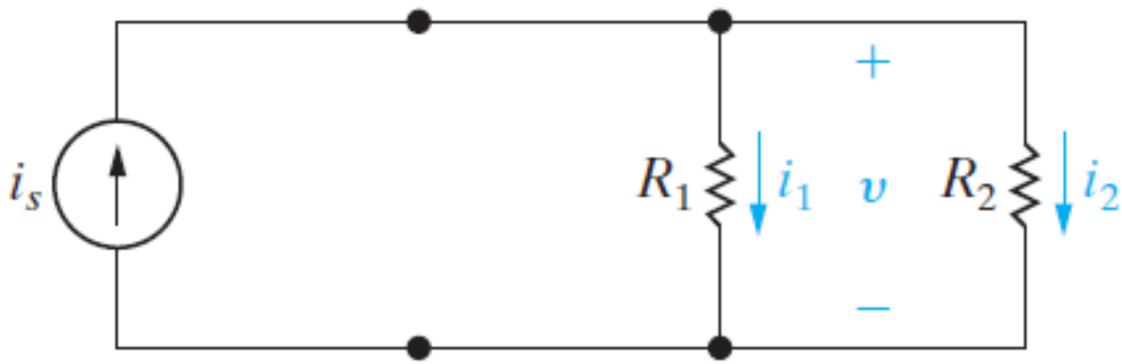


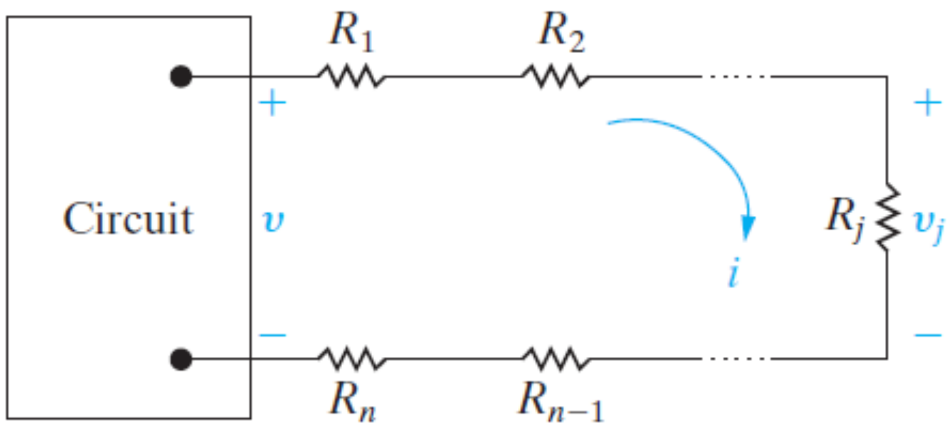
Figure 3.19: The current-divider circuit.

$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s$$

$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$

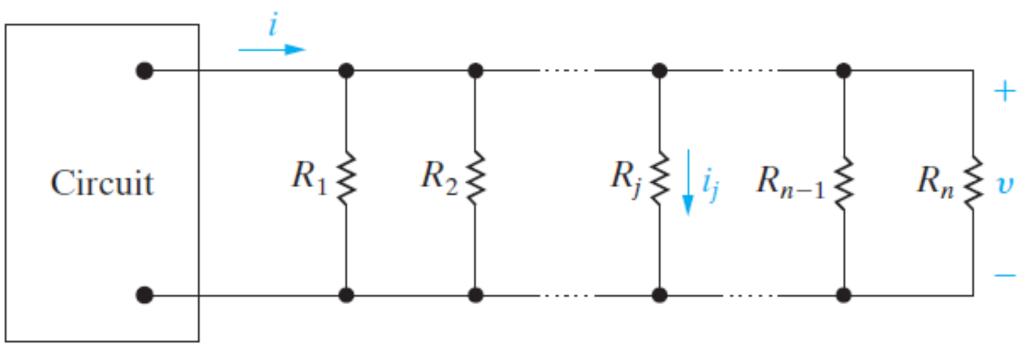
3.4 Voltage/Current Division



$$i = \frac{v}{R_1 + R_2 + \cdots + R_n} = \frac{v}{R_{eq}}$$

$$\boxed{v_j = iR_j = \frac{R_j}{R_{eq}}v}$$

Figure 3.20: Circuit used to illustrate voltage division.



$$v = i(R_1 \| R_2 \| \ldots \| R_n) = iR_{eq}$$

$$\boxed{i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j}i}$$

Figure 3.21: Circuit used to illustrate current division.

3.5 Measuring Voltage and Current

An **ammeter** is an instrument designed to measure current.
A **voltmeter** is an instrument designed to measure voltage.

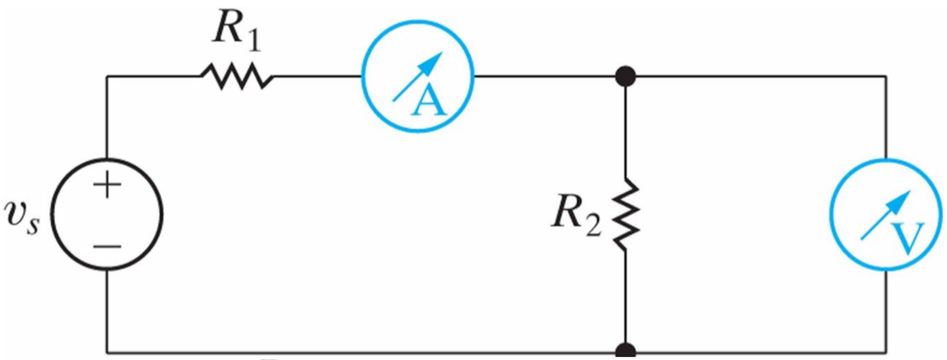


Figure 3.23: An ammeter connected to measure the current in R_1, and a voltmeter connected to measure the voltage across R_2.

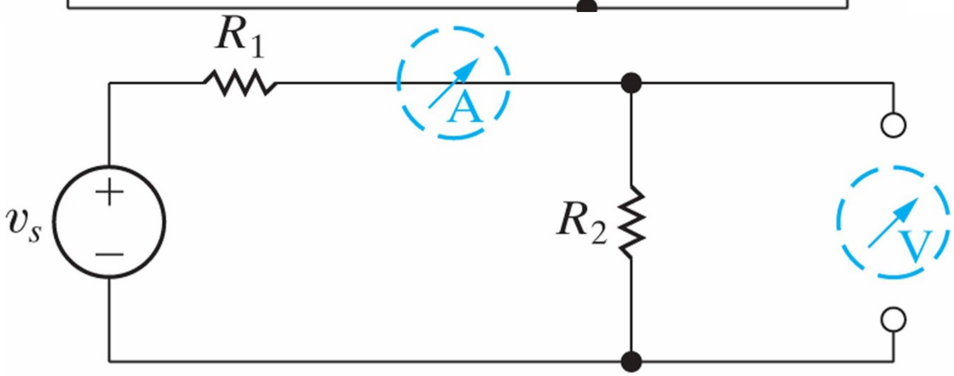


Figure 3.24: A short-circuit model for the ideal ammeter, and an open-circuit model for the ideal voltmeter.

- An **ideal ammeter** has an equivalent resistance of 0 Ω and functions as a short circuit in series with the element whose current is being measured.
- An **ideal voltmeter** has an infinite equivalent resistance and thus functions as an open circuit in parallel with the element whose voltage is being measured.

Digital and Analog Meters

d'Arsonval Meter Movement

d'Arsonval meter movement consists of a movable coil placed in the field of a permanent magnet. When current flows in the coil, it creates a torque on the coil, causing it to rotate and move a pointer across a calibrated scale. *By design, the deflection of the pointer is directly proportional to the current in the movable coil.* The coil is characterized by both a voltage rating and a current rating.

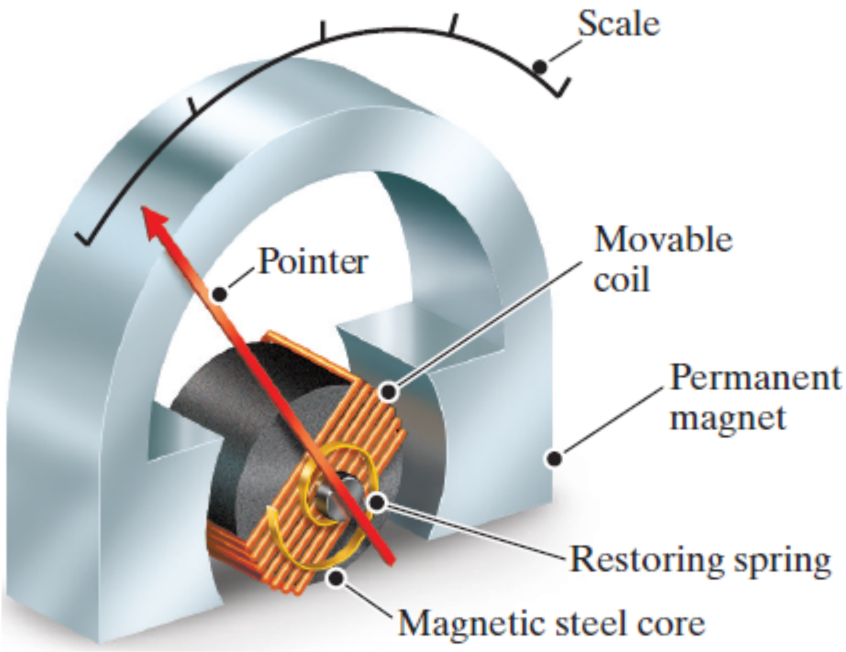


Figure 3.25: A schematic diagram of a d'Arsonval meter movement.

For example, one commercially available meter movement is rated at 50 mV and 1 mA. This means that when the coil is carrying 1 mA, the voltage drop across the coil is 50 mV and the pointer is deflected to its full-scale position.

DC Ammeter/Voltage Circuit

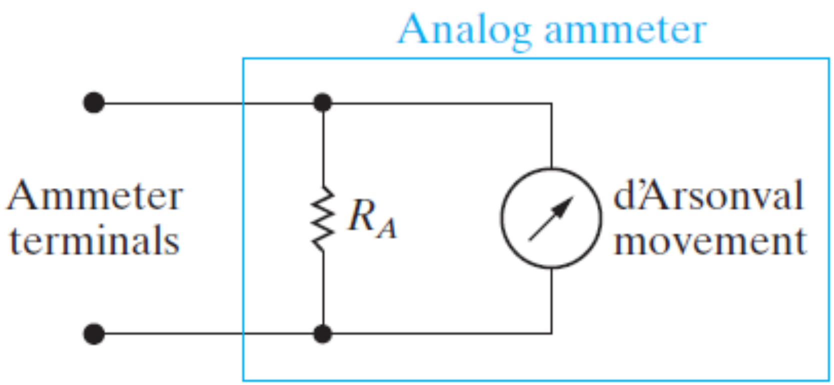


Figure 3.26: An analog ammeter circuit.

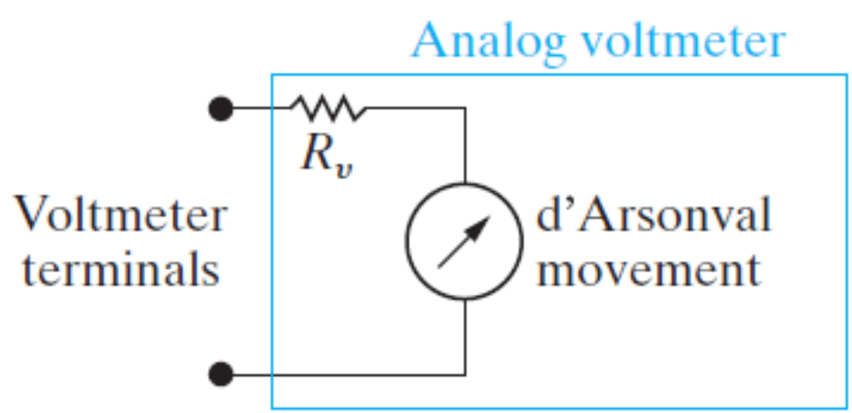


Figure 3.27: An analog voltmeter circuit.

Measuring Resistance: The Wheatstone Bridge

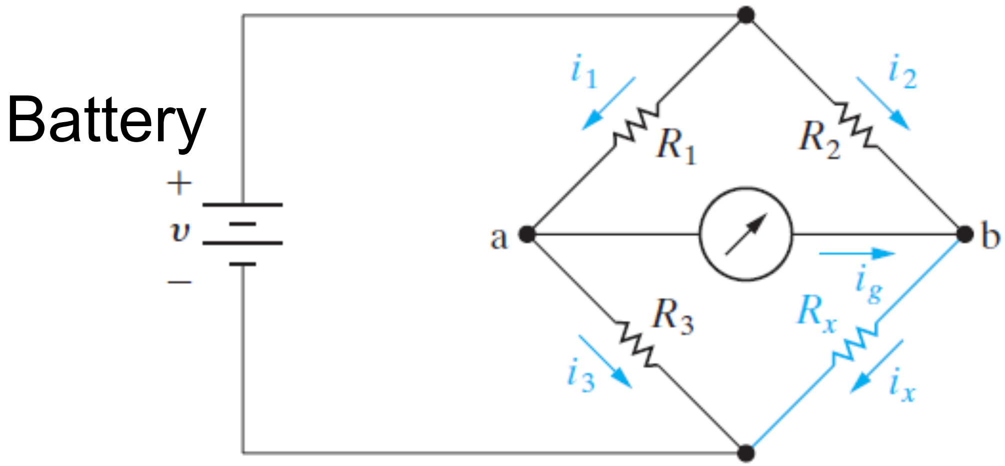


Figure 3.29: A balanced Wheatstone bridge (i_g = 0).

$$i_g = 0$$

$$i_1 = i_3 \qquad i_2 = i_x$$

$$i_1 R_1 = i_2 R_2 \; i_3 R_3 = i_x R_x$$

$$R_1 / R_2 = i_2 / i_1$$

$$R_3 / R_x = i_x / i_3 = i_2 / i$$

$$\boxed{R_x = R_2 R_3 / R_1}$$

In a commercial Wheatstone bridge, R_1 and R_2 consist of decimal values of resistances that can be switched into the bridge circuit.

Pearson

Delta–to–Wye (Pi–to–Tee) Equivalent Circuits

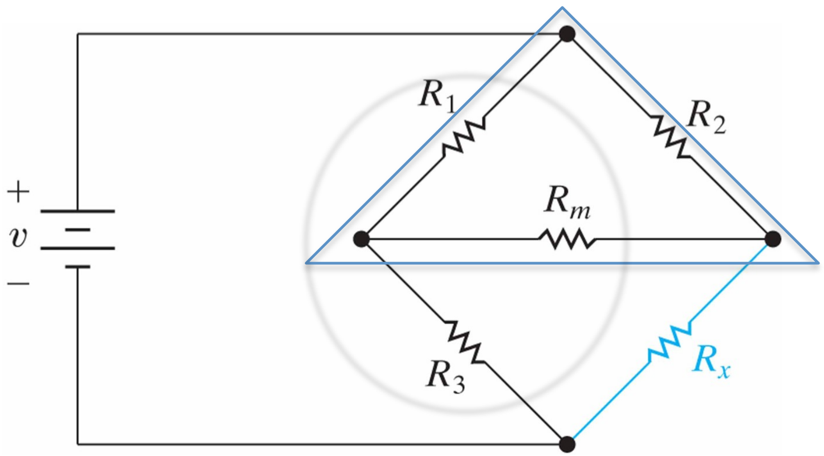


Figure 3.31: A resistive network generated by a Wheatstone bridge circuit.

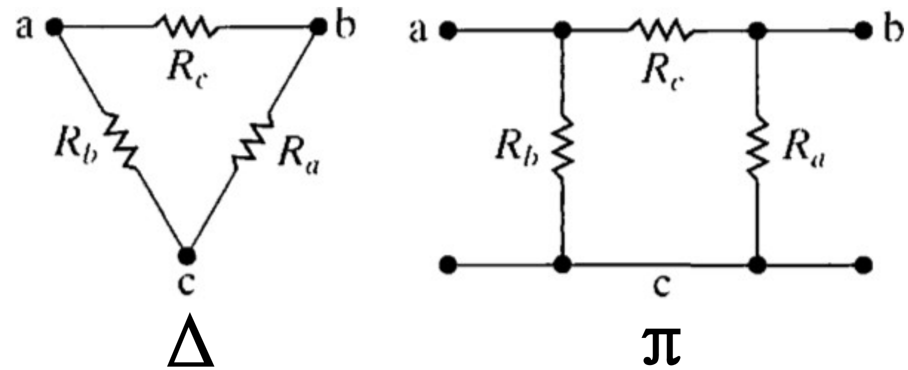


Figure 3.32: A Δ configuration viewed as a π configuration.

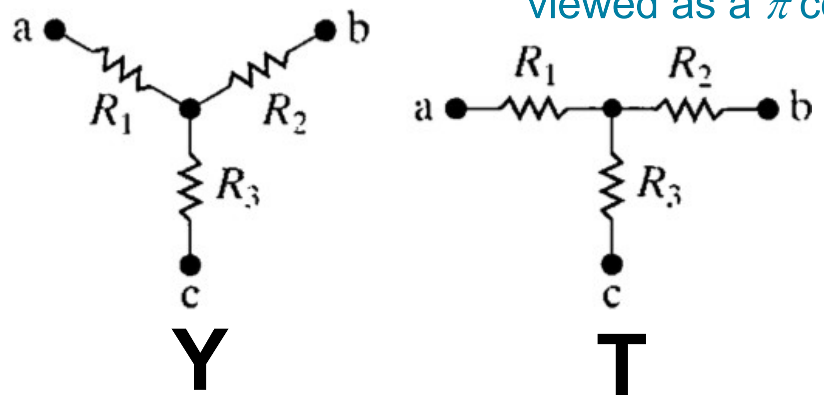


Figure 3.33: A Y structure viewed as a T structure.

The Δ–to–Y Transformation

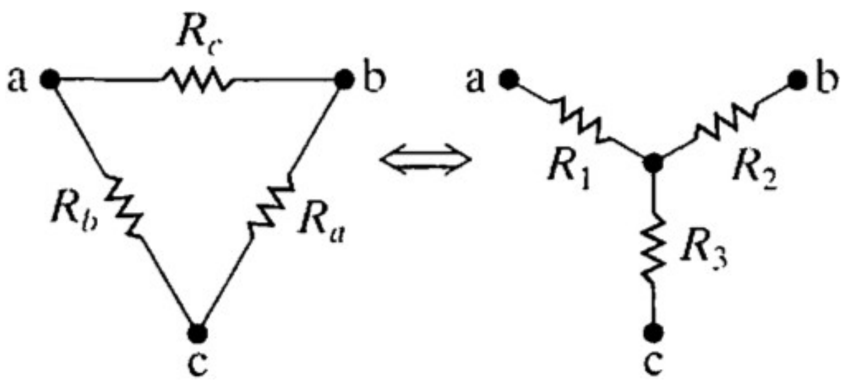


Figure 3.34: The Δ-to-Y transformation.

$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

$$R_{bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3$$

$$R_1 = (R_{ab}+R_{ca}-R_{bc}) / 2 = R_bR_c / (R_a+R_b+R_c)$$

$$R_2 = (R_{ab}+R_{bc}-R_{ca}) / 2 = R_cR_a / (R_a+R_b+R_c)$$

$$R_3 = (R_{bc}+R_{ca}-R_{ab}) / 2 = R_aR_b / (R_a+R_b+R_c)$$

How about Y–to–Δ?

The Y-to-Δ Transformation

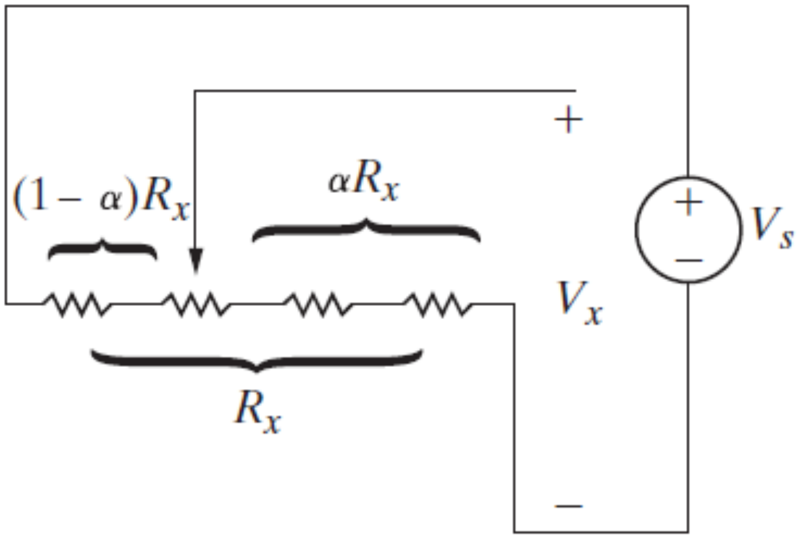
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

Practical Perspective - Resistive Touch Screens

Figure 3.39: The resistive touch screen grid in the *x*-direction.



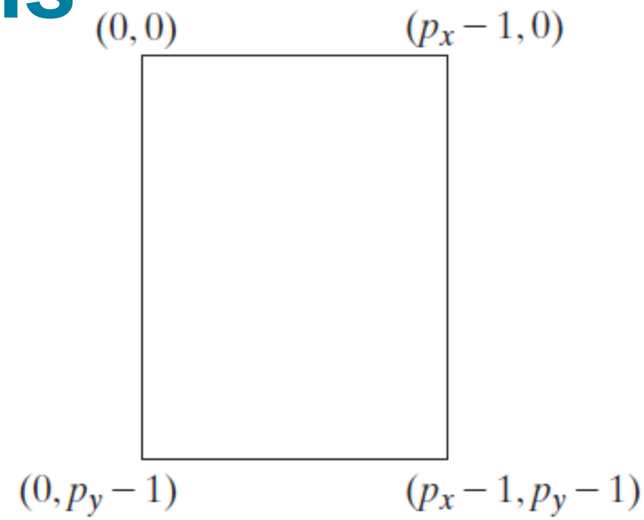


Figure 3.40: The pixel coordinates of a screen with p_x pixels in the *x*-direction and p_y pixels in the *y*-direction.

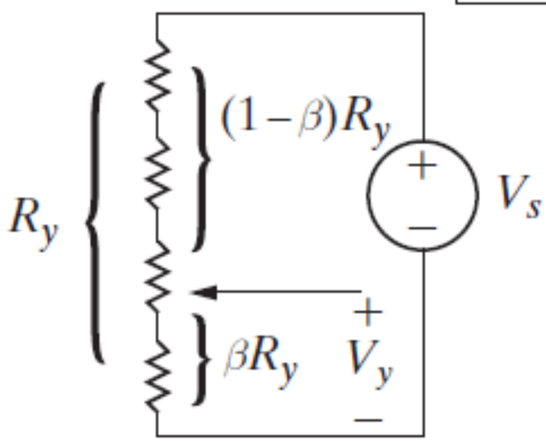


Figure 3.41: The resistive touch screen grid in the *y*-direction.