

1. 1.7 The current entering the upper terminal of [Fig. 1.5](#) is

$$i = 24 \cos 4000t \text{ A}$$

Assume the charge at the upper terminal is zero at the instant the current is passing through its maximum value. Find the expression for  $q(t)$ .

$$\begin{aligned} i &= \frac{dq}{dt} \Rightarrow 24 \cos(4000t) \text{ A} = \frac{dq}{dt} \\ &\Rightarrow dq = 24 \cos(4000t) dt \end{aligned}$$

$$q(t) = 24 \int_0^t \cos(4000x) dx$$

$$q(t) = 24 \left[ \frac{\sin(4000x)}{4000} \right]_0^t$$

$$q(t) = \frac{24}{4000} [\sin(4000t) - \sin(4000 \cdot 0)]$$

$$q(t) = \frac{6}{1000} \sin(4000t) \text{ C}$$

2. 1.8 How much energy is imparted to an electron as it flows through a 6 V battery from the positive to the negative terminal? Express your answer in attojoules.

$$V = \frac{w}{q} \Rightarrow w = Vq, \quad V = 6 \text{ V} \quad q = 1.6022 \times 10^{-19} \text{ C}$$

$$w = (6)(1.6022 \times 10^{-19})$$

$$w = 9.6132 \times 10^{-19} \text{ J}$$

$$w = 0.961 \text{ aJ}$$

1.9 In electronic circuits it is not unusual to encounter currents in the microampere range. Assume a  $35 \mu A$  current, due to the flow of electrons. What is the average number of electrons per second that flow past a fixed reference cross section that is perpendicular to the direction of flow?

$$q = 35 \mu A = 35 \times 10^{-6} A$$

$$\text{Avg # of electrons} = \frac{35 \times 10^{-6} A}{1.6022 \times 10^{-19} C}$$

$$\boxed{\text{Avg # of electrons} = 2.184 \times 10^{14} \text{ electrons/sec}}$$

1.11 The current at the terminals of the element in Fig. 1.5 is

$$\begin{aligned} i &= 0, & t < 0; \\ i &= 40te^{-500t} A, & t \geq 0. \end{aligned}$$

- a) Find the expression for the charge accumulating at the upper terminal.
- b) Find the charge that has accumulated at  $t = 1 \text{ ms}$ .

$$i = \frac{dq}{dt} \Rightarrow q(t) = \int i dt$$

$$q(t) = \int_0^t 40xe^{-500x} dx$$

$$q(t) = 40 \int_0^t x e^{-500x} dx$$

$$q(t) = -\frac{40}{500} \left[ xe^{-500x} + \frac{e^{-500x}}{500} \right]_0^t$$

$$a) \boxed{q(t) = -0.08 \left[ te^{-500t} + \frac{e^{-500t}}{500} - \frac{1}{500} \right] C}$$

Integration

$$\begin{aligned} u &= x & dv &= e^{-500x} \\ du &= 1 & v &= \frac{e^{-500x}}{-500} \end{aligned}$$

$$\int u dv = x \left( \frac{e^{-500x}}{-500} \right) - \int \frac{e^{-500x}}{-500} dx$$

$$\int u dv = \frac{e^{-500x}}{-500} x - \frac{e^{-500x}}{25000}$$

$$= \frac{1}{-500} \left( e^{-500x} x + \frac{e^{-500x}}{500} \right)$$

$$t = 1 \text{ ms}$$

$$q(10^{-3} \text{ sec}) = -0.08 \left[ (10^{-3}) e^{-500(10^{-3})} + \frac{e^{-500(10^{-3})}}{500} - \frac{1}{500} \right]$$

$$b) \boxed{q(10^{-3} \text{ sec}) = 1.44 \times 10^{-5} C}$$

2. 1.13 Two electric circuits, represented by boxes A and B, are connected

as shown in [Fig. P1.13](#). The reference direction for the current  $i$  in the interconnection and the reference polarity for the voltage  $v$  across the interconnection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or vice versa.

1. a)  $i=6 \text{ A}, v=30 \text{ V}$
2. b)  $i=-8 \text{ A}, v=-20 \text{ V}$
3. c)  $i=4 \text{ A}, v=-60 \text{ V}$
4. d)  $i=-9 \text{ A}, v=40 \text{ V}$

$$P = iV$$

a)  $P = (6 \text{ A})(30 \text{ V})$

$P = 180 \text{ Watts}$ , since the power is positive  
the power is flowing from A to B

b)  $P = (-8 \text{ A})(-20 \text{ V})$

$P = 160 \text{ Watts}$ , since the power is positive  
it is flowing from A to B

c)  $P = (4 \text{ A})(-60 \text{ V})$

$P = -240 \text{ Watts}$ , since the power is negative  
it is flowing from B to A

d)  $P = (-9 \text{ A})(40 \text{ V})$

$P = -360 \text{ Watts}$ , since the power is negative  
it is flowing from B to A

1.18 PSPICE MULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for  $t < 0$ . For  $t \geq 0$  they are

$$v = 75 - 75e^{-1000t} \text{ V},$$

$$i = 50e^{-1000t} \text{ mA.}$$

- a) Find the maximum value of the power delivered to the circuit.  
 b) Find the total energy delivered to the element.

$$P = iV = (50e^{-1000t} \times 10^{-3})(75 - 75e^{-1000t})$$

$$P = 3.75e^{-1000t} - 3.75e^{-2000t} \text{ W}$$

$$\frac{dP}{dt} = 7500e^{-2000t} - 3750e^{-1000t}$$

$$2e^{-2000t} - e^{-1000t} = 0$$

$$2 = e^{1000t}$$

$$t = \frac{\ln 2}{1000} = 6.93 \times 10^{-4} \text{ sec}$$

$$P_{\max} = 3.75 e^{-1000(6.93 \times 10^{-4})} - 3.75 e^{-2000(6.93 \times 10^{-4})} \text{ Watts}$$

a)  $\boxed{P_{\max} = 0.937 \text{ Watts}}$

$$W = \int_0^\infty P dt$$

$$W = \int_0^\infty 3.75e^{-1000t} - 3.75e^{-2000t} dt$$

$$= \left[ \frac{3.75e^{-1000t}}{-1000} + \frac{3.75e^{-2000t}}{2000} \right]_0^\infty$$

b)  $\boxed{W = -\frac{3.75}{1000} + \frac{3.75}{2000} = -1.875 \times 10^{-3} \text{ J}}$

1.21 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for  $v < J$ . For  $v \geq J$  they are

$$v = 50e^{-1600t} - 50e^{-400t} \text{ V},$$

$$i = 5e^{-1600t} - 5e^{-400t} \text{ mA.}$$

- a) Find the power at  $v = 625 \mu\text{s}$ .
- b) How much energy is delivered to the circuit element between 0 and  $625 \mu\text{s}$ ?
- c) Find the total energy delivered to the element.

$$\begin{aligned} P_{>i,v} &= (50e^{-1600t} - 50e^{-400t}) [(5e^{-1600t} - 5e^{-400t}) \times 10^{-3}] \\ &= 0.25e^{-3200t} - 0.25e^{-2000t} - 0.25e^{-2000t} + 0.25e^{-800t} \\ &= 0.25(e^{-3200t} - 2e^{-2000t} + e^{-800t}) \\ @ t = 625 \mu\text{s} \quad P &= 0.25(e^{-3200(625 \times 10^{-6})} - 2e^{-2000(625 \times 10^{-6})} \\ &\quad + e^{-800(625 \times 10^{-6})}) \end{aligned}$$

a)  $P = 0.0422 \text{ Watts}$

$$\begin{aligned} w &= \int_0^{625 \mu\text{s}} P dt \\ w &= 0.25 \int_0^{625 \mu\text{s}} e^{-3200t} - 2e^{-2000t} + e^{-800t} dt \\ w &= 0.25 \left[ \frac{e^{-3200t}}{-3200} - \frac{e^{-2000t}}{-2000} + \frac{e^{-800t}}{-800} \right]_0^{625 \mu\text{s}} \\ w &= 0.25 [(-0.00091395) - (-0.0005625)] \end{aligned}$$

b)  $w = 1.21 \times 10^{-5} \text{ J}$

$$\begin{aligned} w &= \int_0^{\infty} P dt = 0.25 \left[ \frac{e^{-3200t}}{-3200} - \frac{e^{-2000t}}{-2000} + \frac{e^{-800t}}{-800} \right]_0^{\infty} \\ &= 0.25 [0 - (-0.0005625)] \end{aligned}$$

$w_{\text{total}} = 1.41 \times 10^{-4} \text{ J}$

1.23 PSPICE MULTISIM The voltage and current at the terminals of the circuit element in **Fig. 1.5** are zero for  $t < 0$  and  $t > 40$  s. In the interval between 0 and 40 s the expressions are

$$\begin{aligned} v &= t(1 - 0.025t) \text{ V}, \quad 0 < t < 40 \text{ s}; \\ i &= 4 - 0.2t \text{ A}, \quad 0 < t < 40 \text{ s}. \end{aligned}$$

- a) At what instant of time is the power being delivered to the circuit element maximum?
- b) What is the power at the time found in part (a)?
- c) At what instant of time is the power being extracted from the circuit element maximum?
- d) What is the power at the time found in part (c)?
- e) Calculate the net energy delivered to the circuit at 0, 10, 20, 30 and 40 s.

$$\begin{aligned} P = iV &= (4 - 0.2t)(t(1 - 0.025t)) = 4t - 0.1t^2 - 0.2t^2 + 0.005t^3 \\ &= 0.005t^3 - 0.3t^2 + 4t \text{ W} \end{aligned}$$

$$\frac{dP}{dt} = 0.015t^2 - 0.6t + 4$$

$$0 = 0.015(t^2 - 40t + 266.67)$$

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 2(1)(266.67)}}{2}$$

$$x_1 = 31.55 \quad x_2 = 8.45$$

$$P(31.55) = -15.40 \text{ W} \quad P(8.45) = 15.37 \text{ W}$$

a)  $\boxed{P_{\max} = 15.37 \text{ W} \text{ @ } 8.45 \text{ seconds}}$

b)  $\boxed{\text{Power @ } t = 8.45 \text{ is } 15.37 \text{ W}}$

c)  $\boxed{\text{@ } t = 31.55 \text{ seconds}}$

d)  $\boxed{\text{@ } t = 31.55, P = -15.40 \text{ W}}$

$$\begin{aligned} W &= \int_0^t P dt = \int_0^t 0.005x^3 - 0.3x^2 + 4x dx \\ &= \left[ \frac{0.005x^4}{4} - \frac{0.3x^3}{3} + \frac{4x^2}{2} \right]_0^t = 0.00125t^4 - 0.1t^3 + 2t^2 \end{aligned}$$

e)  $\boxed{\begin{aligned} w(0) &= 0 \text{ J} & w(30) &= 112.5 \text{ J} \\ w(10) &= 112.5 \text{ J} & w(40) &= 0 \text{ J} \\ w(20) &= 200 \text{ J} \end{aligned}}$

1.25 PSPICE MULTISIM The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for  $t < 0$ . For  $t \geq 0$  they are

$$v = 100e^{-50t} \sin 150t \text{ V}, \\ i = 20e^{-50t} \sin 150t \text{ A.}$$

- a) Find the power absorbed by the element at  $t = 20 \text{ ms}$ .  
 b) Find the total energy absorbed by the element.

$$P = i v = (20e^{-50t} \sin(50t)) (100e^{-50t} \sin(150t)) = 2000 e^{-100t} \sin^2(150t)$$

$$P(20\text{ms}) = 2000 e^{-100(0.02)} \sin^2(150(0.02))$$

a)  $\boxed{P(20\text{ms}) = 5.39 \text{ W}}$

$$W = \int_0^\infty P dt$$

$$W = 2000 \int_0^\infty e^{-100t} \sin^2(150t) dt$$

$$W = 2000 \int_0^\infty e^{-100t} \left[ \frac{1 - \cos(300t)}{2} \right] dt$$

$$W = 1000 \int_0^\infty e^{-100t} - e^{-100t} \cos(300t) dt$$

$$W = 1000 \left[ \frac{e^{-100t}}{-100} - \frac{e^{-100t} (3\sin(300t) - \cos(300t))}{1000} \right]_0^\infty$$

$$W = 1000 \left[ \frac{1}{100} \right] - 1000 \left[ 0 - \frac{1(0-1)}{1000} \right]$$

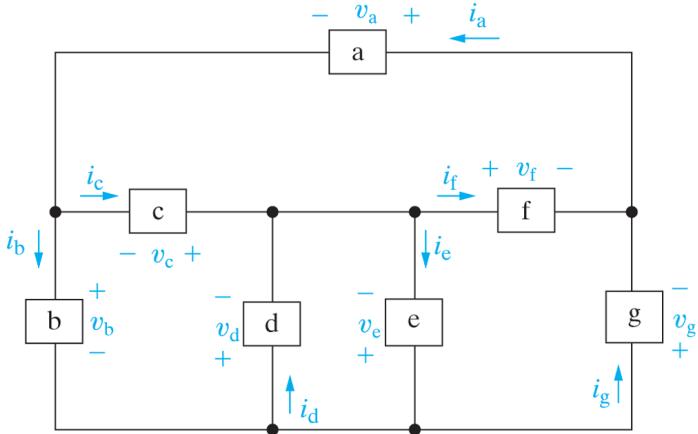
$$W = 10 - 1$$

b)  $\boxed{W = 9 \text{ J}}$

1.32 The current and power for each of the interconnected elements in Fig. P1.32 is measured. The values are listed in Table P1.32

Element	Power (mW)	Current (mA)
a	175	25
b	375	75
c	150	-50
d	-320	40
e	160	20
f	120	-30
g	-660	55

Figure P1.32



- a) Show that the interconnection satisfies the power check.
- b) Identify the elements that absorb power.
- c) Find the voltage for each of the elements in the interconnection, using the values of power and current and the voltage polarities shown in the figure.

a)  $P_{\text{total}} = 0 = 0.175 + 0.375 + 0.150 - 0.320 + 0.160 + 0.120 - 0.660$   
 $= 0$

Since the sum of the powers is zero  
 then the power check is satisfied

b)  $\{a, b, c, e, f\}$  are absorbing power  
 since their power is positive

c)  $P = VI \Rightarrow V = P/I$

$$V_a = \frac{175}{25} = 7V \quad V_e = \frac{-160}{20} = -8V$$

$$V_b = \frac{375}{75} = 5V \quad V_f = \frac{120}{30} = -4V$$

$$V_c = \frac{-150}{-50} = 3V \quad V_g = \frac{-660}{55} = 12V$$

$$V_d = \frac{-320}{40} = -8V$$