Elizabeth Ramkowsky 03.25.2020

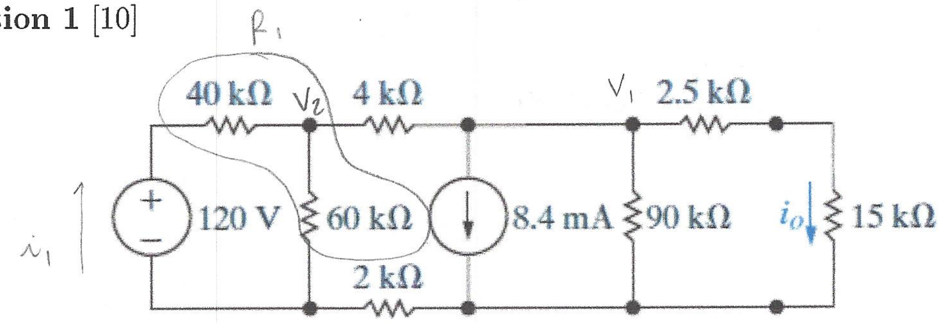
ENGR 2910-101: Circuit Analysis

Homework 9: 03/18/20

Instructor: Leo Silbert

Due: 03/25/20

Question 1 [10]

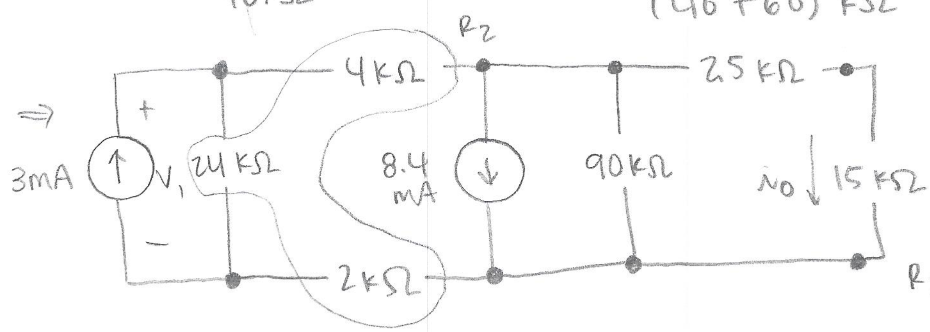


(i) Using several source transformations find the value of the current flowing through the 15 $k\Omega$ resistor. [Hint: start on the left side of the circuit and work your way right.]

(ii) Now that you know this current, work backwards through the original circuit and calculate the following: the voltage drop across the 90 $k\Omega$ and the current flowing through that branch; the current flowing through the 4 $k\Omega$ resistor, the voltage drop across the 60 $k\Omega$ resistor; and the current flowing in the left-hand part of the circuit.

i.

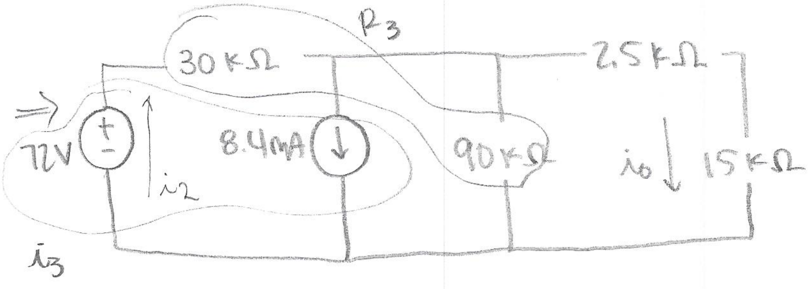
$i_1 = \frac{120V}{40k\Omega} = 3mA$; in parallel $R_1 = \frac{(40k\Omega)(60k\Omega)}{(40+60)k\Omega} = \frac{2400}{100} = 24\ k\Omega$



$V_1 = (3mA)(24k\Omega)$

$= 72$ Volts

in series $R_2 = (24+4+2)k\Omega = 30\ k\Omega$



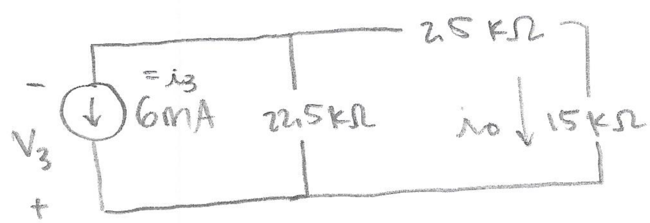
$i_2 = \frac{72V}{30k\Omega} = 2.4\ mA$

in parallel $R_3 = \frac{(30k\Omega)(90k\Omega)}{(30+90)k\Omega} = \frac{2700}{120} = 22.5\ k\Omega$

$i_3 = (8.4mA) - i_2$

$= (8.4 - 2.4)mA$

$= 6mA$



$V_3 = (6mA)(22.5k\Omega)$

$= 135$ volts

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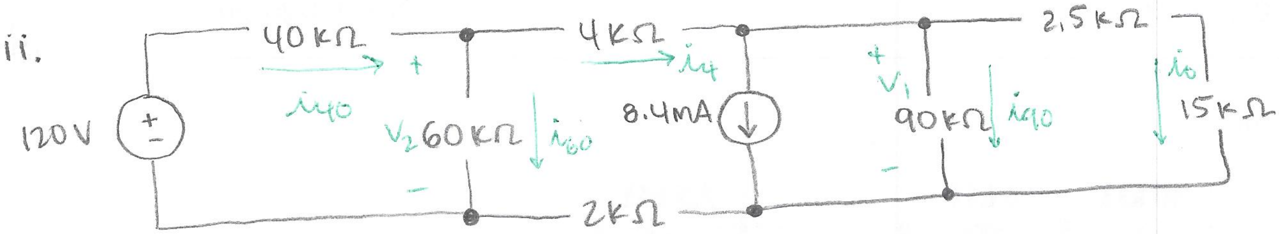
$$\Rightarrow i_o = \frac{\quad - V_3}{(22.5 + 2.5 + 15)\,k\Omega} = \frac{-135\,V}{40\,k\Omega} = \boxed{-3.375\,mA = i_o}$$

ii.



$$V_1 = (-3.375\,mA)(2.5 + 15)\,k\Omega$$
$$= -59.06 \text{ Volts}$$

$$\Rightarrow i_{90} = \frac{V_1}{90\,k\Omega} = \frac{-59.06\,V}{90\,k\Omega} = -0.656\,mA$$

$$i_4 = (8.4 + i_{90} - 3.375)\,mA$$
$$= 4.369\,mA$$

$$V_2 = i_4 (4+2)\,k\Omega + V_1$$
$$= (4.369\,mA)(6\,k\Omega) + (-59.06\,V)$$
$$= -32.846 \text{ Volts}$$

$$i_{60} = \frac{V_2}{60\,k\Omega} = \frac{-32.846\,V}{60\,k\Omega} = -0.547\,mA$$

$$i_{40} = i_{60} + i_4 = (-0.547 + 4.369)\,mA = 3.822\,mA$$

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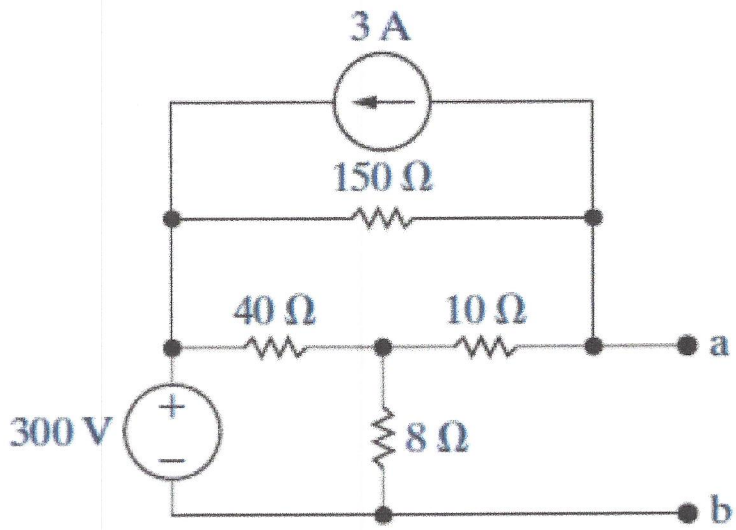
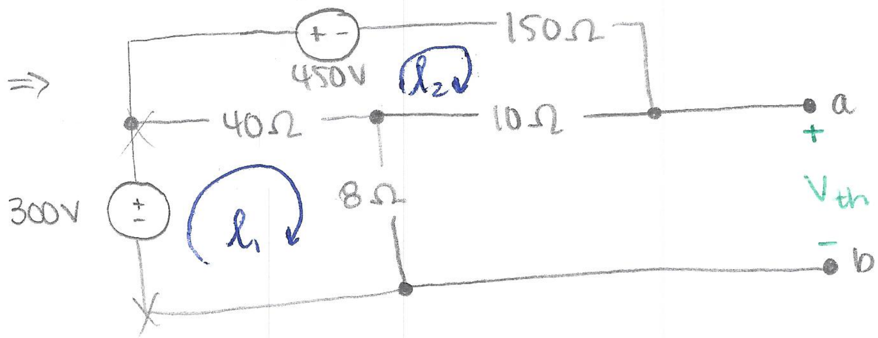
Question 2 [10]
Find the Thévenin equivalent for the following circuit. [Hint: start off by making a source transformation then apply the mesh-current method.]



$V_1 = (3A)(150\Omega) = 450\,V$

$\Rightarrow$



mesh-current i_1: $-300 + 40(i_1 - i_2) + 8(i_1) = 0$

$\Rightarrow 40i_1 - 40i_2 + 8i_1 = 300$

$\Rightarrow 48i_1 - 40i_2 = 300$ ... eq 1

mesh-current i_2: $450 + (150)(i_2) + (10)(i_2) + 40(i_2 - i_1) = 0$

$\Rightarrow 150i_2 + 10i_2 + 40i_2 - 40i_1 = -450$

$\Rightarrow -40i_1 + 200i_2 = -450$ ... eq 2

through solving matrix on calculator:

$$\begin{bmatrix} 48 & -40 & 300 \\ -40 & 200 & -450 \end{bmatrix} \Rightarrow \begin{matrix} i_1 = 5.25\,A \\ i_2 = -1.2\,A \end{matrix}$$

$\Rightarrow V_{TH} = 10i_2 + 8i_1$

$= 10(-1.2A) + 8(5.25A)$

$V_{TH} = 30$ Volts

$$R_{TH} = ((40 \| 8) + 10) \| 150$$

$$= \left(\left(\frac{40 \cdot 8}{48} \right) + 10 \right) \| 150$$

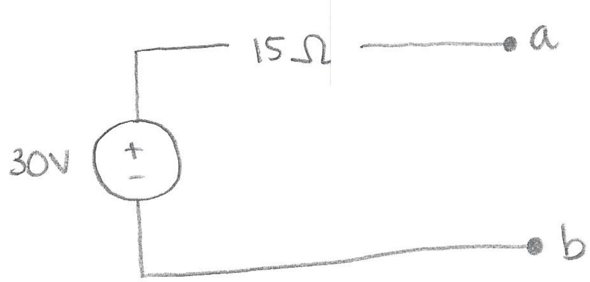
$$= (16.67) \| 150$$

$$= \frac{(16.67)(150)}{150 + 16.67}$$

$$= 15.002\ \Omega$$

$$\Rightarrow R_{TH} = 15\ \Omega$$

$\Rightarrow$ Thevenin Equivelent Circuit :



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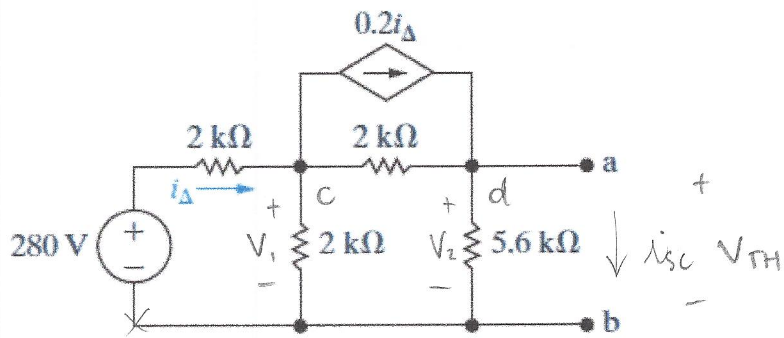
Question 3 [10]

Find the Norton equivalent for the following circuit. [Hint: apply the node-voltage and mesh-current methods.]



$i_\Delta = \dfrac{280 - V_1}{2000\,\Omega}$ $\Rightarrow V_2 = 0$ because a and b is short circuited

KVl node c: $\dfrac{V_1 - 280}{2000\,\Omega} + \dfrac{V_1}{2000} + \dfrac{V_1 - V_2}{2000} = -0.2 i_\Delta$

$\Rightarrow \dfrac{V_1}{2000} - \dfrac{280}{2000} + \dfrac{V_1}{2000} + \dfrac{V_1}{2000} - \dfrac{V_2}{2000} = -0.2 i_\Delta$

$\Rightarrow \dfrac{3V_1}{2000} - \dfrac{V_2}{2000} - \dfrac{280}{2000} = -\dfrac{400 i_\Delta}{2000}$

$\Rightarrow 3V_1 - V_2 - 280 = -400 i_\Delta$

$\Rightarrow 3V_1 - \emptyset - 280 = -400\left(\dfrac{280 - V_1}{2000}\right)$

$\Rightarrow 3V_1 - 280 = \dfrac{-112000}{2000} + \dfrac{400 V_1}{2000}$ $\Rightarrow 2.8V_1 = 224 \Rightarrow V_1 = 80$ Volts

$i_\Delta = \dfrac{280 - (80)}{2000} = \dfrac{200}{2000} = 0.1\text{ A}$

KVl node d: $0.2 i_\Delta + \dfrac{V_1 - V_2}{2000} + \dfrac{V_2}{5600} = i_{sc}$

$\Rightarrow i_{sc} = (0.2)(0.1) + \dfrac{80 - 0}{2000} + \dfrac{0}{5600}$

$= 0.06\text{A} = i_{sc}$

$i_\Delta = \dfrac{280 - V_1}{2000}$

KVl node c to find V_{TH} ∴ calculate norton equivelent resistance:

starting from work on previous page:

$3V_1 - V_2 - 280 = -400 i_\Delta$

$\Rightarrow 3V_1 - V_2 - 280 = -400\left(\dfrac{280 - V_1}{2000}\right)$

$\Rightarrow 2.8V_1 = 224 + V_2$

$\Rightarrow V_1 = \dfrac{224 + V_2}{2.8}$

KVl node d:

$\dfrac{V_2}{5600} + \dfrac{V_2 - V_1}{2000} = 0.2 i_\Delta$

$\Rightarrow \dfrac{V_2 \cdot 5}{5600 \cdot 5} + \dfrac{V_2 \cdot 14}{2000 \cdot 14} - \dfrac{V_1 \cdot 14}{2000 \cdot 14} = 0.2 i_\Delta \Rightarrow \dfrac{5V_2 + 14V_2}{28000} - \dfrac{14V_1}{28000} = 0.2 i_\Delta$

$\Rightarrow -14V_1 + 19V_2 = 5600 i_\Delta$

$\Rightarrow -14V_1 + 19V_2 = 5600\left(\dfrac{280 - V_1}{2000}\right) \Rightarrow -14V_1 + 19V_2 = \dfrac{1568000}{2000} - \dfrac{5600V_1}{2000}$

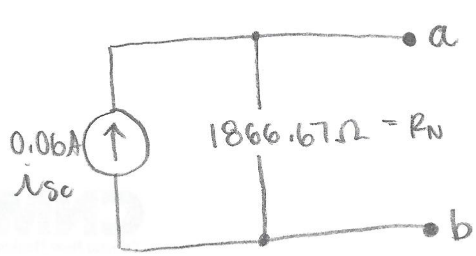
$\Rightarrow -11.2V_1 + 19V_2 = 784$

$\Rightarrow -11.2\left(\dfrac{224 + V_2}{2.8}\right) + 19V_2 = 784 \Rightarrow \dfrac{-2508.8}{2.8} - \dfrac{11.2V_2}{2.8} + 19V_2 = 784$

$\Rightarrow -896 - 4V_2 + 19V_2 = 784 \Rightarrow 15V_2 = 970 \Rightarrow V_2 = 112 \text{ volts} = V_{TH}$

$R_N = \dfrac{V_{TH}}{i_{sc}} = \dfrac{112V}{0.06A} = 1866.67\,\Omega$

Norton Equivelent circuit $\Rightarrow$



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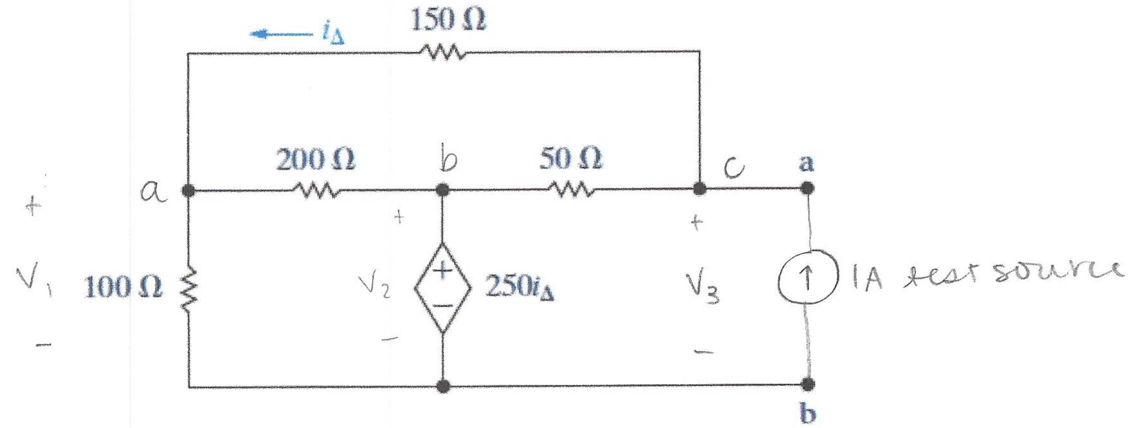
Question 4 [10]

Use the test source method to find the Thévenin resistance. [Hint: use the node-voltage method.]



$$i_\Delta = \frac{V_3 - V_1}{150\,\Omega}$$

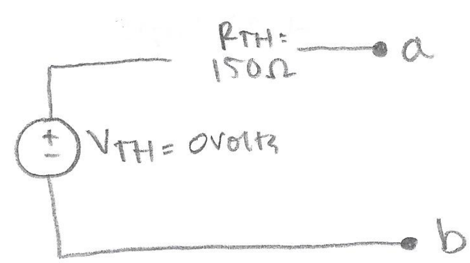
$$V_2 = 250 i_\Delta = 250\left(\frac{V_3 - V_1}{150}\right) = 1.67V_3 - 1.67V_1 = V_2$$

KCl node a:
$$\frac{V_1}{100}\cdot 6 + \frac{V_1 - V_2}{200}\cdot 3 + \frac{V_1 - V_3}{150}\cdot 4 = 0$$
$$\Rightarrow \frac{6V_1}{600} + \frac{3V_1 - 3V_2}{600} + \frac{4V_1 - 4V_3}{600} = 0$$
$$\Rightarrow 6V_1 + 3V_1 - 3V_2 + 4V_1 - 4V_3 = 0$$
$$\Rightarrow 13V_1 - 3V_2 - 4V_3 = 0$$
$$\Rightarrow 13V_1 - 3(1.67V_3 - 1.67V_1) - 4V_3 = 0$$
$$\Rightarrow 13V_1 - 5.01V_3 + 5.01V_1 - 4V_3 = 0$$
$$\Rightarrow 18.01V_1 - 9.01V_3 = 0$$
$$V_1 = 0.5V_3$$

KCl node c:
$$\frac{V_3 - V_2}{50}\cdot\frac{3}{3} + \frac{V_3 - V_1}{150} = 1$$
$$\Rightarrow \frac{3V_3 - 3V_2}{150} + \frac{V_3 - V_1}{150} = 1$$
$$\Rightarrow 4V_3 - 3V_2 - V_1 = 150$$
$$\Rightarrow 4V_3 - 3(1.67V_3 - 1.67V_1) - V_1 = 150$$
$$\Rightarrow 4V_3 - 5.01V_3 + 5.01V_1 - V_1 = 150$$
$$\Rightarrow -1.01V_3 + 4.01V_1 = 150$$
$$\Rightarrow 4.01(0.5V_3) - 1.01V_3 = 150$$
$$\Rightarrow 2V_3 - 1.01V_3 = 150$$
$$\Rightarrow V_3 = 150 \text{ Volts}$$

$$R_{TH} = \frac{V_3}{1A} = \frac{150V}{1A} = 150\,\Omega$$

V_{TH} = 0 volts because of no existing independent source. ⇒



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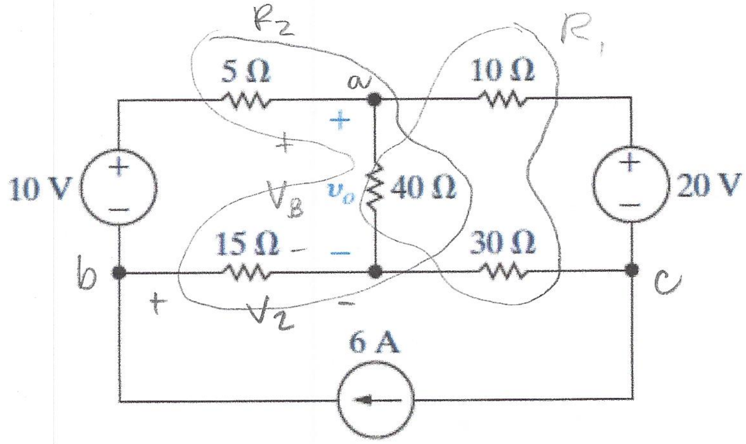
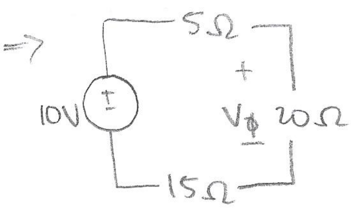
Question 5 [10]

Use the principle of superposition to find the voltage v_o. [Hint: when you analyze the current source, apply the node voltage method choosing the reference node as the node below the 40 Ω resistor.]



$$R_1 = (30+10)\|40 = 40\|40 = \frac{(40)(40)}{40+40} = \frac{1600}{80} = 20\,\Omega$$

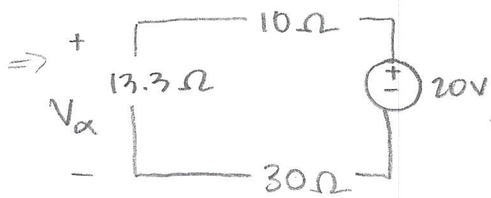
$\Rightarrow$



$$\Rightarrow V_\phi = \left(\frac{20}{(20+15+5)}\right)(10\text{V}) = 5\text{ volts}$$

$$R_2 = (5+15)\|40 = 20\|40 = \frac{(20)(40)}{40+20} = \frac{800}{60} = 13.3\,\Omega$$

$\Rightarrow$



$$\Rightarrow V_\alpha = \left(\frac{13.3}{13.3+10+30}\right)(20\text{V}) = 4.99\text{ V} = 5\text{ volts}$$

KCl node a: $\dfrac{V_B}{40} + \dfrac{(V_B - V_1)\,4}{10\cdot 4} + \dfrac{(V_B - V_2)\,8}{5 \cdot 8} = 0$

$$\Rightarrow \frac{V_B}{40} + \frac{4V_B - 4V_1}{40} + \frac{8V_B - 8V_2}{40} = 0$$

$$\Rightarrow 13V_B - 4V_1 - 8V_2 = 0$$

KCl node b: $\dfrac{(V_2 - V_B)\,3}{5\cdot 3} + \dfrac{V_2}{15} - 6 = 0$

$$\Rightarrow \frac{3V_2 - 3V_B}{15} + \frac{V_2}{15} = 6$$

$$\Rightarrow -3V_B + 4V_2 = 90$$

$$\Rightarrow V_2 = \frac{90 + 3V_B}{4}$$

KCl node c: $\frac{V_1 - V_\beta}{10} \frac{3}{3} + \frac{V_1}{30} + 6 = 0$

$$\Rightarrow \frac{3V_1 - 3V_\beta}{30} + \frac{V_1}{30} = -6$$

$$\Rightarrow 4V_1 - 3V_\beta = -180$$

$$\Rightarrow V_1 = \frac{-180 + 3V_\beta}{4}$$

$$\Rightarrow 13V_\beta - 4V_1 - 8V_2 = 0$$

$$\Rightarrow 13V_\beta - 4\left(\frac{-180 + 3V_\beta}{4}\right) - 8\left(\frac{90 + 3V_\beta}{4}\right) = 0$$

$$\Rightarrow 13V_\beta + \cancel{\frac{720}{4}} - \frac{12V_\beta}{4} - \cancel{\frac{720}{4}} - \frac{24V_\beta}{4} = 0$$

$$\Rightarrow 13V_\beta - 3V_\beta - 6V_\beta = 0$$

$$\Rightarrow 4V_\beta = 0$$

$$\Rightarrow V_\beta = \emptyset \text{ volts}$$

Principle of superposition:

$$V_o = V_\phi + V_\alpha + V_\beta$$

$$= (5 + 5 + 0) \text{ volts}$$

$$\Rightarrow \boxed{V_o = 10 \text{ volts}}$$