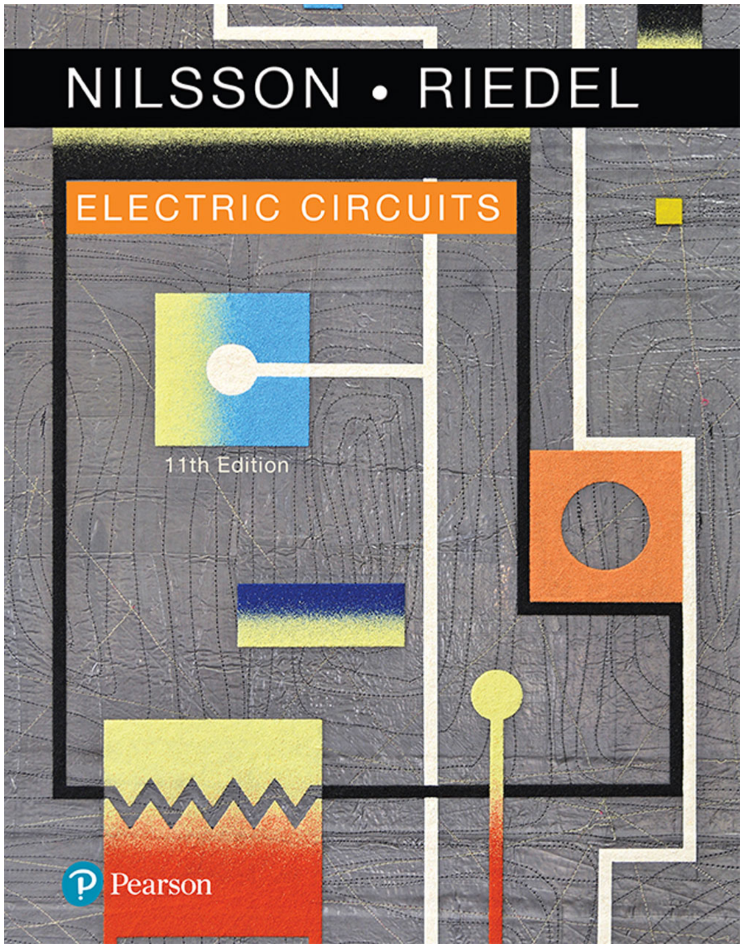
Electric Circuits

Eleventh Edition



Chapter 5

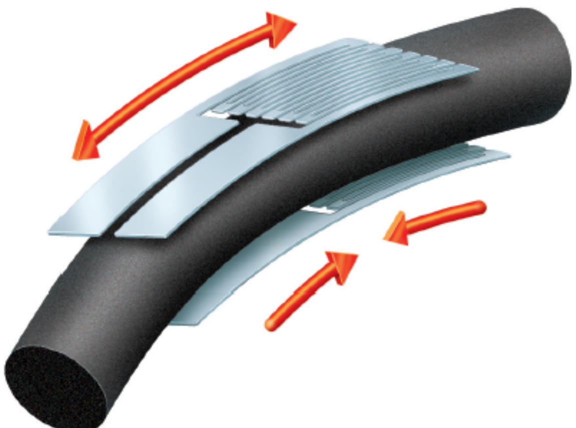
The Operational Amplifier

Learning Objectives

- Operational Amplifier Terminals
- Terminal Voltages and Currents
- The Inverting-Amplifier Circuit
- The Summing-Amplifier Circuit
- The Noninverting-Amplifier Circuit
- The Difference-Amplifier Circuit
- A More Realistic Model for the Operational Amplifier

Practical Perspective- Strain Gages

A strain gage is a grid of thin wires whose resistance changes when the wires are lengthened or shortened.



R: the resistance of the gage at rest;
ΔL/L: is the fractional change in length of the gage (which defines strain);
2: is a typical manufacturer's gage factor;
ΔR: the change in resistance due to the bending of the bar.

$$\boxed{\Delta R = 2R\frac{\Delta L}{L}}$$

The change in resistance experienced by the strain gage is typically much smaller than could be accurately measured by an ohmmeter.

In order to make an accurate measurement of the voltage difference, we use an operational amplifier circuit to amplify, or increase, the voltage difference.

5.1 Operational Amplifier Terminals

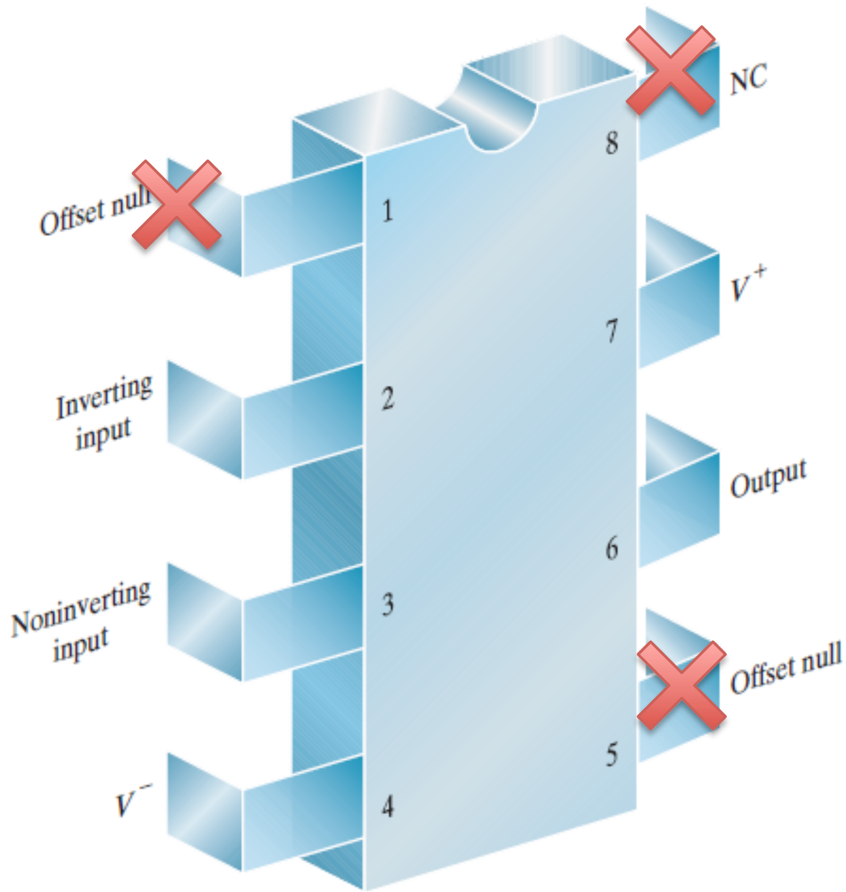


Figure 5.1: The eight-lead DIP package (top view).

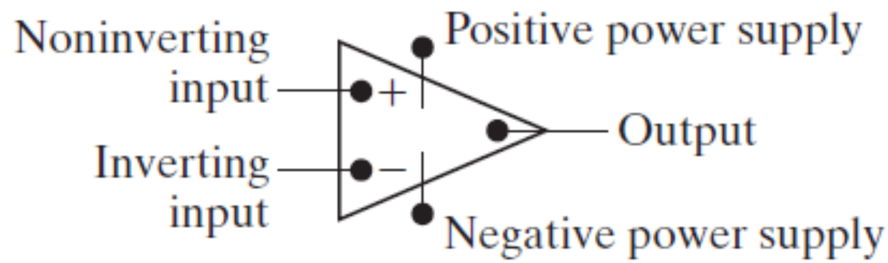


Figure 5.2: The circuit symbol for an operational amplifier (op amp).

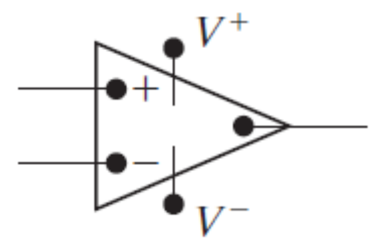


Figure 5.3: A simplified circuit symbol for an op amp.

5.2 Terminal Voltages & Currents

Terminal Voltages

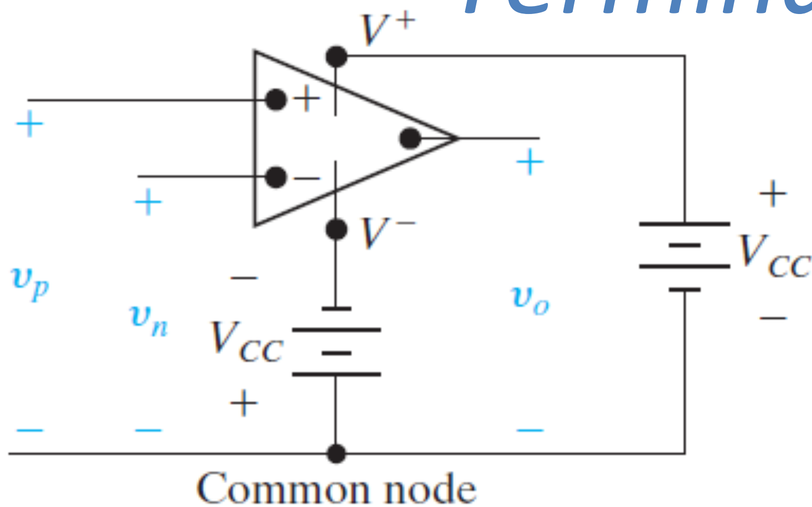


Figure 5.4: Terminal voltage variables.

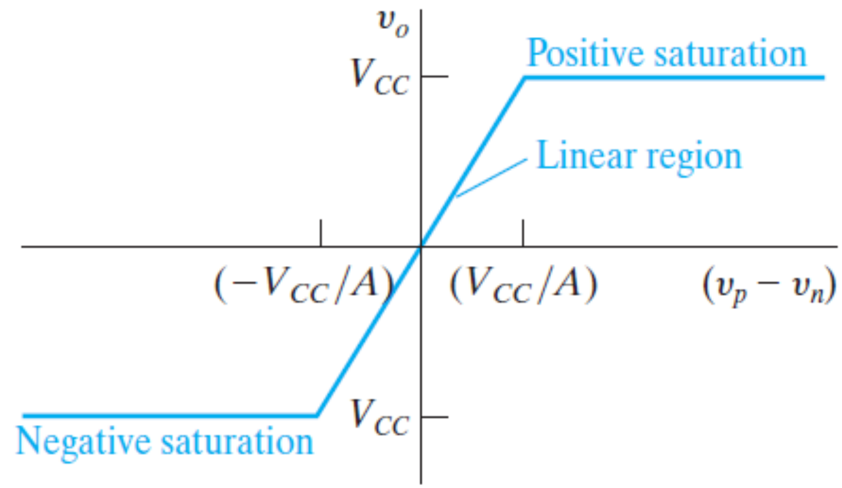


Figure 5.6: The voltage transfer characteristic of an op amp.

$$v_o = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC}, \\ A(v_p - v_n) & -V_{CC} \le A(v_p - v_n) \le +V_{CC}, \\ +V_{CC} & A(v_p - v_n) > +V_{CC}. \end{cases}$$

Saturated

Linear region

Saturated

Ideal Op Amp

$A = +\infty$

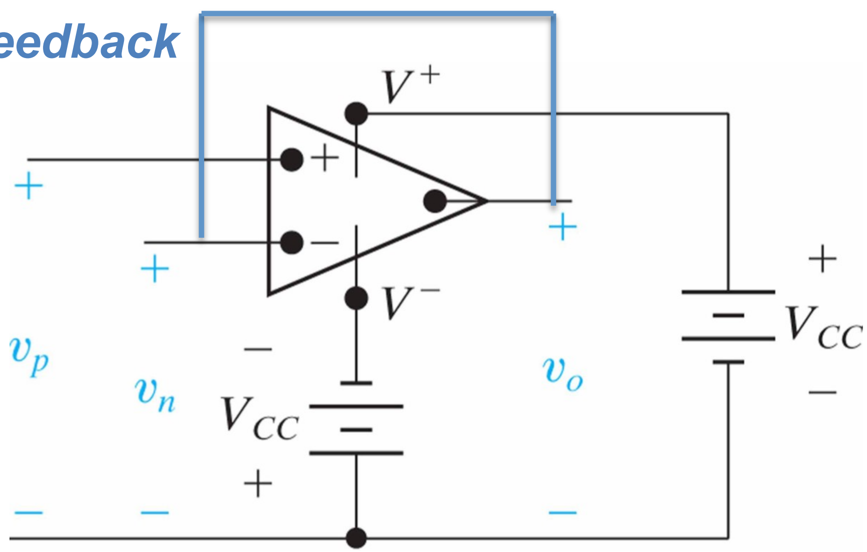
$V_{CC} \leq 20\ \text{V}$

Linear region

$\boxed{v_p = v_n}$ virtual short condition

Maintain the virtual short condition to ensure linear operation:

Negative feedback



Puzzle

- Even if the circuit provides a negative feedback path for the op amp, linear operation is not ensured.

- *So how do we know whether the op amp is operating in its linear region?*

- The answer is: we don't know!

- We first assume linear operation, perform the circuit analysis, and then checking our results for contradictions, if we have $-V_{CC} \leq v_o \leq V_{CC}$, it is in the linear region, otherwise it is saturated.

Terminal Currents

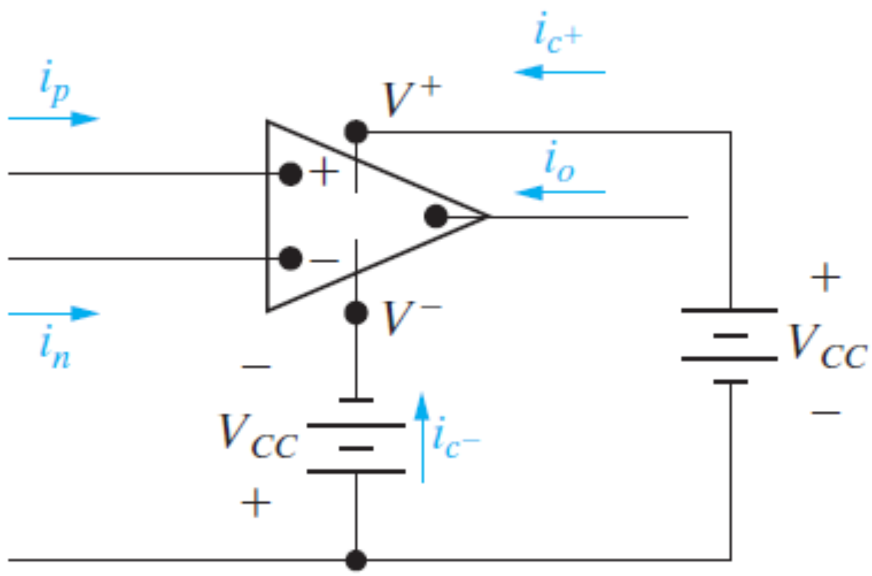


Figure 5.5: Terminal current variables.

Ideally, the equivalent input resistance is infinite, resulting in the current constraint:

$$\boxed{i_p = i_n = 0}$$

Note that the current constraint is not based on assuming the op amp is confined to its linear operating region as was the voltage constraint.

Kirchhoff's current law: $i_p + i_n + i_o + i_{c+} + i_{c-} = 0$

$$\boxed{i_o = -(i_{c+} + i_{c-})}$$

Notes

- The positive and negative power supply voltages do *not have to be equal* in magnitude. In the linear operating region, v_o must lie between the two supply voltages.

- Be aware also that the value of A is *not constant* under all operating conditions. For now, however, we assume that it is.

Example 5.1

The op amp in the circuit is ideal.

a) Calculate v_o, if $v_a = 1$ V and $v_b = 0$ V.

b) If $v_a = 1.5$ V, specify the range of v_b that avoids amplifier saturation.

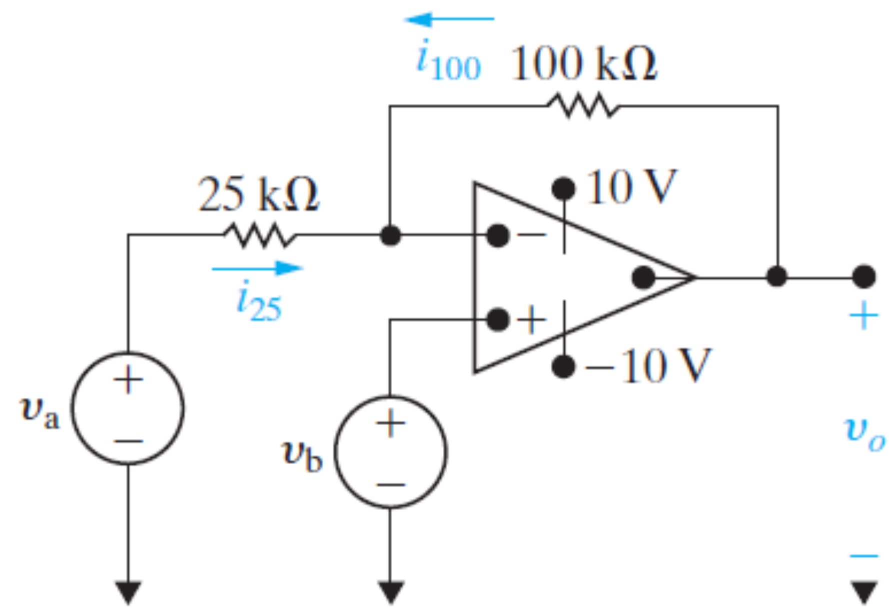


Figure 5.7: The circuit for Example 5.1.

5.3 The Inverting-Amplifier Circuit

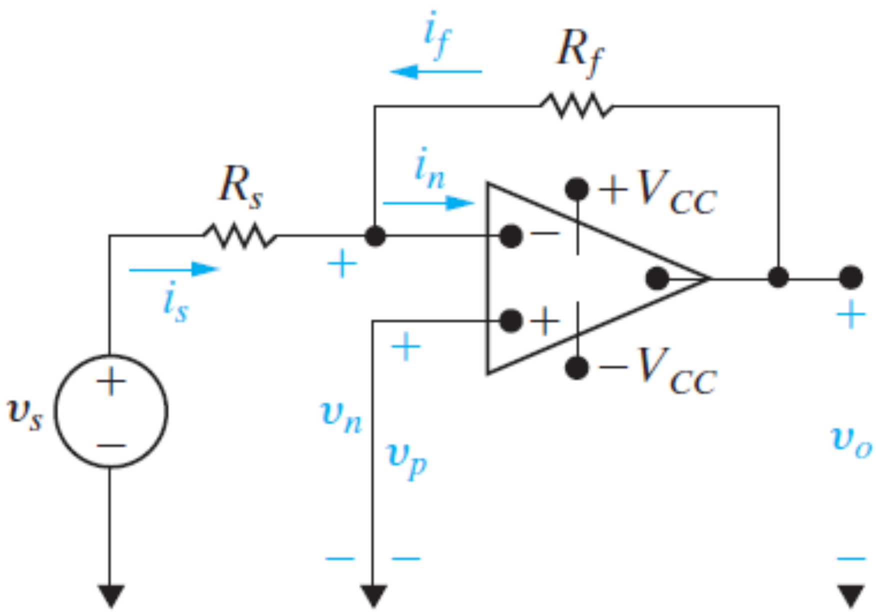


Figure 5.8: An inverting-amplifier circuit.

$$v_n = v_p = 0$$

$$i_s + i_f = i_n = 0$$

$$i_s = \frac{v_s}{R_s} \qquad i_f = \frac{v_o}{R_f}$$

$$\boxed{v_o = -\left(\frac{R_f}{R_s}\right) v_s}$$

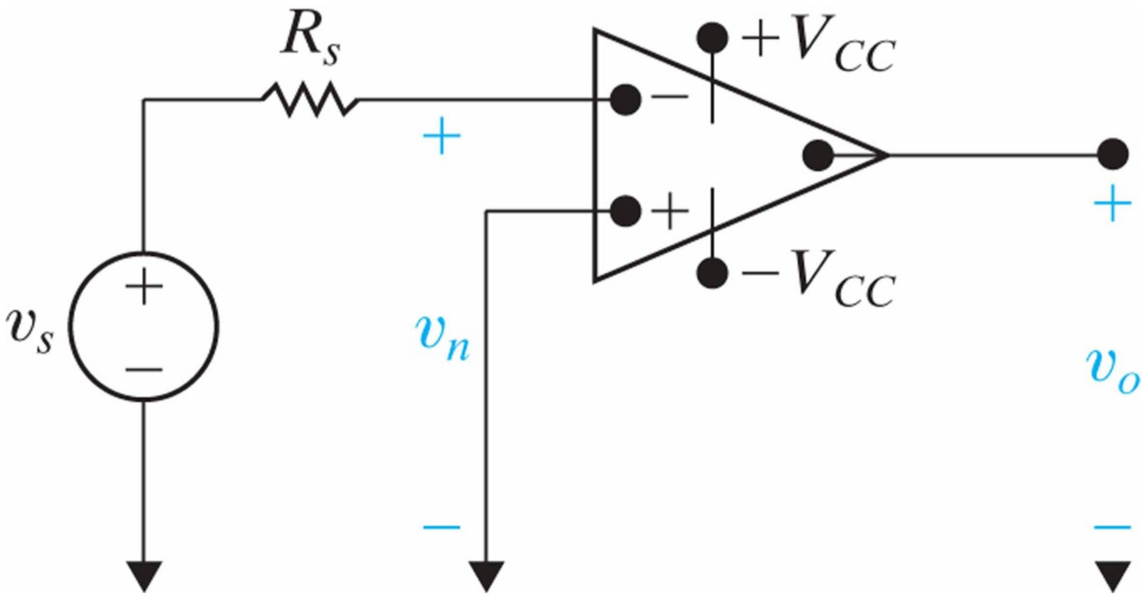
$$|v_o| \leq V_{CC}$$

$$\left|\frac{R_f}{R_s} v_s\right| \leq V_{CC}$$

$$\frac{R_f}{R_s} \leq \left|\frac{V_{CC}}{v_s}\right|$$

Pearson

Open-loop Gain



$$v_o = -Av_n = -Av_s$$

A: open-loop gain

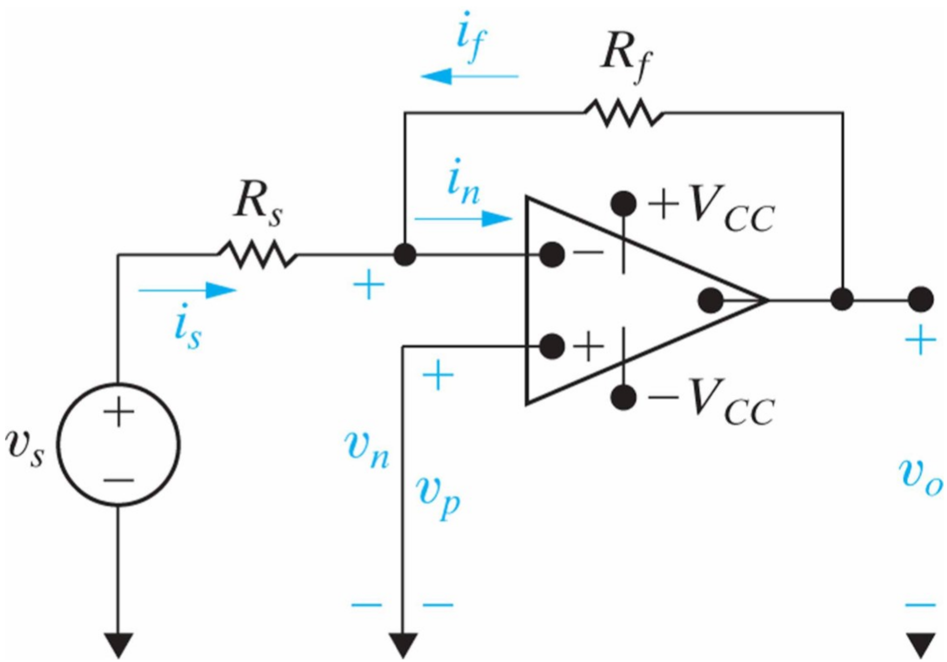
$$|v_s| \leq V_{CC}/A$$

Figure 5.9: An inverting amplifier operating open loop.

Example 5.2

a) Design an inverting amplifier with a gain of 12. Use ±15 V power supplies and an ideal op amp.

b) What range of input voltages, v_s, allows the op amp in this design to remain in its linear operating region?



5.4 The Summing-Amplifier Circuit

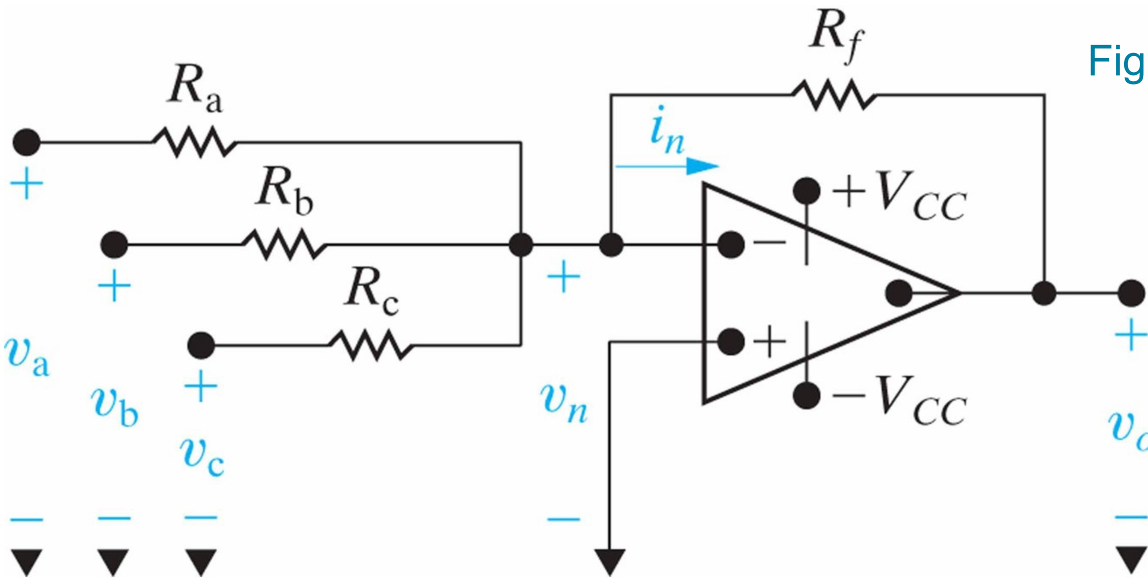


Figure 5.11: A summing amplifier.

$$R_a = R_b = R_c = R_s$$

$$\frac{v_n - v_a}{R_a} + \frac{v_n - v_b}{R_b} + \frac{v_n - v_c}{R_c} + \frac{v_n - v_o}{R_f} + i_n = 0$$

$$i_n = 0 \qquad v_n = v_p = 0$$

$$v_o = -\frac{R_f}{R_s}(v_a + v_b + v_c)$$

$$\boxed{v_o = -\left(\frac{R_f}{R_a}v_a + \frac{R_f}{R_b}v_b + \frac{R_f}{R_c}v_c\right)}$$

Example 5.3

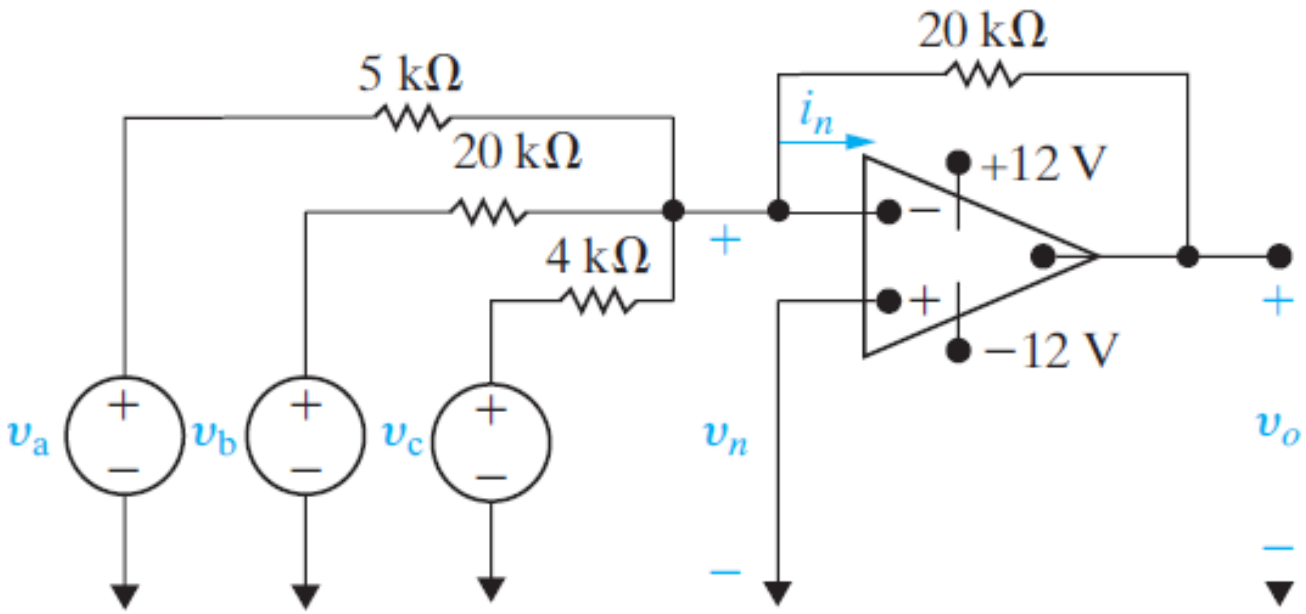


Figure 5.12: The summing amplifier for Example 3.3(a).

5.5 Noninverting-Amplifier Circuit

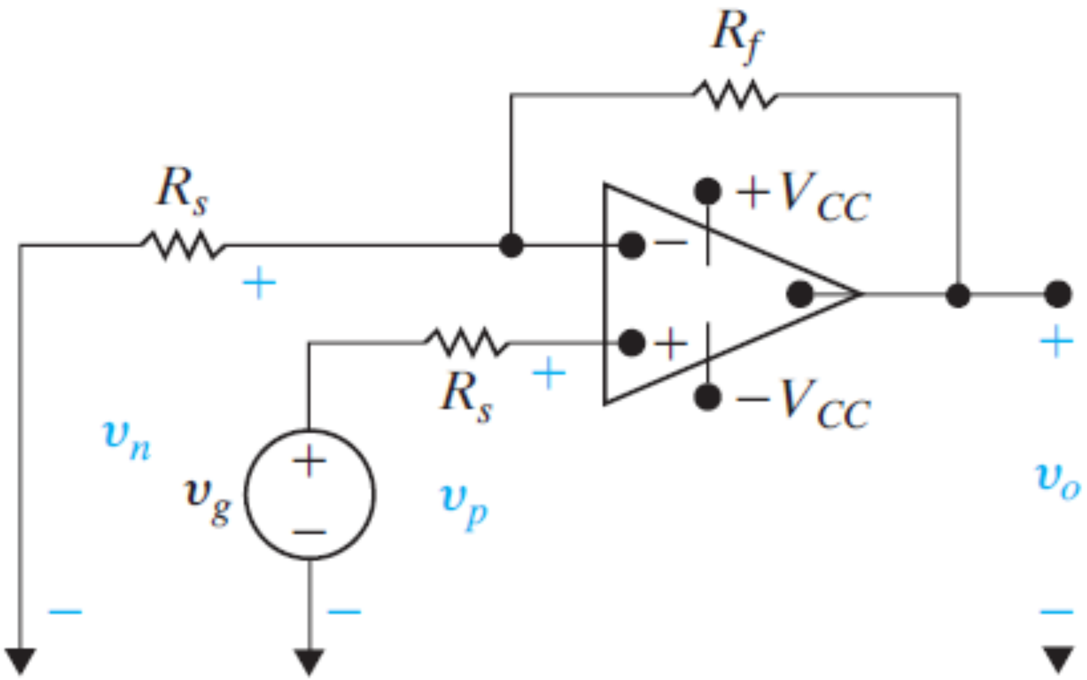


Figure 5.13: A noninverting amplifier.

$$v_n = v_g = \frac{v_o R_s}{R_s + R_f}$$

$$\boxed{v_o = \left(\frac{R_s + R_f}{R_s}\right) v_g}$$

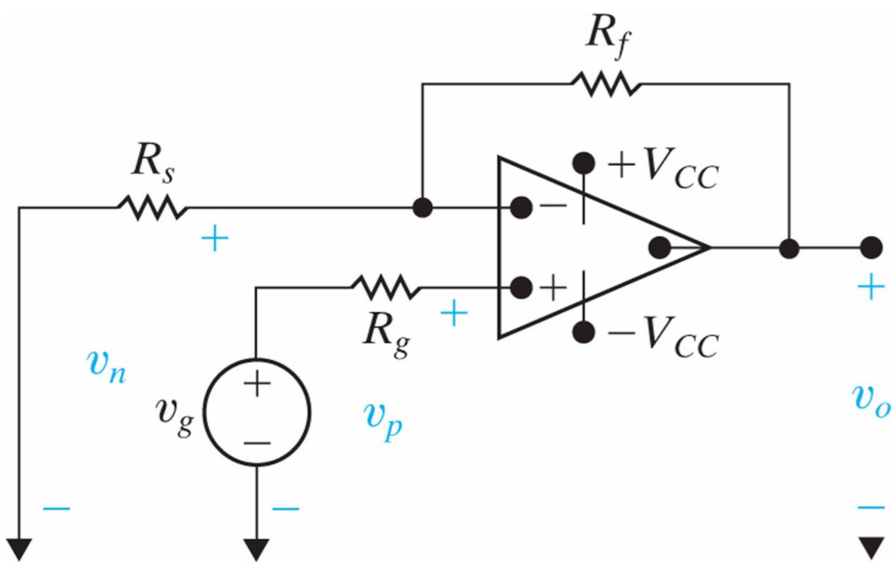
Operation in the linear region requires

$$\frac{R_s + R_f}{R_s} < \left|\frac{V_{CC}}{v_g}\right|$$

Example 5.4

a) Design a noninverting amplifier with a gain of 6. Assume the op amp is ideal.

b) Suppose we wish to amplify a voltage v_g, such that $-1.5\text{ V} \leq v_g \leq 1.5\text{ V}$. What are the smallest power supply voltages that could be used with the resistors selected in part (a) and still have the op amp in this design remain in its linear operating region?



5.6 The Difference-Amplifier Circuit

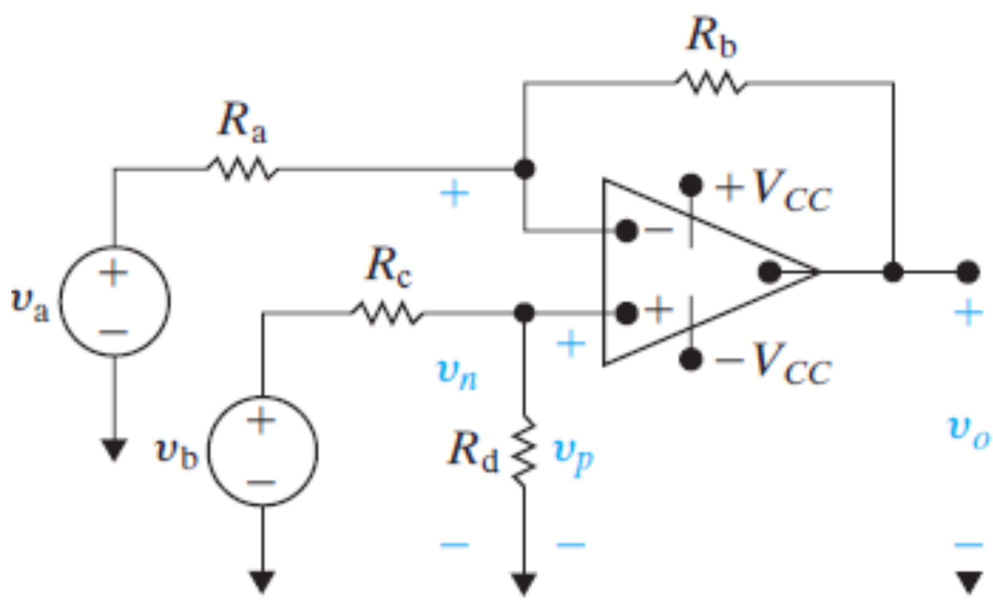


Figure 5.15: A difference amplifier.

$$v_n = v_p = \frac{R_d}{R_c + R_d} v_b$$

$$i_n = i_p = 0$$

$$\frac{v_n - v_a}{R_a} + \frac{v_n - v_o}{R_b} + i_n = 0$$

$$v_o = \frac{R_d\left(R_a + R_b\right)}{R_a\left(R_c + R_d\right)} v_b - \frac{R_b}{R_a} v_a$$

$$\frac{R_a}{R_b} = \frac{R_c}{R_d}$$

$$\boxed{v_o = \frac{R_b}{R_a}\left(v_b - v_a\right)}$$

Example 5.5

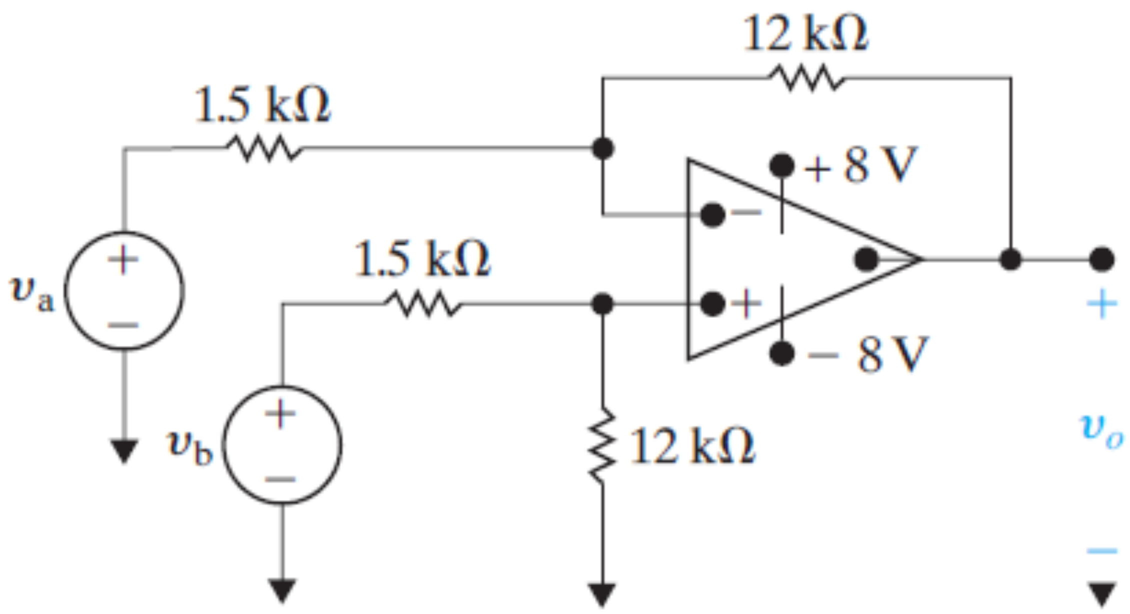


Figure 5.16: The difference amplifier designed in Example 5.4.

The Difference Amplifier- Another Perspective

$$v_{dm} = v_b - v_a$$

$$v_{cm} = (v_a + v_b)/2$$

$$v_a = v_{cm} - \frac{1}{2} v_{dm}$$

$$v_b = v_{cm} + \frac{1}{2} v_{dm}$$

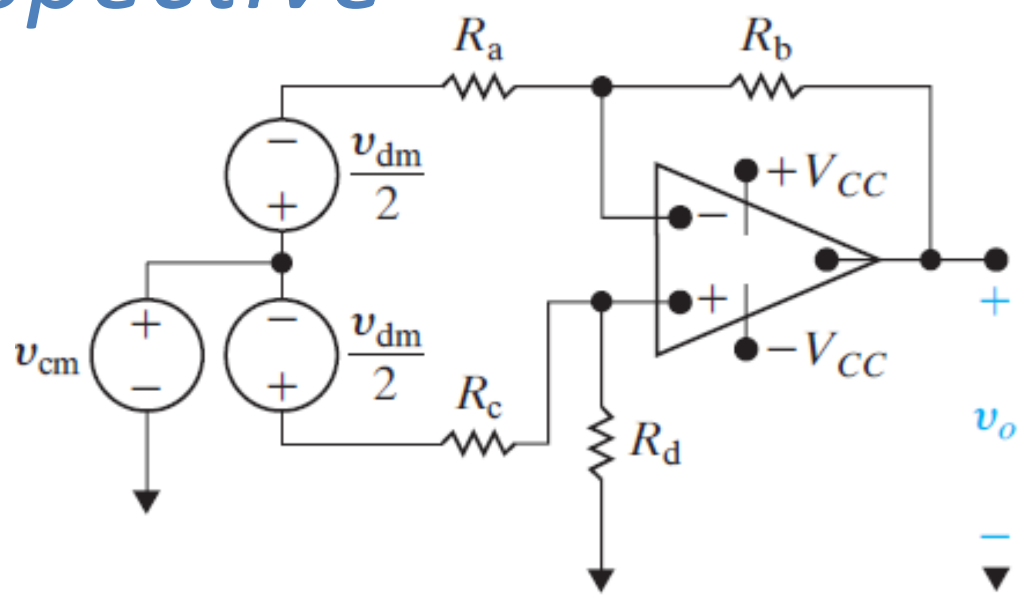


Figure 5.17: A difference amplifier with common mode and differential mode input voltages.

$$v_o = \left[\frac{R_a R_d - R_b R_c}{R_a(R_c + R_d)}\right] v_{cm} + \left[\frac{R_d(R_a + R_b) + R_b(R_c + R_d)}{2R_a(R_c + R_d)}\right] v_{dm}$$

$$= A_{cm} v_{cm} + A_{dm} v_{dm}$$

Insights

- In many applications it is the *differential mode signal that contains the information of interest*, whereas *the common mode signal is the noise* found in all electric signals.

- Thus, we hope,

$$R_a R_d = R_b R_c$$

$$v_o = (0)v_{\mathrm{cm}} + \left(\frac{R_\mathrm{b}}{R_\mathrm{a}}\right)v_{\mathrm{dm}}$$

Common Mode Rejection Ratio

- An ideal difference amplifier has zero common mode gain and nonzero (and usually large) differential mode gain.
- Two factors have an influence on the ideal common mode gain - resistance mismatches or a nonideal op amp.

$$\frac{R_a}{R_b} = (1 - \epsilon)\frac{R_c}{R_d}$$

$$\Downarrow$$

$$R_a = (1 - \epsilon)R_c \quad \text{and} \quad R_b = R_d$$

or

$$R_d = (1 - \epsilon)R_b \quad \text{and} \quad R_a = R_c$$

$$A_{\rm cm} = \frac{R_{\rm a}(1-\epsilon)R_{\rm b} - R_{\rm a}R_{\rm b}}{R_{\rm a}[R_{\rm a} + (1-\epsilon)R_{\rm b}]}$$

$$= \frac{-\epsilon R_{\rm b}}{R_{\rm a} + (1-\epsilon)R_{\rm b}}$$

$$\approx \frac{-\epsilon R_{\rm b}}{R_{\rm a} + R_{\rm b}}.$$

$$A_{\rm dm} = \frac{(1-\epsilon)R_{\rm b}(R_{\rm a} + R_{\rm b}) + R_{\rm b}[R_{\rm a} + (1-\epsilon)R_{\rm b}]}{2R_{\rm a}[R_{\rm a} + (1-\epsilon)R_{\rm b}]}$$

$$= \frac{R_{\rm b}}{R_{\rm a}}\left[1 - \frac{(\epsilon/2)R_{\rm a}}{R_{\rm a} + (1-\epsilon)R_{\rm b}}\right]$$

$$\approx \frac{R_{\rm b}}{R_{\rm a}}\left[1 - \frac{(\epsilon/2)R_{\rm a}}{R_{\rm a} + R_{\rm b}}\right].$$

The **commo mode rejection ratio (CMRR)** can be used to measure how nearly ideal is a difference amplifier. It is defined as the ratio of the differential mode gain to the common mode gain

$$\text{CMRR} = \left|\frac{A_{\rm dm}}{A_{\rm cm}}\right| \approx \left|\frac{1 + R_{\rm b}/R_{\rm a}}{-\epsilon}\right|.$$

5.7 A More Realistic Model - Operational Amplifier

A realistic model includes three modifications to the ideal op amp:

1. finite input resistance, R_i
2. finite open-loop gain, A
3. nonzero output resistance, R_o

Also, the following two assumptions are now invalid:

$$v_n = v_p$$

$$i_n = i_p = 0$$

For the juA 741 op amp, typical values of R_i, A, and R_o are 2 MΩ, 105, and 75 Ω, respectively.

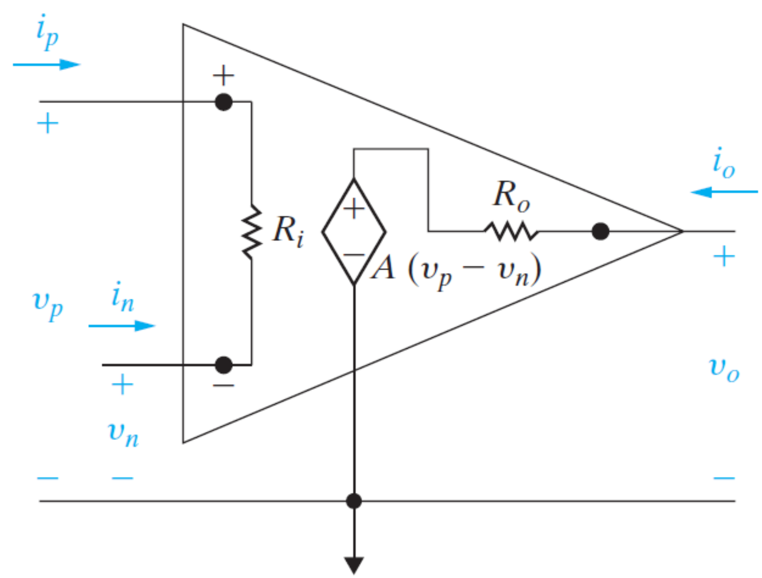


Figure 5.18: An equivalent circuit for an operational amplifier.

Analysis of the More Realistic Op Amp Model

- Take the Inverting-Amplifier Circuit for example:

node a: $$\frac{v_n - v_s}{R_s} + \frac{v_n}{R_i} + \frac{v_n - v_o}{R_f} = 0$$

node b: $$\frac{v_o - v_n}{R_f} + \frac{v_o - A(-v_n)}{R_o} = 0$$

$$v_o = \frac{-A + (R_o/R_f)}{\frac{R_s}{R_f}\left(1 + A + \frac{R_o}{R_i}\right) + \left(\frac{R_s}{R_i} + 1\right) + \frac{R_o}{R_f}} v_s$$

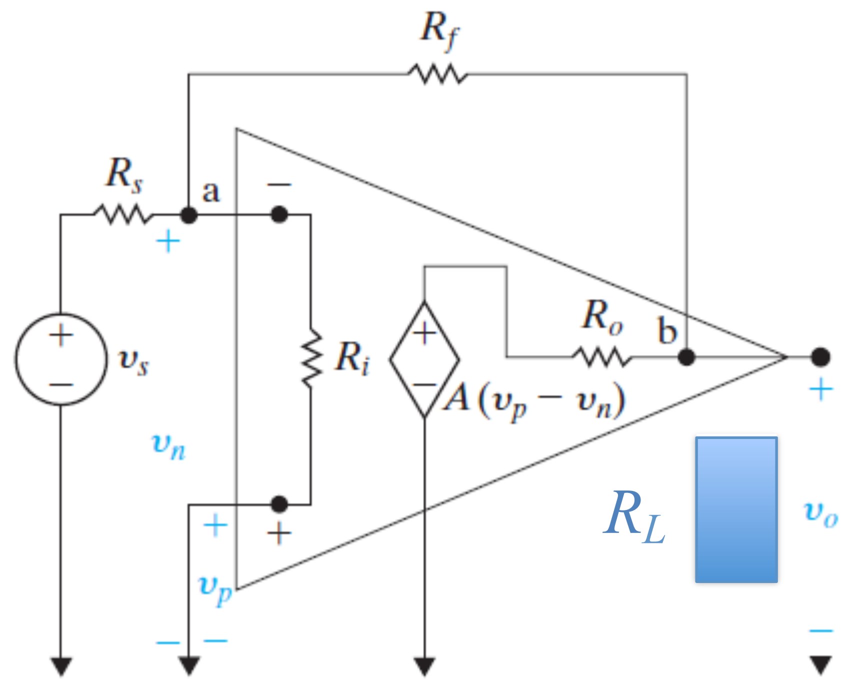


Figure 5.19: An inverting-amplifier circuit.

$R_i \to \infty$, $A \to \infty$, and $R_o \to 0$ $\Rightarrow$ $$v_o = \frac{-R_f}{R_s} v_s$$

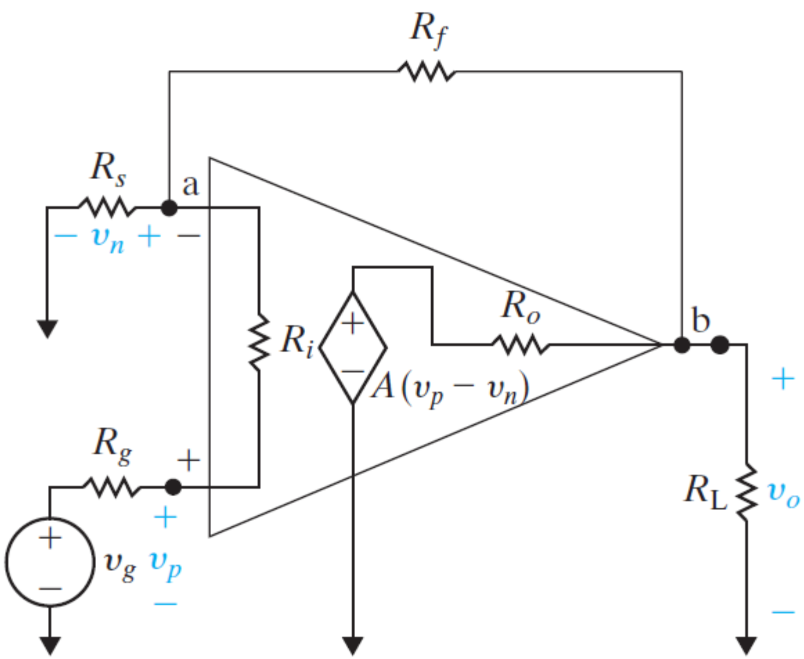


Figure 5.20: A noninverting-amplifier circuit.

Pearson

Practical Perspective - Strain Gages

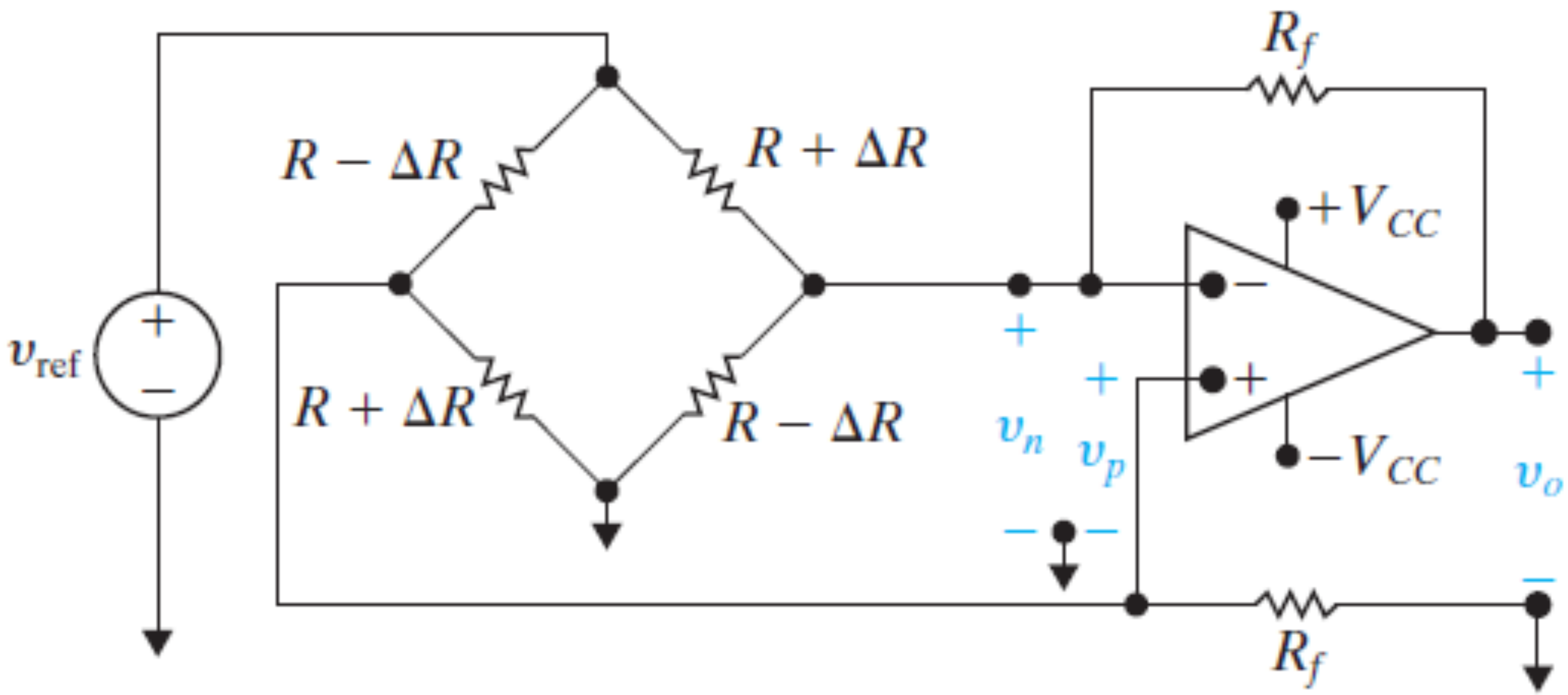


Figure 5.22: An op amp circuit used for measuring the change in strain gage resistance.

The pair of strain gages that are lengthened once the bar is bent have the values $R + \Delta R$ in the bridge feeding the difference amplifier, whereas the pair of strain gages that are shortened have the values $R - \Delta R$.

$$\frac{v_{\text{ref}} - v_n}{R + \Delta R} = \frac{v_n}{R - \Delta R} + \frac{v_n - v_o}{R_f}$$

$$\frac{v_{\text{ref}} - v_p}{R - \Delta R} = \frac{v_p}{R + \Delta R} + \frac{v_p}{R_f}$$

$$v_p = \frac{v_{\text{ref}}}{(R - \Delta R)\left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f}\right)}$$

$$v_o = \frac{R_f(2\Delta R)}{R^2 - (\Delta R)^2} v_{\text{ref}}$$

$(\Delta R)^2 \ll R^2$, so $R^2 - (\Delta R)^2 \approx R^2$

$$\delta = \Delta R / R$$

$$v_o \approx \frac{R_f}{R} 2\delta v_{\text{ref}}$$