ENGR 2910-101: Circuit Analysis

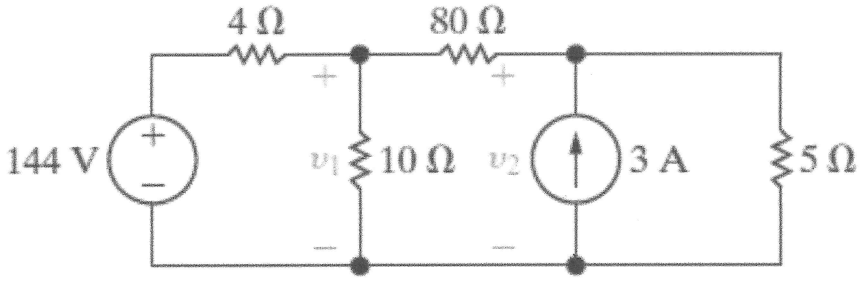
Homework 8: 03/11/20

Instructor: Leo Silbert

Due: 03/18/20

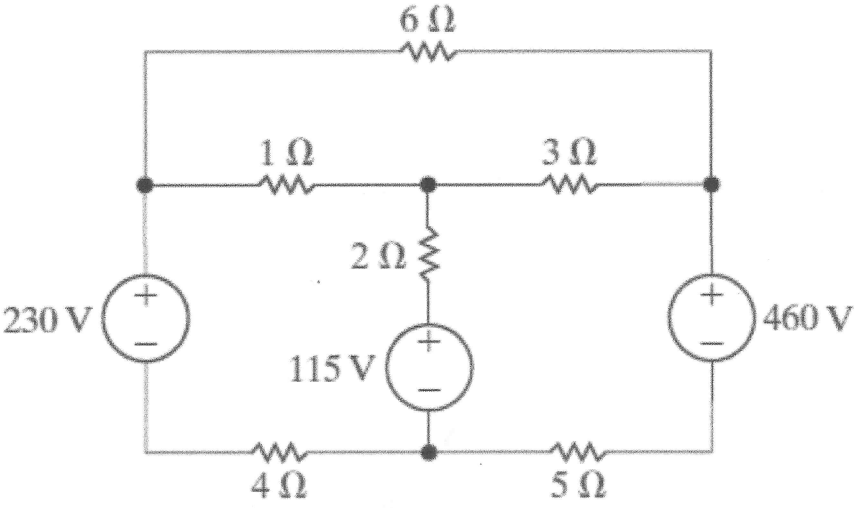
Question 1 [10]

Use the mesh-current method to find v_1 and v_2 in the circuit below.



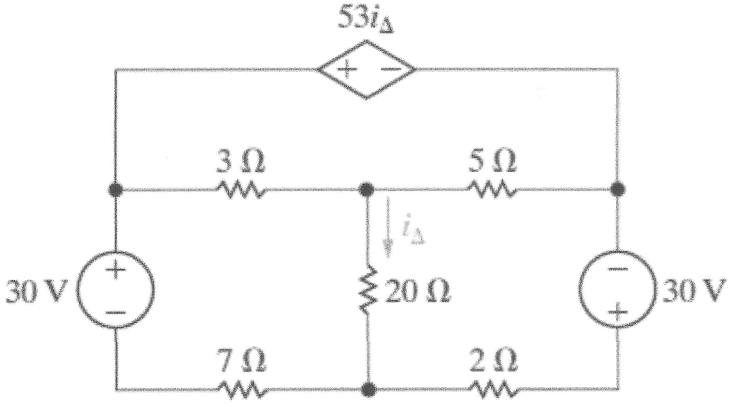
Question 2 [10]

Use the mesh-current method to find the total power developed by the three indepedent voltage sources. Also, explcitly evaluate the power dissipated in the 6 resistors to double check your calculation.



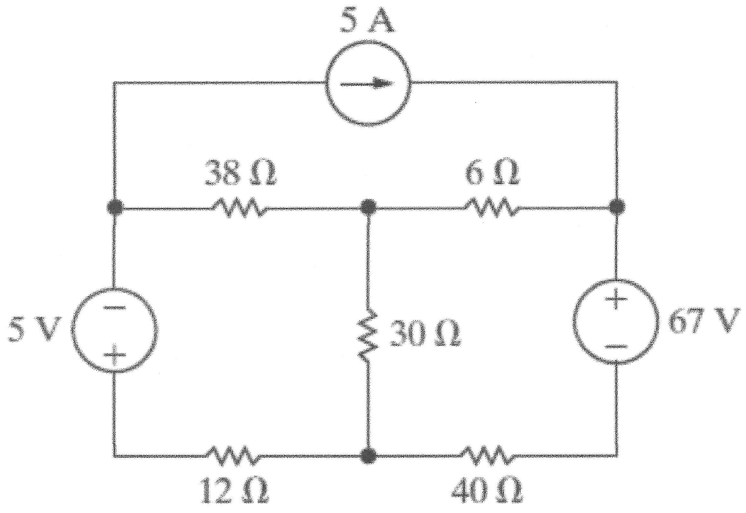
Question 3 [10]

Use the mesh-current method to find the power developed in the three voltage sources.



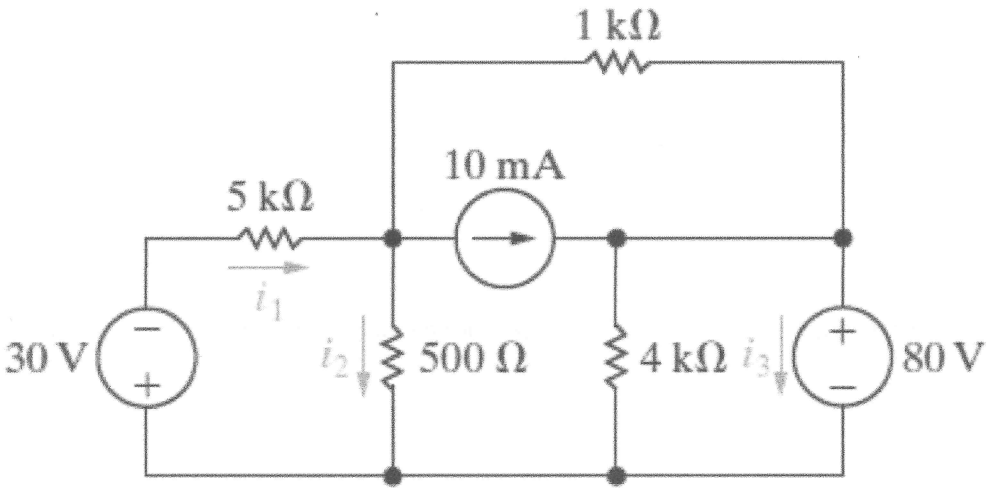
Question 4 [10]

Use the mesh-current method to determine which of the three sources is providing power to the circuit and which are absorbing power. What are the values of the power in each case? Go on to then compute the power dissipated in the resistors to ensure power conversation.



Question 5 [10]

Use the mesh-current method to find the three branch currents identified in the circuit below. [Hint: identify the supermesh.]



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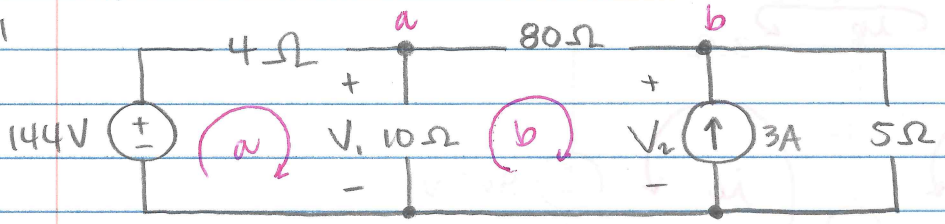
03.18.2020
Engr 2910

Homework #8

#1



KCL node a: $\frac{V_1 - 144}{4} + \frac{V_1}{10} + \frac{V_1 - V_2}{80} = 0$

$\Rightarrow \frac{20V_1}{20} - \frac{144}{4} + \frac{8V_1}{8} + \frac{V_1}{80} - \frac{V_2}{80} = 0$

$\Rightarrow \frac{20V_1}{80} - 36 + \frac{8V_1}{80} + \frac{V_1}{80} - \frac{V_2}{80} = 0$

$\Rightarrow \frac{29V_1}{80} - \frac{V_2}{80} = 36$

$\Rightarrow 29V_1 - V_2 = 2880 \cdots$ eq 1

KCL node b: $\frac{V_2 - V_1}{80} + \frac{16V_2}{16} - 3 = 0$

$\Rightarrow \frac{V_2}{80} - \frac{V_1}{80} + \frac{16V_2}{80} = 3$

$\Rightarrow 17V_2 - V_1 = 240 \cdots$ eq 2

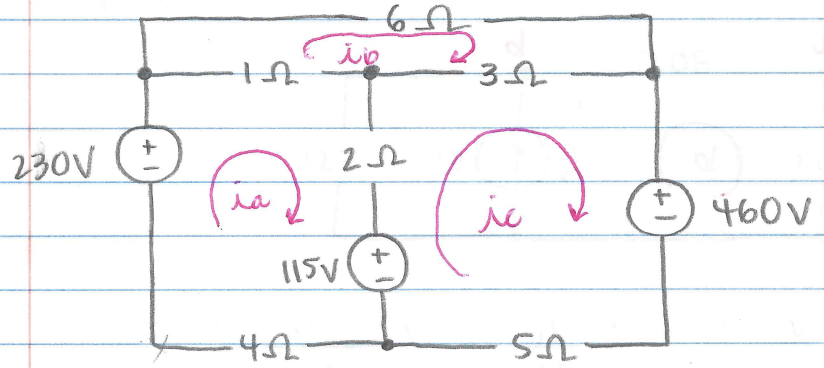
$$29V_1 - V_2 = 2880$$
$$-V_1 + 17V_2 = 240$$

By solving matrix on calculator: $\begin{bmatrix} 29 & -1 & 2880 \\ -1 & 17 & 240 \end{bmatrix}$

$V_1 = 100$ volts ✓

$V_2 = 20$ volts ✓

#2



KCL a: $-230 + (1)(i_a - i_b) + 2(i_a - i_c) + 115 + 4i_a = 0$

$\Rightarrow i_a - i_b + 2i_a - 2i_c + 4i_a = 115$

$\Rightarrow 7i_a - i_b - 2i_c = 115$ ... eq 1

KCL b: $6i_b + 3(i_b - i_c) + (1)(i_b - i_a) = 0$

$\Rightarrow 6i_b + 3i_b - 3i_c + i_b - i_a = 0$

$\Rightarrow 10i_b - 3i_c - i_a = 0$ ... eq 2

KCL c: $-115 + 2(i_c - i_a) + 3(i_c - i_b) + 460 + 5i_c = 0$

$\Rightarrow 2i_c - 2i_a + 3i_c - 3i_b + 5i_c = -345$

$\Rightarrow 10i_c - 2i_a - 3i_b = -345$ ... eq 3

$$\Rightarrow \begin{cases} 7i_a - i_b - 2i_c = 115 \\ -i_a + 10i_b - 3i_c = 0 \\ -2i_a - 3i_b + 10i_c = -345 \end{cases}$$

By solving a 3x4 matrix:

$i_a = 4.4\ A$ ✓

$i_b = -10.6\ A$ ✓

$i_c = -36.8\ A$ ✓

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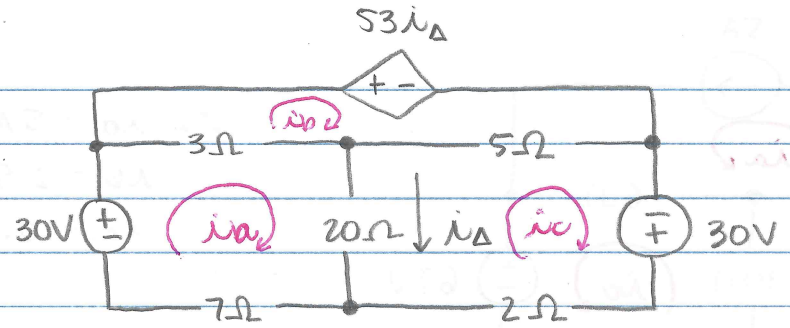
#3



$i_\Delta = i_a - i_c$

KVL b: $53i_\Delta + 5(i_b - i_c) + 3(i_b - i_a) = 0$

$\Rightarrow 53(i_a - i_c) + 5i_b - 5i_c + 3i_b - 3i_a = 0$

$\Rightarrow 53i_a - 53i_c + 5i_b - 5i_c + 3i_b - 3i_a = 0$

$\Rightarrow 50i_a + 8i_b - 58i_c = 0$ ... eq 1

KVL a: $-30 + 3(i_a - i_b) + 20(i_a - i_c) + 7i_a = 0$

$\Rightarrow 3i_a - 3i_b + 20i_a - 20i_c + 7i_a = 30$

$\Rightarrow 30i_a - 3i_b - 20i_c = 30$ ... eq 2

KVL c: $-30 + 2i_c + 20(i_c - i_a) + 5(i_c - i_b) = 0$

$\Rightarrow 2i_c + 20i_c - 20i_a + 5i_c - 5i_b = 30$

$\Rightarrow -20i_a - 5i_b + 27i_c = 30$ ... eq 3

By solving 3×4 matrix:

$i_a = 52\ A$

$i_b = 110\ A$

$i_c = 60\ A$

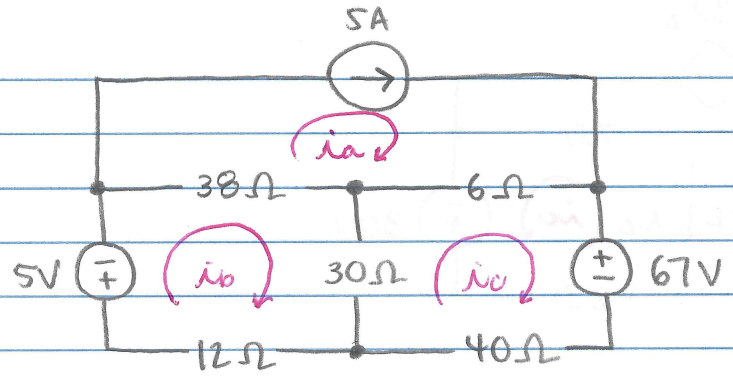
$\Rightarrow i_\Delta = i_a - i_c = 52 - 60 = -8A$

$P_{53i_\Delta} = (-424V)(110A) = -46640$ watts

$P_{30V} = (30V)(52A) = 1560$ watts

$P_{30V} = (30V)(60A) = 1800$ watts

#4



$\therefore$ $i_a = 5A$

$i_b = 2.5A$ ✓

$i_c = 0.5A$ ✓

$i_a = 5A$

KVL b: $5 + 38(i_b - i_a) + 30(i_b - i_c) + 12i_b = 0$

$\Rightarrow 5 + 38i_b - 38(5A) + 30i_b - 30i_c + 12i_b = 0$

$\Rightarrow 38i_b + 30i_b - 30i_c + 12i_b = 185$

$\Rightarrow 80i_b - 30i_c = 185$

$\Rightarrow i_b = \dfrac{185 + 30i_c}{80}$

KVL c: $30(i_c - i_b) + 6(i_c - i_a) + 67 + 40i_c = 0$

$\Rightarrow 30i_c - 30i_b + 6i_c - 6i_a + 67 + 40i_c = 0$

$\Rightarrow 30i_c - 30i_b + 6i_c - 6(5A) + 40i_c = -67$

$\Rightarrow -30i_b + 76i_c = -37$

$\Rightarrow -30\left(\dfrac{185 + 30i_c}{80}\right) + 76i_c = -37$

$\Rightarrow \dfrac{-5550}{80} - \dfrac{900i_c}{80} + 76i_c = -37$

$\Rightarrow \dfrac{-900i_c}{80} + \dfrac{6080i_c}{80} = \dfrac{-2960}{80} + \dfrac{5550}{80}$

$\Rightarrow 64.75i_c = 32.375$

$\Rightarrow i_c = \dfrac{32.375}{64.75} = 0.5A = i_c$

$i_b = \dfrac{185 + 30(0.5A)}{80} = 2.5A = i_b$

KVL a: $-V_{5A} + 6(i_a - i_c) + 38(i_a - i_b) = 0$

$\Rightarrow V_{5A} = 6(5 - 0.5) + 38(5 - 2.5)$

$V_{5A} = 122$ volts

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#4 cont.

$P_{5A} = (122V)(5A) = 610$ watts delivered

$P_{5V} = -(i_b)(5V) = -(2.5A)(5V) = -12.5$ watts absorbed

$P_{67V} = -(i_c)(67V) = -(0.5A)(67V) = -33.5$ watts absorbed

$P_{38\Omega} = (i^2)(R) = (i_b - i_a)^2(38\Omega) = (2.5)^2(38\Omega) = (6.25)(38) = 237.5$ watts absorbed

$P_{12\Omega} = (i^2)(R) = (i_b)^2(12\Omega) = (6.25)(12) = 75$ watts absorbed

$P_{6\Omega} = (i^2)(R) = (i_c - i_a)^2(R) = (-4.5)^2(6) = 121.5$ watts absorbed

$P_{40\Omega} = (i^2)(R) = (0.5)^2(40) = 10$ watts absorbed

$P_{30\Omega} = (i^2)(R) = (2)^2(30) = 120$ watts absorbed

Power delivered = power absorbed

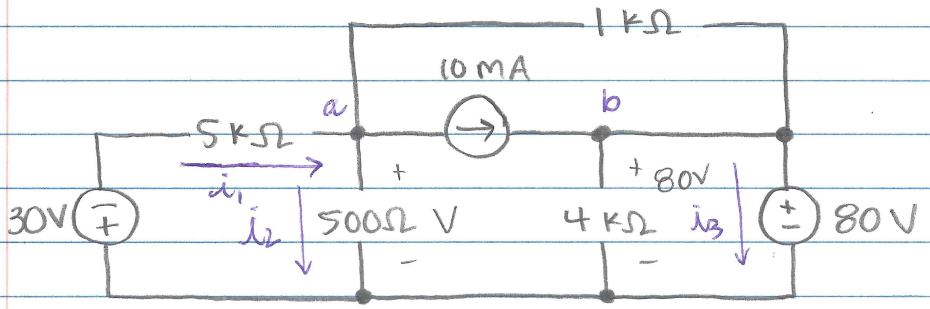
$\Rightarrow P_{5A} = P_{5V} + P_{67V} + P_{38} + P_{12} + P_6 + P_{40} + P_{30}$

$\Rightarrow 610 = (12.5) + (33.5) + 237.5 + 75 + 121.5 + 10 + 120$

$\checkmark\ 610 = 610\ \checkmark$

∴ the 5A power source is providing the power and the 5V and 67V sources are absorbing power.

#5



KCL node a: $\frac{V-(-30)}{5000} + \frac{V}{500} + \frac{(V-80)}{1000} + 0.01 = 0$

$\Rightarrow \frac{V}{5000} + \frac{30}{5000} + \frac{V \cdot 10}{500 \cdot 10} + \frac{V \cdot 5}{1000 \cdot 5} - \frac{80 \cdot 5}{1000 \cdot 5} + \frac{0.01 \cdot 5000}{5000} = 0$

$\Rightarrow \frac{V + 30 + 10V + 5V - 400 + 50}{5000} = 0$

$\Rightarrow \frac{16V}{5000} = \frac{320}{5000} \Rightarrow V = 20$ volts

$\Rightarrow i_1 = \frac{-30 - V}{5000} = \frac{-30-20}{5000} = -0.01\,A = -10\,mA = i_1$

#5 cont. $i_2 = \frac{V}{500} = \frac{20}{500} = 0.04\ A = 40\ mA = i_2$

KCL node b: $-0.01 + \frac{80}{4000} + \frac{(80-V)}{1000} + i_3 = 0$

$\Rightarrow i_3 = 0.01 - \frac{80}{4000} - \frac{80}{1000} + \frac{(20)}{1000}$

$i_3 = -0.07\ A = -70\ mA = i_3$

$\therefore$ $i_1 = -10\ mA$ ✓

$i_2 = 40\ mA$ ✓

$i_3 = -70\ mA$ ✓