ECE 203

Circuits I

Complex Power

Lecture 14-1

Yet another way to talk about power: Complex Power

Recognizes that loads are complex, so the power is complex as well.

S = P + jQ   Has units of VA (volt-amps)

P is average power delivered to the load in Watts

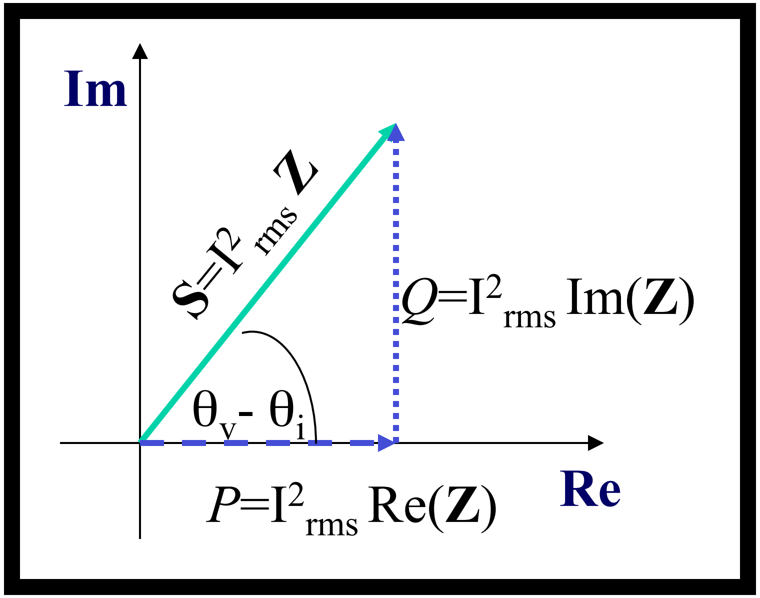
Q is the reactive power (also called quadrature power); units of VAR (volt-amps reactive)

Power Triangle

- The *power triangle* relates *pf angle* to *P* and *Q*

$$\tan(\theta_v - \theta_i) = \frac{Q}{P} = \frac{\text{reactive/quadrature power}}{\text{real/average power}}$$

  - the phasor current that is in phase with the phasor voltage produces the ***real* (*average*) power**
  - the phasor current that is out of phase with the phasor voltage produces the ***reactive* (*quadrature*) power**



Summarizing Complex Power (S)

$$\mathbf{S} = P + jQ = I_{rms}^2 \operatorname{Re}(\mathbf{Z}) + jI_{rms}^2 \operatorname{Im}(\mathbf{Z}) = I_{rms}^2 \mathbf{Z}$$

Complex power (like energy) is conserved, that is, the total complex power supplied equals the total complex power absorbed, $\sum S_i = 0$

Reactive Power	Load	Power Factor	Complex Power
Q is positive	Inductive	Lagging	First quadrant
Q is zero	Resistive	pf = 1	Real valued
Q is negative	Capacitive	Leading	Fourth quadrant

Power Terminology

- *average power*, $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$
- *apparent power* $= V_{rms} I_{rms}$
  - apparent power is expressed in volt-amperes (VA) or kilovolt-amperes (kVA) to distinguish it from average power

Complex Power (S)

- Definition of *complex power*, **S**

$$\boxed{\mathbf{S} = \mathbf{V}_{\mathrm{rms}} \mathbf{I}^{*}_{rms}} = \mathrm{V}_{\mathrm{rms}} \angle \theta_v \ \mathrm{I}_{\mathrm{rms}} \angle -\theta_i$$

$$= \mathrm{V}_{\mathrm{rms}} \mathrm{I}_{\mathrm{rms}} \angle \theta_v - \theta_i$$

$$= \mathrm{V}_{\mathrm{rms}} \mathrm{I}_{\mathrm{rms}} \cos\left(\theta_v - \theta_i\right) + j\,\mathrm{V}_{\mathrm{rms}} \mathrm{I}_{\mathrm{rms}} \sin\left(\theta_v - \theta_i\right)$$

$$= P + jQ$$

  - *P* is the *real* or *average power*
  - *Q* is the *reactive* or *quadrature power*, which indicates temporary energy storage rather than any real power loss in the element; and *Q* is measured in units of volt-amperes reactive, or var

Complex Power (S)

- This is really use of phasor voltage and current rather than just the use of magnitude and rms values
- Complex power is expressed in units of volt-amperes like apparent power
- Complex power has no physical significance; it is a purely mathematical concept
- Note relationships to apparent power and power factor of last section

$$|\mathbf{S}| = V_{rms}\ I_{rms} = \text{apparent power}$$

$$\angle\mathbf{S} = \angle(\theta_v - \theta_i) = \angle\theta_{Z_L} = \text{power factor angle}$$

Real Power (P)

- Alternate expressions for the *real* or *average power* (*P*)

$$P = \mathrm{Re}(\mathbf{S}) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$= |\mathbf{S}| \cos(\theta_Z) = \left(I_{rms} |\mathbf{Z}|\right) I_{rms} \left[\frac{\mathrm{Re}(\mathbf{Z})}{|\mathbf{Z}|}\right]$$

$$= I_{rms}^2 \,\mathrm{Re}(\mathbf{Z})$$

Reactive Power (Q)

- Alternate expressions for the *reactive* or *quadrature power* (Q)

$$Q = \mathrm{Im}(\mathbf{S}) = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$= |\mathbf{S}| \sin(\theta_Z) = (I_{rms} |\mathbf{Z}|) I_{rms} \left[\frac{\mathrm{Im}(\mathbf{Z})}{|\mathbf{Z}|} \right]$$

$$= I_{rms}^2 \,\mathrm{Im}(\mathbf{Z})$$

Summary

AC Power Calculations

$$P = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos(\theta)$$

$$\mathrm{PF} = \cos(\theta)$$

$$\theta = \theta_v - \theta_i$$

$$Q = V_{\mathrm{rms}} I_{\mathrm{rms}} \sin(\theta)$$

Summary

$$\text{apparent power} = V_{\text{rms}} I_{\text{rms}}$$

$$P^2 + Q^2 = \left(V_{\text{rms}} I_{\text{rms}}\right)^2$$

$$P = I_{\text{rms}}^2 R$$

$$P = \frac{V_{R\text{rms}}^2}{R}$$

$$Q = I_{\text{rms}}^2 X$$

$$Q = \frac{V_{X\text{rms}}^2}{X}$$

Power Factor

- Power factor $(0 \le pf \le 1)$

$$pf = \frac{average\ power}{apparent\ power} = \frac{P}{V_{rms}\ I_{rms}}$$

$$= \frac{V_{rms}\ I_{rms}\ \cos(\theta_v - \theta_i)}{V_{rms}\ I_{rms}} = \cos(\theta_v - \theta_i) = \cos(\theta_{Z_L})$$

- A low power factor requires more rms current for the same load power which results in greater utility transmission losses in the power lines, therefore utilities penalize customers that have a low *pf*

Power Factor Angle θ_{ZL}

- *power factor angle* is $\theta_v - \theta_i = \theta_{Z_L}$ (the phase angle of the load impedance)
- *power factor* (*pf*) special cases
  - purely resistive load: $\theta_{Z_L} = 0° \Rightarrow pf=1$
  - purely reactive load: $\theta_{Z_L} = \pm 90° \Rightarrow pf=0$

Power Factor Angle	I/V Lag/Lead	Load Equivalent
$-90° < \theta_{Z_L} < 0°$	Leading	Equivalent RC
$0° < \theta_{Z_L} < 90°$	Lagging	Equivalent RL

Go to examples 14-1.1 thru 14-1.3

Power factor correction

- Most industrial power factors are lagging because most industrial equipment involves electric motors which are very similar to inductors
- Ideally, the pf should be 1; purely resistive
- Although the reactive part of an inductive load does not consume real power, it does require higher input voltage and current to charge and discharge the magnetic fields in the inductor

Power factor correction

- Increased current or voltage leads to increased transmission losses
- So power company provides incentives to make pf as close to 1 as possible.
- Complex power for inductive load:

  $\mathbf{S} = P + jQ_{inductive}$
- Complex power for compensated load:

  $\mathbf{S} = P + jQ_{inductive} - jQ_{capacitive}$

Power factor correction

So, can improve the power factor by adding capacitance to the load.

Trade off: These sorts of capacitors are big and expensive. So have to determine what is the optimum level of power factor correction.

Go to example 14-1.4