ECE 203

Circuits I

Linearity and Superposition

Lecture 7-1

Linear versus Nonlinear Circuits

A circuit is linear if

1) the output is proportional to the input
2) the principle of superposition (additivity) holds

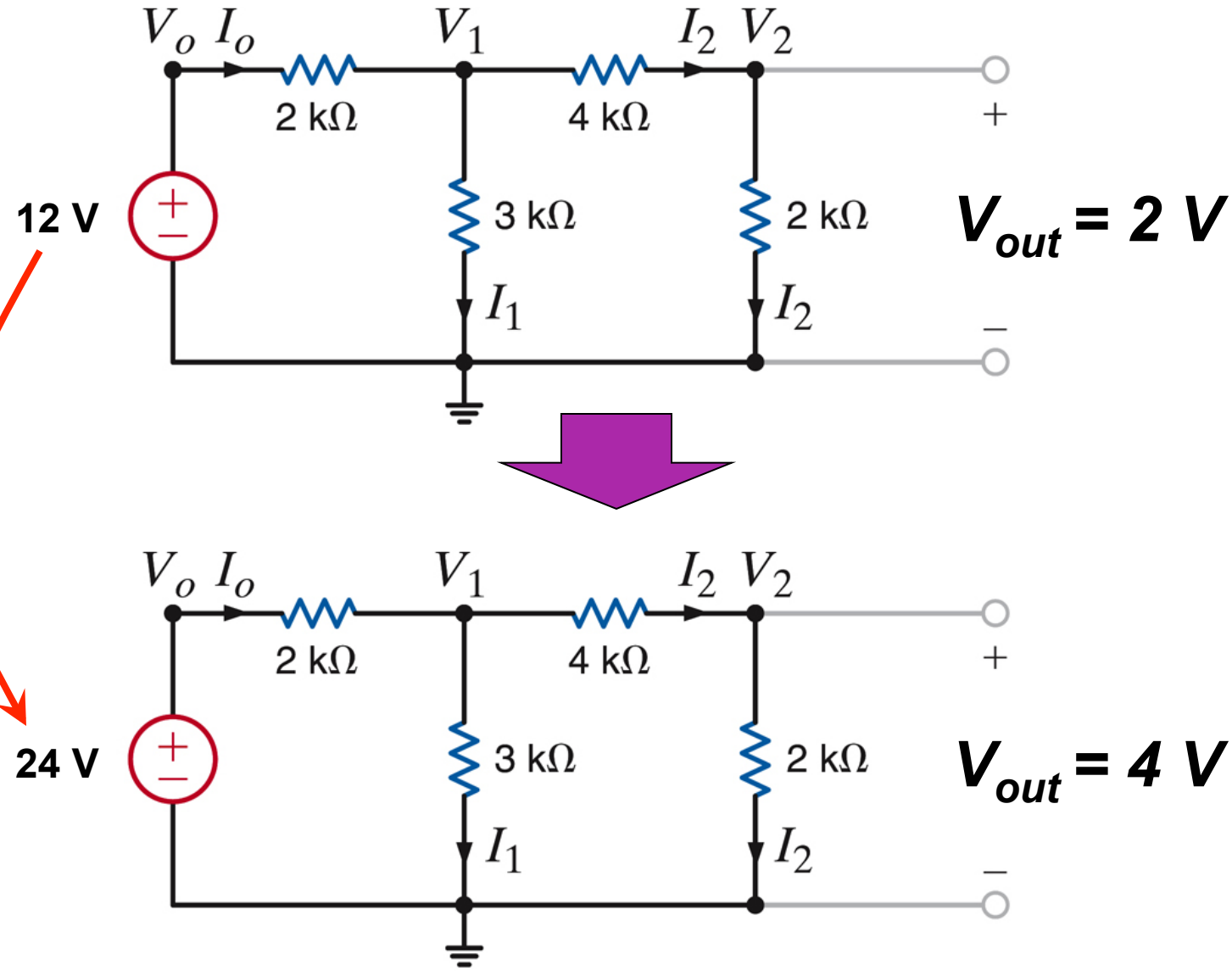
Linear Example: $\mathbf{y} = c\,\mathbf{x}$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2) = c\mathbf{x}_1 + c\,\mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = c\,\mathbf{x}^2$ or $y = \ln(x)$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

A Property of Linearity: Scaling



Note

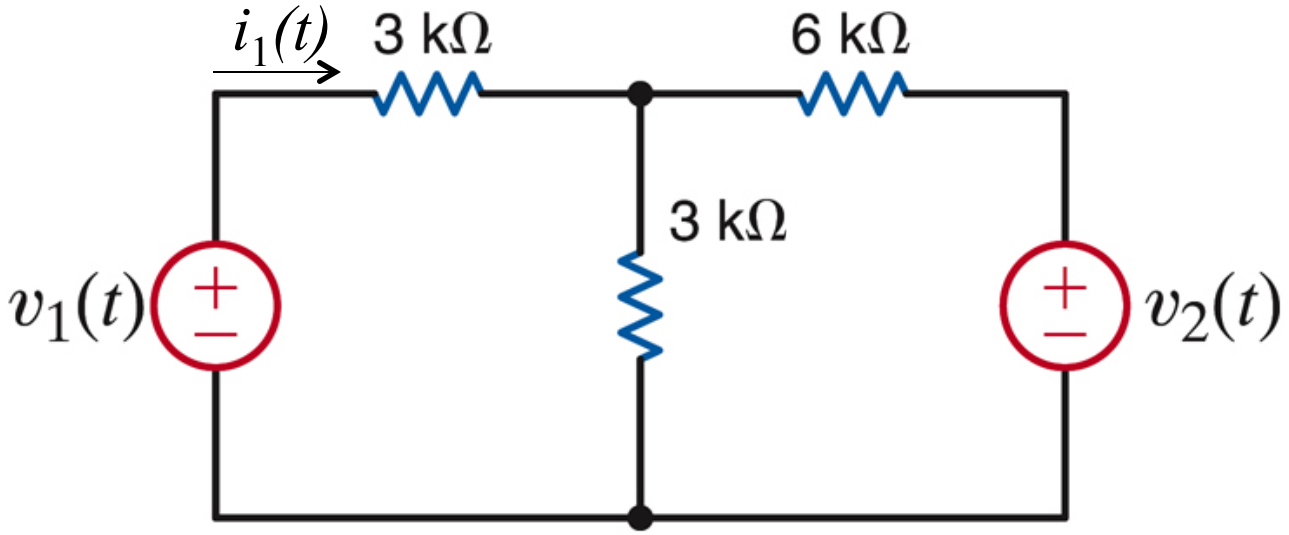
Linearity doesn’t apply to power

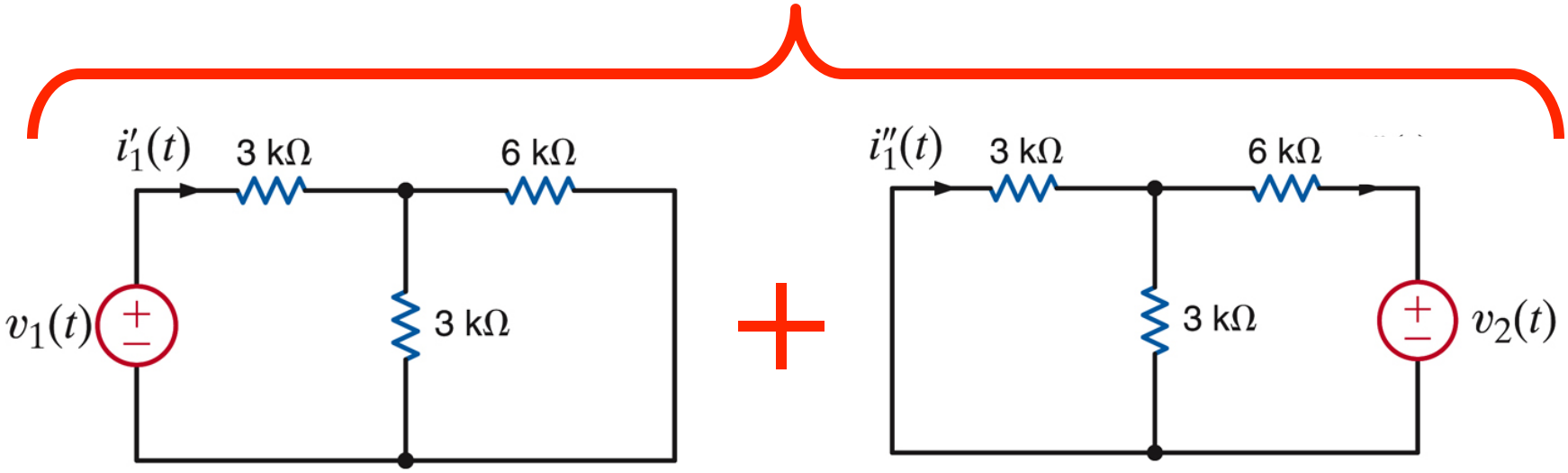
Superposition

An important implication of linearity is that each voltage and current supply in a circuit can be treated independently.

Superposition

Find $i_1(t)$





$$i_1(t) = i_1'(t) + i_1''(t)$$

Superposition

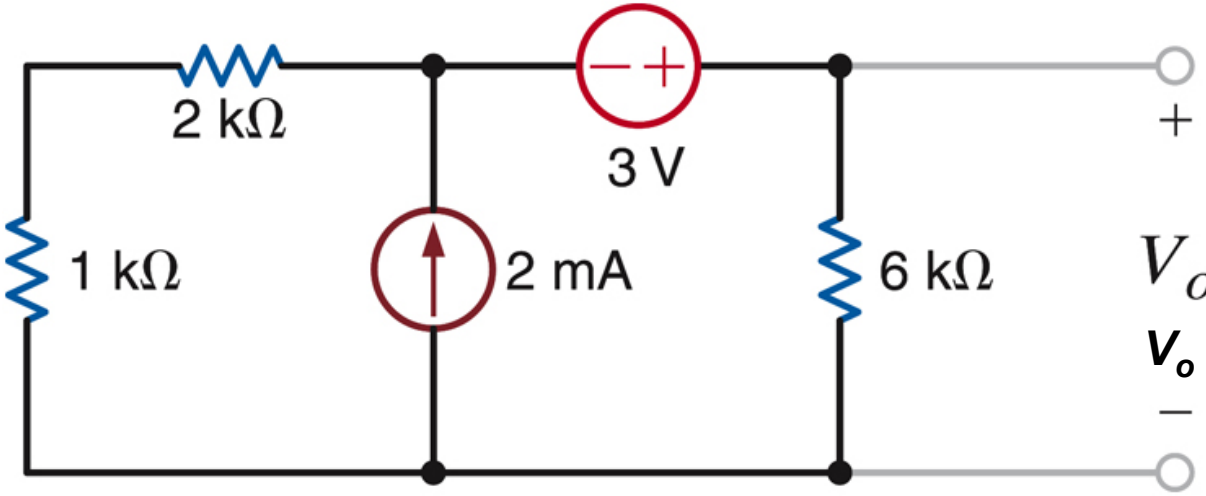
- **In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone**

- **Superposition provide us with the ability to reduce a complicated problem into several easier problems, each containing only a single independent source**

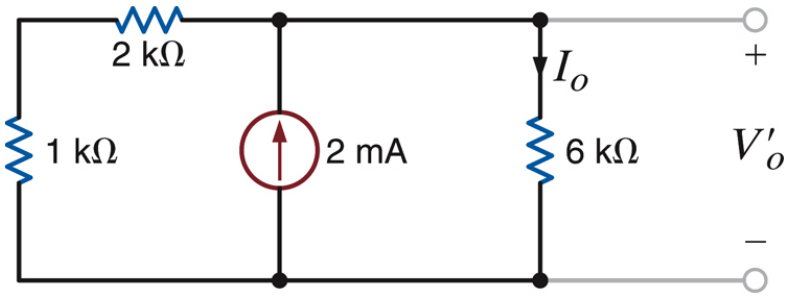
Superposition process

- Choose the voltage or current source you want to work with first.
- Remaining voltage sources are replaced by “short circuits” (V=0)
- Remaining current sources are replaced by “open circuits” (I=0)
- Repeat for each additional voltage or current source in the circuit

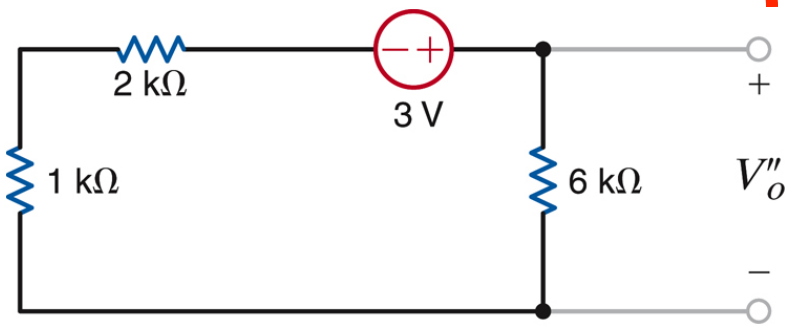
Superposition Example



$V_o = 4+2 = 6$ V

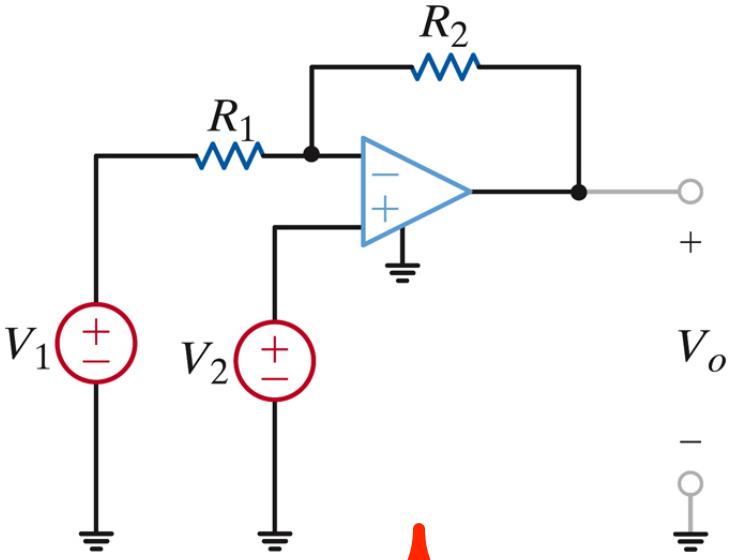


$V'_o = 4$ V

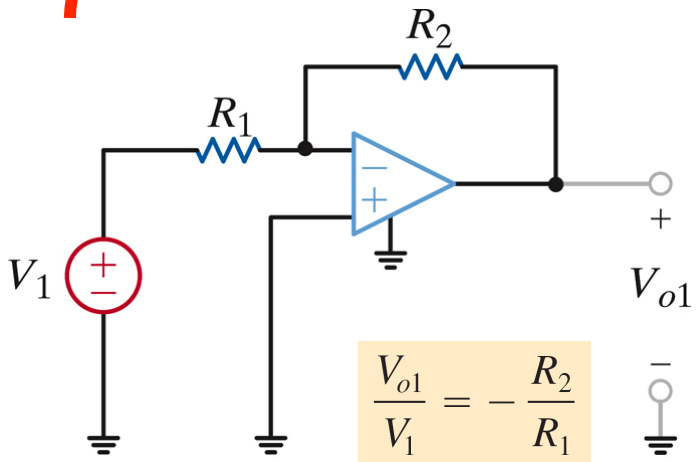


$V''_o = 2$ V

Superposition OP-Amp Example

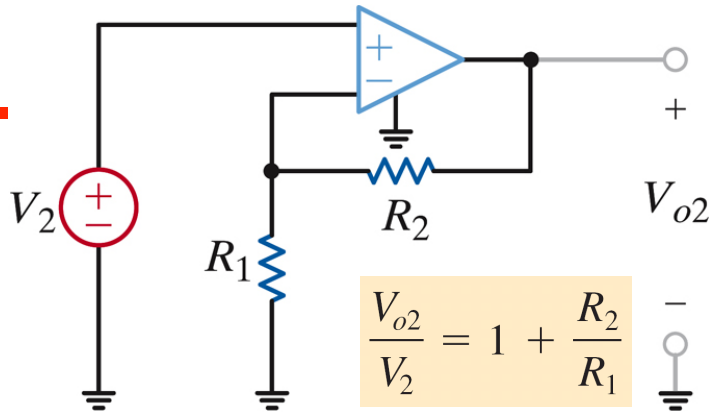


$$V_o = \left[1 + \frac{R_2}{R_1}\right]V_2 - \left[\frac{R_2}{R_1}\right]V_1$$



$$\frac{V_{o1}}{V_1} = -\frac{R_2}{R_1}$$

+



$$\frac{V_{o2}}{V_2} = 1 + \frac{R_2}{R_1}$$

Applying Superposition Technique

- **In a network containing multiple independent sources, each source can be applied independently with the remaining sources turned off.**
- **To turn off a voltage source, replace it with a short circuit, and to turn off a current source, replace it with an open circuit.**
- **When the individual sources are applied to the circuit, all the circuit laws and techniques we have learned, or will soon learn, can be applied to obtain a solution.**
- **The results obtained by applying each source independently are then added together algebraically to obtain a solution.**

More examples of superposition

Go to examples 7-1.1 thru 7-1.3