ECE 203

Circuits I

First Order Transient Circuits

Lecture 10-1

Transient Circuits

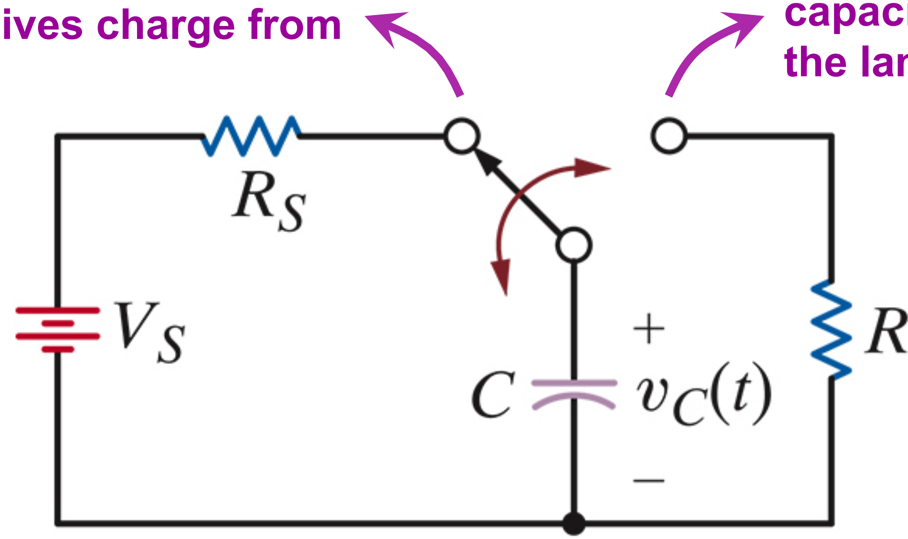
- **The voltage across a capacitor cannot change instantaneously; the current through an inductor cannot change instantaneously.**
- **So, voltages or currents in circuits containing capacitors or inductors do not change instantaneously; we need to determine the transient behavior of these circuits**
- **The application, or removal, of constant sources creates a transient behavior.**
- **In chapter 7 we will discuss:**
  - **FIRST ORDER CIRCUITS**
    - **Circuits that contain a single energy storage element, either a capacitor or an inductor.**
  - **SECOND ORDER CIRCUITS**
    - **Circuits with two energy storage elements in any combination**

An Introduction: Camera's Flash

- **Inductors and Capacitors can store energy. Under suitable conditions, this energy can be released. The rate at which it is released will depend on the parameters of the circuit connected to the terminals of the energy storing element.**

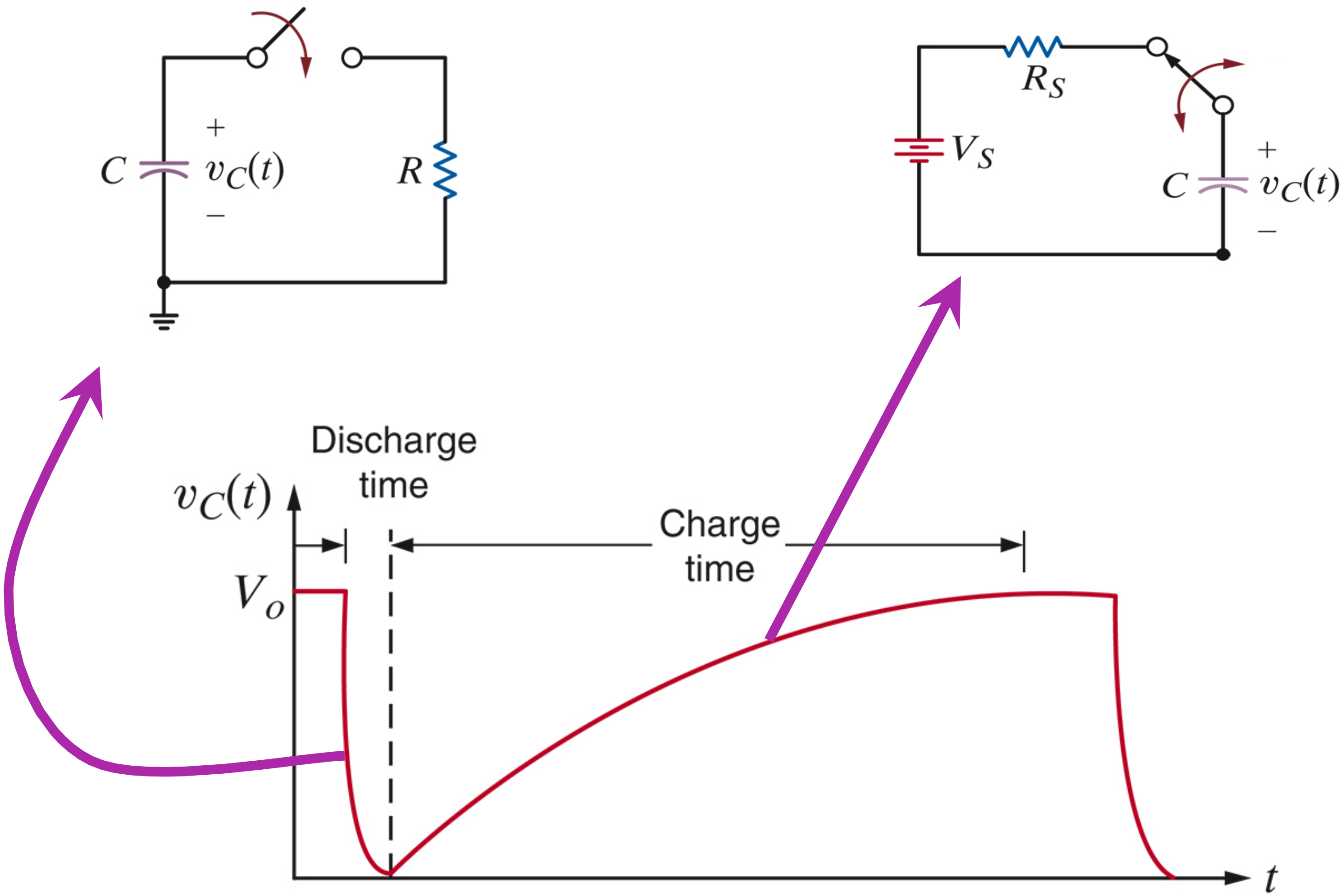
With the switch on the left the capacitor receives charge from the battery.

Switch to the right and the capacitor discharges through the lamp.



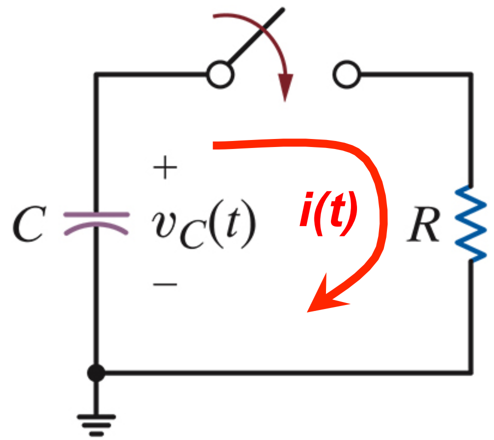
[(R) Xenon lamp]

An Introduction: Camera's Flash



Circuit Analysis During Discharge

The charged capacitor is disconnected from the source and connected to the linear time-invariant resistor with resistance *R* at *t=0*.

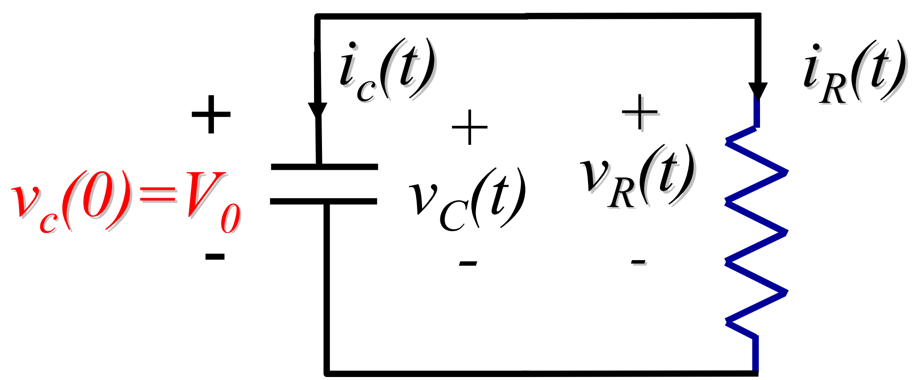


t ≥ 0

Let us describe physically what is going to happen. Because of the charges stored in the capacitor *(Q_0=CV_0)* a current will flow in the direction shown. The charge across the capacitor will decrease gradually and eventually will become zero; the current *i* will do the same. The time required to discharge the capacitor is determined by R. During the process the electric energy stored in the capacitor is dissipated as heat in the resistor.

Circuit Analysis During Discharge

Let us restrict our attention to *$t \geq 0$*, we redraw the RC circuit as shown below. *V_0* along with the positive and negative signs next to the capacitor, specify the magnitude and polarity of the initial voltages.



Kirchoff`'s laws and topology dictate the following equations:

KVL: $$v_C(t) = v_R(t) \qquad t \geq 0$$

KCL: $$i_C(t) + i_R(t) = 0 \qquad t \geq 0$$

Transient Analysis for Simple RC Circuit

KCL indicates that:

$$i_C(t) + i_R(t) = 0$$

We also know the current-voltage relationship for capacitors and resistors:

$$i_C(t) = C\frac{dv_C(t)}{dt} \qquad \text{and} \qquad i_R(t) = \frac{v_C(t)}{R}$$

This results in:

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0 \qquad \text{or} \qquad \frac{dv_C(t)}{dt} + \frac{1}{RC}v_C(t) = 0$$

The solution to this differential equation is:

$$v_C(t) = V_o e^{-t/RC}$$

V_o is the initial voltage on capacitor

General Response in First Order Circuits

Including the initial conditions the model for the capacitor voltage or the inductor current will be shown to be of the form

$$\frac{dx(t)}{dt} + ax(t) = f(t) \qquad\qquad x(t{=}0) = x_0$$

Differential Equation Review:

A fundamental theorem of differential equations states that the general solution to this equation can be written as:

$$x(t) = x_p(t) + x_c(t)$$

Where $x_p(t)$ is the *particular solution*, or *forced response*, and $x_c(t)$ is the *complementary solution* or *natural response*.

$$\frac{dx_p(t)}{dt} + ax_p(t) = f(t)$$

$$\frac{dx_c(t)}{dt} + ax_c(t) = 0$$

General Response in First Order Circuits

Let's assume that $f(t) = A$, i.e. a constant:

$$\frac{dx_p(t)}{dt} + ax_p(t) = A$$

$$\frac{dx_c(t)}{dt} + ax_c(t) = 0$$

The solution to this differential equation is:

$$\begin{cases} x_p(t) = K_1 & \textbf{where} \quad K_1 = \dfrac{A}{a} \\ x_c(t) = K_2 e^{-at} \end{cases}$$

Thus $x(t) = x_p(t) + x_c(t)$

$$= \frac{A}{a} + K_2 e^{-at}$$

K_2 is determined by the initial condition.

First Order Circuits

Any variable in a first-order transient circuit can be written in the form of:

$$y(t) = K_1 + K_2 e^{-\frac{t-t_0}{\tau}} \quad ; \quad t > t_0$$

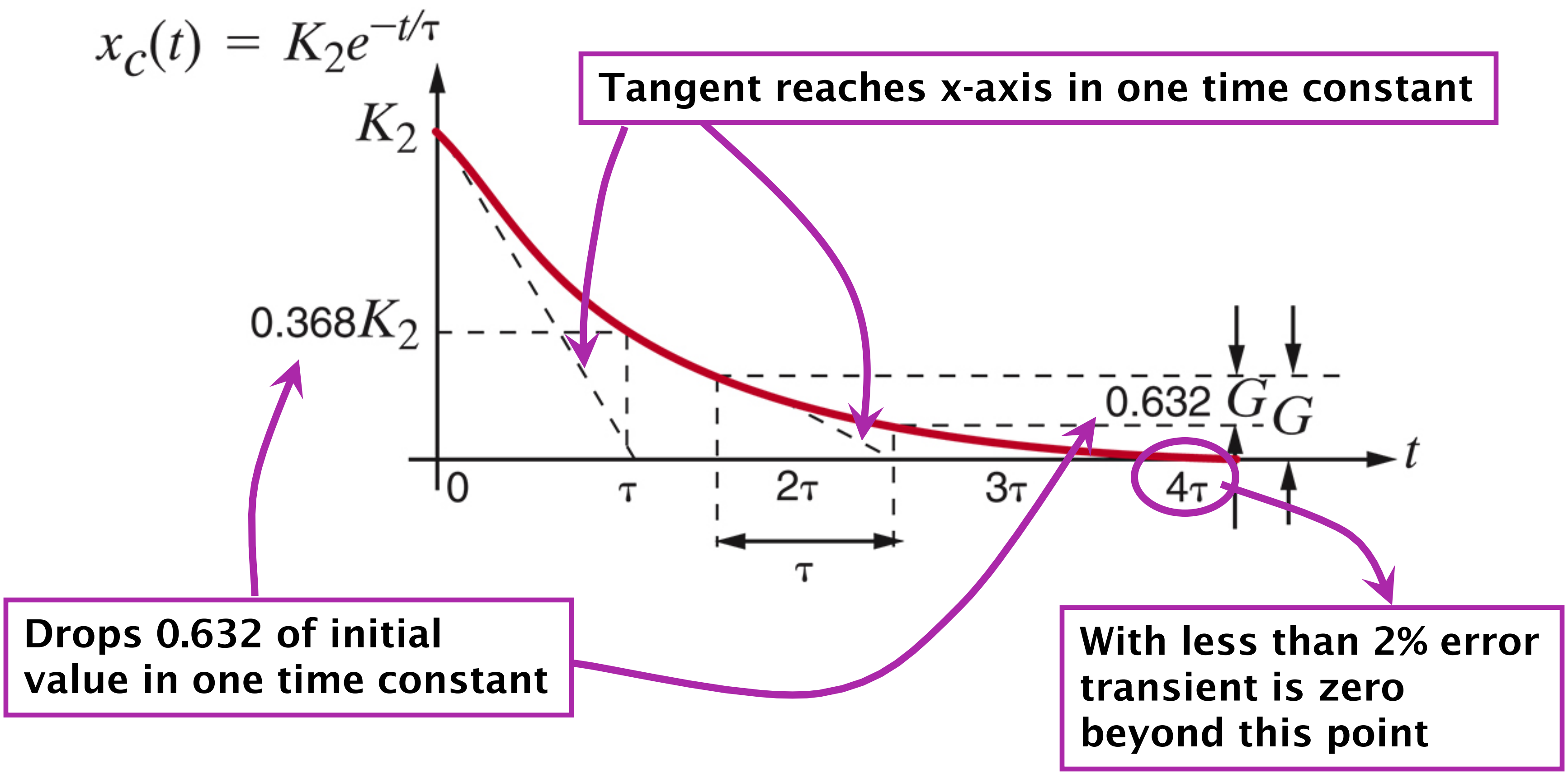
K_1 is called steady-state solution because when $t \to \infty$, $x(t) \to K_1$

τ is called time constant of the circuit. It represents the decay rate of the response.

t_0 **is the time of switching. Often switching occurs at** $t_0 = 0$

Time Constant, τ

Evolution of the transient and interpretation of the time constant



With less than 1% error the transient is negligible after five time constants

The Differential Equation Approach

Conditions:

1. **The circuit has only constant (in other words, DC) independent sources.**
2. **The differential equation for the variable of interest is simple to obtain. Normally using basic analysis tools such as KCL, KVL, or Thevenin equivalent**
3. **The initial condition (or boundary condition) for the differential equation is known, or can be obtained using steady state analysis**

For any variable, *y(t) (either a voltage or a current)* in the circuit, the solution is of the form:

$$\boxed{y(t) = K_1 + K_2 e^{-\frac{t-t_0}{\tau}} \quad ; \quad t > t_0}$$

Solution Strategy: **Use the differential equation and the initial conditions to find the parameters K_1, K_2, and τ**

Solution Strategy

Derive the differential equation for the circuit. It will be in the form of:

$$a_1 \frac{dy}{dt} + a_0 y = f \qquad y(0+) = y_0$$

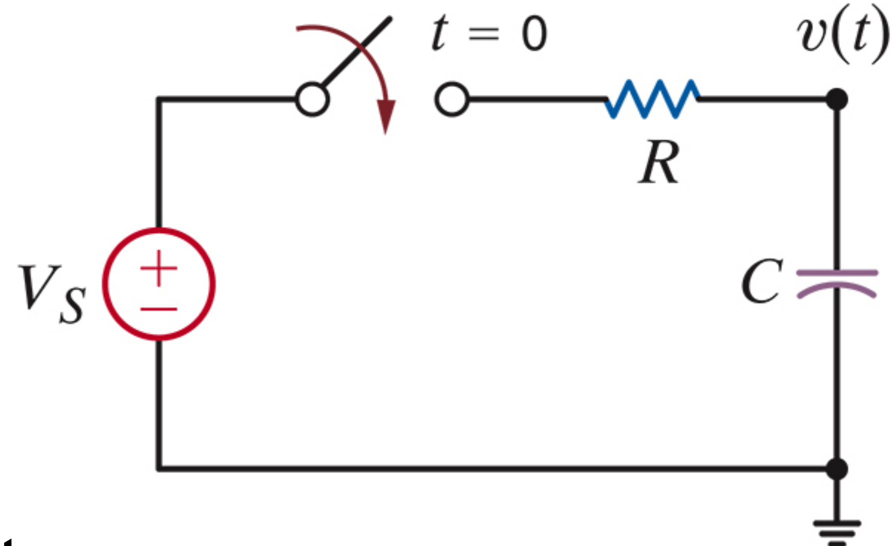
Assume that the form of the solution, y(t), is known. To find the unknown, K_1, K_2, and τ, replace the form of solution into the differential equation and the initial condition.

$$y(t) = K_1 + K_2 e^{-\frac{t}{\tau}}, t > 0 \Rightarrow \frac{dy}{dt} = -\frac{K_2}{\tau} e^{-\frac{t}{\tau}}$$

$$a_1\left(-\frac{K_2}{\tau} e^{-\frac{t}{\tau}}\right) + a_0\left(K_1 + K_2 e^{-\frac{t}{\tau}}\right) = f \quad \begin{cases} a_0 K_1 = f \Rightarrow \boxed{K_1 = \frac{f}{a_0}} \\ \left(-\frac{a_1}{\tau} + a_0\right) K_2 e^{-\frac{t}{\tau}} = 0 \Rightarrow \boxed{\tau = \frac{a_1}{a_0}} \end{cases}$$

$$y(0+) = K_1 + K_2 \Rightarrow \boxed{K_2 = y(0+) - K_1}$$

Example 1 – Find v(t)



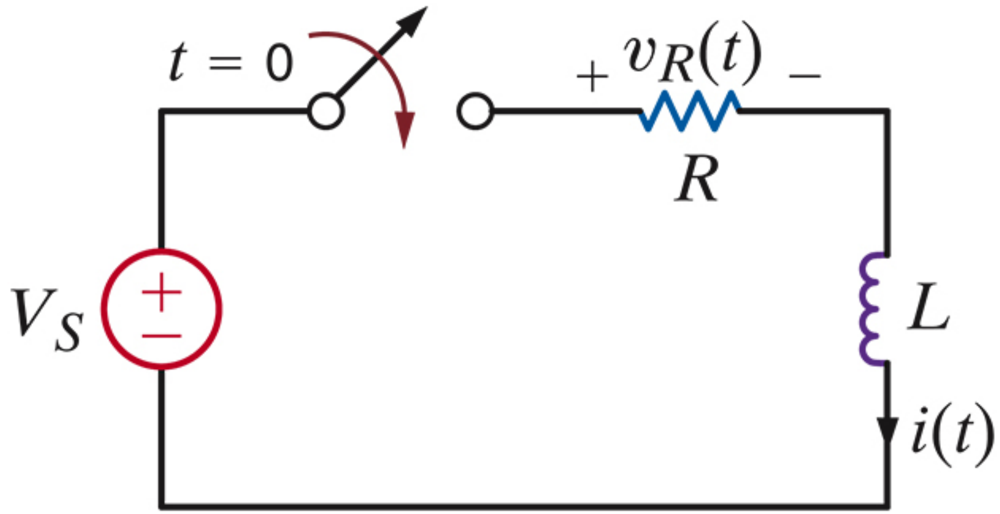
Using KCL at t>0 we get:

$$C\frac{dv(t)}{dt} + \frac{v(t) - V_S}{R} = 0 \quad \Rightarrow \quad \frac{dv(t)}{dt} + \frac{v(t)}{RC} = \frac{V_S}{RC} \quad ; \quad v(t{=}0) = 0$$

Substituting the known parametric solution into the differential equation we get:

$$v(t) = K_1 + K_2 e^{-t/\tau} \quad \Rightarrow \quad \begin{cases} K_1 = V_S \\ \tau = RC \\ K_2 = -V_S \end{cases} \quad \Rightarrow \quad v(t) = V_S - V_S e^{-t/RC}$$

Example 2 – Find $v_R(t)$



Using KVL at t>0 we get:

$$L\frac{di(t)}{dt} + Ri(t) = V_S \quad ; \quad i(t{=}0) = 0$$

Substituting the known form of the solution into the differential equation we get:

$$i(t) = K_1 + K_2 e^{-t/\tau} \Rightarrow \begin{cases} K_1 = V_S/R \\ \tau = L/R \\ K_2 = -V_S/R \end{cases} \Rightarrow i(t) = \frac{V_S}{R} - \frac{V_S}{R} e^{-\frac{R}{L}t}$$