ECE 203

Circuits I

Power revisited

Lecture 13-2

Calculating Power for Sinusoidal Signals

$$v(t) = V_M \cos(\omega t + \theta_v)$$

$$i(t) = I_M \cos(\omega t + \theta_i)$$

$$p(t) = v(t)i(t)$$

Power delivered or absorbed varies with time

$$= V_M I_M \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

Instantaneous power at any time t can be calculated using this expression.

Power calculations continued

From previous expression, using trig identity:

$$p(t) = \frac{V_M I_M}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

Can also calculate instantaneous power using this expression.

Average Power (P)

- To calculate average power, integrate p(t) over one cycle and divide by period

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\ dt = \frac{1}{T}\int_{t_0}^{t_0+T} \left[V_M \cos(\omega t + \theta_v)\right]\left[I_M \cos(\omega t + \theta_i)\right] dt$$

$$= \frac{1}{2} V_M\, I_M \cos(\theta_v - \theta_i)$$

- Can use this to calculate power to entire circuit, or to individual circuit elements
- Recall that passive sign convention says:

  P > 0, power is being absorbed

  P < 0, power is being supplied

Average Power: Special Cases

- **Purely resistive circuit**: $\theta_v = \theta_i$

  $P = \frac{1}{2} V_M I_M$

  The power dissipated in a resistor is

$$P = \frac{1}{2} V_M I_M = \frac{V_M^2}{2R} = \frac{1}{2} I_M^2 R$$

- **Purely reactive circuit**: $\theta_v - \theta_i = \pm 90^o$

  $P = 0$

  - Capacitors and inductors are lossless elements and absorb no average power
  - A purely reactive network operates in a mode in which it stores energy over one part of the cycle and releases it over another part

Average Power Summary

Circuit Element	Average Power
V or I source	$P = ½\, V_M I_M \cos(\theta_v - \theta_i)$
Resistor	$P = ½\, V_M I_M = ½\, I_M^2 R$
Capacitor or Inductor	$P = 0$

Does the expression for the resistor power look identical to that for DC circuits?

Example

- Go to example 13-2.1

Maximum Power Transfer in ac Circuits

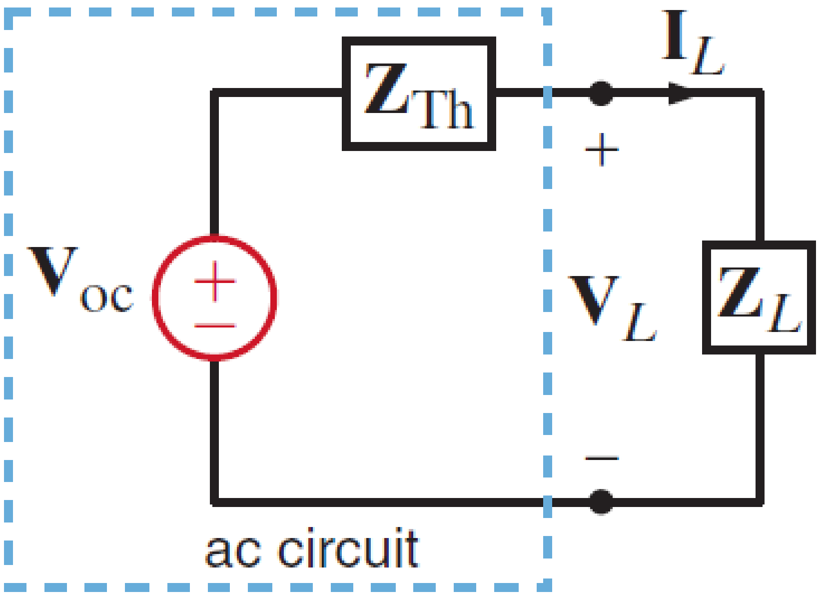
Power delivered to a load:

$$P_L = \frac{1}{2} V_L I_L \cos(\theta_{v_L} - \theta_{i_L})$$

$$\theta_{v_L} - \theta_{i_L} = \theta_{\mathbf{Z}_L}$$

$$Z_L = R_L + jX_L$$

So, $$\cos\theta_{\mathbf{Z}_L} = \frac{R_L}{(R_L^2 + X_L^2)^{1/2}}$$



$$\mathbf{I}_L = \frac{\mathbf{V}_{\text{oc}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L}$$

$$\mathbf{V}_L = \frac{\mathbf{V}_{\text{oc}}\mathbf{Z}_L}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L}$$

$$\mathbf{Z}_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}}$$

$$\mathbf{Z}_L = R_L + jX_L$$

Magnitude of current and voltage at the load:

$$I_L = \frac{V_{\text{oc}}}{\left[\left(R_{\text{Th}} + R_L\right)^2 + \left(X_{\text{Th}} + X_L\right)^2\right]^{1/2}}$$

$$V_L = \frac{V_{\text{oc}}\left(R_L^2 + X_L^2\right)^{1/2}}{\left[\left(R_{\text{Th}} + R_L\right)^2 + \left(X_{\text{Th}} + X_L\right)^2\right]^{1/2}}$$

Finding Max Power Transfer to Load

Magnitude of the power to the load:

$$P_L = \frac{1}{2} \frac{V_{\mathrm{oc}}^2 R_L}{\left(R_{\mathrm{Th}} + R_L\right)^2 + \left(X_{\mathrm{Th}} + X_L\right)^2}$$

R_{Th} and R_L are real and positive; X_{Th} and X_L are imaginary and can be positive or negative.

So P_L will be max when $X_L = -X_{Th}$

Max power transfer continued

$$\boxed{P_L = \frac{1}{2}\frac{V_{\mathrm{oc}}^2 R_L}{(R_L + R_{\mathrm{Th}})^2}}$$

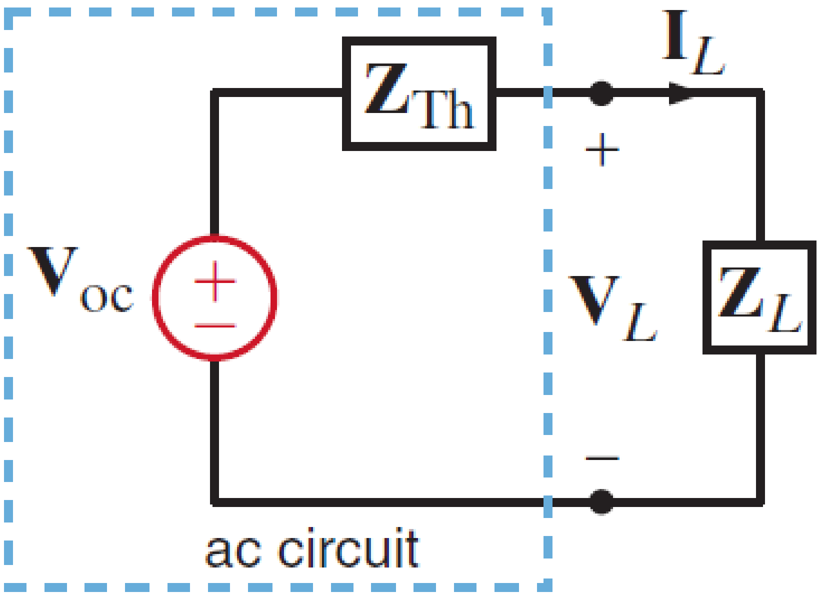
Just like when we had a purely resistive circuit, max power transfer when $R_L = R_{Th}$. So:

$$P_{L,Max} = \frac{1}{8}\frac{V_{oc}^2}{R_{Th}}$$

Impedance for Max Power Transfer

$$\mathbf{Z}_L = R_L + jX_L = R_{\mathrm{Th}} - jX_{\mathrm{Th}} = \mathbf{Z}_{\mathrm{Th}}^*$$

So, max power transfer occurs when the load is equal to the complex conjugate of the Thevenin impedance of the driving circuit.



Maximum Average Power Transfer

To find the load for maximum power transfer:

Step 1. Remove the load $\mathbf{Z}_L$ and find the Thévenin equivalent for the remainder of the circuit.

Step 2. Construct the circuit shown in Fig. 9.6.

Step 3. Select $\mathbf{Z}_L = \mathbf{Z}^*_{\text{Th}} = R_{\text{Th}} - jX_{\text{Th}}$, and then $\mathbf{I}_L = \mathbf{V}_{\text{oc}}/2\,R_{\text{Th}}$ and the maximum average power transfer $= \dfrac{1}{2}\mathbf{I}_L^2 R_{\text{Th}} = \mathbf{V}_{\text{oc}}^2/8\,R_{\text{Th}}$.

Example

- Go to example 13-2.2

Effective values of time-varying signals

We have already discussed how circuits react to dc and sinusoidal ac signals.

Can have a variety of different voltage or current sources exciting a circuit: square waves, triangle waves, …

Would like to talk about the “effective” voltage or current of these signals

Effective values of time-varying signals

Average power:

$$P = \frac{1}{T}\int_{t_o}^{t_o+T} i^2(t)R\,dt$$

Or: $P = I_{eff}^2 R$

So: $I_{eff} = \left[\frac{1}{T}\int_{t_o}^{t_o+T} i^2(t)\,dt\right]^{1/2}$

Effective or RMS Values

- Root-mean-square value (formula reads like the name: rms)

$$\boxed{I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\, dt}} \quad \textit{and} \quad \boxed{V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\, dt}}$$

- For a sinusoid: **$I_{rms} = I_M/\sqrt{2}$**
  - For example, AC household outlets are around 120 Volts-rms

Why RMS Values?

- The effective/rms current allows us to write average power expressions like those used in dc circuits (i.e., $P=I^2R$); that relation is really the basis for defining the rms value
- The average power (P) is

$$P_{source} = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i) = \boxed{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}$$

$$P_{resistor} = \frac{1}{2} V_M I_M = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

- Go to example 13-2.3

Power Factor

- P*ower factor* (0 ≤ *pf* ≤ 1)

$$pf = \frac{average\ power}{apparent\ \ power} = \frac{P}{V_{rms}\ \ I_{rms}}$$

$$= \frac{V_{rms}\ \ I_{rms}\ \ \cos(\theta_v - \theta_i)}{V_{rms}\ \ I_{rms}} = \cos(\theta_v - \theta_i) = \cos(\theta_{Z_L})$$

- A low power factor requires more rms current for the same load power which results in greater utility transmission losses in the power lines, therefore utilities penalize customers with a low *pf*

Power Factor Angle θ_{ZL}

- *power factor angle* is $\theta_v - \theta_i = \theta_{Z_L}$ (the phase angle of the load impedance)
- *power factor* (*pf*) special cases
  - purely resistive load: $\theta_{Z_L} = 0°$ $\Rightarrow$ *pf*=1
  - purely reactive load: $\theta_{Z_L} = \pm 90°$ $\Rightarrow$ *pf*=0

Power Factor Angle	I/V Lag/Lead	Load Equivalent
$-90^\circ < \theta_{Z_L} < 0^\circ$	Leading	Equivalent RC
$0^\circ < \theta_{Z_L} < 90^\circ$	Lagging	Equivalent RL

- Go to example 13-2.4