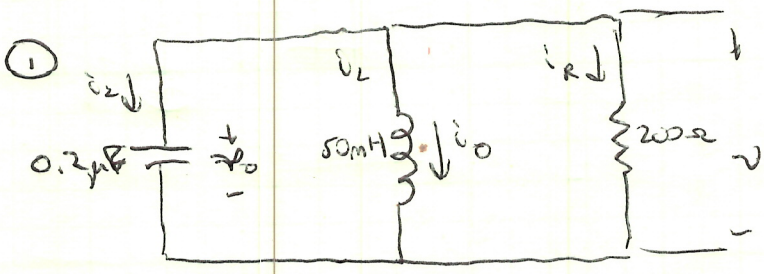
ENGR 2910 - 101
HOMEWORK 9 SOLUTIONS

① 

$v_0 = 15V$

$i_0 = -45mA$

a) $i_R(0) = \frac{15V}{200\Omega} = 75mA$

$i_L(0) = -45mA$

$i_c(0) = -i_L(0) - i_R(0) = -30mA$

b) $\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times10^{-6})} = 12,500$

$\omega_0^2 = \frac{1}{LC} = \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8$

$\alpha^2 > \omega_0^2$: overdamped

$s_{1,2} = -12,500 \pm \sqrt{(12,500)^2 - 10^8}$

$s_1 = -5000 \frac{rad}{s}$ $s_2 = -20,000 \frac{rad}{s}$

$v(t) = A_1 e^{-5000t} + A_2 e^{-20,000t}$

$v(0) = A_1 + A_2 = 15$

$\frac{dv(0)}{dt} = -5000A_1 - 20000A_2 = \frac{(-30\times10^{-3})}{(0.2\times10^{-6})} = -1.5\times10^5 V$

$\Rightarrow A_1 = 10$ $A_2 = 5$

$v(t) = 10e^{-5000t} + 5e^{-20000t}$ V for $t \geq 0$

c) $i_c = C\frac{dv}{dt}$

$= (0.2\times10^{-6})\left[-50,000e^{-5000t} - 100000e^{-20000t}\right]$

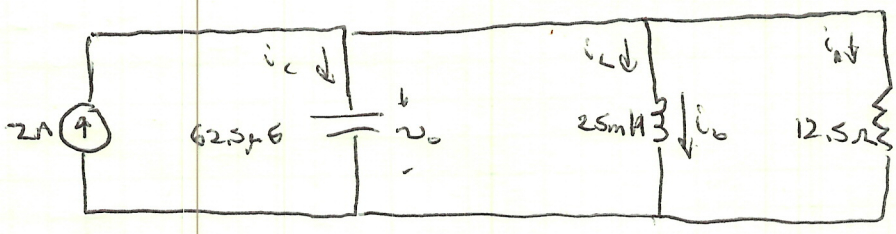
$= -10e^{-5000t} - 20e^{-20000t}$ mA

$i_R = 50e^{-5000t} + 25e^{-20000t}$ mA

$i_L = -i_c - i_R$

$= -40e^{-5000t} - 5e^{-20000t}$ mA for $t \geq 0$

(2)



$i_o = 1\text{mA}$

$v_o = 50\text{V}$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(12.5)(62.5\times10^{-6})} = 640 \text{ rad/s}$$

$$\omega_o = \frac{1}{LC} = \frac{1}{(25\times10^{-3})(62.5\times10^{-6})} = 64\times10^4 \text{ rad}^2/\text{s}^2$$

$\alpha^2 < \omega_o^2$: UNDERDAMPED

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 480 \text{ rad/s}$$

$I_f = 2\text{A}$

$$i_L(t) = 2 + B_1' e^{-640t}\cos(480t) + B_2' e^{-640t}\sin(480t)$$

$$i_L(0) = 2 + B_1' = 1 \Rightarrow B_1' = -1$$

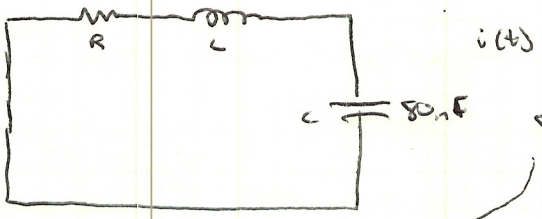
$$\frac{di_L(0)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{v_o}{L}$$

$$= -640(-1) + 480 B_2' = \frac{50}{25\times10^{-3}}$$

$$\Rightarrow B_2' = 2.83$$

$$i_L(t) = 2 - e^{-640t}\cos(480t) + 2.83 e^{-640t}\sin(480t) \text{ A} \quad t \geq 0$$

(3)



$$i(t) = B_1 e^{-2000t}\cos(1500t) + B_2 e^{-2000t}\sin(1500t)$$

$\alpha = 2000 \text{ rad/s}$ $\omega_d = 1500 \text{ rad/s} \Rightarrow \omega_o = 2500 \text{ rad/s}$

$$\alpha = \frac{R}{2L} = 2000$$

$$R = 4000L$$

$$\omega_o^2 = \frac{1}{LC} = 625\times10^9$$

$$L = \frac{1}{(625\times10^9)(80\times10^{-9})}$$

$\boxed{L = 2\text{H}}$ $\boxed{R = 8\text{k}\Omega}$

$$i_L(0) = \boxed{B_1 = 7.5\text{mA}}$$

$$v_L(0) = -(7.5\text{mA})(8\text{k}) = -60\text{V} \; (-30\text{V})$$

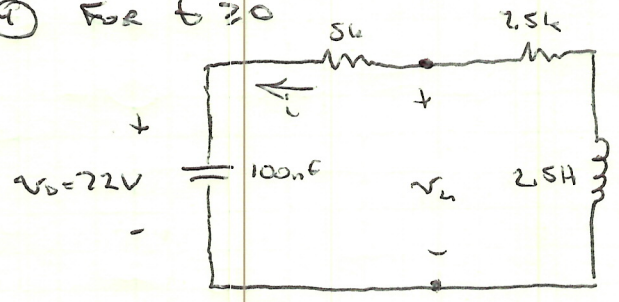
$$= 60 - 30$$

$$= -30\text{V}$$

$$\frac{di(0)}{dt} = 1500 B_1 - 2000 B_2 = \frac{-30}{2}$$

$\boxed{B_2 = 0}$

④ For $t \geq 0$



a) $i(0) = 0 \Rightarrow v_0 = 72V$

b) $v_a(t) = 5000i + \frac{1}{100nF}\int i\,di + 72$

$$\frac{dv_a(t)}{dt} = 5000\frac{di}{dt} + 10^7 i$$

$$\frac{dv_a(t)}{dt} = 5000\frac{di(0)}{dt} + 10^7(i(0))$$

$$= 5000\frac{di(0)}{dt}$$

$$-L\frac{di}{dt} = 72$$

$$\frac{di(0)}{dt} = \frac{-72}{2.5} = -28.8\ A/s$$

$$\Rightarrow \frac{dv_a(0)}{dt} = -144{,}000\ V/s$$

c) $\alpha = \frac{R}{2L} = \frac{12{,}500}{2(2.5)} = 2500\ rad/s$

$\omega_0^2 = \frac{1}{LC} = \frac{1}{2.5(100n)} = 4\times10^6\ rad^2/s^2$ OVERDAMPED

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - (4\times10^6)}$$

$s_1 = -1000 \quad s_2 = -4000$

$$v_a(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$v_a(0) = A_1 + A_2 = 72$$

$$\frac{dv_a(0)}{dt} = -1000A - 4000A = -144{,}000$$

$A_1 = 48 \quad A_2 = 24$

$$v_a(t) = 48e^{-1000t} + 24e^{-4000t} \quad t \geq 0^+$$