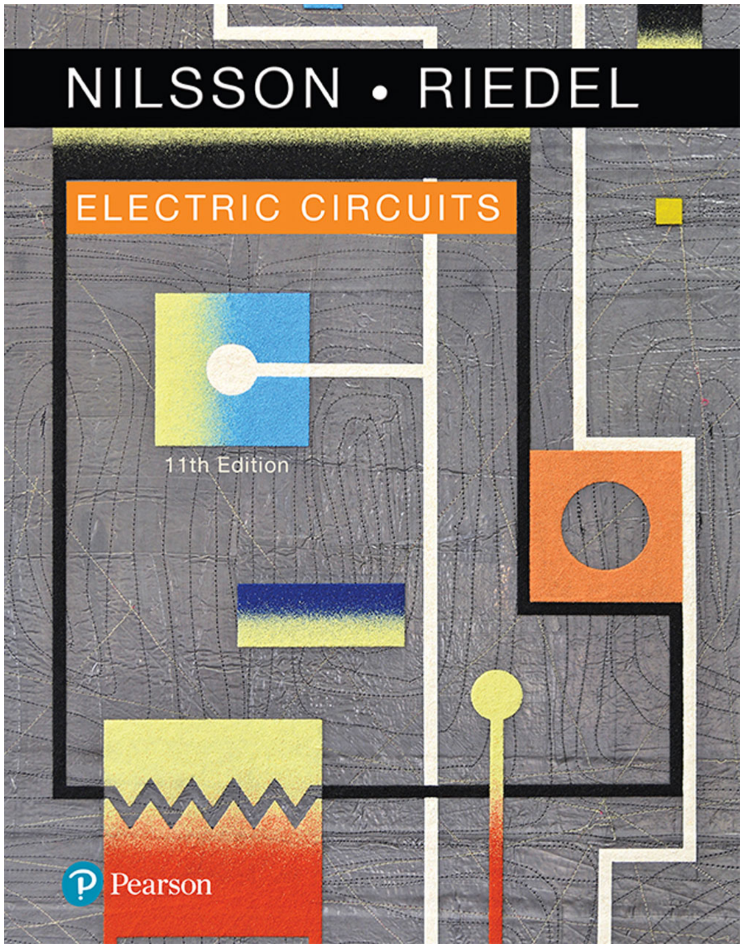
Electric Circuits

Eleventh Edition



Chapter 4

Techniques of Circuit Analysis

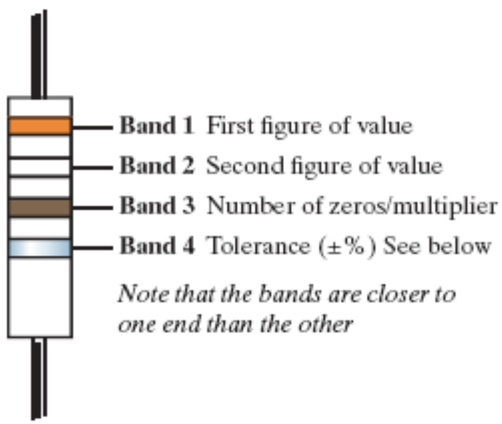
Practical Perspective – Circuits with Realistic Resistors

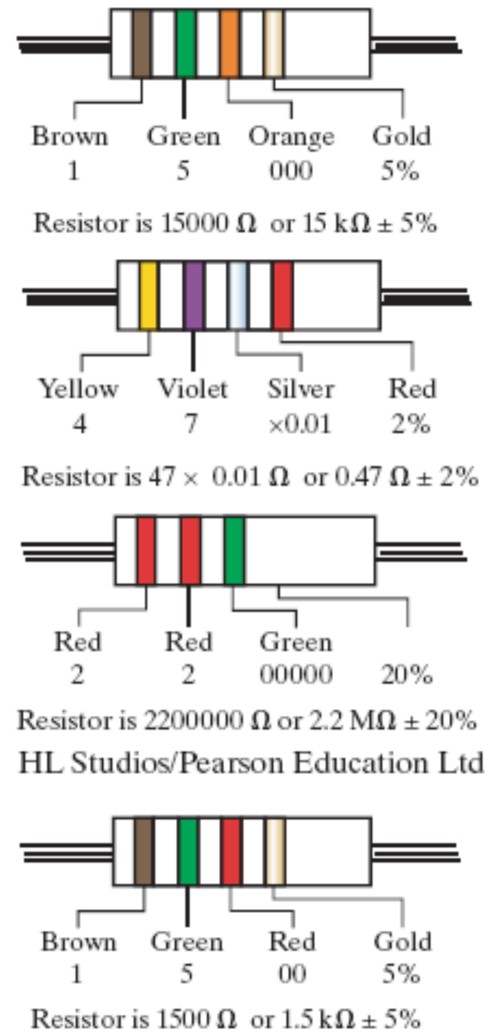
Resistor color code

Band color	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9
Gold	0.1
Silver	0.01

Tolerance color code

Band color	±%
Brown	1
Red	2
Gold	5
Silver	10
None	20





4.1 Terminology

Planar circuits: circuits that can be drawn on a plane with no crossing branches

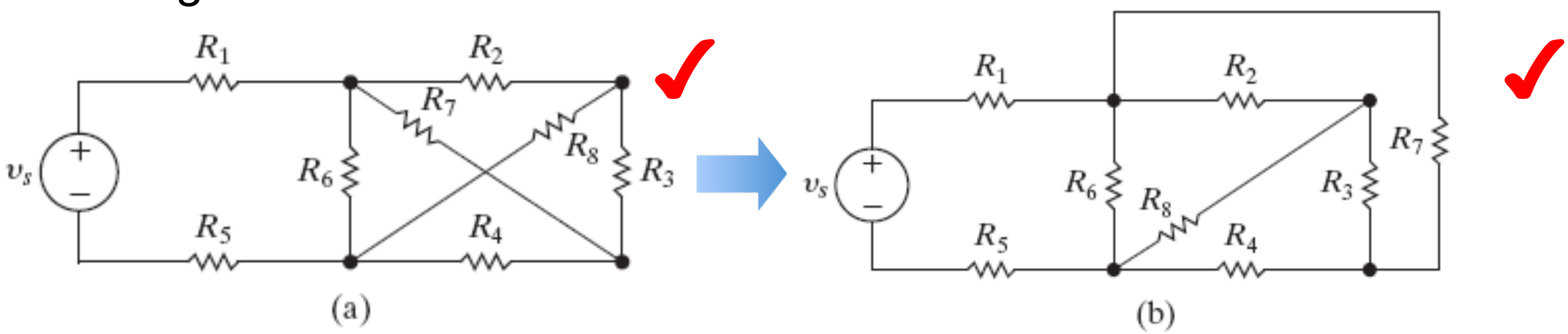


Figure 4.1: (a) A planar circuit. (b) The same circuit redrawn to verify that it is planar.

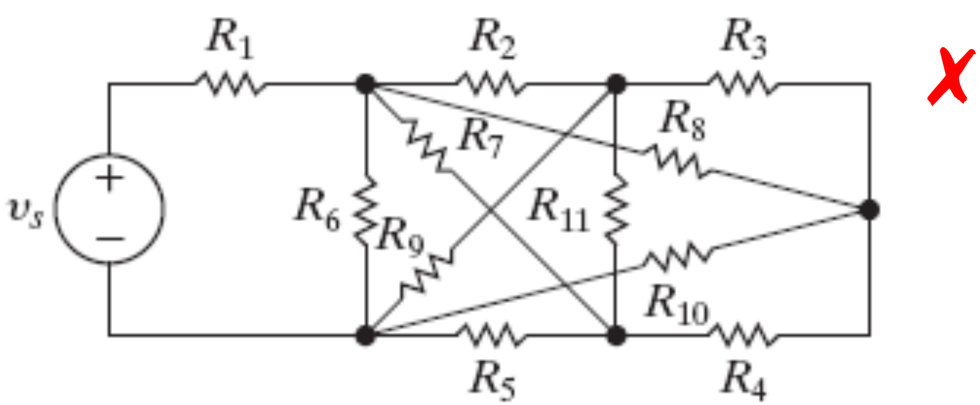


Figure 4.2: A nonplanar circuit.

More Terms for Describing Circuits

Name	Definition	Example from Fig. 4.3
node	A point where two or more circuit elements join	a
essential node	A node where three or more circuit elements join	b
path	A trace of adjoining basic elements with no elements included more than once	$v_1 - R_1 - R_5 - R_6$
branch	A path that connects two nodes	R_1
essential branch	A path which connects two essential nodes without passing through an essen:al node	$v_1 - R_1$
loop	A path whose last node is the same as the starting node	$v_1 - R_1 - R_5 - R_6 - R_4 - v_2$
mesh	A loop that does not enclose any other loops	$v_1 - R_1 - R_5 - R_3 - R_2$
planar circuit	A circuit that can be drawn on a plane with no crossing branches	Fig. 4.3 is a planar circuit Fig. 4.2 is a nonplanar circuit

TABLE 4.1 Terms for Describing Circuits

Simultaneous Equations

- b: number of branches
- n: number of nodes
- Kirchhoff's current law: n-1 independent equations
- Kirchhoff's voltage law: b-(n-1) independent equations

- b_e: number of essential branches
- n_e: number of essential nodes
- Kirchhoff's current law: n_e-1 independent equations
- Kirchhoff's voltage law: b_e-(n_e-1) independent equations

4.2 Node-Voltage Method

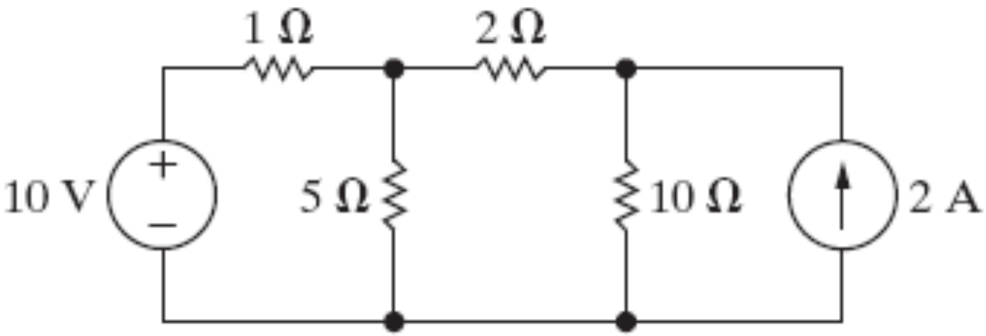


Figure 4.5: A circuit used to illustrate the node-voltage method of circuit analysis.

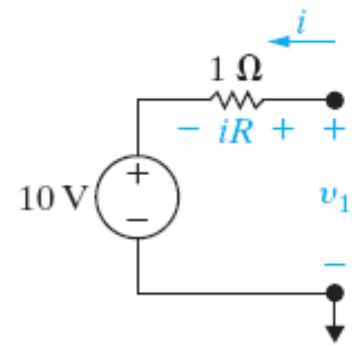


Figure 4.7: Computation of the branch current *i*.

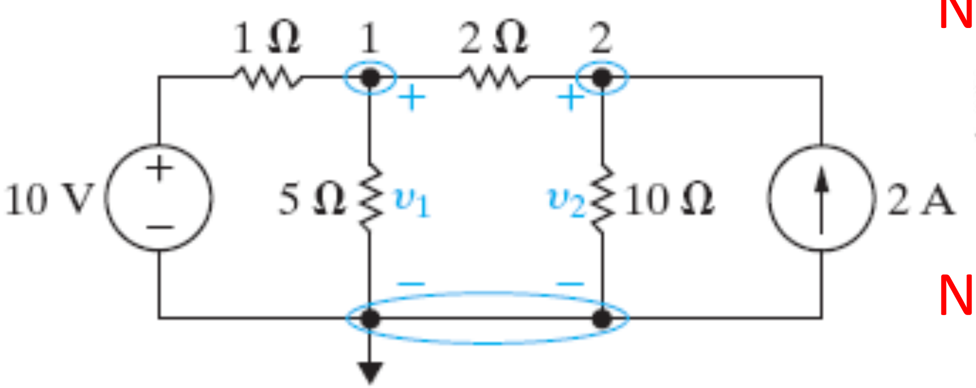


Figure 4.6: The circuit shown in Fig. 4.5 with a reference node and the node voltages.

Node 1:

$$\frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0$$

Node 2:

$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0$$

Pearson

4.3 Node-Voltage Method: Dependent Sources

Supplement the KCL equations with the constraint equation imposed by the dependent source.

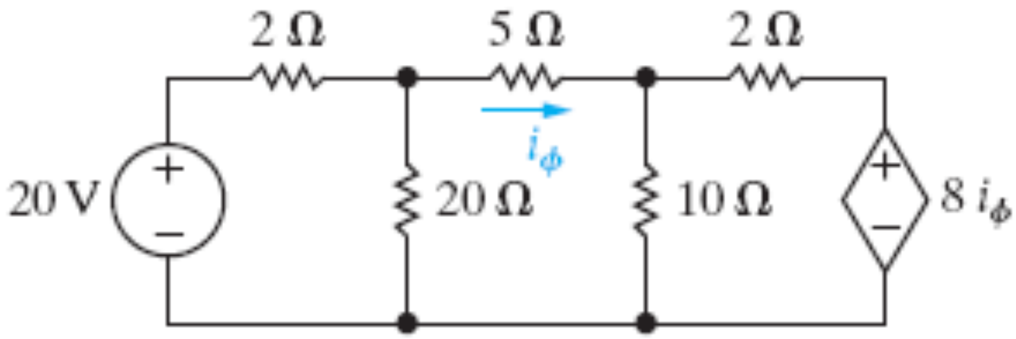


Figure 4.10: The circuit for Example 4.4.

4.4 Node-Voltage Method: Special Cases

Consider the special case in which a voltage drop source is the only element between two essential nodes.

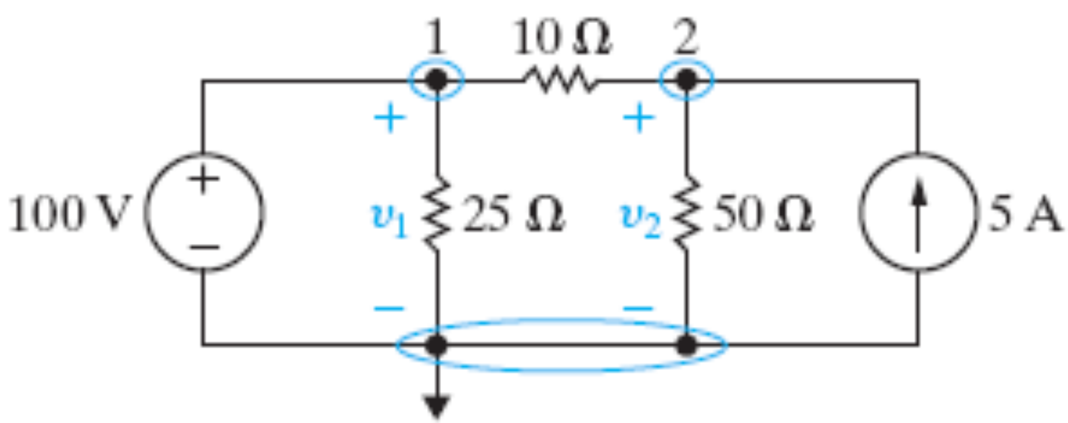


Figure 4.12: A circuit with a known node voltage.

Supernode

When a voltage source is between two essential nodes, we can combine those nodes and the source to form a supernode.

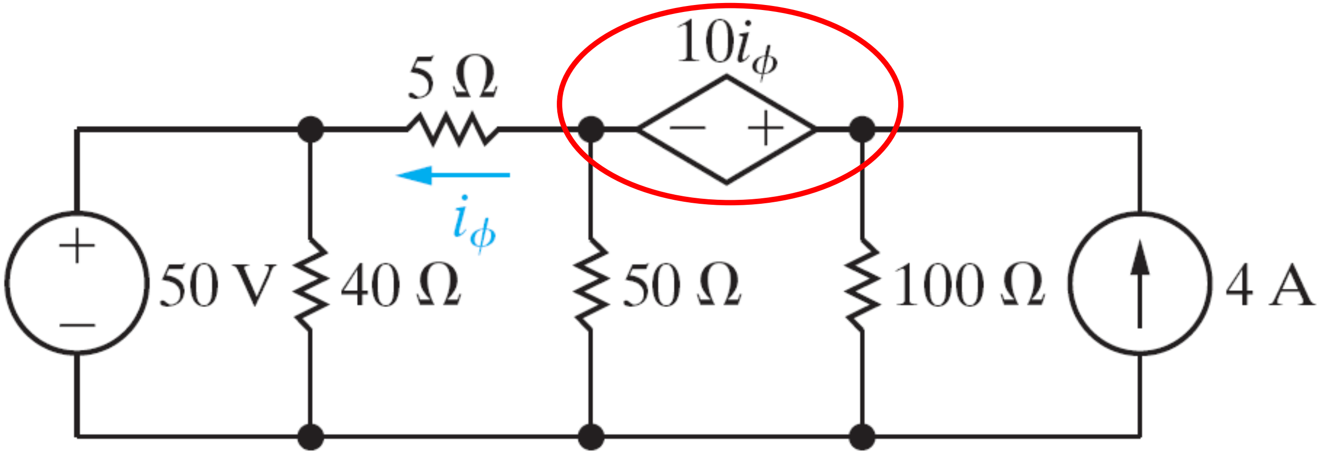


Figure 4.14: A circuit with a dependent voltage source connected between nodes.

4.5 Mesh-Current Method

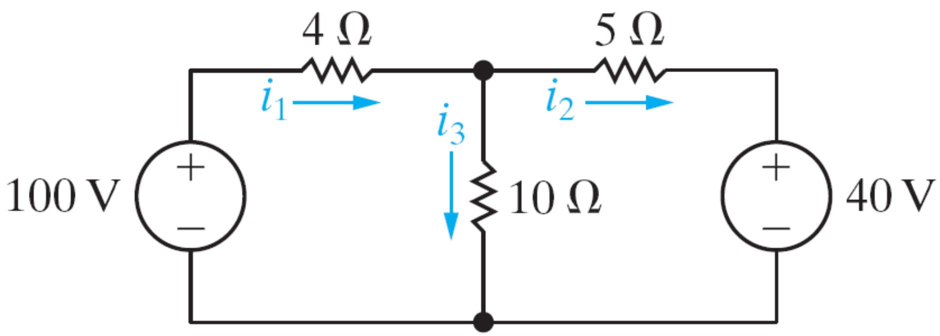


Figure 4.19: A circuit used to illustrate development of the mesh-current method of circuit analysis.

b_e=3 n_e=2 b_e-(n_e-1)=2

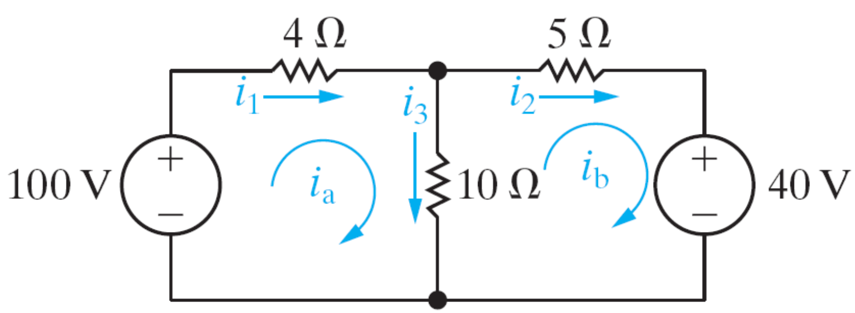


Figure 4.20: Mesh currents i_a and i_b.

$$v_1 = i_a R_1 + (i_a - i_b) R_3$$

$$-v_2 = (i_b - i_a) R_3 + i_b R_2$$

4.6 Mesh-Current Method: Dependent Sources

Supplement the KVL equations with the constraint equations imposed by the dependent sources.

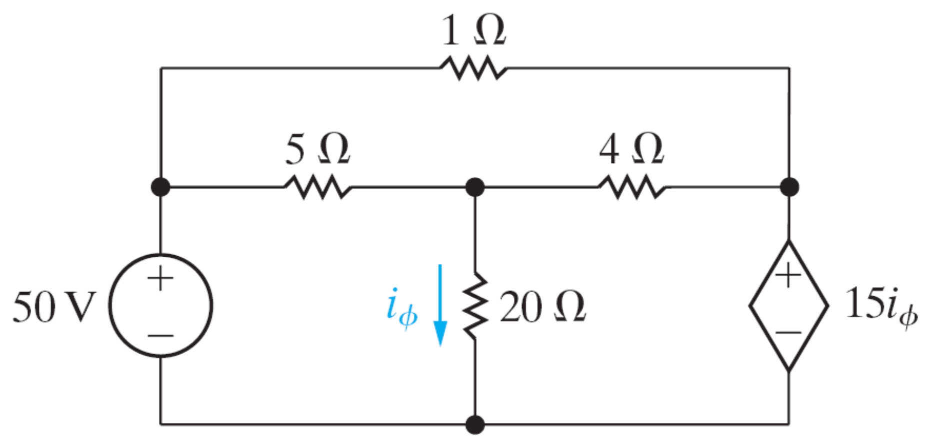


Figure 4.23: The circuit for Example 4.7.

4.7 Mesh-Current Method: Special Cases

Consider the special case in which a current source is in a single mesh, the value of the mesh current is known.

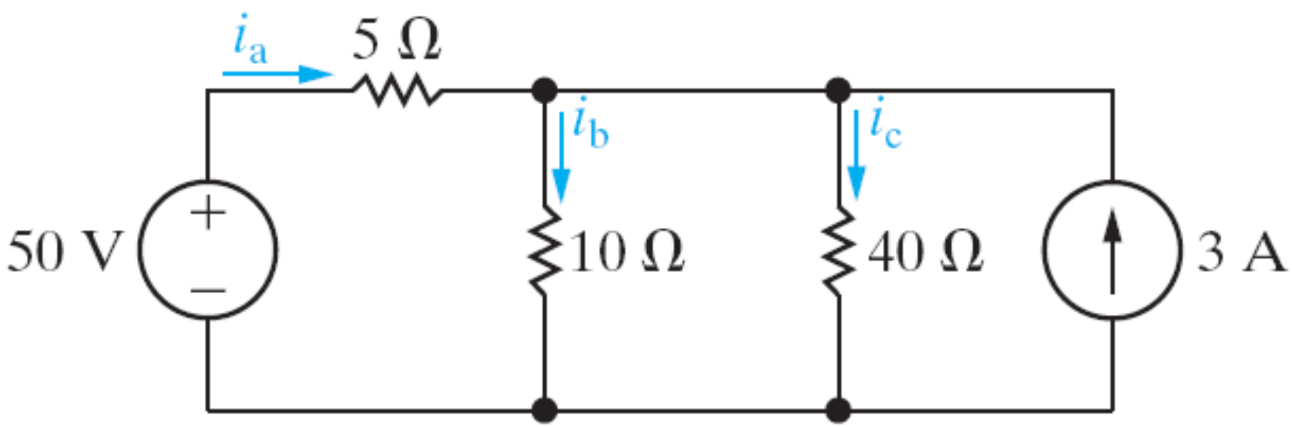


Figure 4.25: The circuit for Example 4.8.

Supermesh

When a current source is shared between two meshes, we can combine these meshes to form a supermesh.

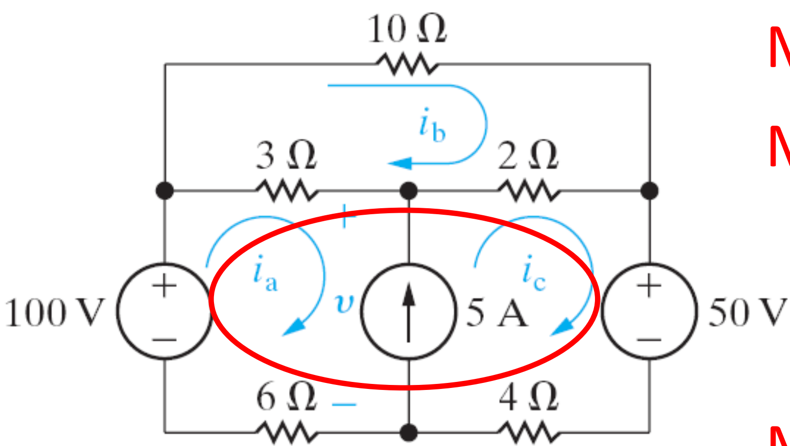


Figure 4.27: A circuit illustrating mesh analysis when a branch contains an independent current source.

Mesh a: $100 = 3(i_a - i_b) + v + 6i_a$

Mesh c: $-50 = 4i_c - v + 2(i_c - i_b).$

$$50 = 9i_a - 5i_b + 6i_c$$

Mesh b: $0 = 3(i_b - i_a) + 10i_b + 2(i_b - i_c)$

Constraint: $i_c - i_a = 5$

$$i_a = 1.75 \text{ A}, \quad i_b = 1.25 \text{ A}, \quad \text{and} \quad i_c = 6.75 \text{ A}.$$

4.8 Node-Voltage vs. Mesh-Current

- Does one of the methods result in *fewer simultaneous equations* to solve?
- Does the circuit *contain supernodes*? If so, using the node-voltage method will permit you to reduce the number of equations to be solved.
- Does the circuit *contain supermeshes*? If so, using the mesh-current method will permit you to reduce the number of equations to be solved.
- Will solving some portion of the circuit give the requested solution? If so, which method is most efficient for solving just the pertinent portion of the circuit?

TABLE 4.3 Steps in the Node-Voltage Method and the Mesh-Current Method

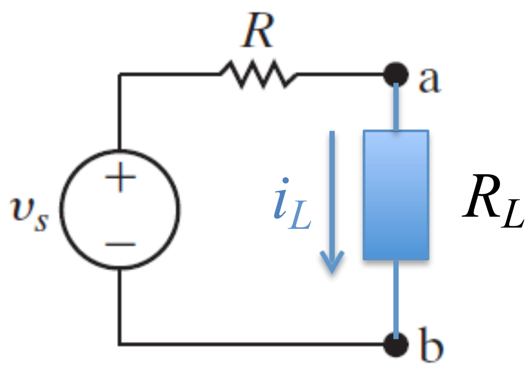
	Node-Voltage Method	Mesh-Current Method
Step 1 Identify nodes/meshes	Identify the essential nodes by circling them on the circuit diagram	Identify the meshes by drawing directed arrows inside each mesh
Step 2 Label node voltages/mesh currents Recognize special cases	• Pick and label a reference node; then label the remaining essential node voltages • If a voltage source is the only component in a branch connecting the reference node and another essential node, label the essential node with the value of the voltage source • If a voltage source is the only component in a branch connecting two nonreference essential nodes, create a supernode that includes the voltage source and the two nodes on either side	Label each mesh current • If a current source is in a single mesh, • If a current source is in a single mesh, label the mesh current with the value of the current source • If a current source is shared by two adjacent meshes, create a supermesh by combining the two adjacent meshes and temporarily eliminating the branch that contains the current source

Pearson

Step 3	Write the following equations:	Write the following equations:
Write the equations	• A KCL equation for any supernodes • A KCL equation for any remaining essential nodes where the voltage is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the node voltages • A constraint equation for each supernode that equates the difference between the two node voltages in the supernode to the voltage source in the supernode	• A KVL equation for any supermeshes • A KVL equation for any remaining meshes where the current is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the mesh currents • A constraint equation for each supermesh that equates the difference between the two mesh currents in the supermesh to the current source eliminated to form the supermesh
Step 4 Solve the equations	Solve the equations to find the node voltages	Solve the equations to find the mesh currents
Step 5 Solve for other unknowns	Use the node voltage values to find any unknown voltages, currents, or powers	Use the mesh current values to find any unknown voltages, currents, or powers

4.9 Source Transformations

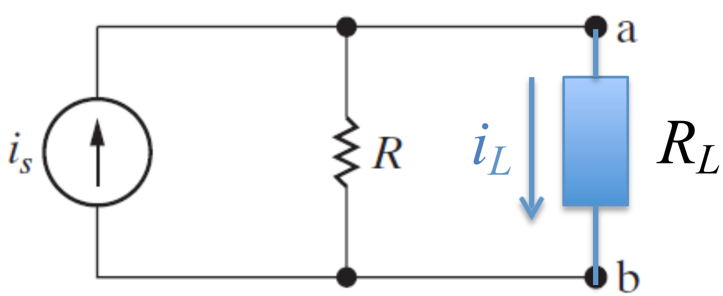
A source transformation allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor.



(a)

$$i_L = \frac{v_s}{R + R_L}$$

$$i_s = \frac{v_s}{R}$$



(b)

$$i_L = \frac{R}{R + R_L} i_s$$

Figure 4.38: Source transformations.

Resistors in series and parallel with sources?

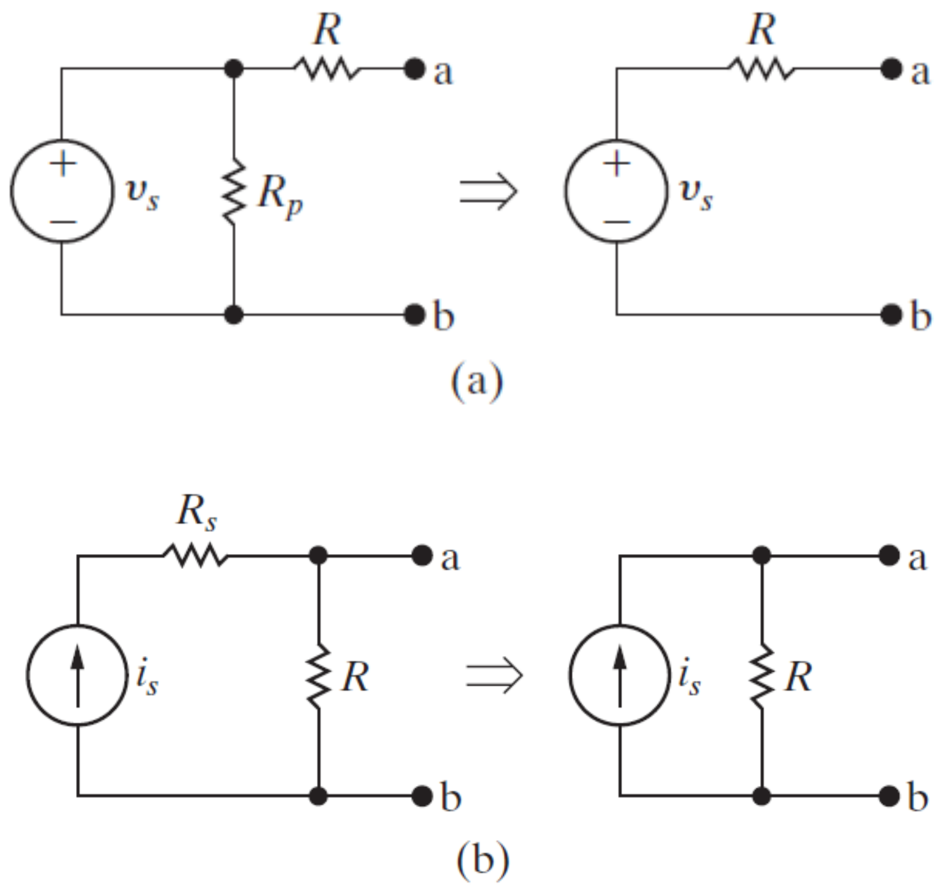


Figure 4.41: (a) Generating a simplified equivalent circuit from a circuit with a resistor in parallel with a voltage source; (b) generating a simplified circuit from a circuit with a resistor in series with a current source.

4.10 Thévenin & Norton Equivalents

- **Thévenin and Norton equivalents** are circuit simplification techniques that focus on terminal behavior and thus are extremely valuable aids in analysis.

- **Thevenin equivalent circuit:** an independent voltage source V_{Th} in series with a resistor R_{Th}, which replaces an interconnection of sources and resistors.

- **Norton equivalent circuit:** consists of an independent current source I_N in parallel with the Norton equivalent resistance, R_N.

Thévenin Equivalent

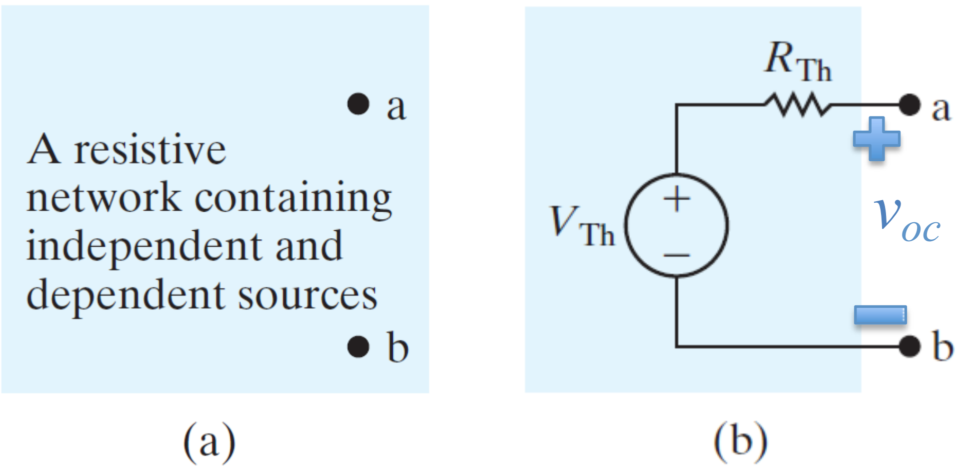
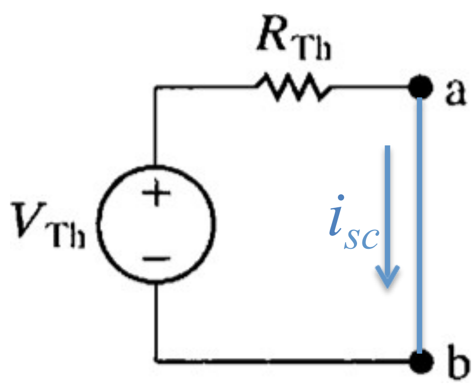


Figure 4.46: (a) A general circuit.
(b)The Thévenin equivalent circuit.

$$V_{Th} = v_{oc}$$

Thévenin voltage V_{Th} equals the **open-circuit voltage** of the original circuit.

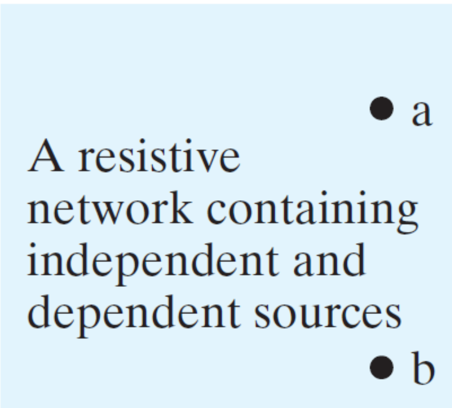
Léon Charles Thévenin
French telegraph engineer



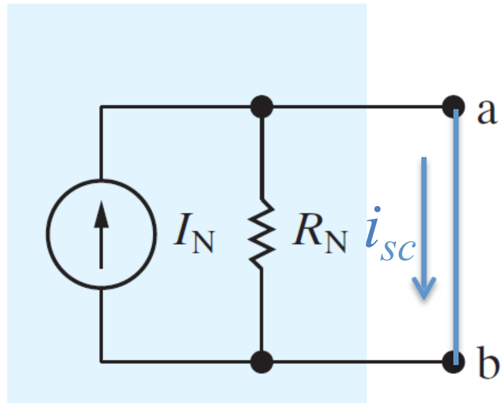
$$R_{Th} = V_{Th}/i_{sc} = v_{oc}/i_{sc}$$

Thévenin resistance R_{Th}: ratio of the **open-circuit voltage** to the **short-circuit current**.

Norton Equivalent



(a)



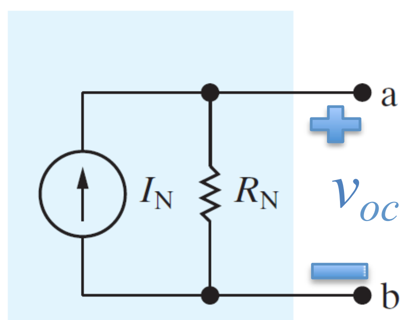
(b)

Figure 4.50: (a) A general circuit. (b)The Norton equivalent circuit.

$$I_N = i_{sc}$$

Norton current I_N equals the short-circuit current of the original circuit.

Edward Lawry Norton
Bell Labs engineer



(b)

$$R_N = v_{oc}/i_{sc} = R_N$$

Norton resistance R_N: ratio of the **open-circuit voltage** to the **short-circuit current.**

4.11 More on Thévenin Equivalent

Finding R_{Th} directly from the circuit for particular cases

***Only** Independent Sources*

1. **Deactivate** independent sources
   - Voltage source -> short circuit
   - Current source -> open circuit
2. Connect open circuit between a-b
3. Compute R_{ab} (= R_{Th}) as seen by terminals a-b

Test Sources

1. Deactivate independent sources
2. Connect test source across a-b:

   test voltage source

   test current source
3. $R_{Th} = v_T/i_T$

Dependent Sources Only

1. Both $V_{Th} = 0$ and $I_{Th} = 0$
2. Equivalent circuit is only R_{Th}
3. Apply test source method

4.12 Maximum Power Transfer

Power Transfer

- The first emphasizes the efficiency of power transfer.
- Power utility systems are a good example of this type because they are concerned with the generation, transmission, and distribution of large quantities of electric power.
- The second basic type of system emphasizes the amount of power transferred.
- Communication and instrumentation systems are good examples because in the transmission of information, or data, via electric signals, the power available at the transmitter or detector is limited

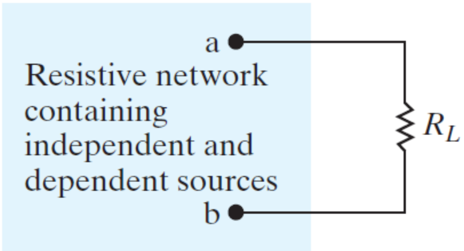


Figure 4.63: A circuit describing maximum power transfer.

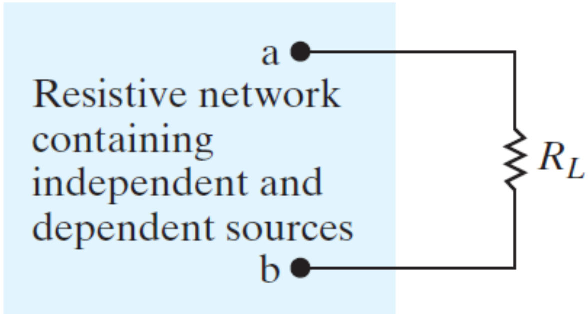


Figure 4.63: A circuit describing maximum power transfer.

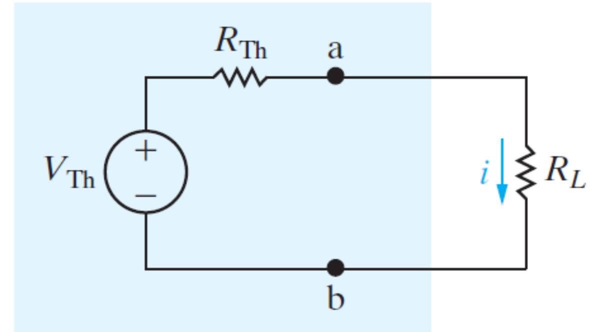


Figure 4.64: A circuit used to determine the value of R_L for maximum power transfer.

$$p = i^2 R_L = \left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} \right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{\mathrm{Th}}^2 \left[\frac{(R_{\mathrm{Th}} + R_{\mathrm{L}}) - 2R_L}{(R_{\mathrm{Th}} + R_L)^3} \right]$$

p is maximized when the derivative is zero, thus

$$\boxed{R_L = R_{\mathrm{Th}}}$$

$$\boxed{p_{\max} = \frac{V_{\mathrm{Th}}^2 R_L}{(2R_L)^2} = \frac{V_{\mathrm{Th}}^2}{4R_L}}$$

4.13 Superposition

A linear system obeys the principle of superposition: the total response is given by the sum of the individual responses.

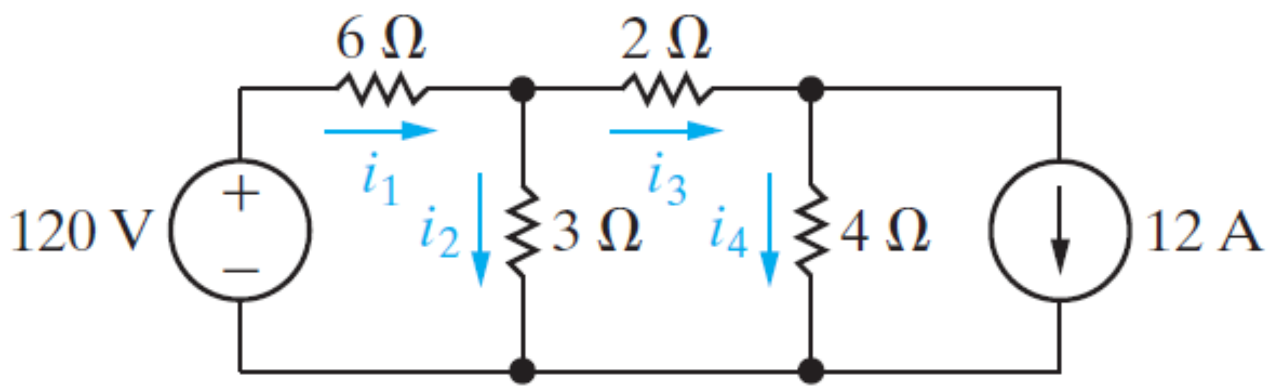


Figure 4.67: A circuit used to illustrate superposition.

In the above example, the branch currents $i_{1\text{-}4}$, derive from a contribution coming from the independent voltage source + a contribution from the independent current source.

Practical Perspective – Circuits with Realistic Resistors

Sensitivity Analysis

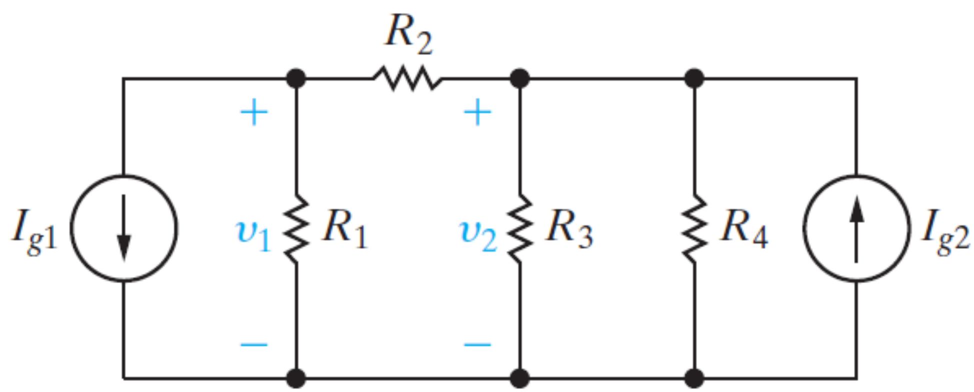


Figure 4.74: Circuit used to introduce sensitivity analysis.

$$v_1 = \frac{R_1\{R_3R_4I_{g2} - [R_2(R_3 + R_4) + R_3R_4]I_{g1}\}}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_1}{dR_1} = \frac{[R_3R_4 + R_2(R_3 + R_4)]\{R_3R_4I_{g2} - [R_3R_4 + R_2(R_3 + R_4)]I_{g1}\}}{[(R_1 + R_2)(R_3 + R_4) + R_3R_4]^2}$$

TABLE 4.2 PSpice Sensitivity Analysis Results

Element Name	Element Value	Element Sensitivity (Volts , Unit)	Normalized Sensitivity (Volts , Percent)
(a) DC Sensitivities of Node VoltageV1			
R1	25	0.5833	0.1458
R2	5	−5.417	−0.2708
R3	50	0.45	0.225
R4	75	0.2	0.15
IG1	12	−14.58	−1.75
IG2	16	12.5	2
(b) Sensitivities of Output V2			
R1	25	0.5	0.125
R2	5	6.5	0.325
R3	50	0.54	0.27
R4	75	0.24	0.18
IG1	12	−12.5	−1.5
IG2	16	15	2.4