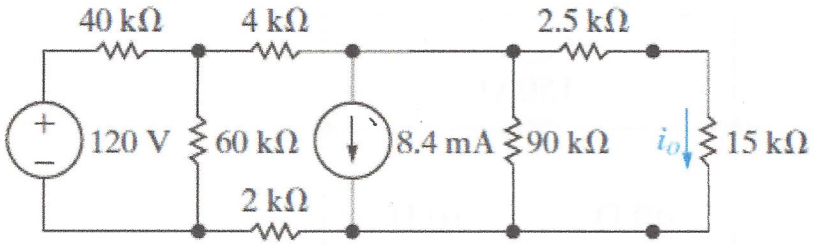
ENGR 2910-101: Circuit Analysis
Homework 9: 03/18/20

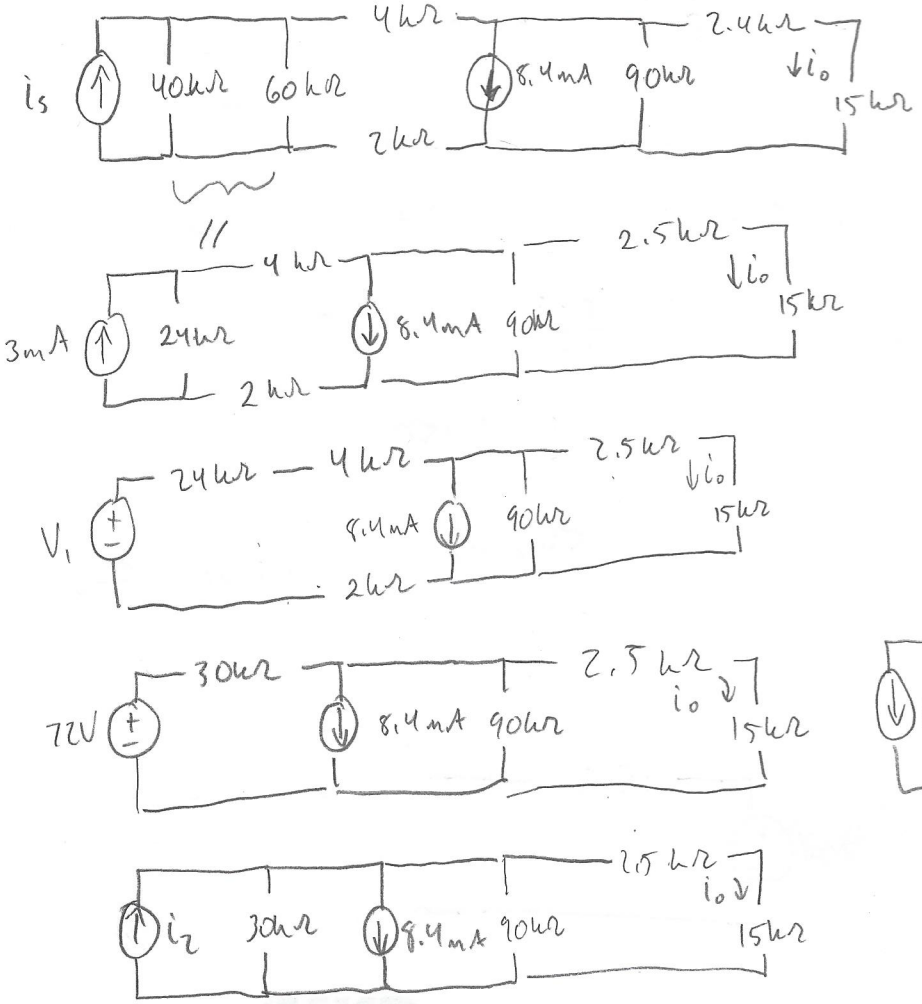
Instructor: Leo Silbert
Due: 03/25/20

Question 1 [10]



(i) Using several source transformations find the value of the current flowing through the 15 $k\Omega$ resistor. [Hint: start on the left side of the circuit and work your way right.]

(ii) Now that you know this current, work backwards through the original circuit and calculate the following: the voltage drop across the 90 $k\Omega$ and the current flowing through that branch; the current flowing through the 4 $k\Omega$ resistor, the voltage drop across the 60 $k\Omega$ resistor; and the current flowing in the left-hand part of the circuit.



$i_1 = \frac{120}{40k} = 3mA$

$R_1 = \frac{40(60)}{100} = 24\,k\Omega$

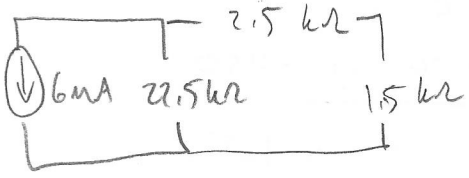
$V_1 = (3m)(24) = 72V$

$R_2 = 24 + 4 + 2 = 30k\Omega$

$i_2 = \frac{72}{30} = 2.4mA$

$R_3 = 30 \,||\, 90 = \frac{30(90)}{120} = 22.5\,k\Omega$

$i_3 = 8.4m - i_2 = 8.4 - 2.4 = 6mA$



$V_3 = (6m)\,22.5k = 135V$

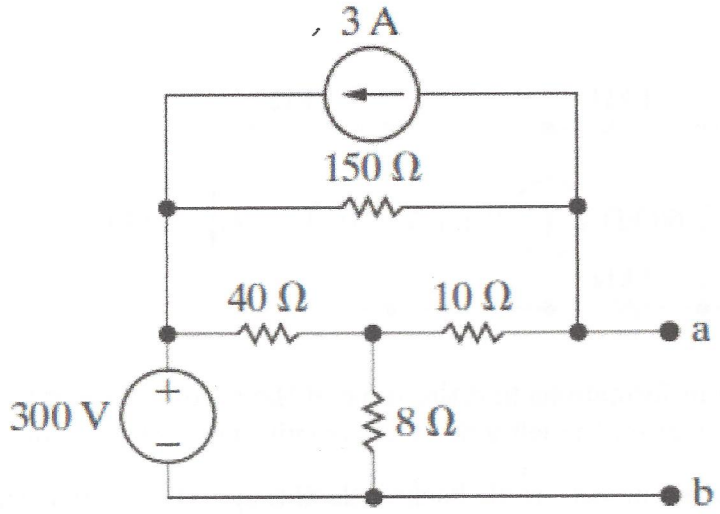
$i_o = -\frac{V_3}{22.5k + 2.5k + 15k} = -\frac{135}{40k} =$

$-3.375mA$

$i_o = -3.375mA$

Question 2 [10]

Find the Thévenin equivalent for the following circuit. [Hint: start off by making a source transformation then apply the mesh-current method.]



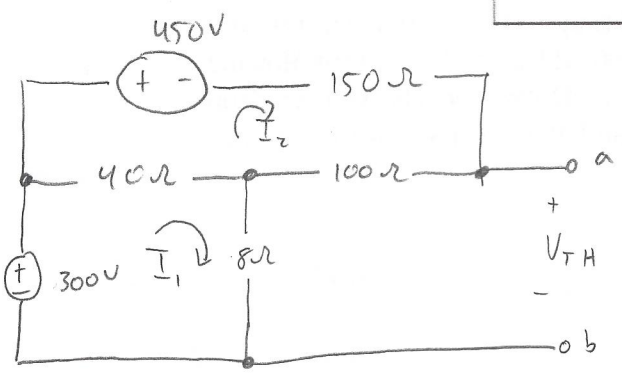
$V = 3(150)$

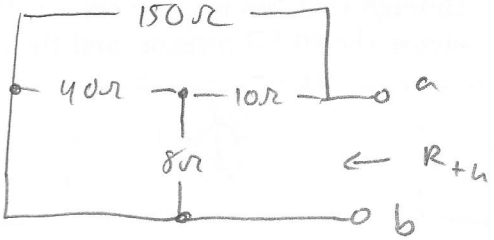
$= 450V$

$V_{TH} = 10I_2 + 8I_1$

$= 10(-1.2) + 8(5.25)$

$= 42 - 12 = 30V$





$R_{TH} = [(40 \| 8) + 10] \| 150$

$= \left(\frac{40 \times 8}{48} + 10\right) \| 150$

$= \frac{50}{3} \| 150$

$= \frac{\frac{50}{3} * 150}{\frac{50}{3} + 150} = 15\Omega = R_{TH}$

· Mesh current eqns:

$-300 + 40(I_1 - I_2) + 8I_1 = 0$

$48I_1 - 40I_2 = 300$

$450 + 150(I_2) + 10(I_2) + 40(I_2 - I_1) = 0$

$-40I_1 + 200I_2 = -450$

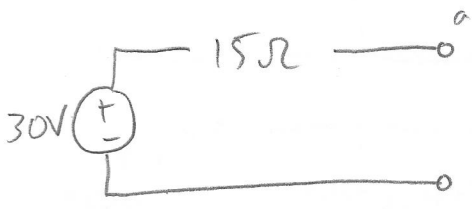
$240I_1 - 200I_2 = 1500$

$-40I_1 + 200I_2 = -450$

$200I_1 = 1050$

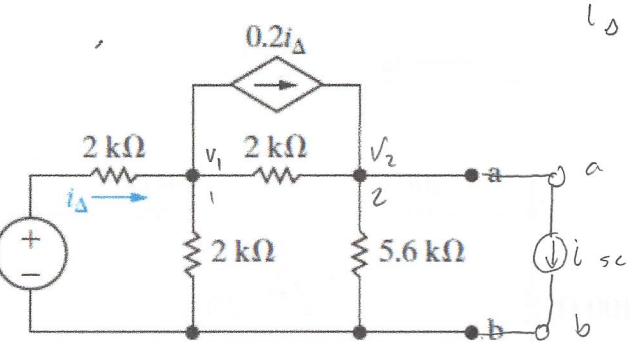
$I_1 = 5.25\ A$

$I_2 = \frac{48I_1 - 300}{40} = -1.2A$



Question 3 [10]

Find the Norton equivalent for the following circuit. [Hint: apply the node-voltage and mesh-current methods.]



$i_\Delta = \frac{280 - V_1}{2000}$ $\quad V_2 = 0V$

KCL node 1

$$\frac{V_1 - 280}{2000} + \frac{V_1}{2000} + \frac{V_1 - V_2}{2000} = -0.2 i_\Delta$$

$$\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) V_1 - \frac{280}{2000} - \frac{V_2}{2000} = -0.2 i_\Delta$$

$$\frac{3V_1 - 280 - V_2}{2000} = -0.2 i_\Delta$$

$$3V_1 - 280 - V_2 = -400 i_\Delta$$

$$3V_1 - 280 - \cancel{V_2}^{0} = -400\left(\frac{280 - V_1}{2000}\right)$$

$$3V_1 - 280 = -56 + 0.2V_1$$

$$2.8V_1 = 224$$

$$V_1 = 80\ V$$

$$i_\Delta = \frac{280 - V_1}{2000}$$

$$= \frac{280 - 80}{2000}$$

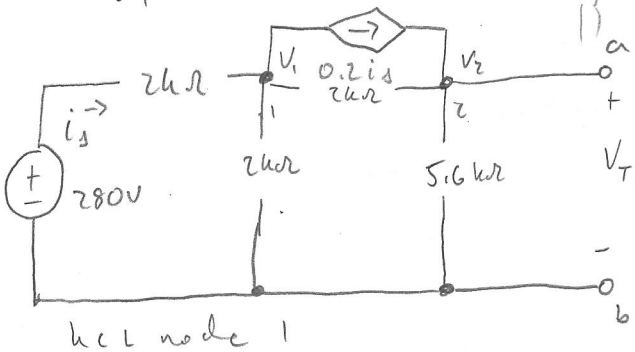
$$= 0.1\ A$$

KCL node 2

$$i_{sc} = 0.2 i_\Delta + \frac{V_1 - V_2}{2000} + \frac{V_2}{5600}$$

$$i_{sc} = 0.2(0.1) + \frac{80 - 0}{2000} + \frac{\cancel{0}^{0}}{5600}$$

$$i_{sc} = 0.06\ A$$



KCL node 2

$$\frac{V_2}{5600} + \frac{V_2 - V_1}{2000} = 0.2 i_\Delta$$

$$-\frac{V_1}{2000} + \left(\frac{1}{5600} + \frac{1}{2000}\right) V_2 = 0.2 i_\Delta$$

$$-\frac{14 V_1}{28000} + \left(\frac{5 + 14}{28000}\right) V_2 = 0.2 i_\Delta$$

$$-14V_1 + 19V_2 = 5600\left(\frac{280 - V_1}{2000}\right)$$

$$-14V_1 + 19V_2 = 784 - 2.8V_1$$

$$-11.2V_1 + 19V_2 = 784$$

$$-11.2\left(\frac{224 + V_2}{2.8}\right) + 19V_2 = 784$$

$$-896 - 4V_2 + 19V_2 = 784$$

$$V_2 = 112\ V$$

$$V_{TH} = 112\ V$$

KCL node 1

$$\frac{V_1 - 280}{2000} + \frac{V_1}{2000} + \frac{V_1 - V_2}{2000} = 0.2(i_\Delta)$$

$$\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{2000}\right) V_1 - \frac{V_2}{2000} - \frac{280}{2000} = 0.2 i_\Delta$$

$$\frac{3V_1 - V_2 - 280}{2000} = -0.2 i_\Delta$$

$$3V_1 - V_2 - 280 = -400\left(\frac{280 - V_1}{2000}\right)$$

$$3V_1 - V_2 - 280 = -56 + 0.2V_1$$

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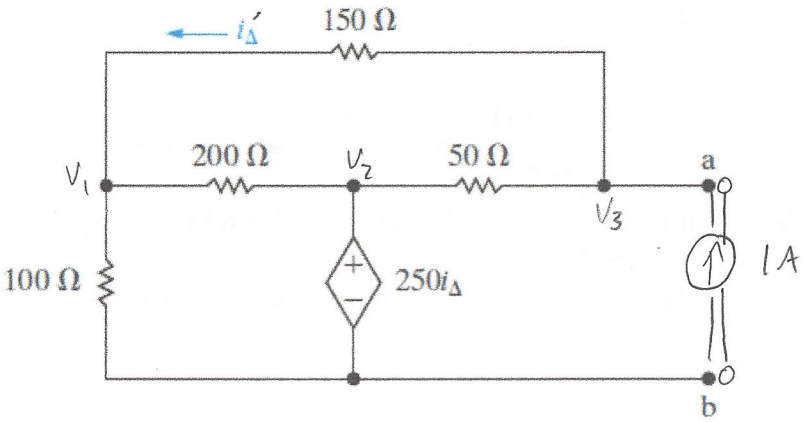
Question 4 [10]

Use the test source method to find the Thévenin resistance. [Hint: use the node-voltage method.]



$$i_\Delta = \frac{V_3 - V_1}{150}$$

$$V_2 = 250\, i_\Delta$$

$$V_2 = 250\left(\frac{V_3 - V_1}{150}\right)$$

$$V_2 = \frac{250V_3 - 250V_1}{150}$$

KCL node 1

$$\frac{V_1}{100} + \frac{V_1 - V_2}{200} + \frac{V_1 - V_3}{150} = 0$$

$$\frac{6V_1 + 3V_1 - 3V_2 + 4V_1 - 4V_3}{600} = 0$$

$$13V_1 - 3V_2 - 4V_3 = 0$$

$$13V_1 - 3\left(\frac{250}{150}V_3 - \frac{250}{150}V_1\right) - 4V_3 = 0$$

$$13V_1 - 5V_3 + 5V_1 - 4V_3 = 0$$

$$18V_1 - 9V_3 = 0$$

$$V_1 = \frac{V_3}{2}$$

KCL node 3

$$\frac{V_3 - V_2}{50} + \frac{V_3 - V_1}{150} = 1$$

$$\frac{3V_3 - 3V_2 + V_3 - V_1}{150} = 1$$

$$-V_1 - 3V_2 + 4V_3 = 150$$

$$-V_1 - 3\left(\frac{250}{150}V_3 - \frac{250}{150}V_1\right) + 4V_3 = 150$$

$$-V_1 - 5V_3 + 5V_1 + 4V_3 = 150$$

$$4V_1 - V_3 = 150$$

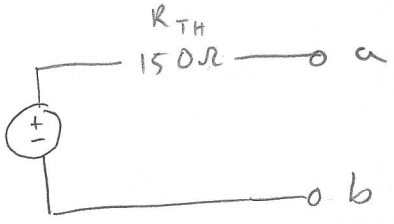
$$4\left(\frac{V_3}{2}\right) - V_3 = 150$$

$$2V_3 - V_3 = 150$$

$$V_3 = 150\ V$$

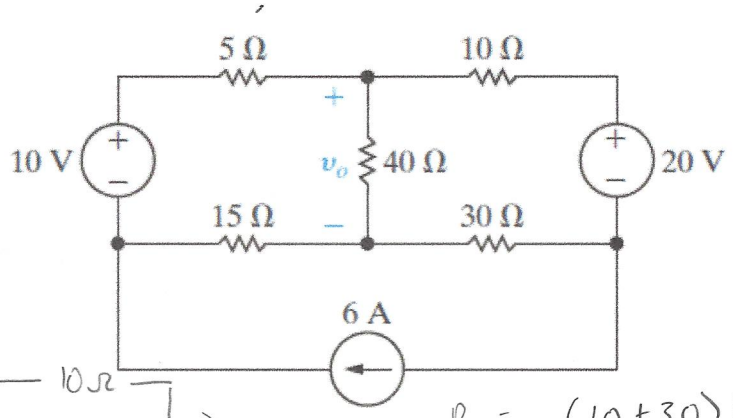
$$R_{TH} = \frac{V_3}{1A} = \frac{150V}{1A} = 150\,\Omega$$

$$V_{TH} = 0\ V$$

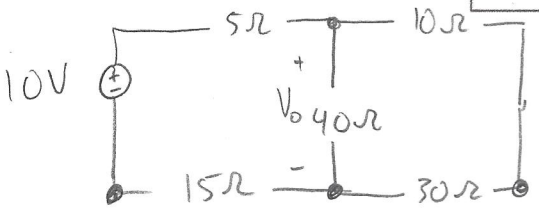


Question 5 [10]

Use the principle of superposition to find the voltage v_o. [Hint: when you analyze the current source, apply the node voltage method choosing the reference node as the node below the 40 Ω resistor.]



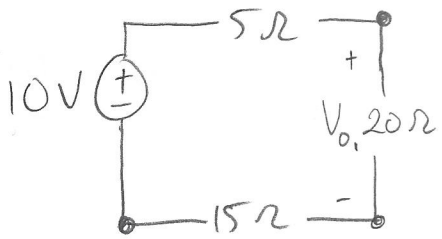
20V & 6A deactivated



) series

$R = (10+30)\|40$

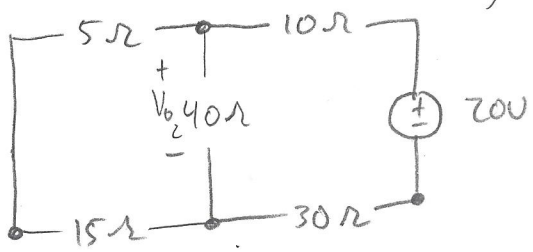
$= 40\|40 = \frac{40\times40}{80} = 20\Omega$

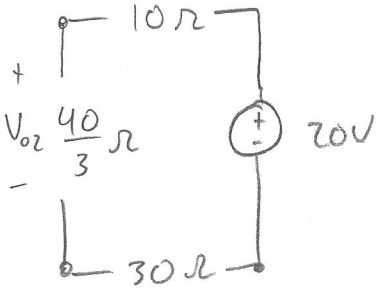


$V_{o_1} = \frac{20}{20+15+5} \times 10 = \frac{20}{40} \times 10 = 5\,V$

$R = (5+15)\|40 = 20\|40 = \frac{20\times40}{60} = \frac{40}{3}\,\Omega$

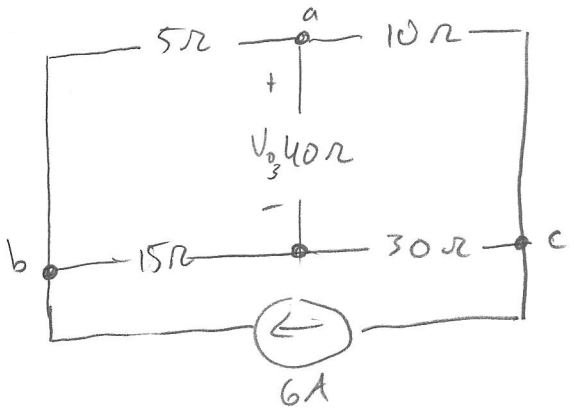
10V & 6A deactivated





$V_{o2} = \frac{\frac{40}{3}}{\frac{40}{3}+30+10} \times 20$

$= \frac{40}{160} \times 20 = 5\,V$



KCL a

$\frac{V_{o3}}{40} + \frac{V_{o3} - V_c}{10} + \frac{V_{o3} - V_b}{5} = 0$

$\frac{V_{o3} + 4V_{o3} - 4V_c + 8V_{o3} - 8V_b}{40} = 0$

$13V_{o3} - 8V_b - 4V_c = 0$

KCL b

$$\frac{V_b - V_{o3}}{5} + \frac{V_b}{15} - 6 = 0$$

$$\frac{3V_b - 3V_{o3} + V_b - 90}{15} = 0$$

$$-3V_{o3} + 4V_b = 90$$

$$V_b = \frac{90 + 3V_o}{4}$$

KCL c

$$\frac{V_c - V_{o3}}{10} + \frac{V_c}{30} + 6 = 0$$

$$\frac{3V_c - 3V_{o3}}{30} + \frac{V_c}{30} + 6 = 0$$

$$-3V_{o3} + 4V_c = -180$$

$$V_c = \frac{-180 + 3V_{o3}}{4}$$

sub eqns in

$$13V_{o3} - 8\left(\frac{90 + 3V_{o3}}{4}\right) - 4\left(\frac{-180 + 3V_{o3}}{4}\right) = 0$$

$$13V_{o3} - 180 - 6V_{o3} + 180 - 3V_{o3} = 0$$

$$4V_{o3} = 0$$

$$V_{o3} = 0V$$

$$V_o = V_{o1} + V_{o2} + V_{o3} = 5 + 5 + 0 = 10V$$

Q 3 continued

$$R_N = \frac{V_{TH}}{i_{sc}} = \frac{11.2\,V}{0.06\,A} = 186.67\,\Omega$$