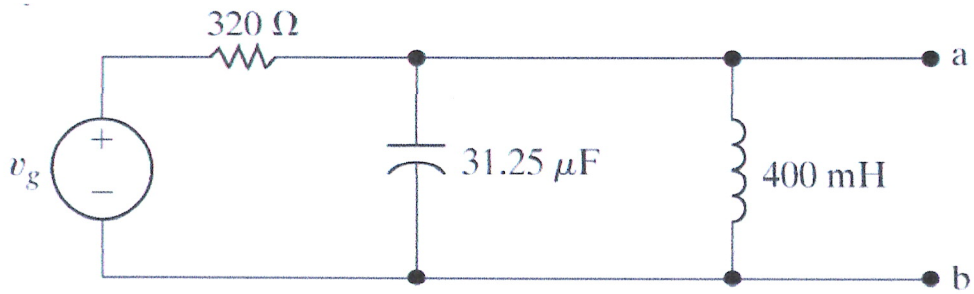
ENGR 2910-101: Circuit Analysis — Instructor: Brian Rashap

Final — Due: 12/08/23

Question 1 [15]

The sinusoidal voltage source in the ciruit below is developing a voltage equal to $50 sin(400t)V$.



(a) Find the Thevenin voltage with respect to terminals a,b. Express in both complex $(a + jb)$ and Phasor form.

(b) Find the Thevenin impedance with respect to terminals a,b (in complex form).

(c) Draw the Thevenin equivalent circuit using a voltage source, resistor, capacitor and/or inductor.

a) $Z_R = 320$

$Z_L = j(400)(0.400) = j160$

$Z_C = \frac{1}{j(400)(31.25 \times 10^{-6})} = -j80$

VOLTAGE DIVIDER

$$V_{ab} = \frac{Z_C \| Z_L}{Z_R + (Z_C \| Z_L)} = \frac{-j80 \| j160}{320 + (-j80 \| j160)} (-j50)$$

$$= \frac{-j160}{320 - j160} (-j50)$$

$$= \frac{-j}{2-j}\left(\frac{2+j}{2+j}\right)(-j50)$$

$$= \frac{1-j2}{5}(-j50)$$

$$V_{TH} = -20 - j10$$

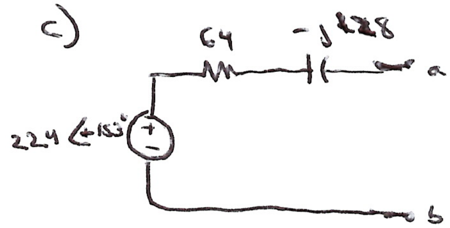
$$= 22.4 \angle -153.43^\circ$$

b) $I_{sc} = \frac{-j50}{320} = \frac{1}{j6.4}$

$$Z_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{-20 - j10}{\left(\frac{1}{j6.4}\right)}$$

$$= 64 - j128$$

c)



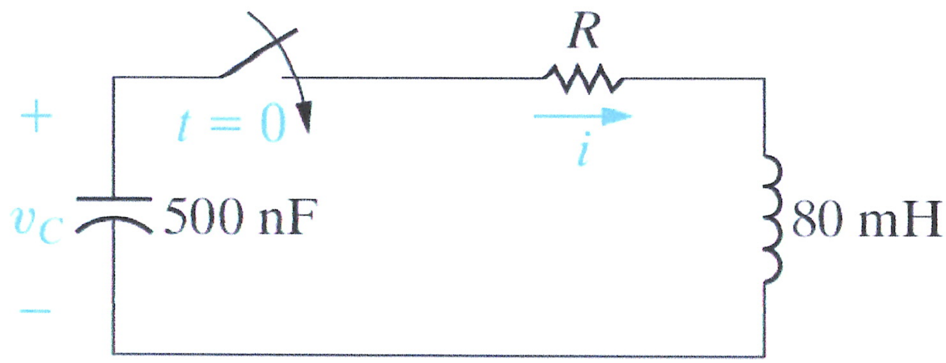
$$Z_{TH} = 64 - j128$$

$$= 64 + \left(\frac{-j}{\frac{1}{128}}\right)$$

↑R ↑C

Question 2 [15]

Find the value of R that makes the below circuit Critically Damped.



$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(0.80)(500\times10^{-6})} = 25\times10^{6}$$

$$\alpha = \frac{R}{2L} = \omega_0 \quad \text{for critically damped}$$

$$R = 2(0.80)(5\times10^{3})$$

$$\boxed{R = 800\ \Omega}$$

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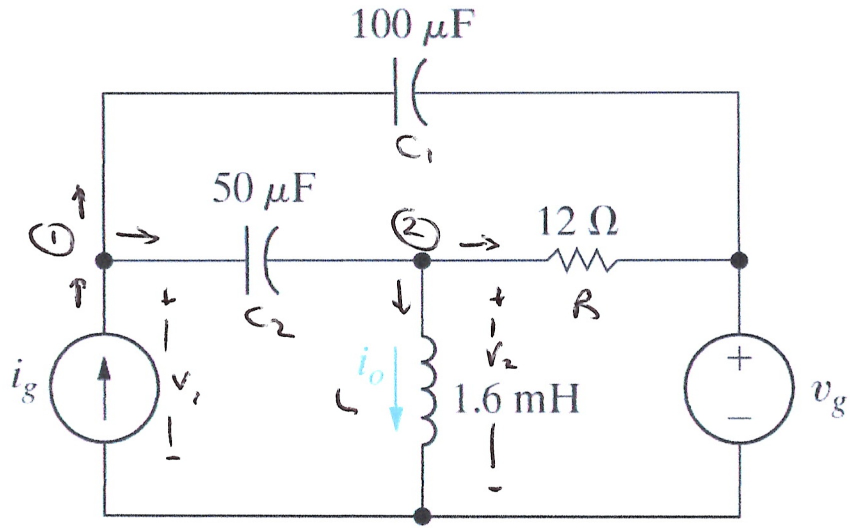
Question 3 [15]

Use the Node-Voltage method to find the matrix representation of V_1 and V_2 if $i_g = 5\cos(2500t)A$ and $v_g = 20\cos(2500t + 90°)V$. You do NOT need to solve for V_1 and V_2, nor reduce the matrix to reduced row echelon form.



$Z_R = 12$

$Z_L = j(2500)(0.0016) = j4$

$Z_{C_1} = \dfrac{1}{j(2500)(100\times10^{-6})} = -j4$

$Z_{C_2} = \dfrac{1}{j(2500)(50\times10^{-6})} = -j8$

$I_g = 5\angle 0°\ A = 5$

$V_g = 20\angle 90°\ V = j20$

① $5 - \dfrac{V_1 - V_2}{-j8} - \dfrac{V_1 - j20}{-j4} = 0$

$5 + \dfrac{jV_1 - jV_2}{8} + \dfrac{jV_1 - 20}{4} = 0$

$j\frac{3}{8}V_1 - j\frac{1}{8}V_2 = 0$

② $\dfrac{V_1 - V_2}{-j8} - \dfrac{V_2}{j4} - \dfrac{V_2 - j20}{12} = 0$

$j\frac{1}{8}V_1 + \left(-\frac{1}{12} + j\frac{1}{8}\right)V_2 = -\frac{5}{3}j$

$$\left[\begin{array}{cc|c} j0.375 & -j0.125 & 0 \\ j0.125 & -0.83+j0.125 & -j1.66 \end{array}\right]$$

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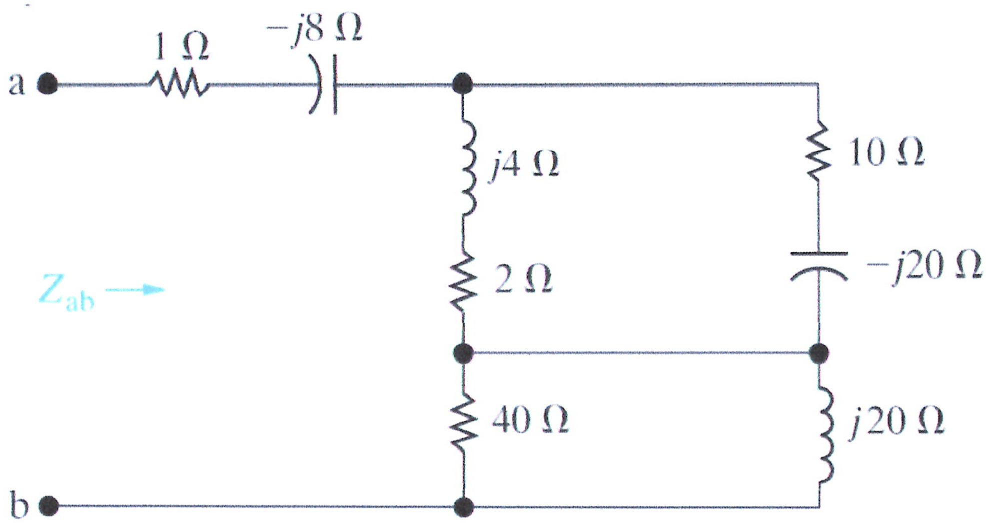
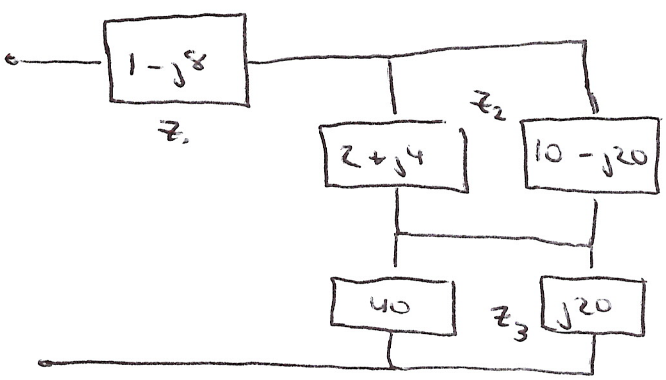
Question 4 [15]

Find Z_{eq} (Z_{ab}) for the circuit below.





$(2+j4)\,||\,(10-j20) \Rightarrow \frac{1}{Z_2} = \frac{1}{2+j4} + \frac{1}{10-j20}$

$\Rightarrow \frac{1}{Z_2} = \frac{2-j4}{20} + \frac{1}{5}\left(\frac{2+j4}{20}\right)$

$= \frac{12-16j}{100}$

$Z_2 = \frac{100}{12-16j} = \frac{100}{400}(12+j16) = 3+j4$

$40\,||\,j20 \Rightarrow \frac{1}{Z_3} = \frac{1}{40} + \frac{1}{j20}$

$= \frac{j}{j40} + \frac{2}{j40}$

$= \frac{2+j}{j40}$

$Z_3 = \frac{j40}{2+j}\cdot\frac{(2-j)}{(2-j)} =$

$= \frac{40}{5}(1+2j)$

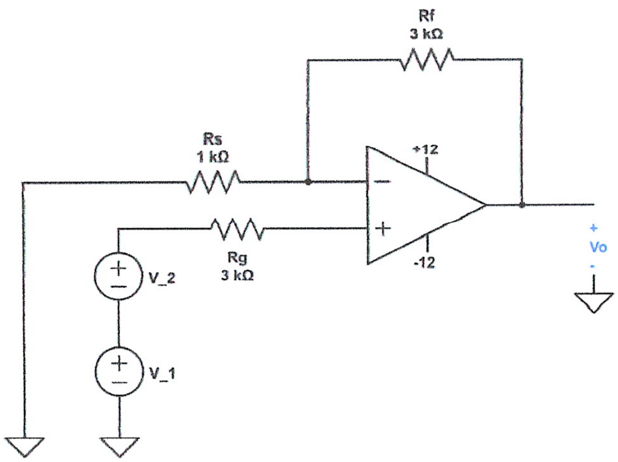
$= 8+16j$

$Z_{ab} = Z_1 + Z_2 + Z_3$

$= (1-j8) + (3+j4) + (8+j16)$

$= 12 + j12\ \Omega$

Question 5 [20]



(a) What is the voltage at the inverting input (v_n) in terms of v_1 and v_2.

(b) Using Kirchhoff's Current Law, what is the equation for currents at the inverting input node?

(c) Using KCL Equation from (b), derive the equation for v_0 as a function of v_1 and v_2. What is the gain of this op amp circuit?

(d) If $v_1 = 1V$ and $v_2 = 3 * \cos\left(\frac{\pi}{2}t\right)$, draw a graph of v_o vs time. Show at least two periods of the output.

a) IDEAL OP AMP

$v_n = v_p = v_1 + v_2$

b) IDEAL OP AMP

$i_n = 0$

KCL: $\frac{v_o - v_n}{3k} - \frac{v_n}{1k} = 0 \Rightarrow \frac{v_o - (v_1 + v_2)}{3k} - \frac{(v_1 + v_2)}{1k} = 0$

c) $\Rightarrow v_o - v_n - 3v_n = 0$

$v_o = 4 v_n = 4(v_1 + v_2)$

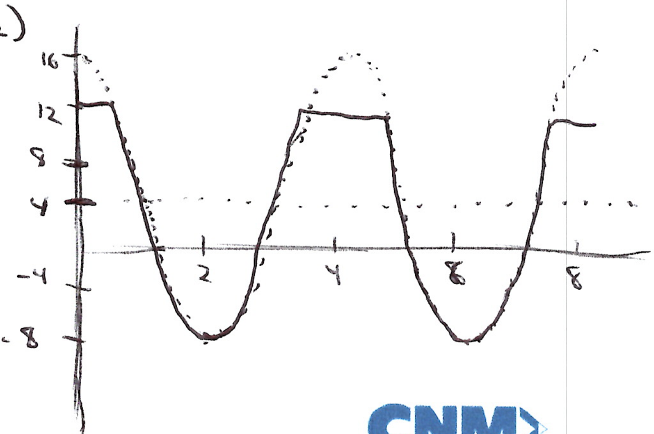
GAIN 4

d) $v_1 + v_2 = 1 + 3\cos\left(\frac{\pi}{2}t\right)$

$\omega = 2\pi f \Rightarrow f = \frac{1}{4}$

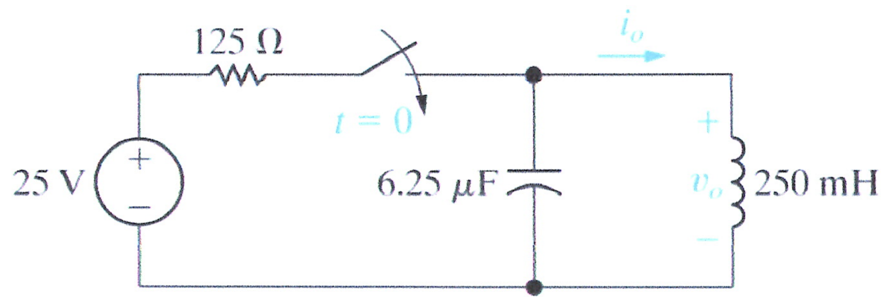
Period $= \frac{1}{f} = 4$ seconds

SATURATION AT $\pm 12V$



Question 6 [20]

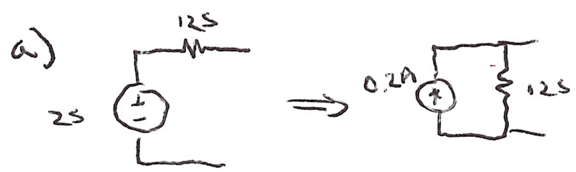
Assume there is no energy stored in the circuit below when the swithc is closed at $t = 0$.

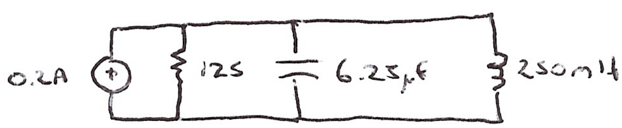


(a) Using the Source Transformation, redraw the circuit as a parallel RLC circuit.

(b) Find $i_0(t)$ for $t \geq 0$.

a)





b) $\alpha = \frac{1}{2RC} = 640$ $\qquad \alpha^2 = 40.96\times10^4$ } UNDER DAMPED

$\omega_0^2 = \frac{1}{LC} = 64\times10^4$

$\omega_d = \sqrt{800^2 - 640^2} = 480$

$I_f = 0.2A$

$i_o(t) = 0.2 + B_1' e^{-400t}\cos(480t) + B_2' e^{-400t}\sin(480t)$

$i_o(0) = 0.2 + B_1' \Rightarrow B_1' = -0.2$

$\frac{di_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = \frac{V_o}{L}$ $\qquad V_o = 0V$

$\Rightarrow -640(-0.2) + 480\, B_2'$

$\Rightarrow B_2' = -0.267$

$i_o(t) = 0.2 - 0.2\, e^{-400t}\cos(480t) - 0.267\, e^{-400t}\sin(480t)$

Extra Credit

Who was Max Salazar (either a factual or humorous answer will be accepted)?