ECE 203

Circuits I

# More 1st Order Transient Circuits

Lecture 10-2

# $1^{st}$ Order Circuits Continued

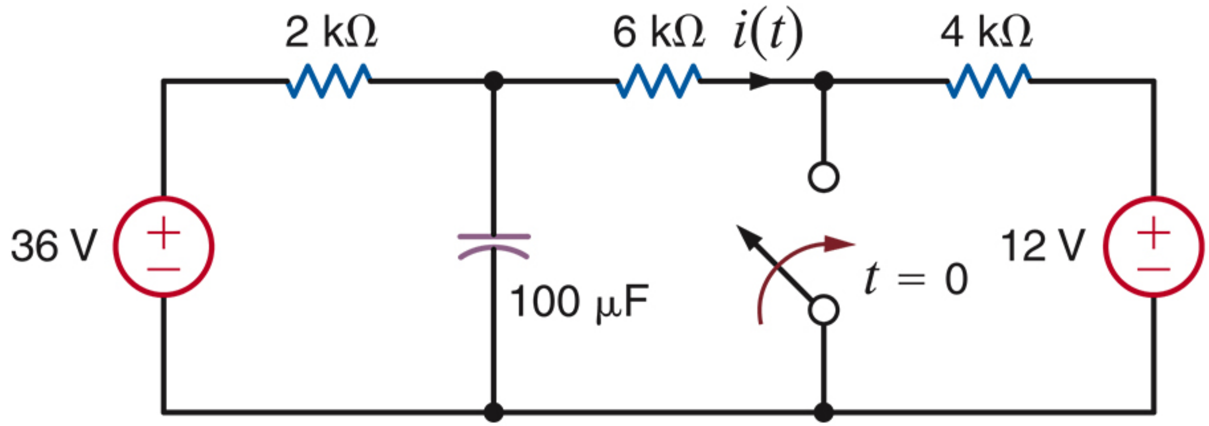
- We've already discussed one method for solving $1^{st}$ order circuits, the "differential equation method"
- Now we will learn the "step-by-step method"

# The Step-by-Step Approach

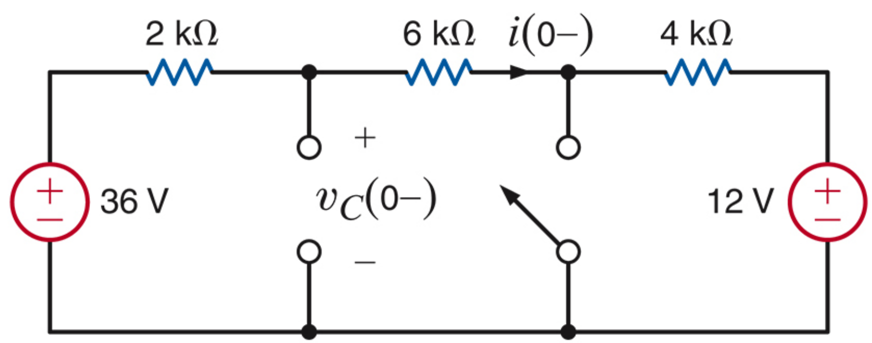
1. Assume that the form of the solution, x(t), is known.
2. Find $v_C(0^-)$ or $i_L(0^-)$, prior to switch action.
3. Considering that $v_C(0^+) = v_C(0^-)$ or $i_L(0^+) = i_L(0^-)$, find $x(0^+)$. This can be done by replacing the capacitor with a voltage source of $v_C(0^+)$ or the inductor with a current source of $i_L(0^+)$ with the switch in the new position.
4. Find $x(\infty)$, by replacing the capacitor with an open circuit or inductor with a short circuit.
5. Find the time constant by obtaining the Thevenin equivalent circuit at the capacitor or inductor after switching.
6. The solution will be:

$$x(t) = x(\infty) + [x(0+) - x(\infty)]e^{-t/\tau}$$

# *Example 1: Step-by-Step Approach*

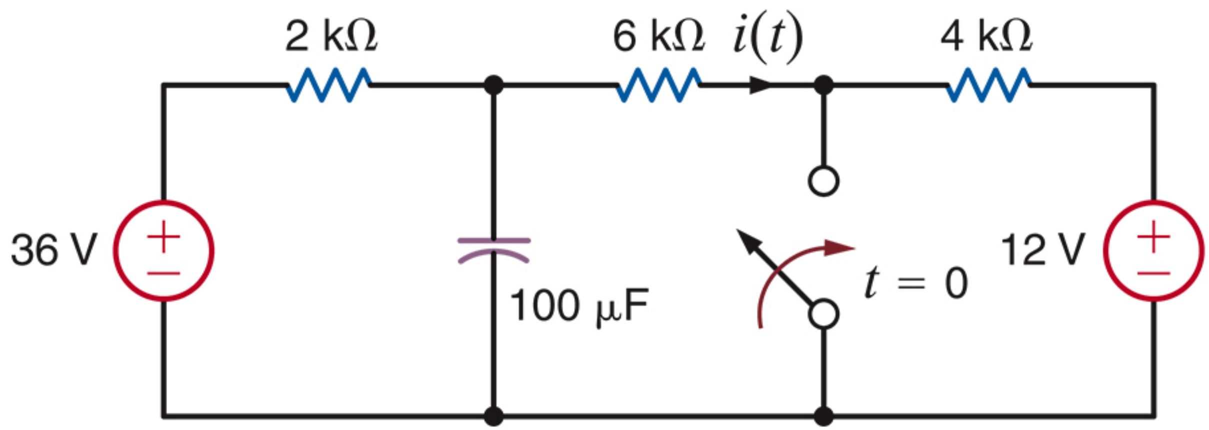



**Step 2 ($t=0^-$):**

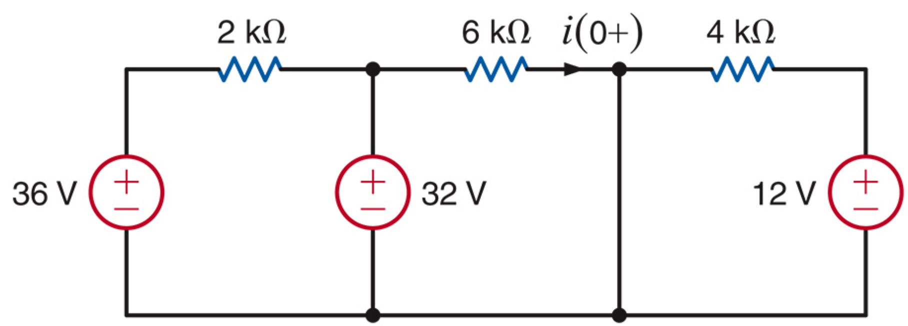



$$\begin{aligned} v_C(0-) &= 36 - (2)(2) \\ &= 32 \text{ V} \end{aligned}$$

# *Example 1: Step-by-Step Approach*

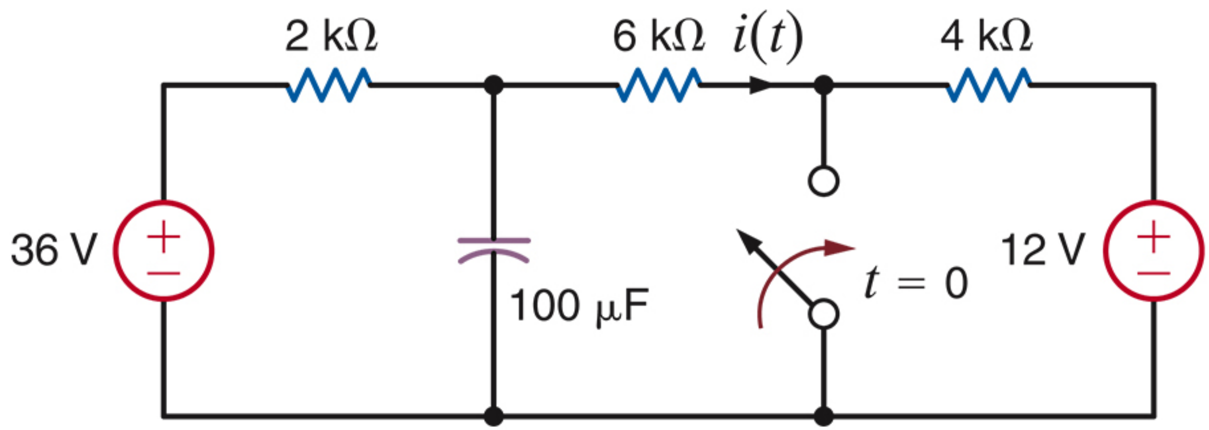



**Step 3 ($t=0^+$):**

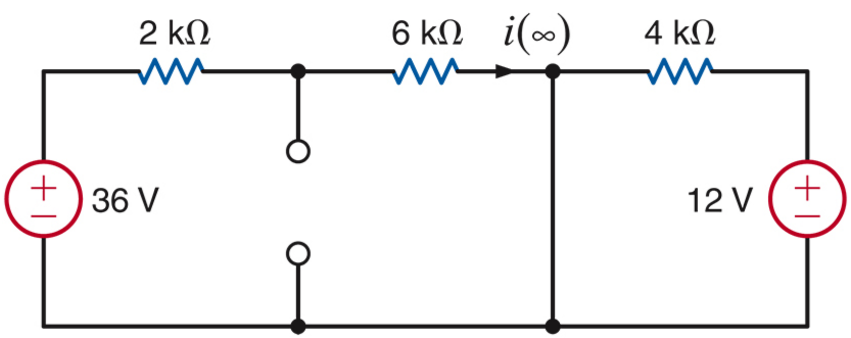



$$i(0+) = \frac{32}{6\text{k}} = \frac{16}{3}\ \text{mA}$$

# *Example 1: Step-by-Step Approach*

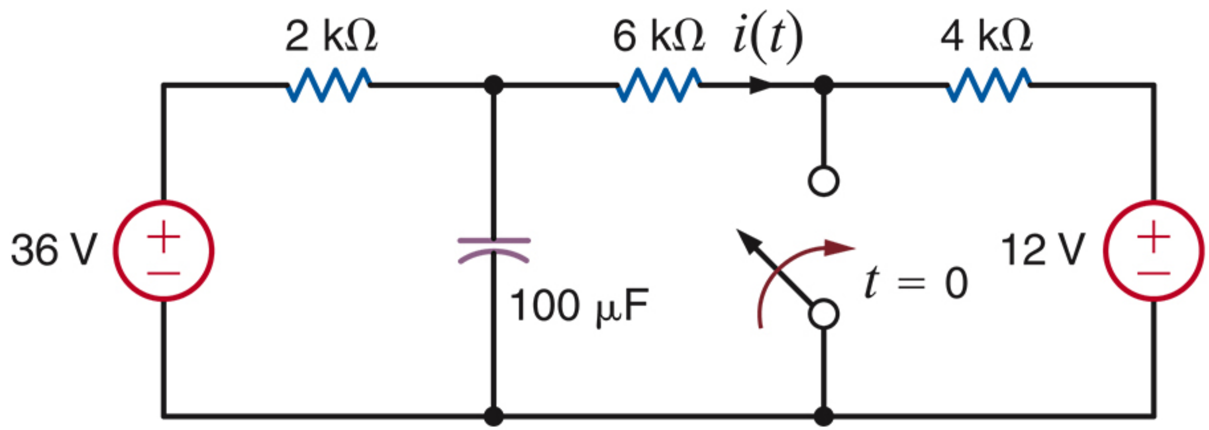



**Step 4 (t=∞):**

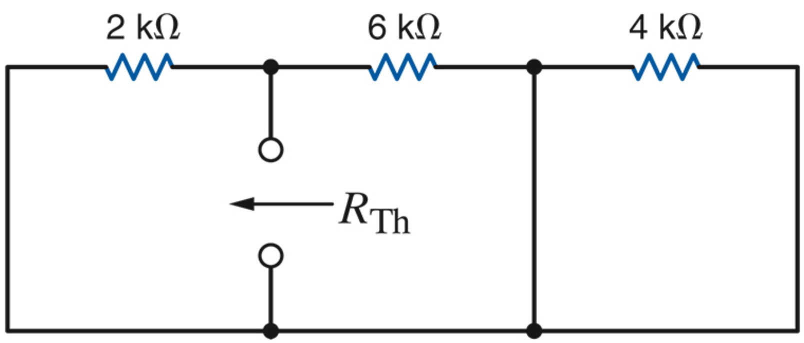



$$
\begin{aligned}
i(\infty) &= \frac{36}{2\text{k} + 6\text{k}} \\
&= \frac{9}{2}\ \text{mA}
\end{aligned}
$$

# *Example 1: Step-by-Step Approach*



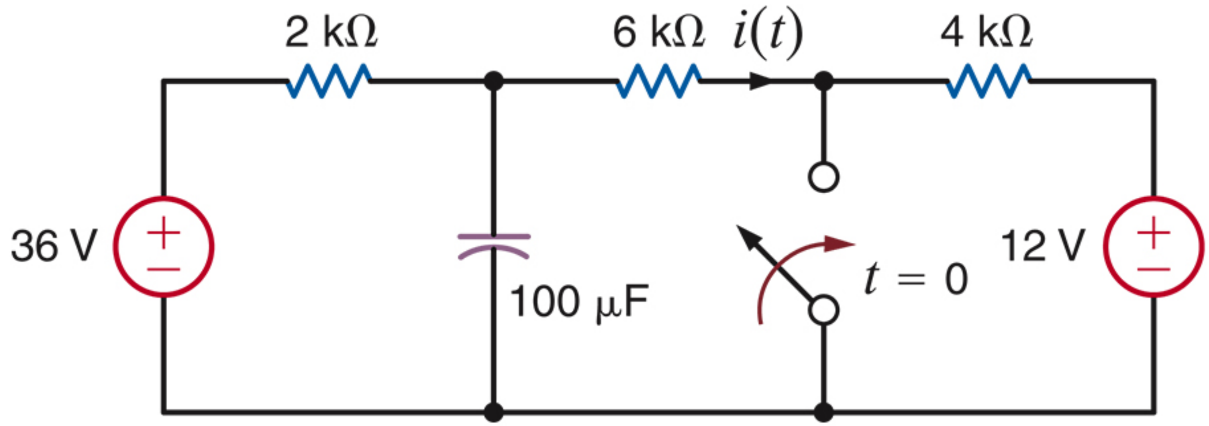

## Step 5 ($R_{TH}$):




$$R_{\mathrm{Th}} = \frac{(2\mathrm{k})(6\mathrm{k})}{2\mathrm{k} + 6\mathrm{k}} = \frac{3}{2}\ \mathrm{k}\Omega$$

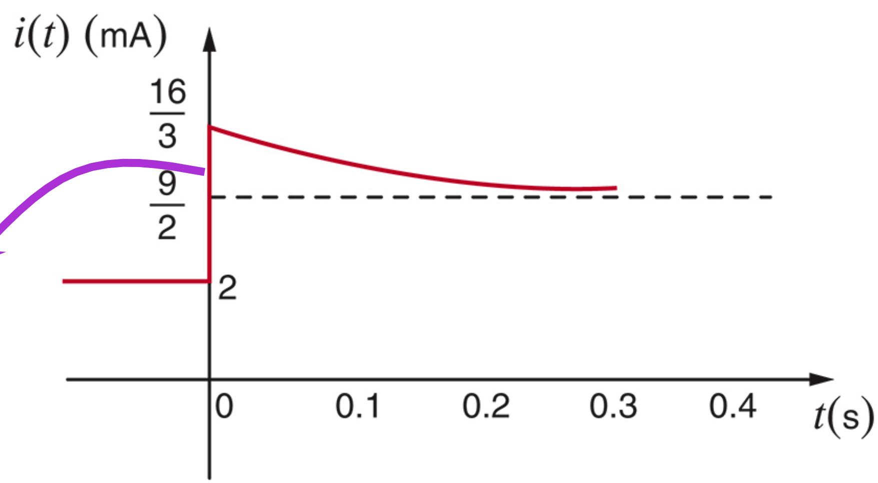
$$\begin{aligned}\tau &= R_{\mathrm{Th}}C \\ &= \left(\frac{3}{2}\right)(10^3)(100)(10^{-6}) \\ &= 0.15\ \mathrm{s}\end{aligned}$$

# Example 1: Step-by-Step Approach



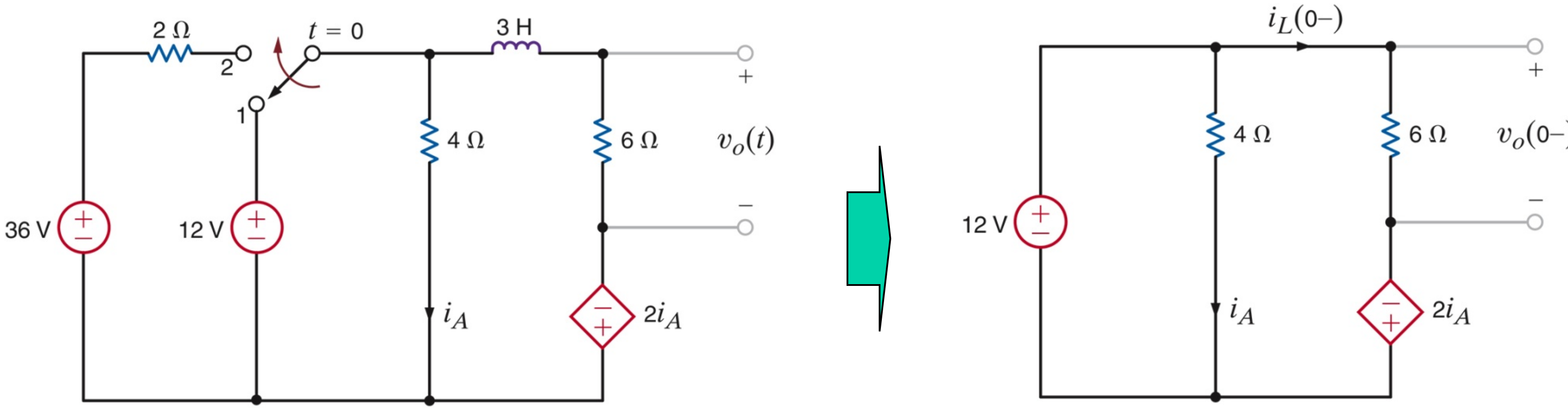

$$x(t) = x(\infty) + [x(0+) - x(\infty)]e^{-t/\tau}$$

$$i(t) = \frac{36}{8} + \frac{5}{6} e^{-t/0.15} \text{ mA}$$




i(t) is not a continuous function.
Is it OK?

# *Example 2: Step-by-Step Approach*

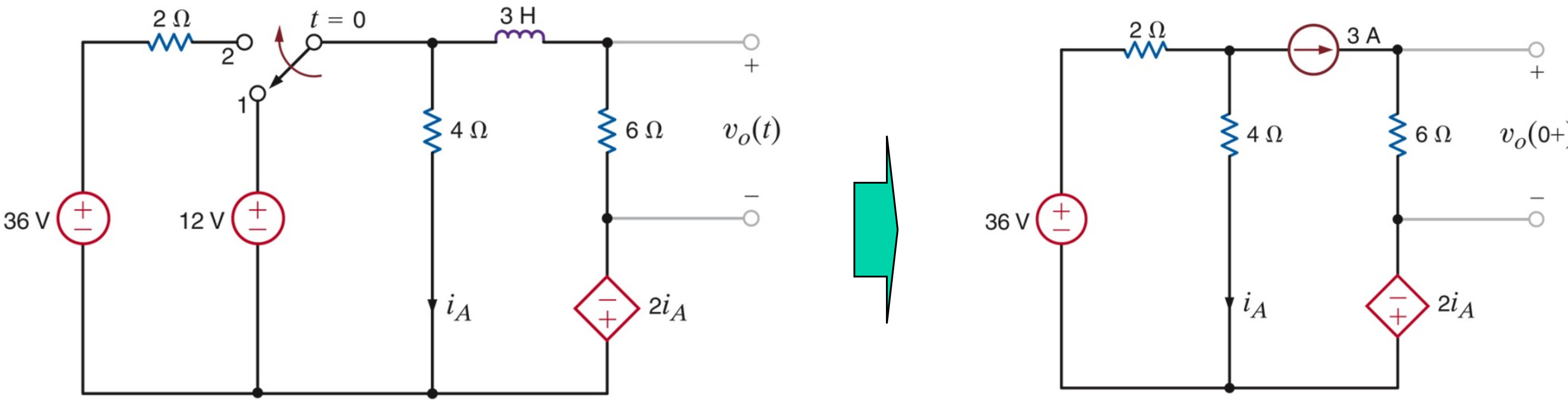



## Step 2 ($t=0^-$)

$$i_A = \frac{12}{4} = 3\ \text{A}$$

$$i_L(0-) = \frac{12 + 2i_A}{6} = \frac{18}{6} = 3\ \text{A}$$

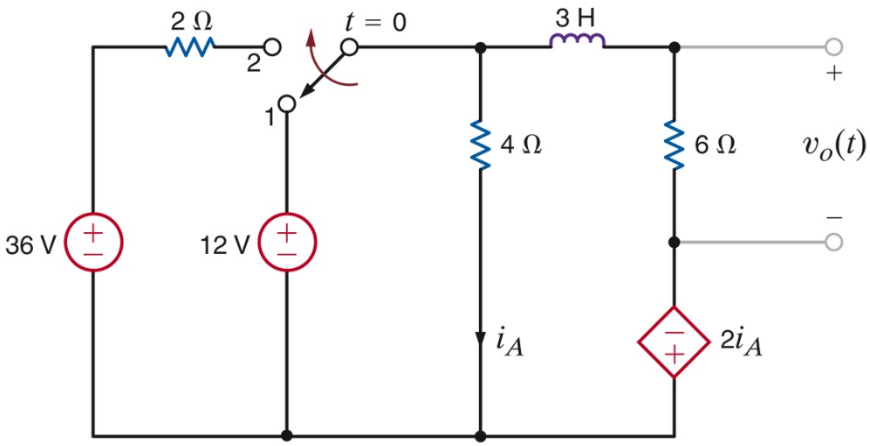
# *Example 2: Step-by-Step Approach*

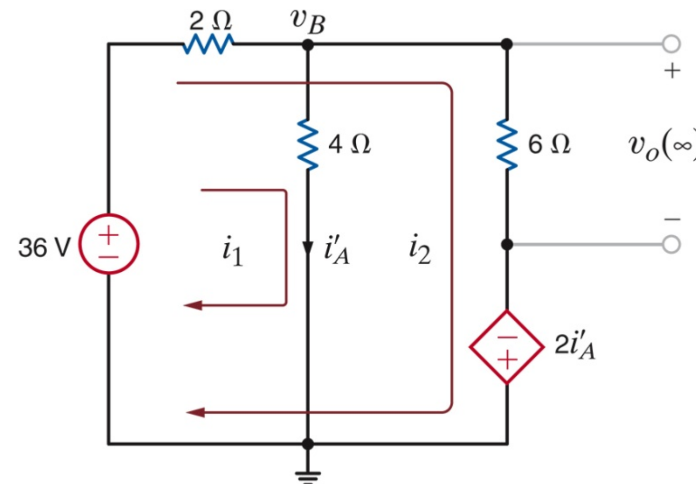



**Step 3 ($t=0^+$)**

$$v_o(0+) = (3)(6) = 18 \text{ V}$$

# Example 2: Step-by-Step Approach




## Step 4 (t=∞)

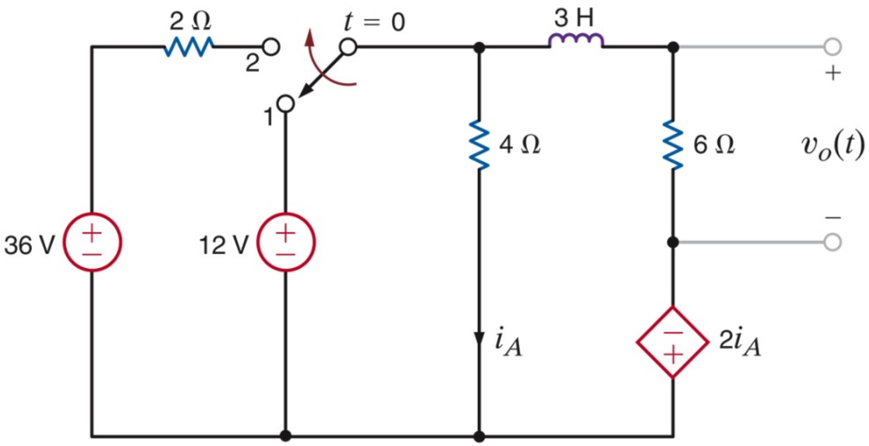
$$\frac{v_B - 36}{2} + \frac{v_B}{4} + \frac{v_B + 2i'_A}{6} = 0$$

$$i'_A = \frac{v_B}{4}$$

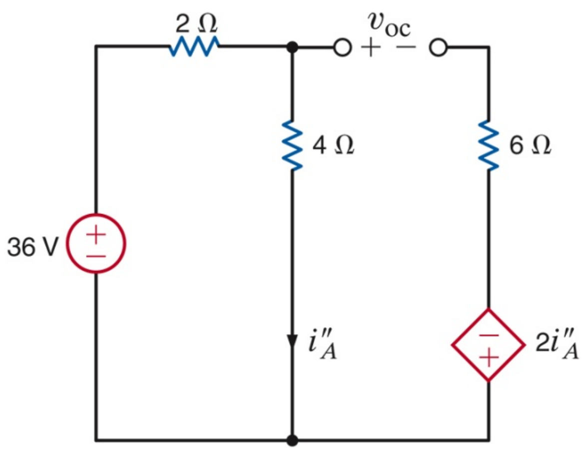
$$v_o(\infty) = v_B + 2i'_A$$

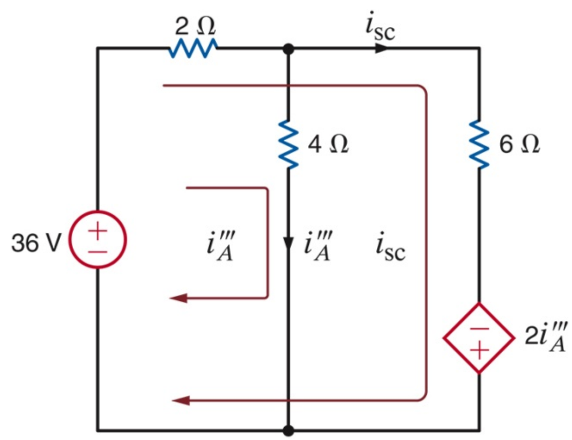
$$v_o(\infty) = 27 \text{ V}$$

# Example 2: Step-by-Step Approach




## Step 5 ($R_{TH}$)




$$R_{\text{Th}} = \frac{v_{\text{oc}}}{i_{\text{sc}}} = \frac{36}{9/2} = 8\ \Omega$$

$$\tau = L/R = 3/8 \text{ sec}$$

$$i''_A = \frac{36}{2+4} = 6 \text{ A}$$

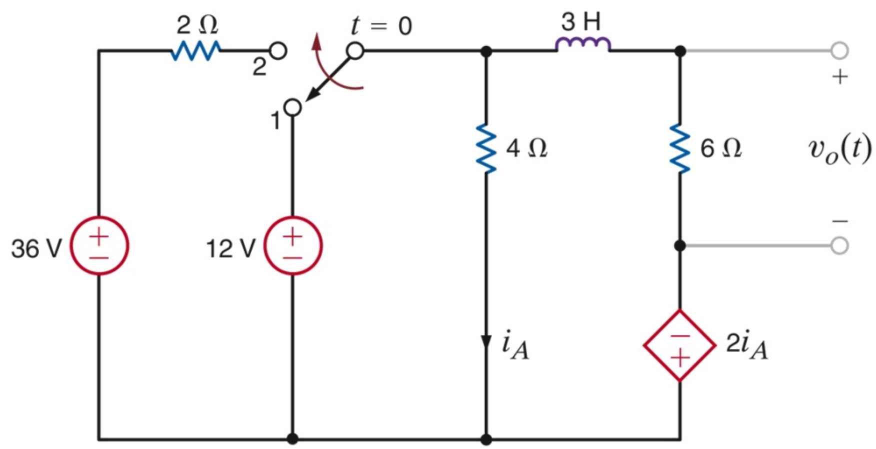
$$v_{\text{oc}} = (4)(6) + 2(6) = 36 \text{ V}$$

$$36 = 2(i'''_A + i_{\text{sc}}) + 4i'''_A$$

$$36 = 2(i'''_A + i_{\text{sc}}) + 6i_{\text{sc}} - 2i'''_A$$

$$i_{\text{sc}} = \frac{9}{2} \text{ A}$$

# Example 2: Step-by-Step Approach




$$x(t) = x(\infty) + [x(0+) - x(\infty)]e^{-t/\tau}$$

$$v_o(t) = 27 - 9e^{-t/(3/8)} \text{ V}$$