HOMEWORK 7 SOLUTIONS

(1) a) Current

$$i(t) = \begin{cases} 0 & t<0 \\ 50t \ \text{A} & 0 \le t \le 5\text{ms} \\ 0.5 - 50t \ \text{A} & 5\text{ms} \le t \le 10\text{ms} \\ 0 & t > 10\text{ms} \end{cases}$$

b) $v = L\frac{di}{dt} = (20\times10^{-3})(50) = 1\text{V}$ $\quad 0 \le t \le 5\text{ms}$

$= (20\times10^{-3})(-50) = -1\text{V}$ $\quad 5\text{ms} \le t \le 10\text{ms}$

$$v(t) = \begin{cases} 0\text{V} & t<0 \\ 1\text{V} & 0 \le t \le 5\text{ms} \\ -1\text{V} & 5\text{ms} \le t \le 10\text{ms} \\ 0\text{V} & t \ge 10\text{ms} \end{cases}$$

$p = vi$

$$p = \begin{cases} 0\ \text{W} & t<0 \\ 50t(1) = 50t\ \text{W} & 0 \le t \le 5\text{ms} \\ (0.5-50t)(-1) = 50t-0.5\ \text{W} & 5\text{ms} \le t \le 10\text{ms} \\ 0\ \text{W} & t > 10\text{ms} \end{cases}$$

$w = \int p\,dt$

$$w = \begin{cases} 0\ \text{J} & 0 < t \\ 25t^2\ \text{J} & 0 \le t \le 5\text{ms} \\ 25t^2 - 0.5t + (2.5\times10^{-3})\ \text{J} & 5\text{ms} \le t \le 10\text{ms} \\ 0\ \text{J} & t > 10\text{ms} \end{cases}$$

(2) a) $i = C\frac{dv}{dt} = (5\times10^{-6})\left[500t(-2500)e^{-2500t} + 500e^{-2500t}\right]$

$= 2.5\times10^{-3}e^{-2500t}(1-2500t)$ A

b) $v(100\mu\text{s}) = 500(100\times10^{-6})e^{-0.25} = 38.94\text{ mV}$

$i(100\mu\text{s}) = (2.5\times10^{-3})e^{-0.25}(1-0.25) = 1.46\text{ mA}$

$p(100\mu\text{s}) = vi = 56.86\ \mu\text{W}$

c) $p > 0$, capacitor is charging. $\Rightarrow$ absorbing power

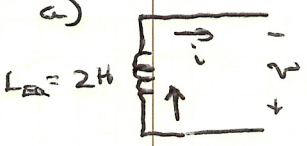
(3) a) $L_2 = \left(\frac{m^2}{k^2 L_1}\right) = \frac{(0.9)^2}{(0.75)^2(0.288)} = 50\,mH$

$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{288}{50}} = 2.4$

b) $P_1 = \frac{L_1}{N_1^2} = 0.2 \times 10^{-6}$ Wb/A

$P_2 = \frac{L_2}{N_2^2} = 0.2 \times 10^{-6}$ Wb/A

(4) a)



$L_{eq} = 2H$

$i(0) = 2A + 4A = 6A$

$i(t) = -\frac{1}{2}\int_0^t 12e^{-t}\,dt + 6$

$= 6e^{-t}\Big|_0^t + 6$

$= 6e^{-t} - 6 + 6$

$i(t) = 6e^{-t}$ A $t > 0$

b) $i_1 = \frac{-1}{3}\int_0^t 12e^{-t}\,dt + 2$

$= 4e^{-t}\Big|_0^t + 2$

$i_1(t) = 4e^{-t} - 2$ A $t \geq 0$

c) $i_2 = \frac{-1}{6}\int_0^t 12e^{-t}\,dt + 4$

$= 2e^{-t}\Big|_0^t + 4$

$i_2(t) = 2e^{-t} + 2A$ $t \geq 0$

d) $p = vi = (12e^{-t})(6e^{-t}) = 72e^{-2t}$ W

$w = \int_0^\infty p\,dt$

$= 72\left(\frac{-1}{2}\right)e^{-2t}\Big|_0^\infty$

$= 36$ J

e) $w = \frac{1}{2}(3)(2)^2 + \frac{1}{2}(6)(4)^2 = 54$ J

f) $w_{trapped} = \frac{1}{2}(3)(-2)^2 + \frac{1}{2}(6)(2)^2 = 18$ J

g) $18\,J + 36\,J = 54\,J$ ✓

(5) a) $v = L_1\frac{di}{dt} + m\frac{di}{dt} + L_2\frac{di}{dt} + m\frac{di}{dt} = (L_1 + L_2 + 2m)\frac{di}{dt}$

$L_{ab} = L_1 + L_2 + 2m$

b) $v = L_1\frac{di}{dt} - m\frac{di}{dt} + L_2\frac{di}{dt} - m\frac{di}{dt} = (L_1 + L_2 - 2m)\frac{di}{dt}$

$L_{ab} = L_1 + L_2 - 2m$