

6.1 PSPICE MULTISIM The current in a  $50\mu\text{H}$  inductor is known to be

$$i_L = 18te^{-10t} \text{ A for } t \geq 0.$$

- Find the voltage across the inductor for  $t > 0$ . (Assume the passive sign convention.)
- Find the power (in microwatts) at the terminals of the inductor when  $t = 200 \text{ ms}$ .
- Is the inductor absorbing or delivering power at  $200 \text{ ms}$ ?
- Find the energy (in microjoules) stored in the inductor at  $200 \text{ ms}$ .
- Find the maximum energy (in microjoules) stored in the inductor and the time (in milliseconds) when it occurs.

a)  $V = L \frac{di}{dt}$

$$\frac{d}{dt}(18te^{-10t}) = 18[-10te^{-10t} + e^{-10t}] = 18e^{-10t}(1-10t)$$

$$V = (50 \times 10^{-6})(18e^{-10t}(1-10t))$$

$$V = 0.9e^{-10t}(1-10t) \text{ mV}$$

b)  $P = Vi$

at  $t = 200 \text{ ms}$

$$i(200 \text{ ms}) = 18(0.2)e^{-2} = 487.2 \text{ mA}$$

$$V(200 \text{ ms}) = 0.9e^{-2}(-1) = -121.8 \mu\text{V}$$

$$P = (-121.8 \mu\text{V})(487.2 \text{ mA}) = -59.34 \mu\text{W}$$

c) Since power is neg the inductor is delivering

d)  $W = \frac{1}{2}Li^2$

$$W = \frac{(50 \times 10^{-6})(487.2 \times 10^{-3})^2}{2}$$

$$W = 5.93 \mu\text{J}$$

e)  $\frac{di_L}{dt} = 0 \Rightarrow t = 0.1 \text{ s}$

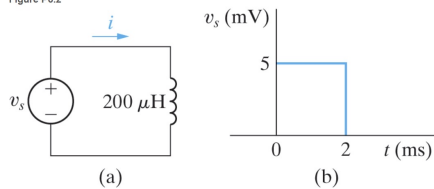
$$i = 18(0.1)e^{-1} = 662.2 \text{ mA}$$

$$W = \frac{(50 \times 10^{-6})(662.2 \times 10^{-3})^2}{2}$$

$$W = 10.96 \mu\text{J}$$

6.2 PSYCH MULTISIM The voltage at the terminals of the  $200\ \mu\text{H}$  inductor in Fig. P6.2(a) is shown in Fig. P6.2(b). The inductor current  $i$  is known to be zero for  $t \leq 0$ .  
a. Derive the expressions for  $i$  for  $t \geq 0$ .  
b. Sketch  $i$  versus  $t$  for  $0 \leq t \leq \infty$ .

Figure P6.2



$$\text{since } v = L \frac{di}{dt} \Rightarrow i = \frac{1}{L} \int v dt + C$$

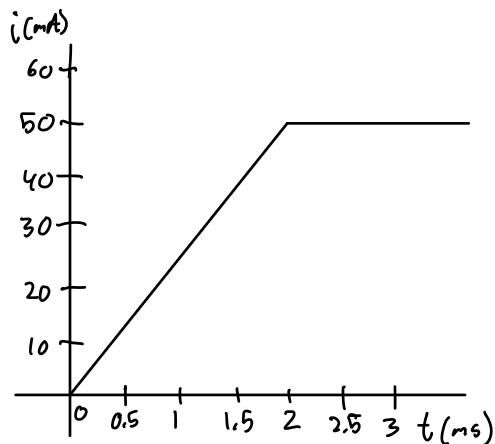
$$a) \quad i = \left( \frac{1}{200 \times 10^{-6}} \right) \int_0^t v dx + C$$

$$b) \quad ① \quad i = \left( \frac{1}{200 \times 10^{-6}} \right) \int_0^t (5 \times 10^{-3}) dx + 0$$

$$i = \frac{(5 \times 10^{-3})t}{200 \times 10^{-6}} = 25t \text{ A}$$

$$② \quad i = \left( \frac{1}{200 \times 10^{-6}} \right) \int_{2 \times 10^{-3}}^t 0 dx + 25(2 \times 10^{-3})$$

$$i = 25(2 \times 10^{-3}) = 50 \text{ mA}$$



6.14 PSYCH MULTISIM The voltage at the terminals of the capacitor in Fig. 6.10 is known to be

$$v = \begin{cases} -10 \text{ V}, & t \leq 0; \\ 40 - 1e^{-1000t}(50 \cos 500t + 20 \sin 500t) \text{ V}, & t \geq 0. \end{cases}$$

Assume  $C = 0.8\ \mu\text{F}$ .

- Find the current in the capacitor for  $t < 0$ .
- Find the current in the capacitor for  $t > 0$ .
- Is there an instantaneous change in the voltage across the capacitor at  $t = 0$ ?
- Is there an instantaneous change in the current in the capacitor at  $t = 0$ ?
- How much energy (in millijoules) is stored in the capacitor at  $t = \infty$ ?

$$a) \quad i = \frac{dv}{dt} \quad \text{since no change in } v$$

$$\boxed{i = 0}$$

$$\begin{aligned} b) \quad \frac{dv}{dt} &= \frac{1}{dt} [40 - 1e^{-1000t}(50 \cos 500t + 20 \sin 500t)] \\ &= [1000e^{-1000t}(50 \cos 500t + 20 \sin 500t)] \\ &\quad - [-e^{-1000t}(-25000 \sin 500t) + 1000 \cos(500t)] \\ &= e^{-1000t} [(50000 - 10000) \cos 500t + (20000 + 25000) \sin 500t] \\ &= e^{-1000t} [40000 \cos(500t) + 45000 \sin(500t)] \\ \boxed{i = C \frac{dv}{dt} = e^{-1000t} [(6.032) \cos(500t) + (0.036) \sin(500t)] \text{ A}} \end{aligned}$$

$$e) \quad w = \frac{1}{2} CV^2 \quad v(\infty) = 40$$

$$w = \frac{1}{2} (0.8 \times 10^{-6}) (40)^2$$

$$\boxed{w = 640 \mu\text{J}}$$

$$c) \quad \boxed{\text{no}}$$

$$d) \quad \boxed{\text{yes}}$$

6.17 A  $20\mu\text{F}$  capacitor is subjected to a voltage pulse having a duration of 1 s. The pulse is described by the following equations:

$$v_c(t) = \begin{cases} 30t^2 \text{ V}, & 0 \leq t \leq 0.5 \text{ s}; \\ 30(t-1)^2 \text{ V}, & 0.5 \text{ s} \leq t \leq 1 \text{ s}; \\ 0 & \text{elsewhere.} \end{cases}$$

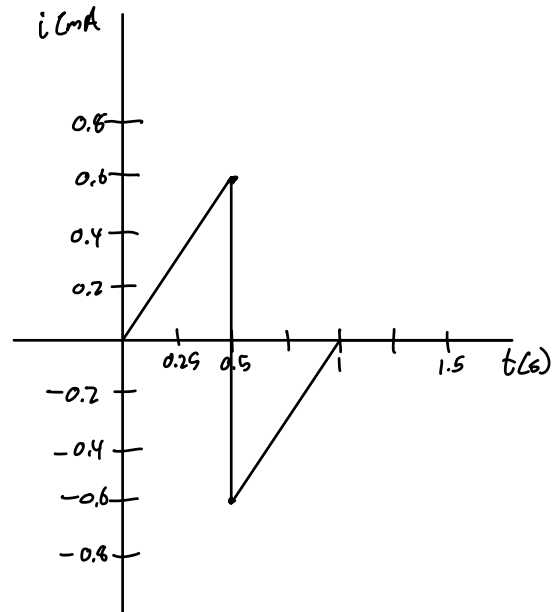
Sketch the current pulse that exists in the capacitor during the 1 s interval.

$$i = C \frac{dv}{dt}$$

$$\begin{aligned} \textcircled{1} \quad i &= (20 \times 10^{-6}) \left( \frac{d}{dt} 30t^2 \right) \\ &= (20 \times 10^{-6}) (60t) = 1.2t \times 10^{-3} \text{ A} \end{aligned}$$

$$\begin{aligned} @t=0, i &= 0 \\ t=0.5, i &= 0.6 \text{ mA} \end{aligned}$$

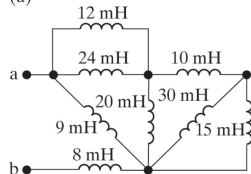
$$\begin{aligned} \textcircled{2} \quad i &= (20 \times 10^{-6}) \left( \frac{d}{dt} 30(t-1)^2 \right) \\ i &= (20 \times 10^{-6}) (60t - 60) = 1.2(t-1) \text{ mA} \\ @t=0.5, i &= -0.6 \text{ mA} \\ t=1, i &= 0 \text{ A} \end{aligned}$$



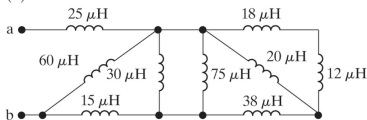
6.23 **PROBLEM** Assume that the initial energy stored in the inductors of Figs. P6.23(a) and (b) is zero. Find the equivalent inductance with respect to the terminals a, b.

Figure P6.23

(a)



(b)



$$a) \textcircled{1} (30 \parallel 15) + 10 \parallel 20 = 10 \text{ mH}$$

$$\textcircled{2} (10 + 8) \parallel 9 = 6 \text{ mH terminal B}$$

$$\textcircled{3} 12 \parallel 24 = 8 \text{ mH terminal A}$$

$$L = 6 \text{ mH} + 8 \text{ mH} = 14 \text{ mH}$$

$$b) \textcircled{1} (12 + 8) = 30 \mu\text{H}$$

$$30 \parallel 20 = 12 \mu\text{H}$$

$$12 + 38 = 50 \mu\text{H}$$

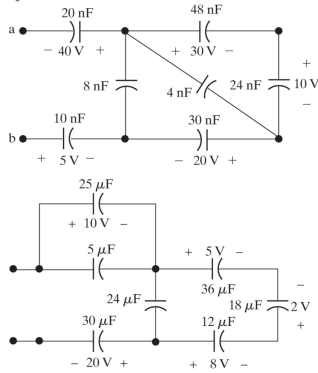
$$30 \parallel 50 \parallel 75 = 15 \mu\text{H}$$

$$\textcircled{2} 15 + 15 = 30 \mu\text{H}$$

$$30 \parallel 60 = 20 \mu\text{H}$$

$$L = 20 \mu\text{H} + 25 \mu\text{H} = 45 \mu\text{H}$$

6.30 Find the equivalent capacitance with respect to the terminals a, b for the circuits shown in Fig. P6.30.



$$a) \frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16} \text{ nF}$$

$$C_2 = 16 + 4 = 20 \text{ nF}$$

$$\frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} = \frac{1}{12} \text{ nF}$$

$$C_4 = 12 + 8 = 20 \text{ nF}$$

$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5} \text{ nF}$$

Capacitance equiv is 5 nF

$$b) \frac{1}{C_1} = \frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \mu\text{F}$$

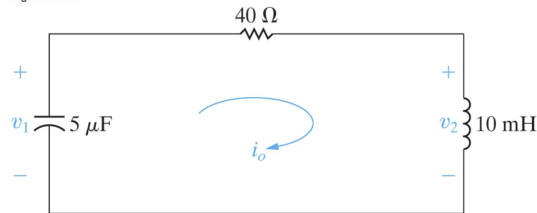
$$C_2 = 6 + 24 = 30 \mu\text{F}$$

$$C_3 = 25 + 5 = 30 \mu\text{F}$$

$$\frac{1}{C_4} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{1}{10}$$

Capacitance equiv is 10 μF

6.35 The current in the circuit in Fig. P6.35 is known to be  $i_o = 5e^{-2000t}(2\cos 4000t + \sin 4000t)$  for  $t \geq 0^+$ . Find  $v_1(0^+)$  and  $v_2(0^+)$ .



$$i_o = 5e^{-2000t}(2\cos 4000t + \sin 4000t)$$

for  $t \geq 0^+$ . Find  $v_1(0^+)$  and  $v_2(0^+)$ .

$$v = \frac{di}{dt} = \frac{d}{dt} 5e^{-2000t}(2\cos 4000t + \sin 4000t)$$

$$\frac{di}{dt} = \left[ 5e^{-2000t}(-8000\sin 4000t + 4000\cos 4000t) \right] - [10000e^{-2000t}(2\cos 4000t + \sin 4000t)]$$

$$\text{for } t=0^+ \quad \frac{di}{dt} = [5(0 + 4000)] - [10000(2 + 0)] = 0$$

$$V_2 = (10 \times 10^{-3}) \frac{di}{dt} = 0 \text{ V}$$

$$V_1 = 40(i_o) + V_2 = 40(10)$$

$$V_1 = 400 \text{ V}$$

$$V_2 = 0 \text{ V}$$