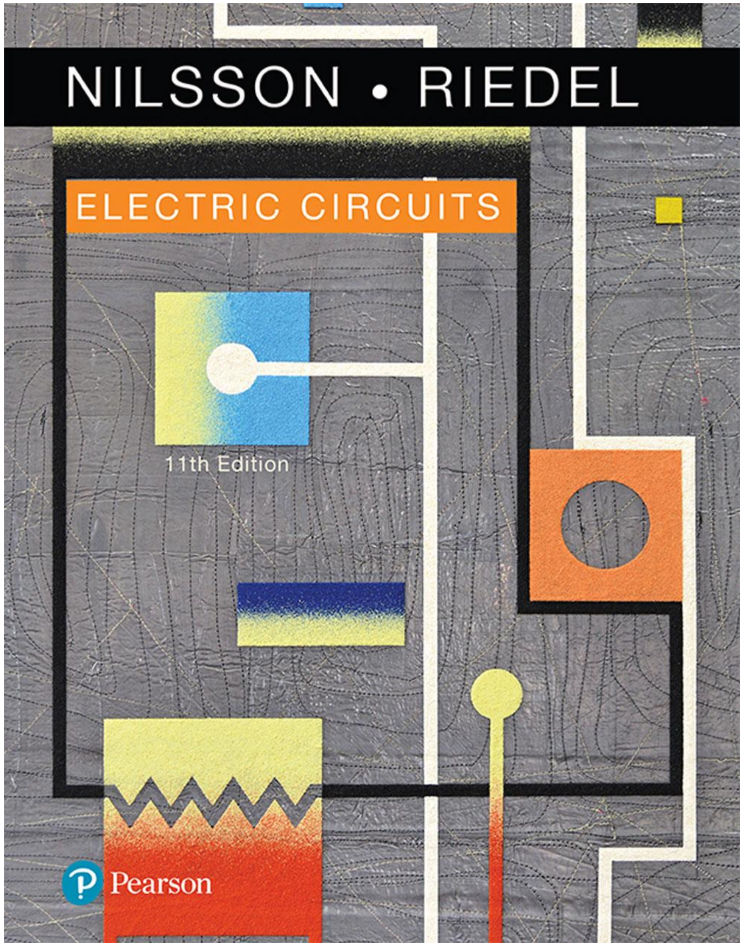
Electric Circuits

Eleventh Edition



Chapter 6

Inductance, Capacitance, and Mutual Inductance

Learning Objectives

- The Inductor
- The Capacitor
- Series-Parallel Combinations of Inductance and Capacitance
- Mutual Inductance
- A Closer Look at Mutual Inductance

Practical Perspective - Capacitive Touch Screens

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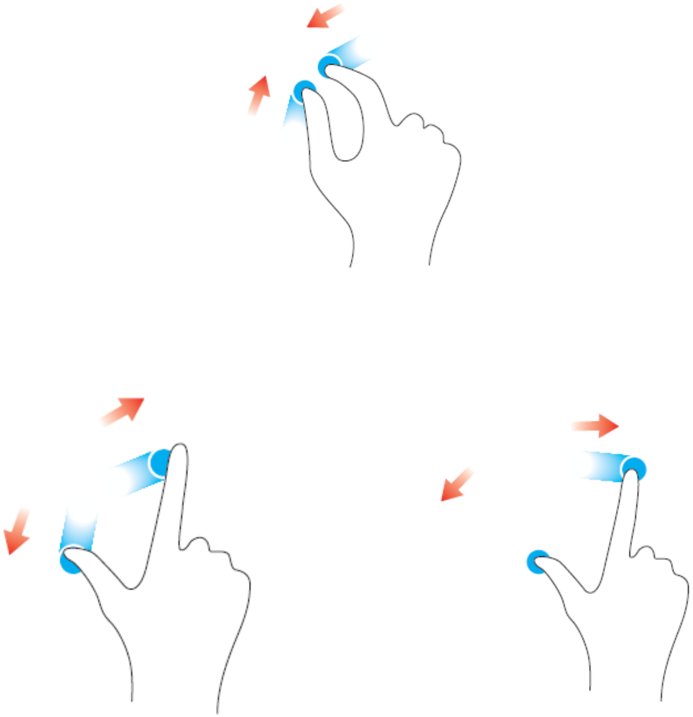
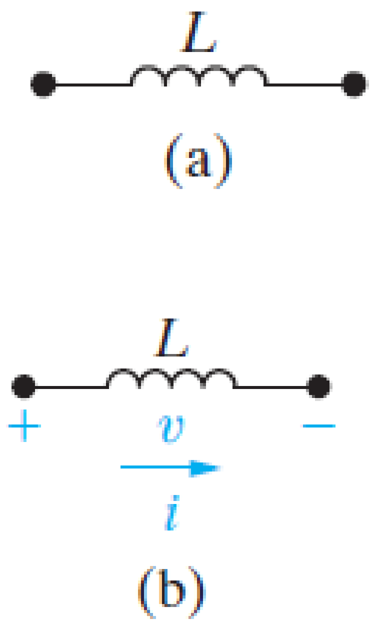


6.1 The Inductor



Inductance is the circuit parameter used to describe an inductor. Inductance is symbolized by the letter L, is measured in Henrys (H), and is represented graphically as a coiled wire - a reminder that inductance is a consequence of a conductor linking a magnetic field.

The Inductor *v*-*i* Equation

$$\boxed{v = L\frac{di}{dt}}$$

Figure 6.1: (a) The graphic symbol for an inductor with an inductance of *L* Henrys. (b) Assigning reference voltage and current to the inductor, following the passive sign convention.

Two Important Observations

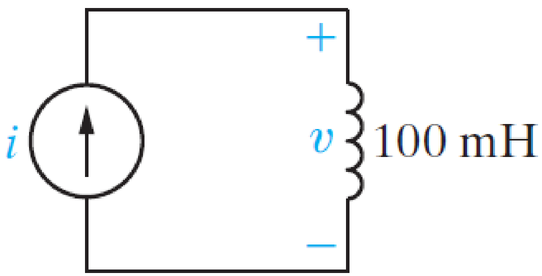
- If the current is constant, the voltage across the ideal inductor is zero. Thus *the inductor behaves as a short circuit in the presence of a constant, or dc, current*.

- Current cannot change instantaneously in an inductor; that is, *the current cannot change by a finite amount in zero time.*

- When someone opens the switch on an inductive circuit in an actual system, the current initially continues to flow in the air across the switch, a phenomenon called *arcing.*

- Arcing must be controlled to prevent equipment damage!

Example 6.1



$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\text{A}, \qquad t > 0$$

The independent current source in the circuit generates zero current for $t < 0$ and a pulse $10te^{-5t}$ A, for $t > 0$.

Figure 6.2: The circuit for Example 6.1.

a) Sketch the current waveform.
b) At what instant of time is the current maximum?
c) Express the voltage across the terminals of the 100 mH inductor as a function of time.
d) Sketch the voltage waveform.
e) Are the voltage and the current at a maximum at the same time?
f) At what instant of time does the voltage change polarity?
g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

Pearson

Solution Example 6.1

a)

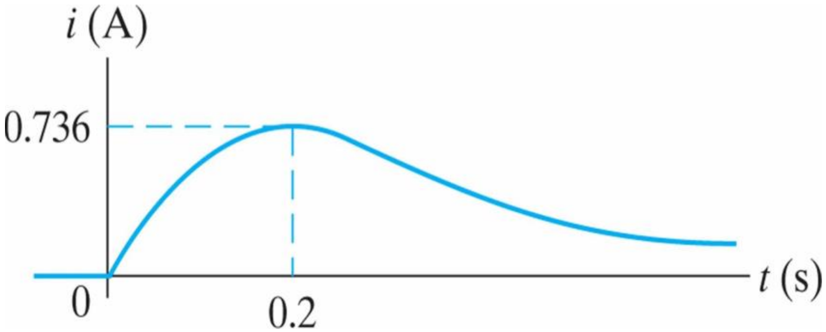


Figure 6.3: The current waveform for Example 6.1.

b)
$$\frac{di}{dt} = 10(e^{-5t} - 5te^{-5t})$$
$$= 10(1 - 5t)e^{-5t}\ A/s, \implies i_{\max}\Big|_{t=\frac{1}{5}\ s}$$

c)
$$v = L\frac{di}{dt} = L10(1 - 5t)e^{-5t}$$
$$= (1 - 5t)e^{-5t}\ V,\ t > 0;\ v = 0,\ t < 0$$

d)

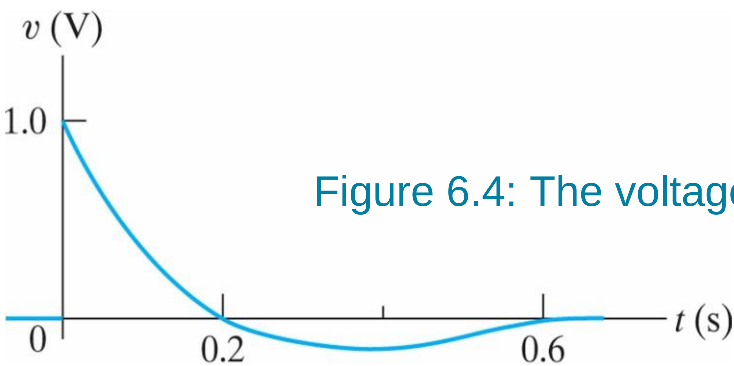


Figure 6.4: The voltage waveform for Example 6.1.

e) No; the voltage is proportional to *di/dt,* not *i.*

f) At 0.2 s, which corresponds to the moment when *di/dt* is passing through zero and changing sign.

g) Yes, at *t* = *0*. Note that voltage can change instantaneously across the terminals of an inductor.

Current in an Inductor in Terms of the Voltage Across the Inductor

Integrate

$$v = L\frac{di}{dt} \Rightarrow v\,dt = L\,di \xrightarrow{\text{Integrate}} L\int_{i(t_0)}^{i(t)} dx = \int_{t_0}^{t} v\,d\tau$$

$$\Rightarrow i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

$t_0 = 0$

Inductor $i - v$ Equation

$$\boxed{i(t) = \frac{1}{L}\int_{0}^{t} v\,d\tau + i(0)}$$

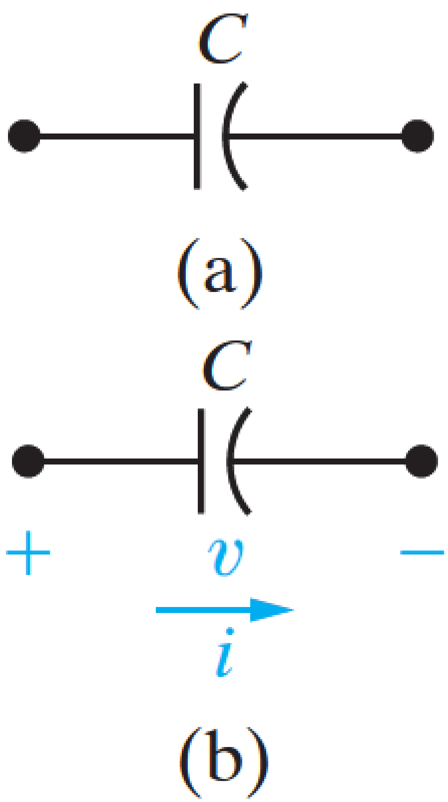
Power and Energy in the Inductor

$$p = vi \quad \Rightarrow \quad p = Li\frac{di}{dt} \quad \Rightarrow \quad p = v\left[\frac{1}{L}\int_{t_0}^{t} v\, d\tau + i(t_0)\right]$$

$$p = \frac{dw}{dt} = Li\frac{di}{dt} \quad \Rightarrow \quad dw = Li\, di \quad \Rightarrow \quad \int_0^w dx = L\int_0^i y\, dy$$

Energy Stored in Inductor $\boxed{w = \frac{1}{2}Li^2}$

6.2 The Capacitor



The circuit parameter of capacitance is represented by the letter C, is measured in Farads (F), and is symbolized graphically by two short parallel conductive plates.

The graphic symbol for a capacitor is a reminder that capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating, material. This condition implies that electric charge is not transported through the capacitor.

1 F is an extremely large quantity of capacitance, practical capacitor values usually lie in the picofarad (pF) to microfarad (μF) range.

Figure 6.10: (a) The circuit symbol for a capacitor. (b) Assigning reference voltage and current to the capacitor, following the passive sign convention.

Capacitor Equations

$$\boxed{i = C\frac{dv}{dt}} \Rightarrow i\,dt = C\,dv \xrightarrow{\text{Integrate}} \int_{v(t_0)}^{v(t)} dx = \frac{1}{C}\int_{t_0}^{t} i\,d\tau$$

Capacitor $i - v$ Equation

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$$

$$\Big\Downarrow \; t_0 = 0$$

Capacitor $v - i$ Equation

$$\boxed{v(t) = \frac{1}{C}\int_{0}^{t} i\,d\tau + v(0)}$$

Power and Energy in the Capacitor

Capacitor Power Equation

$$\boxed{p = vi = Cv\frac{dv}{dt}}$$

$$p = i\left[\frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)\right]$$

$$dw = Cv\,dv$$

$$\int_0^w dx = C\int_0^v y\,dy$$

Energy Stored in Capacitor $\boxed{w = \frac{1}{2}Cv^2}$

Example 6.4

The voltage pulse described by the following equations is imposed across the terminals of a 0.5 μF capacitor:

$$v(t) = \begin{cases} 0, & t \leq 0\,\text{s}; \\ 4t\ \text{V}, & 0\,\text{s} \leq t \leq 1\,\text{s}; \\ 4e^{-(t-1)}\ \text{V}, & t \geq 1\,\text{s}. \end{cases}$$

a) Derive the expressions for the capacitor current, power, and energy.
b) Sketch the voltage, current, power, and energy as functions of time. Specify the time interval when energy is being stored in the capacitor.
c) Specify the time interval when energy is being delivered by the capacitor.
d) Evaluate the integrals and comment on their significance.

$$\int_0^1 p\,dt \quad \text{and} \quad \int_1^\infty p\,dt$$

Solution Example 6.4

a)

$$i = C\frac{dv}{dt} \Rightarrow i = \begin{cases} (0.5 \times 10^{-6})(0) = 0, & t < 0\text{s}; \\ (0.5 \times 10^{-6})(4) = 2\,\mu\text{A}, & 0\text{ s} < t < 1\text{ s}; \\ (0.5 \times 10^{-6})(-4e^{-(t-1)}) = -2e^{-(t-1)}\,\mu\text{A}, & t > 1\text{ s}. \end{cases}$$

$$p = vi = Cv\frac{dv}{dt} \Rightarrow p = \begin{cases} 0, & t \le 0\text{ s}; \\ (4t)(2) = 8t\,\mu\text{W}, & 0\text{ s} \le t < 1\text{ s}; \\ (4e^{-(t-1)})(-2e^{-(t-1)}) = -8e^{-2(t-1)}\,\mu\text{W}, & t > 1\text{ s}. \end{cases}$$

$$w = \frac{1}{2}Cv^2 \Rightarrow w = \begin{cases} 0 & t \le 0\text{ s}; \\ \frac{1}{2}(0.5)16t^2 = 4t^2\mu\text{J}, & 0\text{ s} \le t \le 1\text{ s}; \\ \frac{1}{2}(0.5)16e^{-2(t-1)} = 4e^{-2(t-1)}\,\mu\text{J}, & t \ge 1\text{ s}. \end{cases}$$

b) Energy stored in capacitor when power is positive: 0-1s.

c) Energy delivered by capacitor when power is negative: t >=1 s.

d) The first integral represents energy stored in the capacitor between 0 < t < 1 s; the second integral represents energy returned, or delivered, by the capacitor in the interval 1 s to ∞:

$$\int_0^1 p\,dt = \int_0^1 8t\,dt = 4t^2\Big|_0^1 = 4\ \mu\text{J}$$

$$\int_1^\infty p\,dt = \int_1^\infty (-8e^{-2(t-1)})dt = (-8)\frac{e^{-2(t-1)}}{-2}\Big|_1^\infty = -4\ \mu\text{J}$$

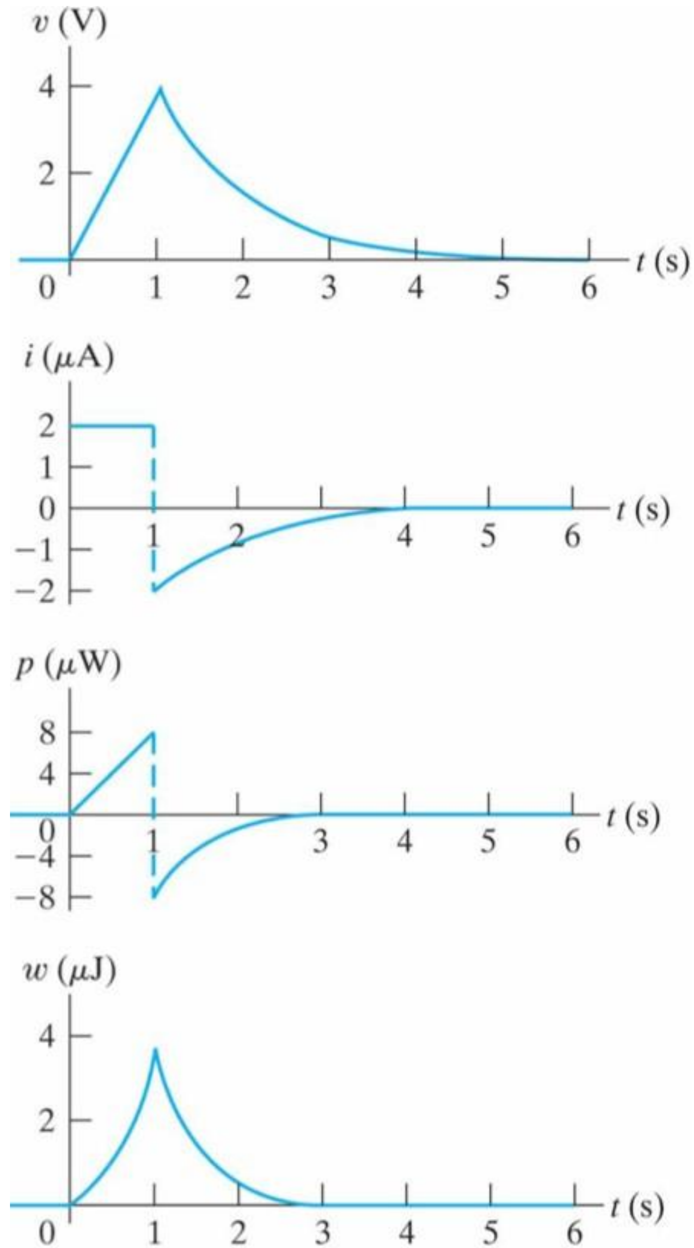


Figure 6.11: The variables *v, i, p*, and *w* versus *t* for Example 6.4.

6.3 Series-Parallel Combinations of Inductance & Capacitance

Series Combination of Inductance

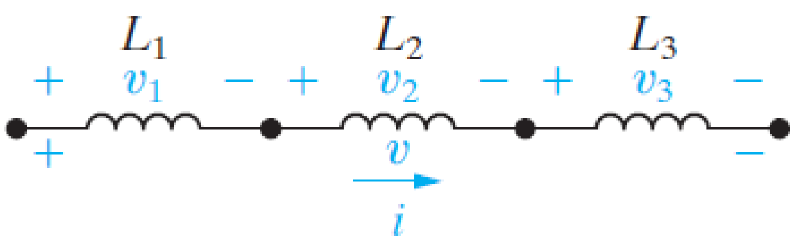


Figure 6.13: Inductors in series.

$$v_1 = L_1\frac{di}{dt} \quad v_2 = L_2\frac{di}{dt} \quad v_3 = L_3\frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n}$$

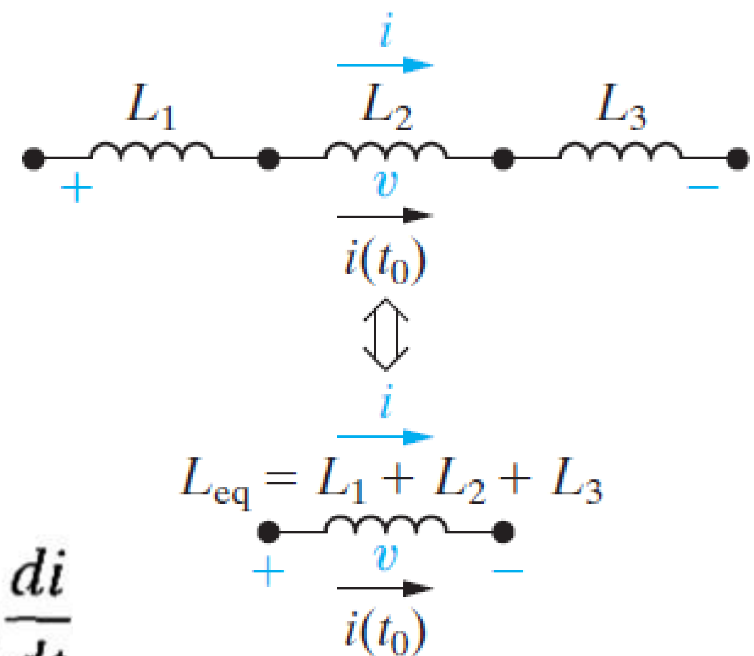


Figure 6.14: An equivalent circuit for inductors in series carrying an initial current $i(t_0)$.

Parallel Combination of Inductance

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,d\tau + i_1(t_0)$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,d\tau + i_2(t_0)$$

$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\,d\tau + i_3(t_0)$$

$$i = i_1 + i_2 + i_3$$

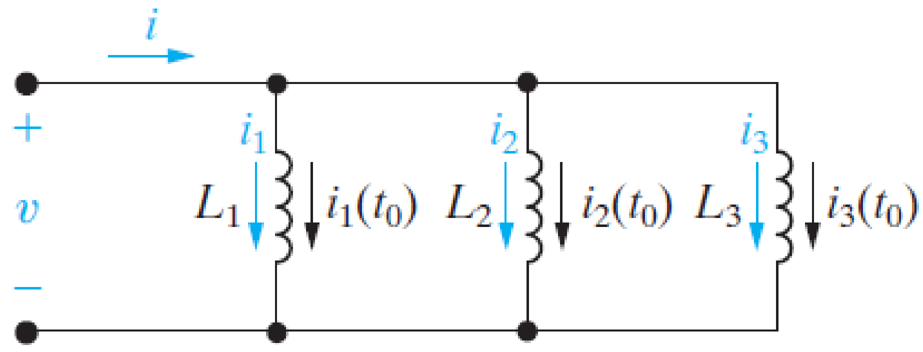
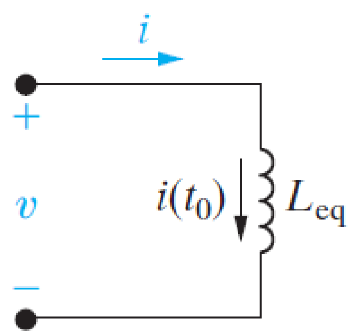


Figure 6.15: Three inductors in parallel.



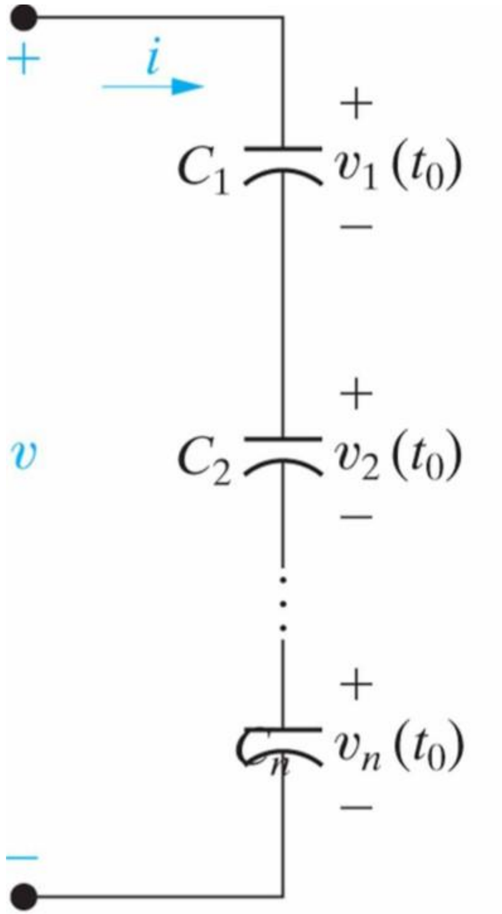
$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

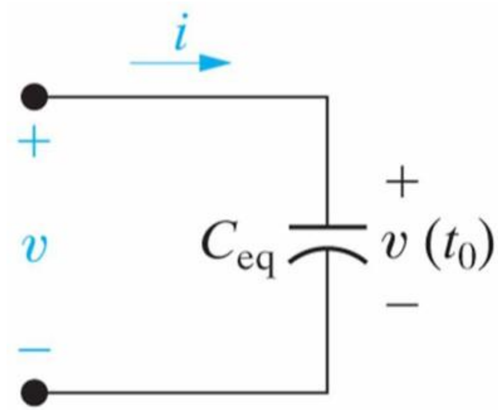
Figure 6.16: An equivalent circuit for three inductors in parallel.

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v\,d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Series Combination of Capacitance



$$v(t) = \frac{1}{C}\int_0^t i\,d\tau + v(0)$$



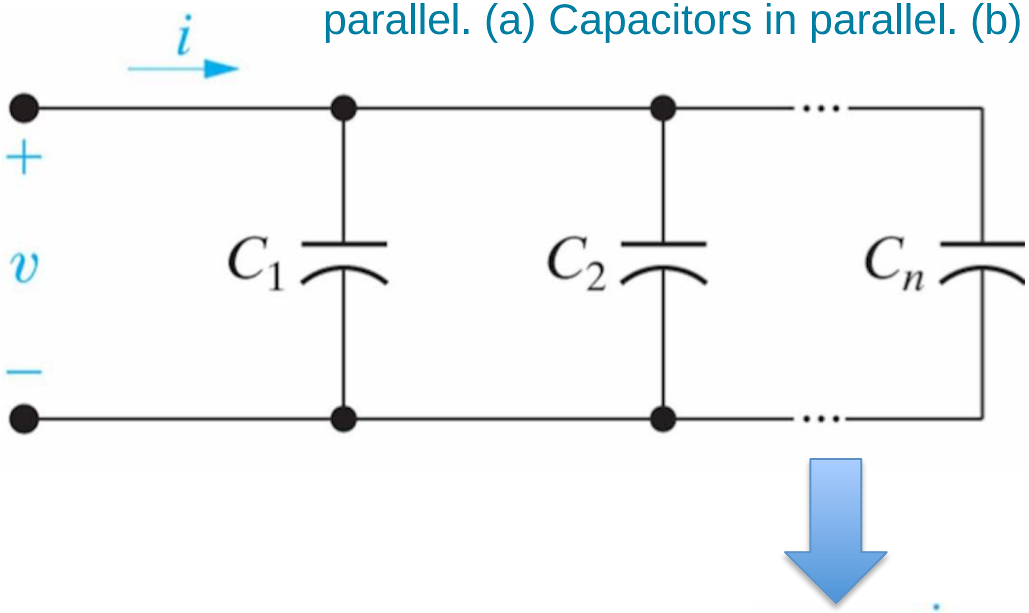
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}}$$

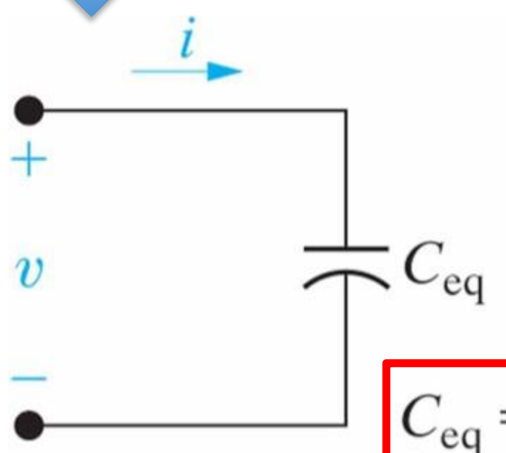
$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

Figure 6.17: An equivalent circuit for capacitors connected in series. (a) The series capacitors. (b) The equivalent circuit.

Parallel Combination of Capacitance

Figure 6.18: An equivalent circuit for capacitors connected in parallel. (a) Capacitors in parallel. (b) The equivalent circuit.





$$i = C\frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2 + \cdots + C_n}$$

Inductor & Capacitor Symmetry

TABLE 6.1 Inductor and Capacitor Duality

	Inductors	**Capacitors**
Primary v-i equation	$v(t) = L\frac{di(t)}{dt}$	$i(t) = C\frac{dv(t)}{dt}$
Alternate v-i equation	$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$	$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$
Initial condition	$i(t_0)$	$v(t_0)$
Behavior with a constant source	If $i(t) = I, v(t) = 0$ and the inductor behaves like a short circuit	If $v(t) = V, i(t) = 0$ and the capacitor behaves like an open circuit
Continuity requirement	$i(t)$ is continuous for all time so $v(t)$ is finite	$v(t)$ is continuous for all time so $i(t)$ is finite
Power equation	$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt}$	$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$
Energy equation	$w(t) = \frac{1}{2}Li(t)^2$	$w(t) = \frac{1}{2}Cv(t)^2$
Series-connected equivalent	$L_{eq} = \sum_{j=1}^{n} L_j$ $i_{eq}(t_0) = i_j(t_0)$ for all j	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ $v_{eq}(t_0) = \sum_{j=1}^{n} v_j(t_0)$
Parallel-connected equivalent	$\frac{1}{L_{eq}} = \sum_{j=1}^{n} \frac{1}{L_j}$ $i_{eq}(t_0) = \sum_{j=1}^{n} i_j(t_0)$	$C_{eq} = \sum_{j=1}^{n} C_j$ $v_{eq}(t_0) = v_j(t_0)$ for all j

Pearson

6.4 Mutual Inductance

L_1, L_2: self-inductances; M: mutual inductance

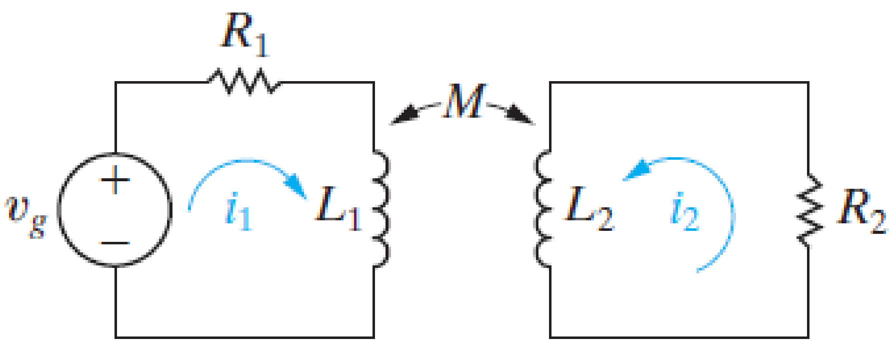


Figure 6.24: Coil currents i_1 and i_2 used to describe the circuit shown in Fig. 6.23.

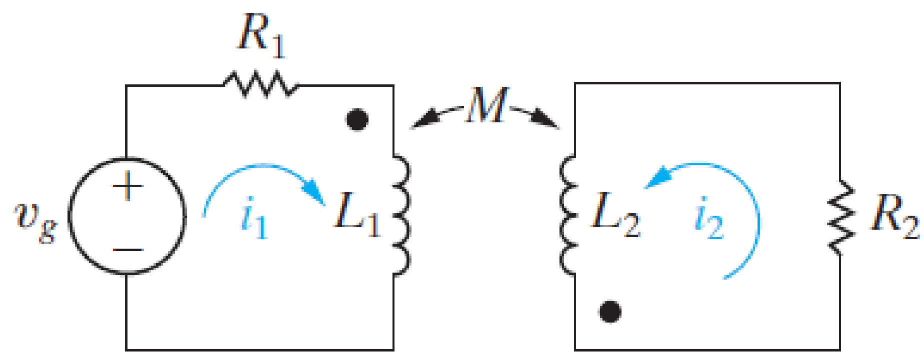


Figure 6.25: The circuit of Fig. 6.24 with dots added to the coils indicating the polarity of the mutually induced voltages.

Dot Convention*:* When the reference direction for a current enters (leaves) the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive (negative) at its dotted terminal.

Mutual Inductance Equations

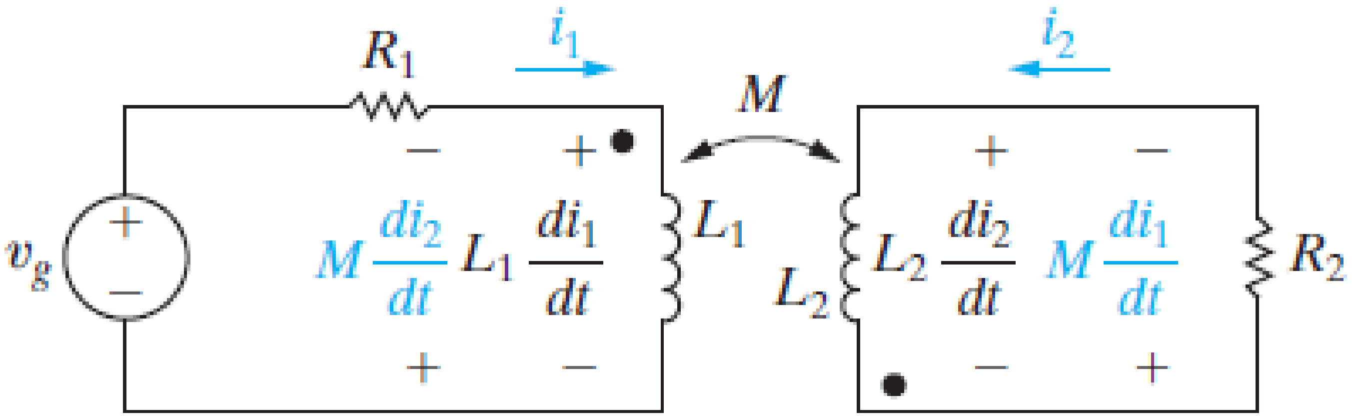


Figure 6.26 The self- and mutually induced voltages appearing across the coils shown in Fig. 6.25.

$$-v_g + i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

$$i_2R_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

Determining Dot Markings

a) Arbitrarily select one terminal - D terminal - of one coil and mark it with a dot.

b) Assign a current into the dotted terminal and label it i_D.

c) Use right hand rule to determine direction of magnetic flux established by i_D inside the coupled coils and label this flux ϕ_D.

d) Arbitrarily pick one terminal of the second coil - terminal A - and assign a current into this terminal, and label it i_A.

e) Use right hand rule to determine direction of flux established by i_A inside the coupled coils and label this flux ϕ_A.

f) Compare directions of the two fluxes ϕ_D and ϕ_A:

- If the fluxes have the same reference direction, place a dot on the terminal of the second coil where the test current (i_A) enters. (In Fig. 6.27, the fluxes ϕ_D and ϕ_A have the same reference direction, and therefore a dot goes on terminal A.)
- If the fluxes have different reference directions, place a dot on the terminal of the second coil where the test current leaves

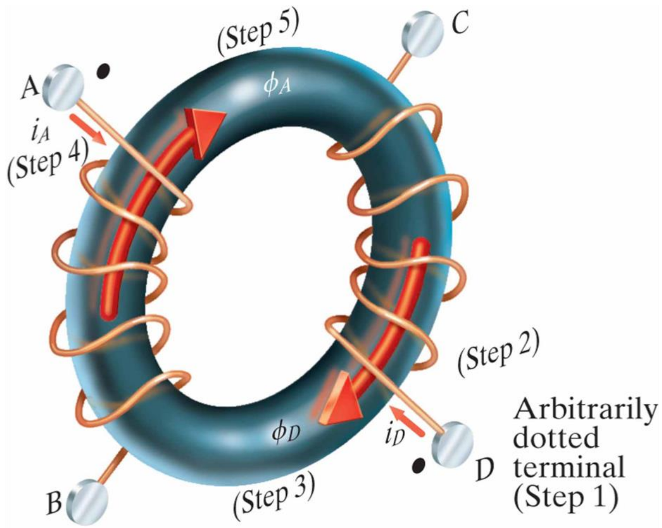
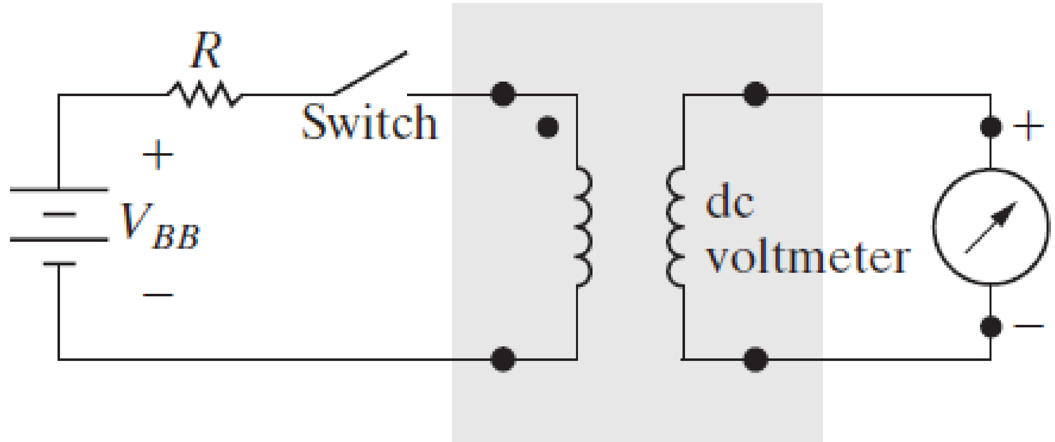


Figure 6.27: A set of coils showing a method for determining a set of dot markings.

Experimental Method

Figure 6.28: An experimental setup for determining polarity marks.



When the switch is closed, voltmeter deflection is observed.

If the momentary deflection is upscale, the coil terminal connected to the positive terminal of the voltmeter receives the polarity mark.

If the deflection is downscale, the coil terminal connected to the negative terminal of the voltmeter receives the polarity mark.

Practical Perspective - Capacitive Touch Screens

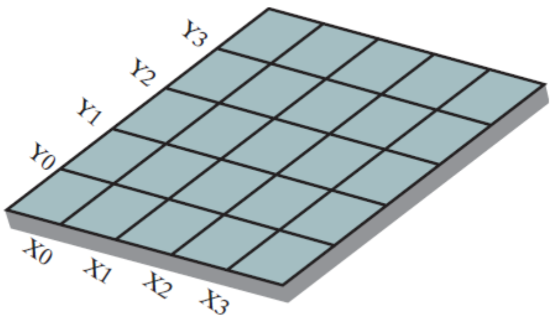


Figure 6.35: Multi-touch screen with grid of electrodes.

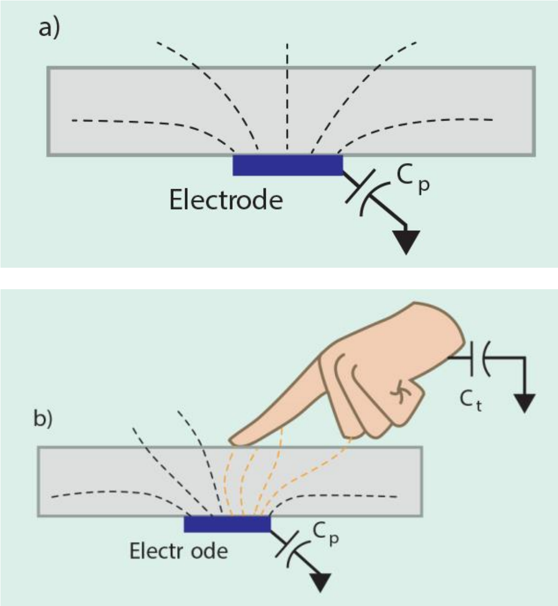


Figure 6.36: (a) Parasitic capacitance between electrode and ground with no touch; (b) Additional capacitance introduced by a touch.

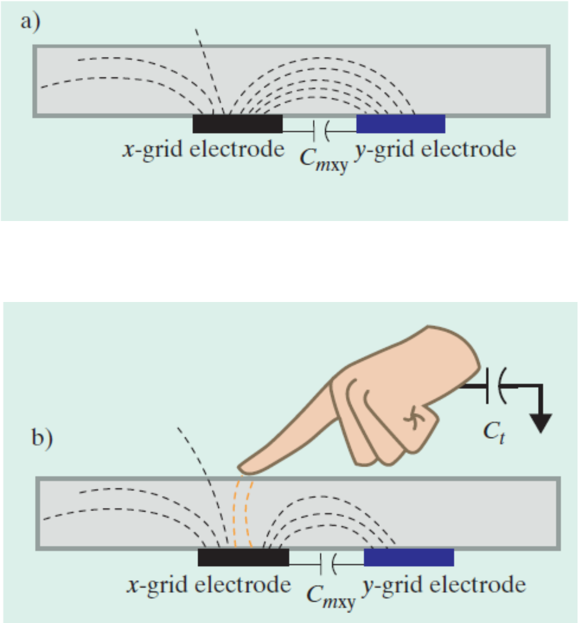


Figure 6.37: (a) Mutual capacitance between an *x*-grid and a *y*-grid electrode; (b) Additional capacitance introduced by a touch.