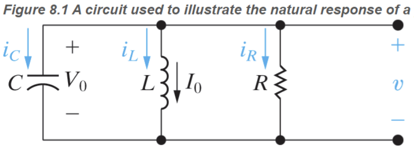
8.2 PSPICE MULTISIM The circuit elements in the circuit in **Fig. 8.1** are $R = 200\ \Omega$, $L = 50\ \text{mH}$, and $C = 0.2\ \mu\text{F}$. The initial inductor current is $-45\ \text{mA}$ and the initial capacitor voltage is 15 V.

a. Calculate the initial current in each branch of the circuit.
b. Find $v(t)$ for $t \geq 0$.
c. Find $i_L(t)$ for $t \geq 0$.

Figure 8.1 A circuit used to illustrate the natural response of a parallel RLC circuit.



a)
$$\boxed{\begin{aligned} i_{R\,\text{initial}} &= \frac{15}{200} = 75\,mA \\ i_{L\,\text{initial}} &= -45\,mA \\ i_{C\,\text{initial}} &= -(-45) - 75 = -30\,mA \end{aligned}}$$

b)
$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$
$$s^2 + \frac{s}{(200)(0.2\times10^{-6})} + \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 0$$
$$s^2 + 25000s + 10^8 = 0$$
$$s = \frac{-25000 \pm \sqrt{(25000)^2 - 4(1)(10^8)}}{2}$$
$$s = -12500 \pm 7500$$
$$s = -5000 \ \ \& \ \ s = -20000$$
$$V = A_1 e^{-5000t} + A_2 e^{-20000t}$$
$$① \rightarrow V(0) = A_1 + A_2 = 15\ V$$
$$V' = -5000A_1 e^{-5000t} + -20000A_2 e^{-20000t}$$
$$V'(0) = -5000A_1 \ -20000A_2$$
$$V'(0) = \frac{i_c(0)}{C} = \frac{-30\times10^{-3}}{0.2\times10^{-6}}$$
$$② \rightarrow -50000A_1 - 20000A_2 = -15\times10^4$$

Solving eq ① + ②

$$A_1 = 10,\ A_2 = 5$$
$$\therefore \boxed{V = 10e^{-5000t} + 5e^{-20000t}\ V}$$

c)
$$i_L = -i_c - i_R$$
$$\rightarrow i_R = 50e^{-5000t} + 25e^{-20000t}\ mA$$
$$\rightarrow i_c = C\frac{dV}{dt}$$
$$i_c = -10e^{-5000t} - 20e^{-20000t}\ mA$$
$$\boxed{i_L = -40e^{-5000t} - 5e^{-20000t}\ mA}$$

8.3 PSPICE MULTISIM The resistance in **Problem 8.2** is increased to $250\ \Omega$. Find the expression for $v(t)$ for $t \geq 0$.

$$\alpha = \frac{1}{2RC} = \frac{1}{2(250)(0.2\times10^{-6})}$$
$$\alpha = 10^4$$
$$\alpha^2 = 10^8 = \omega^2$$
$$V = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}$$

$$V = (A_1 t + A_2)e^{-\alpha t}$$
$$\boxed{V = (75000t + 15)e^{-10000t}\ V}$$

$$i_c(0^+) = -i_L - i_R = 45 - \frac{15}{250} = -15\,mA$$
$$V(0) = A_2 = 15$$
$$V' = A_1\left[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}\right] - \alpha A_2 e^{-\alpha t}$$
$$V'(0) = A_1 - \alpha A_2 = \frac{i_c}{C} = \frac{-15\times10^{-3}}{0.2\times10^{-6}} = -75000$$
$$A_1 = (10^4)(15) - 75000 = 75000$$

8.4 PSPICE MULTISIM The resistance in Problem 8.2 is increased to $312.5\ \Omega$. Find the expression for $v(t)$ for $t \geq 0$.

$$\alpha = \frac{1}{2RC} = \frac{1}{2(312.5)(0.2\times10^{-6})} = 8000$$

$$\omega^2 = 10^8$$

$$s = -8000 \pm \sqrt{8000^2 - 10^8}$$

$$s = -8000 \pm 6000j$$

$$v(t) = A_1 e^{-8000t}\cos(6000t) + A_2 e^{-8000t}\sin(6000t)$$

$$v(0) = A_1 = 15V$$

$$i_C = +45 - \frac{15}{312.5} = -3mA$$

$$v'(0) = -8000A_1 + 6000A_2 = \frac{-3\times10^{-3}}{0.2\times10^{-6}}$$

$$A_2 = \frac{-8000(15) - 15000}{6000}$$

$$A_2 = 17.5V$$

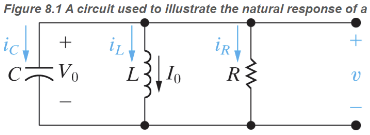
$$\boxed{v(t) = 15e^{-8000t}\cos(6000t) + 17.5e^{-8000t}\sin(6000t)\ V}$$

8.8 The natural voltage response of the circuit in Fig. 8.1 is

$$v(t) = 120e^{-400t}\cos 300t + 80e^{-400t}\sin 300t\ \text{V},$$

when the capacitor is $250\ \mu\text{F}$. Find (a) L; (b) R; (c) V_0; (d) I_0; and (e) $i_L(t)$.

Figure 8.1 A circuit used to illustrate the natural response of a parallel RLC circuit.



$$\alpha = 400 \quad \omega = 300$$

$$\omega_0^2 = \omega^2 + \alpha^2 = 25\times10^4$$

$$\omega_0^2 = \frac{1}{LC} = 25\times10^4$$

$$L = \frac{1}{(25\times10^4)(250\times10^{-6})}$$

a) $\boxed{L = 16mH}$

$$\frac{1}{2RC} = 400$$

$$R = \frac{1}{2(400)(250\times10^{-6})}$$

b) $\boxed{R = 5\Omega}$

c) $\boxed{V_0 = V(0) = 120V}$

$$I_0 = i_L(0)$$

$$i_L(0) = -i_R(0) - i_C(0)$$

$$i_L(0) = -\frac{120}{5} - \left[(250\times10^{-6})(-400(120) + 300(80))\right]$$

$$i_L(0) = -18A$$

d) $\boxed{I_0 = -18A}$

$$i_L(t) = -i_R(t) - i_C(t)$$

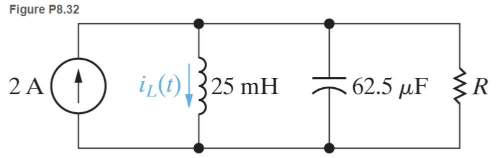
$$i_R(t) = \frac{v(t)}{5} = e^{-400t}(24\cos(300t) + 16\sin(300t))A$$

$$i_C(t) = (250\times10^{-6})\ v'(t) = e^{-400t}(-17\sin(300t) - 6\cos(300t))A$$

e) $\boxed{i_L(t) = e^{-400t}(-18\cos(300t) + \sin(300t))A}$

8.32 PSPICE MULTISIM Assume that at the instant the 2 A current source is applied to the circuit in **Fig. P8.32**, the initial current in the 25 mH inductor is 1 A, and the initial voltage on the capacitor is 50 V (positive at the upper terminal). Find the expression for $i_L(t)$ for $t \geq 0$ if R equals $12.5\ \Omega$.

Figure P8.32



$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(25\times10^{-3})(62.5\times10^{-6})} = 640000$$

$$\alpha^2 = \left(\frac{1}{2RC}\right)^2 = \left(\frac{1}{2(12.5)(62.5\times10^{-6})}\right)^2 = 409600$$

$$\omega_d = \sqrt{640000 - 409600} = \sqrt{230400}$$

$$\omega_d = 480 \text{ rad/s}$$

$$i = 2A$$

$$i_L = 2 + A_1 e^{-640t}\cos(480t) + A_2 e^{-640t}\sin(480t)$$

$$i_L(0) = 2 + A_1 = 1 \Rightarrow A_1 = -1$$

$$\frac{d}{dt} i_L(0) = -640A_1 + 480A_2 \quad + \quad \frac{d}{dt} i_L(0) = \frac{V}{L}$$

$$\therefore \quad -640(-1) + 480A_2 = \frac{50}{25\times10^{-3}}$$

$$A_2 = 2.83$$

$$\boxed{i_L(t) = 2 - e^{-640t}\cos(480t) + 2.83e^{-640t}\sin(480t)\,A}$$

8.33 PSPICE MULTISIM The resistance in the circuit in **Fig. P8.32** is changed to $8\ \Omega$. Find $i_L(t)$ for $t \geq 0$.

$$\omega_0^2 = \frac{1}{LC} = 640000$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(8)(62.5\times10^{-6})} = 1000$$

$$s = -1000 \pm \sqrt{1000^2 - 640000}$$

$$s = -400, \quad s = -1600$$

$$\therefore \quad i_L = 2 + A_1 e^{-400t} + A_2 e^{-1600t}$$

$$i_L(0) = 2 + A_1 + A_2 = 1$$

$$A_1 + A_2 = -1$$

$$\frac{d}{dt} i_L = -400A_1 e^{-400t} - 1600A_2 e^{-1600t}$$

$$\frac{d}{dt} i_L(0) = -400A_1 - 1600A_2$$

Since $\frac{d}{dt} i_L = \frac{V}{L}$,

$$-400A_1 - 1600A_2 = \frac{50}{25\times10^{-3}}$$

Solving for A_1 + A_2

$$A_1 = \frac{1}{3} \quad A_2 = -\frac{4}{3}$$

$$\therefore \boxed{i_L(t) = 2 + \frac{1}{3}e^{-400t} - \frac{4}{3}e^{-1600t}\,A}$$

8.34 PSPICE MULTISIM The resistance in the circuit in **Fig. P8.32** is changed to $10\ \Omega$. Find $i_L(t)$ for $t \geq 0$.

$$\omega_0^2 = 640000$$

$$\alpha = \frac{1}{2(10)(625\times10^{-6})} = 800$$

$$\alpha^2 = 640000$$

$$i_L = 2 + A_1 t e^{-800t} + A_2 e^{-800t}$$

$$i_L(0) = 2 + A_2 = 1$$

$$A_2 = -1$$

$$\frac{d}{dt} i_L = A_1\left[-800 t e^{-800t} + e^{-800t}\right] - 800 e^{-800t} A_2$$

$$\frac{d}{dt} i_L(0) = A_1 - 800 A_2 = \frac{V}{L} = 2000$$

$$A_1 = 2000 - 800$$

$$A_1 = 1200$$

$$\boxed{i_L(t) = 2 + 1200 t e^{-800t} - e^{-800t}\ \text{A}}$$