Circuit Homework #10 Andrew Maradiaga

Q1)

current through inductor @ $t = 0$

$i(0) = 40\,mA$

Current through inductor for $t > 0$

$$i(t) = A_1 e^{-10000t} + A_2 e^{-40000t}$$

$$i(0) = A_1 e^{-10000(0)} + A_2 e^{-40000(0)}$$

$$40 \times 10^{-3} = A_1 + A_2$$

Voltage across inductor

$$V_L(t) = L \frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} = \frac{V_L(t)}{L}$$

$$\frac{di(0)}{dt} = \frac{V_L(0)}{L}$$

$$= \frac{28}{20 \times 10^{-3}}$$

$$= 1400$$

$$\frac{di(t)}{dt} = -10000 A_1 e^{-10000t} - 40000 A_2 e^{-40000(t)}$$

$$\frac{di(0)}{dt} = -10000 A_1 e^{-10000(0)} - 40000 A_2 e^{-40000(0)}$$

$$1400 = -10000 A_1 - 40000 A_2$$

$$14 = -100 A_1 - 400 A_2$$

Solve equation for A_1 and A_2

$A_1 = .1$

$A_2 = -0.06$

current in inductor $t > 0$

$$i(t) = .1\, e^{-10000(t)} - 0.06\, e^{-40000(t)} \text{ A} \quad t \geq 0$$

Voltage across inductor $t > 0$

$$V_L(t) = L \frac{di(t)}{dt}$$

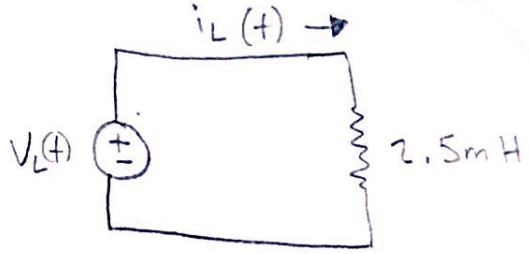
$$= (20 \times 10^{-3})(.1(-10000)\, e^{-10000t} - 0.06(-40000)\, e^{-40000t})$$

$$= -20\, e^{-10000(t)} + 48\, e^{-40000(t)} \text{ V} \quad t \geq 0$$

Q2)

$V_L(t) = 3e^{-4t}$ mV, $0^+ \le t \le 2$ s

$V_L(t) = -3e^{-4(t-2)}$ mV, $2 \le t < \infty$ s



$0 \le t \le 2s$

$$iL = \frac{10^3}{2.5}\int_0^t 3\times10^{-3}\, e^{-4x}\, dx + 1$$

$$= 1.2\,\frac{e^{-4x}}{-4}\Big|_0^t + 1$$

$$= -0.3e^{-4t} + 1.3A \quad 0 \le t \le 2s$$

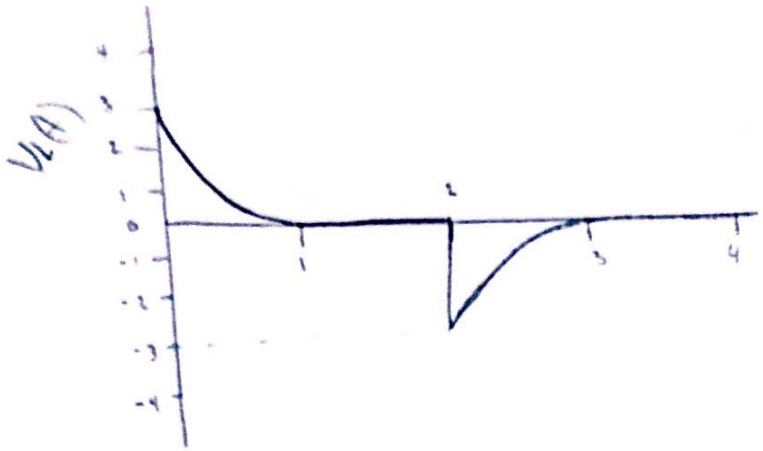
$$i_L(2) = -0.3e^{-8} + 1.3 = 1.3A$$

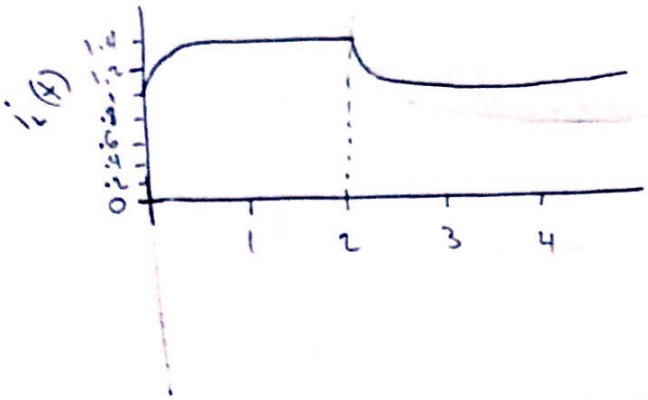
$t \ge 2s$:

$$i_L = \frac{10^3}{2.5}\int_2^t -3\times10^{-3}\, e^{-4(x-2)}\, dx + 1.3$$

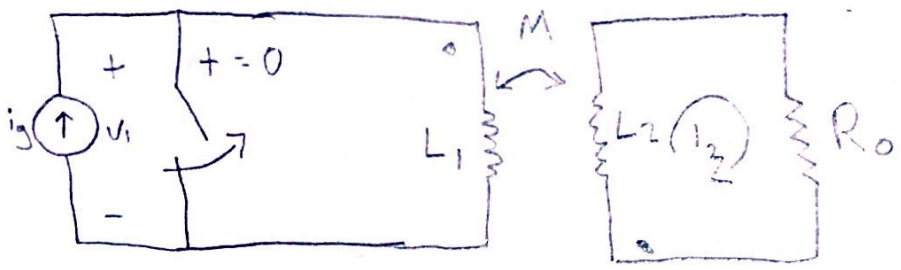
$$= -1.2\,\frac{e^{-4(x-2)}}{-4}\Big|_2^t + 1.3$$

$$= 0.3e^{-4(t-2)} + 1A \quad t \ge 2s$$





Q3)



i) KVL @ loop 2

$$i_2 R_o + L_2 \frac{di_2}{dt} + M \frac{di_g}{dt} = 0$$

$$i_2(10) + (0.2)\frac{di_2}{dt} + (.5)\frac{di_g}{dt} = 0$$

$$0.2\frac{di_2}{dt} + 10 i_2 = -0.5\frac{di_g}{dt}$$

ii)

$$-V_1 + L_1 \frac{di_g}{dt} + M\frac{di_2}{dt} = 0$$

$$V_1 = L_1 \frac{di_g}{dt} + M\frac{di_2}{dt}$$

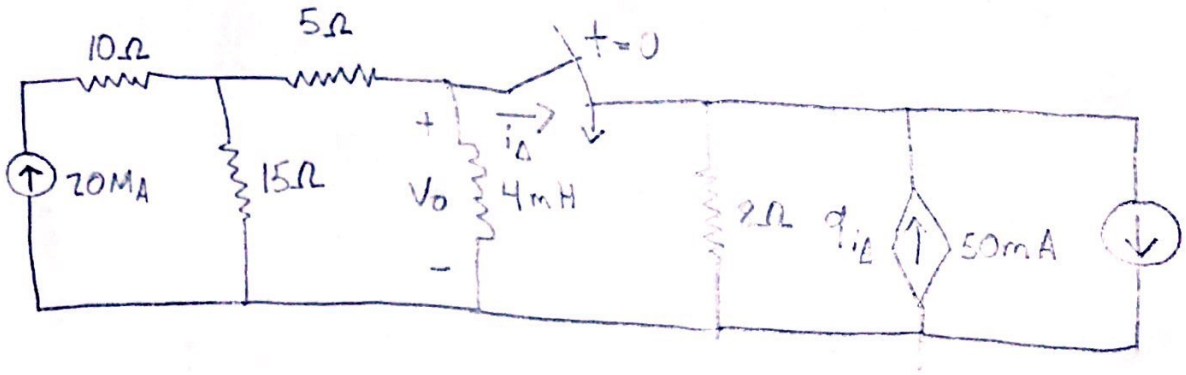
$$V_1 = 5\frac{d(e^{-10t} - 10A)}{dt} + .5\frac{d(625e^{-10t} - 250e^{-50t})}{dt}$$

$$= -50e^{-10t} + .5(-6250e^{-10t} + 12500e^{-50t})\times 10^{-3}$$

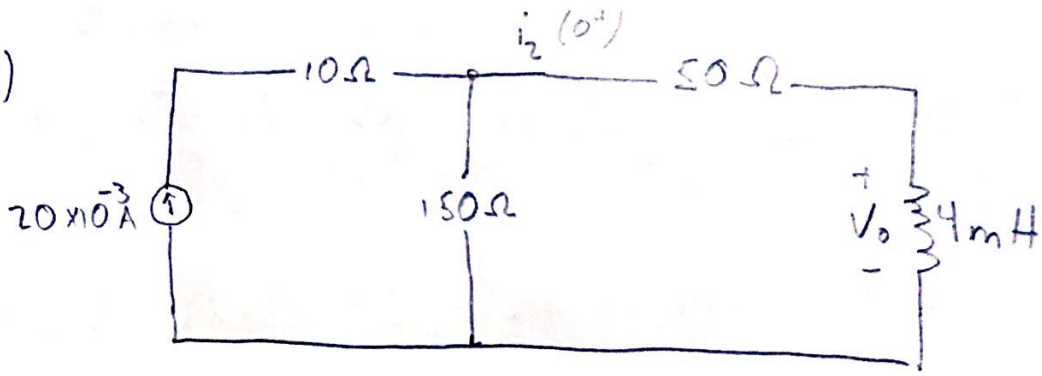
$$= -50e^{-10t} - 3.125e^{-10t} + 6.25e^{-50t}$$

$$= -53.125e^{-10t} + 6.25e^{-50t}$$

Q4.)

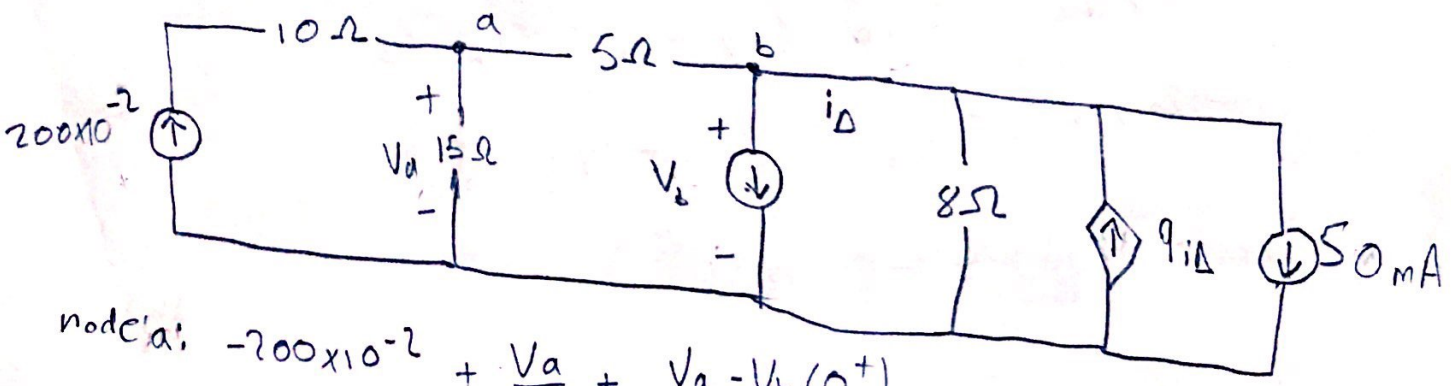


i)



$$i_2(0^+) = \frac{15}{20}\left(2.0\times10^{-2}A\right) = .015A$$

for $t \geq 0$



node 'a': $-200\times10^{-2} + \frac{V_a}{15} + \frac{V_a - V_b(0^+)}{5} = 0$

$$15\left(\frac{V_a}{15}\right) + 15\left(\frac{V_a - V_b(0^+)}{5}\right) = 15\left(2.0\times10^{-2}A\right)$$

$$V_a + 3V_a - 3V_b(0^+) = 30\times10^{-2}A$$

$$4V_a - 3V_b(0^+) = 3.0\times10^{-1}A = 0.3A$$ (1)

$$i_A = \frac{V_b(0^+)}{8} - 9 i_\Delta - 0.050$$

$$10 i_\Delta = \frac{V_b(0^+)}{8} + 0.050 \rightarrow i_\Delta = \frac{V_a(0^+)}{80} + .005$$

---

node b: $\frac{V_b(0^+) - V_a}{5} + .015 + \frac{V_a(0^+)}{8} - 9 i_\Delta + .050 = 0$

$$V_b(0^+) - V_a + .075 + \frac{5V_b(0^+)}{8} - 45 i_\Delta + .25 = 0$$

$$8V_b(0^+) - 8V_a + 0.6 + 5V_b(0^+) - 360 i_\Delta + 2 = 0$$

$$13V_b(0^+) - 8V_a - 4.5 V_b(0^+) - 105 + 206 = 0$$

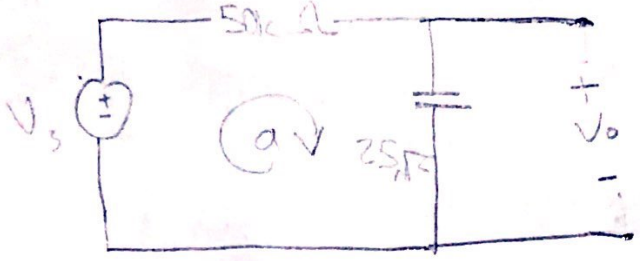
$$-8V_a + 13V_b(0^+) - 4.5 V_b(0^+) = -0.3$$

$$-8V_a + 8.5 V_b(0^+) = -0.8$$

$$V_b = (0^+) = -0.08 V$$

$V_b(\infty) = 0$ inductor is not available.

(Q5)



$$V_s(t) = \begin{cases} 0 & -1 < 0 \text{ ms} \\ 100 & 0 \le t \le 4 \text{ ms} \\ -100 & 4 \text{ ms} \le t \le 8 \text{ ms} \\ 0 & t \ge 8 \text{ ms} \end{cases}$$

KVL@a: $-V_s + R_c \frac{dv}{dt} + V_o = 0$

$$RC \frac{dv}{dt} + V_o = V_s$$

$$RC(Dv) + V_o = V_s$$

$$V_o(DRC + 1) = V_s$$

$$V_o(t) = CF + PI$$

$V_s = 0$

$V_o(DRC + 1) = 0$

$D = \frac{-1}{RC}$

$CF - ke$

PI $= V_s \cdot k e^{-t/RC}$ for

$V_o(t) = k e^{-t/RC} + V_s$

@ $t = 0$, $V_o = 0$

$0 = k e^{0} + V_s$

$V_s = -k \rightarrow k = -V_s$

$V_o(t) = (-V_s) e^{-t/RC} + V_s$

$V_o(t) = V_s - V_s e^{\frac{-t}{RC}}$

$\tau = RC = 50 \times 10^3 (25 \times 10^{-9}) = .00125 = \frac{1}{800}\,s$

$V_s = 100$

$V_o(t) = 100 - 100\, e^{-800t}$

$V_o(4ms) = 100(1 - e^{-800(4 \times 10^{-3})}) \quad (4ms \leq t \leq 8ms)$

$= 95.92V$

$V_o(t) = V_o(\infty) + (V_o(4ms) - V_o(\infty)) e^{-800(t - 4 \times 10^{-3})}$

$V_o(\infty) = -100V$

$V_o(t) = -100 + (95.92 + 100) e^{-800(t - 4 \times 10^{-3})}$

$V_o(t) = -100 + 195.92\, e^{-800t + 3.2}$

For $t \geq 8ms$

$$V_o(t) = V_o(\infty) + (V_o(8ms) - V_o(\infty))e$$

$$V_o(8ms) = -100 + 195.92\, e^{-800(8\times10^{-3})} + 3.2$$

$$= -92.14V$$

$V_o(\infty) = 0 \quad t = \infty$, capacitor is open

$$V_o(t) = 0 + 92.14\left(e^{\frac{-\infty - 8ms}{RC}}\right)$$

$$= -92.14V \quad @ \quad t \geq 8ms$$

$$\begin{cases} 0V & t \leq 0\,ms \\ 100 - 100e^{-800t} & 0\,ms \leq t \leq 4ms \\ -100 + 195.92\, e^{-800t + 3.2} & 4 \leq t \leq 8ms \\ -92.14 & t \geq 8ms \end{cases}$$