Homework 10 | Elijah Stamp

1) i in 20mH inductor =

$i = 4 mA$ $\quad t \leq 0$

$i(t) = (A_1 e^{-10000t} + A_2 e^{-40000t})A \quad t \geq 0$

at $t=0$ v accross the inductor $= 28V$

Find $V(t)$ when $t > 0$

$$V(t) = L\frac{di(t)}{dt}$$

$$= (20\times10^{-3})\frac{d}{dt}(A_1 e^{-10000t} + A_2 e^{-40000t})$$

$$= (20\times10^{-3})(-10000 A_1 e^{-10000t} - 40000 A_2 e^{-40000t})$$

$$= -200A_1 e^{-10000t} - 800A_2 e^{-40000t} = V(t)$$

$V(0) = 28$

$$28 = -200A_1 e^{-10000(0)} - 800A_2 e^{-40000(0)}$$

$$28 = -200A_1 - 800A_2 \quad (1)$$

$i(t) = A_1 e^{-10000t} + A_2 e^{-40000t}$

$i(0) = 4mA$

$i(0) = A_1 e^{-10000(0)} + A_2 e^{-40000(0)}$

$4\times10^{-3} = A_1 + A_2 \quad (2)$

$A_1 = 0.1$

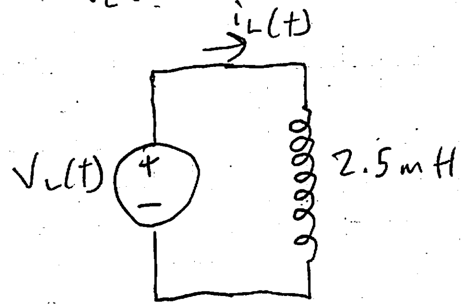
$A_2 = -0.06$

$V(t) = -200A_1 e^{-10000t} - 800A_2 e^{-40000t}$

$$\boxed{V(t) = -20e^{-10000t} + 48e^{-40000t}}$$

2) $V_L(t) = 3e^{-4t}$ mV $\quad 0^+ \leq t \leq 2s$

$V_L(t) = -3e^{-4(t-2)}$ mV $\quad 2 \leq t \leq \infty$ s.



Find $i_L(t)$ $0 \leq t < \infty$

& sketch $V_L(t)$ & $i_L(t)$

$V = L\frac{di}{dt}$

$0 < t < 2s$ $\qquad i = \int \frac{V}{L}$

$$i_L = \frac{10^3}{2.5}\int_0^t 3 \times 10^{-3} e^{-4x} dx + 1$$

$$= 1.2 \frac{e^{-4x}}{-4}\Big|_0^t + 1$$

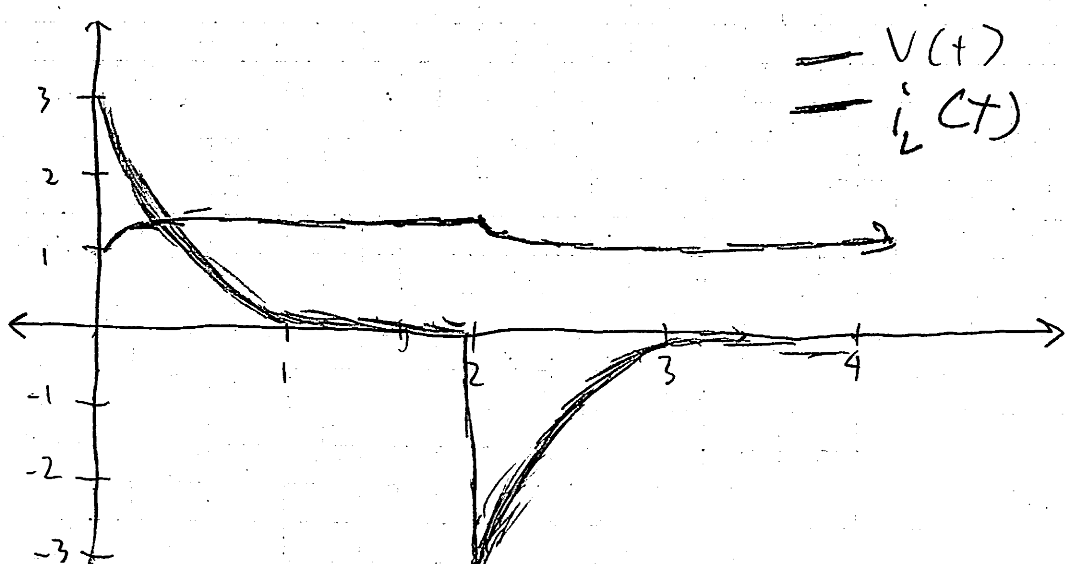
$$= -0.3e^{-4t} + 1.3A \quad 0 \leq t \leq 2s$$

$t \geq 2$

$$i_L = \frac{10^3}{2.5}\int_2^t -3 \times 10^{-3} e^{-4(x-2)} dx + 1.3$$

$$= -1.2 \frac{e^{-4(x-2)}}{-4}\Big|_2^t + 1.3$$

$$= 0.3e^{-4(t-2)} + 1A \quad t \geq 2s$$



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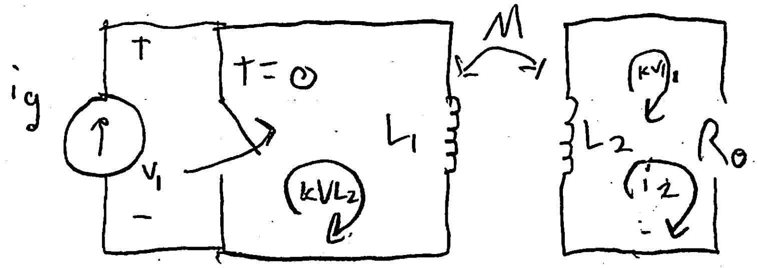
3)



$L_1 = 5\,H$

$L_2 = 0.2\,H$

$M = 0.5\,H$

$R_o = 10\,\Omega$

Find the equation that governs i_2

if $i_g = (e^{-10t} - 10)\,A$ for $t \geq 0$ & $i_2(t) = (625e^{-10t} - 250e^{-50t})\,mA$
what is the voltage V_1

KVL1

$$i_2 R_0 + L\frac{di_2}{dt} + M\frac{di_g}{dt} = 0$$

$$i_2(10) + (0.2)\frac{di_2}{dt} + (0.5)\frac{di_g}{dt} = 0$$

$$\boxed{0.2\frac{di_2}{dt} + 10i_2 = -0.5\frac{di_g}{dt}}$$

$$0.2\frac{di_2}{dt} + 10i_2 = -0.5\frac{d(e^{-10t} - 10)}{dt}$$

$$0.2\frac{di_2}{dt} + 10i_2 = -0.5(-10e^{-10t})$$

$$0.2\frac{di_2}{dt} + 10i_2 = 5e^{-10t}$$

$625e^{-10t} - 250e^{-50t}$ for i_2

$$0.2\frac{di_2}{dt} + 10i_2 = 0.2\frac{d(625e^{-10t} - 250e^{-50t})}{dt} + 10(625e^{-10t} - 250e^{-50t})$$

$$= 0.2(625 \times -10e^{-10t} + 250 \times 50e^{-50t}) + 10(625e^{-10t} - 250e^{-50t})$$

$$= (-1250e^{-10t} + 2500e^{-50t} + 6250e^{-10t} - 2500e^{-50t}) \times 10^{-3}$$

$$= 5000e^{-10t} \times 10^{-3}$$

$$= 5e^{-10t}$$

KVL2

$$-V_1 + L_1\frac{di_g}{dt} + M\frac{di_2}{dt} = 0$$

$$V_1 = 5\frac{d(e^{-10t} - 10)}{dt} + 0.5\frac{d(625e^{-10t} - 250e^{-50t})}{dt}$$

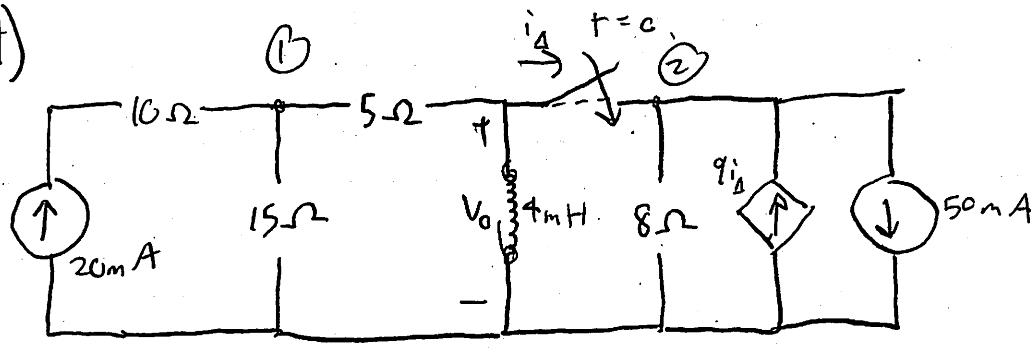
$$= -50e^{-10t} + 0.5(-6250e^{-10t} + 12500e^{-50t}) \times 10^{-3}$$

$$= -50e^{-10t} - 3.125e^{-10t} + 6.25e^{-50t}$$

$$= -53.125e^{-10t} + 6.25e^{-50t}$$

$$\boxed{V_1 = -53.125e^{-10t} + 6.25e^{-50t}\,V}$$

4)



i) Find i through inductor

ii) Find v across 15Ω resistor in terms V_0

iii) Find i_Δ then $V_0(0^+)$

$i_L = 20m \times \frac{15}{15+5} = \boxed{15mA = i_L(0)}$

$0.264V_1 = 20m + 0.2V_0$

$\boxed{V_1 = 75.4m + 0.758V_0}$

$i_\Delta = 0.2(V_1 - V_2)$

$\boxed{i_\Delta = 0.2(V_1 - V_0)}$

node 1

$\frac{V_1}{15} + \frac{V_1 - V_2}{5} - 20m = 0$

$20m = 0.264V_1 - 0.2V_2$ (1)

node 2

$\frac{V_2 - V_1}{5} + \frac{V_2}{8} - 9i_\Delta + 50m = 0$

$-2V_1 + 2.125V_2 = -50m$ (2)

$i_\Delta = \frac{V_1 - V_2}{5}$

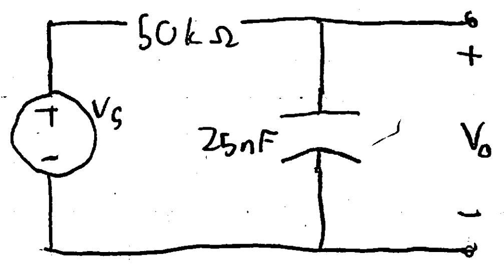
$V_1 = 0.194V_1$

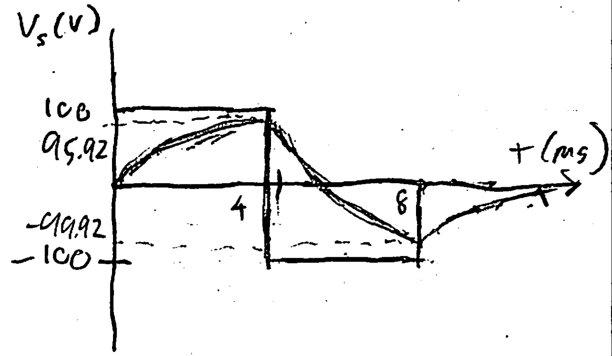
$V_2 = 0.159V$

$\boxed{V_0(0+) = V_2 = 0.159V}$

$\boxed{i_\Delta = 7mA}$ $i_\Delta = \frac{0.194 - 0.159}{5}$

5)





derive the three expressions for $V_o(t)$ for $0 \le t \le 4$

draw V_o & V_s $\quad 4 \le t \le 8$

$t \ge 8$

i) $\underline{0 \le t \le 6}$

$$V_o(t) = V_{(\infty)} + (V_{(0)} - V_{(\infty)})e^{-t/2}$$

$$V_o(t) = 100 + (0 - 100)e^{-t/2} = 100(1 - e^{-t/50\times10^3\times25\times10^{-9}})$$

$$\boxed{V_o(t) = 100(1 - e^{-800t})(u(t) - u(t-4)) \quad 0 \le t \le 6}$$

$\underline{4 \le t \le 8}$

$$V_o(t = 4ms) \Rightarrow 100(1 - e^{-800\times10^{-3}})$$

$$V_o(t) = -100 + [(95.92) + 100]e^{-(t-4)\times800}$$

$$\boxed{V_o(t) = -100 + 195.92e^{-800(t-4)}[u(t-4) - u(t-8)] \quad 4 \le t \le 8}$$

$\underline{t > 8ms}$

$$V_o(t = 8ms) = -100 + 195.92e^{-800(8-4)\times10^{-3}} = -92.01V$$

$$V(t) = V_{\infty} + [V(t = 8ms) - V_{(\infty)}]e^{-t/2}$$

$$\boxed{V(t) = [-92.01 - 0]e^{-800(t-8)}u(t-8) \quad t > 8}$$