Circuits # 10 Homework 10

Edwin Guillen
Circuits

1.) $i = 40\,mA, \quad t \leq 0$

$i(t) = \left(A_1 e^{-10000t} + A_2 e^{-40000t}\right) A \quad t \geq 0$

@ $t = 0$ voltage across inductor is $28V$

$V(t) = L\frac{di(t)}{dt} = (20\times10^{-3})\frac{d}{dt}\left(A_1 e^{-1000t} + A_2 e^{-4000t}\right)$

$= (20\times10^{-3})\left(-10000A_1 e^{-10000t} - 40000A_2 e^{-4000t}\right)$

$V(t) = -200A_1 e^{-10000t} - 800A_2 e^{-40000t}\ V$

$V(0) = 28V$ @ $t = 0$

$28 = -200A_1 e^{-10000(0)} - 800A_2 e^{-40000(0)}$

$28 = -200A_1 - 800A_2$

$i(t) = A_1 e^{-10000t} + A_2 e^{-40000t}$

$i(0) = 40\,mA$

$40\times10^{-3} = A_1 e^{-10000(0)} + A_2 e^{-40000(0)}$

$40\times10^{-3} = A_1 + A_2 \qquad A_1 = 40\times10^{-3} - A_2$

$128 = -200(0.04 - A_2) - 800A_2$

$A_2 = -0.06$

$A_1 = 0.1$

$V(t) = -200(0.1)e^{-10000t} - 800 \ldots$

$\ldots (-0.06)e^{-40000t}\ V$

2.) $V_L(t) = 3e^{-4t}\ mV \quad 0^+ \leq t \leq 2s$

$i(0) = 1A \quad V_L(t) = -3e^{-4(t-2)}\ mV \quad 2s \leq t \leq \infty$

$i_L(t) = \frac{1}{L}\int_0^t V_L(t)\,dt + i(0)$ inductor current for $0^+ \leq t \leq 2s$

$$i_L(t) = \left[\frac{1}{2.5\times10^{-3}}\int_0^t 3\times10^{-3}e^{-4t}\,dt\right] + 1$$

$$= \left[\frac{3\times10^{-3}}{2.5\times10^{-3}}\int_0^t e^{-4t}\,dt\right] + 1 = 1.2\left[\frac{e^{-4t}}{-4}\right]_0^t + 1$$

$$= 1.2\left[\frac{1-e^{-4t}}{4}\right] + 1 = \frac{3}{10}\left(1-e^{-4t}\right] + 1$$

$$= 1.3 - 0.3e^{-4t}\ A$$

$$i_L(2s) = 1.3 - 0.3e^{-4(2)} = 1.2998\ A$$

inductor current for $2s \leq t \leq \infty$

$$i_L(t) = \frac{1}{2.5\times10^{-3}}\int_2^t -3\times10^{-3}e^{-4(t-2)}\,dt + 1.2998$$

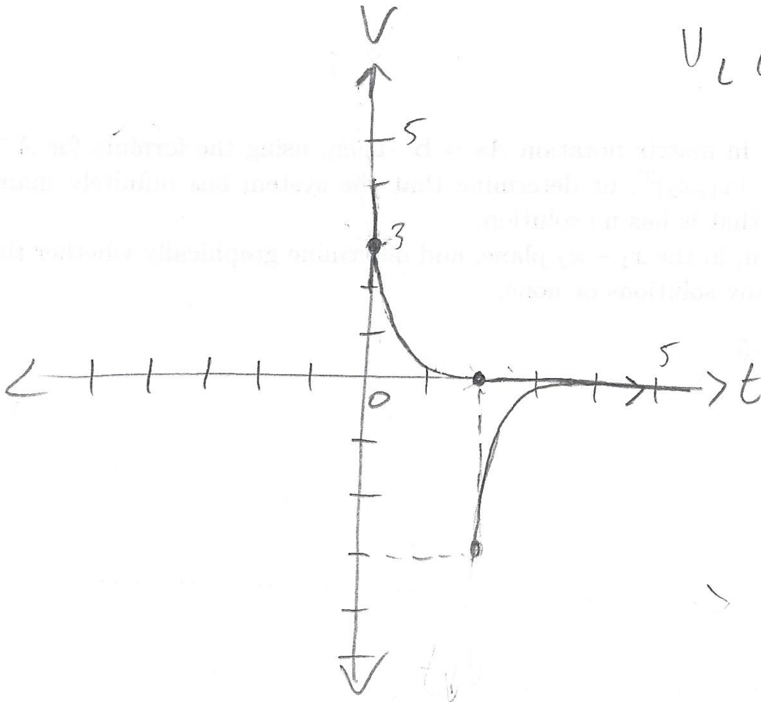
$$= \frac{-3\times10^{-3}}{2.5\times10^{-3}}\int_2^t e^{-4(t-2)}\,dt + 1.2998$$

$$= -1.2\left[\frac{e^{-4(t-2)}}{-4}\right)_2^t + 1.3 = 0.3e^{-4(t-2)} + 1A$$

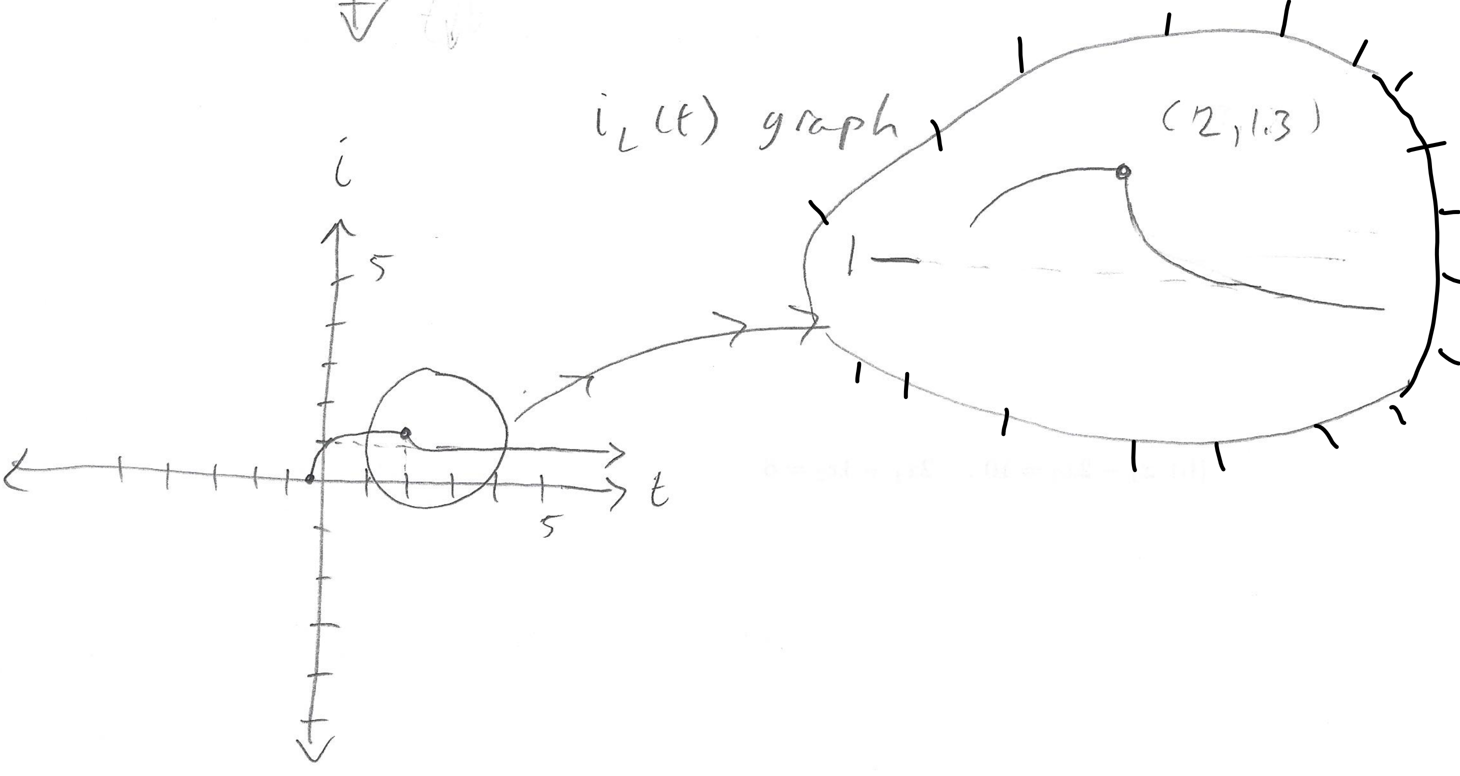
Voltage across inductor

$$V_L(t) = \begin{cases} 3e^{-4t}\ mV & 0^+ \leq t \leq 2s \\ -3e^{-4(t-2)}\ mV & 2s \leq t \leq \infty \end{cases}$$

$v_L(t)$ graph



$i_L(t)$ graph



#3) KVL loop 2

i.) $i_2 R_o + L_2 \frac{di_2}{dt} + M \frac{di_g}{dt} = 0$

$i_2(10) + (0.2)\frac{di_2}{dt} + (0.5)\frac{di_g}{dt} = 0$

$0.2\frac{di_2}{dt} + 10 i_2 = -0.5\frac{di_g}{dt}$

ii.) KVL loop 1

$-V_1 + L_1\frac{di_g}{dt} + M\frac{di_2}{dt} = 0$

$V_1 = L_1\frac{di_g}{dt} + M\frac{di_g}{dt}$ sub in i_2 & i_g

$V_1 = 5\frac{d(e^{-10t} - 10A)}{dt} + 0.5\frac{d(625e^{-10t} - 250e^{-50t})}{dt}$

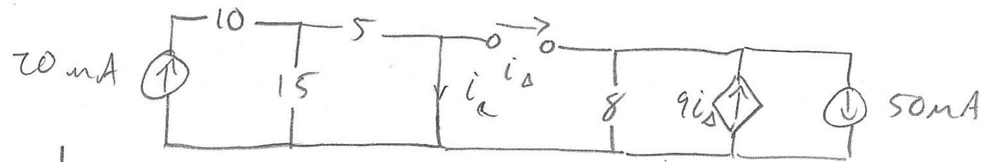
$= -50e^{-10t} + 0.5(-6250e^{-10t} + 12500e^{-50t}) \times 10^{-3}$

$= -50e^{-10} - 3.125e^{-10t} + 6.25e^{-50t}$

$= -53.125e^{-10t} + 6.25e^{-50t}\ V = V_1$

4.) @ $t=0$ switch is opened

i.)



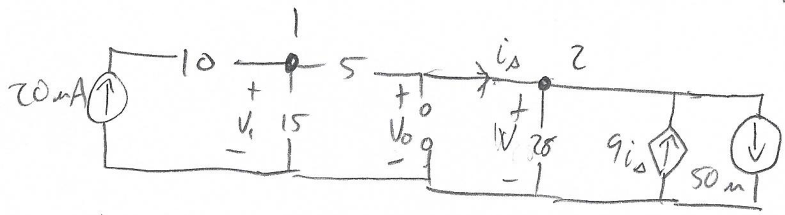
current division

$$i_L(0^-) = 20m \times \frac{15}{15+5} = 15\,mA$$

$$i_L(0^-) = 15\,mA \quad @\ t<0$$

at $t=0^-$ switch is closed

$t=0^+$ inductor opened



KCL node 1

$$\frac{V_1}{15} + \frac{V_1 - V_2}{5} - 20m = 0$$

$$0.264\,V_1 - 0.2\,V_2 = 20m \quad (1)$$

node 2

$$\frac{V_2 - V_1}{5} + \frac{V_2}{8} - 9i_s + 50m = 0$$

$$i_s = \frac{V_1 - V_2}{5} \quad (3)$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{8} - 9\left[\frac{V_1 - V_2}{5}\right] + 50m = 0$$

$$V_1\left[\frac{-1}{5} - \frac{9}{5}\right] + V_2\left[\frac{1}{5} + \frac{1}{8} + \frac{9}{5}\right] = -50m$$

$$-2V_1 + 2.125\,V_2 = -50m$$

solving eq 1 & 2

$V_1 = 0.194\,V,$

$V_2 = 0.159\,V$

$V_0(0^+) = 0.159\,V$

$i_s = 7\,mA$

$0.264V_1 = 20m + 0.2\,V_0$

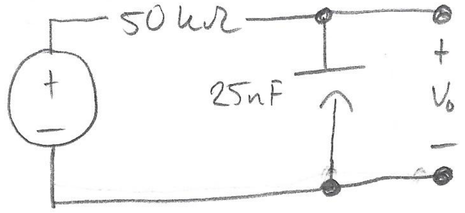
$V_1 = 74.9m + 0.749\,V_0$

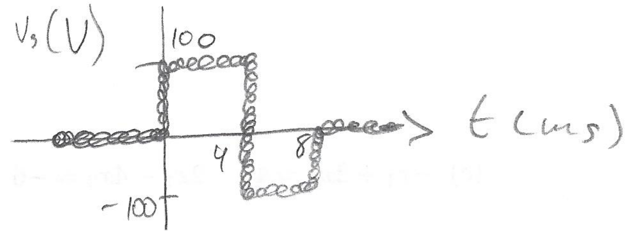
eqn for voltage across $15\,\Omega$

$i_s = 0.2(V_1 - V_2)$

$i_s = 0.2(V_1 - V_0)\,A$

#5)





i.) $V_s = 0\,V$ for $t < 0$

$= 100\,V$ for $0 \le t \le 4ms$

$= -100\,V$ for $4ms \le t \le 8ms$

$= 0\,V$ for $8ms \le t \le \infty$

$V_o(0^+) = 0\,V = V_o(0^-)$

$V_o(\infty) = V_c(\infty) = 100\,V$

$\tau = RC = (25 \times 10^{-9})(50 \times 10^{3}) = 1.25\,ms$

$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]e^{-t/\tau}$

sub values

$V_o(t) = 100 + [0 - 100]e^{-t/1.25\times10^{-3}} = 100 - 100e^{-800t}$

sub $t = 4\,ms$

$= 100 - 100e^{-(800 \times 4 \times 10^{-3})}$

$= 95.92\,V$

For $4ms \le t \le 8ms$

$V_o(4^+) = 95.92V = V_o(4^-)$

$V_o(\infty) = V_c(\infty) = -100\,V$

$V_o(t) = V_o(\infty) + [V_o(4^+) - V_o(\infty)]e^{-\frac{(t-0.004)}{\tau}}$

sub vales

$V_o(t) = -100 + [95.92 - (-100)]e^{-(t-0.004)/1.25\times10^{-3}}$

$V_o(t) = -100 + 195.92 e^{-800(t-0.004)}$ V

$V_o(t)$ @ $t = 8^-$ ms

$V_o(8^-) = -100 + 195.92 e^{-800(0.008-0.004)}$

$= -92.01$ V

For $8ms \leq t \leq \infty$

$V_c(8^-) = V_c(8^+) = -92.01$ V

$V_c(\infty) = 0$

$V_o(t) = V_o(\infty) + [V_o(8^+) - V_o(\infty)] e^{-(t-0.008)/\tau}$

Sub values

$V_o(t) = 0 + [-92.01 - 0] e^{-(t-0.008/(1.25 \times 10^{-3})}$

$= -92.01 e^{-800(t-0.008)}$ V

$V_o(t) = 0$ V $\qquad t < 0$

$= 100 - 100^{-800t}$ V $\qquad 0 \leq t \leq 4$

$= -100 + 195.92 e^{-800(t-0.004)}$ V $\qquad 4 \leq t \leq 8$

$= -92.01 e^{-800(t-0.008)}$ V $\qquad 8 \leq t \leq \infty$