ECE 203

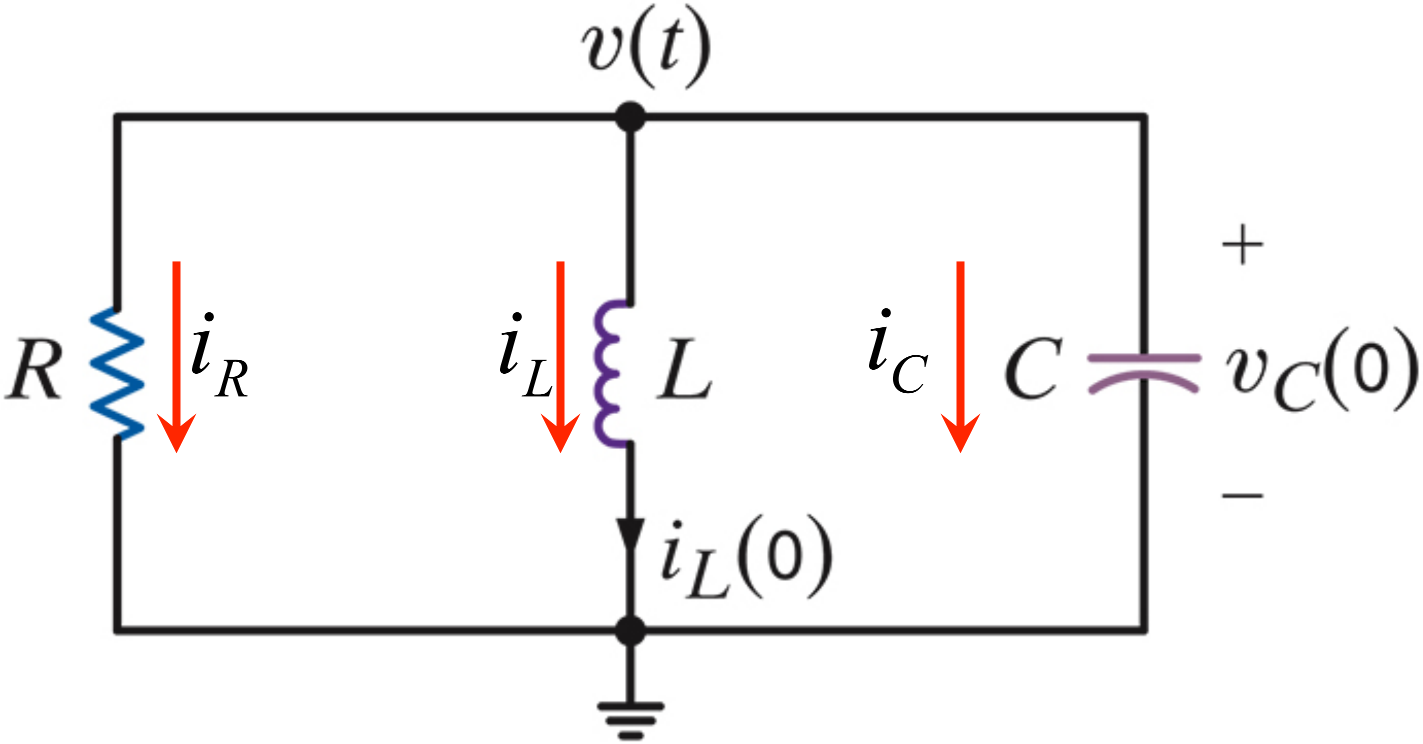
Circuits I

2^{nd} Order Transient Circuits

Lecture 11-2

Example 1

Find v(t) in this circuit:



Initial Conditions:

$$R = 2\Omega, L = 5H, C = \frac{1}{5}F$$

$$i_L(0) = -1A,\ v_C(0) = 4V$$

Example 1: Detailed Analysis

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int_0^t v(x)dx + i_L(0) + C\frac{dv}{dt} = 0$$

STEP 1 MODEL

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

CHARACTERISTIC EQUATION

$$s^2 + 2.5s + 1 = 0$$

STEP 2

$$\Rightarrow \omega_o = 1;\ \varsigma = 1.25$$

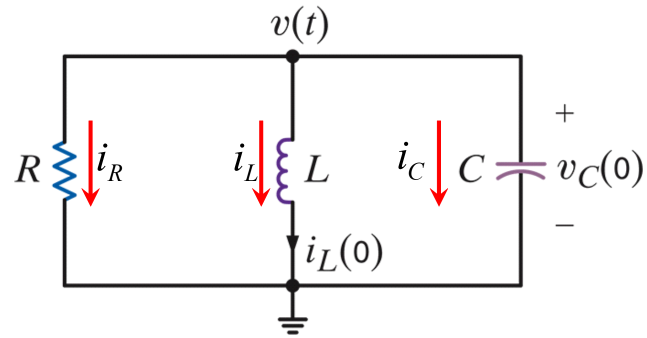
$$s = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2} = \frac{-2.5 \pm 1.5}{2}$$

STEP 3 ROOTS

$$v(t) = K_1 e^{-2t} + K_2 e^{-0.5t}$$

STEP 4 FORM OF SOLUTION

STEP 5: FIND CONSTANTS



To determine the constants we need

$$v(0+);\ \frac{dv}{dt}(0+)$$

IF NOT GIVEN FIND $v_C(0), i_L(0)$

$$v(0+) = v_C(0+) = v_C(0) = 4V$$

KCL AT $t = 0+$

ANALYZE CIRCUIT AT t=0+

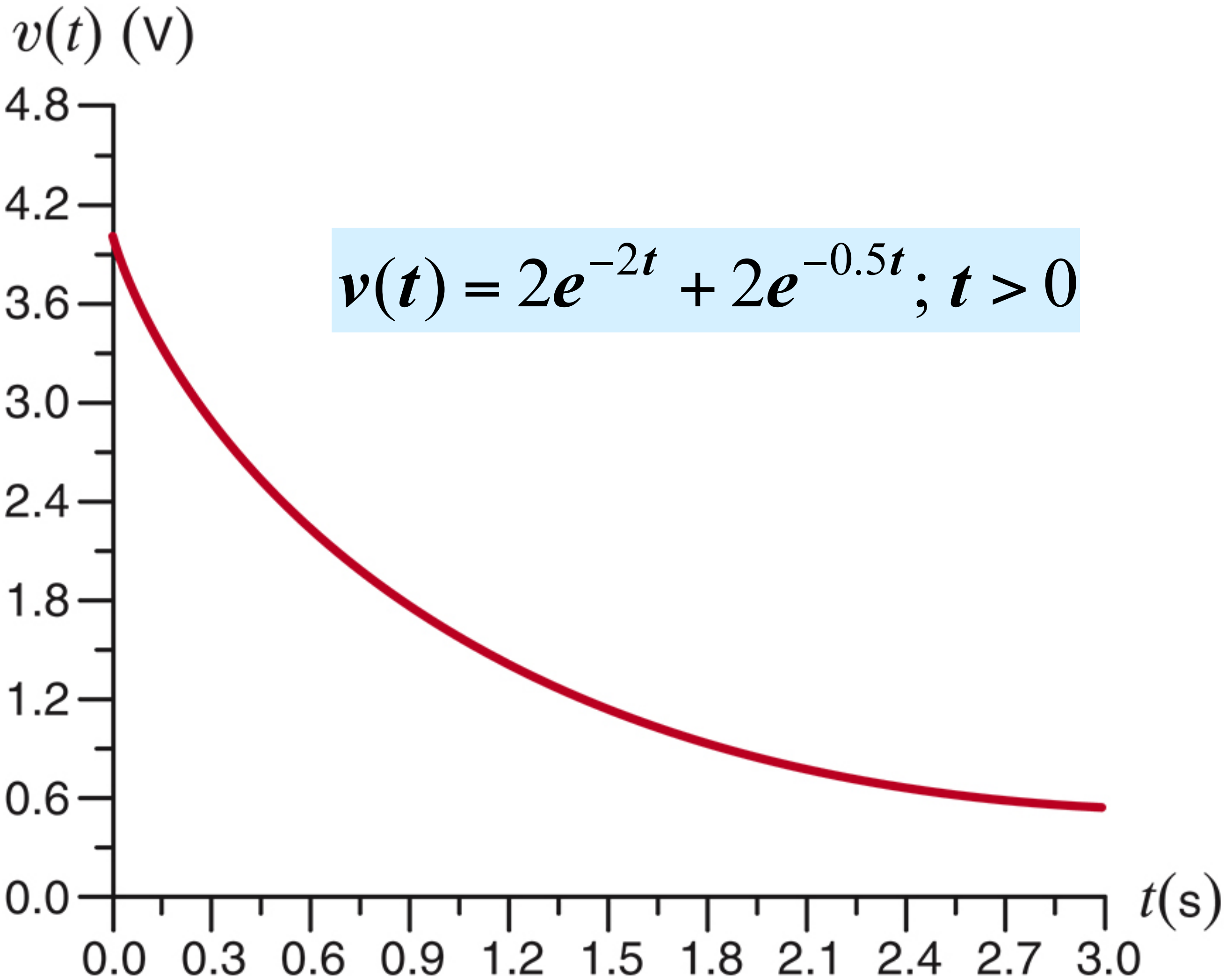
$$\frac{v_C(0+)}{R} + i_L(0+) + C\frac{dv}{dt}(0+) = 0$$

$$\frac{dv}{dt}(0+) = -\frac{4}{2(1/5)} - \frac{(-1)}{(1/5)} = -5$$

$$\left.\begin{array}{r} K_1 + K_2 = 4 \\ -2K_1 - 0.5K_2 = -5 \end{array}\right\} \Rightarrow K_1 = 2;\ K_2 = 2$$

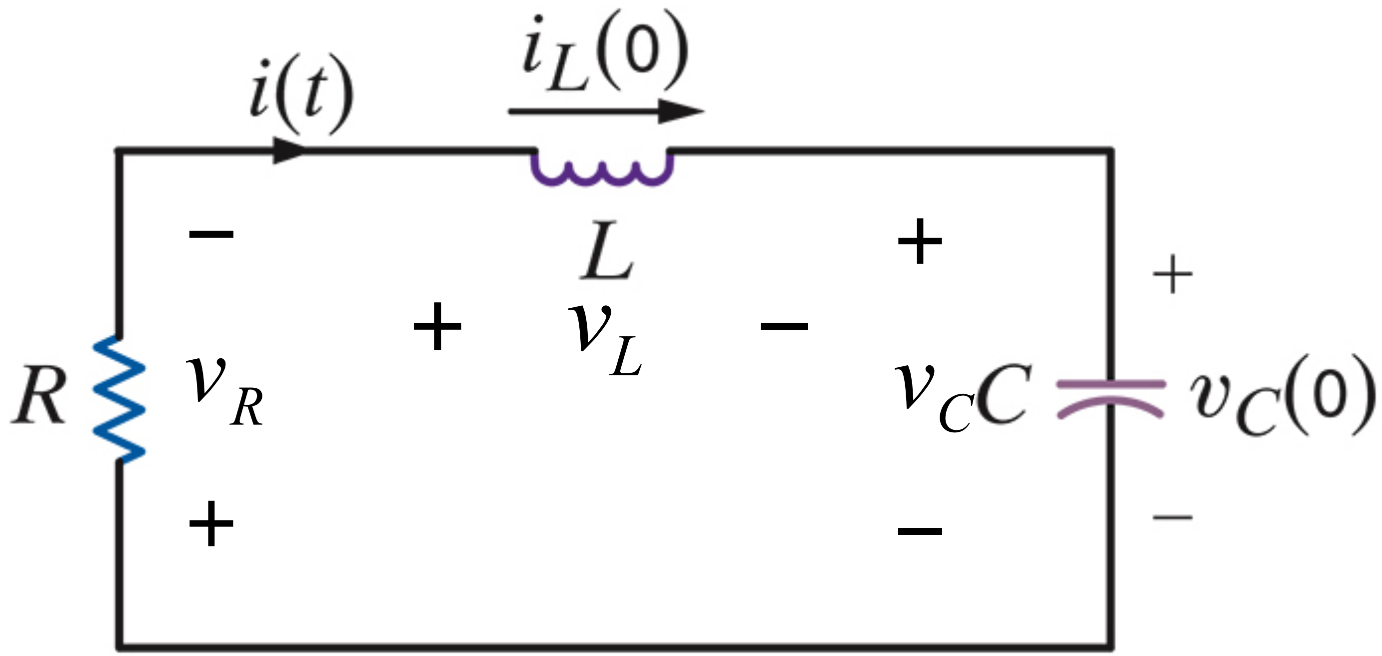
$$v(t) = 2e^{-2t} + 2e^{-0.5t};\ t > 0$$

Example 1: Waveform



Example 2

Find $v_C(t)$ in this circuit:

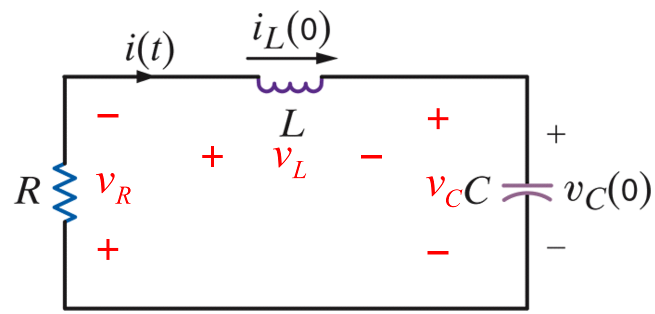


Initial Conditions:

$$\boldsymbol{R = 6\Omega, L = 1H, C = 0.04F}$$

$$\boldsymbol{i_L(0) = 4A; v_C(0) = -4V}$$

Example 2: Detailed Analysis



$$v_R + v_L + v_C = 0$$

$$Ri(t) + L\frac{di}{dt}(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0) = 0$$

STEP 1 MODEL

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt}(t) + \frac{1}{LC}i(t) = 0$$

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt}(t) + 25i(t) = 0$$

Ch. Eq.: $s^2 + 6s + 25 = 0$

STEP 2

$$\omega_o^2 = 25 \Rightarrow \omega_o = 5$$
$$2\varsigma\omega_o = 6 \Rightarrow \varsigma = 0.6$$

STEP 3 ROOTS

roots: $s = \dfrac{-6 \pm \sqrt{36-100}}{2} = -3 \pm j4$

$$i(t) = e^{-3t}(A_1 \cos 4t + A_2 \sin 4t)$$

STEP 4 FORM OF SOLUTION

STEP 5: FIND CONSTANTS

$i(0) = i_L(0) = 4A$ $\quad \Rightarrow A_1 = 4$

TO COMPUTE $\dfrac{di}{dt}(0+)$ $\quad v_L(t) = L\dfrac{di}{dt}(t)$

$$L\frac{di}{dt}(0) = -Ri(0) - v_C(0)$$

$$\Rightarrow \frac{di}{dt}(0+) = -20$$

ANALYZE CIRCUIT AT t=0+

$$\frac{di}{dt}(t) = -3i(t) + e^{-3t}(-4A_1 \sin 4t + 4A_2 \cos 4t)$$

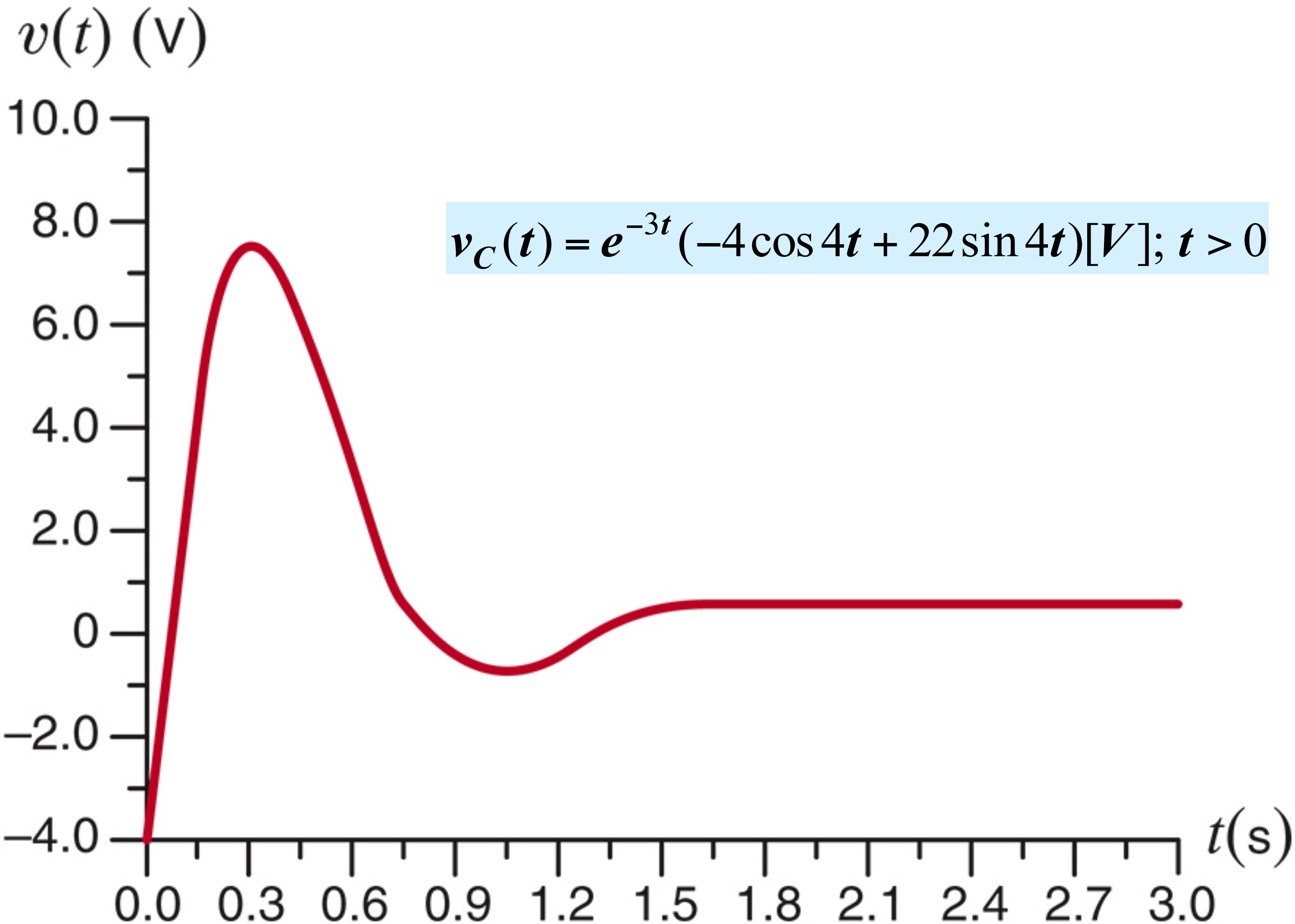
$$@t = 0: \quad -20 = -3\times(4) + 4A_2 \Rightarrow A_2 = -2$$

$$i(t) = e^{-3t}(4\cos 4t - 2\sin 4t)[A];\ t > 0$$

$$v_C(t) = -Ri(t) - L\frac{di}{dt}(t) = v_C(0) + \frac{1}{C}\int_0^t i(x)dx$$

$$v_C(t) = e^{-3t}(-4\cos 4t + 22\sin 4t)[V];\ t > 0$$

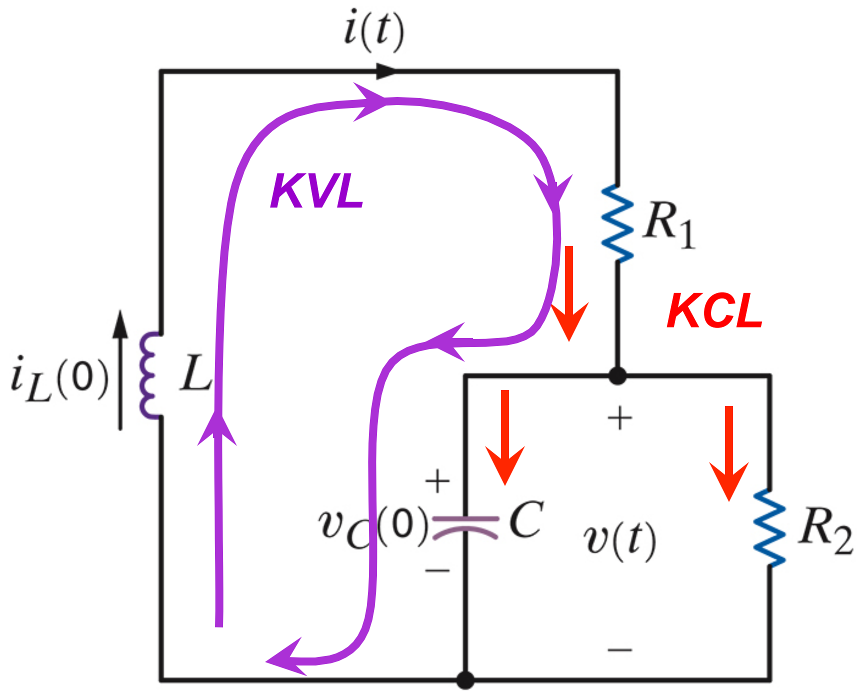
Example 2: Waveform



$$v_C(t) = e^{-3t}(-4\cos 4t + 22\sin 4t)[V];\ t > 0$$

Example 3

Find v(t) in this circuit:



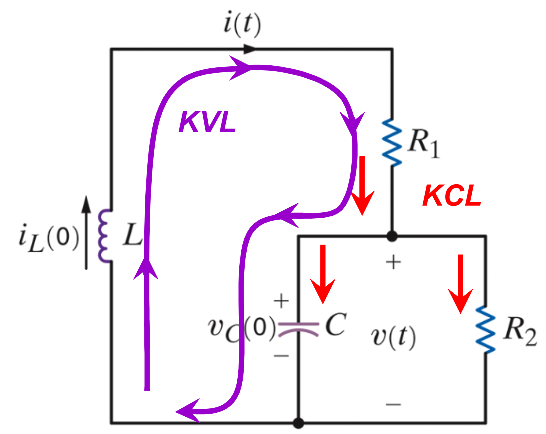
Initial Conditions:

$$R_1 = 10\Omega,\ R_2 = 8\Omega,\ C = 1F,\ L = 2H$$

$$v_C(0) = 1V,\ i_L(0) = 0.5A$$

Example 3: Detailed Analysis

$$\boxed{L\frac{di}{dt}(t) + R_1 i(t) + v(t) = 0}\quad \boxed{i(t) = \frac{v(t)}{R_2} + C\frac{dv}{dt}(t)}$$



$$L\left(\frac{1}{R_2}\frac{dv}{dt}(t) + C\frac{d^2v}{dt^2}\right) + R_1\left(\frac{v(t)}{R_2} + C\frac{dv}{dt}(t)\right) + v(t) = 0$$

$$\frac{d^2v}{dt^2}(t) + \left(\frac{1}{R_2C} + \frac{R_1}{L}\right)\frac{dv}{dt}(t) + \frac{R_1+R_2}{R_2LC}v(t) = 0$$

$$\frac{d^2v}{dt^2}(t) + 6\frac{dv}{dt}(t) + 9v(t) = 0$$

STEP 1 MODEL

ANALYZE CIRCUIT AT t=0+

$$v(0+) = v_c(0+) = 1V$$

Ch. Eq.: $s^2 + 6s + 9 = 0$

STEP 2

$\omega_o = 3,\ 2\varsigma\omega_o = 6 \Rightarrow \varsigma = 1$

KCL AT $t = 0+$

$$i(0) = i_L(0) = \frac{v(0)}{R_2} + C\frac{dv}{dt}(0) \Rightarrow \frac{dv}{dt}(0) = 3$$

Ch. Eq.: $s^2 + 6s + 9 = 0 = (s+3)^2$

STEP 3 ROOTS

$$v(t) = e^{-3t}(B_1 + B_2 t)$$

STEP 4 FORM OF SOLUTION

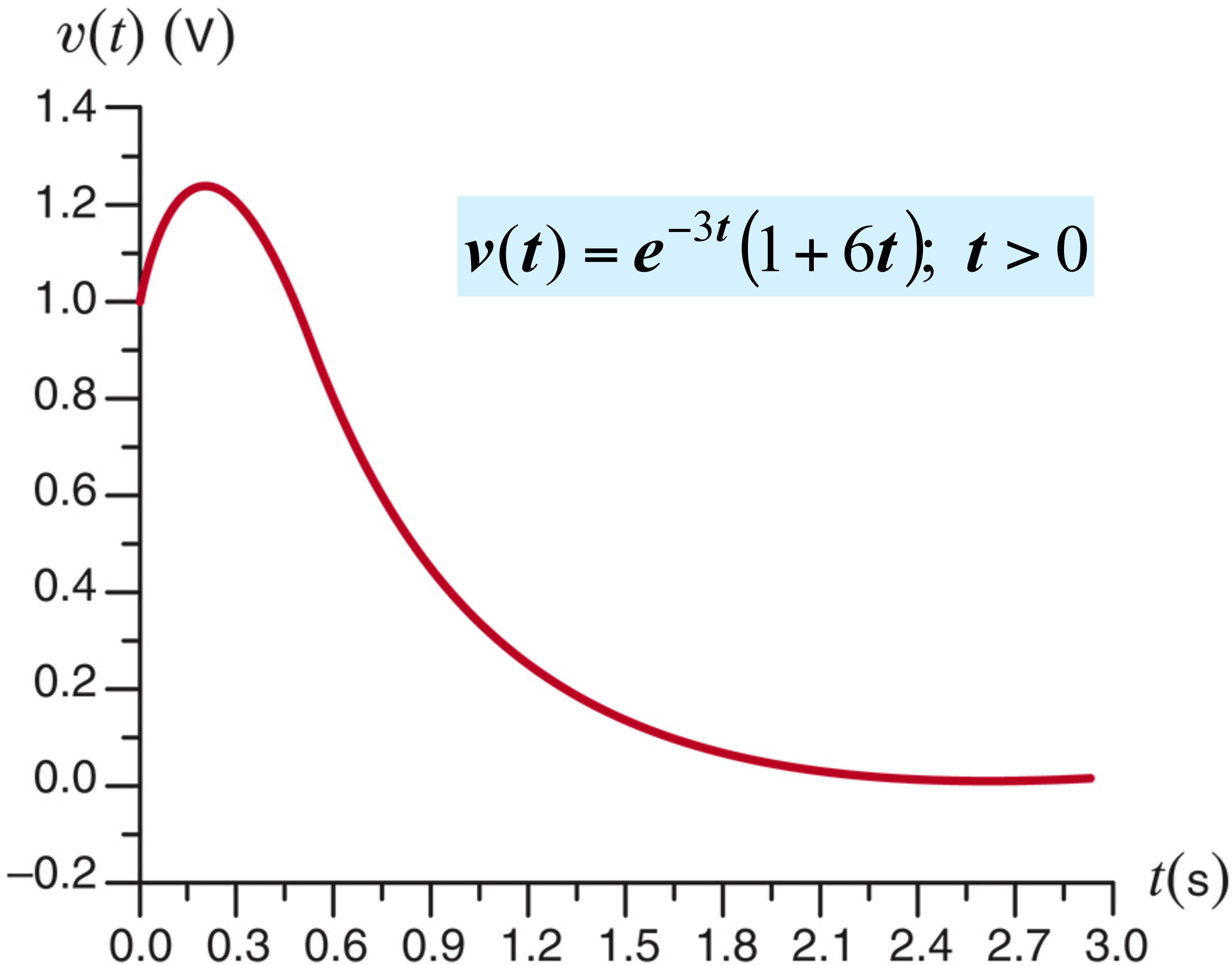
$$v(0) = 1 = B_1$$

$$\frac{dv}{dt}(0) = -3v(0) + B_2 = 3 \Rightarrow B_2 = 6$$

STEP 5: FIND CONSTANTS

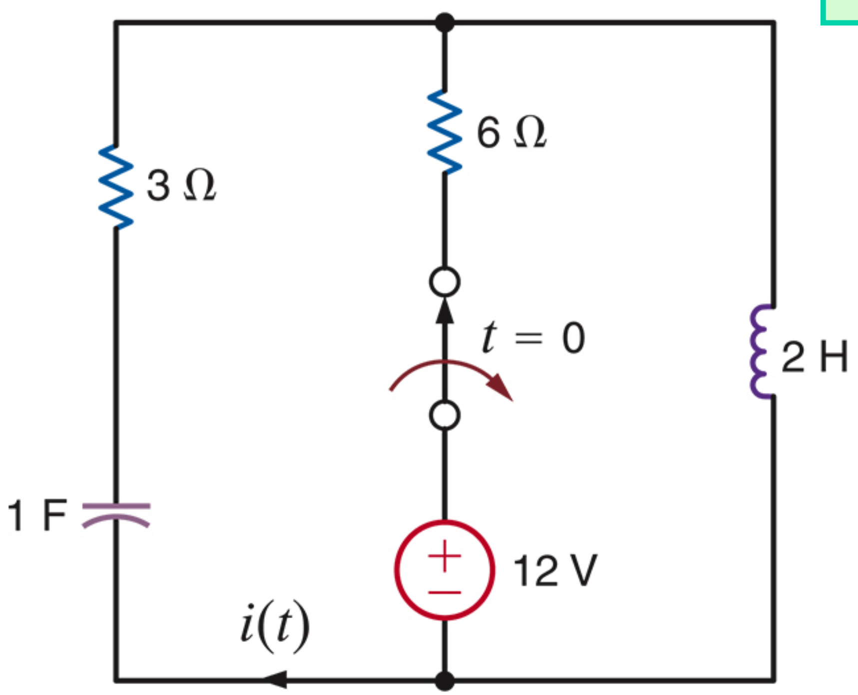
$$v(t) = e^{-3t}(1 + 6t);\ \ t > 0$$

Example 3: Waveform

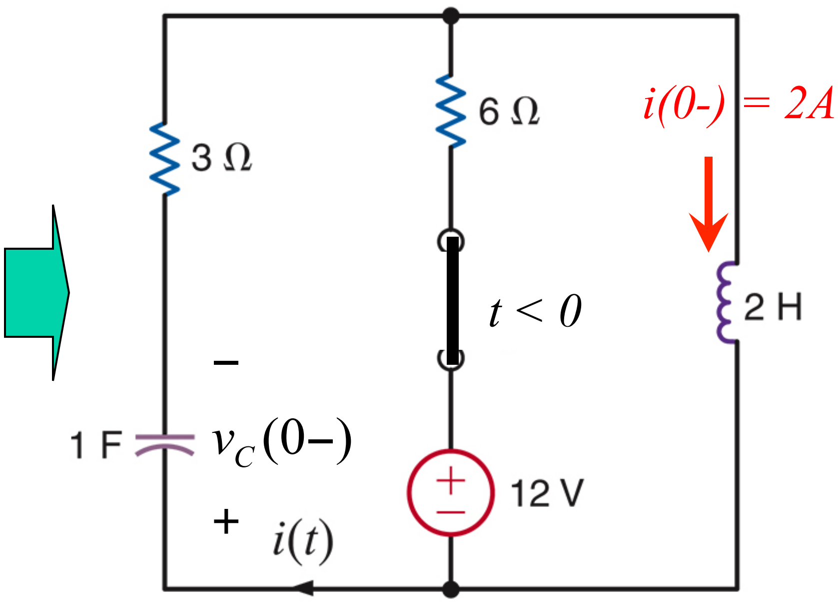


Example 4

Find i(t) in this circuit:



To find initial conditions use steady state analysis for t<0



Initial Conditions:

$$v_C(0) = 0V \quad , \quad i_L(0) = 2A$$

Example 4: Detailed Analysis

Once the switch opens the circuit is RLC series

$$3i(t)+2\frac{di}{dt}(t)+v_C(0)+\int_0^t i(x)dx=0$$

STEP 1 MODEL

$$\frac{d^2i}{dt^2}(t)+\frac{3}{2}\frac{di}{dt}(t)+\frac{1}{2}i(t)=0$$

STEP 2

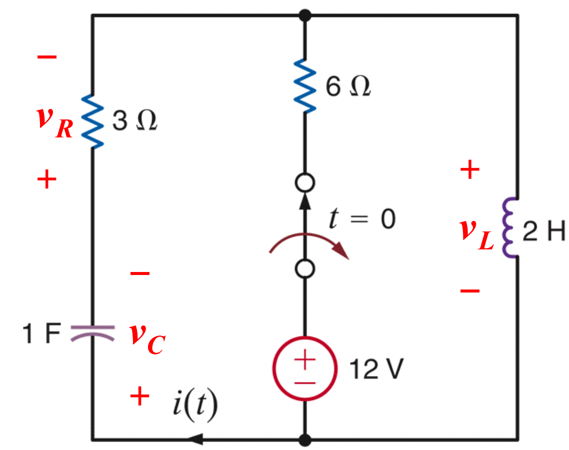
Ch.Eq.: $s^2+1.5s+0.5=0$

roots: $s=-1,-0.5$

STEP 3 ROOTS

$$i(t)=K_1e^{-t}+K_2e^{-\frac{t}{2}};\ t>0$$

STEP 4 FORM OF SOLUTION



ANALYZE CIRCUIT AT t=0+

$$v_C=C\frac{di}{dt} \Rightarrow \frac{di}{dt}(0+)=0$$

STEP 5: FIND CONSTANTS

$$i(0+)=2A$$

$$v_C(0)=0V$$

$$2=K_1+K_2$$

$$0=-K_1-\frac{1}{2}K_2$$

$$i(t)=-2e^{-t}+4e^{-\frac{t}{2}};\ t>0$$