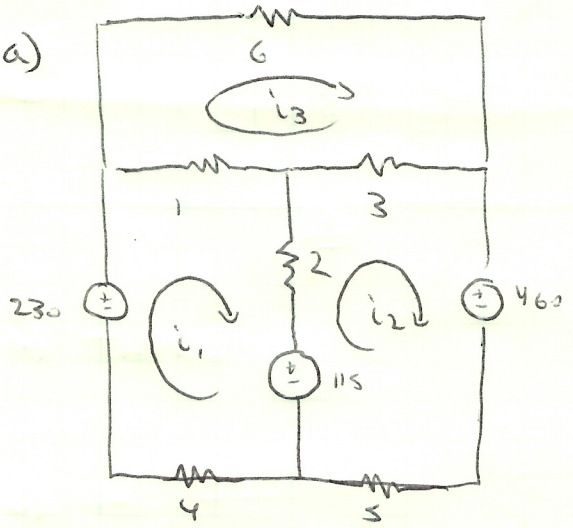
ENGR 2910 CIRCUIT ANALYSIS I
HOMEWORK 5 SOLUTIONS

1)

a)



MESH EQUATIONS

① $230 - (i_1 - i_3) - 2(i_1 - i_2) - 155 - 4i_1 = 0$

② $115 - 2(i_2 - i_1) - 3(i_2 - i_3) - 460 - 5i_2 = 0$

③ $-6i_3 - 3(i_3 - i_2) - (i_3 - i_1) = 0$

$\Downarrow$

$-(1+2+4)i_1 + 2i_2 + i_3 = -115$

$2i_1 - (2+3+5)i_2 + 3i_3 = 345$

$i_1 + 3i_2 - (6+3+1)i_3 = 0$

$$\begin{bmatrix} -7 & 2 & 1 & -115 \\ 2 & -10 & 3 & 345 \\ 1 & 3 & -10 & 0 \end{bmatrix}$$

$i_1 = 4.4\,A$

$i_2 = -36.8\,A$

$i_3 = -10.6\,A$

TOTAL POWER DEVELOPED (or DELIVERED / SUPPLIED)

$P_{tot} = P_{230} + P_{115} + P_{460}$

$= (4.4)(230) + (-36.8 - 4.4)(115) + (36.8)(460)$

$= 1012 \underbrace{- 4738}_{\text{ABSORBED, NOT DELIVERED}} + 16928$

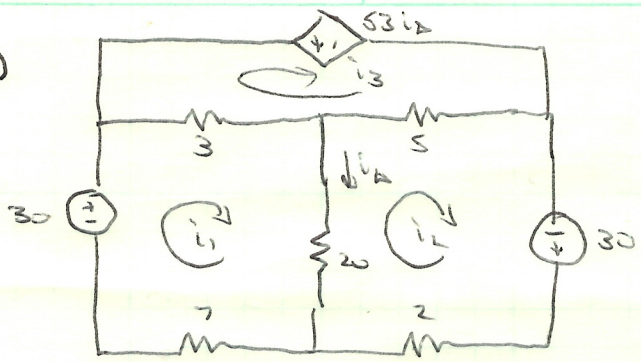
$\boxed{P_{tot} = 17940\ W}$

b) $P_{absorbed}$ or dissipated

$P_1 = (1)(4.4+10.6)^2 = 225$
$P_4 = (4)(4.4)^2 = 77.4$
$P_6 = (6)(10.6)^2 = 674.16$
$P_2 = (2)(4.4+36.8)^2 = 3394.88$
$P_3 = (3)(-10.6+36.8)^2 = 2059.32$
$P_5 = (5)(36.8)^2 = 6771.2$

$\sum P_{abs} = 17940\ W$ (MATCHES)

2)



① $30 - 3(i_1 - i_3) - 20(i_1 - i_2) - 7i_1 = 0$

② $+20(i_2 - i_1) - 5(i_2 - i_3) + 30 - 2i_2 = 0$

③ $-53i_\Delta + 5(i_3 - i_2) - 3(i_3 - i_1) = 0$

CONSTRAINT $i_\Delta = i_1 - i_2$

$$\begin{bmatrix} -30 & +20 & +3 & 0 & -30 \\ +20 & -27 & +5 & 0 & -30 \\ 3 & 5 & -8 & -53 & 0 \\ 1 & -1 & 0 & -1 & 0 \end{bmatrix}$$

$i_1 = 52$ A

$i_2 = 60$ A

$i_3 = 110$ A

$i_\Delta = -8$ A

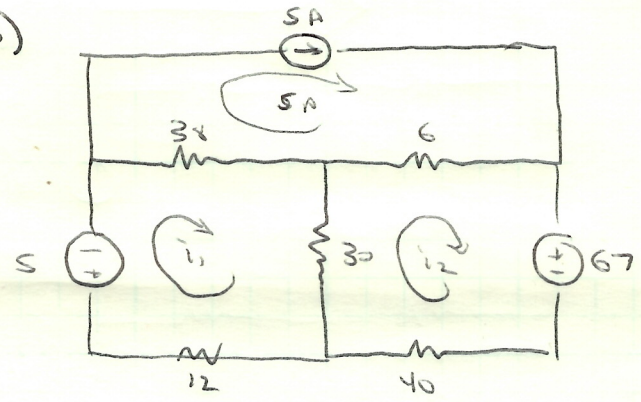
LEFT: $30(52) = 1560$ W

RIGHT: $30(60) = 1800$ W

DEPENDENT SOURCE $= (53)(8)(110) = 46640$

TOTAL = 50 kW

3)



① $-5 - 38(i_1 - 5) - 30(i_1 - i_2) - 12i_1 = 0$

② $-30(i_2 - i_1) - 6(i_2 - 5) + 67 - 40i_2 = 0$

$$\begin{bmatrix} -80 & +30 & -185 \\ 30 & -76 & 37 \end{bmatrix}$$

$i_1 = 2.5A$

$i_2 = 0.5A$

a)

$v_{5A} = 38(2.5-5) + 6(0.5-5)$

$= -112V$

$p_{5A} = -5 v_{5A} = 610W$

b)

$p_5 = 5(-2.5) = -12.5W$

$p_{67} = 67(-0.5) = 33.5W$

} p_5 and p_{67} absorb power

so $p_{delivered} = 610W$

c)

$p_{38} = 38(2.5)^2 = 237.5$

$p_6 = 6(4.5)^2 = 121.5$

$p_{30} = 30(2)^2 = 120$

$p_{12} = 12(2.5)^2 = 75$

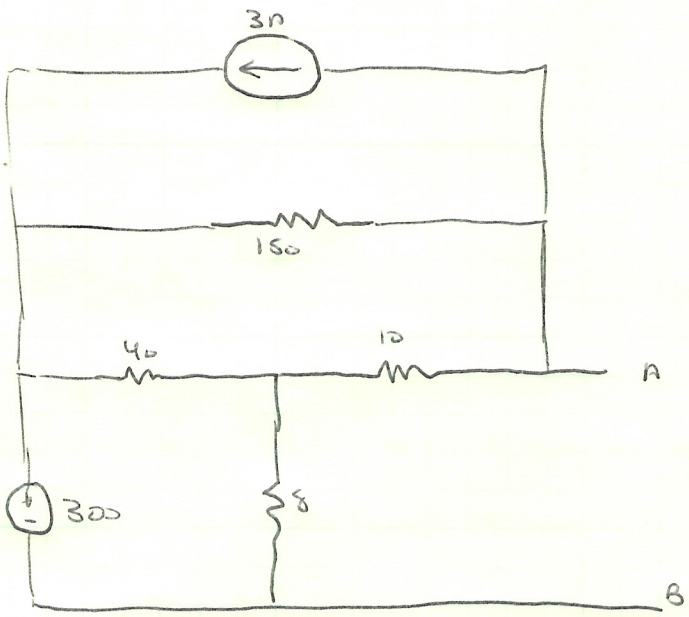
$p_{40} = 40(0.5)^2 = 10$

} $\sum p_{resistor} = 564$

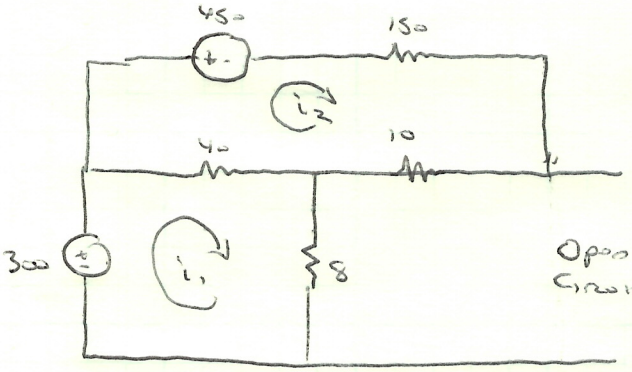
MATCHES

$\sum p_{abs} = 564 + 12.5 + 33.5 = 610W$

4)



⇓ source transformation



Open Circuit

① $300 - 4(i_1 - i_2) - 8i_1 = 0$

② $-45 - 15\,i_2 - 10i_2 = 4(i_2 - i_1) = 0$

⇓

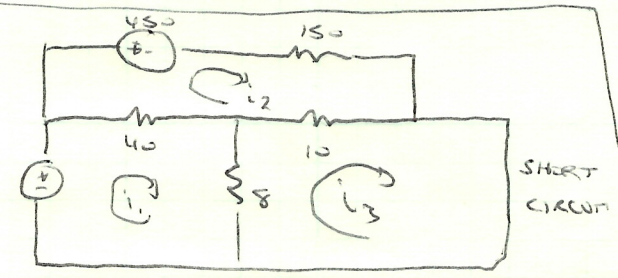
$$\left[\begin{array}{cc|c} 48 & -40 & 300 \\ 40 & -200 & 450 \end{array}\right]$$

⇓

$i_1 = 5.25\ A$
$i_2 = -1.2\ A$

$\Rightarrow V_{TH} = 8(5.25) + 10(-1.2)$

$= 30V$



SHORT CIRCUIT

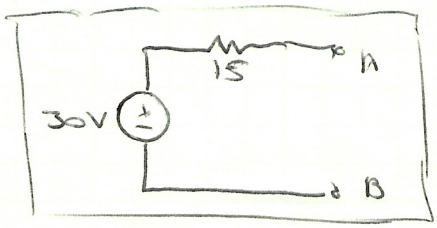
① $300 - 4(i_1 - i_2) - 8(i_1 - i_3) = 0$

② $-45 - 15i_2 - 10(i_2 - i_3) - 4(i_2 - i_1) = 0$

③ $-8(i_3 - i_1) - 10(i_3 - i_2) = 0$

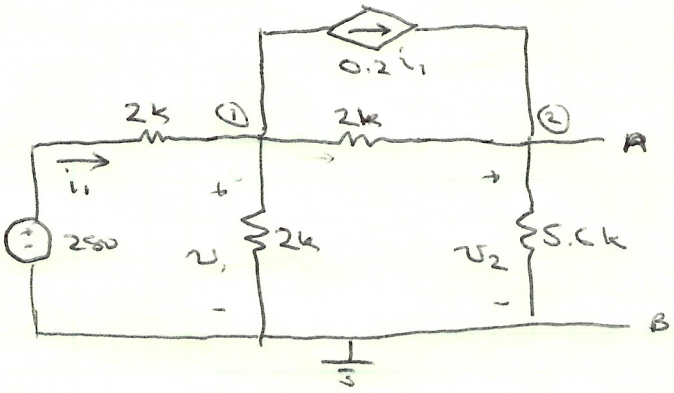
$$\Rightarrow \left[\begin{array}{cccc} 48 & -40 & -8 & 300 \\ 40 & -200 & +10 & 450 \\ 8 & 10 & -18 & 0 \end{array}\right]$$

$\Rightarrow i_3 = 2A$



$$R_{TH} = \frac{V_{TH}}{i_{sc}} = \frac{V_{TH}}{i_3} = \frac{30}{2} = 15\,\Omega$$

⑤



NODE VOLTAGE

$$① \quad \frac{280 - v_1}{2000} - \frac{v_1}{2000} - \frac{v_1 - v_2}{2000} + 0.2 i_\Delta = 0$$

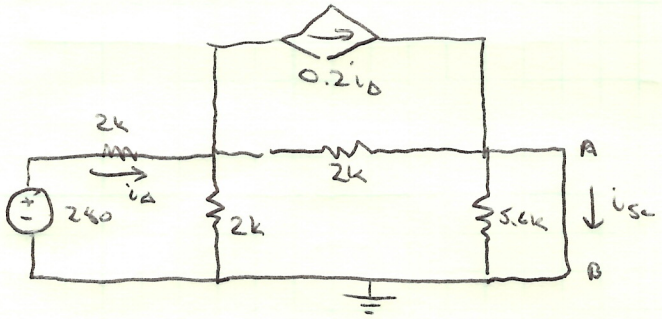
$$② \quad \frac{v_1 - v_2}{2000} - \frac{v_2}{5600} + 0.2 i_\Delta = 0$$

CONSTRAINT $\quad i_\Delta = \frac{280 - v_1}{2000}$

$$\begin{bmatrix} -\frac{3}{2000} & \frac{1}{2000} & -0.2 \\ \frac{1}{2000} & -\left(\frac{1}{2000} + \frac{1}{5600}\right) & +0.2 \\ \frac{1}{2000} & 0 & 1 \end{bmatrix} \begin{bmatrix} -280/2000 \\ 0 \\ 280/2000 \end{bmatrix} \Rightarrow \begin{aligned} v_1 &= 120 \\ v_2 &= 112 \\ i_\Delta &= 0.08\,A \end{aligned}$$

$$\boxed{V_{TH} = V_2 = 112V}$$

SHORT CIRCUIT CURRENT



MESH. CURRENT

$$280 - 2000 i_\Delta + 2000 (i_\Delta - i_{sc}) = 0$$
$$-2000 (i_{sc} - 0.2 i_\Delta) - 2000 (i_{sc} - i_\Delta) = 0$$

$$\begin{bmatrix} 4000 & -2000 \\ -2400 & 4000 \end{bmatrix} \begin{bmatrix} 280 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} i_\Delta &= 0.1\,A \\ i_{sc} &= 0.06A \end{aligned} \Bigg\} R_{TH} = \frac{V_{TH}}{i_{sc}} = 1866.67\,\Omega$$