CHAPTER 2: VECTORS

• Vectors vs. Scalars

Vectors:

1) Require a magnitude (size) AND a direction to be fully defined

2) Examples are:

position, displacement, velocity, acceleration, force, momentum, torque

3) • Can add & subtract vectors
   • Can multiply by a scalar or a vector
   • Can divide by a scalar but not by another vector

Scalars:

1) Only requires a magnitude (size) to be fully defined

2) Examples are:

temperature, distance, speed, mass, time, length, volume, energy

3) Can be added, subtracted, multiplied, and divided

• I moved the block $\underbrace{\text{5 meters}}_{\text{magnitude}}$ scalar

• I moved the block $\underbrace{\text{5 meters}}_{\text{magnitude}}$ to the $\underbrace{\text{right.}}_{\text{direction}}$ vector

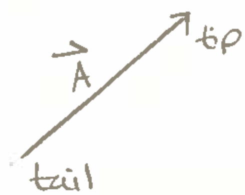
Because vectors have a direction, we must apply vector algebra

Let's start by using a graphical method to get a qualitative understanding.

VECTOR ALGEBRA

a. Graphical Representation

We graphically represent a vector with an arrow

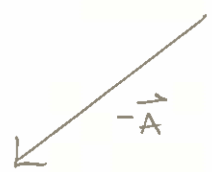


$|\vec{A}| = A$

length of arrow = magnitude of vector

direction of arrow = direction of vector

To represent the negative of a vector



simply flip the tail to the other side

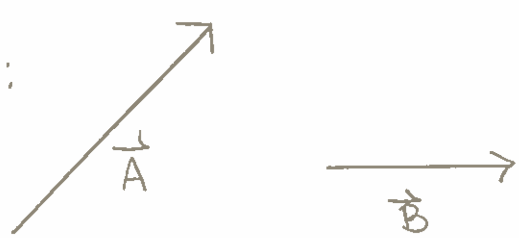
b. Graphical Addition and Subtraction of Vectors

- there are 2 graphical methods that can be used to add vectors.

Method 1: Tip-to-tail
Method 2: Parallelogram

Example:

Given:



Find: $\vec{R} = \vec{A} + \vec{B}$ where $\vec{R}$ = vector sum or resultant

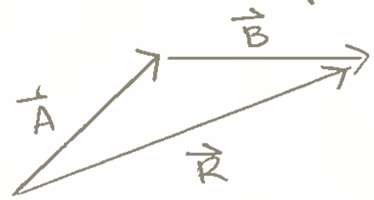
Method 1: Tip-to-tail

Steps: 1) layout the vector $\vec{A}$ to a convenient scale

2) layout vector $\vec{B}$ to the same scale with its tail at the head of vector $\vec{A}$

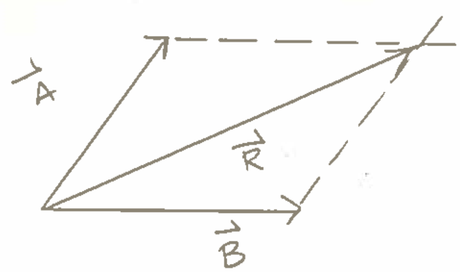
3) make the vector sum by drawing a line from the tail of $\vec{A}$ to the tip of $\vec{B}$

* connect tail of $\vec{R}$ to the tail of 1st vector
connect tip of $\vec{R}$ to the tip of last vector



Method 2: Parallelogram

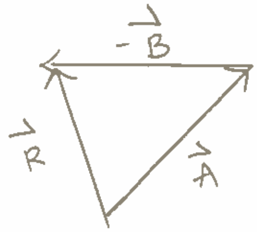
Steps: 1) take vector $\vec{A}$ and $\vec{B}$ and place them tail-to-tail so that they form 2 sides of a parallelogram
2) Draw two other sides of parallelogram
3) vector $\vec{R}$ is then drawn from the tail of the intersecting vectors to the tip



Use Law of Sines/cosines to find sides/angles

Subtraction of vectors: Find $\vec{R} = \vec{A} - \vec{B} \Rightarrow \vec{R} = \vec{A} + (-\vec{B})$

tip-to-tail



Vector Addition Properties:

1) commutative law: order does not matter

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

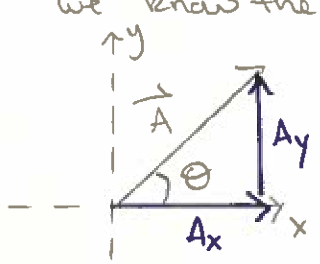
2) "Grouping" does not matter: associative law

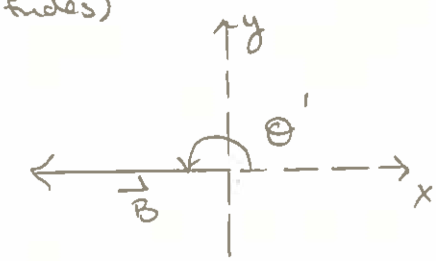
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

c. <u>Exact Addition of Vectors</u>: Component Method

Graphical methods are useful to visualize vectors, but can be difficult to get an exact answer.

Let's start by looking at 2 vectors $\vec{A}$ and $\vec{B}$ (assume we know the magnitudes)





To fully define a vector, there are two forms:

1) Magnitude and Direction (polar coordinates)

2) Component Form (rectangular components)
(cartesian coordinates)

The first step is to define a coordinate system

Measure the angle with respect to the +x-axis

1) Magnitude and Direction

$$\vec{A} = A\ @\ \theta$$

$$\vec{B} = B\ @\ \theta'$$

magnitude (B) direction (θ')

2) Component Form

- Use the directions of the coordinate system to express info about the vector.

$\vec{A} = A_x$ along $+x$-axis $+$ A_y along $+y$-axis

In component form you are always travelling along the direction of one of your axes.

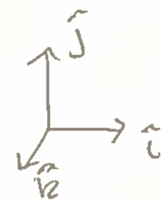
Convenient to define unit vectors whose only purpose is to specify direction

1) length of 1
2) unitless

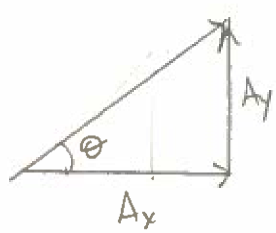
$\hat{\imath}, -\hat{\imath} \Rightarrow$ along $\pm x$ direction

$\pm\hat{\jmath} \Rightarrow$ along $\pm y$ direction

$\pm\hat{k} \Rightarrow$ along $\pm z$-direction



$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ $\qquad$ $\vec{B} = -B\hat{\imath}$



To go from MAD $\rightarrow$ components

$A_x = A\cos\theta$

$A_y = A\sin\theta$

To go from components $\rightarrow$ MAD

$\vec{A} = \sqrt{A_x^2 + A_y^2}$ $\qquad$ $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

often, vectors are reported in MAD form, but you can only add vectors in component form.

$\vec{R} = \vec{A} + \vec{B} = A_x\hat{\imath} + A_y\hat{\jmath} - B\hat{\imath}$

$\vec{R} = (A_x - B)\hat{\imath} + A_y\hat{\jmath}$

Rules for Exact Addition of Vectors:

1) If given in polar coordinates, resolve each vector into its components using trig.

2) Add the components for each coordinate axis

3) the sums obtained are the <u>components</u> of the resultant vector

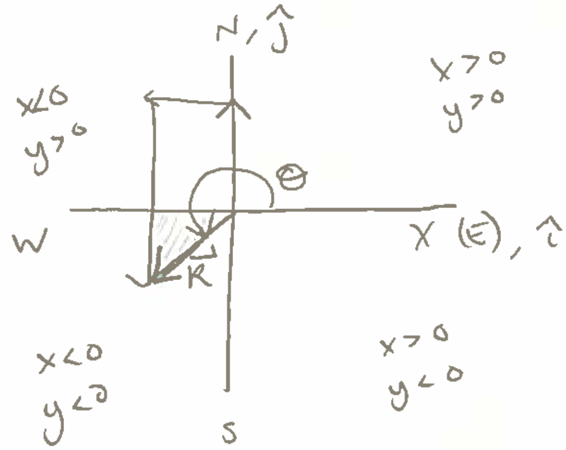
4) Reconstruct the vector into polar coordinates using Pythagorean's theorem and tangent relationship.

Example

Find displacement in MAD and component form:

beginning at the origin, you drive North 3 miles, then head west for 2 miles, then south for 5 miles

Solution

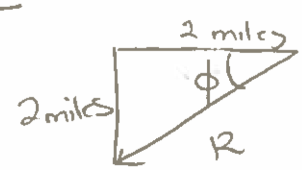


component form:

$$\vec{R} = 3\text{ miles}\,\hat{j} + 2\text{ miles}\,(-\hat{\imath}) + 5\text{ miles}\,(-\hat{j})$$

$$\vec{R} = 2\text{ miles}\,(-\hat{\imath}) + 2\text{ miles}\,(-\hat{j}) \quad \text{or} \quad -2\text{ miles}\,\hat{\imath} - 2\text{ miles}\,\hat{j}$$

$$|\vec{R}| = \sqrt{(2\text{ miles})^2 + (2\text{ miles})^2}$$



$$|\vec{R}| = \sqrt{8}$$

$$\phi = \tan^{-1}\left(\frac{2}{2}\right) = 45^\circ$$

$$\theta = 180^\circ + 45^\circ = 225^\circ$$

$$\vec{R} = 2\sqrt{2}\text{ miles} \text{ @ } 225^\circ$$