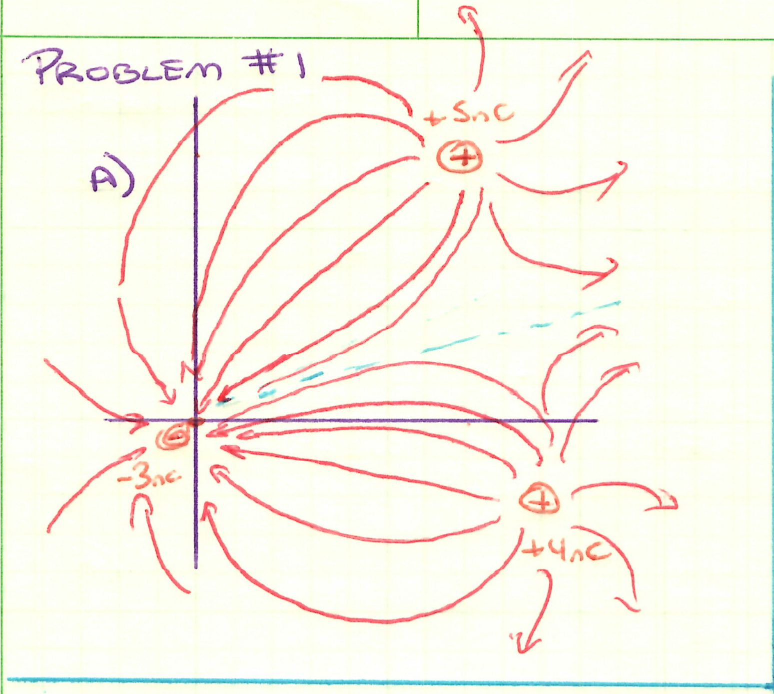
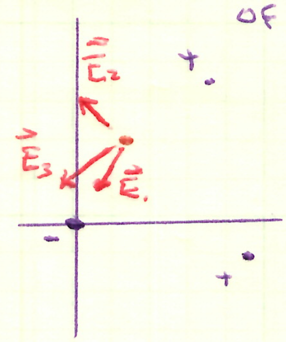
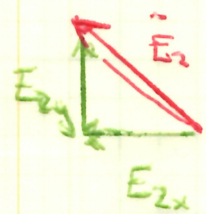
PROBLEM #1

A)



B) SET UP - SUPERPOSITION OF $\vec{E}$ FROM EACH CHARGE





BREAK EACH VECTOR INTO x & y COMPONENTS

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{3\times10^{-9}\,C}{(5\,m)^2}\right) = 8.99\times10^9\,\frac{N\cdot m^2}{C^2}\left(\frac{3\times10^{-9}\,C}{25\,m^2}\right) = 1.08\,N/C$$

$\theta_1 = 180° + \tan^{-1}\left(\frac{4}{3}\right) = 233$ POINTS TOWARDS NEGATIVE CHARGE

$$E_{1x} = E_1\cos\theta_1 = -0.65\,N/C$$
$$E_{1y} = E_1\cos\theta_1 = -0.86\,N/C$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{4\times10^{-9}\,C}{(6\sqrt{2}\,m)^2}\right) = 8.99\times10^9\,\frac{N\cdot m^2}{C^2}\left(\frac{4\times10^{-9}\,C}{72\,m^2}\right) = 0.50\,N/C$$

$$\theta_2 = 180° - \tan^{-1}\left(\frac{6}{6}\right) = 135°$$

$$E_{2x} = -0.35\,N/C$$
$$E_{2y} = +0.35\,N/C$$

$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{|q_3|}{r_3^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{5\times10^{-9}\,N/C}{(5\,m)^2}\right) = 8.99\times10^9\,\frac{N\cdot m^2}{C^2}\left(\frac{5\times10^{-9}\,C}{25\,m^2}\right) = 1.73\,N/C$$

$$E_{3x} = -1.38\,N/C$$
$$E_{3y} = -1.04\,N/C$$

$$\theta_3 = 180° + \tan^{-1}\left(\frac{3}{4}\right) = 216°$$

$$\vec{E} = (E_{1x} + E_{2x} + E_{3x})\hat{i} + (E_{1y} + E_{2y} + E_{3y})\hat{j}$$

$$\boxed{= -2.38\,\hat{i} - 1.55\,\hat{j}\ N/C}$$

c) $\vec{E} = -2.38\,\hat{i} - 1.55\,\hat{j}$

$$E = \sqrt{(-2.38)^2 + (-1.55)^2}$$

$$\boxed{E = 2.84\ \text{N/C}}$$

$$\theta = 180° + \tan^{-1}\left(\frac{-1.55}{-2.38}\right)$$

$$\boxed{\theta = 213°}$$

D) $\vec{F} = q\vec{E}$

$$F = (1.602 \times 10^{-19}\ \text{C})(2.84\ \text{N/C})$$

$$= 4.55\text{N}$$

$\theta = 213° - 180°$ FORCE ON ELECTRON IS IN THE OPPOSITE DIRECTION OF $\vec{E}$

$$\boxed{\vec{F} = 4.55\ \text{N} \ @\ 33°}$$

PROBLEM #2

A) CHARGE DENSITY = TOTAL CHARGE DIVIDED BY LENGTH

$$\lambda = \frac{Q}{2\pi r} = \frac{(-31.5 \times 10^{-6}\ \text{C})}{2\pi\ (0.1\text{m})}$$

$$\lambda = -5.01 \times 10^{-5}\ \text{C/m}$$

B) $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{r^2}\,\hat{r}$

$$dl = R\, d\theta$$

$$r = \sqrt{z^2 + R^2}$$

$$\cos\phi = \frac{z}{\sqrt{z^2+R^2}} \Rightarrow \hat{r} = \frac{z}{\sqrt{z^2+R^2}}\,\hat{z}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi} \frac{\lambda R\, d\theta}{z^2+R^2}\,\frac{z}{\sqrt{z^2+R^2}}\,\hat{z}$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{2\pi R \lambda z}{(z^2+R^2)^{3/2}}\,\hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_{tot} z}{(z^2+R^2)^{\frac{3}{2}}} \hat{z}$$

$$= \left(8.99\times10^{9} \frac{N\cdot m^2}{C^2}\right) \frac{(-31.5\times10^{-6}\,C)(5\times10^{-2}\,m)}{\left((5\times10^{-2}\,m)^2 + (10^{-1}\,m)^2\right)^{\frac{3}{2}}} \hat{z}$$

$$\boxed{\vec{E} = -1.01\times10^{7}\, \hat{z}\ \frac{N}{C}}$$

PROBLEM #3

$$\Phi = \int \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} \quad \text{for uniform fields and flat surfaces}$$
$$= EA\cos\theta \quad \text{where } \theta \text{ is angle between } \hat{r} + \hat{n}$$

A) FRONT

$$A_{FRONT} = (0.01\,m)(0.02\,m) = 2\times10^{-4}\,m^2$$

$$\vec{A}_{FRONT} = 2\times10^{-4}\,m^2\ \hat{i}$$

$$\Phi_{FRONT} = \left(3\tfrac{N}{C}\right)(-\hat{i})\cdot(2\times10^{-4}\,m^2)\hat{i} + \underbrace{\left(8\tfrac{N}{C}\right)(\hat{j})\cdot(2\times10^{-4}\,m^2)(\hat{i})}_{\hat{j}\cdot\hat{i}=\emptyset}$$

$$\boxed{\Phi_{FRONT} = -6\times10^{-4}\ \frac{N\cdot m^2}{C}}$$

B) TOP

$$\hat{n} = \hat{k}$$
$$\vec{E}\cdot A\hat{k} = \emptyset$$
$$\boxed{\Phi_{TOP} = \emptyset\ \frac{N\cdot m^2}{C}}$$

C) RIGHT

$$A_{RIGHT} = (0.0141\,m)(0.04\,m) = 5.66\times10^{-4}\,m^2$$

$$\Phi_{RIGHT} = \left(8\tfrac{N}{C}\right)(5.66\times10^{-4}\,m^2)\cos(45^\circ)$$

$$\boxed{\Phi_{RIGHT} = 3.2\times10^{-3}\ \frac{N\cdot m^2}{C}}$$

D) TWO METHODS

1) TOP = -BOTTOM | LEFT = -RIGHT | FRONT = -BACK

$$\Phi_{TOT} = T + B_o + L + R + F + B_a = \emptyset$$

OR

2) GAUSS'S LAW $\quad q_{enc} = \emptyset \Rightarrow \boxed{\Phi_{TOT} = \emptyset}$

PROBLEM 4

A) Inside the inner diameter

$r = 0.2m$

$$\Phi = \oint \vec{E}\, d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow 4\pi r^2 E = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q_{enc}}{r^2}$$

$$= 8.99\times10^{-9}\ \frac{N\cdot m^2}{C^2}\left(\frac{45\times10^{-9}\ C}{.04\ m^2}\right)$$

$$\boxed{\vec{E} = (1.01\times10^{5})\hat{r}\ N/C}$$

B) INSIDE THE CONDUCTING SPHERE (SHELL)

$$\boxed{\vec{E} = \emptyset}$$

C) OUTSIDE THE SHELL

$\vec{E}$ outside a conducting sphere is the same as that of a point charge at its center

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

$$E = \left(8.99\times10^{+9}\ \frac{N\cdot m^2}{C^2}\right)\frac{105\times10^{-9}\ C}{(.55\ m)^2}$$

$$\vec{E} = (3.120\times10^{4})\hat{r}\ N/C$$

D) From B: Φ inside shell $= \emptyset$

$$\Phi = \emptyset = \frac{q_{enc}}{\epsilon_0} = \frac{Q_{center} + Q_{inner}}{\epsilon_0}$$

$$Q_{inner} = -Q_{center} = -45\times10^{-9}\ C$$

$$\sigma = \frac{Q_{inner}}{A} = \frac{45\times10^{-9}\ C}{4\pi(.25\ m)^2}$$

$$\boxed{\sigma_{inner} = 57.3\ \frac{nC}{m^2} = 5.73\times10^{-8}\ C/m^2}$$

PROBLEM #5

A) $\Delta U + \Delta K = 0$ CONSERVATION OF ENERGY

$$(U_f - U_i) + (K_f - K_i) = 0$$

$K_f = \emptyset$ because $v_f = \emptyset$ at turn around point

$$-qEx - \left(-qE\frac{d}{2}\right) = \frac{1}{2} m_p v_i^2$$

$$qE\left(\frac{d}{2} - x\right) = \frac{1}{2} m_p v_i^2$$

$$x = \frac{1}{2}\left(d - \frac{m_p v_i^2}{qE}\right)$$

$$x = \frac{1}{2}\left(0.2\text{m} - \frac{(1.687\times10^{-27}\text{kg})(10{,}000 \tfrac{\text{m}}{\text{s}})^2}{(1.602\times10^{-19}\text{C})(150 \text{N/C})}\right)$$

$\boxed{x = 0.096 \text{ m from positive plate}}$

B) ELECTRON IS ACCELERATED IN OPPOSITE DIRECTION TO ELECTRIC FIELD, SO IT WILL BE ACCELERATED TOWARDS THE POSITIVE PLATE AND EITHER

1) BOUNCE OFF
2) PASS THROUGH

PROBLEM #6

A) $C = \frac{Q}{V}$ $\quad V = \frac{Q}{C} = \frac{(0.114\text{ C})}{(475\times10^{-6}\text{ F})}$

$\boxed{V = 240\text{ V}}$

B) $C = \frac{Q}{V} = \frac{Q}{\left(Qd/\epsilon_0 A\right)} = \epsilon_0 \frac{A}{d}$

$$A = \frac{Cd}{\epsilon_0} = \frac{(4.75\times10^{-6})(1.87\times10^{-4})}{(8.85\times10^{-12})}\text{ m}^2$$

$\boxed{\approx 10{,}000 \text{ m}^2}$

C) $V = Ed$ $\quad E = \frac{V}{d} = \frac{240\text{V}}{(1.87\times10^{-4}\text{m})} = \boxed{1.28\times10^{6}\ \text{N/C}}$

D) $\sigma = \frac{Q}{A} = \frac{(.114\text{C})}{10{,}000\text{m}^2} = \boxed{1.14\times10^{-6}\ \text{C/m}^2}$

EXTRA CREDIT

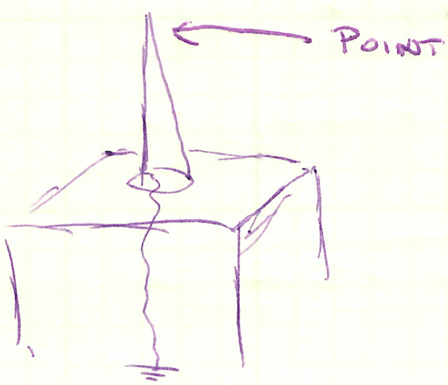
A) THE LIGHTNING ROD WAS INVENTED INDEPENDANTLY BY

BENJAMIN FRANKLIN OF USA IN 1752

and

PROCOP DIVIS OF CZECH REPUBLIC IN 1752

B)



C) SMALL RADIUS OF CURVATURE OF POINT LEADS TO LARGE CHARGE DENSITY WHICH ATTRACTS THE LIGHTNING AWAY FROM THE BUILDING