Quantum Math

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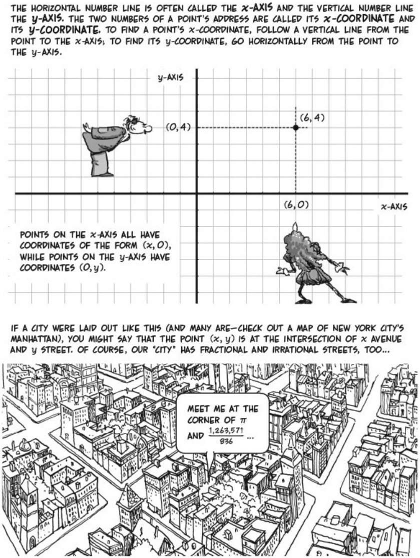
Algebra

Algebra Overview

- Functions
- Transformations
- Polynomials
- Rational Functions
- Exponentials and Logarithms

Cartesian Coordinates

Some text



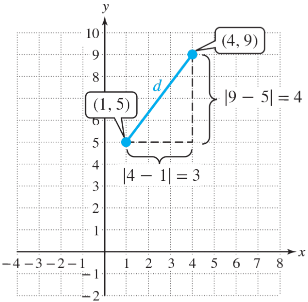
Measuring Distance - Pythagorean Theorem

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

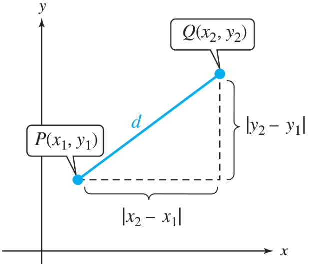
For example:

$$d^2 = 3^2 + 4^2$$
$$d^2 = 9 + 16 = 25$$
$$d = \sqrt{25} = 5$$



More generally for two points $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Noting that $| a | = (a)^2$:

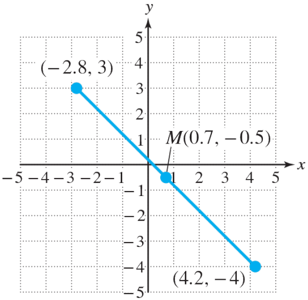
Midpoints and Intercepts

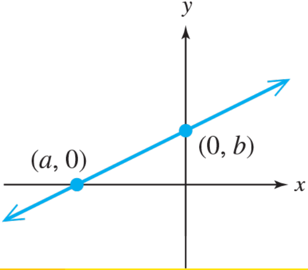
Midpoint:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_2+y_1}{2}\right)$$

Intercepts:

Two key features of a graph are where the graph intersects the x and y axes, the x-intercept and y-intercept, respectively.





The Circle

A circle is a set of all points that are equidistant from a fixed point called the center (h, k). The distance from any point on the circele to the center is called the radius (r)

$r = \sqrt{(x-h)^2 + (y-k)^2}$

Equation of a circle:

Standard form: $(x-h)^2 + (y-k)^2 = r^2$

Expand binomials:

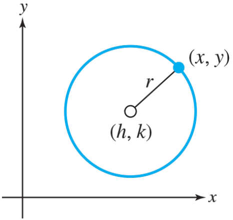
$x^2 - hx + h^2 + y^2 - ky + k^2 - r^2 = 0$

General form:

$x^2 + y^2 - hx - ky + (h^2 + k^2 - r^2) = 0$

or

$x^2 + y^2 + Ax + By + C = 0$

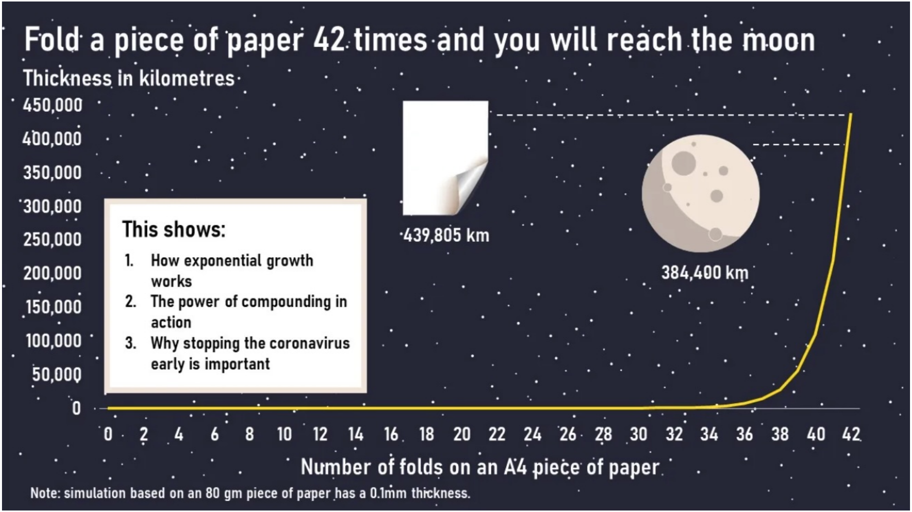


Exponential Functions

- Linear growth - a constant rate of change, that is,a constant number by which the output increased for each unit increase in input.
- Exponential growth - increase based on a constant multiplicative rate of change over equal increments of time, that is, a percent increase of the original amount over time.

x	$f(x) = 2^x$	$g(x) = 2x$
0	1	0
1	2	2
2	4	4
3	8	6
4	16	8
5	32	10
6	64	12

Origami to the Moon

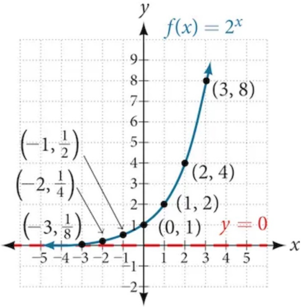


What about Negative Exponents

For example, $f(x) = 2^x$:

The general form of an exponential function is $f(x) = ab^x$, where a is any non-zero number and b is an positive number not equal to 1.

- If $b > 1$ the function grows at a rate proportional to its size.
- If $0 < b < 1$ the function decays at a rate proportional to its size.



x	-3	-2	-1	0	1	2	3
$f(x) = 2^x$	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$

Scientific (SI) Prefixes

The Metric System Prefixes				
Prefix	**Label**	**Decimal Value**	**Scientific**	**Colloquial**
yocto	y	0.000 000 000 000 000 000 000 001	10^{-24}	septillionth
zepto	z	0.000 000 000 000 000 000 001	10^{-21}	sextillionth
atto	a	0.000 000 000 000 000 001	10^{-18}	quintillionth
femto	f	0.000 000 000 000 001	10^{-15}	quadrillionth
pico	p	0.000 000 000 001	10^{-12}	trillionth
nano	n	0.000 000 001	10^{-9}	billionth
micro	µ	0.000 001	10^{-6}	millionth
milli	m	0.001	10^{-3}	thousandth
centi	c	0.01	10^{-2}	hundredth
deci	d	0.1	10^{-1}	tenth
---	---	1	10^{0}	one
deka	da	10	10^{1}	ten
hecto	h	100	10^{2}	hundred
kilo	k	1 000	10^{3}	thousand
mega	M	1 000 000	10^{6}	million
giga	G	1 000 000 000	10^{9}	billion
tera	T	1 000 000 000 000	10^{12}	trillion
peta	P	1 000 000 000 000 000	10^{15}	quadrillion
exa	E	1 000 000 000 000 000 000	10^{18}	quintillion
zetta	Z	1 000 000 000 000 000 000 000	10^{21}	sextillion
yotta	Y	1 000 000 000 000 000 000 000 000	10^{24}	septillion

e - an interesting aside

The letter e represents the irrational number:

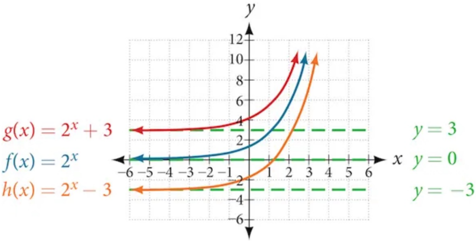
$$e = (1 + \frac{1}{n})^n \tag{1}$$

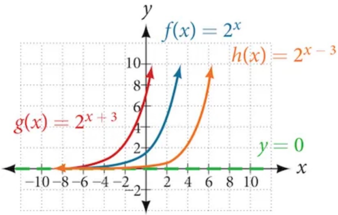
as n increases without bound.

The number e is used as a base for many real-world exponential models. To work with base e, we use the approximation, $e \approx 2.718282$. The constant was named by the Swiss mathematician Leonhard Euler (1707–1783) who first investigated and discovered many of its properties.

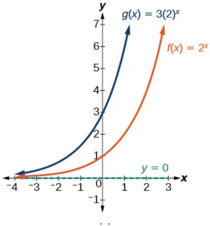
Graphing Exponentials

Shifts:

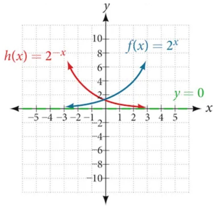




Stretch:



Flip:

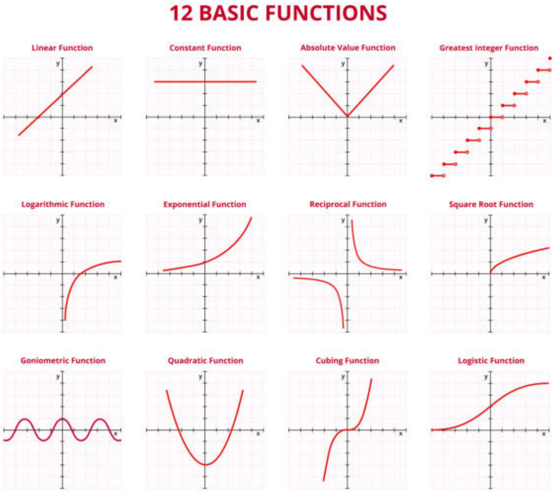


Trigonometry

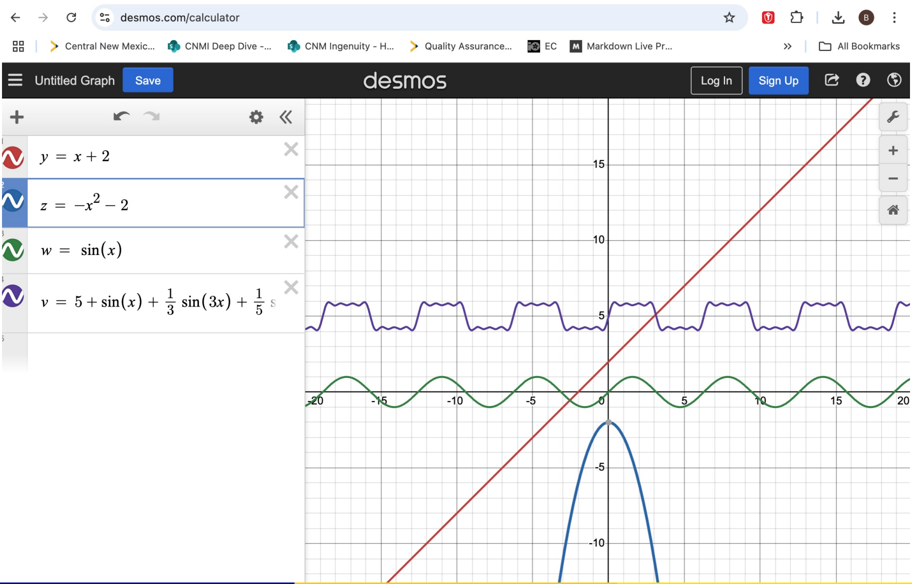
Algebraic Functions

An algebraic function provides a "y-value" for every "x-value"

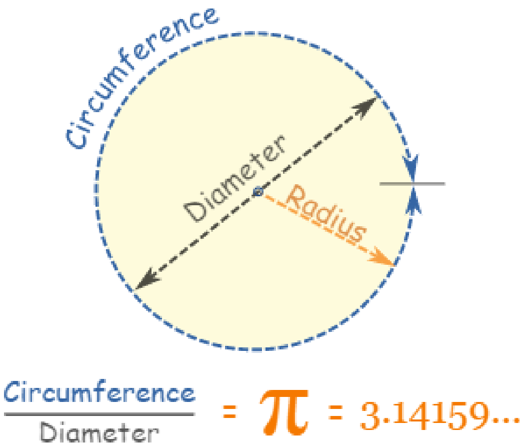
- Linear: $y = x + 2$
- Quadratic: $y = x^2$
- Periodic: $y = sin(x)$



More Desmos Fun

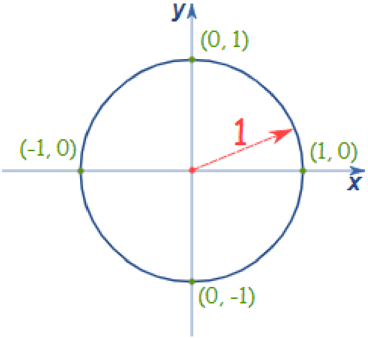


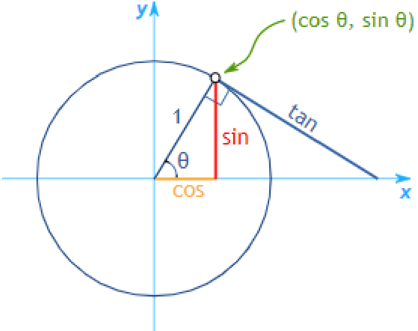
Pi (π)


Circumference
Diameter
Radius
Circumference / Diameter = π = 3.14159...

Unit Circle and Trigonometric Functions

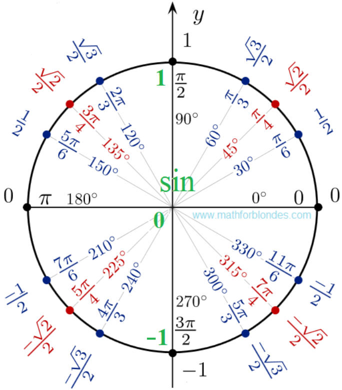
The Unit Circle is a circle with a radius of 1.

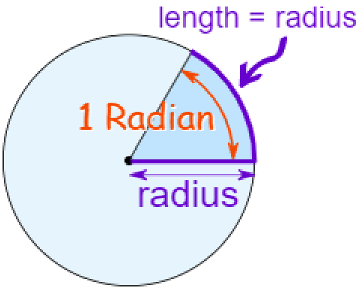




The Unit Circle can be used to map out the trigonometric values of sine, cosine, and tangent.

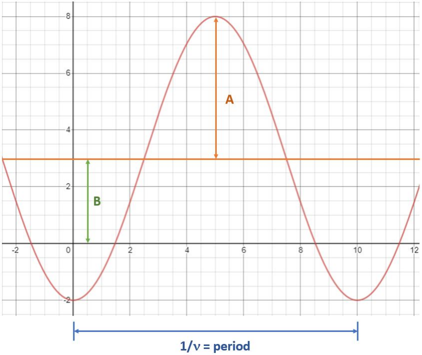
Unit Circle and the Value of *sin*(θ)





- *sin*(θ) is the y-value of the point on the Unit Circle at angle θ.
- In our trig functions, θ is measured in radians (rad), not degrees.
- 360 degrees $= 2\pi$ radians.

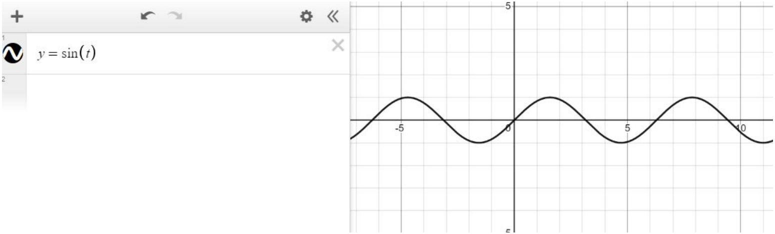
Sine Waves

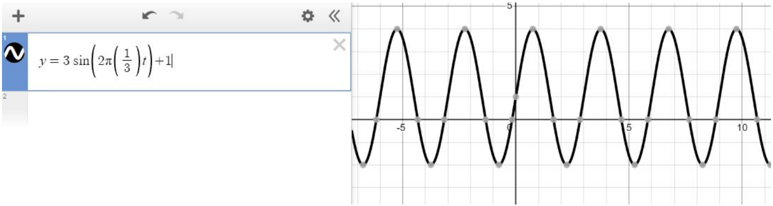


$$y = A * sin(2 * \pi * \nu * t) + B$$

where A = amplitude, B = offset, ν = frequency = $\frac{1}{period}$, and t = time in seconds.

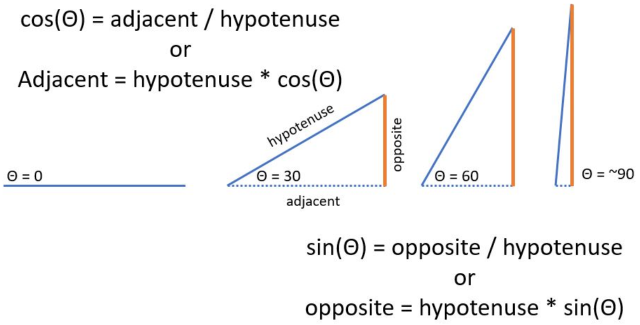
Using Desmos (desmos.com/calculator)





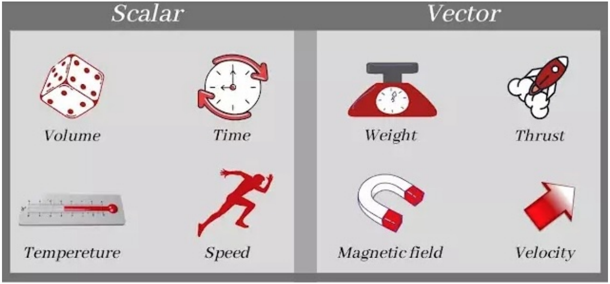
SOH CAH TOA

- sin = opposite over hypotenuse
- cos = adjacent over hypotenuse
- tan = opposite over adjacent

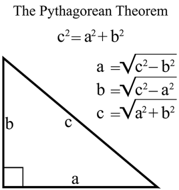


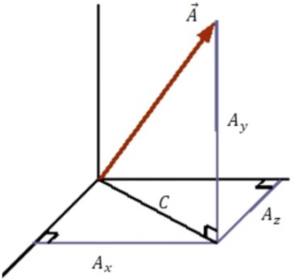
Scalars and Vectors

Scalars are quantities that are fully described by a magnitude (or numerical value) alone.
Vectors are quantities that are fully described by both a magnitude and a direction.



Pythagorean Theorem in 3 Dimensions





To add orthogonal (at right angles to each other) vectors in 3 Dimensions:

- $C = \sqrt{A_x^2 + A_y^2}$
- $A_{total} = \sqrt{C^2 + A_z^2}$
- $A_{total} = \sqrt{A_x^2 + A_y^2 + A_z^2}$