

# Quantum Math

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# Algebra



# Algebra Overview

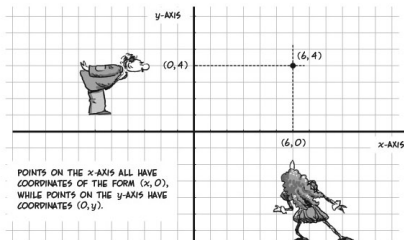
- Functions
- Transformations
- Polynomials
- Rational Functions
- Exponentials and Logarithms



# Cartesian Coordinates

Some text

THE HORIZONTAL NUMBER LINE IS OFTEN CALLED THE  $x$ -AXIS AND THE VERTICAL NUMBER LINE THE  $y$ -AXIS. THE TWO NUMBERS OF A POINT'S ADDRESS ARE CALLED ITS  $x$ -COORDINATE AND ITS  $y$ -COORDINATE. TO FIND A POINT'S  $x$ -COORDINATE, FOLLOW A VERTICAL LINE FROM THE POINT TO THE  $x$ -AXIS; TO FIND ITS  $y$ -COORDINATE, GO HORIZONTALLY FROM THE POINT TO THE  $y$ -AXIS.



IF A CITY WERE LAID OUT LIKE THIS (AND MANY ARE—CHECK OUT A MAP OF NEW YORK CITY'S MANHATTAN), YOU MIGHT SAY THAT THE POINT  $(x, y)$  IS AT THE INTERSECTION OF  $x$  AVENUE AND  $y$  STREET. OF COURSE, OUR "CITY" HAS FRACTIONAL AND IRRATIONAL STREETS, TOO...





# Measuring Distance - Pythagorean Theorem

Pythagorean Theorem:

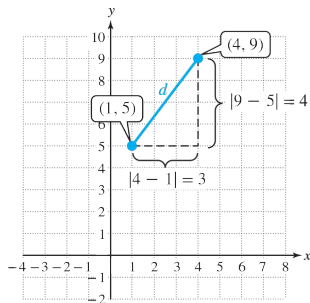
$$a^2 + b^2 = c^2$$

For example:

$$d^2 = 3^2 + 4^2$$

$$d^2 = 9 + 16 = 25$$

$$d = \sqrt{25} = 5$$

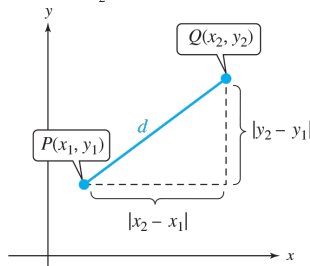


More generally for two points

$P(x_1, y_1)$  and  $Q(x_2, y_2)$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Noting that  $|a| = (a)^2$ :



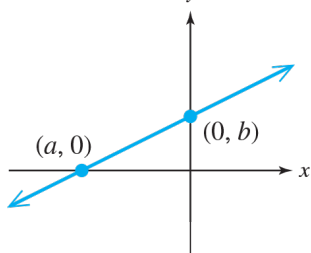
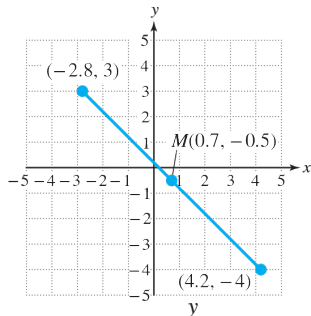
# Midpoints and Intercepts

Midpoint:

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Intercepts:

Two key features of a graph are where the graph intersects the x and y axes, the x-intercept and y-intercept, respectively.





# The Circle

A circle is a set of all points that are equidistant from a fixed point called the center  $(h, k)$ . The distance from any point on the circle to the center is called the radius  $(r)$

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

Equation of a circle:

Standard form:  $(x - h)^2 + (y - k)^2 = r^2$

Expand binomials:

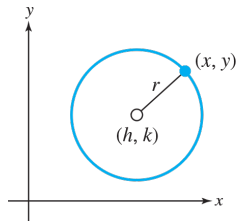
$$x^2 - hx + h^2 + y^2 - ky + k^2 - r^2 = 0$$

General form:

$$x^2 + y^2 - hx - ky + (h^2 + k^2 - r^2) = 0$$

or

$$x^2 + y^2 + Ax + By + C = 0$$





# Exponential Functions

- Linear growth - a constant rate of change, that is, a constant number by which the output increased for each unit increase in input.
- Exponential growth - increase based on a constant multiplicative rate of change over equal increments of time, that is, a percent increase of the original amount over time.

$x$	$f(x) = 2^x$	$g(x) = 2x$
0	1	0
1	2	2
2	4	4
3	8	6
4	16	8
5	32	10
6	64	12





# Origami to the Moon

## Fold a piece of paper 42 times and you will reach the moon

Thickness in kilometres

450,000

400,000

350,000

300,000

250,000

200,000

150,000

100,000

50,000

0

**This shows:**

1. How exponential growth works
2. The power of compounding in action
3. Why stopping the coronavirus early is important



0.439,805 km



384,400 km

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42

Number of folds on an A4 piece of paper

Note: simulation based on an 80 gm piece of paper has a 0.1mm thickness.

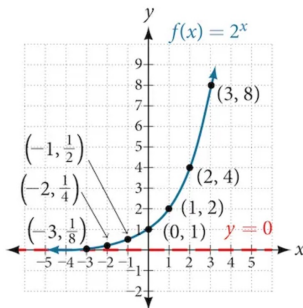


# What about Negative Exponents

For example,  $f(x) = 2^x$ :

The general form of an exponential function is  $f(x) = ab^x$ , where  $a$  is any non-zero number and  $b$  is a positive number not equal to 1.

- If  $b > 1$  the function grows at a rate proportional to its size.
- If  $0 < b < 1$  the function decays at a rate proportional to its size.



$x$	-3	-2	-1	0	1	2	3
$f(x) = 2^x$	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$



# Scientific (SI) Prefixes

## The Metric System Prefixes

Prefix	Label	Decimal Value	Scientific	Colloquial
yocto	y	0.000 000 000 000 000 000 001	$10^{-24}$	septillionth
zepto	z	0.000 000 000 000 000 000 001	$10^{-21}$	sextillionth
atto	a	0.000 000 000 000 000 001	$10^{-18}$	quintillionth
femto	f	0.000 000 000 000 001	$10^{-15}$	quadrillionth
pico	p	0.000 000 000 001	$10^{-12}$	trillionth
nano	n	0.000 000 001	$10^{-9}$	billionth
micro	$\mu$	0.000 001	$10^{-6}$	millionth
milli	m	0.001	$10^{-3}$	thousandth
centi	c	0.01	$10^{-2}$	hundredth
deci	d	0.1	$10^{-1}$	tenth
--	--	1	$10^0$	one
deka	da	10	$10^1$	ten
hecto	h	100	$10^2$	hundred
kilo	k	1 000	$10^3$	thousand
mega	M	1 000 000	$10^6$	million
giga	G	1 000 000 000	$10^9$	billion
tera	T	1 000 000 000 000	$10^{12}$	trillion
peta	P	1 000 000 000 000 000	$10^{15}$	quadrillion
exa	E	1 000 000 000 000 000 000	$10^{18}$	quintillion
zetta	Z	1 000 000 000 000 000 000 000	$10^{21}$	sextillion
yotta	Y	1 000 000 000 000 000 000 000 000	$10^{24}$	septillion



## e - an interesting aside

The letter  $e$  represents the irrational number:

$$e = \left(1 + \frac{1}{n}\right)^n \quad (1)$$

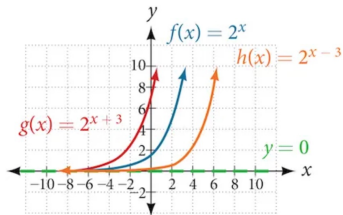
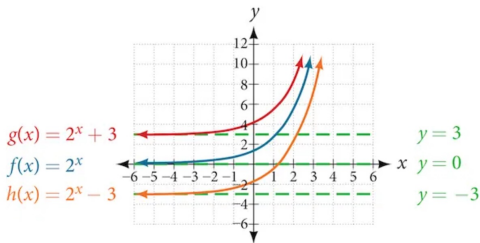
as  $n$  increases without bound.

The number  $e$  is used as a base for many real-world exponential models. To work with base  $e$ , we use the approximation,  $e \approx 2.718282$ . The constant was named by the Swiss mathematician Leonhard Euler (1707–1783) who first investigated and discovered many of its properties.

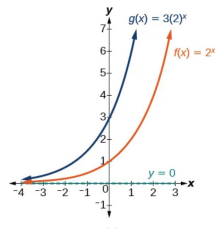


# Graphing Exponentials

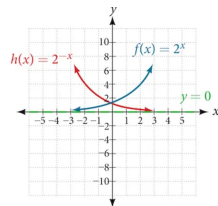
Shifts:



Stretch:



Flip:



# Trigonometry

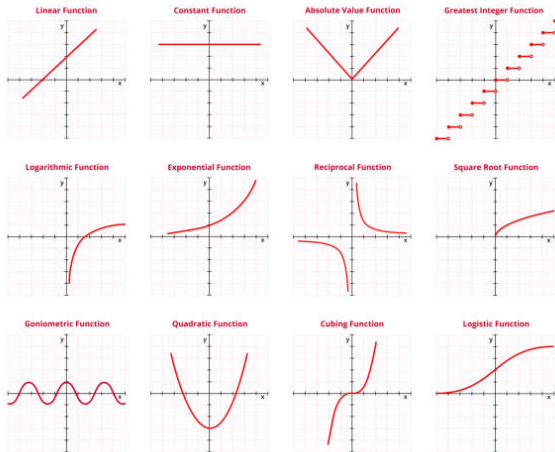


# Algebraic Functions

## 12 BASIC FUNCTIONS

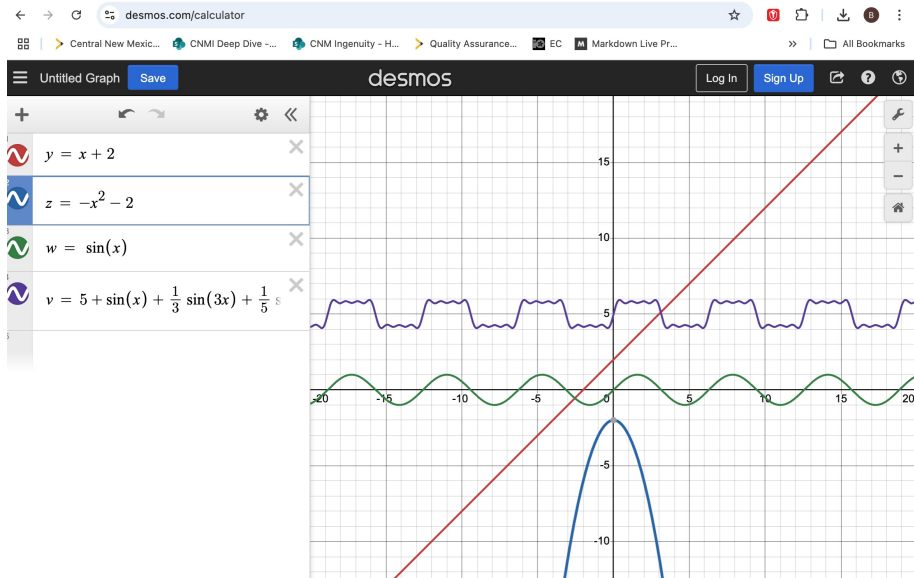
An algebraic function provides a "y-value" for every "x-value"

- Linear:  $y = x + 2$
- Quadratic:  $y = x^2$
- Periodic:  $y = \sin(x)$





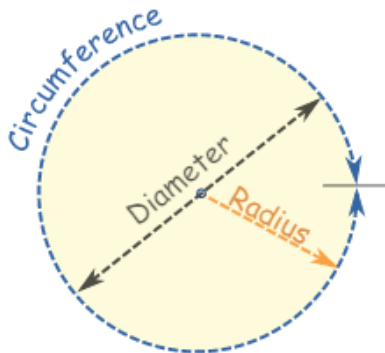
# More Desmos Fun







# Pi ( $\pi$ )

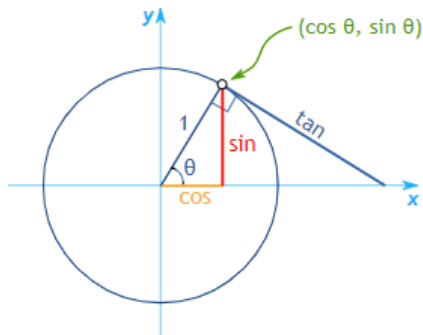
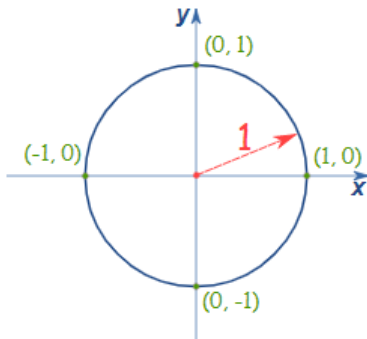


$$\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14159...$$



# Unit Circle and Trigonometric Functions

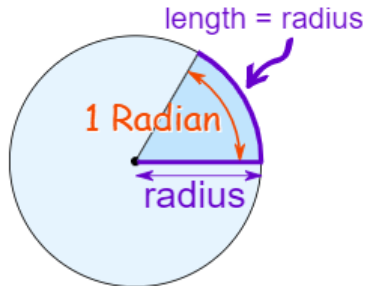
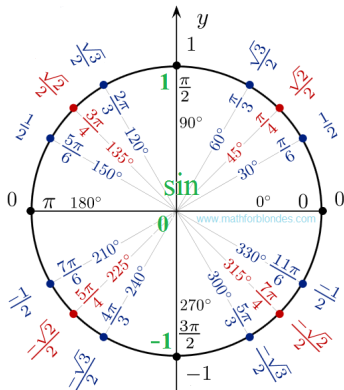
The Unit Circle is a circle with a radius of 1.



The Unit Circle can be used to map out the trigonometric values of sine, cosine, and tangent.



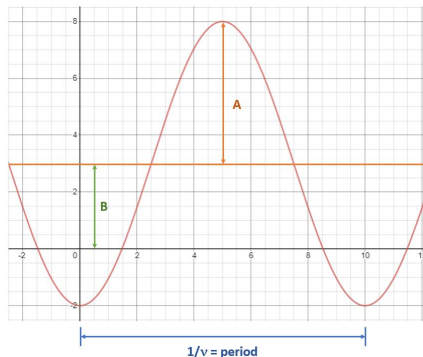
# Unit Circle and the Value of $\sin(\theta)$



- $\sin(\theta)$  is the y-value of the point on the Unit Circle at angle  $\theta$ .
- In our trig functions,  $\theta$  is measured in radians (rad), not degrees.
- $360$  degrees =  $2\pi$  radians.



# Sine Waves

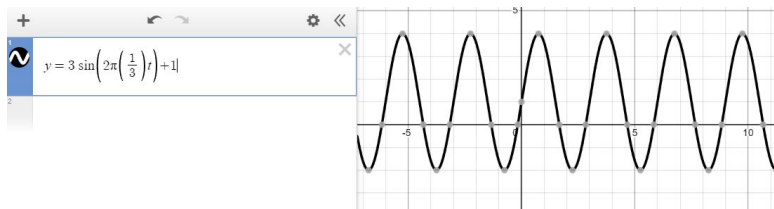
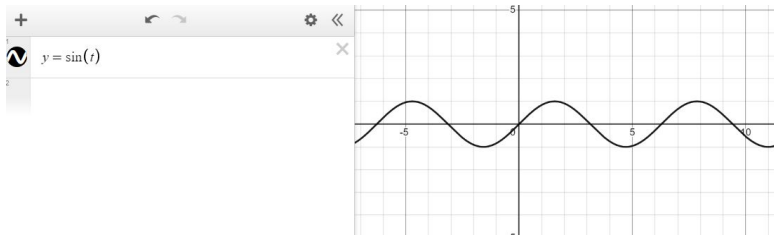


$$y = A * \sin(2 * \pi * \nu * t) + B$$

where  $A$  = amplitude,  $B$  = offset,  $\nu$  = frequency =  $\frac{1}{\text{period}}$ ,  
and  $t$  = time in seconds.



# Using Desmos (desmos.com/calculator)





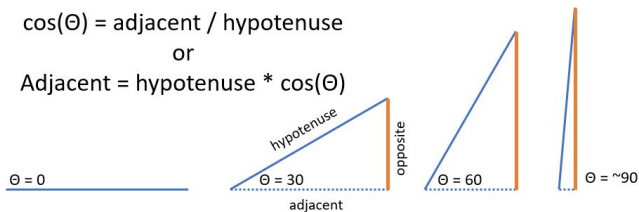
# SOH CAH TOA

- $\sin = \text{opposite over hypotenuse}$
- $\cos = \text{adjacent over hypotenuse}$
- $\tan = \text{opposite over adjacent}$

$$\cos(\Theta) = \text{adjacent} / \text{hypotenuse}$$

or

$$\text{Adjacent} = \text{hypotenuse} * \cos(\Theta)$$



$$\sin(\Theta) = \text{opposite} / \text{hypotenuse}$$

or





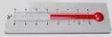



$$\text{opposite} = \text{hypotenuse} * \sin(\Theta)$$



# Scalars and Vectors

Scalars are quantities that are fully described by a magnitude (or numerical value) alone.

Vectors are quantities that are fully described by both a magnitude and a direction.

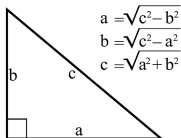
<i>Scalar</i>		<i>Vector</i>	
			
<i>Volume</i>	<i>Time</i>	<i>Weight</i>	<i>Thrust</i>
			
<i>Temperature</i>	<i>Speed</i>	<i>Magnetic field</i>	<i>Velocity</i>



# Pythagorean Theorem in 3 Dimensions

The Pythagorean Theorem

$$c^2 = a^2 + b^2$$



$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

$$c = \sqrt{a^2 + b^2}$$

To add orthogonal (at right angles to each other) vectors in 3 Dimensions:

- $C = \sqrt{A_x^2 + A_y^2}$
- $A_{total} = \sqrt{C^2 + A_z^2}$
- $A_{total} = \sqrt{A_x^2 + A_y^2 + A_z^2}$

