Quantum Math

Brian Rashap

August 2025

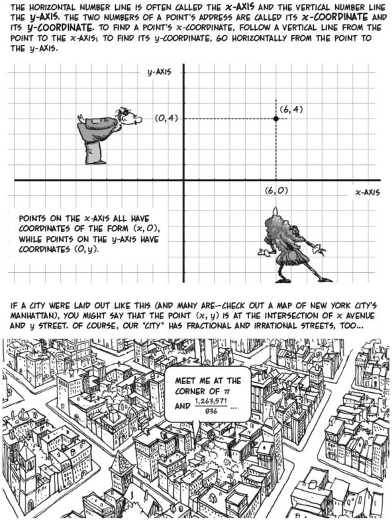
Algebra

Algebra Overview

- Functions
- Transformations
- Polynomials
- Rational Functions
- Exponentials and Logarithms

Cartesian Coordinates

Cartesian coordinates is a system of describing the position of points in space using perpendicular axis lines that meet at a point called the origin. Any given point's position can be described based on its distance from the origin along each axis.



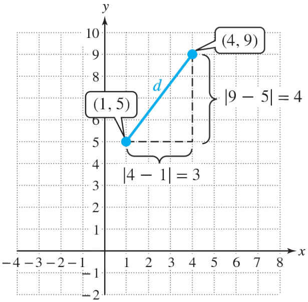
Measuring Distance - Pythagorean Theorem

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

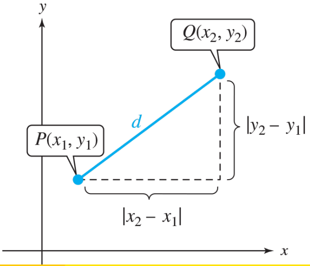
For example:

$$d^2 = 3^2 + 4^2$$
$$d^2 = 9 + 16 = 25$$
$$d = \sqrt{25} = 5$$



More generally for two points $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Noting that $\mid a \mid = (a)^2$:

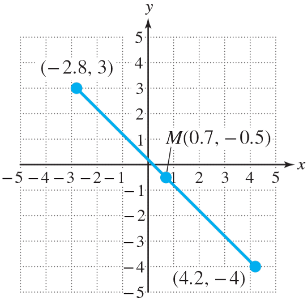
Midpoints and Intercepts

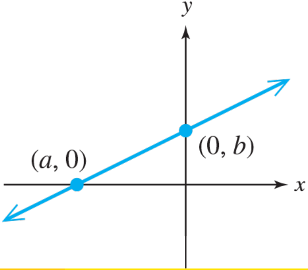
Midpoint:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_2+y_1}{2}\right)$$

Intercepts:

Two key features of a graph are where the graph intersects the x and y axes, the x-intercept and y-intercept, respectively.





The Circle

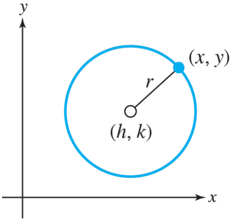
A circle is a set of all points that are equidistant from a fixed point called the center (h, k). The distance from any point on the cirecle to the center is called the radius (r)

$r = \sqrt{(x-h)^2 + (y-k)^2}$

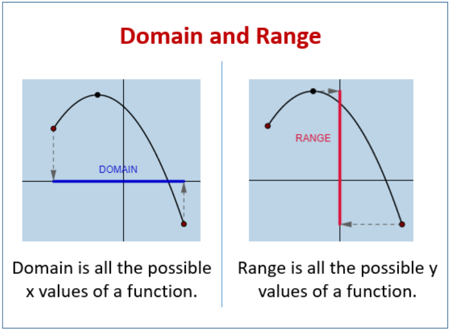
Equation of a circle:
Standard form: $(x-h)^2 + (y-k)^2 = r^2$

Expand binomials:
$x^2 - hx + h^2 + y^2 - ky + k^2 - r^2 = 0$

General form:
$x^2 + y^2 - hx - ky + (h^2 + k^2 - r^2) = 0$
or
$x^2 + y^2 + Ax + By + C = 0$



Domain and Range



A set of ordered pairs (x, y) is called a relation in x and y.

- The set of x-values in the ordered pairs is called the domain of the relations.
- The set of y-values in the ordered pairs is called the range of the relations.

Linear Equations with Two Variables

A linear equation in variables x and y can be written in the standard form:

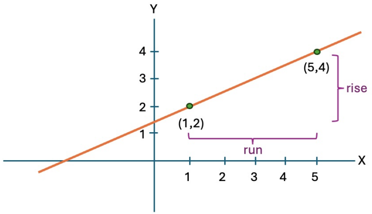
$$Ax + By = C \tag{1}$$

However, it is more common to see it in slope-intercept form:

$$y = mx + b \tag{2}$$

where, m is the slope and b is the y-intercept

Linear Conversion - Slope and Y-Intercept



$$y = mx + b$$

where m is slope and b y-intercept.

For example, given two points:

- $(x_1, y_1) = (1, 2)$
- $(x_2, y_2) = (5, 4)$

Find slope

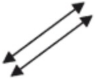
- $m = \frac{rise}{run} = \frac{4-2}{5-1} = \frac{1}{2}$

Find y-intercept

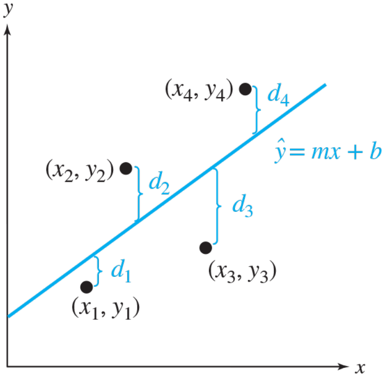
- $y_1 = m * x_1 + b$
- $b = y_1 - (m * x_1)$
- $b = 2 - (\frac{1}{2} * 1) = 1\frac{1}{2}$

Use this to find the conversion from Celsius to Fahrenheit.

Parallel and Perpendicular Lines

	Relationship with Slopes (m)	Line 1	Line 2
Parallel Lines "Equal Slopes" 	$m_1 = m_2$	$\frac{1}{3}$	$\frac{1}{3}$
		5	5
		$-\frac{2}{7}$	$-\frac{2}{7}$
Perpendicular Lines "Opposite Reciprocal Slopes" 	$m_1 = -\frac{1}{m_2}$	$\frac{1}{3}$	$-\frac{3}{1}$
		5	$-\frac{1}{5}$
		$-\frac{2}{7}$	$\frac{7}{2}$

Linear Regression



Consider a set of data:
$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)$

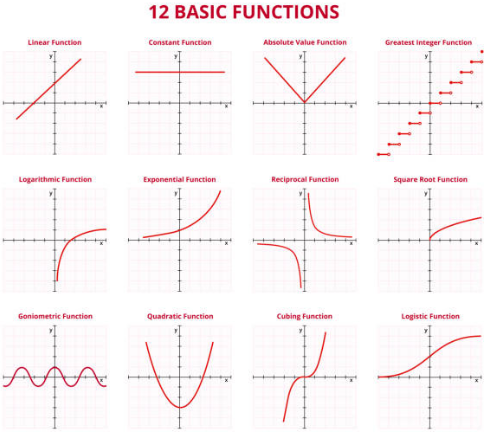
- The least-squares regression line $\hat{y} = mx + b$, is a unique line that minimizes the sum of the squared vertical deviations from the the observed data points to the line.

Use this to find the conversion from Celsius to Fahrenheit.

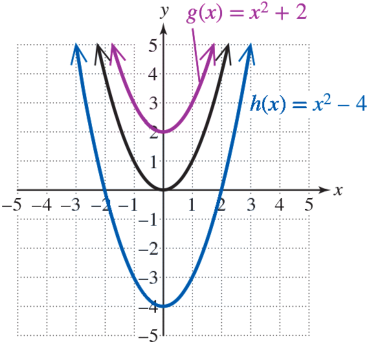
Recognizing Functions

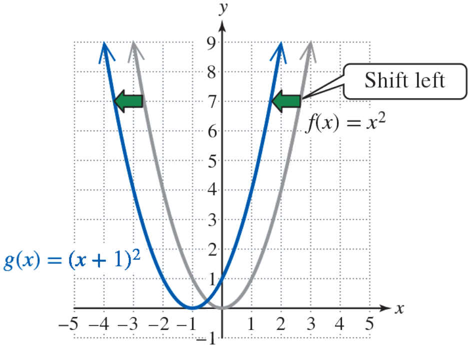
An algebraic function provides a "y-value" for every "x-value"

- Linear: $y = x + 2$
- Quadratic: $y = x^2$
- Periodic: $y = sin(x)$

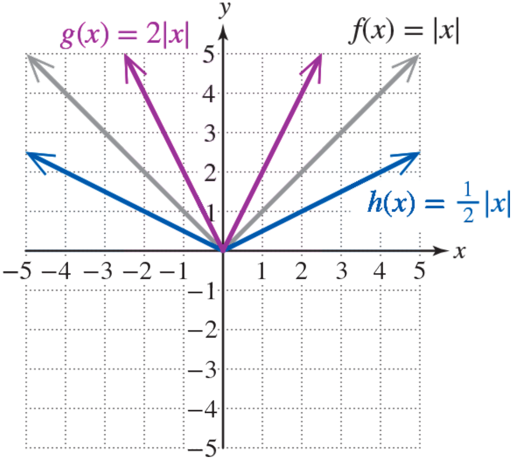


Vertical and Horizontal Shifts

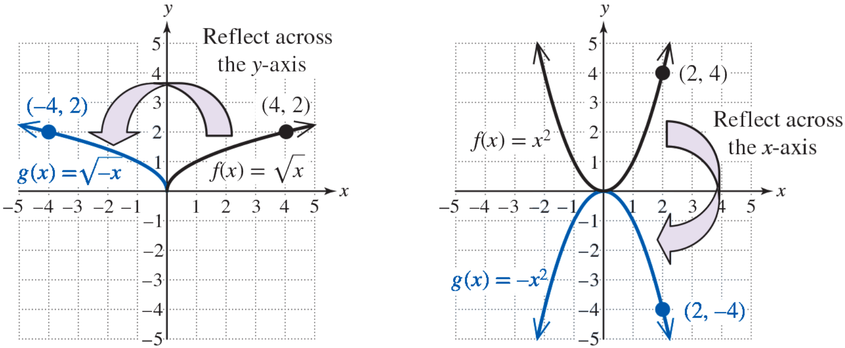




Shrink and Expand



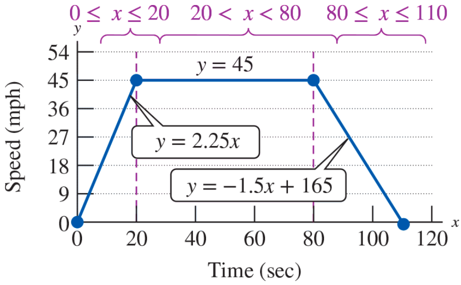
X and Y Reflections



Summary - Transformations of Functions

Transformation	Effect on the Graph of f	Changes to Points on f
Vertical translation (shift) $y = f(x) + k$ $y = f(x) - k$	Shift upward k units Shift downward k units	Replace (x, y) by $(x, y + k)$. Replace (x, y) by $(x, y - k)$.
Horizontal translation (shift) $y = f(x - h)$ $y = f(x + h)$	Shift to the right h units Shift to the left h units	Replace (x, y) by $(x + h, y)$. Replace (x, y) by $(x - h, y)$.
Vertical stretch/shrink $y = a[f(x)]$	Vertical stretch (if $a > 1$) Vertical shrink (if $0 < a < 1$) Graph is stretched/shrunk vertically by a factor of a.	Replace (x, y) by (x, ay).
Horizontal stretch/shrink $y = f(a \cdot x)$	Horizontal shrink (if $a > 1$) Horizontal stretch (if $0 < a < 1$) Graph is shrunk/stretched horizontally by a factor of $\frac{1}{a}$.	Replace (x, y) by $\left(\frac{x}{a}, y\right)$.
Reflection $y = -f(x)$ $y = f(-x)$	Reflection across the x-axis Reflection across the y-axis	Replace (x, y) by $(x, -y)$. Replace (x, y) by $(-x, y)$.

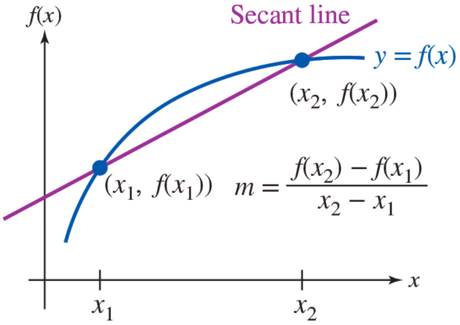
Piece-Wise Functions



$$f(x) = \begin{cases} 2.25x & \text{for } 0 \le x \le 20 \\ 45 & \text{for } 20 < x < 80 \\ -1.5x + 165 & \text{for } 80 \le x \le 100 \end{cases}$$

Rate of Change

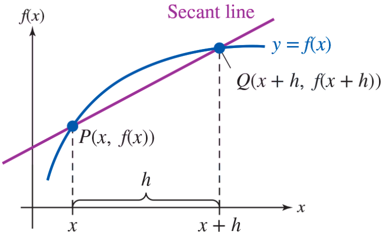
Given points (x_1, y_1) and (x_2, y_2) as points on the graph of a function $f()$, if $f()$ is defined on the interval $[x_1, x_2]$, then the average rate of change is the slope of the secant[1] line containing $(x_1, f(x_1))$ and $(x_2, f(x_2))$.



[1] Secante comes from the latin secare meaning "to cut."

Difference Quotient

Suppose we choose a value x from the domain of $f()$ and a second value $x + h$, where $h \neq 0$, but very small.



The difference quotient[2].

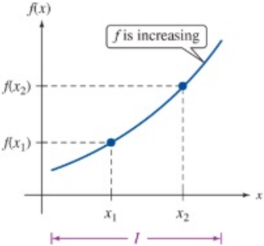
$$m = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h} \tag{3}$$

[2] The difference quotient is important to calculus, where the exact rate of change at a point is given by $\lim_{h \to 0}(m)$

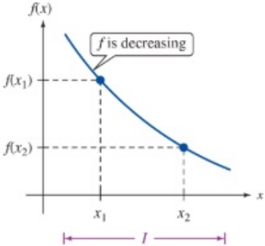
Increaing, Decreaing, Constant

Suppose that I is an interval contained within the domain of a function f.

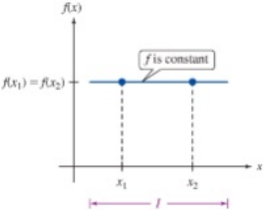
- f is increasing on I if $f(x_1) < f(x_2)$ for all $x_1 < x_2$ on I.
- f is decreasing on I if $f(x_1) > f(x_2)$ for all $x_1 < x_2$ on I.
- f is constant on I if $f(x_1) = f(x_2)$ for all x_1 and x_2 on I.



For all $x_1 < x_2$ on I,
$f(x_1) < f(x_2)$



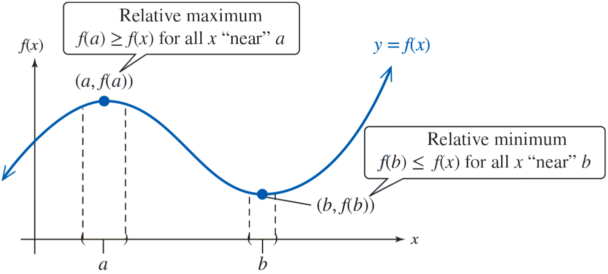
For all $x_1 < x_2$ on I,
$f(x_1) > f(x_2)$



For all x_1 and x_2 on I,
$f(x_1) = f(x_2)$

Local Minima and Maxima

- $f(a)$ is a relative maximum of f if there exists an open interval[3] containing a such that $f(a) \geq f(x)$ for all x in the interval.
- $f(b)$ is a relative minimum of f if there exists an open interval[4] containing a such that $f(b) \leq f(x)$ for all x in the interval.

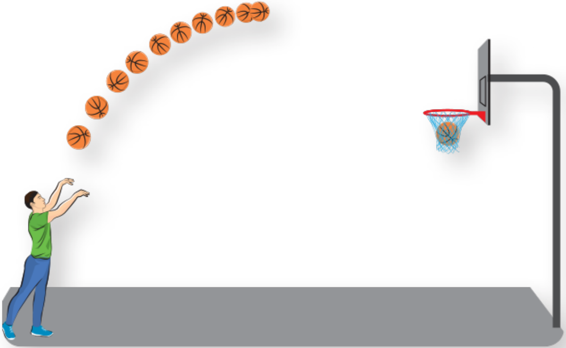


[3] An open interval is an interval in which the endpoints are not included.
[4] An open interval is an interval in which the endpoints are not included.

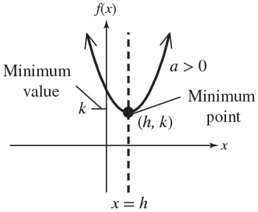
Operations on Functions

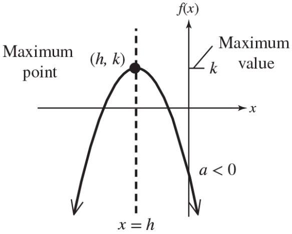
Quadratic Function

A quadratic function is often used as a model for the projectile motion. This is the motion followed by an object influenced by an initial force and by the force of gravity.



Quadratic Function - Vertex Form





Quadratic Function:
$f(x) = ax_2 + bx + c(a \neq 0)$
By completing the square, it can be expressed in vertex form: $f(x) = a(x - h)^2 + k$

- The graft of f(x) is a parabola with vertex (h, k)
- If $a > 0$ the parabola opens upward and minimum value is k.
- If $a > 0$ the parabola opens downward and maximum value is k.
- The axis of symmetry is $x = h$.

Polynomial Functions

Definition of a Polynomial Function

Let $a_n, a_{n-1}, a_{n-2}, \ldots, a_0$ represent real numbers and $n, n-1, n-2, \ldots, 0$ represent whole numbers. Then a function defined by

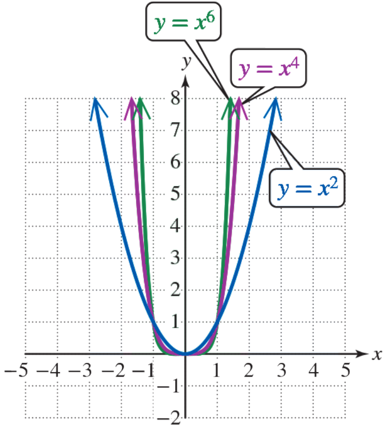
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0$$

is called a **polynomial function**.

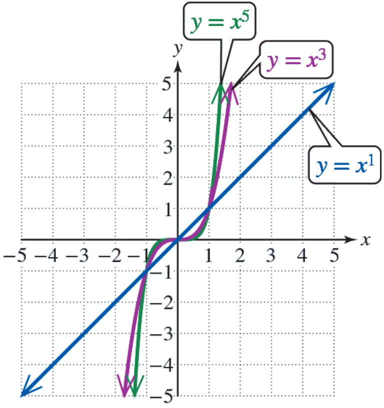
The term $a_n x^n$ is called the **leading term**, the coefficient a_n is the **leading coefficient**, and the exponent n is the **degree** of the polynomial function.

Even and Odd Exponents

Even:



Odd:



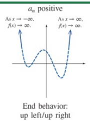
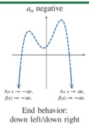
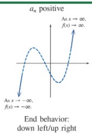
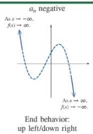
Polynomial End Behavior

The Leading Term Test

Consider a polynomial function given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0.$$

As $x \to \infty$ or as $x \to -\infty$, the graph of f will eventually have no more turns and will become forever increasing or forever decreasing. Thus, the graph of f far to the left and far to the right will follow the general behavior of $y = a_n x^n$.

***n* is even**		***n* is odd**	
			

Polynomial Finding Zeros

Rational Functions

Definition of a Rational Function

Let $p(x)$ and $q(x)$ be polynomials where $q(x) \neq 0$. A function f defined by

$$f(x) = \frac{p(x)}{q(x)} \text{ is called a } \textbf{rational function}.$$

Note: The domain of a rational function is all real numbers excluding the real zeros of $q(x)$.

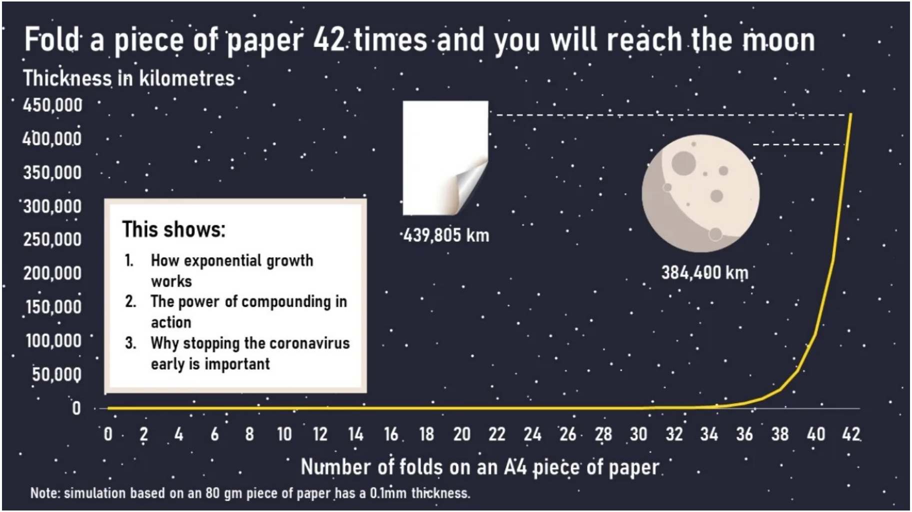
Function	**Factored Form**	**Domain**
$f(x) = \frac{1}{x}$	$f(x) = \frac{1}{x}$	$\{x \mid x \neq 0\}$ $(-\infty, 0) \cup (0, \infty)$
$g(x) = \frac{5x^2}{2x^2 + 5x - 12}$	$g(x) = \frac{5x^2}{(2x-3)(x+4)}$	$\left\{x \middle\vert x \neq \frac{3}{2}, x \neq -4\right\}$ $\left(-\infty, -4\right) \cup \left(-4, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$
$k(x) = \frac{x+3}{x^2+4}$	$k(x) = \frac{x+3}{x^2+4}$	$\mathbb{R}$ $(-\infty, \infty)$

Exponential Functions

- Linear growth - a constant rate of change, that is,a constant number by which the output increased for each unit increase in input.
- Exponential growth - increase based on a constant multiplicative rate of change over equal increments of time, that is, a percent increase of the original amount over time.

x	$f(x) = 2^x$	$g(x) = 2x$
0	1	0
1	2	2
2	4	4
3	8	6
4	16	8
5	32	10
6	64	12

Origami to the Moon

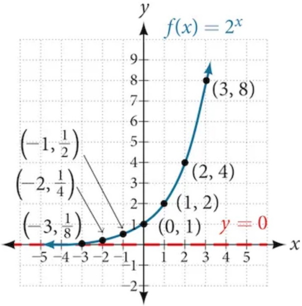


What about Negative Exponents

The general form of an exponential function is $f(x) = ab^x$, where a is any non-zero number and b is an positive number not equal to 1.

- If $b > 1$ the function grows at a rate proportional to its size.
- If $0 < b < 1$ the function decays at a rate proportional to its size.

For example, $f(x) = 2^x$:



x	-3	-2	-1	0	1	2	3
$f(x) = 2^x$	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$

Scientific (SI) Prefixes

The Metric System Prefixes

Prefix	Label	Decimal Value	Scientific	Colloquial
yocto	y	0.000 000 000 000 000 000 000 001	10^{-24}	septillionth
zepto	z	0.000 000 000 000 000 000 001	10^{-21}	sextillionth
atto	a	0.000 000 000 000 000 001	10^{-18}	quintillionth
femto	f	0.000 000 000 000 001	10^{-15}	quadrillionth
pico	p	0.000 000 000 001	10^{-12}	trillionth
nano	n	0.000 000 001	10^{-9}	billionth
micro	μ	0.000 001	10^{-6}	millionth
milli	m	0.001	10^{-3}	thousandth
centi	c	0.01	10^{-2}	hundredth
deci	d	0.1	10^{-1}	tenth
---	---	1	10^{0}	one
deka	da	10	10^{1}	ten
hecto	h	100	10^{2}	hundred
kilo	k	1 000	10^{3}	thousand
mega	M	1 000 000	10^{6}	million
giga	G	1 000 000 000	10^{9}	billion
tera	T	1 000 000 000 000	10^{12}	trillion
peta	P	1 000 000 000 000 000	10^{15}	quadrillion
exa	E	1 000 000 000 000 000 000	10^{18}	quintillion
zetta	Z	1 000 000 000 000 000 000 000	10^{21}	sextillion
yotta	Y	1 000 000 000 000 000 000 000 000	10^{24}	septillion

e - an interesting aside

The letter e represents the irrational number:

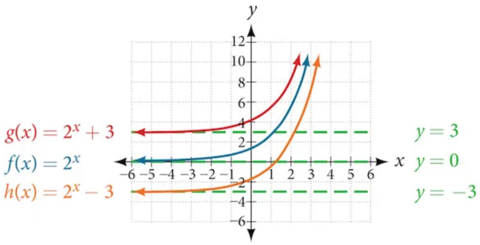
$$e = (1 + \frac{1}{n})^n \tag{4}$$

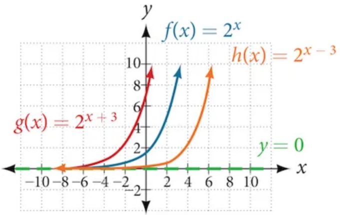
as n increases without bound.

The number e is used as a base for many real-world exponential models. To work with base e, we use the approximation, $e \approx 2.718282$. The constant was named by the Swiss mathematician Leonhard Euler (1707–1783) who first investigated and discovered many of its properties.

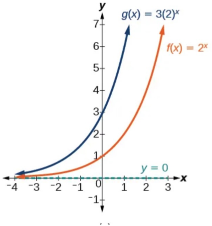
Graphing Exponentials

Shifts:

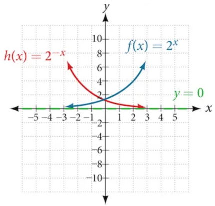




Stretch:



Flip:



Logarithmic Functions

Definition of a Logarithmic Function

If x and b are positive real numbers such that $b \neq 1$, then $y = \log_b x$ is called the **logarithmic function base *b***, where

$$y = \log_b x \text{ is equivalent to } b^y = x$$

Notes:

- Given $y = \log_b x$, the value y is the exponent to which b must be raised to obtain x.
- The value of y is called the **logarithm**, b is called the **base**, and x is called the **argument**.
- The equations $y = \log_b x$ and $b^y = x$ both define the same relationship between x and y. The expression $y = \log_b x$ is called the **logarithmic form**, and $b^y = x$ is called the **exponential form**.

Natural Log

Definition of Common and Natural Logarithmic Functions

- The logarithmic function base 10 is called the **common logarithmic function**. The common logarithmic function is denoted by $y = \log x$. Notice that the base 10 is not explicitly written; that is, $y = \log_{10} x$ is written simply as $y = \log\ x$.
- The logarithmic function base e is called the **natural logarithmic function**. The natural logarithmic function is denoted by $y = \ln x$; that is, $y = \log_e x$ is written as $y = \ln x$.

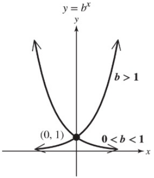
Logarithmic Functions

Logarithmic form: $y = log_b x$		Exponential form: $b^y = x$
$log_2 16 = 4$	$\Longleftrightarrow$	$2^4 = 16$
$log_1 0 \frac{1}{100} = -2$	$\Longleftrightarrow$	$10^{-2} = \frac{1}{100}$
$log_7 1 = 0$	$\Longleftrightarrow$	$7^0 = 1$

Graphing Exponential and Logarithmic Functions

Graphs of Exponential and Logarithmic Functions

Exponential Functions



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

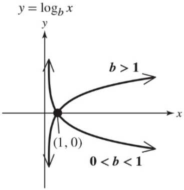
Horizontal asymptote: $y = 0$

Passes through $(0,\ 1)$

If $b > 1$, the function is increasing.

If $0 < b < 1$, the function is decreasing.

Logarithmic Functions



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

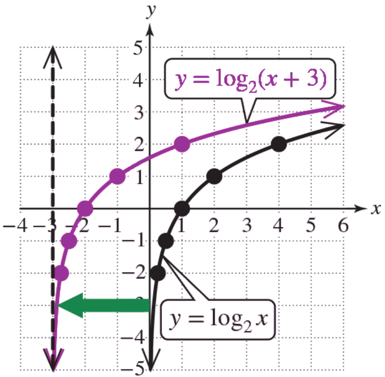
Vertical asymptote: $x = 0$

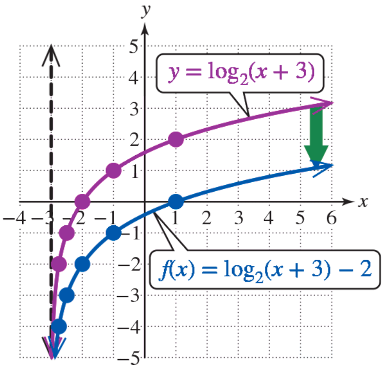
Passes through $(1, 0)$

If $b > 1$, the function is increasing.

If $0 < b < 1$, the function is decreasing.

Logarithmic Transforms





Logarithm Rules

- Product Property

$$log_b(xy) = log_b(x) + log_b(y) \tag{5}$$

- Quotient Property

$$log_b(\frac{x}{y}) = log_b(x) - log_b(y) \tag{6}$$

- Power Property

$$log_b x^p = p \cdot log_b x \tag{7}$$

Properties of Logarithms

Properties of Logarithms

Let b, x, and y be positive real numbers where $b \neq 1$, and let p be a real number. Then the following properties of logarithms are true.

1. $\log_b 1 = 0$

2. $\log_b b = 1$

3. $\log_b b^p = p$

4. $b^{\log_b x} = x$

5. $\log_b (xy) = \log_b x + \log_b y$ **Product property**

6. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$ **Quotient property**

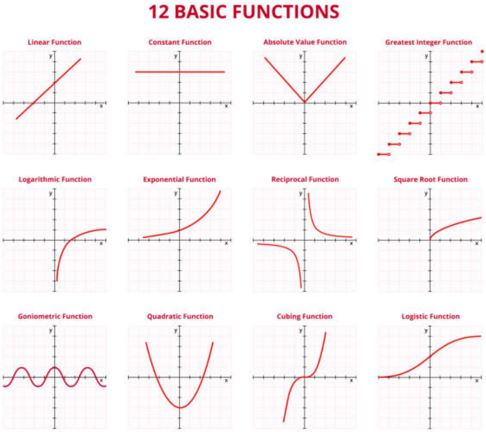
7. $\log_b x^p = p\log_b x$ **Power property**

Trigonometry

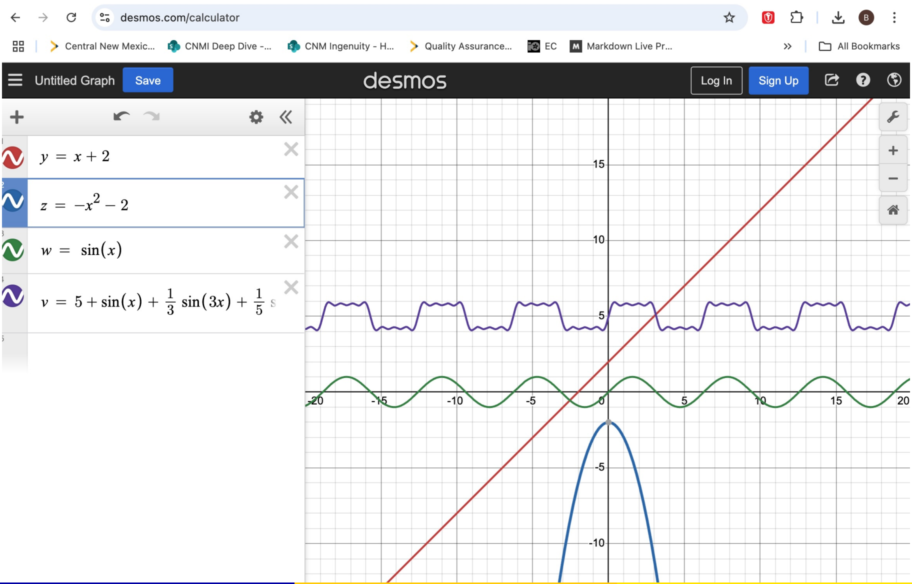
Algebraic Functions

An algebraic function provides a "y-value" for every "x-value"

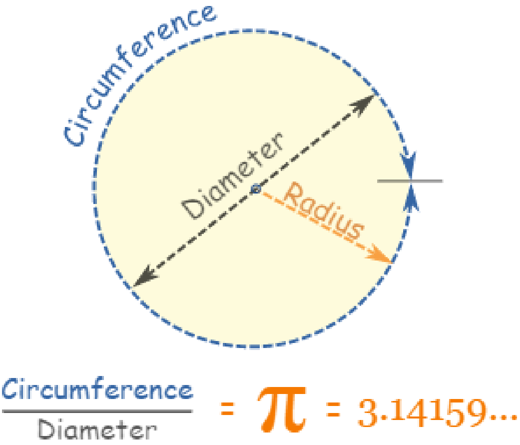
- Linear: $y = x + 2$
- Quadratic: $y = x^2$
- Periodic: $y = sin(x)$



More Desmos Fun

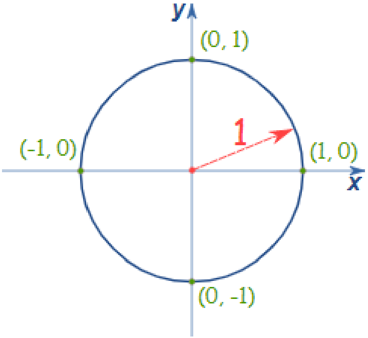


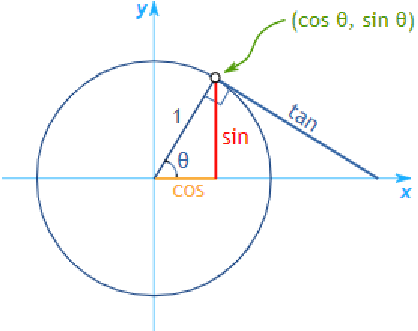
Pi (π)


Circumference
Diameter
Radius
Circumference / Diameter = π = 3.14159...

Unit Circle and Trigonometric Functions

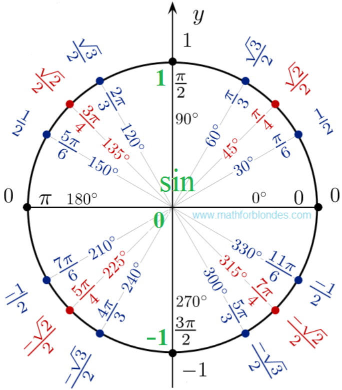
The Unit Circle is a circle with a radius of 1.

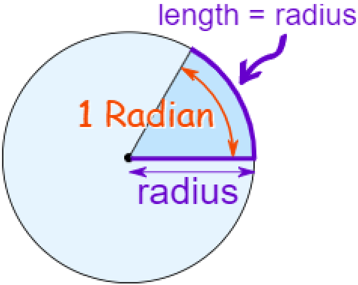




The Unit Circle can be used to map out the trigonometric values of sine, cosine, and tangent.

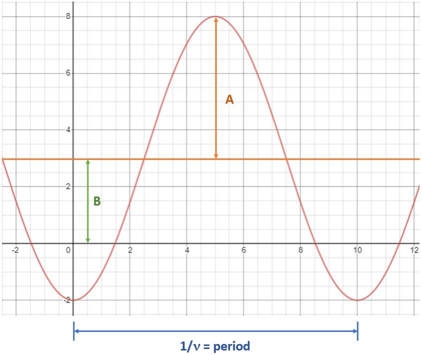
Unit Circle and the Value of $sin(\theta)$





- $sin(\theta)$ is the y-value of the point on the Unit Circle at angle θ.
- In our trig functions, θ is measured in radians (rad), not degrees.
- 360 degrees $= 2\pi$ radians.

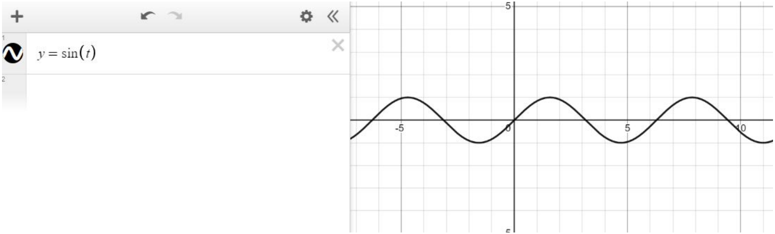
Sine Waves

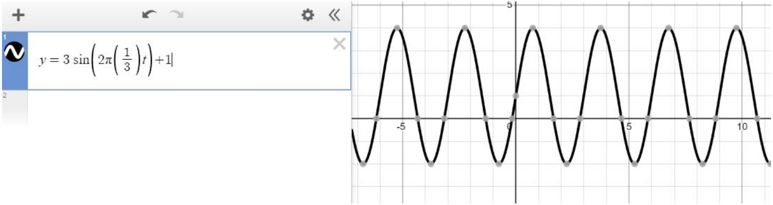


$$y = A * sin(2 * \pi * \nu * t) + B$$

where A = amplitude, B = offset, ν = frequency = $\frac{1}{period}$, and t = time in seconds.

Using Desmos (desmos.com/calculator)

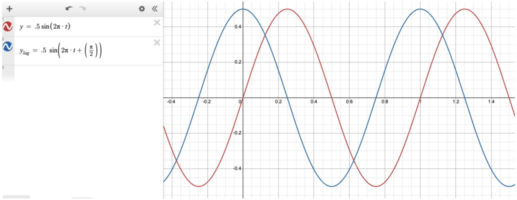




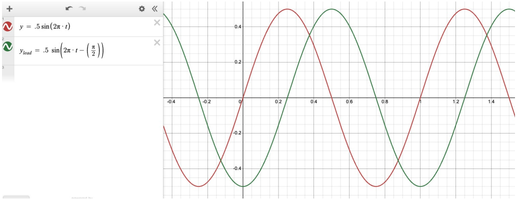
Phase Shift

The sine wave can be shifted relative to each other by adding in a phase shift (ϕ), which will shift the wave to the left or right.

Blue lags Red:

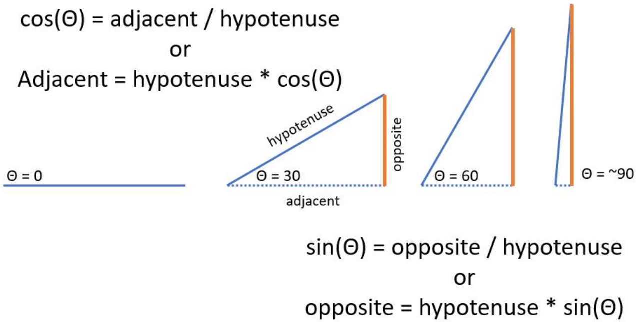


Green leads Red:



SOH CAH TOA

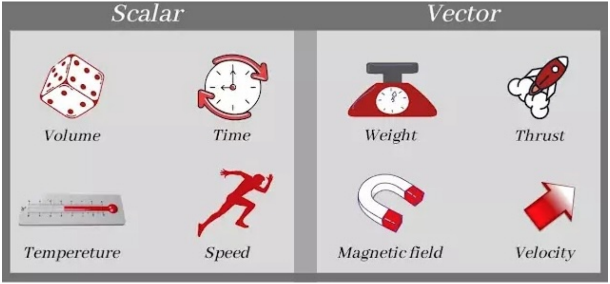
- sin = opposite over hypotenuse
- cos = adjacent over hypotenuse
- tan = opposite over adjacent



Vectors

Scalars and Vectors

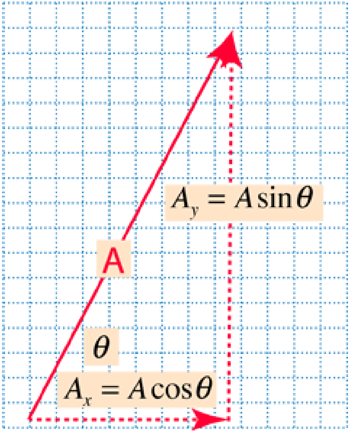
Scalars are quantities that are fully described by a magnitude (or numerical value) alone.
Vectors are quantities that are fully described by both a magnitude and a direction.



Vector Components

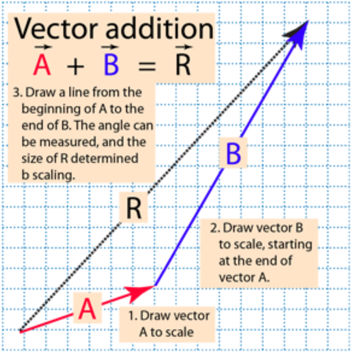
Finding the components of a vector involves forming a right triangle and using trigonometry's SOH-CAH-TOA

- $A_x = Acos(\theta)$
- $A_y = Asin(\theta)$



Graphical Vector Addition

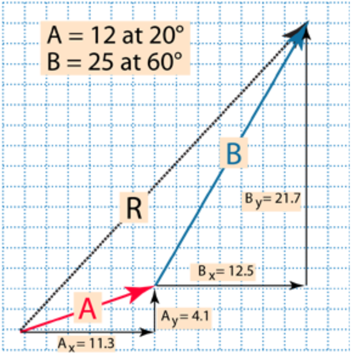
Adding two vectors A and B graphically can be visualized like two successive walks, with the vector sum being the vector distance from the beginning to the end point. Representing the vectors by arrows drawn to scale, the beginning of vector B is placed at the end of vector A. The vector sum R can be drawn as the vector from the beginning to the end point.



Vector Components

Finding the components of a vector involves forming a right triangle and using trigonometry's SOH-CAH-TOA

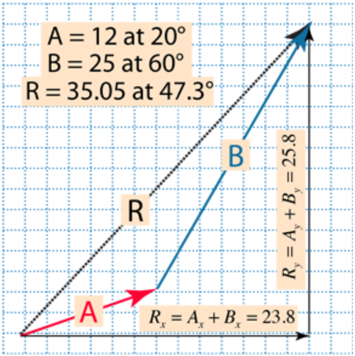
- Add the X components
  - $A_x = 12cos(20^\circ) = 11.3$
  - $B_x = 25cos(60^\circ) = 12.5$
  - $R_x = A_x + B_x = 23.8$
- Add the Y components
  - $A_y = 12sin(20^\circ) = 4.1$
  - $B_y = 25sin(60^\circ) = 21.7$
  - $R_y = A_y + B_y = 25.8$



Polar Form

After finding the components, the result can be placed in Polar Form:

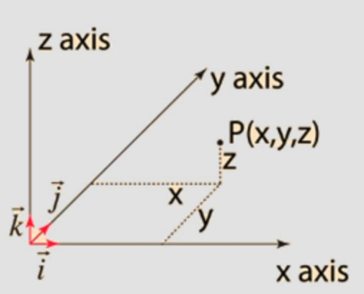
- $R_x = 23.8$
- $R_y = 25.8$
- $R = \sqrt{R_x^2 + R_y^2} = 35.05$
- $\theta_R = tan^{-1}(\frac{R-y}{R_x}) = 47.3^\circ$



Unit Vectors in 3 Dimensions

Vectors of unit length (i.e., length equals 1) can be used to specify the direction of vector quantities in various coordinate systems.
In Cartesian coordinates, it is typical to use i, j, and k to represent the unit vectors in the x, y, and z directions, respectively:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \qquad (8)$$



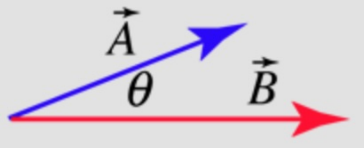
Dot (Scalar) Product

The dot (or inner or scalar) product of two vectors can be constructed by taking the component of the first vector in the direction of the second vector and by multiplying it by the second vector's magnitude.

$$\vec{A} \cdot \vec{B} = ABcos(\theta) \tag{9}$$

or

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_y \tag{10}$$

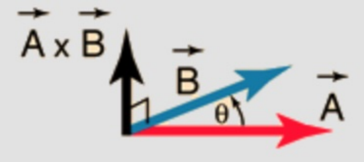


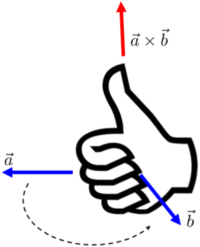
Cross (Vector) Product

The magnitude of the cross (or outer or vector) product of vectors can be constructed by taking the product of the magnitude of the vectors times the sine of the angle between them.

$$\vec{A} \times \vec{B}_{magnitude} = ABsin(\theta) \quad (11)$$

and the direction is given by the right hand rule.





In terms of unit vectors:

$$\vec{A} \times \vec{B} = \vec{i}(A_y B_z - A_z B_y) + \vec{j}(A_z B_x - A_x B_z) + \vec{k}(A_x B_y - A_y B_x) \quad (12)$$

Bonus - Cross Product - Determinant Form

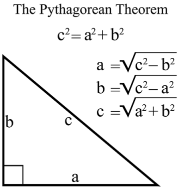
THe cross product can be compactly stated in teh form of a determinant:

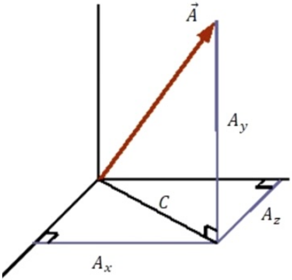
$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Which can be expanded to

$$\vec{A} \times \vec{B} = \vec{i}(A_y B_z - A_z B_y) + \vec{j}(A_z B_x - A_x B_z) + \vec{k}(A_x B_y - A_y B_x)$$

Pythagorean Theorem in 3 Dimensions



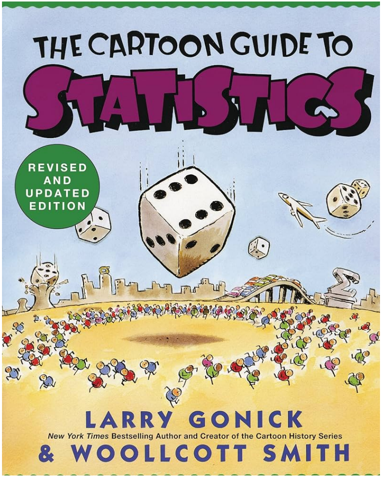


To add orthogonal (at right angles to each other) vectors in 3 Dimensions:

- $C = \sqrt{A_x^2 + A_y^2}$
- $A_{total} = \sqrt{C^2 + A_z^2}$
- $A_{total} = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Probability and Statistics

Probability and Statistics



- Data Analysis: The gathering, display, and summary of data
- Probability: The laws of chance (inside and outside of a casino)
- Statistical Inference: the science of drawing statistical conclusions from specific data using the laws of probability

A Tale as Old as Time

Gambling is as old as mankind, so it seems that probability should be almost as old. But, the realization that one could predict an outcome to a certain degree of accuracy was unconceivable until the 16^{th} century. In order to make a profit, underwriters were in need of dependable guidelines by which a profit could be expected, while the gambler was interested in predicting the possibility of gain.

Rich Guys Gambling

Known as the “Father of Probability”, Gerolamo Cardano was an Italian mathematician, physician, and gambler who first talked about probability. Cardano’s fascination with games of chance led him to write the first book dedicated to probability, “Liber de Ludo Aleae” (Book on Games of Chance), published in 1564. Cardano introduced concepts like odds and probabilities in this work, providing a framework for analyzing the likelihood of different outcomes in dice rolls and other games.

Probability

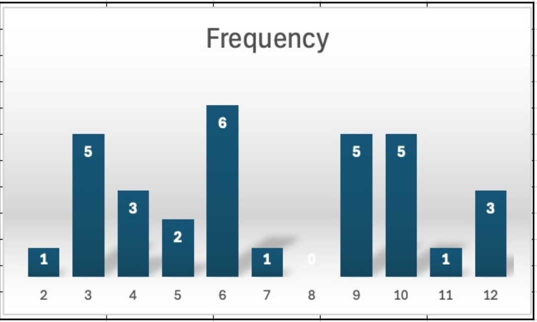
The probability of an event expresses the likelihood of the event outcome

$$P(event) = \frac{\#\text{ of favorable outcomes}}{\#\text{ of all possibly outcomes}}$$

Frequency Tables and Histograms

Roll two dice and add them together, repeat.

Dice Roll	Frequency
2	1
3	5
4	3
5	2
6	6
7	1
8	0
9	5
10	5
11	1
12	3



Assignment: Tenzi

Role two dice 12 times, after each role:

- Record the sum
- Record the running average (total up to this point divided by number of roles)

After the twelfth role:

- Create a histogram of the sums
- Create a line chart of the running averages

Summary Statistics

- Mean (or average): add the totals and divide by number of samples

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

or

$$\bar{x} = \frac{\sum_{i=1}^{n}(x_i)}{n}$$

- Median: middle sample in the ordered distribution
  - Odd: median is the value of middle sample
  - Even: median is the average of the two middle samples
- Mode: the value the appears most often

Dice Roll	Frequency	Total
2	1	2
3	5	15
4	3	12
5	2	10
6	6	36
7	1	7
8	0	0
9	5	45
10	5	50
11	1	11
12	3	36
SUM	**32**	**224**

$$\text{average} = \bar{x} = 7$$
$$\text{median} = 6$$
$$\text{mode} = 6$$

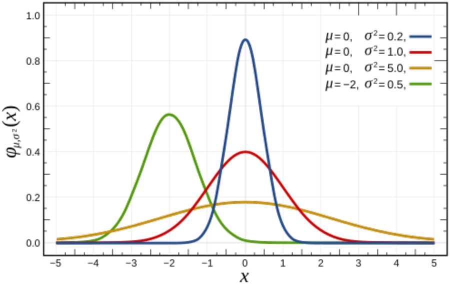
Summary Statistics: Variation

1-8								9-16								17-24								25-32							
2	3	3	3	3	3	4	4	4	5	5	6	6	6	6	6	6	7	9	9	9	9	9	10	10	10	10	10	11	12	12	12

- Interquartile Range (IRQ): Spread of the middle 50% of the data
  - Find the median
  - Find the first quartile (Q1): median of the lower half.
  - Find the third quartile (Q3): median of the upper half.
  - IRQ = Q3 - Q1 = 6
- Standard Deviations (σ):

$$\sigma = \sqrt{\frac{\sum_{i=0}^{n}(x_i - \bar{x})^2}{n}} = 3.06$$

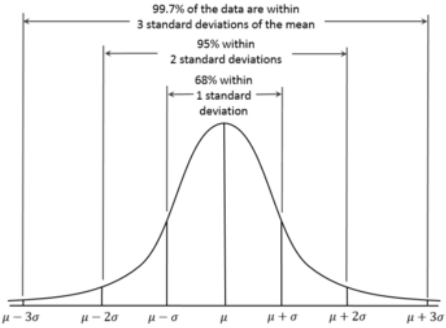
Gaussian Distribution



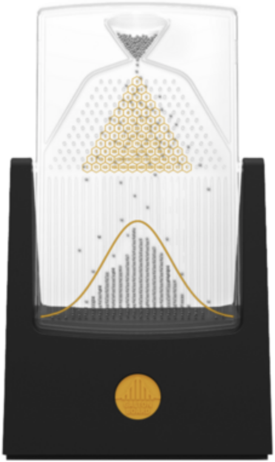
Probability Distribution Function $f(x)$:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$

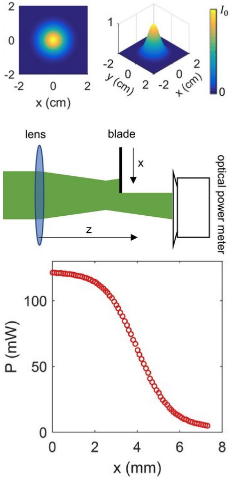
Gaussian Distribution: 3σ



Galton Baord



Assignment: Measuring Bean Profile

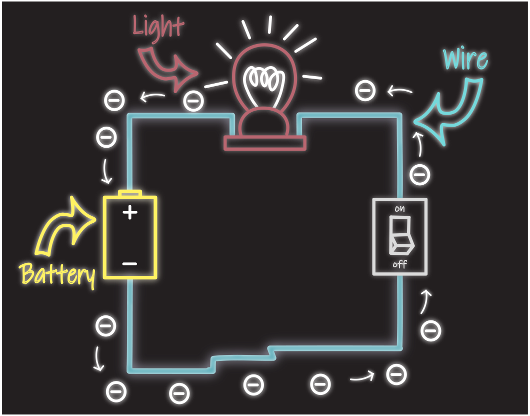


The laser beam has a Gaussian profile.

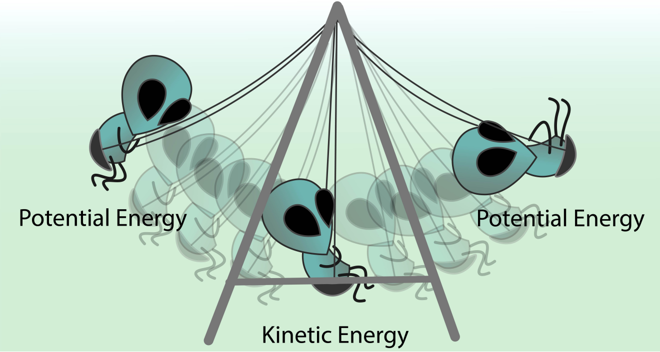
1. Direct a laser beam at a power meter, setting the correct wavelength
2. Moving the knife edge across the beam path, record the drop in power vs distance.
3. Setup a telescope to expand the beam size by 2-5 times.
4. Repeat the knife edge process.
5. Plot the results of both sets of measurements on paper and in excel.

Some Physics

Introduction to Electrical Circuits



Energy



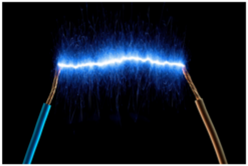
- Kinetic Energy - energy of motion
- Potential Energy - energy stored in an object

Electrical Circuit Terms

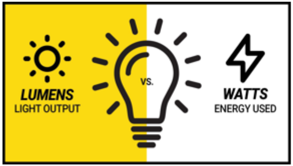
Voltage is **electric potential energy per unit charge** (V = J/C)

The Sub-atomic Particles

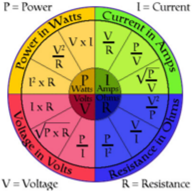
Relative size	Name	Mass (Kg)	Charge (C)
	Proton	1.67×10^{-27}	$+1.602 \times 10^{-19}$
	Neutron	1.67×10^{-27}	0
	Electron	9.11×10^{-31}	-1.602×10^{-19}



Electric current is the **rate of charge flow** (A = C/s)



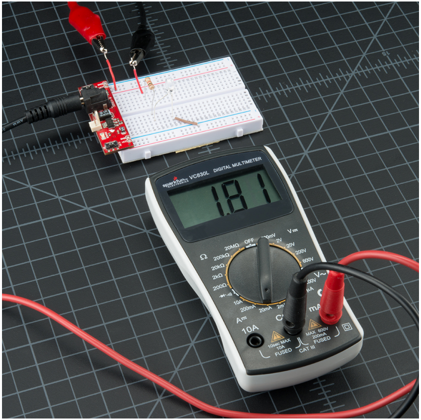
Power is the rate of doing work or **the rate of using energy**. (W = J/S)



Power = Voltage x Current

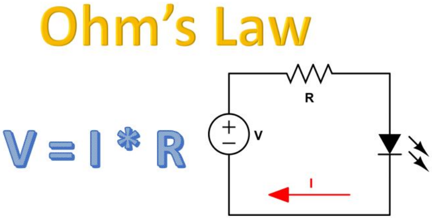
Energy is the **amount of power** produced or consumed **over a given time**. J = W x s

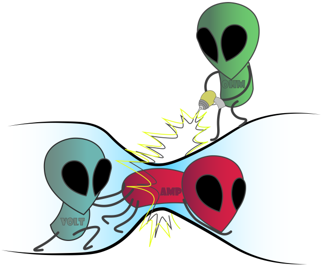
Measuring Voltage, Current, and Resistance



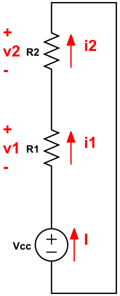
Ohm's Law

Georg Ohm (16 March 1789 – 6 July 1854) was a German physicist and mathematician. As a school teacher, Ohm began his research with the new electrochemical cell, invented by Italian scientist Alessandro Volta. Ohm found that there is a direct proportionality between the potential difference (voltage) applied across a conductor and the resultant electric current. This relationship is known as Ohm's law:





A word about circuit notation



Subscripts will be used to denote quanties (voltage, currect, etc) for different elements:

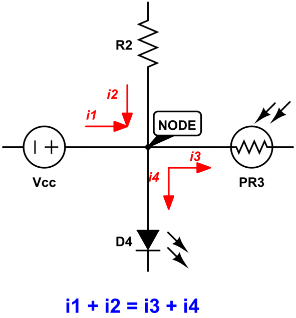
- i1 or i_1 is the current through Resistor 1 (R_1)
- i2 or i_2 is the current through Resistor 2 (R_2)
- v2 or v_2 is the votage across Resistor 2 (R_2)
- I is the current delivered by the power supply
- V_{cc} (common collector [a] voltage) is the notation will will use for $3.3V$ from the Particle

[a] Common Collector is a term for certain parts of transistor circuits. We will learn about Transistors in Lesson 11

Kirchhoff's First Law

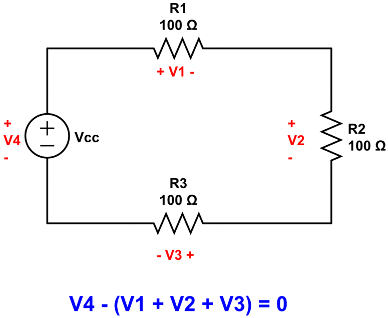
Gustav Robert Kirchhoff (12 March 1824 – 17 October 1887) was a German physicist who contributed to the fundamental understanding of electrical circuits. His first law:

> In an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node



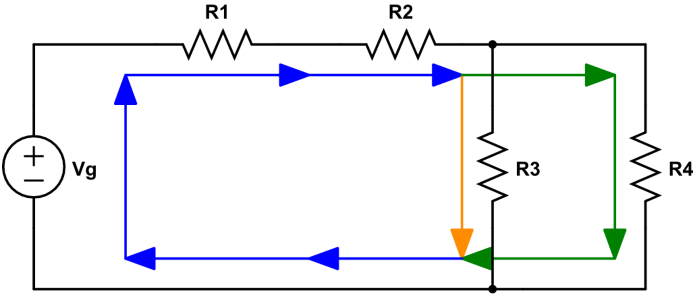
Kirchhoff's Second Law

The directed sum of the potential differences (voltages) around any closed loop is zero.



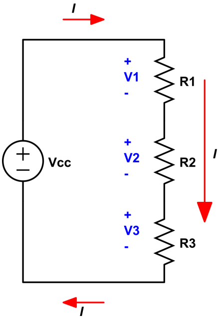
Kirchhoff's Second Law

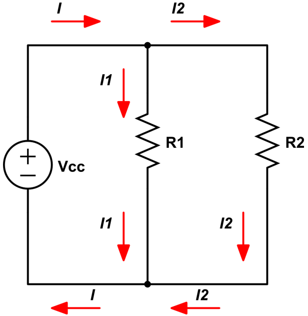
Resistors in Series and Parallel



How many nodes? How many loops?

Resistors in Series and Parallel

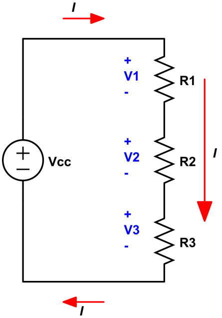




$$R_{eq} = R_1 + R_2 + R_3$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Resistors in Series



Node Law: $I = I_1 = I_2 = I_3$

Loop Law:
$V_{cc} - (V_1 + V_2 + V_3) = 0$

Rearranging the Loop Law:

$$V_{cc} = V_1 + V_2 + V_3 \quad (13)$$

Using Ohm's Law:

$$V_{cc} = IR_1 + IR_2 + IR_3 \quad (14)$$

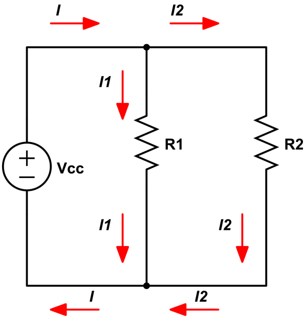
Using the Distributive Property:

$$V_{cc} = I(R_1 + R_2 + R_3) \quad (15)$$

Gives the Equivalent Resistance:

$$\boxed{R_{eq} = R_1 + R_2 + R_3} \quad (16)$$

Resistors in Parallel



Node Law: $I = I_1 + I_2$

Loop Law: $V_{cc} = V_1 = V_2$

$$I = I_1 + I_2 \tag{17}$$

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2} \tag{18}$$

$$I = \frac{V_{cc}}{R_1} + \frac{V_{cc}}{R_2} \tag{19}$$

$$\frac{V_{cc}}{R_{eq}} = V_{cc}(\frac{1}{R_1} + \frac{1}{R_2}) \tag{20}$$

$$\boxed{\frac{1}{R_{eq}} = (\frac{1}{R_1} + \frac{1}{R_2})} \tag{21}$$