6. Transistor current gain (β or h_{fe})

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Objective

Measure the current gain, $h_{fe} = \beta$ characteristic of an NPN transistor.

Background

We saw how the voltage and current of a 2-terminal electronic device are not independent. Their relationships are defining characteristics of the device. Resistors have linear voltage-current relationships, LEDs are closer to "hockysticks", for V < V_f $i \sim 0$, and above V_f the current increases somewhat exponentially. Bipolar junction transistors (BJT) have 3 terminals that typically form 2 circuits with a shared emitter. The 3 common regimes of a BJT are shown in Fig. 1. In this lab, we'll use the Arduino to measure base and collector currents and compute their ratio. In the "active regime", this ratio is a constant called Hfe = $\beta = I_C/I_B$.

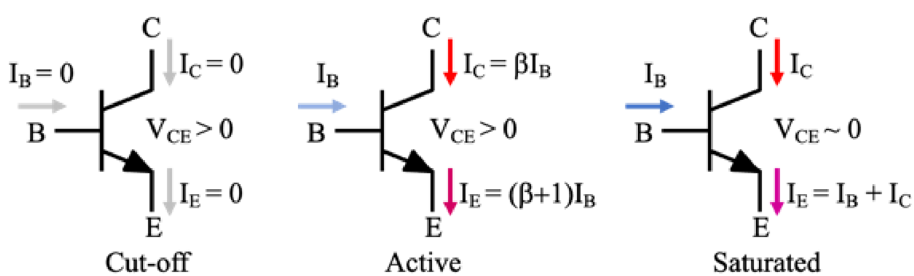
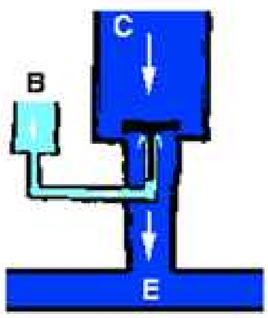


Figure 1. Three common states of a NPN BJT.

The value of β is a characteristic of the transistor, and relatively independent of the circuit. Many factors affect β, including intrinsic geometry, doping and structural composition, and temperature. Unlike resistors, who's values are specified within a narrow range, e.g., R is within 1% or 5% of its rated value, the range of β is often wide, e.g., 40-400. When using transistors, robust designs don't rely on a particular value but incorporate negative feedback or other mechanisms to make the circuit's behaviors constant for any β greater than a minimum value. Some DMMs can measure β directly, more reliable measurements are from dedicated testors like https://peak.retronics-uk.com/product/dca55/, and back in the early days of electronics we used "curve tracer" machines, e.g., https://en.wikipedia.org/wiki/Semiconductor_curve_tracer. Today we'll measure β with our LM317 power supply, Arduino, and MATLAB.

This document doesn't attempt to explain transistor theory, you'll have to look elsewhere for that. E.g., I found this explanation websurfing and it "clicked" or resonated with me. But maybe that's because I had previously read a dozen other explanations that didn't make as much sense, but left me more receptive to this one ...



Thousands of textbooks have been written to explain electronics and I haven't found a single one that can explain the operation of a transistor. They all make it seem so complicated!

Let's see if I can do better. Here is a picture of a transistor. My transistor runs on water current. You see there are three openings which I have labelled "B" (Base), "C" (Collector) and "E" (Emitter) for convenience. By an amazing coincidence, these also happen to be the names used by everyone else for the three connections of a transistor!

We provide a reservoir of water for "C" (the "power supply voltage") but it can't move because there's a big black plunger thing in the way which is blocking the outlet to "E". The reservoir of water is called the "supply voltage". If we increase the amount of water sufficiently, it will burst our transistor just the same as if we increase the voltage to a real transistor. We don't want to do this, so we keep that "supply voltage" at a safe level.

If we pour water current into "B" this current flows along the "Base" pipe and pushes that black plunger thing upwards, allowing quite a lot of water to flow from "C" to "E". Some of the water from "B" also joins it and flows away. If we pour even more water into "B", the black plunger thing moves up further and a great torrent of water current flows from "C" to "E".

I like the analogy for a BJT, although of course the "black plunger thing" can more accurately be described with (more complicated) motionless solid state physics. Key phenomenology is that a little base-to-emitter current controls a much larger collector-emitter current and β characterizes that current gain. We'll make a test circuit with two resistors Rb in series with the base and Rc in series with the collector, which serve to both limit and transduce the base and collector currents into easily measurable voltages ($i = \Delta V/R$) according to Ohm's Law.

Methods

Hardware Interface

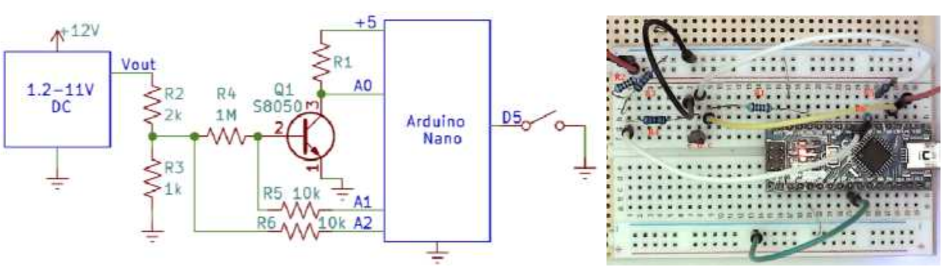


Figure 2. Schematic and breadboard of the β measuring circuit.

1. R2 and R3 are voltage dividers reducing the LM317's output by 1/3. Dividing LM317's voltage by 3 ensures Arduino isn't damaged measuring V>5V, but also allows measuring transistor behavior as it becomes active ~ 0.5V, below the LM317's minimum output ~ 1.25V.
2. R5 and R6 provide current limiting protection to the Arduino in case the voltage divider fails, a wire shorts, etc. Analog inputs are high impedance and draw little current (normally) so R5-6 have (normally) no effect on measurements.
3. R4 = Rb turns the varying LM317 voltage (divided by ≈ 3) into a varying current into the base of Q1. Base current from Ohm's Law is ib = (A2-A1) / R4. The high value of $R4 = 1M\Omega$ lets us explore the transistor's behavior for small i_B, before it saturates.
4. R1 = Rc both limits and enables measurement of the resulting collector current as a voltage, ic = (vref-A0) / R1. Reducing R1 < ~500Ω risks burning out the Arduino, which provides current through R1 up to ~5V/R1 when the transistor is saturated. I have destroyed Arduinos from sinking >10mA for long durations. Repeat measurements with a few values of R1, e.g., 1k, 4.7k, 10k, 20k.
5. Transistor Q1 is an S8050 [1]. You also may have S8550 (PNP) transistors in your kit – they won't work in this circuit. Carefully read the part number and look up a datasheet for it's pinout. (A trick that usually works is set your DVM to Diode Test mode and measure all permutations of the pins. You'll get two similar diode V_f readings ~ 0.6V. The common terminal is the base, if it's the postive DMM lead then the transistor is NPN (the base is the 'P'). Typically $V_{BC} < V_{BE}$ although the difference maybe small.
6. Measure resistor values with your DMM ohmmeter before connecting them in the circuit. Use clip leads to connect the resistors to your DMM. Try holding the 1MΩ against the DMM leads with your two hands, and you'll probably measure a value considerably less than 1M, because your body's finite resistance is in parallel with the 1M DUT. Shorting the clip leads measures the loop resistance of the leads, which if not zero can be subtracted from other measurements for a more precise result.
7. Measure `vref`, Arduino's reference voltage on pin 3, using jumper wires and clip leads to avoid shorting to adjacent pins.

Software Interface

Same Arduino code as prior experiments, except we need to measure a 3rd voltage (generally we'd need to measure 2 voltages on either side of each resistor, but one side of Rc is at Vref which we'll measure once with the DVM, and = 1023ADU always). When D5 is grounded, data is printed to the Serial Monitor ~5 times per second. Ground D5, then slowly vary the LM317 voltage, unground D5, and copy the Serial Monitor data into MATLAB.

```
void setup()
{
  Serial.begin(9600);
  pinMode(5, INPUT_PULLUP);
  analogRead(A0); // initialize vref
}

void loop()
{
  if (!digitalRead(5)) // ground D5 to run
  {
    Serial.print(analogRead(A0));
    Serial.print('\t');
    Serial.print(analogRead(A1));
    Serial.print('\t');
    Serial.println(analogRead(A2));
  }
  delay(200); // milliseconds
```

Figure 3. Arduino source code measures and prints 3 voltages approximately 5 times per second. Note the last print statement calls a different print function that terminates the line).

Data Interface

Copy data from the Serial Monitor and paste into MATLAB variable `data`. A first look at the raw data is often useful. E.g.,

```
vref = 4.93; % volts on aref pin
adu2volts = vref / 1023;
R2 = 1990; % ohms
R3 = 999; % ohms

Rb = 1.014e6; % ohms, measured with dmm
Rc = 1003; % R1, ohms, measured with dmm -- try a few other (larger) values
data = [ ... % A0, A1, A2
% paste your Serial Monitor Data below, trim duplicate rows at start & end
1023 119 96
1023 121 103
1022 121 108
1021 122 116
1021 122 120
1020 122 126
1019 122 132
1017 123 140
1016 122 148
1015 123 152
1013 123 163
1010 124 175
1008 124 184
1006 124 192
1004 125 202
1002 125 212
998 126 226
995 127 240
992 128 253
989 128 265
989 128 270
987 128 277
984 128 288
981 128 301
977 129 317
975 129 327
973 129 340
970 129 352
967 129 363
964 130 377
960 129 392
955 130 413
950 131 435
944 131 461
936 131 493
928 132 531
916 132 578
904 132 627
897 132 662
```

```
884 133 716
873 133 759
862 133 808
858 134 824
] * adu2volts;
plot(data,'d-');
xlabel('sample #'); ylabel('Volts');
legend('A0','A1','A2','location','best'); grid;
```

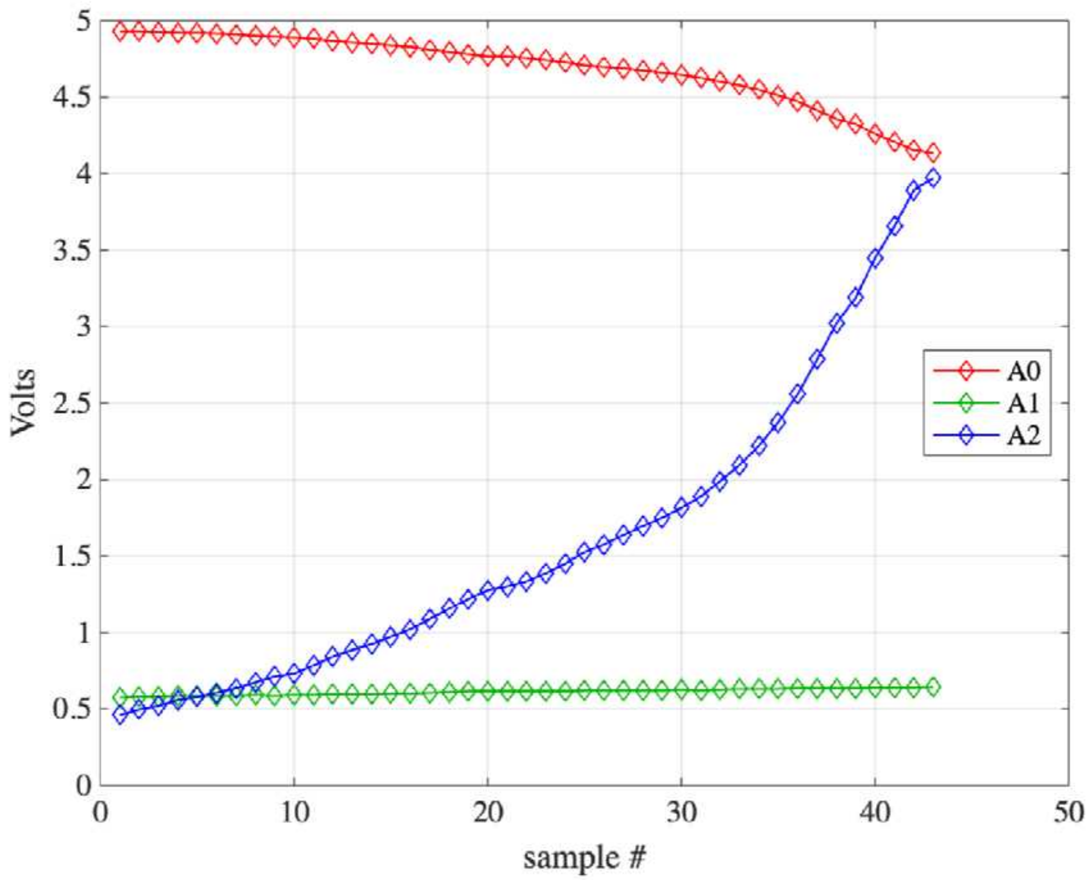


Figure 4. Raw data from the Arduino.

Not a lot to quantify here because sample # is arbitrary. But if the curves aren't moving consistently or reveal random noise, then troubleshoot and repeat. Vbe = A1 (green) hardly moves, acting like a forward biased diode with $V_f \sim 0.6$V. Even though Vbe is nearly constant, ib is increasing, indicated by increasing A2 (blue). If the transistor is active, then the collector current should increase – causing more voltage drop across Rc, V = icRc which causes Vce to drop according to KVL: Vref = Vce + icRc – so if the 2nd term increases, the first one must proportionally decrease, and the red curve above does that. So far so good. To measure characteristics of the transistor (and not how smoothly one turns the LM317's pot), compute the key currents – let's rename the columns first so the equations make more sense. I also changed the current units from amps to more convenient mA or μA. Draw the schematic in Fig. 2 and refer to it as you read the next block of code.

```
vce = data(:,1); % collector-emitter voltage
vbe = data(:,2); % base-emitter voltage
vin = data(:,3);
ib = (vin-vbe)/Rb * 1e6; % base current in microAmps (uA)
ic = (vref-vce)/Rc * 1e3; % collector current in mA
plot(ib, ic,'d-')
grid; xlabel('base current/\muA');
ylabel('collector current/mA')
title(sprintf('R_C = %.0f\\Omega', Rc)); % why \\?
```

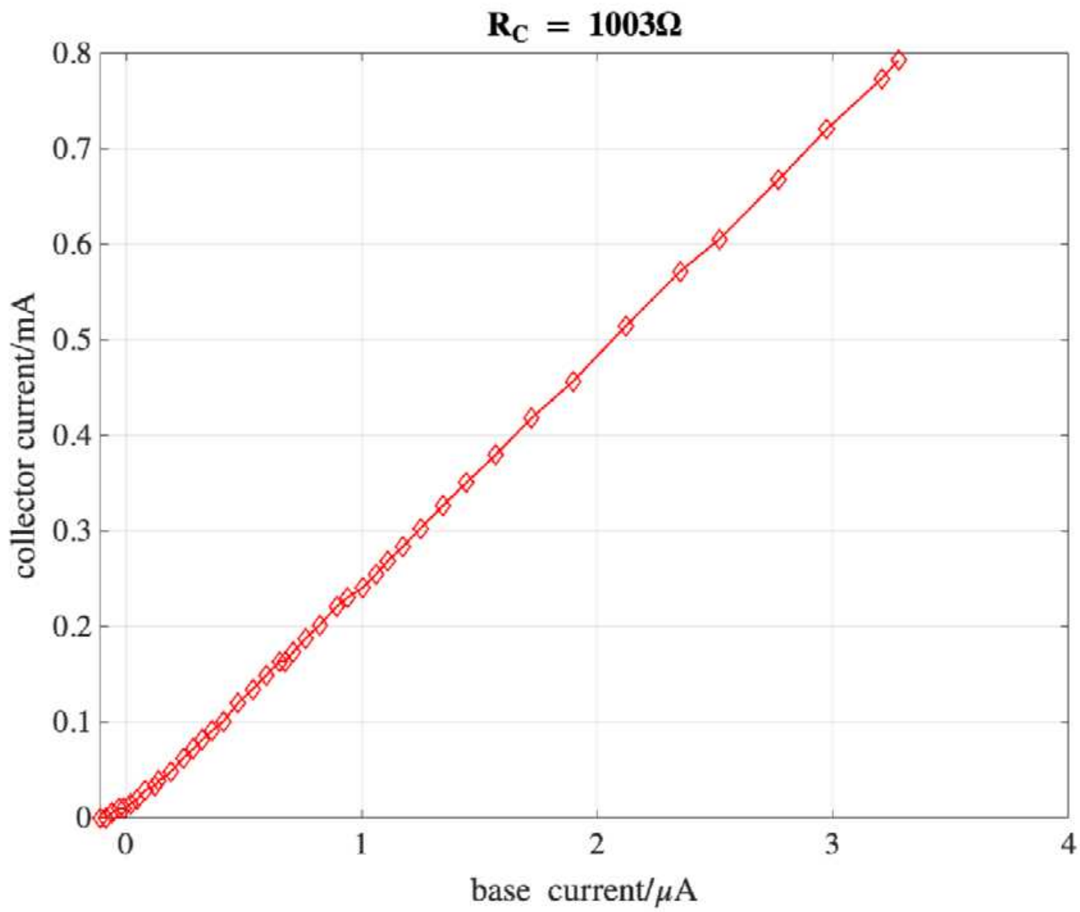


Figure 5. Collector vs. base currents.

Do a visual estimate of the slope before computing it with `polyfit.` $\Delta i_c/\Delta i_b = .55mA/2.8\mu A = 196$

We can tell MATLAB to exclude the lowest few points (where the transistor maybe not fully "active"), and perhaps the largest data where the transistor might be saturated, by creating a Boolean index variable that's true when e.g., ib > .1uA AND ic < 95% of its maximum value. Also since the slope has units of mA/uA, it has to be multiplied by 1000 uA/1mA = 1.

```
indx = ib > .1 & ic < .95 * max(ic);
p = polyfit(ib(indx), ic(indx), 1);
```

```
fprintf('beta = %.1f\n', p(1)*1e3)
```

```
beta = 240.0
```

If it's not consistent with your visual estimate, troubleshoot why not. Rename `data` and `beta` and repeat with another value of R1 = Rc.

Finally combine the multiple data sets into one graph.

Results

```
ib1k = ib; ic1k = ic; beta(1) = p(1)*1e3;
Rc = 4700; % ohms measured with dvm
data = [...
1023 124 88
1023 124 89
1020 126 104
1005 126 125
993 128 138
971 128 161
950 128 180
935 129 196
921 128 209
907 128 221
897 128 232
895 128 233
886 128 241
861 128 265
821 128 302
790 129 329
762 129 356
742 129 374
709 129 404
677 129 434
645 129 463
597 129 508
548 129 554
498 129 600
449 129 645
417 130 675
391 129 695
363 130 721
320 130 761
281 130 799
] * vref / 1023; % now in volts
vce = data(:,1); % collector-emitter voltage
vbe = data(:,2); % base-emitter voltage
vin = data(:,3);
ib = (vin-vbe)/Rb * 1e6; % base current in uA
ic = (vref-vce)/Rc * 1000; % collector current in mA
plot(ib, ic,'d-')
grid; xlabel('base current/\muA');
ylabel('collector current/mA')
```

```
title(sprintf('Rc = %.0f\\Omega', Rc)); % why \\?
```

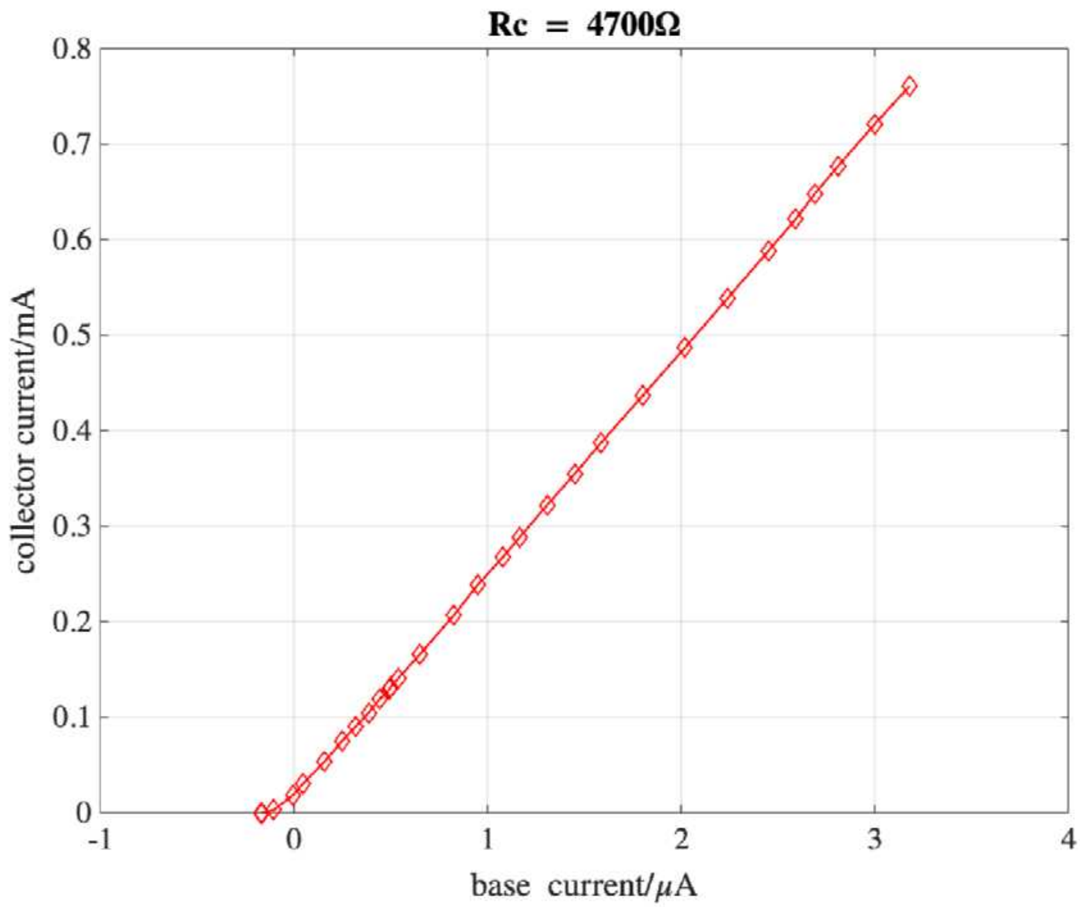


Figure 6. Switching to $Rc = 4.7k\Omega$ appears to change nothing.

Did I mess up? No, the data points have moved around, but the min and max appear the same, the beta is:

```
indx = ib > .1 & ic < .95 * max(ic);
p = polyfit(ib(indx), ic(indx), 1); % help polyfit for more info
fprintf('beta = %g\n', p(1)*1e3)
```

```
beta = 234.815
```

Very consistent. This suggests that the collector resistor doesn't affect this curve – the slope is indeed a *characteristic of the transistor*. What if we increase Rc ... such that say at ic ~0.5mA (around 1/2 the range above), ic*Rc ~ vref –> Rc = Vref / 0.5mA ~ 10kohm.

```
ib5k = ib; ic5k = ic; beta(2) = p(1)*1e3; % save the former results
vref = 4.76; % volts on aref pin -- the following measurements are on another day.

Rc = 9860; % new Rc, ohms measured with DMM
data = [...  % A0 A1 A2
1022 123 90
```

```
1000 124 112
993 124 117
966 124 131
949 124 138
932 124 147
911 124 157
889 124 167
871 124 176
843 124 189
810 124 203
763 124 223
716 124 243
661 124 268
607 124 291
528 124 325
443 124 361
357 124 397
299 124 422
182 124 474
145 124 490
68 124 524
40 124 547
30 124 583
27 124 614
25 124 639
23 124 671
21 124 708
20 124 734
19 124 748
18 125 764
18 124 793
18 124 797
] * vref/1023; % volts
vce = data(:,1); % collector-emitter voltage
vbe = data(:,2); % base-emitter voltage
vin = data(:,3);
ib = (vin-vbe)/Rb * 1e6; % base current in uA
ic = (vref-vce)/Rc * 1000; % collector current in mA
plot(ib, ic,'d-')
grid; xlabel('base current/\muA');
ylabel('collector current/mA')
title(sprintf('Rc = %.1fk\\Omega', Rc/1000)); % why \\?
```

Figure 7. Collector vs. base currents with $Rc = 10k\Omega$, which causes the BJT to saturate around $i_C = .43mA$.

```
indx = ib > .1 & ic < .95 * max(ic);
p = polyfit(ib(indx), ic(indx), 1); % help polyfit for more info
fprintf('beta = %.1f\n', p(1)*1e3)
```

```
beta = 237.5
```

The plateau value is saturation. The collector-emitter resistance approaches zero and ic ~ Vref / Rc =

```
vref / Rc * 1000 % mA
```

```
ans = 0.4828
```

Compare to the graph:

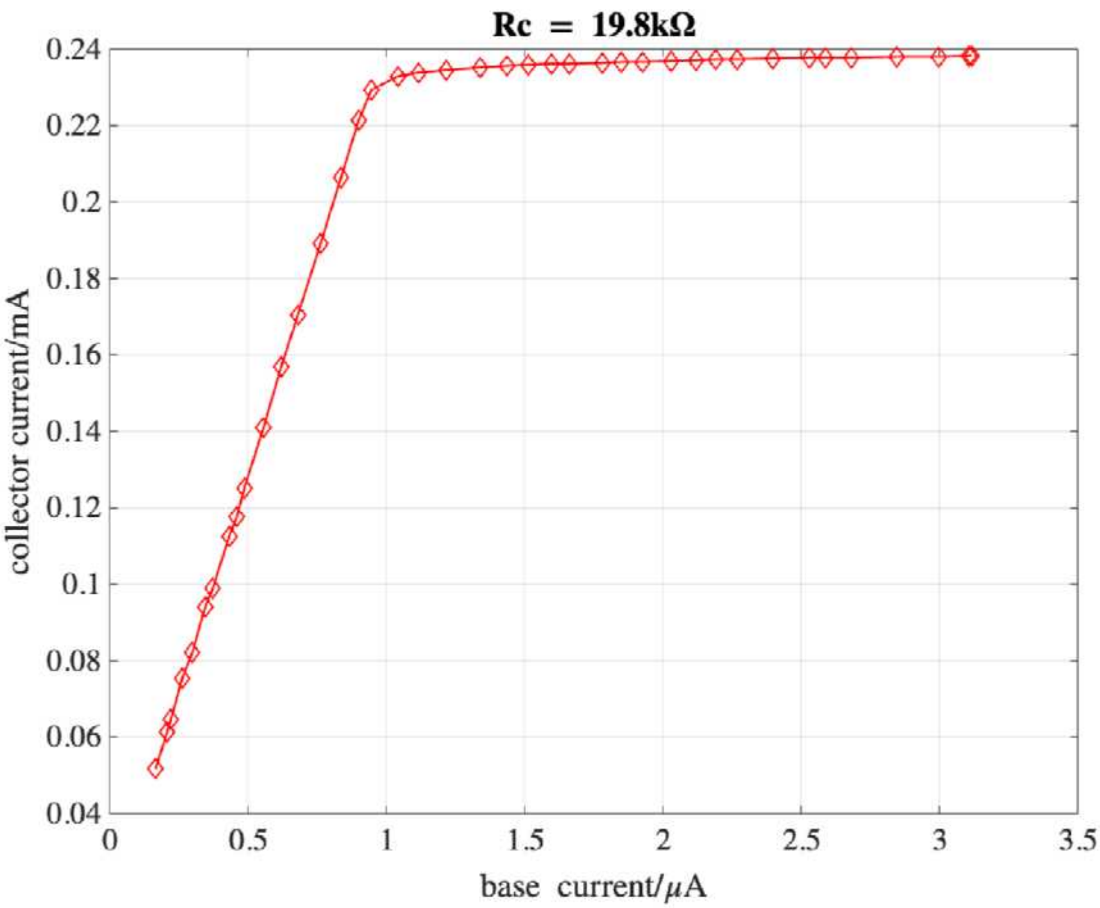
```
max(ic)
```

```
ans = 0.4743
```

One more resistor value:

```
ib10k = ib; ic10k = ic; beta(3) = p(1)*1e3; % save the former results
Rc = Rc+9940; % the old 10k + another 10k in series
data = [ ...
803 120 156
762 120 165
748 120 168
702 120 177
673 120 185
623 120 195
602 120 201
544 119 213
522 119 219
490 120 226
423 119 240
355 119 254
298 119 267
218 119 285
145 119 301
81 119 315
47 119 325
32 119 346
28 119 362
25 119 384
22 119 411
20 119 432
19 119 449
18 119 467
18 119 481
17 118 506
16 118 521
16 118 538
15 118 560
14 119 581
13 119 596
13 119 613
12 118 640
11 119 670
11 119 683
11 119 703
10 119 739
```

```
10 119 772
9 119 796
9 119 798
9 120 799
] * vref/1023; % volts
vce = data(:,1); % collector-emitter voltage
vbe = data(:,2); % base-emitter voltage
vin = data(:,3);
ib = (vin-vbe)/Rb * 1e6; % base current in uA
ic = (vref-vce)/Rc * 1000; % collector current in mA
plot(ib, ic,'d-')
grid; xlabel('base current/\muA');
ylabel('collector current/mA')
title(sprintf('Rc = %.1fk\\Omega', Rc/1000)); % why \\?
```



The assymptote appears consistent with:

```
vref / Rc * 1000 % mA
```

```
ans = 0.2404
```

```
max(ic)
```

```
ans = 0.2383
```

```
indx = ib > .1 & ic < .95 * max(ic);
p = polyfit(ib(indx), ic(indx), 1); % help polyfit for more info
fprintf('beta = %.1f\n', p(1)*1e3)
```

```
beta = 230.3
```

Let's superimpose these plots:

```
ib20k = ib; ic20k = ic; beta(4) = p(1)*1e3; % for consistency
plot(ib1k,ic1k,'r+-',ib5k,ic5k,'gx--',ib10k,ic10k,'bd-.',ib20k,ic20k,'co:');
xlabel('base current/\muA'); ylabel('collector current/mA')
legend('1k\Omega','5k','10k','20k','location','best');
title('varying R_C'); axis tight; grid;
```

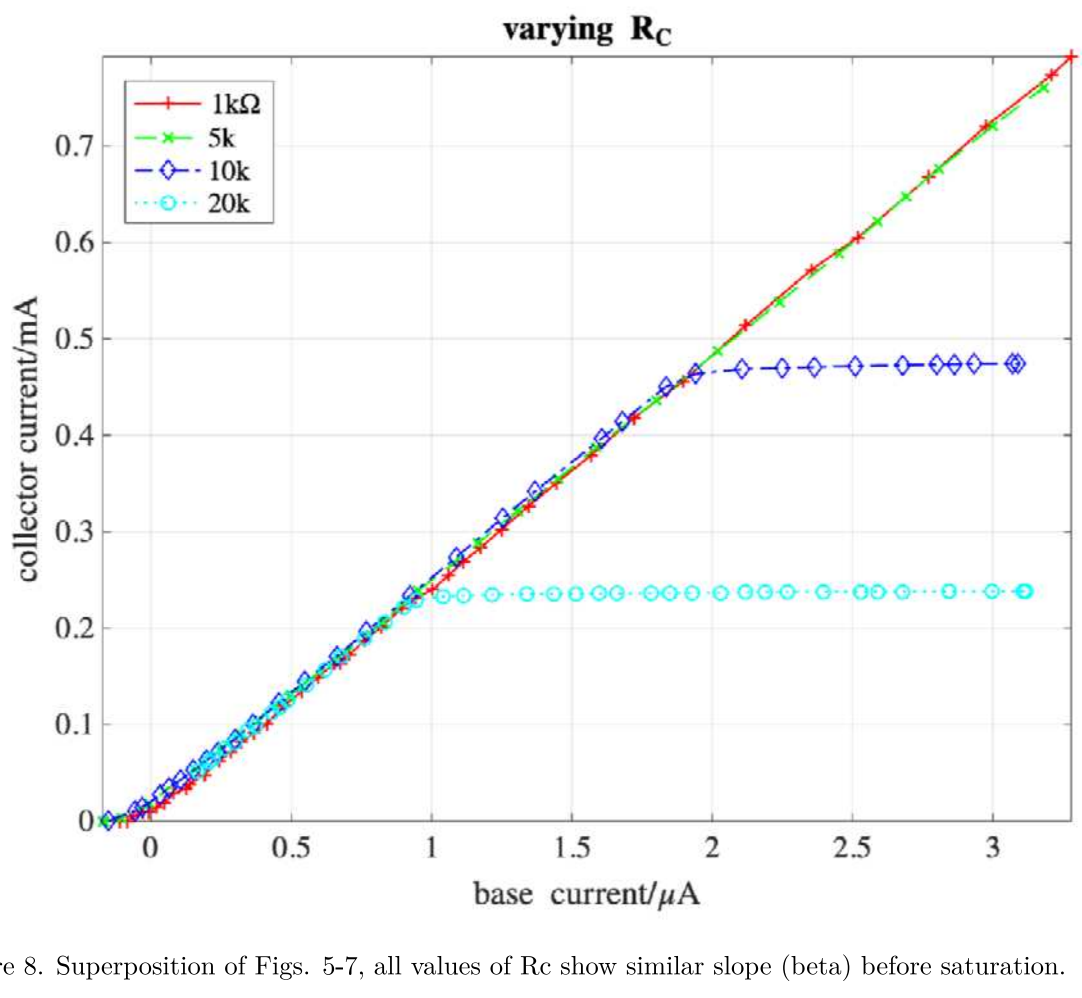


Figure 8. Superposition of Figs. 5-7, all values of Rc show similar slope (beta) before saturation.

The value of β for this transistor was:

```
fprintf('beta = %.0f ± %.0f\n', mean(beta), std(beta))
```

```
beta = 236 ± 4
```

Discussion

If your results look radically different, then troubleshoot, e.g., `plot(data)` might reveal a misconnected AIN. Did you fail to update Rc's value when computing ic? MATLAB facilitates asking questions of your data, e.g., what is `max(data)` and `min(data)`, but your task is to answer whether those results are reasonable or indicative of a bug.

These curves aren't i-v curves, but ic vs ib curves. This makes sense since transistors have 3 terminals, and hence 2 circuits (with a common emitter), and BJTs are current-controlled-current devices (contrasted to FETs, where (typically) gate-source voltage controls drain-source current). At the bottom left of Figs. 5-8, the transistor is in **cut-off**, with both currents ~0. Next the BJT enters the **active region** characterized by $i_C = \beta i_B$, with linear, constant slope = β, until the BJT becomes **saturated**, as happened with Rc = 10k at ic ~ .45mA. In the saturated regime, ic is limited by the rest of the circuit (`vref/Rc`) and not the transistor's internal characteristics.

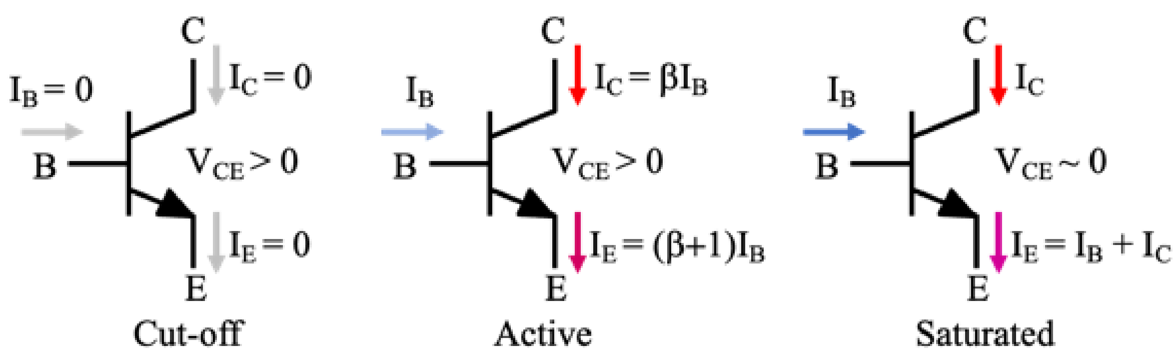


Figure 9. Three states of a BJT (Fig. 1).

I encourage you to measure a second NPN BJT, I wouldn't be surprised if its β were significantly different (I've measured between ≈ 175-260).

What other observations or lessons did you learn?

Automated data collection beats moving voltmeter leads around and manually copying its display to paper and spreadsheets. Arduino measured and printed ~440 voltages for this activity in seconds (soon we'll be measuring at its top rate ~8500 voltages/second but we slowed it down with `delay(200)`. In subsequent labs, we'll continue automating more aspects of data collection and analysis. First the manual copy/paste operation will be eliminated by establishing a serial connection between MATLAB and Arduino. Manually varying the LM317 voltage can be eliminated too with a (3-transistor) circuit we'll build controlled via Arduino's `analogWrite()` function. At that point, this entire lab could be abstracted into a single MATLAB script or function. Just plug a transistor into the breadboard and run a simple function `measureBeta` that you could write.

The Digital Abstraction

Digital circuits use the transistor's cut-off and saturated states to represent the Boolean states of true and false, 1 and 0, or visa-versa. But to transition from either of these states to the other, a transistor must pass through its active region. In other words, a transistor can't go from 0 to 1 without momentarily passing through 0.5 (and 0.6, 0.7321, etc. – except those "analog values" don't exist in a Boolean world!). So **the voltages and currents in digital circuits are actually continuous and analog**!! What then makes "digital" circuits digital? I found the answer really surprising .... see if you can find a good explanation. Hint: it has to do with *time*.

References

[1] https://web.mit.edu/6.101/www/reference/2N2222A.pdf