3D Metric Fields

Optimizing Frame Fields in a new Metric

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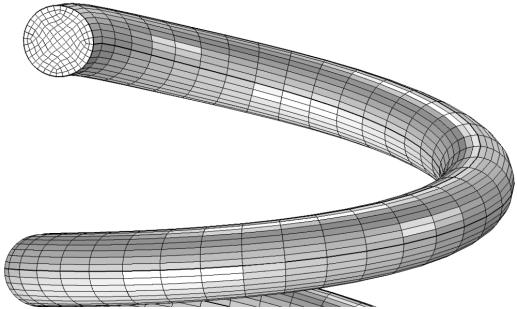
Bachelor thesis
September 20, 2023

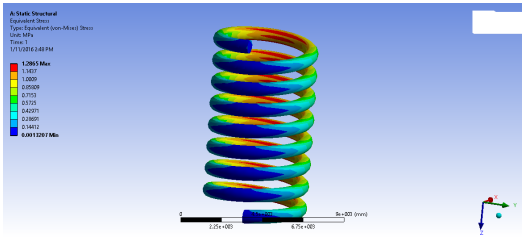
Presentation Overview

Representation of geometry

Geometry needs to be represented in computers for applications



(a) Hexahedral mesh of spring ©COMSOL



(b) Stress simulation on spring ©ANSYS

Meshes out of hexahedral elements are prefered in practice

Integer-Grid Maps

Key Idea:
Search for map $\phi : \mathcal{M} \subset \mathbb{R}^3 \rightarrow \mathbb{R}^3$ which maps the object to a 3D grid

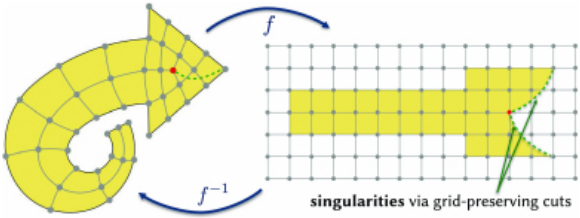


Figure: Integer-Grid map [Pietroni et al., 2022]

Direct search is infeasible $\rightarrow$ hard mixed-integer non-convex optimization problem

Search for Integer-Grid Maps

We want:

$$\phi : \mathcal{M} \to \mathbb{R}^3$$

Idea: Take the Jacobian

$$\nabla\phi : \mathcal{M} \to \mathbb{R}^{3\times 3}$$

and search for an approximation $F \approx \nabla\phi$. We call $F : \mathcal{M} \to \mathbb{R}^{3\times 3}$ a frame field. A frame locally represents the deformed edges of a cube.

Frame Fields

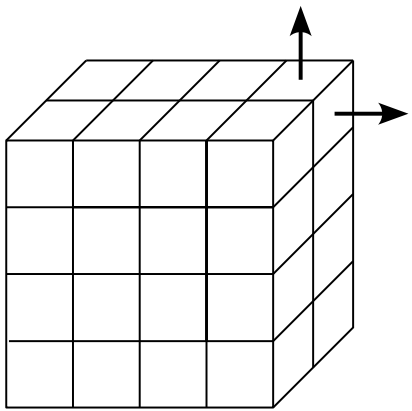
Think of F as three vector fields $F_i : \mathbb{R}^3 \to \mathbb{R}^3$

$$F = \begin{pmatrix} | & | & | \\ F_1 & F_2 & F_3 \\ | & | & | \end{pmatrix}$$

Goals for our frame field:

- Smoothness
- Boundary Alignment: One column should match with surface normal

Why?



Frame Fields

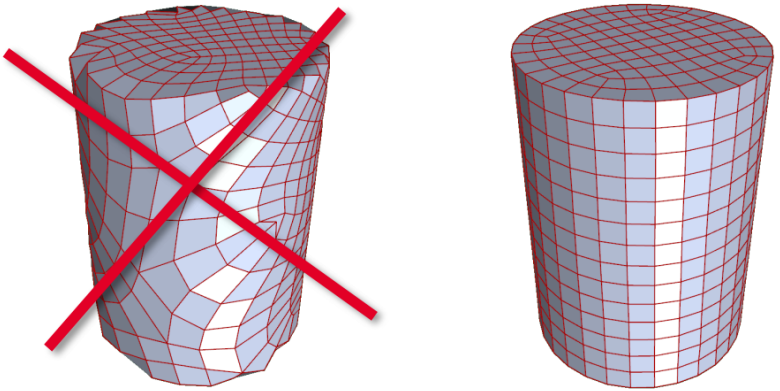


Figure: Left: Missing Boundary Alignment, not smooth. Right: Smooth + Boundary Aligned ©CGG Bern, David Bommes

Optimization Approach

- Goal: Smooth Frame Field + Boundary Aligned
- How to measure smoothness?

Dirichlet Energy

$$E(F) = \int_{\mathcal{M}} ||\nabla F||^2 \tag{1}$$

Controlling frames

- For orthonormal frame field $F^T F = \mathrm{Id}$, extracted elements are unit cubes
- Add metric field g for control of frames
- g-orthonormality $F^T g F = \mathrm{Id}$

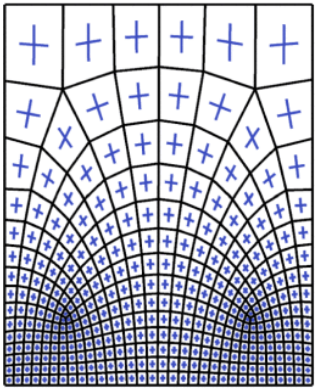


Figure: Under new metric g, lengths at the top are the same lengths as at the bottom [Fang et al., 2023]

Measuring Smoothness in new Metric

Definitions & Examples

Definition

A prime number is a number that has exactly two divisors.

Example

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (three divisors: 1, 2, and 4).

You can also use the `theorem`, `lemma`, `proof` and `corollary` environments.

Theorem, Corollary & Proof

Theorem (Mass–energy equivalence)

$E = mc^2$

Corollary

$x + y = y + x$

Proof.

$\omega + \phi = \epsilon$ □

Equation

$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{2}$$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Slide without title.

Citing References

An example of the `\cite` command to cite within the presentation:

This statement requires citation [**?**].

References


Pietroni et al. (2022)
Hex-Mesh Generation and Processing: A Survey
ACM Transaction on Graphics


Fang et al. (2023)
Metric-Driven 3D Frame Field Generation
IEEE Transactions on Visualization and Computer Graphics.

Acknowledgements

Smith Lab

- Alice Smith
- Devon Brown

Cook Lab

- Margaret
- Jennifer
- Yuan

Funding

- British Royal Navy
- Norwegian Government

The End

Questions? Comments?