Position determination using Accelerometers

Introduction

Quite often the possibility of using accelerometers to track position is investigated. We present in this application note the considerations that must be made for using accelerometers in position determination.

Fundamentals:

Ideally the position (x) of a particle at any time (t) can be determined from the time dependant acceleration of that particle. In the following equation the integration constants have been explicitly included.

$$x(t) = \int_0^t \int_0^t a(t)\,dt\,dt + \int_0^t v_0\,dt + x_0 \qquad (1)$$

where

- a is the acceleration of the object,
- v_0 is the initial velocity of the object,
- x_0 is the initial position of the object.

The acceleration term in Equation 1 can be measured directly using accelerometers. However, there are associated errors in these sensor measurements. Care must be taken in analyzing the impact of these sensor errors. In the following section we will consider the error that impact the double integration to the greatest extent.

Integration of sensor errors:

A simplified model of the output of an accelerometer (measured voltage) is as follows.

$$V_m(t) = S(1+\delta S)a(t) + V_b + \delta V_b \qquad (2)$$

where

- S is the sensitivity,
- δS is the sensitivity error (including temperature, ratiometric, cross axis errors),
- V_b is the voltage bias (0g offset),
- δV_b is the voltage bias error (including temperature, ratiometric, noise errors), and
- a is the acceleration to be measured

Writing this in terms of an apparent measured acceleration we have

$$a_m(t) = \frac{V_m(t) - V_b}{S} = a(t) + \delta S a(t) + \frac{\delta V_b}{S} \qquad (3)$$

36 Thornwood Dr. - Ithaca, NY 14850
tel: 607-257-1080 — fax: 607-257-1146
www.kionix.com — info@kionix.com

Assuming that the initial position and velocity are zero ($x_0 = 0; v_0 = 0$), substituting Equation 3 into Equation 1 and integrating twice to obtain the position as a function of time yields:

$$x(t) = \int_0^t \int_0^t a_m(t)dtdt = \int_0^t \int_0^t a(t)dtdt + \delta S \int_0^t \int_0^t a(t)dtdt + \frac{\delta V_b}{S} \int_0^t \int_0^t dtdt \qquad (4)$$

The first term is Equation 4 yields the true position data that you want to determine. The second and third terms are the integration of the intrinsic sensor errors. The error from these terms can become egregious very quickly since they increase quadratically with time. For example, for a ±2g sensor operated at V_{dd} = 3.3 V with a 1% voltage bias error (16.5 mV) and a sensitivity of 660mV/g, the coefficient on the third term is 0.245 m/s^2. In 1 second this contributes 0.1225 m (= 12.25 cm) of position error, and in 5 seconds the error has ballooned to over 3 meters. The voltage bias error term reaches approximately 1 km in about 1.5 minutes.

It is clear that care must be taken when attempting the above double integration. In many applications this short-term drift renders the double integration useless without an independent position check. In the medium term (many hours) the above errors can be bounded through the use of filters and control loops. However, the errors are still large without very high precision sensors or secondary position checks/corrections.

Additional information on minimizing accelerometer errors can be found in Application Note:AN012 Accelerometer Errors.

Theory of Operation

Kionix MEMS linear tri-axis accelerometers function on the principle of differential capacitance. Acceleration causes displacement of a silicon structure resulting in a change in capacitance. A signal-conditioning CMOS technology ASIC detects and transforms changes in capacitance into an analog output voltage which is proportional to acceleration. These outputs can then be sent to a micro-controller for integration into various applications. Kionix technology provides for X, Y and Z-axis sensing on a single, silicon chip. One accelerometer can be used to enable a variety of simultaneous features including, but not limited to:

- Drop force modeling for warranty management
- Hard disk drive shock protection
- Tilt screen navigation
- Theft, man-down, accident alarm
- Image stability, screen orientation
- Computer pointer
- Navigation, mapping
- Game playing

For product summaries, specifications, and schematics, please refer to the Kionix accelerometer product sheets at http://www.kionix.com/sensors/accelerometer-products.html.

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