# Lec-32

$$F = \bar A\bar B\bar C D + A\bar B\bar C D + \bar A\bar B C\bar D + A\bar B C\bar D + \bar A B C\bar D + A\bar B\bar C\bar D + \bar A\bar B\bar C\bar D$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | | 1 |
| 01 | | | | 1 |
| 11 | | | | |
| 10 | 1 | 1 | | 1 |

$\bar A\bar B C\bar D$ ✓

$\bar A B C\bar D$ ✓

$\boxed{ABCD}$

$\bar B\bar C + \bar B\bar D + \bar A C\bar D$

$\bar B C + \bar B\bar D + \bar A B C\bar D$

$\bar A\bar B C\bar D + \bar A B C\bar D$

$= \bar A C\bar D(\bar B + B) = \bar A C\bar D.(1)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | | 1 | | 1 |
| 11 | | | | |
| 10 | 1 | 1 | 1 | 1 |

$\bar A\bar B$

$\bar B$

$A\bar B$

$\bar A\bar B + A\bar B$

$= \bar B(A + \bar A) = \bar B$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | | 1 |
| 01 | | | | |
| 11 | | | | |
| 10 | 1 | 1 | | 1 |

$\bar B\bar D + \bar B\bar C D + \bar A C\bar D$

2
4
8
16
32

$2^n$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

$$f = \bar{A}B + A\bar{B}$$

$$f = A \oplus B$$

$$f = \bar{A}B$$

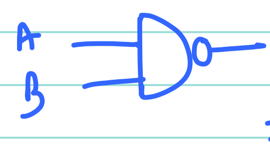
$$\bar{f} = \overline{\bar{A}B} = \bar{\bar{A}} + \bar{B} = A + \bar{B}$$

NOR implementation as Home work

## NAND and NOR gates:

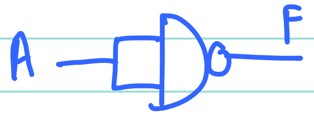
- AND
- OR
- NOT

→ Universal gates.

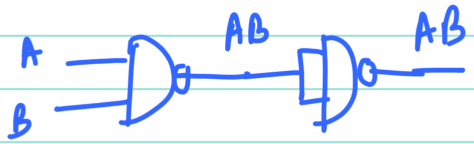



$= \overline{AB}$

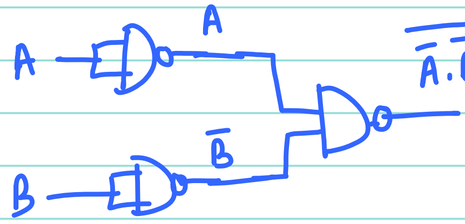
| A B | F |
|---|---|
| 0 0 | 1 |
| 0 1 | 1 |
| 1 0 | 1 |
| 1 1 | 0 |




$F = \bar{A}$ NOT




$\overline{AB}$ $AB$ AND



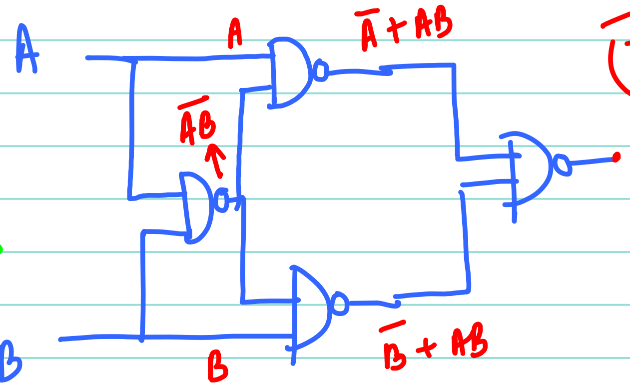

$\bar{A}$ $\bar{B}$

$\overline{\bar{A} . \bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A + B$

OR

Ex-OR $A \oplus B$ using NAND gates.

$(\bar{A}B + A\bar{B})$

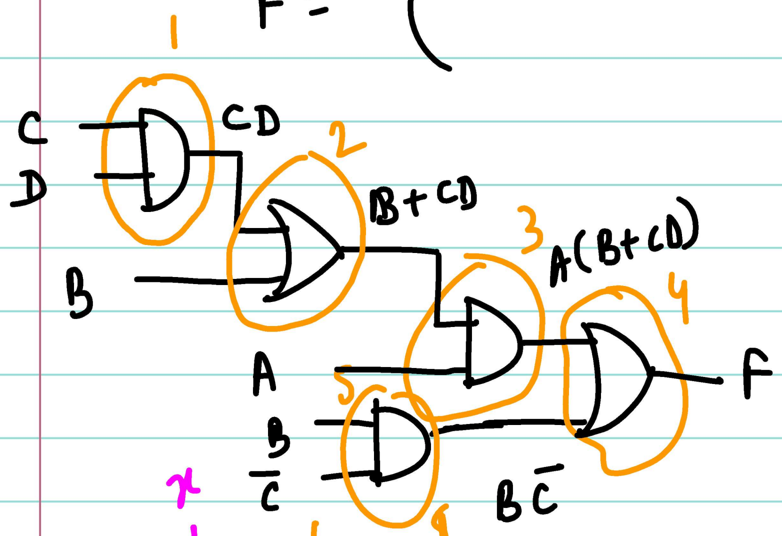



A, $\overline{AB}$, B, $\bar{A} + AB$, $\bar{B} + AB$

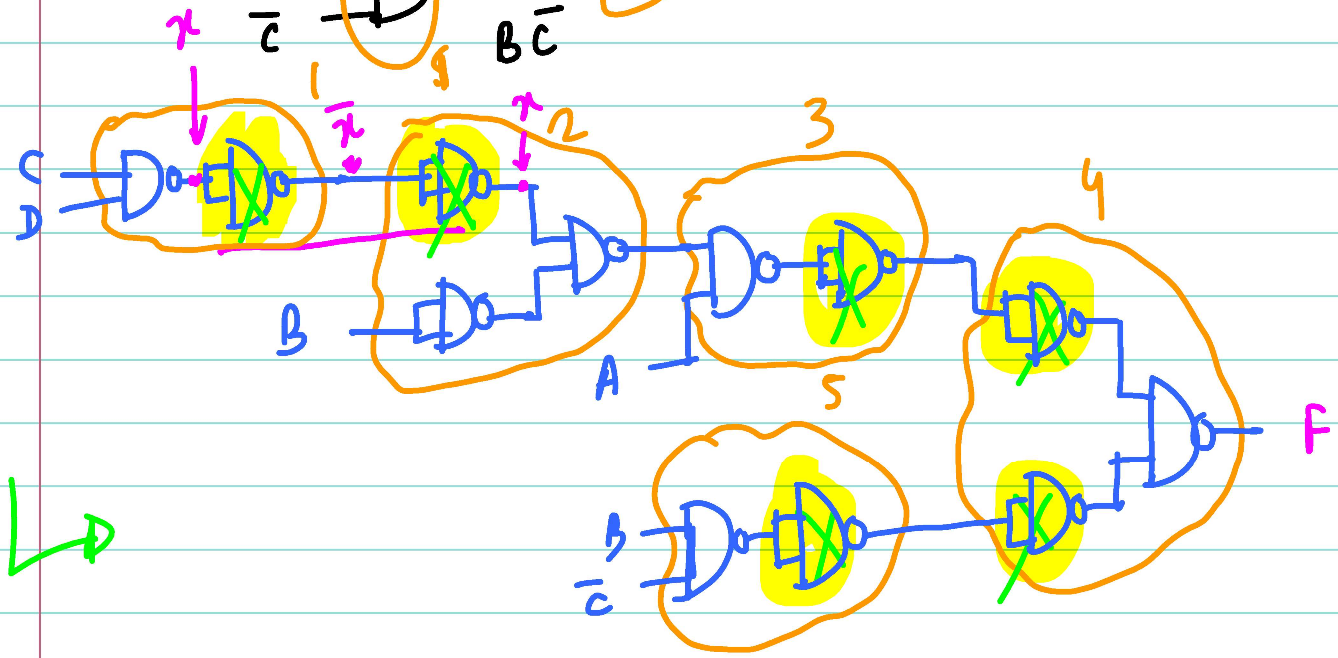
Ex-OR

$$\overline{(\bar{A} + AB) . (\bar{B} + AB)}$$
$$= \overline{\bar{A} + AB} + \overline{\bar{B} + AB}$$
$$= \bar{\bar{A}} . \overline{AB} + \bar{\bar{B}} . \overline{AB}$$
$$= A . \overline{AB} + B(\overline{AB})$$
$$= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$$
$$= A\bar{B} + \bar{A}B$$
$$= A \oplus B$$

$F = (\qquad\qquad\qquad)$




$AB + ACD + B\bar{C}$

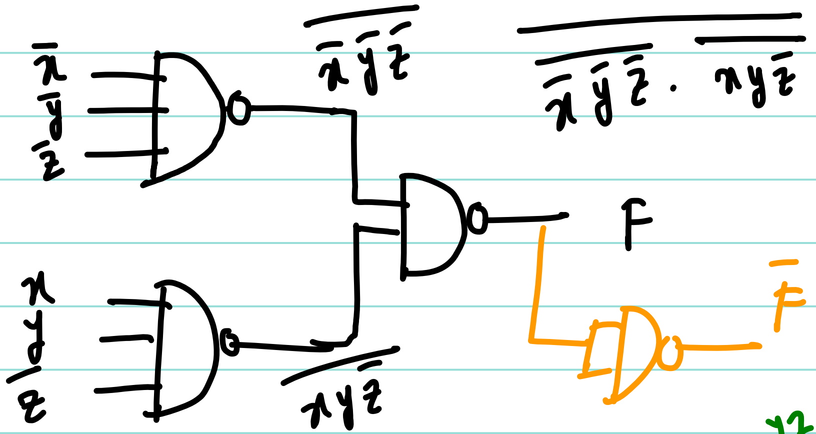


# NAND Implementation:

→ $F(x,y,z) = \sum(0,6)$

SOP Form for NAND implementation.

$F(x,y,z) = \bar{x}\bar{y}\bar{z} + xy\bar{z}$




$\overline{\overline{\bar{x}\bar{y}\bar{z}} \cdot \overline{xy\bar{z}}} = \bar{x}\bar{y}\bar{z} + xy\bar{z}$

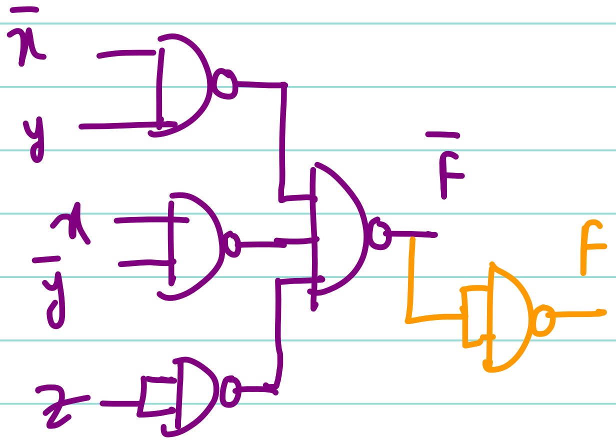
$\overline{F(x,y,z)}$

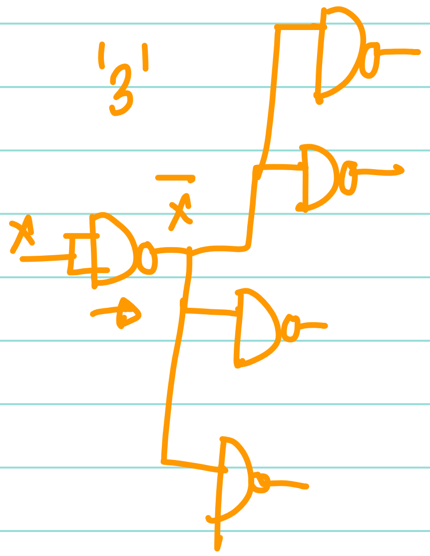
$(\ +\ ) \cdot (\ +\ )(\ \ )$

| x \ yz | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |

$(\ ) + (\ ) + (\ )$

$\bar{F} = z + \bar{x}y + x\bar{y}$ SOP



# NOR implementation.

## POS form

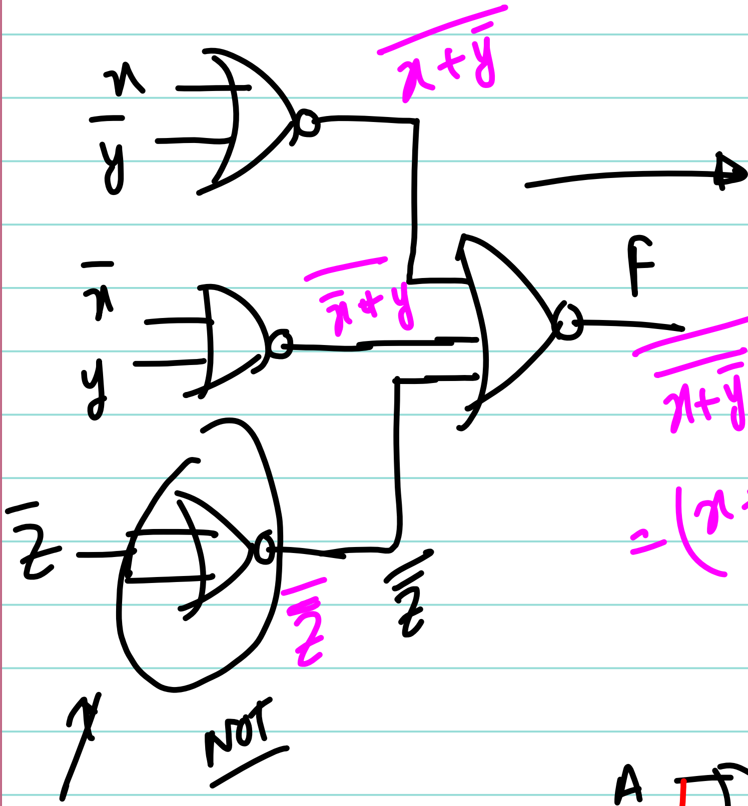
~~F = x~~  $F(x,y,z) = \sum(0,6)$

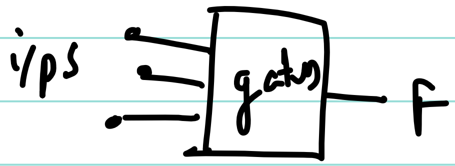
$\bar{F} = \bar{x}y + x\bar{y} + z$

$\bar{\bar{F}} = F = \overline{\bar{x}y + x\bar{\bar{y}} + z}$

$F = (x+\bar{y})\cdot(\bar{x}+y)\cdot\bar{z}$ ✓

POS form

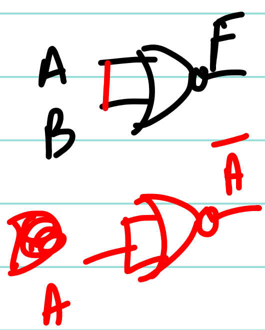



$\overline{\overline{x+\bar{y}} + \overline{\bar{x}+y} + \bar{\bar{z}}}$

$= (x+\bar{y})(\bar{x}+y)\cdot\bar{z}$ ✓

## NOR gate

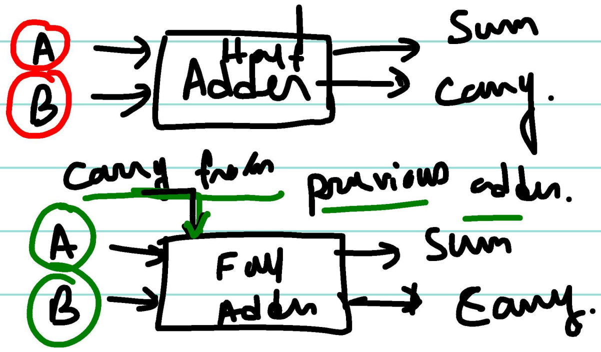



| AB | F |
|---|---|
| 00 | 1 |
| 01 | 0 |
| 10 | 0 |
| 11 | 0 |

# Combinational Logic

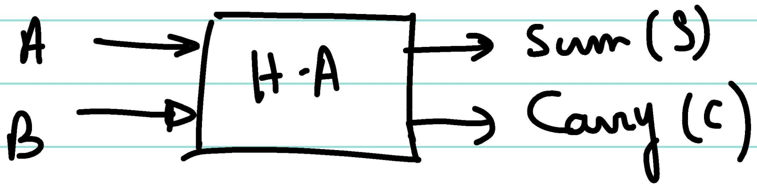
Adder (single-bit) → Half-Adder

Adder (single-bit) → Full-Adder




| | | | | | |
|---|---|---|---|---|---|
| | 1 | 0 | 1 | 1 | → 11 |
| | 0 | 1 | 0 | 1 | → 5 |
| 1 | 0 | 0 | 0 | 0 | → 16 |

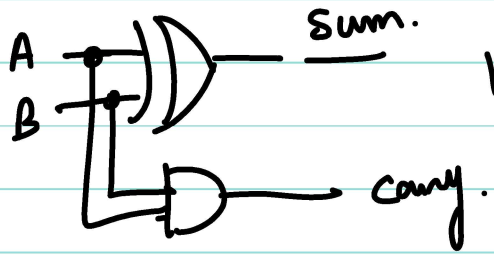
## Half-Adder:




| A | B | S | C |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$$S = \bar{A}B + A\bar{B} = A \oplus B$$

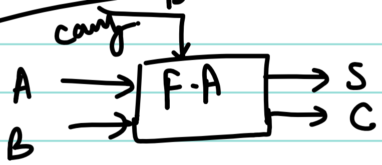
$$C = AB$$




Half-Adder

Full - Adder

carry previous

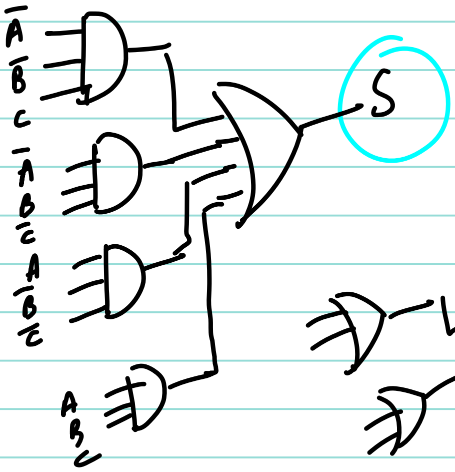



| A | B | C | S | C |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

S

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |

$S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

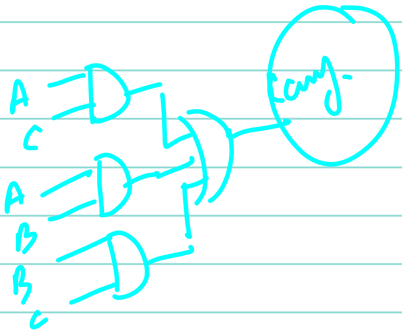




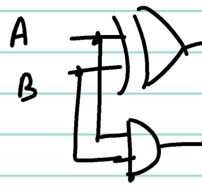
| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 |

Carry

Carry = AC + AB + BC



Full-Adder using two half-adder circuit and an OR gate




Sum = $A \oplus B$

Carry = $AB$.

} Half - Adder

$(\bar{A}\bar{B} + AB)$

$F_1 = A \odot B$

$A \odot B = \bar{F}$

| A B | F | $F_1$ |
|---|---|---|
| 0 0 | 0 | 1 |
| 0 1 | 1 | 0 |
| 1 0 | 1 | 0 |
| 1 1 | 0 | 1 |

Full-Adder Sum = $\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

Carry = $AC + AB + BC$

Sum = $\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

$= C(\bar{A}\bar{B} + AB) + \bar{C}(\bar{A}B + A\bar{B})$

$= C(\overline{A \oplus B}) + \bar{C}(A \oplus B)$

$A \oplus B = A\bar{B} + \bar{A}B$

$A \oplus B = x$

$= C\bar{x} + \bar{C}x$

$= C \oplus x = C \oplus A \oplus B = A \oplus B \oplus C$

Carry = $AC + AB + BC$


$(A \oplus B)C + AB$

$x + x + x = x$

$= (\bar{A}B + A\bar{B})C + AB(C + \bar{C})$

$= \bar{A}BC + A\bar{B}C + ABC + AB\bar{C}$

$= \bar{A}BC + A\bar{B}C + ABC + ABC + ABC + AB\bar{C}$

$= BC(\bar{A} + A) + AC(\bar{B} + B) + AB(C + \bar{C})$

$= BC + AC + AB$