INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR,
End-Sem, Autumn Semester 2022, Department of Electrical Engineering

Date of Examination: 22/11/2022 (AN) **Duration: 3 Hrs**
Subject. No: EE21201 **Subject Name: Signals and Systems**
Department: Electrical Engineering **TOTAL MARKS: 50**
Specific Chart, graph paper etc. required: No **Special Instruction: None.**

ANSWER ALL THE QUESTIONS

1. a) Determine the fundamental period for the discrete-time signal $x[n] = e^{jm\frac{2\pi}{N}n}$ where m and N are positive integers. (2M).
   b) Determine whether the continuous-time system given as $y(t) = \cos(4t)x(t)$ is i) memoryless, ii) time-invariant, iii) linear, iv) causal, v) BIBO-stable. Here, $x(t)$ and $y(t)$ are the input and output respectively. ($5 \times 1 = 5$M).

2. a) Let $x(t)$ be a periodic signal whose Fourier series co-efficients are given as $a_k = \begin{cases} 2, & \text{when } k = 0 \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise} \end{cases}$. Using Fourier series properties, determine if i) $x(t)$ is real, ii) $x(t)$ is even, iii) $\frac{dx(t)}{dt}$ is even. ($3 \times 2 = 6$M).
   b) Given a discrete-time LTI system with impulse response $h[n] = \left(\frac{1}{2}\right)^{|n|}$, find the Fourier series representation of the output $y[n]$ of the system when its input is $x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$, $\delta[n]$ being the unit impulse signal. (6M).

3. a) A causal and stable continuous-time LTI system S has frequency response $H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$. Determine its impulse response $h(t)$. (4M).
   b) Compute the discrete-time Fourier transform of $x[n] = \begin{cases} n, & \text{for } -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$. (2M).

4. Consider the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$. Let the initial distribution at $t = 0$ be $u(0, x) = e^{-x^2}$. Find the solution profile $u(t, x)$. (8M).

5. Let $x[n]$ be a discrete signal with autocorrelation defined as

$$\phi_{xx}[n] := \sum_{k=-\infty}^{\infty} x[k]x[n + k].$$

Find the impulse response of the system with x as the input and ϕ_{xx} as the output. Find the relation between the $Z-$transforms of the input signal, the output signal and the system $(4 + 3 = 7\text{M})$.

6. Let $x(t) = \sin(\omega_0 t)$ which is sampled using sampling frequency $\omega_s > 2\omega_0$. Let $\sin(nT)$ denote the discrete samples with $T = \frac{2\pi}{\omega_s}$. Find a filter $h(t)$ such that $\cos\omega_0(t - t_0) = \sum_{n=-\infty}^{\infty} \sin(nT)h(t - nT)$ where $t_0 \in \mathbb{R}, t_0 \neq 0$ (10M).