INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR,
Mid-Sem, Autumn Semester 2022, Department of Electrical Engineering

Date of Examination: 26/09/2022 (FN) | **Duration: 2 Hrs**
Subject. No: EE21201 | **Subject Name: Signals and Systems**
Department: Electrical Engineering | **TOTAL MARKS: 30**
Specific Chart, graph paper etc. required: No | **Special Instruction: None.**

ANSWER ALL THE QUESTIONS

1.

(a) Find if the following signals are periodic or not. If periodic, also find their fundamental periods. (2M)

i. $x[n] = \cos\frac{\pi}{8}n^2 + \sin\frac{\pi}{3}n.$

ii. $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2.$

(b) Find if the following systems are invertible or not. If invertible, also find their inverses. (2M)

i. $y[n] = nx[n]$

ii. $y(t) = \int_{-\infty}^{t} x(\tau) \cdot d\tau.$

(c) Suppose the signal $x(t) = u(t+0.5) - u(t-0.5)$ (where $u(t)$ is the unit step signal) is applied to the LTI system given by its impulse response $h(t) = e^{j\omega_0 t}$. Find a value of ω_0 such that its response $y(t)$ for $x(t)$ is 0 at $t = 0$. (3M)

2.

(a) Find the Fourier series representation for the signal $x(t)$ which is periodic with period 4 and $x(t) = \begin{cases} \sin \pi t & 0 \le t \le 2 \\ 0 & 2 < t \le 4 \end{cases}$. (4M)

(b) Consider the causal discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related as $y[n] - \frac{1}{4}y[n-1] = x[n]$. Find the Fourier series representation of the output $y[n]$ when input is $x[n] = \cos\left(\frac{\pi}{4}n\right) + 2\cos\left(\frac{\pi}{2}n\right)$. (3M)

(c) Consider a discrete-time LTI system with impulse response $h[n]$ given as $h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$.

Find the Fourier series co-efficients of the output $y[n]$ when the input $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$ and $\delta[n]$ is the unit impulse sequence. (3M)

3. Find the Fourier transforms of the following

(a) $x(t) = t.\text{sgn}(t)$ where $\text{sgn}(t) = 1$ for $t > 0$, $\text{sgn}(t) = -1$ for $t < 0$ and $\text{sgn}(t) = 0$ for $t = 0$. (4M)

(b) $x(t) = \int_{-\infty}^{t} \frac{\sin^2(\pi\tau)}{\pi\tau^2} d\tau$. (4M)

(c) $x[n] = \sum_{k=-\infty}^{n} a^{|k|}$ where $0 < a < 1$. (5M)