Signals and systems End Sem (30M), Time: 1.5hrs

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1. Let $x(t)$ be a continuous time signal sampled by an impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ such that $x_p(t) = x(t)p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$. Let $T = 1$.

   (a) Suppose the discrete samples $x_p(t)$ are delayed by n_0 and the impulse response of the ideal low pass filter is delayed by t_0. Find the reconstructed signal from the discrete samples using the low pass filter with delay mentioned above. (2M)

   (b) Construct a system with an impulse response g such that the reconstructed signal $x_r(t) = x_p(t) * g(t)$ is $x_r(t) = \frac{d}{dt}(x(t - t_0))$. (3M)

   (c) Suppose $x(t)$ and $y(t)$ are two band limited signals such that $X(j\omega)$ and $Y(j\omega)$ are both zero for $|\omega| > \omega_M$. Suppose both $x(t)$ and $y(t)$ are sampled with a sampling frequency ω_s such that the condition for sampling theorem is satisfied to obtain discrete samples $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$ and $y_p(t) = \sum_{n=-\infty}^{\infty} y(nT)\delta(t - nT)$ where $T = \frac{2\pi}{\omega_s}$. Explain how to obtain $x(t) * y(t)$ from the discrete samples $x(nT), y(nT)$. (2M)

2. Consider a real and causal system with impulse response h with no singularity at $t = 0$. Let $H(j\omega) = H_R(j\omega) + jH_I(j\omega)$ be its continuous time Fourier transform. Show that

$$H(j\omega) = \frac{1}{j\pi} \int_{-\infty}^{\infty} \frac{H(j\eta)}{\omega - \eta} d\eta.$$

   Find an expression for $H_R(j\omega)$ in terms of $H_I(j\omega)$ and one for $H_I(j\omega)$ in terms of $H_R(j\omega)$. (6+3=9M)

3. Find the Laplace transform with the ROC of $x(t) = \frac{e^{t/2}}{n!} \frac{d^n}{dt^n}(t^n e^{-t})u(t)$. (6M)

4. Show that

$$\sum_{n=-\infty}^{\infty} e^{-inT\omega} = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{T}).$$

   (8M)