Signals & systems End sem (Sol^n & hints)

Q.1 a) $y(n) = x(n - n_0)$

$h(t) \rightarrow$ ideal LPF

$x_p(t - n_0) * h(t - t_0) \rightarrow$ reconstr. sig.

$= x(t - t_0 - n_0)$ (2M)

b) $x_2(t) = \frac{d}{dt}(x(t - t_0))$

$x_p(t) * h(t - t_0) = x(t - t_0)$

(1M)

$\frac{d}{dt} x(t - t_0) = x_p(t) * \frac{d}{dt} h(t - t_0)$

(1M)

$\Rightarrow g(t) = \frac{dh(t - t_0)}{dt}$

h → ideal LPF (1M)

c) ~~$z[n] = x[n]$~~

$$z(nT) = x(nT) * y(nT)$$

$$z_p(t) = \sum_n z(nT)\,\delta(t - nT)$$

$$z_p(t) * h(t) = x(t) * y(t)$$

↓

ideal LPF ($\mathrm{sinc}(t)$) (2M)

Q.2 $h(t) = h(t)u(t)$

$$\Rightarrow H(j\omega) = \frac{1}{2\pi} H(j\omega) * \left\{ \frac{1}{j\omega} + \pi\delta(\omega) \right\}$$

(2M)

$$\Rightarrow H(j\omega) = \frac{1}{2} H(j\omega) + \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{H(j\Omega)}{\omega - \Omega}\, d\Omega$$

$$\Rightarrow H(j\omega) = \frac{1}{\pi j} \int_{-\infty}^{\infty} \frac{H(j\Omega)}{\omega - \Omega}\, d\Omega$$

(4M)

$$\Rightarrow H_R(j\omega) + j H_I(j\omega) = \frac{1}{\pi j} \int_{-\infty}^{\infty} \frac{H_R(j\Omega) + j H_I(j\Omega)}{\omega - \Omega}\, d\Omega$$

$$\Rightarrow H_R(j\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{H_I(j\Omega)}{\omega - \Omega}\, d\Omega \quad (1.5M)$$

$$H_I(j\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{H_R(j\Omega)}{\omega - \Omega}\, d\Omega \quad (1.5M)$$

Q.3

$$\frac{t^n e^{-t}}{n!} u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^{n+1}} \quad \text{ROC: } \sigma > -1$$

(2M)

$$\frac{1}{n!}\frac{d^n}{dt^n}\frac{t^n e^{-t}}{n!} u(t) \xleftrightarrow{\mathcal{L}} \frac{s^n}{(s+1)^{n+1}} \quad \sigma > -1$$

(2M)

$$\Rightarrow \frac{e^{t/2}}{n!}\frac{d^n}{dt^n}\frac{t^n e^{-t}}{1} u(t) \xleftrightarrow{\mathcal{L}} \frac{(s-\frac{1}{2})^n}{(s+\frac{1}{2})^{n+1}} \quad \sigma > -\frac{1}{2}$$

(2M)

Q.4

$$\delta(t) \xleftrightarrow{F} 1$$

$$\delta(t-nT) \xleftrightarrow{F} e^{-inT\omega}$$

$$\Rightarrow \sum_n \delta(t-nT) = \sum_n e^{-inT\omega}$$ (Four. transf)

(4M)

(Four. series)

$$\sum_n \delta(n-nT) = \frac{2\pi}{T}\sum_k \delta\left(\omega - \frac{2\pi k}{T}\right)$$

(2M)

$$\Rightarrow \sum_n e^{-inT\omega} = \frac{2\pi}{T}\sum_k \delta\left(\omega - \frac{2\pi k}{T}\right)$$

(2M)