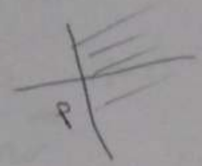
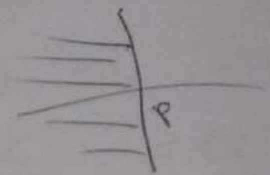
ROC ctd.

Types of ROC

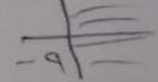
ϕ, $\mathbb{C}$, , , 

(i) (ii) (iii) (iv) (v)

<u>causal sig.</u> : ROC can be (i), (ii) or (iii) but ($\cancel{iv}$).

ex. $x(t) = e^{t^2} u(t)$, ROC $= \phi$

$x(t) \to$ finite dur.^{n} ⇒ ROC $= \mathbb{C}$

$x(t) = e^{-at} u(t)$ ROC : 

supp. x is causal (or rt. sided) & $\sigma_0 \in$ ROC

$$\Rightarrow \int_{-\infty}^{\infty} |x(t) e^{-\sigma_0 t}| \, dt < \infty$$

$$\int_{0}^{\infty} |x(t)| e^{-\sigma_0 t} \, dt < \infty$$

$\Rightarrow$ if $\sigma_1 > \sigma_0$,

$$\int_{0}^{\infty} |x(t)| e^{-\sigma_1 t} \, dt < \infty$$

$\Rightarrow$ all $\sigma > \sigma_0 \in$ ROC

however, if $\hat{\sigma} < \sigma_0$, then

$$\int_0^\infty |x(t)| e^{-\sigma_0 t} dt < \infty \not\Rightarrow \int_0^\infty |x(t)| e^{-\hat{\sigma} t} dt < \infty$$

$\therefore$ causal (rt. sided signals) cannot have ROC of type (iv) or (v)

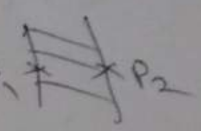
for ROC of type (iv), $\sigma \to -\infty \in$ ROC

& $e^{-\sigma t}$ becomes unbdd. exp. fast

if $\sigma \to -\infty \Rightarrow \int_0^\infty |x(t)| e^{-\sigma t} dt \to \infty$

unless $x(t)$ is of fin. dur.^{n}.

for ROC of type (v), there are at least 2 poles at: p_1, p_2



$\therefore \; x = \underbrace{x_1}_{\text{pole at } p_1} + \underbrace{x_2}_{\text{pole at } p_2} + *$

$\not\Downarrow$

$x_1 \to$ rt. sided

$x_2 \to$ cannot be rt. sided.

$\Rightarrow x$ is not rt. sided.

pf. ~~(sketch)~~.

ROC of type iii) $\not\Rightarrow$ $x(t)$ is rt. sided.

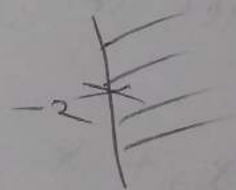
ex. $x(t) = \underbrace{e^{-t^2}}_{\text{both sided}} + \underbrace{e^{-2t}u(t)}_{\text{rt. sided}}$

$$X(s) = \int_{-\infty}^{\infty} e^{-(t^2+st)}\,dt + \frac{1}{s+2}$$

$$\int_{-\infty}^{\infty} e^{-(t^2+st)}\,dt = \sqrt{\pi}\, e^{\frac{s^2}{4}}$$

$\frac{1}{s+2}$: ROC : $Re(s) > -2$ (R_2)

(R_1) ROC $= \mathbb{C}$

$\therefore R_1 \cap R_2 =$ 

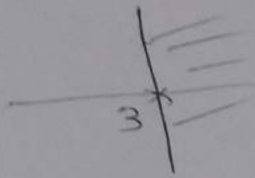
but $x(t)$ is not rt. sided.

anticausal : ROC can be i), ii) or (iv)

$\not\Leftarrow$

Both sided: ROC can be (i), (ii) or (iii), (iv) or (v)

$$x(t)= e^{3t}u(t)+ e^{-3t}u(-t)$$

ROC = 

$\cap$ 

$= \{\phi\}$

$x(t) = \delta(t)$, ROC $= \mathbb{C}$

~~$x(t) = e$~~

$$x(t)= e^{-t^2} + e^{-2t}u(t)$$

$$x(t) = e^{-t^2} + e^{2t}u(-t)$$