Tutorial 8

Formal Language and Automata Theory
(PDA and PDA-CFG Equivalence)

March 9, 2023

Question 1

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(i) a set to push w on the stack
$Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, z\}$, $F = \{q_2\}$
$\delta(q_0, a, a) = \{(q_0, aa)\}$
$\delta(q_0, b, a) = \{(q_0, ba)\}$
$\delta(q_0, a, b) = \{(q_0, ab)\}$
$\delta(q_0, b, b) = \{(q_0, bb)\}$
$\delta(q_0, a, z) = \{(q_0, az)\}$
$\delta(q_0, b, z) = \{(q_0, bz)\}$

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$\delta(q_0, \epsilon, a) = \{(q_1, a)\}$
$\delta(q_0, \epsilon, b) = \{(q_1, b)\}$

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$\delta(q_1, a, a) = \{(q_1, \epsilon)\}$
$\delta(q_1, b, b) = \{(q_1, \epsilon)\}$
and finally $\delta(q_1, \epsilon, z) = \{(q_2, z)\}$

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$\delta(q_0, a, z) = \{(q_0, az)\}$
$\delta(q_0, a, a) = \{(q_0, aa)\}$

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$\delta(q_0, b, a) = \{(q_1, \epsilon)\}$
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$\delta(q_1, b, z) = \{(q_2, z)\}$
$\delta(q_1, \epsilon, a) = \{(q_2, \epsilon)\}$
and finally $\delta(q_2, \epsilon, z) = \{(q_2, \epsilon)\}$

Question 3

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$S \rightarrow aSbb|a$

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Construct an NPDA that accepts the language generated by a grammar with productions:
$S \to aSbb|a$
Hint: The language generated by the grammar is $\{a^n b^{2n-2} : n \geq 1\}$.

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Construct an NPDA that accepts the language generated by a grammar with productions:

$S \rightarrow aSbb|a$

Solution: The corresponding automaton will have

$Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $\Gamma = \{S, A, B, z\}$, $F = \{q_2\}$

The transitions are:

$\delta(q_0, \epsilon, z) = \{(q_1, Sz)\}$ [First, the start symbol S is put on the stack by]

$\delta(q_1, a, S) = \{(q_1, SA), (q_1, \epsilon)\}$

$\delta(q_1, b, A) = \{(q_1, B)\}$

$\delta(q_1, b, B) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, z) = \{(q_2, \epsilon)\}$

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Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.

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Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.
Hint: Let $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ be a DFA that recognizes R and $M = (Q_M, \Sigma, \Gamma, \delta_M, p_0, F_M)$ be a PDA that recognizes C. How do we combine both machines ?

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Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.
Hint: The machines N and M are combined to construct a PDA M' that recognizes $C \cap R$. This will show that $C \cap R$ is context-free. How do we show that ?

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Solution: A state of M' will be a pair of states (p, q) with p a state of M and q a state of N. M' will simultaneously keep track of a state that M could be in after reading the symbols seen so far and a state that N could be in after reading these symbols. The formal definition is:

$$M' = (Q_M \times Q_N, \Sigma, \Gamma, \delta_{M'}, (p_0, q_0), F_M \times F_N)$$

The transition function $\delta_{M'}$ is defined by
$\delta_{M'}((p, q), a, x) = \{((p', q'), y) \mid (p', y) \in \delta_M(p, a, x) \text{ and } \delta_N(q, a) = q'\}$
for all $p \in Q_M, q \in Q_N, a \in \Sigma$ and $x \in \Gamma_\varepsilon$ and

$$\delta_{M'}((p, q), \varepsilon, x) = \left\{ \left((p', q), y \right) \mid (p', y) \in \delta_M(p, \varepsilon, x) \right\}$$

for all $p \in Q_M, q \in Q_N$ and $x \in \Gamma_\varepsilon$.