statistically valid conclusions about the simulation's results. We concentrated on output analysis for both terminating and steady state simulations.

19-6 EXERCISES

19-1. Extension of Example 19-1.

(a) Flip a coin 100 times. How many heads to do you observe?

(b) How many times do you observe two heads in a row? Three in a row? Four? Five?

(c) Find 10 friends and repeat (a) and (b) based on a total of 1000 flips.

(d) Now simulate coin flips via a spreadsheet program. Flip the simulated coin 10,000 times and answer (a) and (b).

19-2. Extension of Example 19-2. Throw n darts randomly at a unit square containing an inscribed circle. Use the results of your tosses to estimate π. Let $n = 2^k$ for $k = 1, 2, \ldots, 15$, and graph your estimates as a function of k.

19-3. Extension of Example 19-3. Show that $\hat{I}_n$, defined in equation 19-3, is unbiased for the integral I, defined in equation 19-2.

19-4. Other extensions of Example 19-3.

(a) Use Monte Carlo integration with $n = 10$ observations to estimate $\int_0^2 \frac{1}{2\pi} e^{-x^2/2} dx$. Now use $n = 1000$. Compare to the answer that you can obtain via normal tables.

(b) What would you do if you had to estimate $\int_0^{10} \frac{1}{2\pi} e^{-x^2/2} dx$?

(c) Use Monte Carlo integration with $n = 10$ observations to estimate $\int_0^1 \cos(2\pi x)\, dx$. Now use $n = 1000$. Compare to the actual answer.

19-5. Extension of Example 19-4. Suppose that 10 customers arrive at a post office at the following times:

3 4 6 7 13 14 20 25 28 30

Upon arrival, customers queue up in front of a single clerk and are processed in a first-come-first-served manner. The service times corresponding to the arriving customers are as follows:

6.0 5.5 4.0 1.0 2.5 2.0 2.0 2.5 4.0 2.5

Assume that the post office opens at time 0, and closes its doors at time 30 (just after customer 10 arrives), serving any remaining customers.

(a) When does the last customer finally leave the system?

(b) What is the average waiting time for the 10 customers?

(c) What is the maximum number of customers in the system? When is this maximum achieved?

(d) What is the average number of customers in line during the first 30 minutes?

(e) Now repeat parts (a)–(d) assuming that the services are performed last-in-first-out.

19-6. Repeat Example 19-5, which deals with an (s, S) inventory policy, except now use order level $s = 6$.

19-7. Consider the pseudorandom number generator $X_i = (5X_{i-1} + 1) \bmod (16)$, with seed $X_0 = 0$.

(a) Calculate X_1 and X_2, along with the corresponding PRNs U_1 and U_2.

(b) Is this a full-period generator?

(c) What is X_{150}?

19-8. Consider the "recommended" pseudorandom number generator $X_i = 16807\, X_{i-1} \bmod (2^{31} - 1)$, with seed $X_0 = 1234567$.

(a) Calculate X_1 and X_2, along with the corresponding PRNs U_1 and U_2.

(b) What is $X_{100,000}$?

19-9. Show how to use the inverse transform method to generate an exponential random variable with rate $\lambda = 2$. Demonstrate your technique using the PRN $U = 0.75$.

19-10. Consider the inverse transform method to generate a standard normal (0,1) random variable.

(a) Demonstrate your technique using the PRN $U = 0.25$.

(b) Using your answer in (a), generate an N (1,9) random variable.

19-11. Suppose that X has probability density function $f(x) = |x/4|,\ -2 < x < 2$.

(a) Develop an inverse transform technique to generate a realization of X.

(b) Demonstrate your technique using $U = 0.6$.

(c) Sketch out $f(x)$ and see if you can come up with another method to generate X.

19-12. Suppose that the discrete random variable X has probability function

$$p(x) = \begin{cases} 0.35 & \text{if } x = -2.5, \\ 0.25 & \text{if } x = 1.0, \\ 0.40 & \text{if } x = 10.5, \\ 0, & \text{otherwise.} \end{cases}$$

As in Example 19-12, set up a table to generate realizations from this distribution. Illustrate your technique with the PRN $U = 0.86$.

19-13. The Weibull (α, β) distribution, popular in reliability theory and other applied statistics disciplines, has CDF

$$F(x) = \begin{cases} 1 - e^{-(x/\alpha)^\beta} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show how to use the inverse transform method to generate a realization from the Weibull distribution.

(b) Demonstrate your technique for a Weibull (1.5,2.0) random variable using the PRN $U = 0.66$.

19-14. Suppose that $U_1 = 0.45$ and $U_2 = 0.12$ are two IID PRNs. Use the Box–Müller method to generate two N (0,1) variates.

19-15. Consider the following PRNs:

0.88 0.87 0.33 0.69 0.20 0.79 0.21
0.96 0.11 0.42 0.91 0.70

Use the Central Limit Theorem method to generate a realization that is approximately standard normal.

19-16. Prove equation 19-4 from the text. This shows that the sum of n IID exponential random variables is Erlang. *Hint*: Find the moment-generating function of Y, and compare it to that of the gamma distribution.

19-17. Using two PRNs, $U_1 = 0.73$ and $U_2 = 0.11$, generate a realization from an Erlang distribution with $n = 2$ and $\lambda = 3$.

19-18. Suppose that $U_1, U_2, \ldots, U_n$ are PRNs.

(a) Suggest an easy inverse transform method to generate a sequence of IID Bernoulli random variables, each with success parameter p.

(b) Show how to use your answer to (a) to generate a binomial random variate with parameters n and p.

19-19. Use the acceptance–rejection technique to generate a geometric random variable with success probability 0.25. Use as many of the PRNs from Exercise 19-15 as necessary.

19-20. Suppose that $Z_1 = 3$, $Z_2 = 5$, and $Z_3 = 4$ are three batch means resulting from a long simulation run. Find a 90% two-sided confidence interval for the mean.

19-21. Suppose that $\mu \in [-2.5, 3.5]$ is a 90% confidence interval for the mean cost incurred by a certain inventory policy. Further suppose that this interval was based on five independent replications of the underlying inventory system. Unfortunately, the boss has decided that she wants a 95% confidence interval. Can you supply it?

19-22. The yearly unemployment rates for Andorra during the past 15 years are as follows:

6.9 8.3 8.8 11.4 11.8 12.1 10.6 11.0
9.9 9.2 12.3 13.9 9.2 8.2 8.9

Use the method of batch means on the above data to obtain a two-sided 95% confidence interval for the mean unemployment. Use five batches, each consisting of three years' data.

19-23. Suppose that we are interested in steady state confidence intervals for the mean of simulation output $X_1, X_2, \ldots, X_{10000}$. (You can pretend that these are waiting times.) We have conveniently divided the run up into five batches, each of size 2000; suppose that the resulting batch means are as follows:

100 80 90 110 120

Use the method of batch means on the above data to obtain a two-sided 95% confidence interval for the mean.

19-24. The yearly total snowfall figures for Siberacuse, NY, during the past 15 years are as follows:

100 103 88 72 98 121 106 110 99
162 123 139 92 142 169

(a) Use the method of batch means on the above data to obtain a two-sided 95% confidence interval for the mean yearly snowfall. Use five batches, each consisting of three years' data.

(b) The corresponding yearly total snowfall figures for Buffoonalo, NY (which is down the road from Siberacuse), are as follows:

90 95 72 68 95 110 112 90 75
144 110 123 81 130 145

How does Buffoonalo's snowfall compare to Siberacuse's? Just give an eyeball answer.

(c) Now find a 95% confidence interval for the difference in means between the two cities. *Hint*: Think common random numbers.

19-25. Antithetic variates. Suppose that $X_1, X_2, \ldots, X_n$ are IID with mean μ and variance σ^2. Further suppose that $Y_1, Y_2, \ldots, Y_n$ are also IID with mean μ and variance σ^2. The interesting trick here is that we will also assume that $Cov(X_i, Y_i) < 0$ for all i. So, in other words, the observations *within* one of the two sequences are IID, but they are negatively correlated *between* sequences.

(a) Here is an example showing how can we end up with the above scenario using simulations. Let $X_i = -\ln(U_i)$ and $Y_i = -\ln(1 - U_i)$, where the U_i are the usual IID uniform (0,1) random variables.

i. What is the distribution of X_i? Of Y_i?

ii. What is $Cov(U_i, 1 - U_i)$?