Q1:

By Arden's theorem

Step-01:

Form a equation for each state-

- $q1 = \in + q1.a + q3.a$ ……(1)
- $q2 = q1.b + q2.b + q3.b$ ……(2)
- $q3 = q2.a$ …….(3)

Step-02:

Bring final state in the form $R = Q + RP$.

Using (3) in (2), we get-

$q2 = q1.b + q2.b + q2.a.b$

$q2 = q1.b + q2.(b + a.b)$ …….(4)

Using Arden's Theorem in (4), we get-

$q2 = q1.b.(b + a.b)^*$ …….(5)

Using (5) in (3), we get-

$q3 = q1.b.(b + a.b)^*.a$ …….(6)

Using (6) in (1), we get-

$q1 = \in + q1.a + q1.b.(b + a.b)^*.a.a$

$q1 = \in + q1.(a + b.(b + a.b)^*.a.a)$ …….(7)

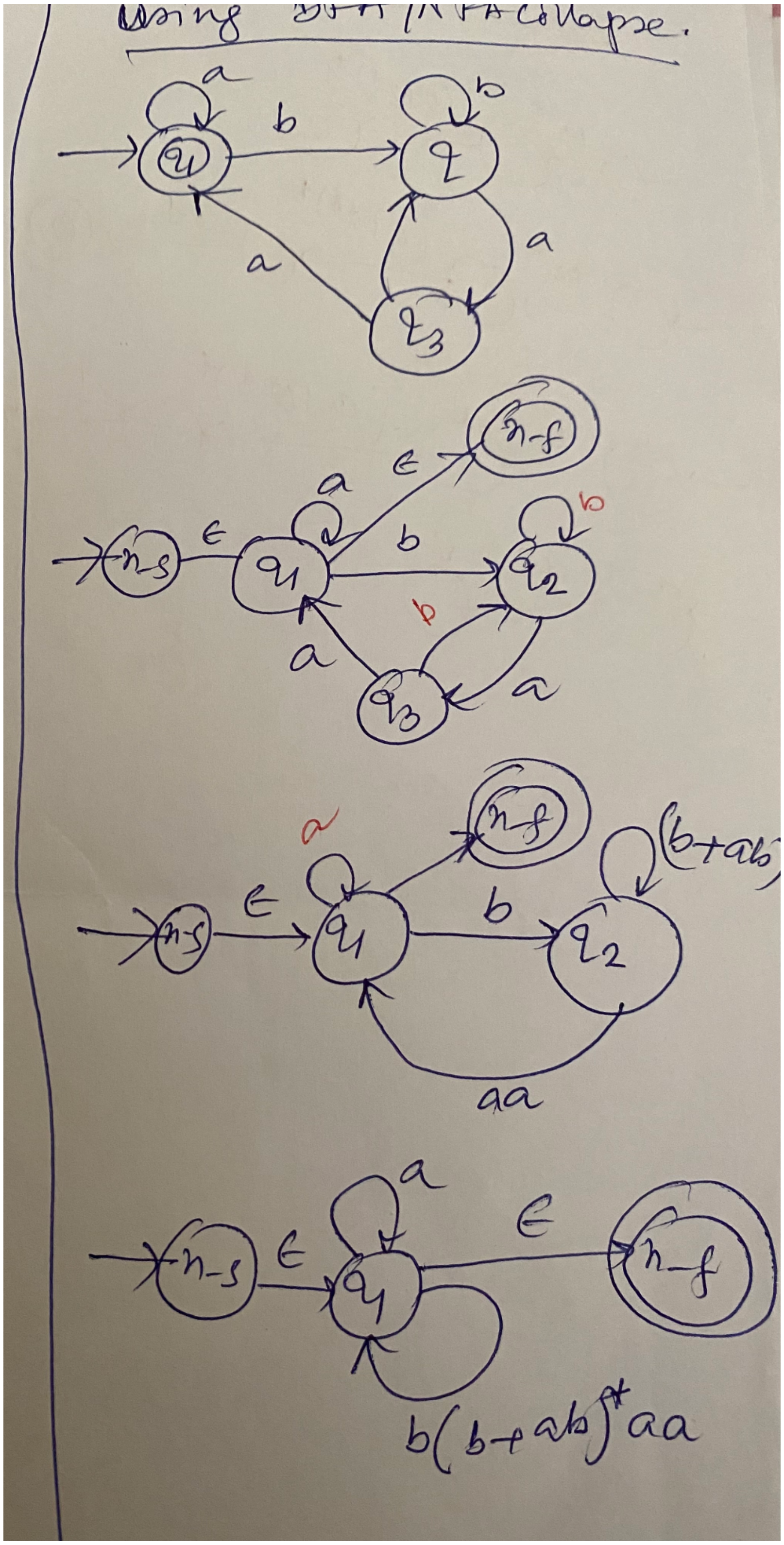
Using Arden's Theorem in (7), we get-

$q1 = \in.(a + b.(b + a.b)^*.a.a)^*$

$q1 = (a + b.(b + a.b)^*.a.a)^*$

Thus, Regular Expression for the given DFA = $(a + b(b + ab)^*aa)^*$

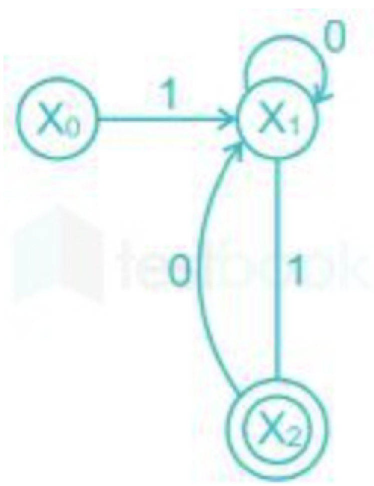
By DFA collapse:



Q2: (l+_)(l+d+_)*

Q3:

Equivalent FA is:



Hence regular expression is:

1(0 + 10)*1

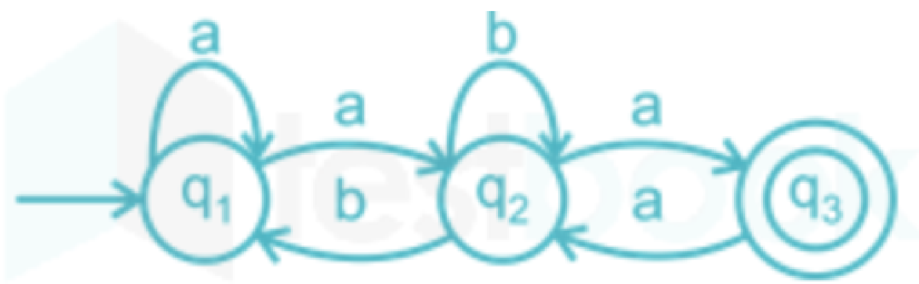
Or 10*1(00*1)*

Q4:

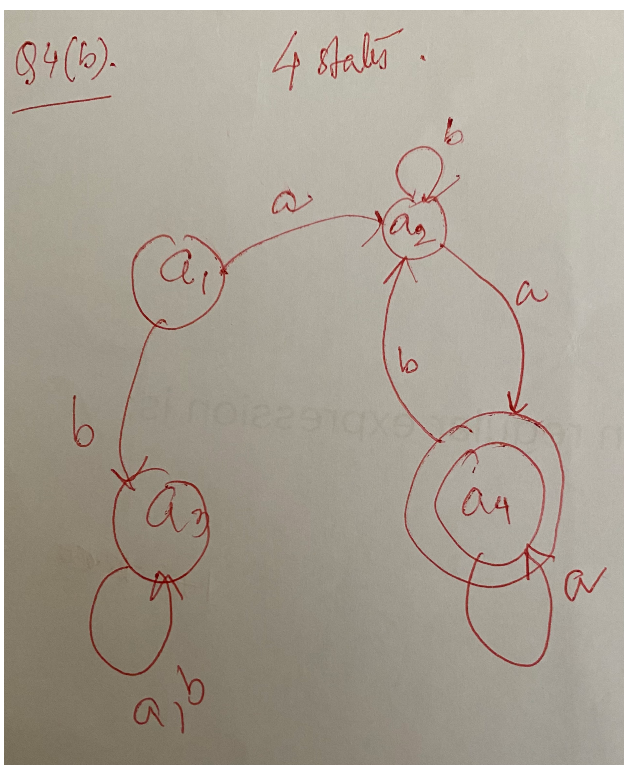
Answer: 3

Explanation:

The NFA for the given regular expression is:



The equivalent DFA has 4 states as follows:



Q5:

Answer: Let $A_3 = A_1 \cup A_2$, and we need to show that $L(G_3) = A_3$. To do this, we need to prove that $L(G_3) \subseteq A_3$ and $A_3 \subseteq L(G_3)$. To show that $L(G_3) \subseteq A_3$, first consider any string $w \in L(G_3)$. Since $w \in L(G_3)$, we have that $S_3 \overset{*}{\Rightarrow} w$. Since the only rules in R_3 with S_3 on the left side are $S_3 \to S_1 \mid S_2$, we must have that $S_3 \Rightarrow S_1 \overset{*}{\Rightarrow} w$ or $S_3 \Rightarrow S_2 \overset{*}{\Rightarrow} w$. Suppose first that $S_3 \Rightarrow S_1 \overset{*}{\Rightarrow} w$. Since $S_1 \in V_1$ and we assumed that $V_1 \cap V_2 = \emptyset$, the derivation $S_1 \overset{*}{\Rightarrow} w$ must only use variables in V_1 and rules in R_1, which implies that $w \in A_1$. Similarly, if $S_3 \Rightarrow S_2 \overset{*}{\Rightarrow} w$, then we must have that $w \in A_2$. Thus, $w \in A_3 = A_1 \cup A_2$, so $L(G_3) \subseteq A_3$.

To show that $A_3 \subseteq L(G_3)$, first suppose that $w \in A_3$. This implies $w \in A_1$ or $w \in A_2$. If $w \in A_1$, then $S_1 \overset{*}{\Rightarrow} w$. But then $S_3 \Rightarrow S_1 \overset{*}{\Rightarrow} w$, so $w \in L(G_3)$.

Similarly, if $w \in A_2$, then $S_2 \overset{*}{\Rightarrow} w$. But then $S_3 \Rightarrow S_2 \overset{*}{\Rightarrow} w$, so $w \in L(G_3)$. Thus, $A_3 \subseteq L(G_3)$, and since we previously showed that $L(G_3) \subseteq A_3$, it follows that $L(G_3) = A_3$; i.e., the CFG G_3 generates the language $A_1 \cup A_2$.