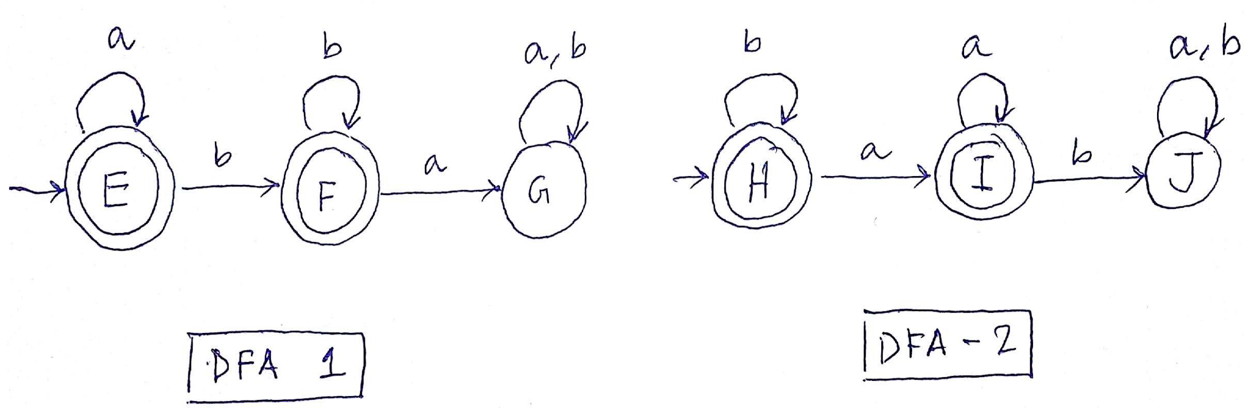
DFA and NFA

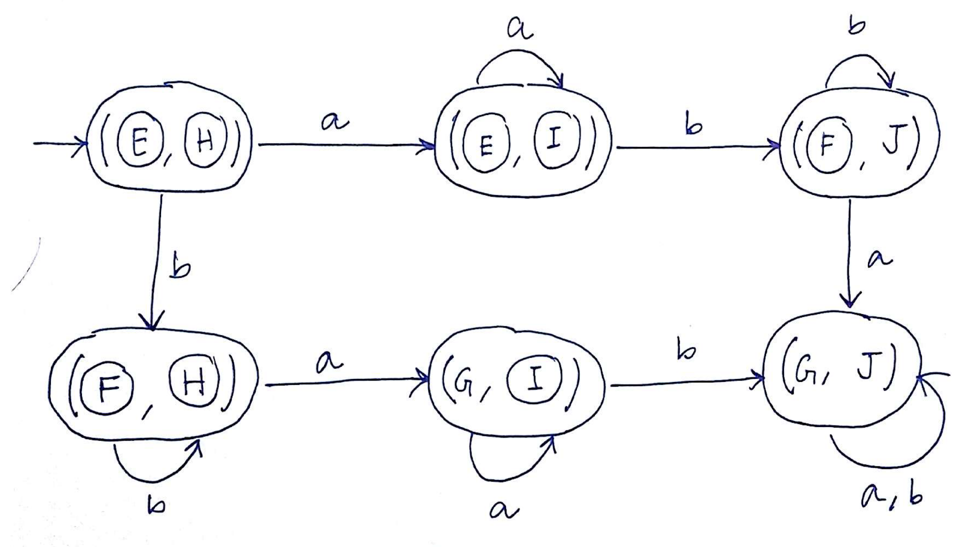
25 Jan 2017

Instructions : Write the answers to the problems neatly in loose sheets with your name and roll number. Submit to the TA at the end of the class.

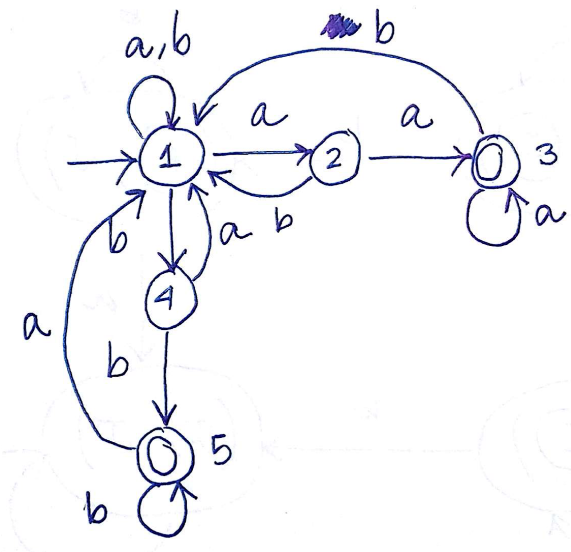
1. Compute the product of DFA 1 and DFA 2.

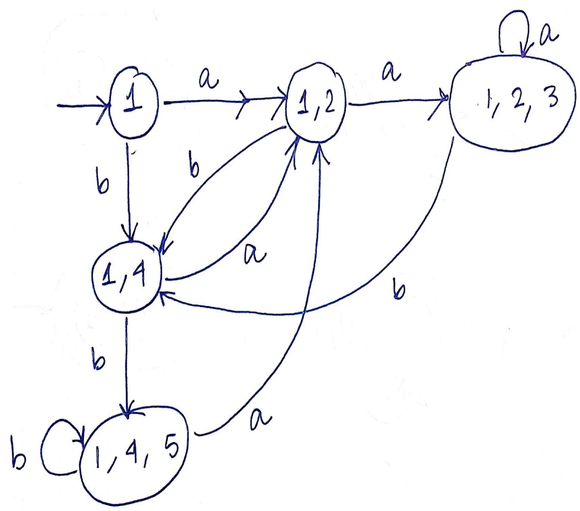


Solution



2. Let A be the set of all strings that end with two consecutive a's or two consecutive b's. Draw an NFA for the language A. Convert the NFA into a DFA.
Solution





3. For any language L over Σ, the *prefix closure* of L is defined as

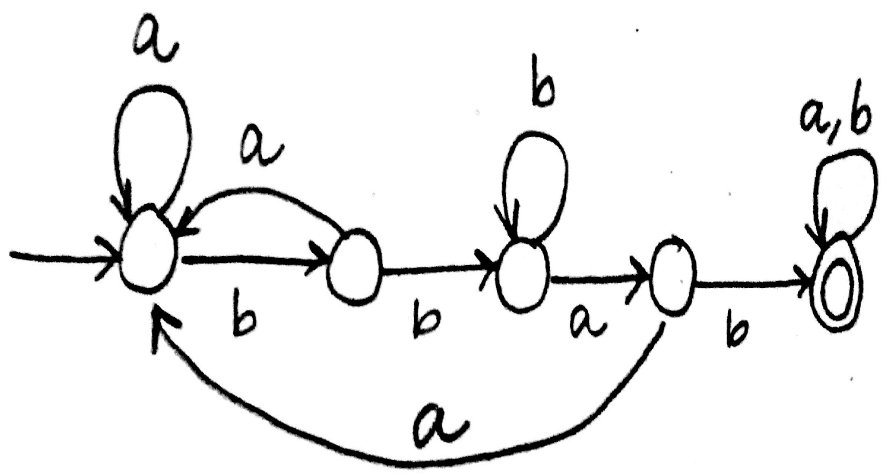
$$Pre(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$$

Prove that if L is regular then so is $Pre(L)$.

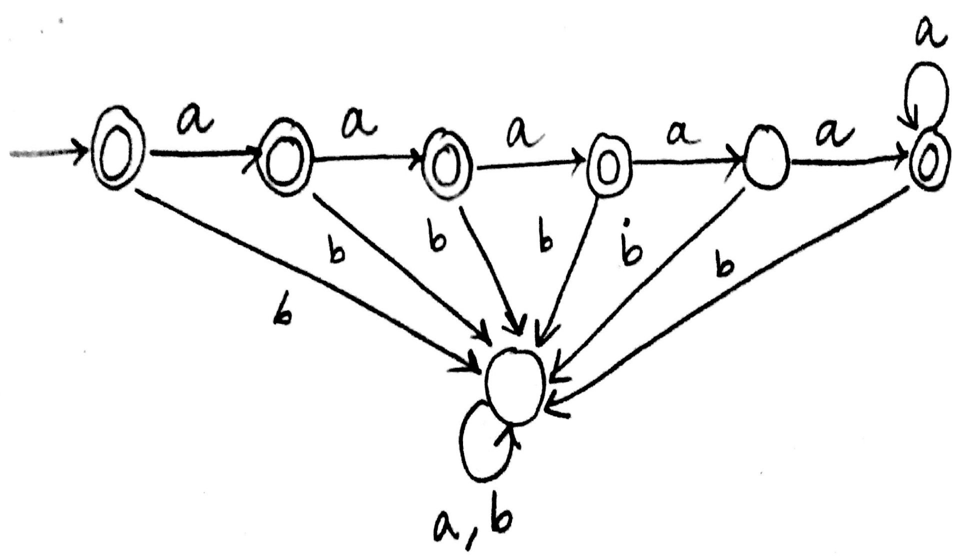
Solution Suppose L is recognized by a DFA D. We need to build a new DFA D' from D. D' has exact same states as D, but for each state s of D, if there's an accepting state reachable from s, then the corresponding state in D' will be an accepting state. And D' will recognize $Pre(L)$.

4. The corresponding DFA are:

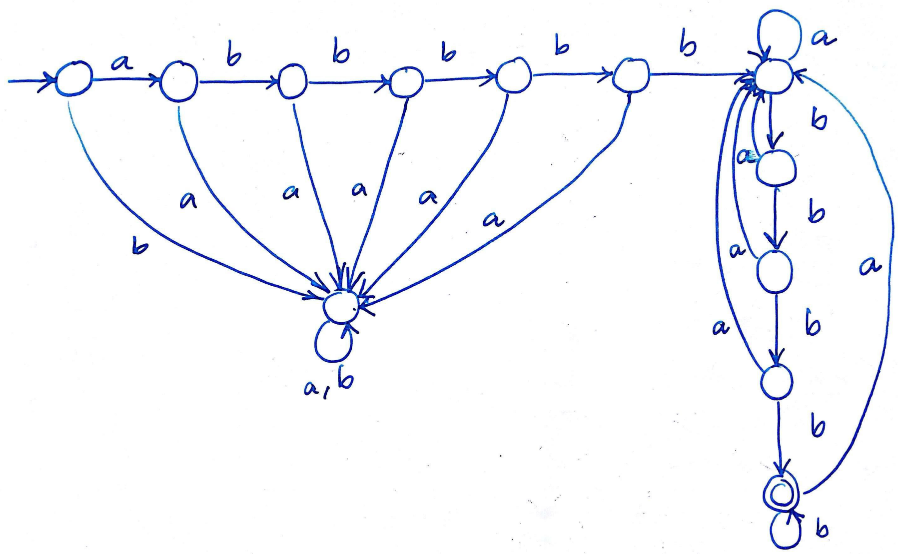
   (a) Language: L is the set of all strings containing $bbab$ as a substring.
   Following is the DFA:



   (b) Language: $L = \{a^n \mid n \neq 4\}$
   Following is the DFA:



   (c) Language: $L = \{ab^5wb^4 \mid w \in \{a,b\}^*\}$
   Following is the DFA:



5. RTP $\forall L_1, L_2$, $(L_1L_2)^R = L_2^R L_1^R$.
Solution Let $\sigma \in (L_1L_2)^R$. Hence $\sigma^R \in L_1L_2$. Hence $\sigma^R = xy$ such that $x \in L_1$ and $y \in L_2$. Now $\sigma = (xy)^R = y^R x^R \in L_2^R L_1^R$.
Let $\sigma \in L_2^R L_1^R$. Hence $\sigma = xy$ such that $x \in L_2^R$ and $y \in L_1^R$. Then $x^R \in L_2$ and $y^R \in L_1$. Hence $\sigma^R = y^R x^R \in L_1L_2$. Hence $\sigma \in (L_1L_2)^R$. Thus proved.

6. The corresponding grammars are:.

   (a) Language: $L_1 = \{a^n b^m \,|\, n \geq 0, m > n\}$
   Solution
   $S \to Ab$
   $A \to \lambda$
   $A \to aAb$
   $A \to Ab$

   (b) Language: $L_2 = \{a^n b^{2n} \,|\, n \geq 0\}$
   Solution
   $S \to \lambda$
   $S \to aSbb$

   (c) Language: $L_1 \backslash \overline{L}$ where $L = \{a^n b^{n-3} \,|\, n \geq 3\}$
   Since $L_1 = \{a^n b^m \,|\, n \geq 0, m > n\}$, so $L_1 \backslash \overline{L} = \emptyset$. Thus, it represents an empty language.
   Solution $S \to \lambda$.