Languages and Grammars

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1. Let $\Sigma = \{a, b\}$. Find a grammar that generates the language $l = \{a^n b^{n-3} | n \geq 3\}$.
   Solution The set of production rules P for the grammar would be
   $$P = \{S \to aaaA, A \to aAb|\lambda\}$$
   where the set of non-terminals is $V = \{S, A\}$ and S is the usual start symbol.

2. Give the description of the language generated by $S \to aSb|bSa|a$.
   Solution
   $$L = \{waw' \mid w,\, w' \in \{a, b\}^* \text{ and } w[i] \neq w'[|w'| - i + 1] \text{ for any i}\}$$

3. Let $\Sigma = \{a, b\}$. Find a grammar that generates the language $L = \{w|n_a(w) = 2n_b(w)\}$.
   Solution
   The production rules for the grammar would be
   $$\begin{aligned} S &\to AaAaAbA|AaAbAaA|AbAaAaA \\ A &\to AaAaAbA|AaAbAaA|AbAaAaA|\lambda \end{aligned}$$
   where the set of non-terminals is $V = \{S, A\}$ and S is the usual start symbol.

4. Show that the grammars $S \to SS|aSb|bSa|a$ and $S \to aSb|bSa|\lambda$ are not equivalent.
   Solution Let us call the grammars G_1 and G_2. Note that every string $\sigma \in L(G_2)$ is of length $2n$ for some n, i.e. all strings in $L(G_2)$ are of even length. This is because a derivation of any length k using G_2 generates strings of length $2k - 2$ (the last step in the derivation will apply $S \to \lambda$). However G_1 can generate odd length strings, e.g. $S \to aSb \to aab$.