Markov Chain (n-step TPM)

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1. For a cascade of binary communication channels, let $P(X_0 = 1) = \alpha$ and $P(X_0 = 0) = 1 - \alpha,\ \alpha \geq 0$, and assume that $a = b$. Compute the probability that a one was transmitted, given that a one was received after n th stage, i.e., compute $P(X_0 = 1|X_n = 1)$.

2. Let the state of the system are running system, system is under repair or system is idle. We observe the system only when it changes state. Define X_n as the state of the system after the n th state change, so that:

$$X_n = \begin{cases} 0 \ , & \text{if system is running} \\ 1 \ , & \text{if system is under repair} \\ 2 \ , & \text{if system is idle} \end{cases}$$

   Assume that the matrix P is

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

   Draw the state diagram and compute the matrix P^n.

3. Define the vector $z-$transform (generating function):

$$G_{\mathbf{p}}(z) = \sum_{n=0}^{\infty} \mathbf{p}^{(n)} z^n.$$

   Show that:

   (a) $G_{\mathbf{p}}(z) = \mathbf{p}^{(0)}[I - zP]^{-1}$, where I is the identity matrix. Thus P^n is the coefficient of z^n in the matrix power series expansion of $(I - zP)^{-1}$.

   (b) Using the above result, give an alternate proof for finding P^n in the two state Markov chain problem discussed in class.

4. Using Chapman-Kolomogorov equation and the principle of mathematical induction, show that $\sum_j p_{ij}^{(n)} = 1,\ \forall i$.