Tutorial 5

Formal Language and Automata Theory

February 9, 2023

Question 1

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SOLUTION: Select $s = 0^N 1^N 0^{2N} \in L$. We have that that $s = xyz$, for some x, y, and z, $y \in 0^+$ and $xy^i z \in L$ for all $i \geq 0$. But, letting $i = 0$, observe that $xz = 0^n 1^N 0^{2N}$, where $n < N$. This string is not in L, a contradiction.

Question 2

Use the pumping lemma to show that the following languages are not regular. With $\Sigma = \{0, 1, +, =\}$, $\mathsf{L} = \{\mathsf{x} = \mathsf{y} + \mathsf{z} : \mathsf{x}$, y, and z are binary numbers and x is the sum of y and z$\}$, e.g. $11 = 11 + 0 \in L$.

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HINT 2: Suppose for contradiction that L were regular. Let N be the pumping length, as guaranteed by the pumping lemma. Consider the string s which is $1^N = 1^N + 0$. By the pumping lemma, s can be partitioned into $s = xyz$ where $|xy| \leq N$, $|y| \geq 1$, and $xy^iz \in L_c$ for all $i \geq 0$. *Now, lets try to check the constraints ??*

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Use the pumping lemma to show that the following languages are not regular. With $\Sigma = \{0, 1, +, =\}$, L $= \{$x = y + z : x, y, and z are binary numbers and x is the sum of y and z$\}$, e.g. $11 = 11 + 0 \in L$.

SOLUTION: Suppose for contradiction that L were regular. Let N be the pumping length, as guaranteed by the pumping lemma. Consider the string s which is $1^N = 1^N + 0$. By the pumping lemma, s can be partitioned into $s = xyz$ where $|xy| \leq N$, $|y| \geq 1$, and $xy^iz \in L_c$ for all $i \geq 0$. But y then falls within the initial run of 1's and so xy^0z is a string $1^n = 1^N + 0$ for some $n < N$. This string is not in L, a contradiction.

Question 3

Use the pumping lemma to show that the following languages are not regular. $L = \{ww \mid w \in \{0,1\}^*\}$.

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Use the pumping lemma to show that the following languages are not regular. $L = \{ww \mid w \in \{0,1\}^*\}$.

HINT 2: Suppose for contradiction that L were regular. Let N be the pumping length, as guaranteed by the pumping lemma. *What can be an example that can be used?* There exists a N(pumping length) from pumping lemma Choose $s = 0^N 1 0^N$. For any x, y, z such that $s = xyz$ and $\mid y \mid \geq\ 1$ and $\mid xy \mid \leq\ N$, so y contains only 0s. *Show that for some i contradiction happens?*

Question 3

$L = \{ww \mid w \in \{0,1\}^*\}$.
Solution:

Choose some i such that $xy^iz \notin L$. Here, we choose $i = 2$.
$xy^2z = xyyz = 0^{N+|y|}10^N$,, but $\mid y \mid \neq 0$, so this string is not in L, contradicting the pumping lemma. Thus L is not regular.

Question 4

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Question 4

Use the pumping lemma to show that the following languages are not regular.
Palindromes over $\{0,1\}$.

HINT 2: Suppose that the set of palindromes were regular. Let n be the value from the pumping lemma. Consider the string $s = 0^N 110^N$. s is clearly a palindrome and $| w | \geq N$. *By the pumping lemma there must exist strings x,y and z satisfying the constraints?*

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Use the pumping lemma to show that the following languages are not regular.
Palindromes over $\{0,1\}$.

SOLUTION:
Let x,y and z be such that $s = xyz, \mid xy \mid \leq N$ and $\mid y \mid \geq 1$. Because $\mid xy \mid \leq N$, xy is entirely contained in the 0^N at the start of s. So x and y consist entirely of zeroes, i.e $x = 0^i$ and $y = 0^j$. Then z must look like $0^k 110^N$, where $i + j + k = N$.
Now consider xz. By the pumping lemma, xz must be in the language. But $xz = 0^i 0^k 110^N$. This is just $0^{i+k} 110^N$. Since $\mid y \mid \geq 1$, we know that $\mid j \mid \geq 1$, so $\mid i + k \mid \leq N$. This means that xz is not a palindrome because the number of zeros on the two ends don't match.
This means that the set of palindromes doesn't satisfy the pumping lemma and, thus, the set of palindromes cannot be regular.

Question 5

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HINT 1: Assume L is regular. From the pumping lemma there exists a p such that every $w \in L$ such that $|w| \geq p$ can be represented as xyz with $|y| = 0$ and $|xy| \leq p$. *What can be an example that can be used?*

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HINT 2: Assume L is regular. From the pumping lemma there exists a p such that every $w \in L$ such that $|w| \geq p$ can be represented as xyz with $|y| = 0$ and $|xy| \leq p$. Let us choose $b^p a^p$. *Now, how do we check the constraints?*

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TRAILING-COUNT as any string s followed by a number of a's equal to the length of s.

SOLUTION: Assume L is regular. From the pumping lemma there exists a p such that every $w \in L$ such that $|w| \geq p$ can be represented as xyz with $|y| = 0$ and $|xy| \leq p$. Let us choose $b^p a^p$. Its length is $2p \geq p$. Since the length of xy cannot exceed p, y must be of the form b^k for some $k > 0$. From the pumping lemma $b^{p-k} a^p$ must also be in L but it is not of the right form. Hence the language is not regular.