Induction on Strings

Instruction : Write the answers to the problems neatly in loose sheets with your name and roll number. Submit to the TA at the end of the class.

1. A palindrome can be defined as a string that reads the same forward and backward, or by the following definition.

   (a) ϵ is a palindrome.

   (b) If a is any symbol, then the string a is a palindrome.

   (c) If a is any symbol and x is a palindrome, then axa is a palindrome.

   (d) Nothing is a palindrome unless it follows from (a) through (c).

   Prove by induction that the two definitions are equivalent.

2. The strings of balanced parenthesis can be defined in at least two ways.

   (a) A string w over alphabet $\{(,)\}$ is balanced if and only if:

      i. w has an equal number of ('s as)'s, and

      ii. any prefix of w has at least as many ('s as)'s.

   (b)

      i. ϵ is balanced.

      ii. If w is a balanced string, then (w) is balanced.

      iii. If w and x are balanced strings, then so is wx.

      iv. Nothing else is a balanced string.

   Prove by induction on the length of a string that definitions (a) and (b) define the same class of strings.