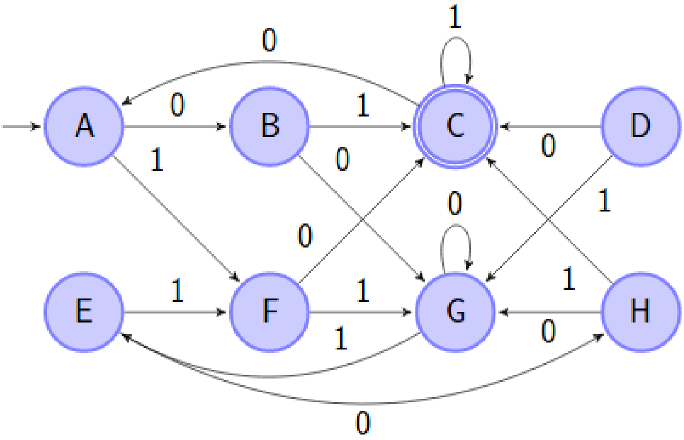
Tutorial 6

Formal Language and Automata Theory

February 13, 2023

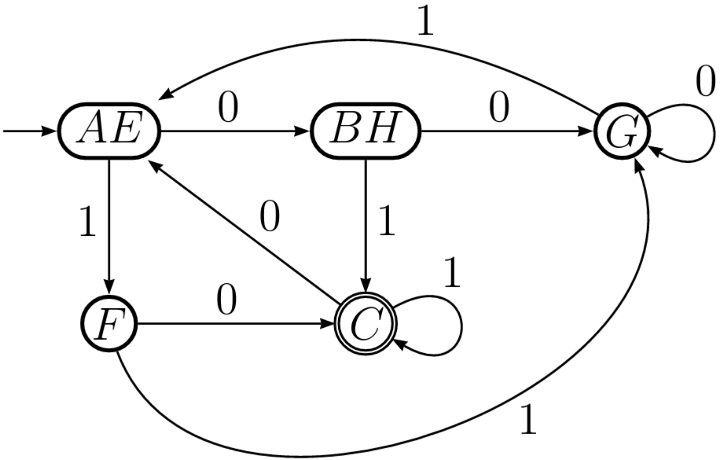
Question 1

Minimize the following DFA



Solution

HINT: Generate state transition diagram and equivalence classes to minimize it


AE
BH
G
F
C
0
0
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1
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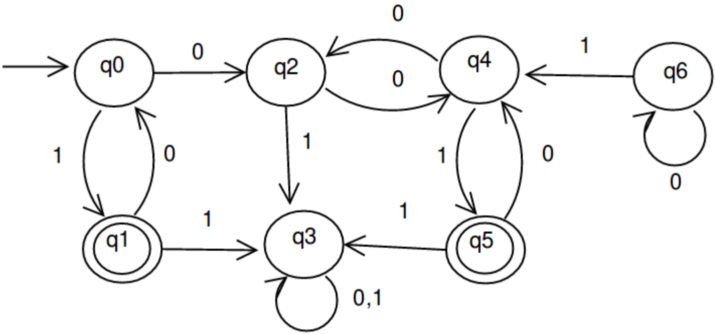
1
1

Question 2

Minimize the following DFA



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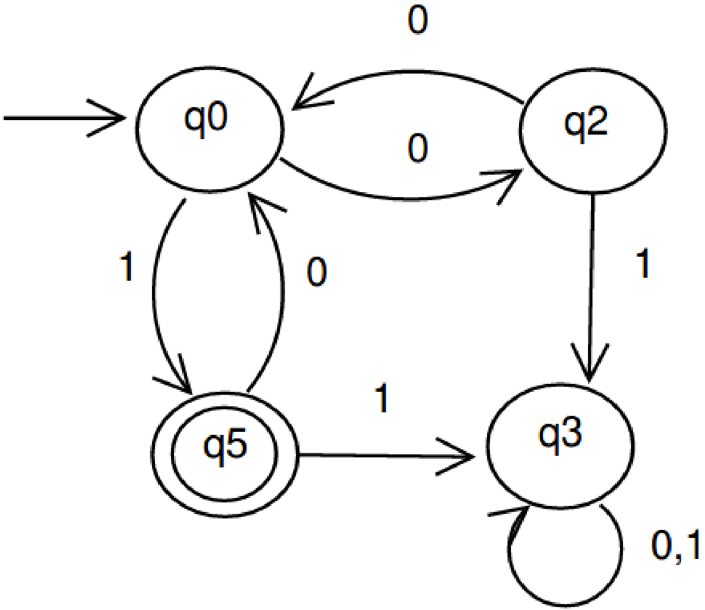
0
q0
q2
0
1
0
1
1
q5
q3
0,1

Question 3

Apply the concept of Myhill-Nerode relation to show that the following languages are not regular.
(a)$\{ww | w \in \Sigma^*\}$

Solution

HINT1: Consider any Myhill-Nerode relation $\equiv$ with respect to the language $\{ww | w \in \Sigma^*\}$. Let a^k and a^m be two arbitrary members of the set, where k and m are positive integers and $k \neq m$.

By right congruence, if $a^k \equiv a^m$ then $a^k ba^k b \equiv a^m ba^k b$. But $a^k ba^k b$ is in the given language $\{ww | w \in \Sigma^*\}$ while $a^m ba^k b$ is not. So, a^k and a^m are in two separate equivalence classes and it is not the case that $a^k \equiv a^m$. Since this is true for any k and m, the number of partition classes for any Myhill-Nerode relation $\equiv$ with respect to the language $\{ww | w \in \Sigma^*\}$ is infinite. Hence the language $\{ww | w \in \Sigma^*\}$ is nonregular.

(b) $\{a^{n^2} | n \geq 0\}$

Solution

HINT1: Consider a^{m^2}, a^{k^2} with $k > m$. Now, if $a^{m^2} \equiv a^{k^2}$ then $a^{m^2+2m+1} = a^{(m+1)^2} \equiv a^{k^2+2m+1}$ by right congruence.

HINT2: But $k^2 < k^2 + 2m + 1 < k^2 + 2k + 1 = (k+1)^2$. Hence, $a^{k^2+2m+1} \not\subset L$ but $a^{(m+1)^2} \subset L$. Hence, it must be the case that there is no Myhill-Nerode relation such that $a^{m^2} \equiv a^{k^2}$.

Question 4

Provide CFG for the following languages.

(a) $L = \{a^n b^m c^k \mid k = |n - m|\}$

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HINT: Split the problem into two cases: $n = k + m$ and $m = k + n$. Solve both cases separately. Then combine using a single starting state.

Question 4

Solution:

(a) $L = \{a^n b^m c^k \mid k = |n - m|\}$

First case:
$S_1 \rightarrow aS_1c|S_3|\epsilon$
$S_3 \rightarrow aS_3b|\epsilon$
Second case:
$S_2 \rightarrow aS_2bS_4|\epsilon$
$S_4 \rightarrow bS_4c|\epsilon$
Combine:
$S \rightarrow S_1|S_2$

Question 4

(b) $L = \{a^n b^m c^k \mid n = m \, or \, m \leq k\}$

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HINT 1: For the first case $n = m$ and k is arbitrary. Try to solve this independently.

HINT 2: In second case, n is arbitrary and $m \leq k$. Try to solve this independently. Finally, Combine both grammars with a single starting state.

Question 4

Solution:
(b) $L = \{a^n b^m c^k \mid n = m \, or \, m \leq k\}$

First case:
$S_1 \rightarrow AC$
$A \rightarrow aAb|\epsilon$
$C \rightarrow Cc|\epsilon$
Second case:
$S_2 \rightarrow BD$
$B \rightarrow aB|\epsilon$
$D \rightarrow bDc|E$
$E \rightarrow Ec|\epsilon$
Combine:
$S \rightarrow S_1|S_2$

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(c) $L = \{w \in \{a, b\}^*\}$, where the length of w is odd and its middle symbol is a

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HINT 1: There exists 4 possible combination of first and last symbols. Try to derive all four scenarios.

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HINT 1: There exists 4 possible combination of first and last symbols. Try to derive all four scenarios.

HINT 2: Single-length string is also possible.

Question 4

Solution:
(c) $L = \{w \in \{a, b\}^*\}$, where the length of w is odd and its middle symbol is a

$S \rightarrow a|aSa|aSb|bSa|bSb$

Question 4

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HINT 1: Divide the problem into 2 parts. Define rules for x.

HINT 2: Try to define w independently such that w^R ends with x.

Question 4

Solution:

(d) $L = \{w\#x\}$, where w^R is a substring for x for $w, x \in \{a, b\}^*$

$S \rightarrow WX$
$W \rightarrow 0W0|1W1|\#X$
$X \rightarrow 0X|1X|\epsilon$