Set-1
Sol^n

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067) — Code: Abxw1037x6cFep

Name: — Class Test-1

Roll No. — Marks:15 — Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Suppose a village has three soft drink shops (shop 1, shop 2, shop 3) along the main road in the sequence $\to$ shop 1 $\to$ shop 2 $\to$ shop 3 $\to$. A customer decides after every pint whether he moves to the next soft drink shop on the left or on the right, and he chooses each option with the same probability. If there is no soft drink shop in this direction, he stays where he is for another pint. Let X_n is the ~~position~~ (shop number) of customer on nth pint. (a) Determine the transition probability P_{22}. (b) Determine the long run probability that customer will be in shop 1.	(a) $P_{22} = 0$ (b) $\pi_1 = \frac{1}{3}$	1 1
2.	For which of the following transition probability matrices is the corresponding Markov chain irreducible? $(a)\ \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \end{array}$ ✓, $(b)\ \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.1 & 0.0 & 0.9 \\ 0.6 & 0.1 & 0.3 \\ 0.7 & 0.0 & 0.3 \end{pmatrix} \end{array}$ ✗, $(c)\ \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$ ✗ (i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b), (v) both (a) and (c), (vi) both (b) and (c), (vii) all (viii) None.	(i) Only (a)	1
3.	Consider a Markov chain with state space S$=\{0,1,2,3,4,5\}$ and tpm given by $\mathbf{P} = \begin{array}{c c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0.3 & 0 & 0.2 & 0.2 & 0.3 & 0 \\ 0.3 & 0 & 0 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 \end{pmatrix} \end{array}$ (a) Identify the classes of markov chain.(b) Determine the recurrent states (c) Determine the transient states.	(a) $\{0\}, \{1\}, \{2,3\}, \{4,5\}$ transient (c) recurrent (b)	1 1 1
4.	Consider a DTMC $\{X_n\}$, with state space $S = \{1,2,3\}$, and tpm $\mathbf{P} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 1/3 & 2/3 \\ 2/5 & 3/5 & 0 \\ 1/4 & 3/4 & 0 \end{pmatrix} \end{array}$ and initial distribution $P(X_0 = 1) = 2/5,\ P(X_0 = 2) = 1/5$ and $P(X_0 = 3) = 2/5$. Compute (a) $P(X_1 = 2, X_2 = 2, X_3 = 1 \mid X_0 = 1)$; (b) $P(X_3 = 1 \mid, X_2 = 2, X_1 = 2, X_0 = 1)$; (c) $P(X_1 = 3, X_2 = 2, X_3 = 1)$; (d) period of state 1	(a) $2/25 = 0.08$ (b) $2/5 = 0.4$ (c) $2/25 = 0.08$ (d) period 1 i.e. aperiodic	1 1 1 1
5.	A state i of the Markov chain of state space S, is called recurrent if the probability of ever returning to state i i.e. f_{ii}^* is	$= 1$ (equal to 1)	1
6.	The tpm $\begin{pmatrix} 0 & 1 \\ 0.5 & 0.5 \end{pmatrix}$ is regular? (Yes/No)	Yes	1
7.	John is in jail and has 3 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If John bets A dollars, he wins A dollars with probability 0.7 and loses A dollars with probability 0.3. Find the probability that he wins 8 dollars before losing all of his money if (a) he bets 1 dollar each time (timid strategy). (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). (c) Which strategy gives ~~Smith~~ John the better chance of getting out of jail?(timid/bold)	(a) 0.9223 (b) 0.637 (c) timid	1 1 1

Set-2

Sol^n

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067) Code: Abxw1037x6dFep

Name: Class Test-1

Roll No. Marks:15 Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Suppose a village has three soft drink shops (shop 1, shop 2, shop 3) along the main road in the sequence $\to$ shop 2 $\to$ shop 1 $\to$ shop 3 $\to$. A customer decides after every pint whether he moves to the next soft drink shop on the left or on the right, and he chooses each option with the same probability. If there is no soft drink shop in this direction, he stays where he is for another pint. Let X_n is the ~~position~~ (shop number) of customer on nth pint. (a) Determine the transition probability P_{22}. (b) Determine the long run probability that customer will be in shop 2.	(a) $\frac{1}{2}$ (b) $\frac{1}{3}$	1 1
2.	For which of the following transition probability matrices is the corresponding Markov chain irreducible? $(a)\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \end{array}$, $(b)\begin{pmatrix} 0.1 & 0.0 & 0.9 \\ 0.6 & 0.1 & 0.3 \\ 0.7 & 0.0 & 0.3 \end{pmatrix}$, $(c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b), (v) both (a) and (c), (vi) both (b) and (c), (vii) all (viii) None.	(i) only (a)	1
3.	Consider a Markov chain with state space S= $\{0,1,2,3,4,5\}$ and tpm given by $\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0.3 & 0 & 0.2 & 0.2 & 0.3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.3 & 0 & 0 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 \end{pmatrix} \end{array}$ (a) Identify the classes of markov chain.(b) Determine the recurrent states (c) Determine the transient states.	(a) $\{1\}, \{0,2,3,4,5\}$ recurrent (b) transient (c)	1 1 1
4.	Consider a DTMC $\{X_n\}$, with state space $S = \{1,2,3\}$, and tpm $\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 2/5 & 3/5 & 0 \\ 0 & 1/3 & 2/3 \\ 1/4 & 3/4 & 0 \end{pmatrix} \end{array}$ and initial distribution $P(X_0 = 1) = 2/5$, $P(X_0 = 2) = 1/5$ and $P(X_0 = 3) = 2/5$. Compute (a) $P(X_1 = 2, X_2 = 2, X_3 = 1 \mid X_0 = 1)$; (b) $P(X_3 = 1 \mid, X_2 = 2, X_1 = 2, X_0 = 1)$; (c) $P(X_1 = 3, X_2 = 2, X_3 = 1)$; (d) period of state 1	(a) 0 (b) 0 (c) 0 (d) period 1 i.e, aperiodic	1 1 1 1
5.	A state i of the Markov chain of state space S, is called transient if the probability of ever returning to state i i.e. f_{ii}^* is	<1 (less than 1)	1
6.	The tpm $\begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix}$ is regular? (Yes/No)	NO	1
7.	John is in jail and has 2 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If John bets A dollars, he wins A dollars with probability 0.7 and loses A dollars with probability 0.3. Find the probability that he wins 8 dollars before losing all of his money if (a) he bets 1 dollar each time (timid strategy). (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). (c) Which strategy gives ~~Smith~~ John the better chance of getting out of jail?(timid/bold)	(a) 0.8172 (b) 0.49 (c) timid	1 1 1

Set-3

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067)		Code: Abyz1037x6eFep	
Name:		Class Test-1	
Roll No.		Marks:15	Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Suppose a village has three soft drink shops (shop 1, shop 2, shop 3) along the main road in the sequence $\rightarrow$ shop 3 $\rightarrow$ shop 2 $\rightarrow$ shop 1 $\rightarrow$. A customer decides after every pint whether he moves to the next soft drink shop on the left or on the right, and he chooses each option with the same probability. If there is no soft drink shop in this direction, he stays where he is for another pint. Let X_n is the ~~position~~ (shop number) of customer on nth pint. (a) Determine the transition probability P_{22}. (b) Determine the long run probability that customer will be in shop 3.	(a) 0 (b) $\frac{1}{3}$	1 1
2.	For which of the following transition probability matrices is the corresponding Markov chain irreducible? $(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $(b)\begin{pmatrix} 0.1 & 0.0 & 0.9 \\ 0.6 & 0.1 & 0.3 \\ 0.7 & 0.0 & 0.3 \end{pmatrix}$, $(c)\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$ (i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b), (v) both (a) and (c), (vi) both (b) and (c), (vii) all (viii) None.	(i) only (a)	1
3.	Consider a Markov chain with state space S= $\{0,1,2,3,4,5\}$ and tpm given by $\mathbf{P} = \begin{array}{c c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0.3 & 0 & 0.2 & 0.2 & 0.3 & 0 \\ 0.3 & 0 & 0 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0.4 & 0.2 & 0.4 & 0 \end{pmatrix} \end{array}$ (a) Identify the classes of markov chain.(b) Determine the recurrent states (c) Determine the transient states.	(a) $\{0\}, \{1\}, \{2,3,4,5\}$ transient (c) recurrent (b)	1 1 1 1
4.	Consider a DTMC $\{X_n\}$, with state space $S = \{1,2,3\}$, and tpm $\mathbf{P} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 2/5 & 3/5 & 0 \\ 0 & 1/3 & 2/3 \\ 1/4 & 3/4 & 0 \end{pmatrix} \end{array}$ and initial distribution $P(X_0 = 1) = 2/5$, $P(X_0 = 2) = 1/5$ and $P(X_0 = 3) = 2/5$. Compute (a) $P(X_1 = 2, X_2 = 2, X_3 = 1 \mid X_0 = 1)$; (b) $P(X_3 = 1 \mid, X_2 = 2, X_1 = 2, X_0 = 1)$; (c) $P(X_1 = 3, X_2 = 2, X_3 = 1)$; (d) period of state 1	(a) 0 (b) 0 (c) 0 (d) period = 1 i.e. aperiodic	1 1 1 1 1
5.	A state i of the Markov chain of state space S, is called recurrent if the probability of ever returning to state i i.e. f_{ii}^* is	= 1 (equal to 1)	1
6.	The tpm $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ is regular? (Yes/No)	No	1
7.	John is in jail and has 3 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If John bets A dollars, he wins A dollars with probability 0.6 and loses A dollars with probability 0.4. Find the probability that he wins 8 dollars before losing all of his money if (a) he bets 1 dollar each time (timid strategy). (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). (c) Which strategy gives ~~Smith~~ John the better chance of getting out of jail?(timid/bold)	(a) 0.7323 (b) 0.504 (c) timid	1 1 1

Set→4

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067)		Code: Abql1037x6fFep	
Name:		Class Test-1	
Roll No.		Marks:15	Time: 1 hour
Q.No.	**Question**	**Answer**	**Marks**
1.	Suppose a village has three soft drink shops (shop 1, shop 2, shop 3) along the main road in the sequence → shop 1 → shop 3 → shop 2 →. A customer decides after every pint whether he moves to the next soft drink shop on the left or on the right, and he chooses each option with the same probability. If there is no soft drink shop in this direction, he stays where he is for another pint. Let X_n is the ~~position~~ (shop number) of customer on nth pint. (a) Determine the transition probability P_{22}. (b) Determine the long run probability that customer will be in shop 1.	(a) $\frac{1}{2}$ (b) $\frac{1}{3}$	1 1
2.	For which of the following transition probability matrices is the corresponding Markov chain irreducible? $(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $(b)\begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.1 & 0.0 & 0.9 \\ 0.6 & 0.1 & 0.3 \\ 0.3 & 0.7 & 0.0 \end{pmatrix} \end{array}$, $(c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b), (v) both (a) and (c), (vi) both (b) and (c), (vii) all (viii)None.	(iv) both (a) and (b)	1
3.	Consider a Markov chain with state space S= $\{0, 1, 2, 3, 4, 5\}$ and tpm given by $\mathbf{P} = \begin{array}{cc} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0.3 & 0 & 0.2 & 0.2 & 0.3 & 0 \\ 0 & 0 & 0.4 & 0.2 & 0.4 & 0 \\ 0.3 & 0 & 0 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.2 & 0.8 \end{pmatrix} \end{array}$ (a) Identify the classes of markov chain.(b) Determine the recurrent states (c) Determine the transient states.	(a) {0,2}, {1}, {3}, {4,5} transient (c) recurrent (b)	1 1 1.
4.	Consider a DTMC $\{X_n\}$, with state space $S = \{1, 2, 3\}$, and tpm $\mathbf{P} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 1/3 & 2/3 \\ 1/4 & 3/4 & 0 \\ 2/5 & 3/5 & 0 \end{pmatrix} \end{array}$ and initial distribution $P(X_0 = 1) = 2/5$, $P(X_0 = 2) = 1/5$ and $P(X_0 = 3) = 2/5$. Compute (a) $P(X_1 = 2, X_2 = 2, X_3 = 1 \mid X_0 = 1)$; (b) $P(X_3 = 1 \mid, X_2 = 2, X_1 = 2, X_0 = 1)$; (c) $P(X_1 = 3, X_2 = 2, X_3 = 1)$; (d) period of state 1	(a) $\frac{1}{16} = 0.0625$ (b) $1/4 = 0.25$ (c) $1/25 = 0.04$ (d) period = 1 i.e., aperiodic	1 1 1 1
5.	A state i of the Markov chain of state space S, is called transient if the probability of ever returning to state i i.e. f_{ii}^* is	< 1 (less than 1)	1
6.	The tpm $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is regular? (Yes/No)	No	1
7.	John is in jail and has 2 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If John bets A dollars, he wins A dollars with probability 0.6 and loses A dollars with probability 0.4. Find the probability that he wins 8 dollars before losing all of his money if (a) he bets 1 dollar each time (timid strategy). (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy). (c) Which strategy gives ~~Smith~~ John the better chance of getting out of jail?(timid/bold)	(a) 0.5781 (b) 0.36 (c) timid	1 1 1

Stochastic Process / Stochastic Process & Simulatio
(MA41017 / MA60067)

Set - 1

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(1)

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & \left[\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{array}\right] & \\ 3 & & & \end{array}$$

$$P_{22} = 0$$

$\pi_1 = \frac{1}{3}$ since doubly stochastic tpm

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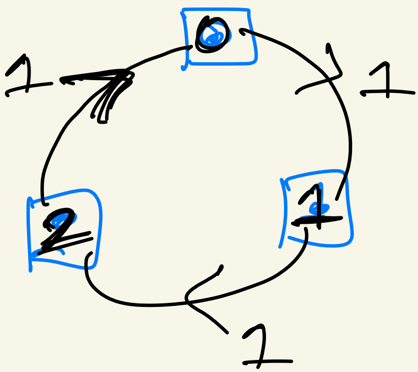
(2) Only (a) is irreducible



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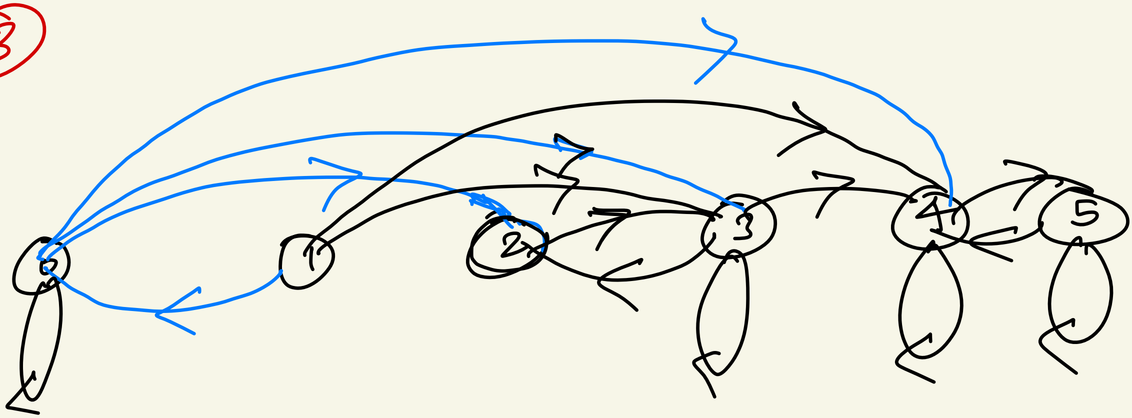
③



(a) classes are Markov Chain

$\{0\}, \{1\}, \{2,3\}, \{4,5\}$

(b) Recurrent state : $\{4,5\}$

c) Transient states: $\{0\}, \{1\}, \{2,3\}$

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④

(a) $P(X_1 = 2, X_2 = 2, X_3 = 1 \mid X_0 = 1)$

$= P(X_1 = 2 \mid X_0 = 1) \cdot P(X_2 = 2 \mid X_1 = 2)$

$\cdot P(X_3 = 1 \mid X_2 = 2)$

$= \frac{1}{3} \times \frac{3}{5} \times \frac{2}{5} = \frac{2}{25} = 2/25 = 0.08$

(b) $P(X_3=1 \mid X_2=2, X_1=2, X_0=1)$

$= P(X_3=1 \mid X_2=2)$

$= \frac{2}{5} = 0.4$

(c) $P(X_1=3, X_2=2, X_3=1)$

$= P(X_1=3) \cdot P(X_2=2 \mid X_1=3)$

$\cdot\, P(X_3=1 \mid X_2=2)$

$= \sum_{k=1}^{3} P(X_0=k) \cdot P(X_1=3 \mid X_0=k) \cdot$

$P(X_2=2 \mid X_1=3) \cdot P(X_3=1 \mid X_2=2)$

$= \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix} \cdot \begin{bmatrix} 2/3 \\ 0 \\ 0 \end{bmatrix} \times \frac{3}{4} \times \frac{2}{5}$

$= \frac{2}{5} \times \frac{2}{3} \times \frac{3}{4} \times \frac{2}{5} = \frac{2}{25} = 0.08$

(d) Period of state 1

$= \gcd\{n : p_{11}^{(n)} > 0\}$

$= \gcd\{2, 3, 4, \dots\}$

$= 1$

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(5) $f_{ii}^{*} = 1$

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(6) $P^2 = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

Yes, tpm is regular.

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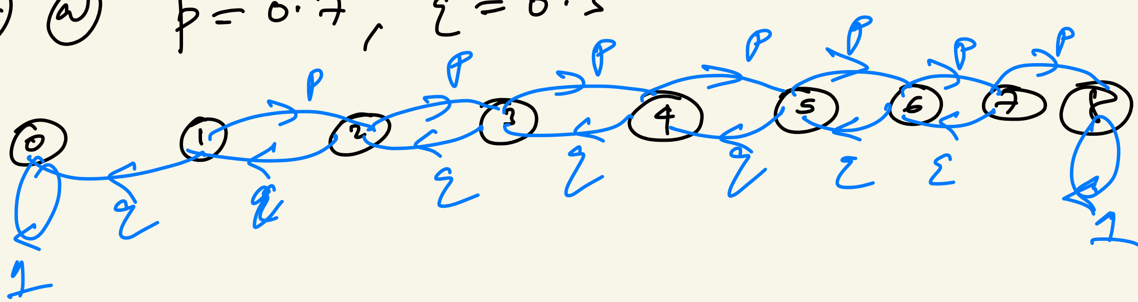
(7) (a) $p = 0.7$, $q = 0.3$



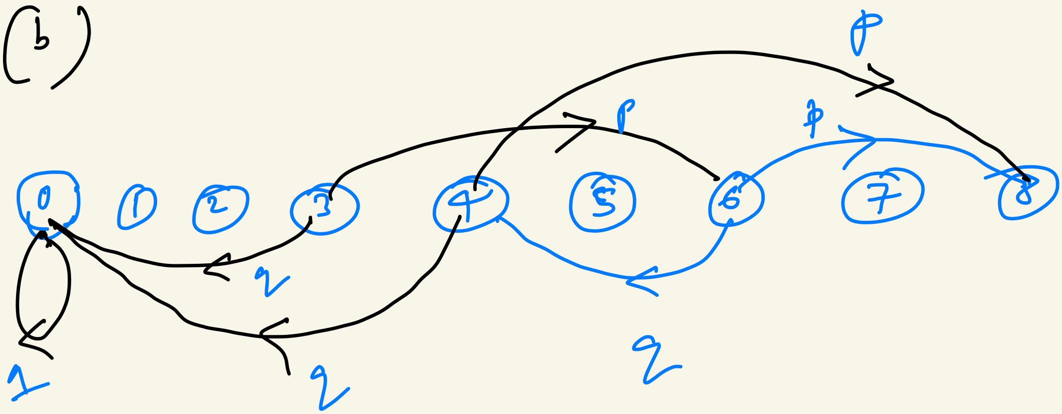
Gambler's ruin problem

$i = 3$, $N = 8$, $\frac{q}{p} = \frac{3}{7} \neq 1$

$$P_3 = \frac{1-\left(\frac{q}{p}\right)^i}{1-\left(\frac{q}{p}\right)^N} = \frac{1-\left(\frac{3}{7}\right)^3}{1-\left(\frac{3}{7}\right)^8} = \frac{0.9213}{0.9989}$$

$$= 0.9223$$

(b)



Probability of ultimate absorption in state 8, starting from state i, $i = 1, 2, \dots, 7$

$$u_3 = p \cdot u_6$$

$$u_4 = p$$

$$u_6 = p + q \cdot u_4$$

$$\Longrightarrow \quad u_4 = p$$

$$u_6 = p + q \cdot p$$

$$u_3 = p(p + qp)$$

If $p = 0.7 \Longrightarrow u_3 = 0.7(0.7 + 0.7 \times 0.3) = 0.637$

(c) timid.