Chapter 3 (Answers to Exercises)

3.1.1 0.

3.1.2 0.12, 0.12.

3.1.3 0.03.

3.1.4 0.02, 0.02.

3.1.5 0.025, 0.0075.

3.2.1 **(a)** $\mathbf{P}^2 = \begin{Vmatrix} 0.47 & 0.13 & 0.40 \\ 0.42 & 0.14 & 0.44 \\ 0.26 & 0.17 & 0.57 \end{Vmatrix}$.

(b) 0.13.

(c) 0.16.

3.2.2

n	0	1	2	3	4
$\Pr\{X_n = 0 \mid X_0 = 0\}$	1	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$

3.2.3 0.264, 0.254.

3.2.4 0.35.

3.2.5 0.27, 0.27.

3.2.6 0.42, 0.416.

3.3.1

$$\mathbf{P} = \begin{array}{c|ccccc} & -1 & 0 & 1 & 2 & 3 \\ \hline -1 & 0 & 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0 & 0.3 & 0.3 & 0.4 \\ 1 & 0.3 & 0.3 & 0.4 & 0 & 0 \\ 2 & 0 & 0.3 & 0.3 & 0.4 & 0 \\ 3 & 0 & 0 & 0.3 & 0.3 & 0.4 \end{array}$$

3.3.2 $P_{ii} = \left(\frac{i}{N}\right)p + \left(\frac{N-i}{N}\right)q$;

$P_{i,i+1} = \left(\frac{N-i}{N}\right)p$;

$P_{i,i-1} = \left(\frac{i}{N}\right)q$.

3.3.3

$$\mathbf{P} = \begin{array}{c|ccccc} & -1 & 0 & 1 & 2 & 3 \\ \hline -1 & 0 & 0 & 0.1 & 0.4 & 0.5 \\ 0 & 0 & 0 & 0.1 & 0.4 & 0.5 \\ 1 & 0.1 & 0.4 & 0.5 & 0 & 0 \\ 2 & 0 & 0.1 & 0.4 & 0.5 & 0 \\ 3 & 0 & 0 & 0.1 & 0.4 & 0.5 \end{array}$$

3.3.4

$$\begin{array}{c|cccccc} & -2 & -1 & 0 & 1 & 2 & 3 \\ \hline -2 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ -1 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 1 & 0.2 & 0.3 & 0.4 & 0.1 & 0 & 0 \\ 2 & 0 & 0.2 & 0.3 & 0.4 & 0.1 & 0 \\ 3 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \end{array}$$

3.3.5

$$\mathbf{P} = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 2 & 0 & 1 & 0 \end{array}$$

3.4.1 $v_{03} = 10$.

3.4.2 **(a)** $u_{10} = \frac{1}{4}$;

(b) $v_1 = \frac{5}{2}$.

3.4.3 **(a)** $u_{10} = \frac{40}{105}$;

(b) $v_1 = \frac{10}{3}$.

3.6.1 **(a)** $u_{35} = \frac{3}{5}$;

(b) $u_{35} = \left[1 - \left(\frac{q}{p}\right)^3\right] \Big/ \left[1 - \left(\frac{q}{p}\right)^5\right]$.

3.6.2 $u_{10} = 0.65$.

3.6.3 $v = 2152.777\ldots$.

3.6.4 $v_1 = 2.1518987$.

3.7.1 $\mathbf{W} = \begin{Vmatrix} \frac{20}{11} & \frac{25}{11} \\ \frac{10}{11} & \frac{40}{11} \end{Vmatrix}$.

(a) $u_{10} = \frac{9}{22}$;

(b) $w_{11} = \frac{20}{11}$; $w_{12} = \frac{25}{11}$.

3.7.2 $\mathbf{W} = \begin{Vmatrix} \frac{100}{79} & \frac{70}{79} \\ \frac{30}{79} & \frac{100}{79} \end{Vmatrix}$.

(a) $u_{10} = \frac{30}{79}$;

(b) $w_{11} = \frac{100}{79}$; $w_{12} = \frac{70}{79}$.

3.8.1 $M(n) = 1$, $V(n) = n$.

3.8.2 $\mu = b + 2c$; $\sigma^2 = b + 4c - (b + 2c)^2$.

3.8.3

n	1	2	3	4	5
u_n	0.5	0.625	0.695	0.742	0.775

3.8.4 $M(n) = \lambda^n$, $V(n) = \lambda^n\left(\frac{1-\lambda^n}{1-\lambda}\right)$, $\lambda \neq 1$.

3.9.1

n	1	2	3	4	5
u_n	0.333	0.480	0.564	0.619	0.658

$u_\infty = 0.82387$.

3.9.2 $\varphi(s) = p_0 + p_2 s^2$.

3.9.3 $\varphi(s) = p + qs^N$.

3.9.4 $\frac{\varphi(s) - \varphi(0)}{1 - \varphi(0)}$.

Chapter 4 (Answers to Exercises)

4.1.1 $\pi_0 = \frac{10}{21}$, $\pi_1 = \frac{5}{21}$, $\pi_2 = \frac{6}{21}$.

4.1.2 $\pi_0 = \frac{31}{66}$, $\pi_1 = \frac{16}{66}$, $\pi_2 = \frac{19}{66}$.

4.1.3 $\pi_1 = \frac{3}{13}$.

4.1.4 2.94697.

4.1.5 $\pi_0 = \frac{10}{29}$, $\pi_1 = \frac{5}{29}$, $\pi_2 = \frac{5}{29}$, $\pi_3 = \frac{9}{29}$.

4.1.6 $\pi_0 = \frac{5}{14}$, $\pi_1 = \frac{6}{14}$, $\pi_2 = \frac{3}{14}$.

4.1.7 $\pi_0 = \frac{140}{441}$, $\pi_1 = \frac{40}{441}$, $\pi_2 = \frac{135}{441}$, $\pi_3 = \frac{126}{441}$.

4.1.8 $\pi_u = \frac{4}{17}$.

4.1.9 $\pi_0 = \frac{2}{7}, \pi_1 = \frac{3}{7}, \pi_2 = \frac{2}{7}$.

4.1.10 $\pi_{\text{late}} = \frac{17}{40}$.

4.2.1 $\pi_s = \frac{8}{9}$.

4.2.2 One facility: $\Pr\{\text{Idle}\} = \frac{q^2}{1+p^2}$;

Two facilities: $\Pr\{\text{Idle}\} = \frac{1}{1+p+p^2}$.

4.2.3

(a)

p	0	0.02	0.04	0.06	0.08	0.10
AFI	0.10	0.11	0.12	0.13	0.14	0.16
AOQ	0	0.018	0.036	0.054	0.072	0.090

(b)

p	0	0.02	0.04	0.06	0.08	0.10
AFI	0.20	0.23	0.27	0.32	0.37	0.42
AOQ	0	0.016	0.032	0.048	0.064	0.080

4.2.4

p	0.05	0.10	0.15	0.20	0.25
R_1	0.998	0.990	0.978	0.962	0.941
R_2	0.998	0.991	0.981	0.968	0.952

4.2.5 $\pi_A = \frac{1}{5}$.

4.2.6 $\pi_0 = \frac{1}{3}$.

4.2.7 **(a)** 0.6831;

(b) $\pi_1 = \pi_2 = \frac{10}{21}, \pi_3 = \frac{1}{21}$;

(c) $\pi_3 = \frac{1}{21}$.

4.2.8 $\pi_3 = \frac{8}{51}$.

4.3.1 $\left\{n \geq 1; P_{00}^{(n)} > 0\right\} = \{5, 8, 10, 13, 15, 16, 18, 20, 21, 23, 24, 25, 26, 28, \ldots\}$

$d(0) = 1, P_{5,7}^{(37)} = 0, P_{i,j}^{(38)} > 0$ for all i, j.

4.3.2 Transient states: $\{0, 1, 3\}$.
Recurrent states: $\{2, 4, 5\}$.

4.3.3 **(a)** $\{0, 2\}, \{1, 3\}, \{4, 5\}$;
(b) $\{0\}, \{5\}, \{1, 2\}, \{3, 4\}$.

4.3.4 $\{0\}, d = 1$;
$\{1\}, d = 0$;
$\{2, 3, 4, 5\}, d = 1$.

4.4.1 $\pi_k = p^k / \left(1 + p + p^2 + p^3 + p^4\right)$ for $k = 0, \ldots, 4$.

4.4.2 **(a)** $\pi_0 = \frac{1449}{9999}$.

(b) $m_{10} = \frac{8550}{1449}$.

4.4.3 $\pi_0 = \pi_1 = 0.2, \pi_2 = \pi_3 = 0.3$.

4.5.1 $\lim P_{00}^{(n)} = \lim P_{10}^{(n)} = 0.4$;
$\lim P_{20}^{(n)} = \lim P_{30}^{(n)} = 0$;
$\lim P_{40}^{(n)} = 0.4$.

4.5.2 **(a)** $\frac{3}{11}$, **(e)** $\frac{3}{11}$,
(b) 0, **(f)** X,
(c) $\frac{2}{33}$, **(g)** $\frac{1}{3}$,
(d) $\frac{2}{9}$, **(h)** $\frac{4}{27}$.