Assignment (P.P.)

$N(t) = \#$ of customers arriving $(0,t] \sim P.P.(\lambda)$

$\lambda = 2$ customers per min.

1. $P(N(5)=8) = \dfrac{e^{-\lambda\times 5}(\lambda\times 5)^8}{8!} = \dfrac{e^{-10}(10)^8}{8!} = 0.1125$

2. $E(N(3.2)) = \lambda t = 2\times 3.2 = 6.4$

3. $P(N(1)>2) = 1 - P(N(1)=0) - P(N(1)=1) - P(N(1)=2)$

$$= 1 - \sum_{i=0}^{2} \frac{e^{-2\times 1}\, 2^i}{i!} = 0.3233$$

4. $\begin{array}{r} 6.7 \\ -\,3.3 \\ \hline 3.4 \end{array}$ $\qquad P(N(3.4)=4)$

5. $\begin{array}{r} 17.8 \\ -16.0 \\ \hline 1.8 \end{array}$ $\qquad \lambda t = 2\times 1.8 = 3.6$

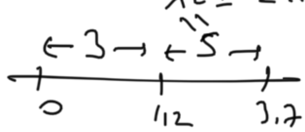
6. $P(N(0,12.2]=7 \mid N(0,8)=5)$

$$= \frac{P(N(0,8)=5,\ N[8,12.2) = 7-5)}{P(N(0,8)=5)}$$

$$= \frac{P(N(8)=5)\ \ P(N(4.2)=2)}{P(N(8)=5)}$$

$$= \frac{e^{-2\times 4.2}(2\times 4.2)^2}{2!} = 0.0079$$

7. $3+5=8$ $\qquad \lambda t = 2\times 2.5 = 5$



8. $P\left(N\left(\frac{18}{60}\right)=2\right) = \dfrac{e^{-\frac{18}{60}\times 2}\left(\frac{18}{60}\times 2\right)^2}{2!}$

$$= \frac{e^{-3/5}(3/5)^2}{2!}$$

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9. time 2 , $\lambda = 2$

$\lambda t = 2 \times 2 = 4$

10.

$\leftarrow 2 \rightarrow \leftarrow 3 \rightarrow \leftarrow 3 \rightarrow$

0 | 1 | 2.2 | 3.7

$\lambda t = 2 \times 1$ $\qquad$ $\lambda t = 1.5 \times 2 = 3$

$2 + 3 + 3 = 8$

11. $P(0.2 < T_3 < 1.3) = F(1.3) - F(0.2)$

$= \cancel{1} - e^{-1.3 \times 2} - \cancel{1} + e^{-0.2 \times 2}$

$= e^{-0.4} - e^{-2.6}$

$= 0.5960$

12. $\frac{1}{\lambda} = \frac{1}{2} = 0.5$

13. $P(T_4 > 1.7) = e^{-2 \times 1.7} = 0.0334$

14. $P(T_{11} < 2.1 \mid T_{11} > 1.3) = P(T_{11} < 0.8)$

$= 1 - e^{-0.8 \times 2} = 0.798$

15. $0.7 + E(T_{11}) = 0.7 + 0.5 = 2.1$

16. indep $\begin{cases} K \sim \exp(\lambda_1 = \frac{1}{10}) \\ B \sim \exp(\lambda_2 = \frac{1}{7}) \end{cases}$ $\qquad B \overset{\text{pdf}}{\sim} f_B$

$P(K > B) = \int_0^\infty P(K > B \mid B = t)\, f_B(t)\, dt$

$= \int_0^\infty P(K > t) \times \frac{1}{7} e^{-t/7}\, dt$

$= \int_0^\infty e^{-t/10} \times \frac{1}{7} e^{-t/7}\, dt$

$= \frac{1}{7} \int_0^\infty e^{-\frac{17}{70}t}\, dt = \frac{1}{7} \times \frac{70}{17} = \frac{10}{17}$

$= 0.5882$

17. $T = \min(T_1, T_2, T_3) \sim \exp(\alpha_1 + \alpha_2 + \alpha_3 = 0.75)$

since indep $T_i \sim \exp(\alpha_i)$ $i = 1, 2, 3$

$N(t) \sim P.P.(\overset{0.75}{\overset{\shortparallel}{\lambda}})$

$P(T > 1) = P(N(1) = 0)$

$= e^{-0.75}$

$= 0.4724$

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