1. A rat is put into the linear maze a shown below:

0 Shock	1	2 Rat is here	3	4	5 food

At each step the rat moves to the right with probability 3/4 and to the left with probability 1/4. What is the probability that the rat finds the food before getting shocked? Answer 0.892

2. Consider the Markov chain whose TPM is given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 0.4 & 0 & 0.6 \\ 0 & 0 & 1 \end{array} \right\| \end{array}$$

Starting in 1, determine the probability that the Markov chain ends in state 0. Answer 0.4

3. The probability of the thrower winning in the dice game called "craps" is $p = 0.49$. Suppose Player A is the thrower and begins the game with \$ 5, and Player B, his opponent, begins with \$ 10. What is the probability that Player A goes bankrupt before Player B? Assume that the bet is \$ 1 per round.

4. Define the Gambler's ruin problem. Also derive the probability $P_i,\ i = 0,\ 1,\ \ldots, N$ that starting with i units, the gamblers fortune will eventually reach N units before reaching 0.

5. Let state space be $\{1, 2\}$. Classify the states (i.e., check whether Markov chain is ergodic or not) and compute the stationary distribution of $\{X_n\}$ and the limits $\lim_{n\to\infty} P(X_n = 1)$ where the initial distribution is $\mathbf{p}^{(0)} = (1/4, 3/4)$ for the cases when the TPM is

(i) $\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$ (ii) $\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

6. Suppose gambler has the initial capital of N dollars and aims to win (extra) M dollars and then stop playing. Of course, if he goes bankrupt before achieving the goal, he also stop playing. He find a slot machine which gives "odds" of p of winning a dollar for every dollar one put in (and with probability $q = 1 - p$ the dollar is lost). We assume that successive games are independent. (i) What is the probability that the gambler will win the desired extra M dollar before he goes bankrupt? (ii) If the slot machine was fair (p=q=1/2), what would be the probability of winning extra 10 dollar when starting with dollar 50.

7. An NCD system has the discount classes: E_0 (no discount), E_1 (20% discount) and E_2 (40% discount). Movement in the system is determined by the rule whereby one steps back one discount level (or stays in E_0) with one claim in a year, and returns to a level of no discount if more than one claim is made. A claim-free year results in a step up to a higher discount level (or one remains in class E_2 if already there). This NCD system has only three discount classes as shown below:

NCD class	E_0	E_1	E_2
% discount	0	20	40
annual preminum	100	80	60

If we suppose that for anyone in this scheme the probabilities of one claim in a year is 0.2 while the probability of two or more claims is 0.1. Find (i) the TPM for this system (ii) In long run, what proportion of time is the process in each of the three discount classes. (iii) Find the average annual premium paid.

8. A stock price stays constant for 1 unit of time and then it may either go up 1 unit with probability 0.4 or go down 1 unit with probability 0.6. Suppose the current price is 50. Find the probability that the price will rise up to 60 before falling to 40.