Chapter 5 (Answer to Excercises)

5.1.1 **(a)** e^{-2};
(b) e^{-2}.

5.1.2 $(p_k/p_{k-1}) = \lambda/k, \quad k = 0, 1, \ldots$.

5.1.3 $\Pr\{X = k|N = n\} = \binom{n}{k} p^k (1-p)^{n-k}, \quad p = \frac{\alpha}{\alpha+\beta}$.

5.1.4 **(a)** $\frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, \ldots$;
(b) $\Pr\{X(t) = n+k|X(s) = n\} = \frac{[\lambda(t-s)]^k e^{-\lambda(t-s)}}{k!}$.
$E[X(t)X(s)] = \lambda^2 ts + \lambda s$.

5.1.5 $\Pr\{X = k\} = (1-p)p^k$ for $k = 0, 1, \ldots$ where $p = 1/(1+\theta)$.

5.1.6 **(a)** e^{-12};
(b) Exponential, parameter $\lambda = 3$.

5.1.7 **(a)** $2e^{-2}$;
(b) $\frac{64}{3}e^{-6}$;
(c) $\binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$;
(d) $\frac{32}{3}e^{-4}$.

5.1.8 **(a)** $5e^{-2}$;
(b) $4e^{-4}$;
(c) $\frac{1-3e^{-2}}{1-e^{-2}}$.

5.1.9 **(a)** 4;
(b) 6;
(c) 10.

5.2.1

k	0	1	2
(a)	0.290	0.370	0.225
(b)	0.296	0.366	0.221
(c)	0.301	0.361	0.217

5.2.2 Law of rare events, e.g., (a) Many potential customers who could enter store, small probability for each to actually enter.

5.2.3 The number of distinct pairs is large; the probability of any particular pair being in sample is small.

5.2.4 $\Pr\{\text{Three pages error free}\} \approx e^{-12}$.

5.3.1 e^{-6}.

5.3.2 **(a)** $e^{-6} - e^{-10}$;
(b) $4e^{-4}$.

5.3.3 $\frac{1}{4}$.

5.3.4 $\binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 = \frac{80}{243}$.

5.3.5 $\binom{n}{m}\left(\frac{t}{T}\right)^m\left(1-\frac{t}{T}\right)^{n-m}, \quad m = 0, 1, \ldots, n.$

5.3.6 $F(t) = \left(1 - e^{-\lambda t}\right)^n$.

5.3.7 $t + \frac{2}{\lambda}$.

5.3.8 $\binom{12}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^7$.

5.3.9 $\Pr\{W_r \le t\} = 1 - \sum_{k=0}^{r-1} \frac{(\lambda t)^k e^{-\lambda t}}{k!}$.

5.4.1 $\frac{1}{n+1}$.

5.4.2 $\frac{1}{4}$.

5.4.3 $\frac{5}{2}$.

5.4.4 See equation (5.23).

5.4.5 $\left[1 - \frac{1-e^{-\alpha}}{\alpha}\right]^5$.

5.5.1 0.9380.

5.5.2 0.05216.

5.5.3 0.1548.

5.6.1 0.0205.

5.6.2 Mean $= \frac{\lambda t}{\theta}$, Variance $= \frac{2\lambda t}{\theta^2}$.

5.6.3 $\frac{e^{-\lambda G(z)t} - e^{-\lambda t}}{1 - e^{-\lambda t}}$.

5.6.4 **(a)** $\frac{1}{9}$;

(b) $\frac{11}{27}$.

5.6.5 $\Pr\{M(t) = k\} = \frac{\Lambda(t)^k e^{-\Lambda(t)}}{k!}$, where $\Lambda(t) = \lambda \int_0^t [1 - G(u)]\,du$.