Tutorial 4

Formal Language and Automata Theory

February 2, 2023

Question 1

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Hint 2: Think of how you can reverse the order of each production rules

Question 1

Provide an algorithm for converting a left linear grammar to a right linear grammar.

Solution:

Our algorithm assumes that the left linear grammar doesn't have any rule with the start symbol on the right hand side. If the left linear grammar has a rule with the start symbol S on the right hand side, simply add this rule: $S_0 \implies S$ to the given grammar and use the algorithm on the modified grammar with start symbol S_0

- Let S denote the start symbol
- Let A, B denote non-terminal symbols
- Let p denote zero or more terminal symbols
- Let ϵ denote the empty symbol

Provide an algorithm for converting a left linear grammar to a right linear grammar.

Solution:

1. If the left linear grammar has a rule $S \to p$, then make that a rule in the right linear grammar
2. If the left linear grammar has a rule $A \to p$, then add the following rule to the right linear grammar: $S \to pA$
3. If the left linear grammar has a rule $B \to Ap$, add the following rule to the right linear grammar: $A \to pB$
4. If the left linear grammar has a rule $S \to Ap$, then add the following rule to the right linear grammar: $A \to p$

Question 2

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Hint 2: Try to prove (i) $BB \subseteq B$ if $B = B+$ holds, and (ii) $B = B+$ if $BB \subseteq B$ holds, from the closure property

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Solution:

If part : Assume that $B = B+$ and show that $BB \subseteq B$. For every language $BB \subseteq B+$ holds, so if B = B+, then $BB \subseteq B$.

Only If part : Assume that $BB \subseteq B$ and show that $B = B+$. For every language $B \subseteq B+$, so we need to show only $B+ \subseteq B$. If $w \in B+$, then $w = x_1 x_2 ... x_k$ where each $x_i \in B$ and $k \geq 1$. Because $x_1, x_2 \in B$ and $BB \subseteq B$, we have $x_1 x_2 \in B$. Similarly, because $x1x2$ is in B and x_3 is in B, we have $x_1 x_2 x_3 \in B$. Continuing in this way $x_1 x_2 ... x_k \in B$. Hence $w \in B$, and so we may conclude that $B+ \subseteq B$

Question 3

The symmetric difference of two sets S1 and S2 is defined as:

$$S1 \theta S2 = \{x : x \in S1 \mid x \in S2, \text{ but x is not in both S1 and S2}\}$$

Show that the family of regular languages is closed under symmetric difference

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Hint 1: Try to express in terms of a combination of set operations

Hint 2: Apply closure properties of different set operations

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Show that the family of regular languages is closed under symmetric difference

Solution:

Let S1 and S2 be regular sets

S1 θ S2 = (S1 or S2) and (not (S1 and S2))

$= (S1 \cup S2) \cap \overline{(S1 \cap S2)}$

Since regular sets are closed under union, intersection, and complement, the symmetric difference of two sets is also regular

Question 4

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I. $L = \{w : n_a(w) \text{ and } n_b(w) \text{ are both even}\}$

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I. $L = \{w : n_a(w) \text{ and } n_b(w) \text{ are both even}\}$
Hint 1: First consider the strings with only a or only b. In both cases, the number of a's and b's will be even.

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Find the regular grammars for the following languages on $\{a, b\}$:
I. $L = \{w : n_a(w) \text{ and } n_b(w) \text{ are both even}\}$
Hint 2: Think how you can merge the production rules for an even number of a's and even number of b's.

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I. $L = \{w : n_a(w) \text{ and } n_b(w) \text{ are both even}\}$
solution:

$$q_0 \rightarrow aq_1 | bq_2 | \lambda$$

$$q_1 \rightarrow bq_3 | aq_0$$

$$q_2 \rightarrow aq_3 | bq_0$$

$$q_3 \rightarrow aq_2 | bq_1$$

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Solution:

$$S \rightarrow aaA$$
$$A \rightarrow aA|B$$
$$B \rightarrow bbbC$$
$$C \rightarrow bC|\lambda$$

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Hint 1: Write the sub-string containing at least 4 a's.

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Find the regular expressions for the following languages on $\{a, b\}$:
I. $L = \{a^n b^m : n \geq 4, m \leq 3\}$
Hint 2: What are the possible ways less than 3 b's can occur in the strings generated by this grammar?

Question 5

Find the regular expressions for the following languages on $\{a, b\}$:
I. $L = \{a^n b^m : n \geq 4, m \leq 3\}$
Solution:
Generate 4 or more $a's$, follows by the requisite number of $b's$. Hence,
$aaaaa^*(\lambda + b + bb + bbb)$

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II. $L = \{w \in \{a, b\}^*$: w has exactly one pair of consecutive a's$\}$
Hint 1: How exactly 2 consecutive a's can occur in the strings? This part is the easiest.

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Find the regular expressions for the following languages on $\{a, b\}$:
II. $L = \{w \in \{a, b\}^*$: w has exactly one pair of consecutive a's$\}$
Hint 2: How substrings containing no consecutive 2 a's can occur before or after 'aa'?

Question 5

Find the regular expressions for the following languages on $\{a, b\}$:
II. $L = \{w \in \{a, b\}^*$: w has exactly one pair of consecutive a's$\}$
Solution:
$(b + ab)^* aa(b + ba)^*$

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Find the regular expressions for the following languages on $\{a, b\}$:
III. $L = \{w \in \{a, b\}^*$: w has at least one pair of consecutive a's$\}$

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III. $L = \{w \in \{a, b\}^*$: w has at least one pair of consecutive a's$\}$
Hint 1: Again, how exactly 2 consecutive a's can come in the strings?

Question 5

Find the regular expressions for the following languages on $\{a, b\}$:
III. $L = \{w \in \{a, b\}^*$: w has at least one pair of consecutive a's$\}$
Hint 2: Any number of a's and b's can occur before and after the consecutive a's

Question 5

Find the regular expressions for the following languages on $\{a, b\}$:
III. $L = \{w \in \{a, b\}^*$: w has at least one pair of consecutive a's$\}$
Solution:
$(a + b)^* aa(a + b)^*$

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IV. $L = \{w : n_a(w) mod 3 = 0\}$

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IV. $L = \{w : n_a(w) mod 3 = 0\}$
Hint 1: How many ways 3 a's can occur in a string of a and b.

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Find the regular expressions for the following languages on $\{a, b\}$:
IV. $L = \{w : n_a(w) mod 3 = 0\}$
Hint 2: Number of a's can be 0,3,6,9,...

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Find the regular expressions for the following languages on $\{a, b\}$:
IV. $L = \{w : n_a(w) mod 3 = 0\}$
Solution:
$b^* + (b^*ab^*ab^*ab^*)^*$

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V. All strings that do not end with aa

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Find the regular expressions for the following languages on $\{a, b\}$:
V. All strings that do not end with aa
Hint 1: What are the possible substrings of length 2 consisting of a and b?

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Find the regular expressions for the following languages on $\{a, b\}$:
V. All strings that do not end with aa
Hint 2: Any combination of a and b can occur in the string except the last 2 positions.

Question 5

Find the regular expressions for the following languages on $\{a, b\}$:
V. All strings that do not end with *aa*
Solution:
$\lambda + a + b + (a + b)^*(ab + ba + bb)$