Tutorial 7

Formal Language and Automata Theory

March 2, 2023

Question 1

Show that a regular language cannot be inherently ambiguous.

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HINT 2: Use DFA

Question 1 Solution

Let L be a regular language, and suppose there exists an ambiguous grammar G that generates L.
We can construct a DFA (deterministic finite automaton) M that recognizes L.
Using the DFA M, we can construct a right-regular grammar G' that generates L.
Since every right-regular grammar is unambiguous, G' is unambiguous.
Therefore, G can be transformed into G' by converting it into a DFA and then into a right-regular grammar, without losing any strings that G generates.
Hence, L cannot be inherently ambiguous.

Question 2a

Show that following languages are not context-free using pumping lemma
(a) $L = \{a^n b^j c^k \ : \ k > n, k > j\}$

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HINT 2: Try to apply the pumping lemma: write s as $s = uvwxy$ such that $|vwx| \leq p$, and for all $i \geq 0, uv^i wx^i y \in L$. Show the contradiction

Question 2a Solution

(a) $L = \{a^n b^j c^k \ : \ k > n, k > j\}$

Solution:

We assume that L is context-free. Then, let p be the pumping length given by the pumping lemma for L.

Consider the string $s = a^p b^{p+1} c^{p+2}$, which is in L.

By the pumping lemma, we can write s as $s = uvwxy$ such that $|vwx| \leq p$, and for all $i \geq 0, uv^i wx^i y \in L$.

Let vwx does not have any c, (e.g., $a^4 b^5 c^6$) then the string $uv^3 wx^3 y$ will have atleast $p + 2$ a's or b's.

Therefore, $uv^3 wx^3 y$ is not in L, which contradicts the pumping lemma assumption that for all $i \geq 0, uv^i wx^i y \in L$.

Question 2b

Show that following languages are not context-free using pumping lemma
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HINT 1: Consider an example string $s \in L$, and represent the indices with the pumping length
HINT 2: Try to apply the pumping lemma: write s as $s = uvwxy$ such that $|vwx| \leq p$, and for all $i \geq 0, uv^iwx^iy \in L$. Show the contradiction (Think, when $i = 0$)

Question 2b Solution

(b) $L = \{a^{n!} \ : \ n \geq 0\}$

Solution:

let m be the pumping length given by the pumping lemma for L

we pick $a^{m!} (= uvwxy)$. Obviously, whatever the decomposition is, it must be of the form $v = a^k, x = a^l$.

Then $w0 = uwy$ (pump down) has length $m!$ - $(k + l)$.

This string is in L only if $m!$ - $(k + l) = j!$ for some j. But this is impossible, since with $k + l \leq m, m!$-$(k + l) >$ (m - 1)!.

Question 3

Prove that the following language is ambiguous and also provide its unambiguous counterpart.

$S \rightarrow if\ \ A\ then\ S\ else\ S | if\ \ A\ then\ S | print\ A$

$A \rightarrow true | false$

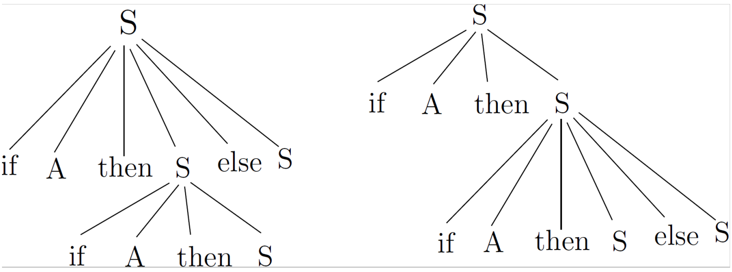
Solution

Hint 1:
Construct the derivation tree. If a string has more than one derivation tree, then the grammar is ambiguous.

Hint 2:

Consider the input string and try to construct the derivation tree.

if false then if false then print true else print false



Since two different derivation trees exist for the same input string, the grammar is ambiguous.

The respective unambiguous counterpart is-
$S \rightarrow S^{'} | if\ A\ then\ S | print\ A$
$S^{'} \rightarrow if\ A\ then\ S^{'}\ else\ S | print\ A$
$A \rightarrow true | false$

Question 4a

Convert the following grammar into Chomsky Normal Form.

$S \rightarrow AACD$
$A \rightarrow aAb|\epsilon$
$C \rightarrow aC|a$
$D \rightarrow aDa|bDb|\epsilon$

Solution

Hint 1:

In this case, $P^{'}$ becomes:

$S \rightarrow AACD|ACD|AAC|CD|AC|aC|a$

$A \rightarrow aAb|ab|\epsilon$

$C \rightarrow aC|a$

$D \rightarrow aDa|bDb|aa|bb|\epsilon$

Hint 2: Eliminate ϵ productions. The nullable variables are A and D. So remove those productions.

$$
\begin{aligned}
S &\rightarrow AACD|ACD|AAC|CD|AC|C \\
A &\rightarrow aAb|ab \\
C &\rightarrow aC|a \\
D &\rightarrow aDa|bDb|aa|bb
\end{aligned}
$$

Hint 3:

Eliminate unit-production. We remove the unit production $S \to C$ replacing it by $S \to aC|a$

$S \rightarrow AACD|ACD|AAC|CD|AC|aC|a$
$A \rightarrow aAb|ab$
$C \rightarrow aC|a$
$D \rightarrow aDa|bDb|aa|bb$

Hint 4:

Restrict the right side of production to single terminals or strings of two or more variables.

$$S \rightarrow AACD|ACD|AAC|CD|AC|X_aC|a$$
$$A \rightarrow X_aAX_b|X_aX_b$$
$$C \rightarrow X_aC|a$$
$$D \rightarrow X_aDX_a|X_bDX_b|X_aX_a|X_bX_b$$
$$X_a \rightarrow a$$
$$X_b \rightarrow b$$

Final Step to CNF: There are six productions whose right sides are too long.

$S \to AT_1|AU_1|AV_1|CD|AC|X_aC|a$

$T_1 \to AT_2$

$T_2 \to CD$

$U_1 \to CD$

$V_1 \to AC$

$A \to X_aW_1|X_aX_b$

$W_1 \to AX_b$

$C \to X_aC|a$

$D \to X_aY_1|X_bZ_1|X_aX_a|X_bX_b$

$Y_1 \to DX_a$

$Z_1 \to DX_b$

$X_a \to a$

$X_b \to b$

Question 4b

Convert the following grammar into CNF:

$S \rightarrow aXbX$

$X \rightarrow aY|bY|\epsilon$

$Y \rightarrow X|c$

Solution

Step1:

$S \rightarrow aXbX|abX|aXb|ab$

$X \rightarrow aY|bY|a|b|\epsilon$

$Y \rightarrow X|c$

Step2:

$S \rightarrow aXbX|abX|aXb|ab$

$X \rightarrow aY|bY|a|b$

$Y \rightarrow X|c$

Step3:

$S \rightarrow aXbX|abX|aXb|ab$
$X \rightarrow aY|bY|a|b$
$Y \rightarrow aY|bY|a|b|c$

Final Step:

$S \rightarrow EF|AF|EB|AB$

$X \rightarrow AY|BY|a|b$

$Y \rightarrow AX|BY|a|b|c$

$E \rightarrow AX$

$F \rightarrow BX$

$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow c$