Tutorial 9

Formal Language and Automata Theory

March 16, 2023

Question 1

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HINT 1: The class of CFLs is closed under the concatenation operation
Can this property be useful here?

Question 1

We defined the rotational closure of language A to be RC(A) $=$ $\{yx \mid xy \in A\}$. Show that the class of CFLs is closed under rotational closure.

HINT 2: Consider a string in the language A. *Apply RC property there?*

Question 1

We defined the rotational closure of language A to be RC(A) = $\{yx \mid xy \in A\}$. Show that the class of CFLs is closed under rotational closure.

SOLUTION:

Consider a string $s = xy$ in the language A. It can be formed from the concatenation of two languages X and Y, such that $x \in X$ and $y \in Y$. As the language $A = X.Y$ is a CFL, the languages X and Y will also be CFLs. The rotational closure of the string $s = xy$ will be $RC(s) = RC(xy) = yx$. It can be formed by concatenating Y and X.

$RC(s) = Y.X$

As both Y and X are context-free languages, the language $RC(s)$ is a context-free language for any string $s \in A$. The rotational closure has been proven for the class of CFLs.

Question 2

Show that the class of CFLs is not closed under NOEXTEND.
$NOEXTEND(P) = \{w \in P \mid \text{w is not a proper prefix of any string in P }\}$

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HINT 1: Can the Union property of CFLs be used in this case?

Question 2

Show that the class of CFLs is not closed under NOEXTEND.
$NOEXTEND(P) = \{w \in P \mid \text{w is not a proper prefix of any string in P}\}$

HINT 2: Assume strings $P_1 = \{x^a y^b z^c \mid a \neq b, a, b, c \geq 1\}$,
$P_2 = \{x^a y^b z^b \mid a, b \geq 1\}$, and $P = P_1 \cup P_2$

Question 2

SOLUTION: Consider the language $P = P_1 \cup P_2$, where $P_1 = \{x^a y^b z^c \mid a \neq b, a, b, c \geq 1\}$ and $P_2 = \{x^a y^b z^b \mid a, b \geq 1\}$.
Consider the string $x^a y^b z^c \in P_1$, the given string is not in NOEXTEND(P), since the extension of the string $x^a y^b z^{c+1}$, also belongs to P.
Now the string $x^a y^b z^b$ is considered. Any extension of this string in P should belong to P_1. Hence this string will not exist in NOEXTEND(P), if and only if an extension of it belongs to P_1 and $a \neq b$. Therefore, the string of the form $x^a y^a z^a$ belongs to NOEXTEND(P). Hence NOEXTEND(P) $= \{x^a y^a z^a \mid a \geq 1\}$. We now see that P is a CFL but NOEXTEND(P) is not. Therefore from the above explanation, it can be said that the context-free language are not closed under NOEXTEND operation.

Question 3

Using CYK Algorithm, verify whether the string $w = abbb$ belongs to the language generated by the grammar: $S \rightarrow AB,\ A \rightarrow BB|a,\ B \rightarrow AB|b$.

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Hint 1: Convert the grammar to CNF form

Question 3

Using CYK Algorithm, verify whether the string $w = abbb$ belongs to the language generated by the grammar: $S \rightarrow AB,\ A \rightarrow BB|a,\ B \rightarrow AB|b$. The grammar is already in the CNF form.

Hint 2: We need to fill the table with non-terminals. Start with substrings of length 1.

Solution:

The CNF :$S \rightarrow AB,\ A \rightarrow BB|a,\ B \rightarrow AB|b$

Table: Step 1

a	b	b	b
A			
	B		
		B	
			B

Question 3

Solution:

The CNF :$S \rightarrow AB,\ A \rightarrow BB|a,\ B \rightarrow AB|b$

Table: Step 2

a	b	b	b
A			
S,B	B		
	A	B	
		A	B

Question 3

Solution:

The CNF :$S \to AB, \; A \to BB|a, \; B \to AB|b$

Table: Step 3

a	b	b	b
A			
S,B	B		
A	A	B	
	S,B	A	B

Question 3

Solution:

The CNF :$S \rightarrow AB,\ A \rightarrow BB|a,\ B \rightarrow AB|b$

Table: Step 4

a	b	b	b
A			
S,B	B		
A	A	B	
S,B	S,B	A	B

Question 4 (Submit to the TAs)

Consider the following context-free grammar for the English language

$$S- > NP\ VP$$
$$NP- > Det\ Nom|PropN$$
$$Nom- > Adj\ Nom|N$$
$$VP- > V\ Adj|V\ NP|V\ S|V\ NP\ PP$$
$$PP- > P\ NP$$
$$PropN- >'Buster'|'Chatterer'|'Joe'$$
$$Det- >'the'|'a'$$
$$N- >'bear'|'squirrel'|'tree'|'fish'|'log'|'dog'|'man'|'park'$$
$$Adj- >'angry'|'frightened'|'little'|'tall'$$
$$V- >'chased'|'saw'|'said'|'thought'|'was'|'put'$$
$$P- >'in'$$

Using CYK Algorithm, verify whether "*the dog saw a man in the park*" is a valid English sentence.

Question 4

Solution: The CNF form of the given grammar is:

$$
\begin{aligned}
S- &> NP\ VP \\
NP- &> Det\ Nom | 'Buster' | 'Chatterer' | 'Joe' \\
Nom- &> Adj\ Nom | 'bear' | 'squirrel' | 'tree' | 'fish' | 'log' | 'dog' | 'man' | 'park' \\
VP- &> V\ Adj | V\ NP | V\ S | V1\ PP \\
V1- &> V\ NP \\
PP- &> P\ NP \\
Det- &> 'the' | 'a' \\
Adj- &> 'angry' | 'frightened' | 'little' | 'tall' \\
V- &> 'chased' | 'saw' | 'said' | 'thought' | 'was' | 'put' \\
P- &> 'in'
\end{aligned}
$$

Question 4

Solution:

the	dog	saw	a	man	in	the	park
Det							
NP	Nom						
-	-	V					
-	-	-	Det				
S	-	VP, V1	NP	Nom			
-	-	-	-	-	P		
-	-	-	-	-	-	Det	
S	-	VP	-	-	PP	NP	Nom

Since the left bottom cell contains the start symbol S, the given sentence is a valid English sentence.

Homework

Prove or disprove:

1. CFL is closed under substitution.
2. CFL is closed under inverse homomorphism.

Upload the solutions of the above problems in a single PDF in the following link by Monday (20.3.2023): `https://drive.google.com/drive/folders/1XyJ71KrYsby7ty1dvjJm3ziW8stsYjoz?usp=share_link`

Note: Name of the file should be *roll_no.pdf*