Set-1

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067) Code: Abxw1037x6cFep

Name: Class Test-2

Roll No. Marks:17 Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Buses arrive at a park according to a Poisson process of rate 5 per hour beginning at 8 a.m. Each bus carries between 1 to 10 visitors of equal probability. Find the expected number and the variance of visitors who arrive at the park by 10 a.m.	(i) 55 (ii) 385	
2.	Suppose customers arrive in a store according to a Poisson process of rate $\lambda = 2$ customers per hour. The store opens at 8 a.m. Find (a) P(5 customers arrive before 10 a.m. and total 6 arrivals by 11 a.m.); (b) Suppose each customer has a probability of 0.4 to make a purchase. Find the probability that a purchase occurs by 10 a.m.	(i) 0.0423 (ii) 0.7981	
3.	Consider a CTMC with three states 1, 2, 3 and transition rates $q_{12} = q_{13} = 2,\ q_{21} = 4, q_{23} = 3, q_{31} = q_{32} = 5$. Determine the generator Q (i.e., Q-matrix).	$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} -4 & 2 & 2 \\ 4 & -7 & 3 \\ 5 & 5 & -10 \end{bmatrix} \end{array}$	
4.	Consider a birth and death process with birth rates $\lambda_i = 2,\ i = 0, 1, 2, \ldots$, and death rates $\mu_i = 3,\ i = 1, 2, \ldots$. Determine the expected time to go from state 0 to state 2.	$\frac{7}{4} = 1.75$	
5.	Mr. Smith runs a one-person, unisex hair salon. He finds that customers seem to arrive according to a Poisson process with a mean arrival rate of 4 per hour. Because of his excellent reputation, customers were always willing to wait. The data further showed that customer processing time (aggregated female and male) was exponentially distributed with an average of 10 min. (a) The average number of customers in the shop is (b) The average number of customers waiting for a haircut (c) The percentage of time an arrival can walk right in without having to wait at all (d) The expected waiting time of a customer is	(a) 2 (b) $\frac{4}{3} = 1.33$ (c) 33.34% (d) $\frac{1}{3} = 0.33$	
6.	For M/M/1 queueing system the steady state solution exist if $\rho = \lambda/\mu$ is	$\rho < 1$	
7.	In CTMC with state space $\{1, 2, \ldots, n\}$; $$\sum_{j=1; j \neq i}^{n} q_{ij} = -v_i$$ (True/False)	False	
8.	On campus building, there are 2 offices of similar sizes with identical air conditioners. The electrical grid supplies electric energy to the air conditioners whose thermostats turn on and off in each individual office as needed to maintain each office's temperature at the desired level of 76^o Fahrenheit, independent of the other offices. Suppose that a thermostat remains on or off for exponential amounts of times with means 2 and 3, respectively. Let $X(t)$ denote the number of thermostats on at time t. (a) Whether $X(t)$ is ctmc? (Yes/No) (b) Find State space of $X(t)$; (c) Find the birth and death rates. (d) Find the long run probability that there will be 2 thermostats on at the same time.	(a) Yes (b) $\{0, 1, 2\}$ (c) $0 \underset{\mu_1 = 1/3}{\overset{\lambda_0 = 1}{\rightleftarrows}} 1 \underset{\mu_2 = 2/3}{\overset{\lambda_1 = 1/2}{\rightleftarrows}} 2$ (d) $\Pi_2 = 9/25 = 0.36$	
9.	Poisson process is a birth and death process. (True/False)	True	

Set-2

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067) Code: Abxx1037x6cFep

Name: Class Test-2

Roll No. Marks:17 Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Buses arrive at a park according to a Poisson process of rate 4 per hour beginning at 8 a.m. Each bus carries between 1 to 10 visitors of equal probability. Find the expected number and the variance of visitors who arrive at the park by 10 a.m.	(i) 44 (ii) 308	
2.	Suppose customers arrive in a store according to a Poisson process of rate $\lambda = 1$ customers per hour. The store opens at 8 a.m. Find (a) P(5 customers arrive before 10 a.m. and total 6 arrivals by 11 a.m.); (b) Suppose each customer has a probability of 0.4 to make a purchase. Find the probability that no purchase occurs by 10 a.m.	(a) 0.0133 (b) $e^{-0.8} = 0.449$	
3.	Consider a CTMC with three states 1, 2, 3 and transition rates $q_{12} = q_{13} = 1$, $q_{21} = 4, q_{23} = 2, q_{31} = q_{32} = 3$. Determine the generator Q (i.e., Q-matrix).	$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & -2 & 1 & 1 \\ 2 & 4 & -6 & 2 \\ 3 & 3 & 3 & -6 \end{array}$	
4.	Consider a birth and death process with birth rates $\lambda_i = 2$, $i = 0, 1, 2, \ldots$, and death rates $\mu_i = 4$, $i = 1, 2, \ldots$. Determine the expected time to go from state 0 to state 2.	2	
5.	Mr. Smith runs a one-person, unisex hair salon. He finds that customers seem to arrive according to a Poisson process with a mean arrival rate of 4 per hour. Because of his excellent reputation, customers were always willing to wait. The data further showed that customer processing time (aggregated female and male) was exponentially distributed with an average of 3 min. (a) The average number of customers in the shop is (b) The average number of customers waiting for a haircut (c) The percentage of time an arrival can walk right in without having to wait at all (d) The expected waiting time of a customer is	(a) $\frac{1}{4}$ (b) $\frac{1}{20}$ (c) 80% (d) $\frac{1}{80}$	
6.	For M/M/c queueing system the steady state solution exist if $\lambda/c\mu$ is	$\frac{\lambda}{c\mu} < 1$	
7.	In CTMC with state space $\{1, 2, \ldots, n\}$; $$\sum_{j=1}^{n} q_{ij} = v_i$$ (True/False)	False	
8.	On campus building, there are 2 offices of similar sizes with identical air conditioners. The electrical grid supplies electric energy to the air conditioners whose thermostats turn on and off in each individual office as needed to maintain each office's temperature at the desired level of 76^o Fahrenheit, independent of the other offices. Suppose that a thermostat remains on or off for exponential amounts of times with means 3 and 2, respectively. Let $X(t)$ denote the number of thermostats on at time t. (a) Whether $X(t)$ is ctmc? (Yes/No) (b) Find State space of $X(t)$; (c) Find the birth and death rates. (d) Find the long run probability that there will be 2 thermostats on at the same time.	(a) Yes (b) $\{0, 1, 2\}$ (c) $\lambda_0 = \frac{2}{3}, \lambda_1 = \frac{1}{3}$ $\mu_1 = \frac{1}{2}, \mu_2 = 1$ (d) $\pi_2 = \frac{4}{25}$	
9.	Pure Birth process is a CTMC. (True/False)	True	

Set 3

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067) Code: Abxy1037x6cFep

Name: Class Test-2

Roll No. Marks:17 Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Buses arrive at a park according to a Poisson process of rate 5 per hour beginning at 8 a.m. Each bus carries between 1 to 5 visitors of equal probability. Find the expected number and the variance of visitors who arrive at the park by 10 a.m.	(i) 30 (ii) 110	
2.	Suppose customers arrive in a store according to a Poisson process of rate $\lambda = 2$ customers per hour. The store opens at 8 a.m. Find (a) P(4 customers arrive before 10 a.m. and total 5 arrivals by 11 a.m.); (b) Suppose each customer has a probability of 0.4 to make a purchase. Find the probability that a purchase occurs by 10 a.m.	(i) 0.0528 (ii) 0.7981	
3.	Consider a CTMC with three states 1, 2, 3 and transition rates $q_{12} = q_{13} = 4,\ q_{21} = 3, q_{23} = 4, q_{31} = q_{32} = 3$. Determine the generator Q (i.e., Q-matrix).	$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} -8 & 4 & 4 \\ 3 & -7 & 4 \\ 3 & 3 & -6 \end{pmatrix} \end{array}$	
4.	Consider a birth and death process with birth rates $\lambda_i = 2,\ i = 0, 1, 2, \ldots$, and death rates $\mu_i = 3,\ i = 1, 2, \ldots$. Determine the expected time to go from state 0 to state 3.	$\frac{33}{8} = 4.125$	
5.	Mr. Smith runs a one-person, unisex hair salon. He finds that customers seem to arrive according to a Poisson process with a mean arrival rate of 4 per hour. Because of his excellent reputation, customers were always willing to wait. The data further showed that customer processing time (aggregated female and male) was exponentially distributed with an average of 5 min. (a) The average number of customers in the shop is (b) The average number of customers waiting for a haircut (c) The percentage of time an arrival can walk right in without having to wait at all (d) The expected waiting time of a customer is	(a) $\frac{1}{2} = 0.5$ (b) $\frac{1}{6} = 0.1667$ (c) 66.6% (d) $\frac{1}{24} = 0.0416$ hrs.	
6.	For M/M/1 queueing system the steady state solution exist if $\rho = \lambda/\mu$ is	$\rho < 1$	
7.	In CTMC with state space $\{1, 2, \ldots, n\}$; $$\sum_{j=1; j\neq i}^{n} q_{ij} = v_i$$ (True/False)	True	
8.	On campus building, there are 2 offices of similar sizes with identical air conditioners. The electrical grid supplies electric energy to the air conditioners whose thermostats turn on and off in each individual office as needed to maintain each office's temperature at the desired level of 76^o Fahrenheit, independent of the other offices. Suppose that a thermostat remains on or off for exponential amounts of times with means 1 and 3, respectively. Let $X(t)$ denote the number of thermostats on at time t. (a) Whether $X(t)$ is ctmc? (Yes/No) (b) Find State space of $X(t)$; (c) Find the birth and death rates. (d) Find the long run probability that there will be 1 thermostat on.	(a) Yes (b) $\{0, 1, 2\}$ (c) $\lambda_0 = 2,\ \lambda_1 = 1$ $\mu_1 = \frac{1}{3},\ \mu_2 = \frac{2}{3}$ (d) $\pi_1 = \frac{3}{8} = 0.375$	
9.	Poisson process is NOT a CTMC. (True/False)	False	

Set 4

Stochastic Process/Stochastic Process and Simulation (MA41017/MA60067)		Code: Abxz1037x6cFep	
Name:		Class Test-2	
Roll No.		Marks:17	Time: 1 hour

Q.No.	Question	Answer	Marks
1.	Buses arrive at a park according to a Poisson process of rate 4 per hour beginning at 8 a.m. Each bus carries between 1 to 5 visitors of equal probability. Find the expected number and the variance of visitors who arrive at the park by 10 a.m.	(i) 24 (ii) 88	
2.	Suppose customers arrive in a store according to a Poisson process of rate $\lambda = 1$ customers per hour. The store opens at 8 a.m. Find (a) P(4 customers arrive before 10 a.m. and total 5 arrivals by 11 a.m.); (b) Suppose each customer has a probability of 0.6 to make a purchase. Find the probability that no purchase occurs by 10 a.m.	(a) 0.0332 0.301 (b)	
3.	Consider a CTMC with three states 1, 2, 3 and transition rates $q_{12} = q_{13} = 4,\ q_{21} = 2, q_{23} = 2, q_{31} = q_{32} = 2$. Determine the generator Q (i.e., Q-matrix).	$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} -8 & 4 & 4 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{pmatrix} \end{array}$	
4.	Consider a birth and death process with birth rates $\lambda_i = 1,\ i = 0, 1, 2, \ldots$, and death rates $\mu_i = 4,\ i = 1, 2, \ldots$. Determine the expected time to go from state 0 to state 3.	27	
5.	Mr. Smith runs a one-person, unisex hair salon. He finds that customers seem to arrive according to a Poisson process with a mean arrival rate of 4 per hour. Because of his excellent reputation, customers were always willing to wait. The data further showed that customer processing time (aggregated female and male) was exponentially distributed with an average of 2 min. (a) The average number of customers in the shop is (b) The average number of customers waiting for a haircut (c) The percentage of time an arrival can walk right in without having to wait at all (d) The expected waiting time of a customer is	(a) $\frac{2}{13} = 0.153$ (b) $\frac{4}{195} = 0.0205$ (c) 86.6 % (d) $\frac{2}{390} = 0.005$ hr	
6.	For M/M/c queueing system the steady state solution exist if $\lambda/c\mu$ is	$\frac{\lambda}{c\mu} < 1$	
7.	In CTMC with state space $\{1, 2, \ldots, n\}$; $$\sum_{j=1}^{n} q_{ij} = 0$$ (True/False)	True	
8.	On campus building, there are 2 offices of similar sizes with identical air conditioners. The electrical grid supplies electric energy to the air conditioners whose thermostats turn on and off in each individual office as needed to maintain each office's temperature at the desired level of 76^o Fahrenheit, independent of the other offices. Suppose that a thermostat remains on or off for exponential amounts of times with means 1 and 2, respectively. Let $X(t)$ denote the number of thermostats on at time t. (a) Whether $X(t)$ is ctmc? (Yes/No) (b) Find State space of $X(t)$; (c) Find the birth and death rates. (d) Find the long run probability that there will be 2 thermostats on at the same time.	(a) Yes (b) $\{0,1,2\}$ (c) $\lambda_0 = 2, \lambda_1 = 1$ $\mu_1 = \frac{1}{2}, \mu_2 = 1$ (d) $\Pi_2 = \frac{4}{9} = 0.444$	
9.	Pure Birth process is NOT a CTMC. (True/False)	False	

Set - 1

① $N(t) \sim$ Poisson process ($\lambda = 5$ per hour)

$X(t)$ = Number of Visitors who arrive between 8 am & 10 am

$$X(t) = \sum_{i=1}^{N(t)} Y_i$$

$$P(Y_i = x) = \begin{cases} \frac{1}{10}, & i = 1, 2, \cdots, 10 \\ 0, & \text{otherwise} \end{cases}$$

$$E(Y_i) = \frac{11}{2}, \quad E(Y_i^2) = \frac{77}{2}$$

$$E(X(t)) = \lambda t \, E(Y_1)$$

$$E(X(2)) = 5 \times 2 \times \frac{11}{2} = 55$$

$$Var(X(t)) = \lambda \cdot t \, E(Y_1^2)$$

$$= 5 \times 2 \times \frac{77}{2} = 385.$$

② ⓐ $\lambda = 9$ Customer per hour.

$$P(N(8,10) = 5, N(8,11) = 6)$$

$$= P(N(2)=5,\ N(3)=6)$$

$$= P(N(2)=5) \cdot P(N(1)=1)$$

$$= \frac{e^{-4} \cdot 4^5}{5!} \cdot \frac{e^{-2} \cdot 2^1}{1!} = 0.0423.$$

(b) $P(\text{no purchage by 10 am})$

$$= E\left[P(\text{no percharge by 10 am} \mid N(2))\right]$$

$$= E\left((0.6)^{N(2)}\right)$$

$$= \sum_{n=0}^{\infty} (0.6)^n \cdot \frac{e^{-2\times 2} 4^n}{n!} = e^{-4} \sum_{n=0}^{\infty} \frac{(2.4)^n}{n!}$$

$$= 0.2018$$

$\therefore$ Probability of purchase by 10 am

$$= 1 - 0.2019 = 0.7981$$

3

$$Q = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & -4 & 2 & 2 \\ 2 & 4 & -7 & 3 \\ 3 & 5 & 5 & -10 \end{array}$$

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4 $E(T_0) = \frac{1}{\lambda_0} = \frac{1}{2}$

$$E(T_i) = E(E(T_i \mid I_i))$$

$$E(T_i) = \frac{1}{5} + \frac{3}{5}E(T_{i-1}) + \frac{3}{5}E(T_i)$$

$$\frac{2}{5}E(T_i) = \frac{1}{5} + \frac{3}{5}E(T_{i-1})$$

$$\Rightarrow E(T_i) = \frac{1}{2} + \frac{3}{2}E(T_{i-1})$$

$$E(T_0) + E(T_1) = \frac{1}{2} + \frac{1}{2} + \frac{3}{2} \times \frac{1}{2} = \frac{7}{4}$$

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5 $\lambda = 4$ per hour, $\frac{1}{\mu} = 10$ min $= \frac{1}{6}$ hr

$$\rho = \frac{\lambda}{\mu} = \frac{4}{6} = \frac{2}{3}$$

ⓐ $L = \dfrac{\rho}{1-\rho} = \dfrac{2/3}{1/3} = 2$

ⓑ $L_q = \dfrac{\rho^2}{1-\rho} = \dfrac{4}{9} \times 3 = \dfrac{4}{3}$

ⓒ $P(N=0) = 1-\rho = 1-\frac{2}{3} = \frac{1}{3}$

ⓓ $W_s = \dfrac{1}{\mu} = \dfrac{1}{6}$

$W_q = \dfrac{\rho \cdot W_s}{1-\rho} = \dfrac{2}{3} \times \dfrac{1}{6} \times 3 = \dfrac{1}{3}$

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⑥ $\rho \leq 1$

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⑦ $\displaystyle\sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ij} = v_i$

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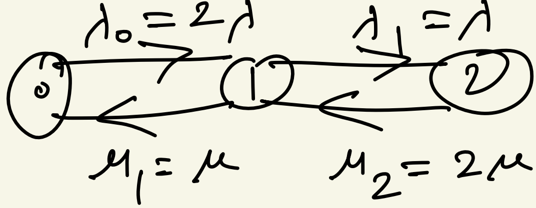
⑧



$\lambda = \frac{1}{2}$

$\mu = \frac{1}{3}$

(a) yes

(b) $\{0, 1, 2\}$

(c) $\lambda_0 = 1, \ \lambda_1 = \frac{1}{2}, \ \mu_1 = \frac{1}{3}, \ \mu_2 = \frac{2}{3}$

(d)
$$S = 1 + c_1 + c_2$$
$$= 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0}{\mu_1} \cdot \frac{\lambda_1}{\mu_2}$$
$$= 1 + 3 + 3 \times \frac{3}{4}$$
$$= 1 + 3 + \frac{9}{4}$$
$$= \frac{25}{4}$$

$$\pi_0 = \frac{1}{S} = \frac{4}{25}$$

$$\pi_1 = c_1 \cdot \pi_0 = 3 \times \frac{4}{25} = \frac{12}{25}$$

$$\pi_2 = c_2 \cdot \pi_0 = \frac{9}{\cancel{4}} \times \frac{\cancel{4}}{25} = \frac{9}{25}$$ ✓

(g) True