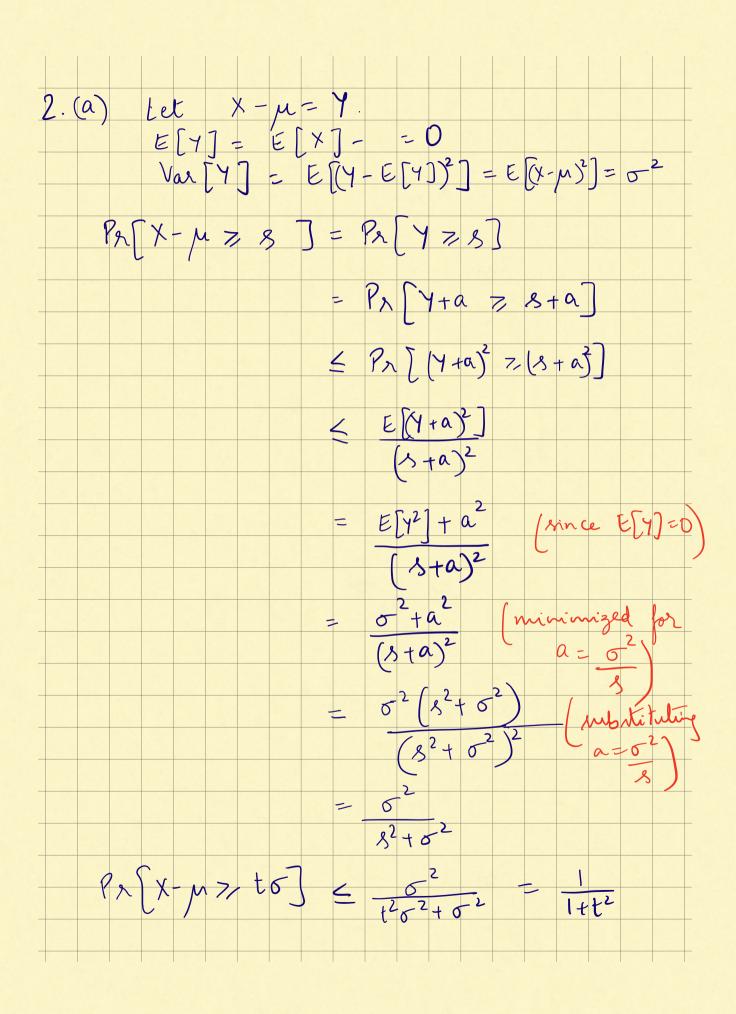
(a)
$$n$$
 balls $\rightarrow n$ bins

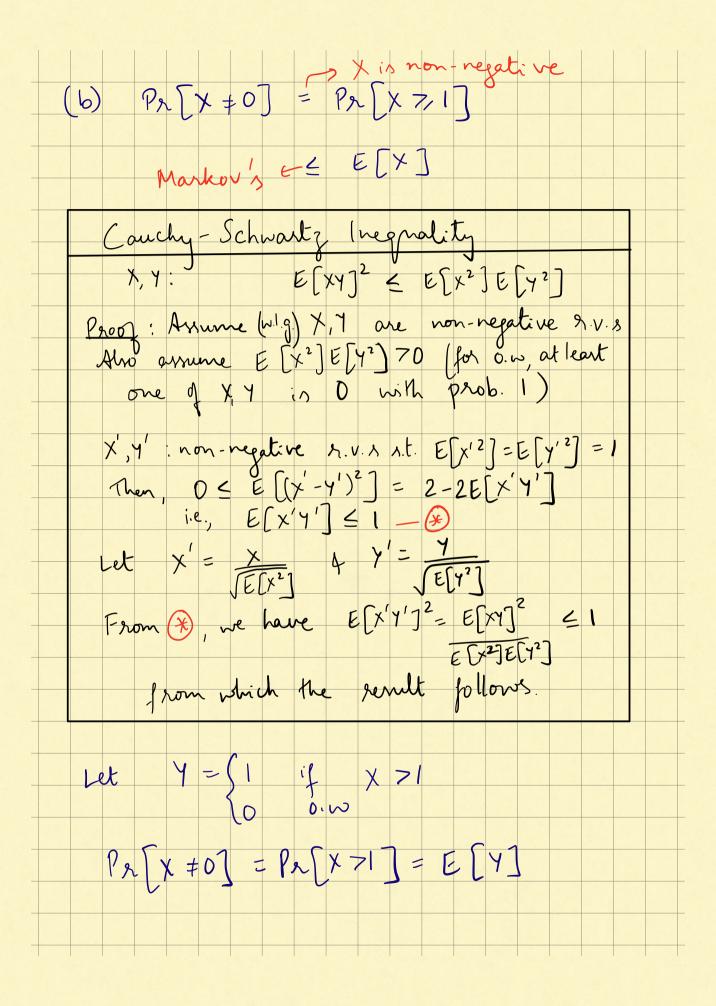
 $X_i = \begin{cases} if & im bin is empty. \\ 0 & otherwise \end{cases}$

Let $X = \underbrace{\sum}_{i=1}^{n} X_i$
 $empty bins = (1-1)^n$
 $empty bins = (1-1$



(b) Pa[X-M1 = t6] = Pr [X-u > to] + Pr [X-u <- to] The inequality in part (a) is better
than thebyther's inequality as

1 2 1
1+t² t² (C) But for 2-rided tail bounds, Chebyrlev's inequality is always better than part (b) unless t < 1. 3. (a) $P_{\Lambda}[X=0] \leq P_{\Lambda}[X-E[X]] = [X]$ Chebyehev's E[x]² den. of E[x]-E[x]²
Iariance



By Canchy-Schwartz inequality, E[Y]E[X2] = E[X]2 i.e., E[Y]-PA[X71] >, E[X]² 7. (a) Discurred in class (b) Consider the im now $\vec{a}_i^T = (\hat{a}_{c_i}, q_{i_2}, ..., q_{i_m})$ Let k denote the no. of is in \vec{a}_i . Case 1: k = Jaminn Then irrespective of b, $|\vec{a}_i \vec{b}| = |c_i| \le \sqrt{4m \ln n}$ Care 2: le > /4m/n n Then there are k non-zero terms in $C_i = \sum_{j=1}^{n} a_{ij} b_j$ All the non-zero learny are independent grabability 1.

Uning part (a), we have $\frac{4mln}{2k}$ Pr (|C;| > $\frac{4mln}{k}$) \(\frac{2}{k} \) \(\frac{2}{k} \ Pr[1A610 > V4mlnn] - Pa () (C1 > V4mlan) $\leq n \cdot \frac{2}{n^2} = \frac{2}{n}$ 8. (a) Assume n is even White $x \in \{0,13^n \text{ as } (x',x^2)\}$ where $x',x^2 \in \{0,13^{n/2}\}$ Let TT be a permutation s.t. $TT(x', 0') = (0'')^2 x'$ for all $x' \in \{\alpha_1\}^{3/2}$ Length of the growte from (x,0) to $(\ddot{0},\dot{x}) = n$ Number of possible choices for x = 2All the 2^n paths pass through $(\ddot{0},\ddot{0})$.

are only noutgoing edges $\begin{pmatrix} 0^{n/2} & 0^{n/2} \end{pmatrix}.$ at least 2 /n line steps required to rend all the parkets from vertices (x', \tilde{o}^2) Refer to the text-book (Motwani-Raghavan) for a detailed discussion. (b) [Only a hint. If you cannot solve it before mid-sem, I will share the solution] Y: no. of rounds before every bin
is non-empty. In the coupon collector's problem, we do not care whether a bin is empty or non-empty of in each round only player throws a boll in a bix. [4] is certainly much smaller compared to the expected no rounds in the coupon collector's problem which is O(nlogn) Given that n balls are thorown instead of 1 E[4] = Ollogn)

It turns out that E[7] has that follows all logarithms are to f(0) = 1 $f(i+1) = 2^{f(i)}$ for i = 71a fined n, the mallest KEIN o. of time we take logarithm of repeatedly for the result to $k = \log^{+} n = \left\{ 1 + \log^{+} (\log n) \right\}$ Now show the following: After round i and before round it!,
if Mi is the expected no. of empty
bins then,

