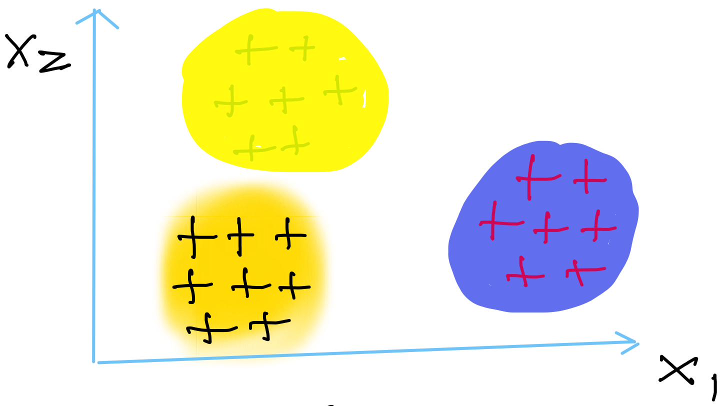
Unsupervised Learning

$\Longrightarrow$ $x_1 \dots x_N \longrightarrow f(x)$ (representation)



CLUSTERING

Distance:-
1. Symmetry: $D(a,b) = (b,a)$
2. Reflexivity: $D(a,a) = 0$
3. Triangular Inequality
$D(a,b) + D(b,c) \geq D(a,c)$

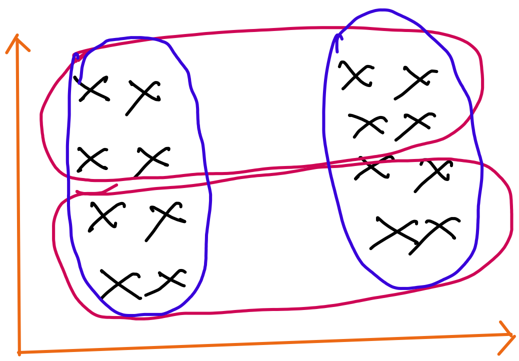
$\propto \frac{1}{\text{distance}}$

Cohesion $\rightarrow$ high (Intra-Cluster)

Separation $\rightarrow$ high (Inter-Cluster)

Scatter Coefficient

$SC = \frac{\text{avg Intra-Cluster dist}}{\text{avg Inter-Cluster dist}} \downarrow$

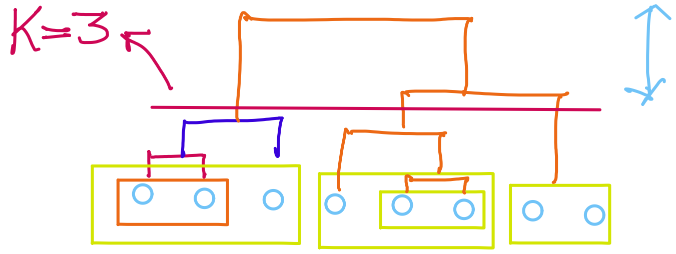


Q. How many Clusters?

Hierarchial Clustering

Divisive (Top-Down)

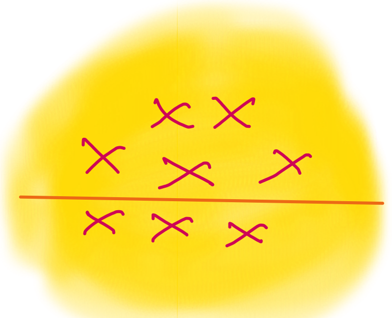
Agglomerative (Bottom-Up)



Computation:- $^{n}C_2 + {}^{n-1}C_2 + \dots + {}^{2}C_2$

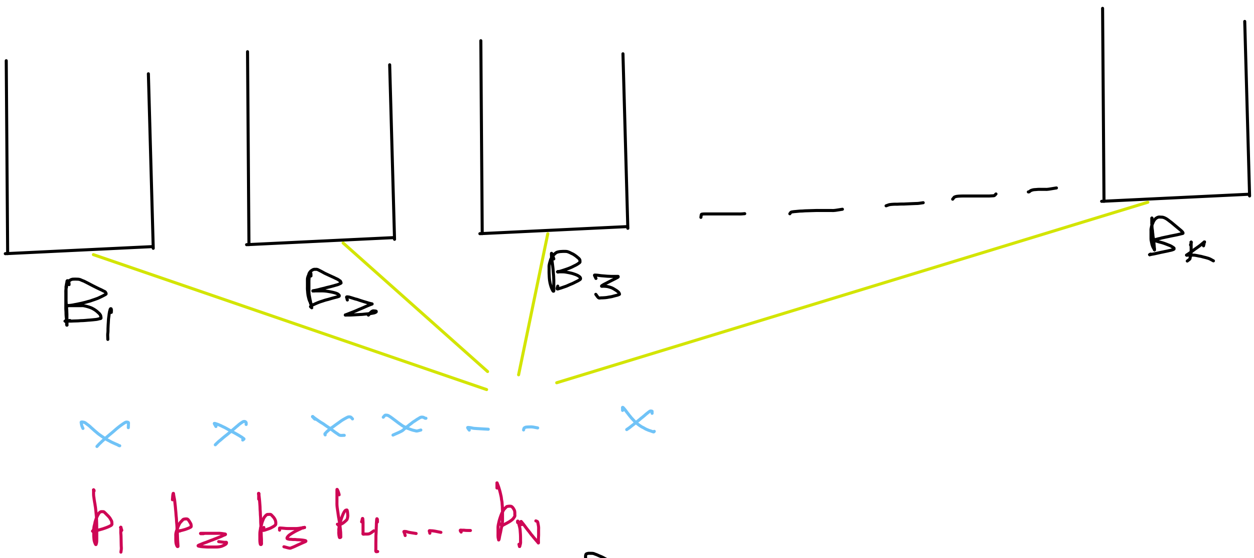
$= O(n^3)$

Computationally complex

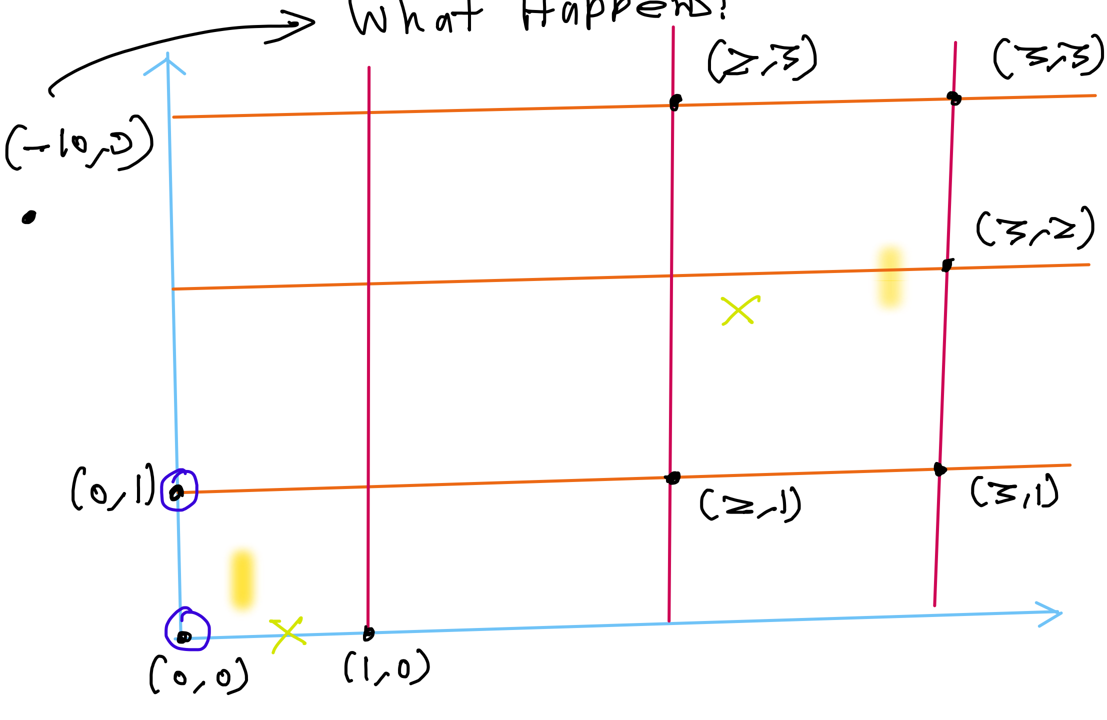


$$({}^{n}C_1 + {}^{n}C_2 + \dots) = O(2^n)$$

Partitional Clustering:- "K-Means"



What Happens?



$B_1 \rightarrow (0,0)\ (0,1) \rightarrow (0.5,0)$

$B_2 \rightarrow (0,1),(2,1),(3,1),(3,2),(3,3),(2,3) \rightarrow (2.167, 1.83)$

$B_1' \rightarrow (0,0)\ (1,0),(0,1) \rightarrow (0.33, 0.33)$

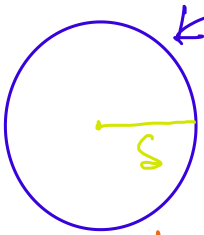
$B_2' \rightarrow (2,1),(3,1),(3,2),(3,3),(2,3),(2.6,2) \rightarrow (2.6,2)$

$O(N) + O(K) + O(NK) * \#Itr$

$K=2 \Rightarrow K=3$ $\quad K=?$

Density based Clustering (DBSCAN)

dense points



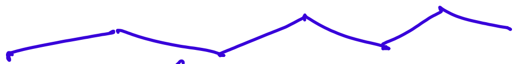
$\geq MAX$

Border Point

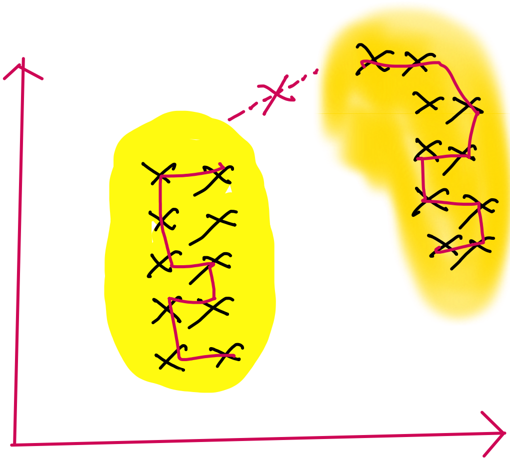
$x_j \leq \epsilon\ x_i$

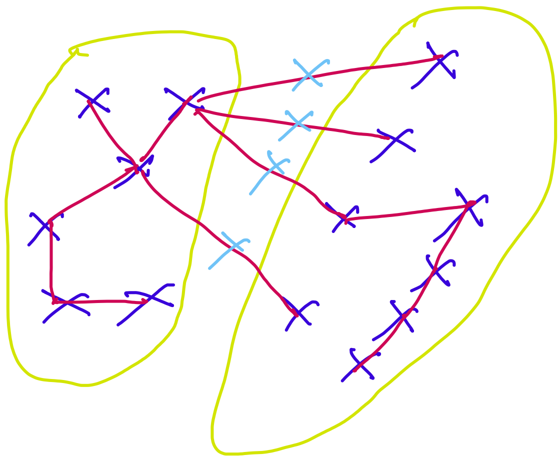
▷ Connectedness:- $x_i \leftarrow$ dense

(Path Connect)



$x_i \leq \epsilon$

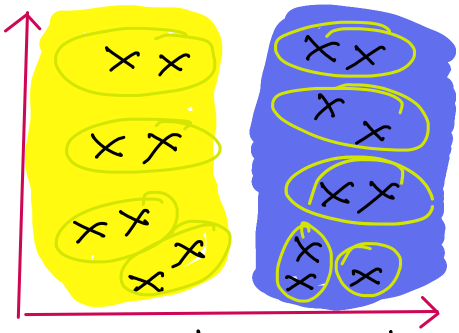




1) Form MST

2) Release longer edges

Hybrid → Aggl
→ K-Means



$K=2$

$K' >> K$

$\Rightarrow$

① Centroid (Avg dist) → Avg linkage AC

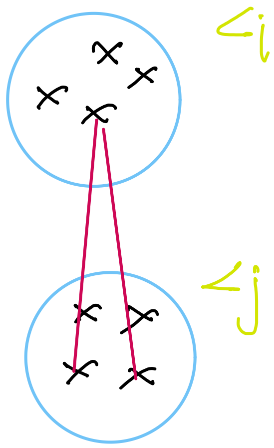
② Min dist → Single linkage AC

③ Max dist → Complete linkage AC

Silhoutte Coefficient

$$\text{dissimilarity} \propto \frac{1}{Coh} = \frac{1}{|C_i|-1} \sum_{\substack{K \in C_i \\ K \neq i}} d(i,K)$$

$a(i)$



Separation

$$b(i) = \min_{K} \left\{ \frac{1}{|C_j|} \sum_{j \in C_K} d(i,j) \right\}$$

$$SC(i) = \frac{b(i) - a(i)}{\max\{b(i), a(i)\}}$$

① $a(i) > b(i)$

$SC(i) = \frac{b(i)}{a(i)} - 1$

$a(i) >> b(i)$

$SC(i) \rightarrow -1$

② $a(i) = b(i)$

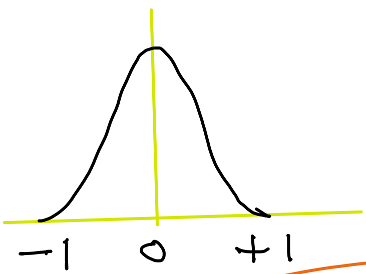
$SC(i) = 0$

③ $a(i) < b(i)$

$1 - \frac{a(i)}{b(i)}$

$a(i) << b(i)$

$SC(i) \rightarrow 1$



$\tilde{S}(i) =$ mean of all $S(i)$

$SC = \max_{K} \tilde{S}(C_K)$

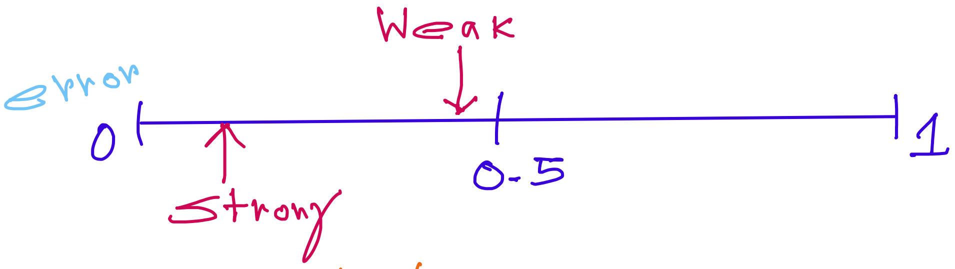
$\boxed{\text{Is it over the Cluster or the dataset?}}$

Netflix Problem:-

↳ Neural Net → Regularization

→ Boosting (Ensemble)

→ Cross-Validation



$f(x) \to +1$

$\quad\;\;\; \to -1$

Different Learners

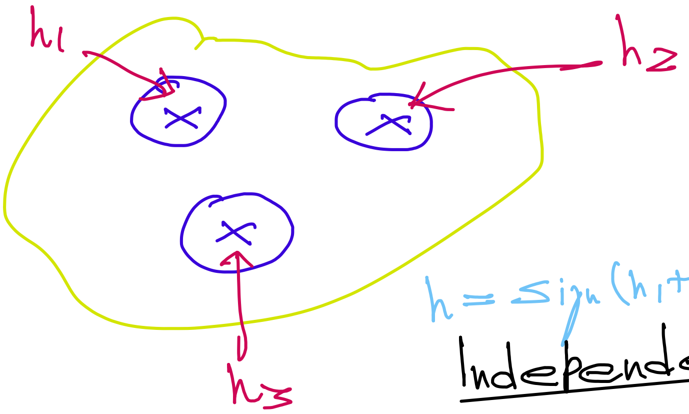
↳ Various Algo (BP, SVM, Per, BL)

↳ Diff types

↳ Diff act

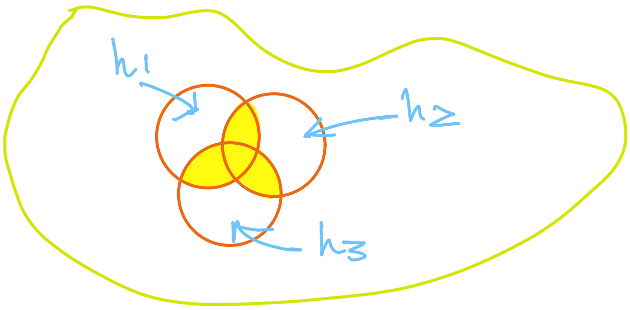
→ Regularize

→ Dataset



$h = \text{sign}(h_1 + h_2 + h_3)$

Independence

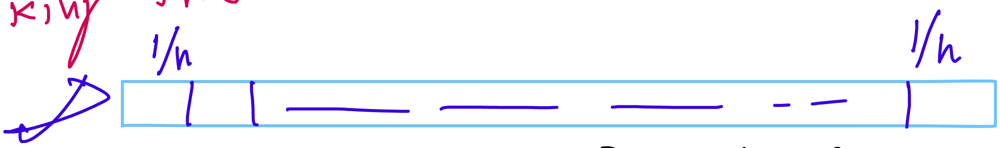


$$Var(d) = \frac{1}{K^2} \sum_i [Var(d_i)]$$

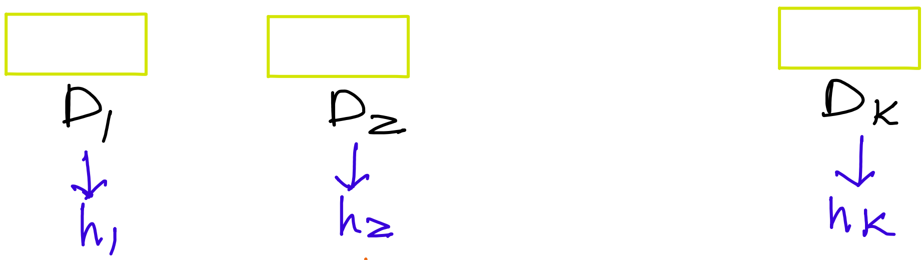
$$Var(d) = \frac{1}{K^2} \left[\sum_i Var(d_i) + \sum_i \sum_j Cov(d_i d_j) \right]$$

① Making diff Learners

② Making them Independent

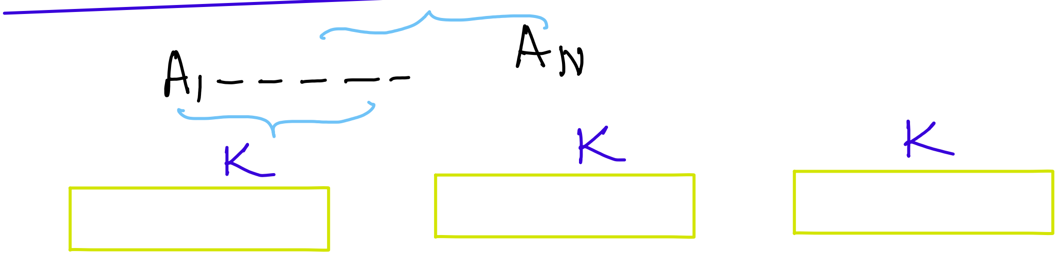


BAGGING

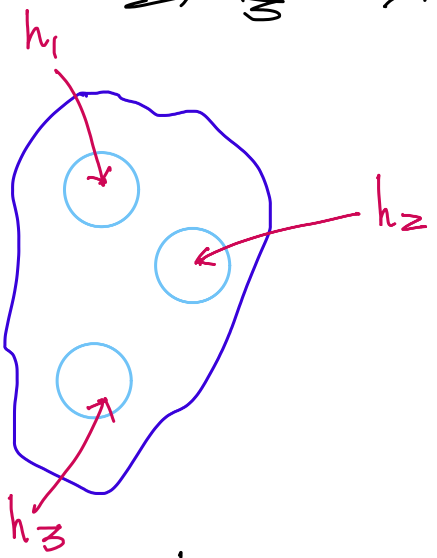


Random Subsampling:-

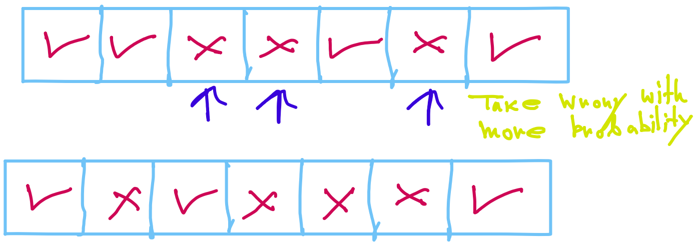
Random Forest:-



$D \to D_1 \to h_1$
exaggeration
$D_2 \to h_2$
$D_3 \to h_3$

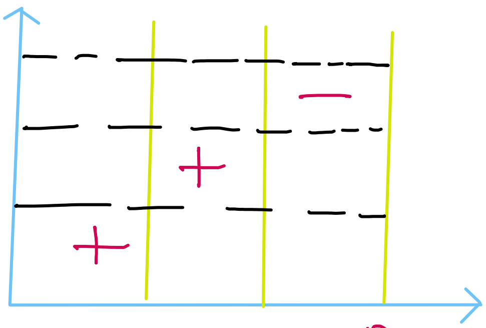


① Weak Learner
② Independent

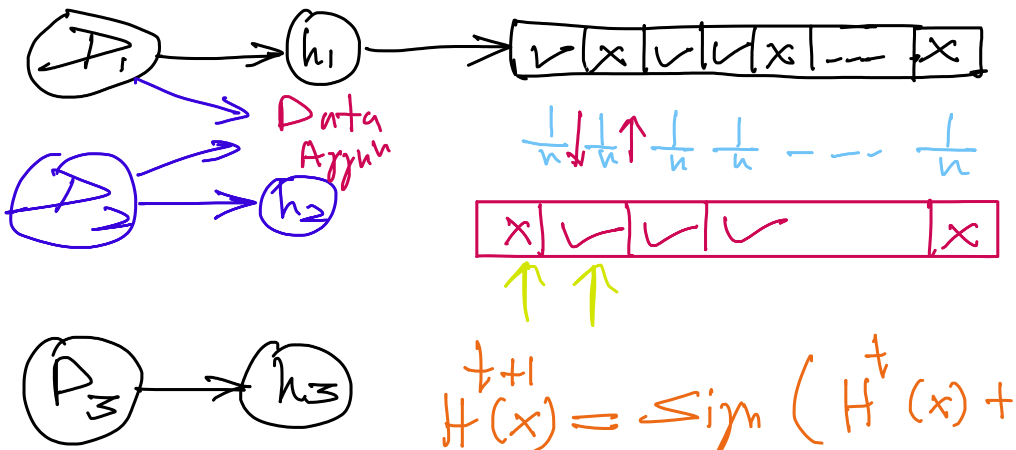


AdaBoost

$$H(x) = \mathrm{sign}(\alpha_1 h_1(x) + \alpha_2 h_2(x) + \alpha_3 h_3(x))$$



$\geq N$ Des St



$$H^{t+1}(x) = \mathrm{sign}\left(H^{t}(x) + \alpha^{t+1} h^{t+1}(x)\right)$$

$d_i \to w_i^{(1)} = \frac{1}{N}$

$\downarrow \mathcal{D}^{(t)}$

$h^{(t)}$ such that min $\varepsilon^{(t)}$ $\to$ Pick α_{t+1}

$\downarrow \mathcal{D}$

new w_i^{t+1}

$t+1$

$H^t(x) \to \alpha^{t+1}$

$H^t(x) \to w_i^{(t+1)} \to h^{t+1}(x)$

$$H^{(t+1)}(x) = H^t(x) + \alpha^{t+1} h^{t+1}(x)$$

$\epsilon^t = \sum_{i \in WR} w_i^{(t)}$ $\qquad (1-\epsilon^t) = \sum_{i \in COR} w_i^{(t)}$ $\qquad \sum_i w_i^{(t)} = 1$

Wrong

Exp Loss:- $\quad w_i^{t+1} \leftarrow \frac{w_i^{(t)}}{Z} e^{-\alpha^t h^t(x) y(x)}$ $\quad (h^t(x) = \pm 1,\ y(x) = \pm 1)$

Correct decision, weight decreases for that point and ....

$$= \frac{w_i^{(t)}}{Z} \begin{cases} e^{-\alpha(t)} & COR \\ e^{\alpha(t)} & WR \end{cases}$$

α^{t+1}? $\qquad \sum_i w_i^{t+1} = 1$

$$\frac{\partial}{\partial \alpha(t)} \left[\sum_{i \in WR} w_i^{(t)} e^{\alpha(t)} + \sum_{i \in COR} w_i^{(t)} e^{-\alpha(t)} \right] = 0$$

$$\Rightarrow \frac{1}{Z} \epsilon^{(t)} e^{\alpha(t)} + (1-\epsilon^{(t)}) e^{-\alpha(t)} = 0$$

$$\alpha^{(t)} = \frac{1}{2} \ln\left(\frac{1-\epsilon^{(t)}}{\epsilon^{(t)}} \right)$$

$$Z = 2\sqrt{\epsilon^t (1-\epsilon^t)}$$

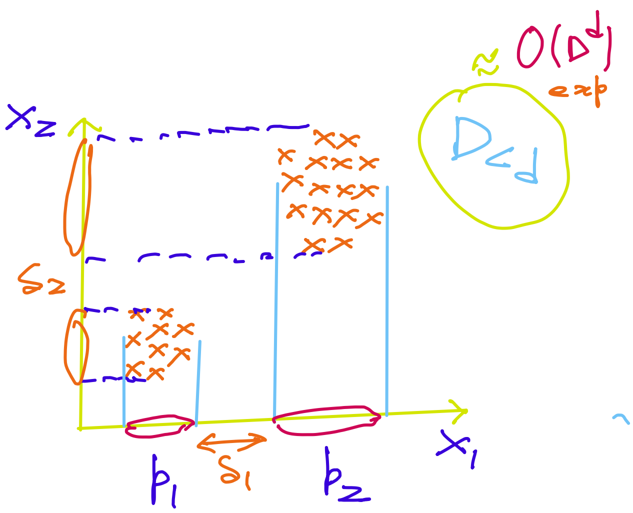
$$\Longrightarrow \frac{w_i^{(t)}}{2} \begin{cases} \frac{1}{1-\epsilon^t} & COR \\ \frac{1}{\epsilon^t} & WR \end{cases}$$

Dimensionality Reduction

▷ dim $\rightsquigarrow$ $1 \le d \le D$ dim

① Evaluate

② Project



Kullback Liebler Divergence (KL Div)

$$KL(p_1 \| p_2) = \sum_{x \in \mathcal{X}} p_1(x) \log \frac{p_1(x)}{p_2(x)} \quad \Big| \quad p_1(x) \log \frac{p_1(x)}{p_2(x)} + p_2(x) \log_2 \frac{p_2(x)}{p_1(x)}$$

$\{x_1, x_2, x_3, \dots, x_D\}$

$\boxed{A}$ Forward Search

$\{x_{k_1}\} \to D$

$\{x_{k_1}, x_{k_2}\} \to D-1$

$\vdots$

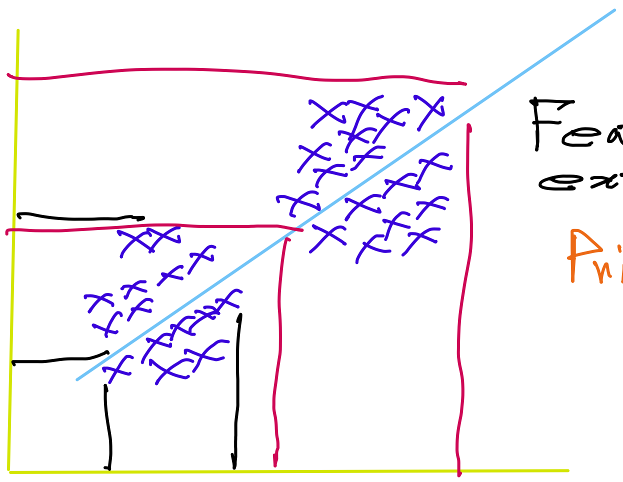
$\{x_{k_1}, x_{k_2}, \dots x_{k_d}\} \to D-(d-1)$

$O(dD)$

$\boxed{b}$ Backward Search

$(\{x_{k_1}, \dots x_{k_{D-1}}\} - 1)$

Feature Selection



Feature extraction

Principal Component

PCA:- (1) Mean $\bar{x}_1, \bar{x}_2$

(2) Covar

$$\begin{array}{cc} & \begin{matrix} x_1 & \cdots & x_N \end{matrix} \\ \begin{matrix} x_1 \\ \vdots \\ x_N \end{matrix} & \begin{bmatrix} \square & & \\ & \ddots & \\ & & \square \end{bmatrix} \end{array}$$

$N \times N$

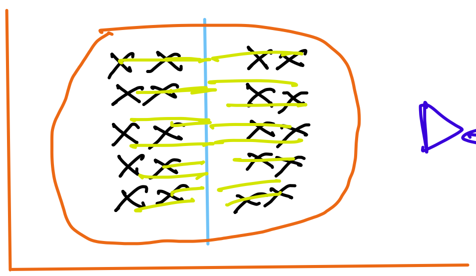
$$Var(x_1) = \frac{\sum_D (x_1 - \bar{x}_1)^2}{N}$$

$$Covar(x_1, x_2) = \frac{\sum_D (x_1 - \bar{x}_1)(x_2 - \bar{x}_2)}{N}$$

(3) Try to find eigenval λ

$Cx = \lambda x$

$\hookrightarrow \underbrace{\lambda_1, \lambda_2, \dots \lambda_d}_{\text{Top d eigval}} \to$ EVs



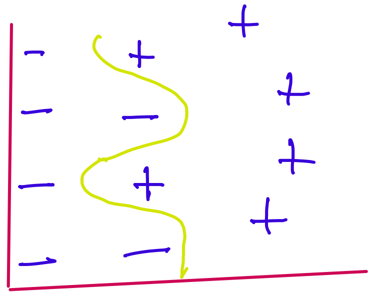
Demerit of PCA

LDA SVD (eff)

$$J = \frac{(m_1 - m_2)^2}{\sigma_1^2 + \sigma_2^2}$$

SVM:-

$$\mathbb{E}[E_{out}] \le \frac{\mathbb{E}[\#SV]}{N-1}$$



Kernel Trick:-

Primal:-

$\min \frac{1}{2} w^T w \quad$ s.t $\quad y_i(w^T x_i + b) \ge 1$

Dual:-

$\min \mathcal{L} = \frac{1}{2} w^T w + \sum \alpha_i \left(1 - y_i(w^T x_i + b)\right)$

$\alpha_i \ge 0$

$$\mathcal{L}(\alpha) = \sum_n \alpha_n - \frac{1}{2} \sum_n \sum_m y_n y_m \alpha_n \alpha_m (x_m x_n)$$

Q.P $\quad \alpha_n > 0$

$\sum_{n \in SV} \alpha_n y_n = 0 \qquad W = \sum_{n \in SV} \alpha_N y_N X_N$

b $\quad X_m \in SV$

$W^T x_m + b = 1$

$b = 1 - W^T x_m$

$$h(x) = \text{sign}(W^T x + b)$$

$$H = \begin{bmatrix} & | & \\ — & y_i y_j x_i x_j & — \\ & | & \end{bmatrix} \quad x_n \cdot x_m$$

$Z = \phi(x)$

$Z = (1, x_1, x_2, x_1^2, x_2^2, x_1 x_2)$

$Z' = (1, x_1', x_2', x_1'^2, x_2'^2, x_1' x_2')$

$$Z \cdot Z' = (1 + x_1 x_1' + x_2 x_2' + x_1'^2 x_1^2 + x_2'^2 x_2^2 + x_1 x_1' x_2 x_2')$$

$K(x, x')$

$$K(x, x') = (1 + x^T x')^2 \qquad (1, x_1, x_2) \quad (1, x_1', x_2')$$

$$= (1 + x_1 x_1' + x_2 x_2')^2$$

$$= (1 + x_1^2 x_1'^2 + x_2^2 x_2'^2 + 2 x_1 x_1' + 2 x_2 x_2' + 2 x_1 x_2 x_1' x_2')$$

$Z = (1, x_1^2, x_2^2, \sqrt{2} x_1, \sqrt{2} x_2, \sqrt{2} x_1 x_2)$

$Z' = (1, x_1'^2, x_2'^2, \sqrt{2} x_1', \sqrt{2} x_2', \sqrt{2} x_1' x_2')$

$K(x, x') = (1 + x^T x')^Q \leftarrow$ Poly Kernel

$$= (1 + x_1 x_1' + \cdots + x_d x_d')^2$$

$$\exp\left(Q \log\left(\frac{d}{\ }\right)\right)$$

$$K(x,x') = \exp(-\gamma(x-x')^2) = (e^{-x^2})(e^{-x'^2})(e^{2xx'})$$

$$= (e^{-x^2})(e^{-x'^2}) \sum_{k=0}^{\infty} \sqrt{\frac{2^k}{k!}}\,(x)^k\,(x')^k$$

RBF

K(x,x')

Construction

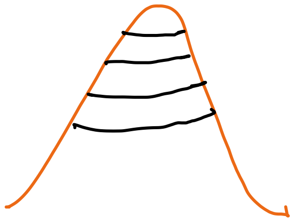
Math

Mercer's

Symmetric

Positive Semidef

$$h(x) = e^{-\gamma\underbrace{\|x-x_m\|^2}_{\text{radial}}}$$



$$h(x) = \sum_{m=1}^{N} \underbrace{w_m e^{-\gamma\|x-x_m\|^2}}_{\text{basis } f^n}$$

(x_1,y_1) ...... (x_N,y_N)

$$\sum_{m=1}^{N} w_m e^{-\gamma\|x_n-x_m\|^2} = y_N$$

N eq

N unknown

$$\underbrace{\begin{bmatrix} e^{-\gamma\|x_1-x_1\|^2} & \cdots & e^{-\gamma\|x_1-x_N\|^2} \\ \vdots & & \vdots \\ e^{-\gamma\|x_N-x_1\|^2} & \cdots & e^{-\gamma\|x_N-x_N\|^2} \end{bmatrix}}_{\phi} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

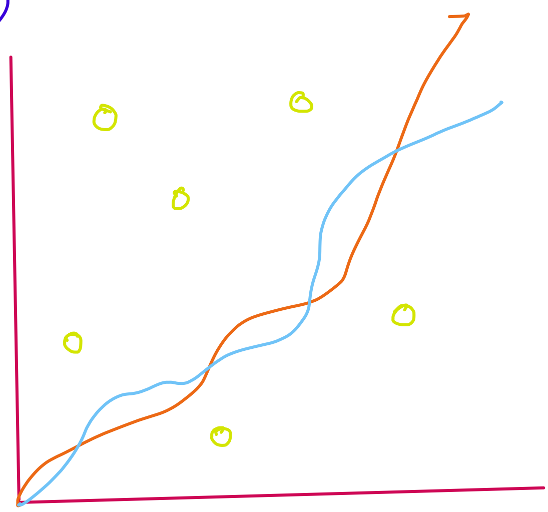
$$w = \phi^{-1} y$$

K-Means:- $\sum_{K=1}^{K}\sum_{x\in S_K} \underbrace{(x-\mu_K)^2}_{\text{RBF}}$

Nearest Neighbour
↳ hard boundary

$\mu_K - \mu_K' \Longrightarrow$ (Lloyd's algo)

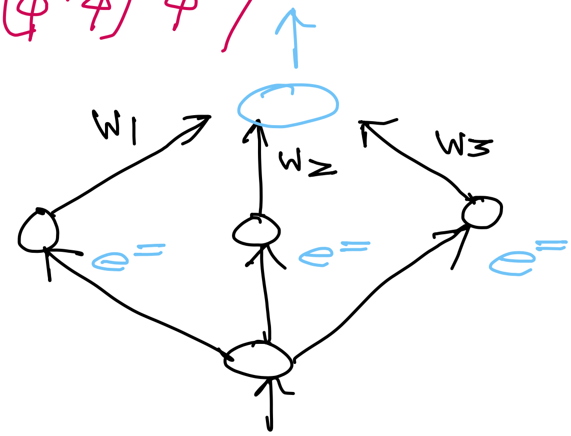
SVM



$$\left[\quad\right]_{K\times N}\left[\quad\right]_{N\times 1} = \left[\quad\right]$$

$W = (\phi^T\phi)^{-1}\phi^T y$

Semi Sup



Occam's Razor:-

↳ Simple means what?

→ Simple is better, why?

$H = \{h_1, \ldots, h_M\}$

h = final model   Poly(Q)

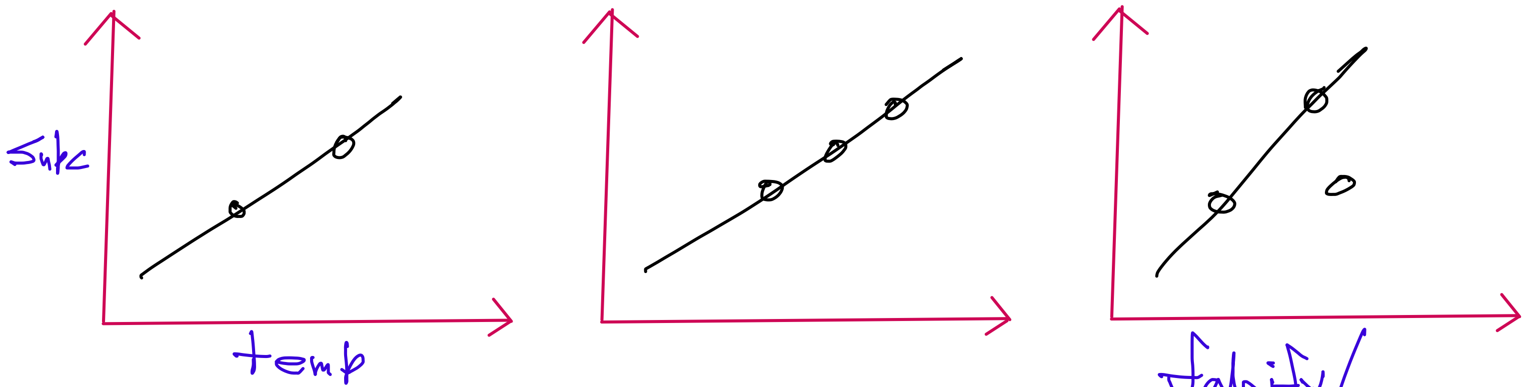
| Simple model (h) fits all → |

Simple
↳ Representation of H

h: MDL, bits to represent

H: vc-dim, Entropy

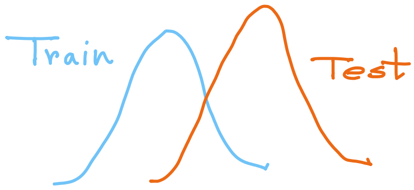
h: → l bits ⇒ $2^l = |H|$



Falsify/
Justify

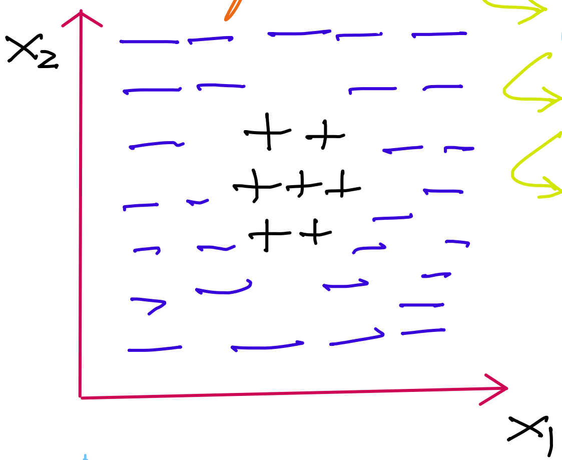
Sampling Bias :- 1948 T vs D

D beats T



Credit Approval :-

Snooping :-

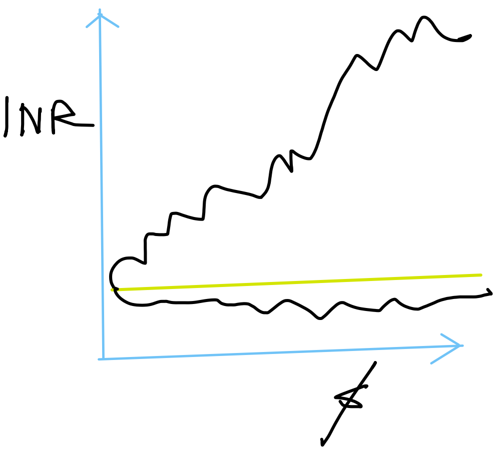


$(1, x_1, x_2)$

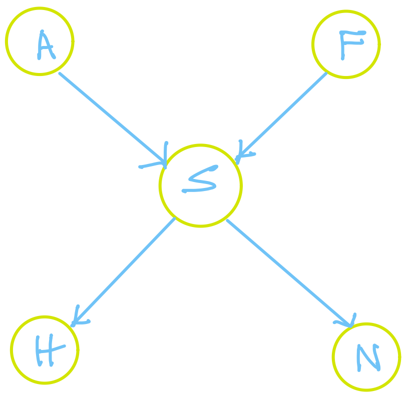
$\to (1, x_1^2, x_2^2)$

$\to (1, x_1^2 + x_2^2)$

$\to (x_1^2 + x_2^2 - 0.6)$



$\mathcal{D} \to$ Norm $\to \mathcal{D}_{Train}$, $\mathcal{D}_{Test}$



K	A	F	S	H	N
1	1	0	1	1	0
2	0	1	1	0	1
3	1	1	0	1	0
4	0	0	1	1	1
5	1	0	1	0	0
6	1	1	0	1	1

$$\hat{\mathbb{P}}(s \mid F, A) = \theta_{s|ij} = \frac{\sum_{k=1}^{K} \delta(S_k = 1, F_k = i, A_k = j)}{\sum_{k=1}^{K} (F_k = i, A_k = j)}$$

$\theta_{s|01} = \frac{2}{2} = 1$

$\theta_{s|00} = 1$

$\theta_{s|10} = \frac{1}{2}$

$P(a, f, s, h, n)$

$= P(a)\, P(f)\, P(s \mid a, f)\, P(h \mid s)\, P(h \mid s)$

$$\arg\max_{\theta_{s|ij}} \sum_{k=1}^{K} \log P(s_k, a_k, f_k, h_k, h_k, \theta_{ij})$$

$$\equiv \arg\max_{\theta_{s|ij}} \sum_{k=1}^{K} \Big[\log P(a_k) + \log P(f_k) + \log P(s_k|a_k, f_k) + \log P(h_k|s_k) + \log P(h_k|s_k)\Big]$$

$$\frac{\partial}{\partial \theta_{s|ij}} = 0$$

$$X^{(P_0)} = \{A, F, H, N\}$$

$$Z_{(P_0)} = S$$

$$\hat{\theta}_{s|ij} = \frac{\sum_{k=1}^{K} \delta(F_k = i, A_k = j) * \mathbb{E}[S_k = 1]}{\sum_{k=1}^{K} \delta(F_k = i, A_k = j)}$$

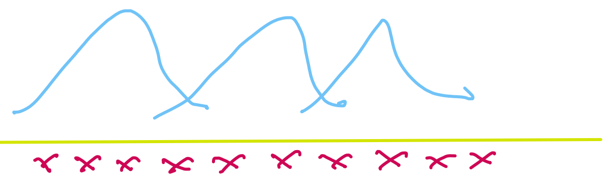
$$\mathbb{E}[S_k = 1] = \frac{P(S_k = 1 | a_k, f_k, h_k, h_k, \theta)}{P(S_k = 1 | a_k, f_k, h_k, h_k, \theta) + P(S_k = 0 | a_k, f_k, h_k, h_k, \theta)}$$

exp-max algo

E-M

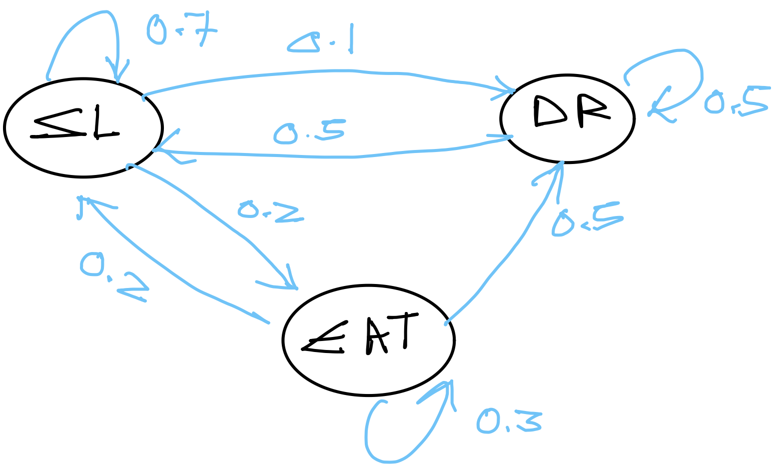
Chow-Liu
- → Inf-th (KL Div)
- → Tree structure

$$\frac{\partial}{\partial \theta} \arg\max_{\theta} \mathbb{E}_{Z|X,\theta}\big[\log(X|Z, \theta)\big]$$



$$P(x_1, \dots x_N) = \prod_{i=1}^{n} P(x_i)$$

$$P(x_i | z_j) \cdot \frac{P(z = j)}{N(\sigma_j, \mu_j)}$$



$$\begin{tikzcd}
\arrow[r] & Y_1 \arrow[r] \arrow[d] & Y_2 \arrow[r] \arrow[d] & Y_3 \arrow[r] \arrow[d] & \cdots \\
& X_1 & X_2 & X_3 &
\end{tikzcd}$$

HMM

Stat:-

Init dist:- $\mathbb{P}(y_1)$

Tran dist:- $\mathbb{P}(y_t \mid y_{t-1})$

(Markov) $P(y_t \mid y_{1 \ldots t-1}) = P(y_t \mid y_{t-1})$

▷ Emit dist $P(x_t \mid y_t)$ → cont

→ multinomial (Discont)

$$P(x_{1 \ldots t}\ y_{1 \ldots t})$$

$$= P(y_1) \prod_{t=2}^{N} P(y_t \mid y_{t-1}) \prod_{t=1}^{N} P(x_t \mid y_t)$$

Monitor:- $\mathbb{P}(y_t \mid x_{1 \ldots t})$

Predict: $\mathbb{P}(y_{t+k} \mid x_{1 \ldots t})$, $k \geq 1$

Hindsight:- $\mathbb{P}(y_k \mid x_{1 \ldots t})$

$$M_t = P(y_t \mid x_{1 \ldots t}) \propto P(x_t \mid y_t\ x_{1 \ldots t})\, P(y_t \mid x_{1 \ldots t-1})$$

Bayes Cond

$$= P(x_t \mid y_t) \sum_{y_{t-1}} P(y_t\, y_{t-1} \mid x_{1 \ldots t-1})$$

Prob Marg

$$= P(x_t \mid y_t) \sum P(y_t \mid y_{t-1}) \cdot \underbrace{P(y_{t-1} \mid x_{1 \ldots t-1})}_{M_{t-1}}$$

$$\begin{tikzcd}
y_1 \arrow[r] \arrow[d] & y_2 \arrow[r] \arrow[d] & y_3 \arrow[r] \arrow[d] & y_4 \arrow[d] \\
x_1 & x_2 & x_3 & x_4
\end{tikzcd}$$

Forward algo

$$P_{t+k} = \mathbb{P}(y_{t+k} \mid x_{1 \ldots t}) \quad k \geq 1$$

$$= \sum_{y_{t+k-1}} P(y_{t+k}\ y_{t+k-1} \mid x_{1 \ldots t})$$

$$= \sum_{y_{t+k-1}} P(y_{t+k}|y_{t+k-1}) \underbrace{P(y_{t+k-1}|x_{1...t})}_{P_{t+k-1}}$$

Forward Pass algo

$$H_k = P(y_k|x_{1...t})$$

$$= P(\underset{a}{y_k}|\underset{b}{x_{1..k}}\ \underset{c}{x_{k+1...t}})$$

Bayes Cond

$$\propto P(x_{k+1...t}|y_k\ \cancel{x_{1..k}})\ \underbrace{P(y_k|x_{1..k})}_{M_k}$$

$$P(\underset{a}{x_{k+1...t}}|\underset{b}{y_k})$$

(Fwd-Back Algo)

$$= \sum_{y_{k+1}} P(y_{k+1}\ x_{k+1...t}|y_k)$$

$$= \sum_{y_{k+1}} P(x_{k+1...t}|y_{k+1})\ P(y_{k+1}|y_k)$$

$$= \sum_{y_{k+1}} P(x_{k+1}|y_{k+1})\ P(x_{k+2...t}|y_{k+1})\ P(y_{k+1}|y_k)$$

Explainability:- $\arg\max_{y_{1...t}} P(y_{1...t}|x_{1...t})$

Vitrebi Algo (Dynamic Prog)

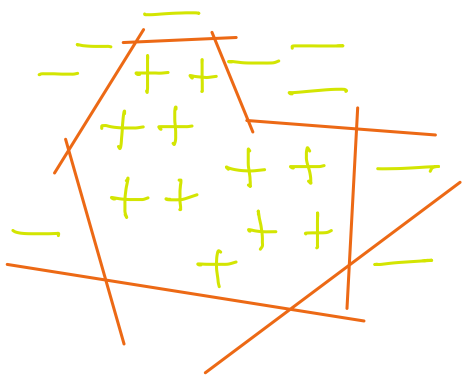
$$\max_{y_{1...t}} P(y_{1...t}|x_{1...t})$$

$$= \max_{y_t} P(x_t|y_t)\ \max_{y_{1...t-1}} P(y_{1...t}|x_{1...t-1})$$

$$\propto \max_{y_{t-1}} P(y_t|y_{t-1})\ P(x_{t-1}|y_{t-1}) \cdots \underbrace{P(y_{t-1}|x_{1..t-1})}_{M_{t-1}}$$

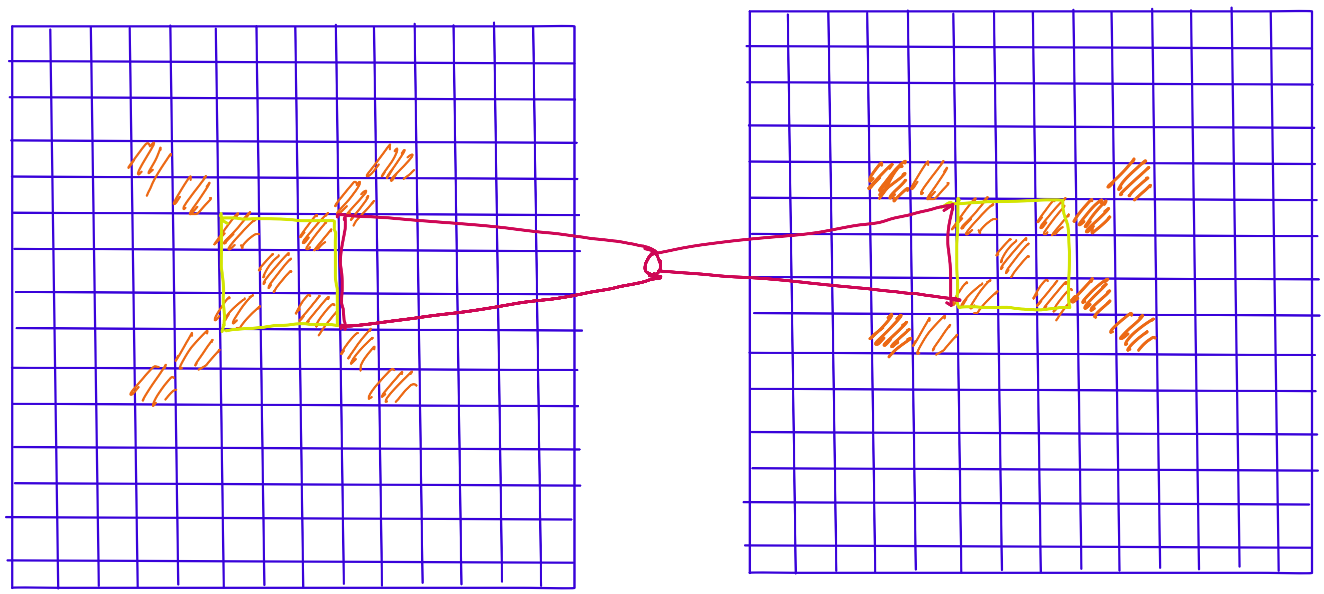
$$\max_{y_{1..t-2}} \underbrace{P(y_{1...t-1}|x_{1...t-2})}_{\varepsilon_{t-1}}$$

ANN:



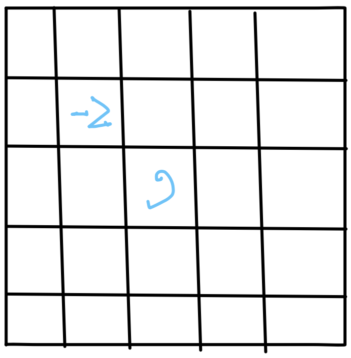
Spatial Info

DNN

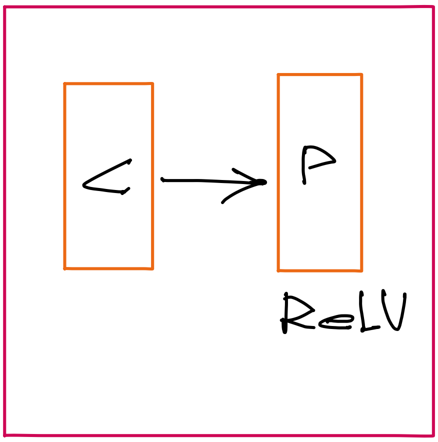


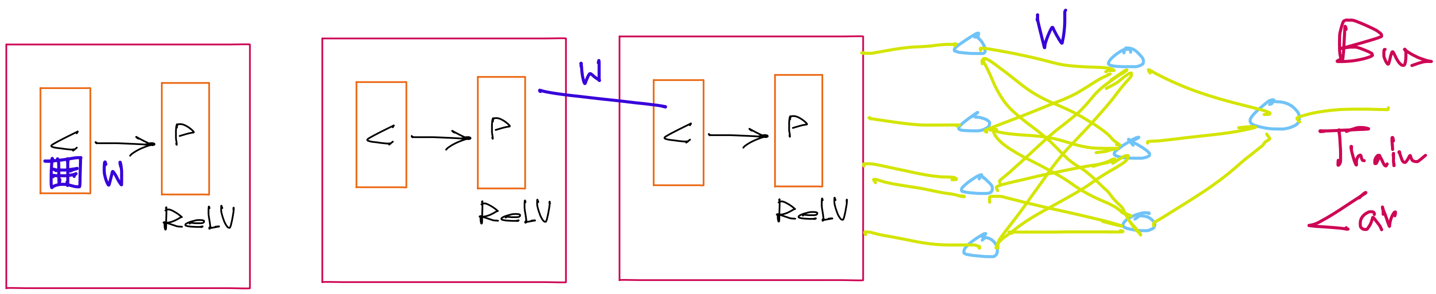
1	-1	1
-1	1	-1
1	-1	1

"Convolution"

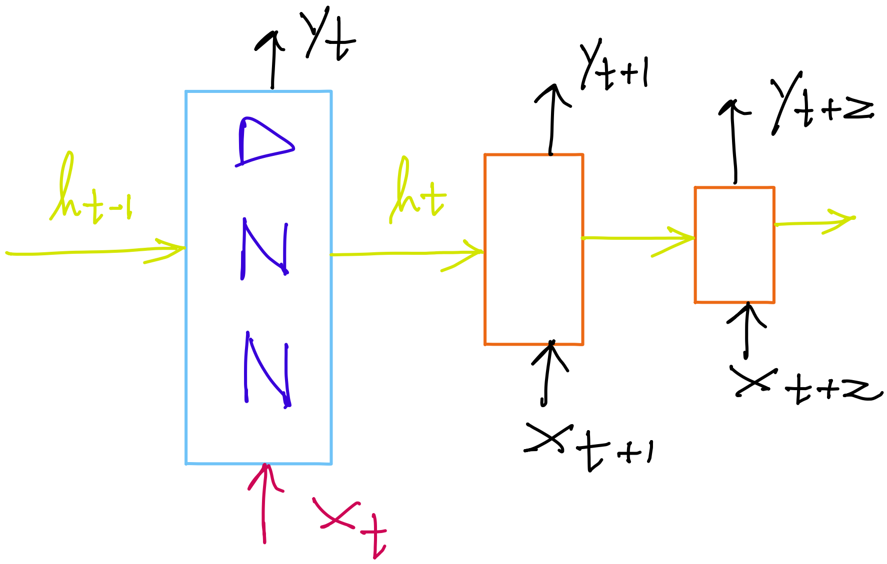


ReLU





CNN



Diminishing Gradient

$$\theta'(s) = \theta(s)\,(1-\theta(s))$$

Back P through time

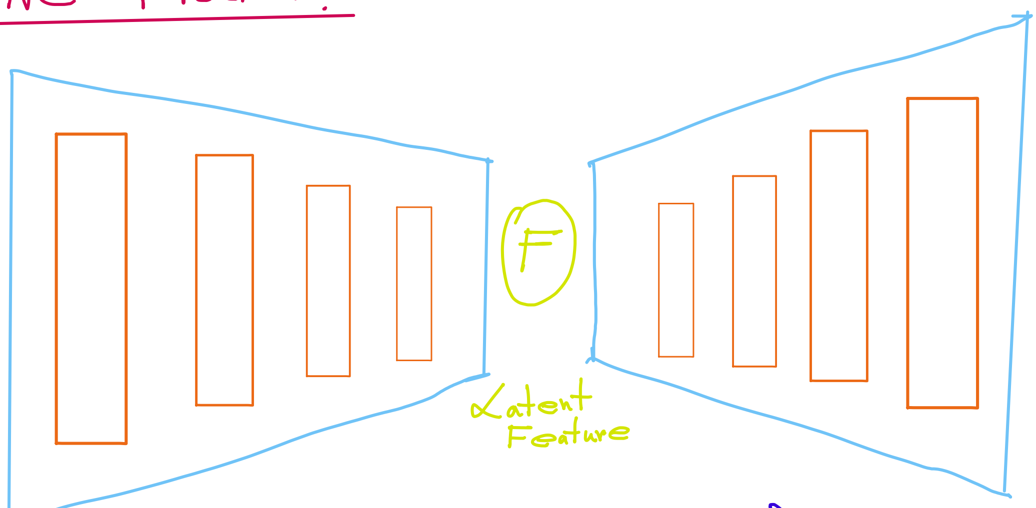
RNN

$$y_t = \tanh(W x_t + W' h_t)$$

$$h_t = f(W'' x_t + W''' h_{t-1})$$

Generative Model :-

$\boxed{2}$



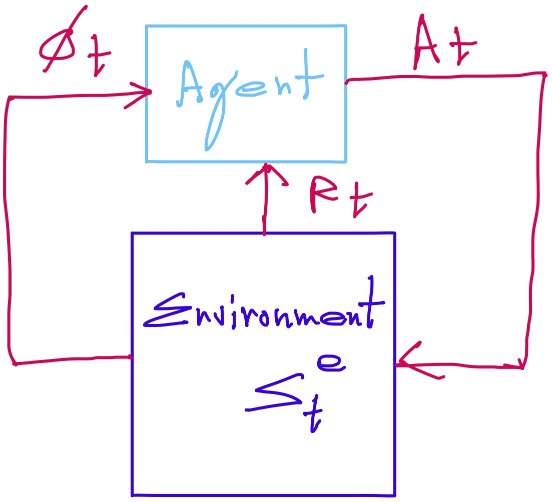
Compressing

Reg

VAR

"Auto encoder"

Reinforcement Learning



Sequential Dec making

$$O^t = S_t^a = S_t^R$$

$$P[S_{t+1} \mid S_{1 \ldots t}] = P[S_{t+1} \mid S_t]$$

$\boxed{\text{Markov Prop}}$

- Policy
  $\pi : S \to A$
  - Det $\pi(a|s) = 1$
  - Stochastic $0 \leq \pi(a|s) \leq 1$

- Value f^n :-

$$V_\pi(s) = \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right]$$

Goal max cummulative Reward

- Model :-

Trans Prob :-

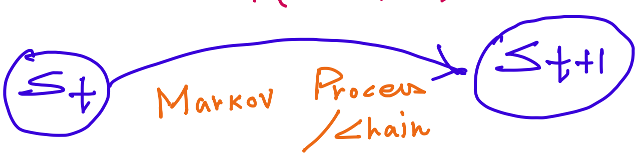
Reward Prob :-

Model Based & Model free } Planning vs Learning

Value Based vs Policy Based

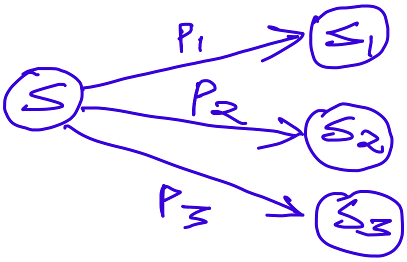
Exploit vs Exploration

$P(S_{t+1} \mid S_t)$



$M : \langle S, P \rangle \longrightarrow M : \langle S, P, R, \gamma \rangle$

$0 \leq \gamma \leq 1$

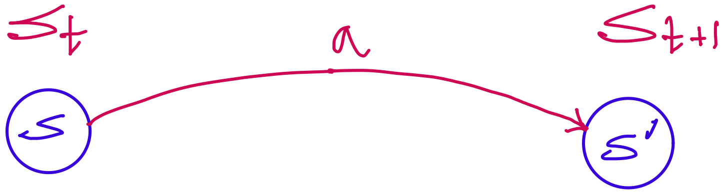


$$P_{ss'} = \mathbb{P}[S_{t+1} = s' \mid S_t = s]$$

$$R_s = \mathbb{E}[R_{t+1} \mid S_t = s]$$

$$V_\pi(s) = \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right]$$

$$= \mathbb{E}\left[R_{t+1} + \gamma V_\pi(s')\right]$$



$P^a_{ss'} = p(s,a,s')$

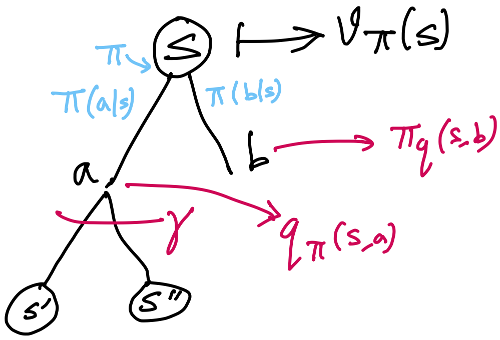
$= \mathbb{P}[S_{t+1}=s' \mid S_t=s, A_t=a]$

$R^a_s = r(s,a)$

$= E[R_{t+1} \mid S_t=s, A_t=a]$

Markov Decision Process

$\langle S, A, P, R, \gamma \rangle$



$$V_\pi(s) = \sum \pi(a|s)\, q_\pi(s|a)$$

$$q_\pi(s,a) = R^a_s + \gamma \left[\sum_{s'} P^a_{ss'} v_\pi(s')\right]$$

$$V_\pi(s) = \sum \pi(a|s) \left[R^a_s + \gamma \sum P^a_{ss'} V_\pi(s')\right]$$

$$q_\pi(s,a) = R^a_s + \gamma \sum_{s'} P^a_{ss'} \sum \pi(a'|s)\, q_\pi(s',a')$$

Bellman Exp eq^n $\longrightarrow$

$$V_*(s) = \max_\pi V_\pi(s)$$

$$q_*(s,a) = \max_\pi q_\pi(s,a)$$

$\pi \to V_\pi \to \pi' \to V_{\pi'}$

"evaluate" $\longrightarrow$ "Improving"

Bellman Opt Eqn

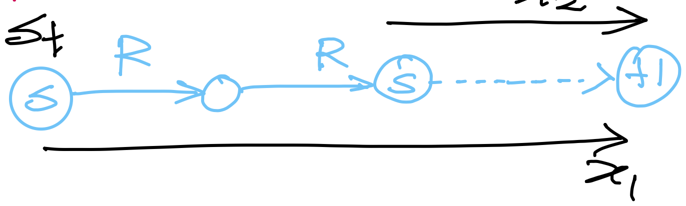
$$V_*(s) = \max_a q_*(s,a)$$

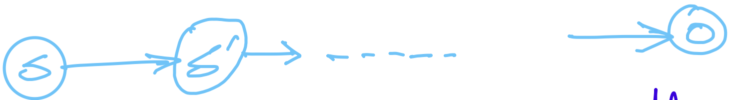
$$q_*(s,a) = R^a_s + \gamma \left[\sum_{s'} P^a_{ss'} V_*(s')\right]$$

$$V_*^{(K)}(s) \leftarrow \max_a q_*^{(K-1)}(s,a)$$

$$= \max_a \left[R^a_s + \gamma \sum P^a_{ss'} V_*^{(K-1)}(s')\right]$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{K-1} R_{t+K}$$





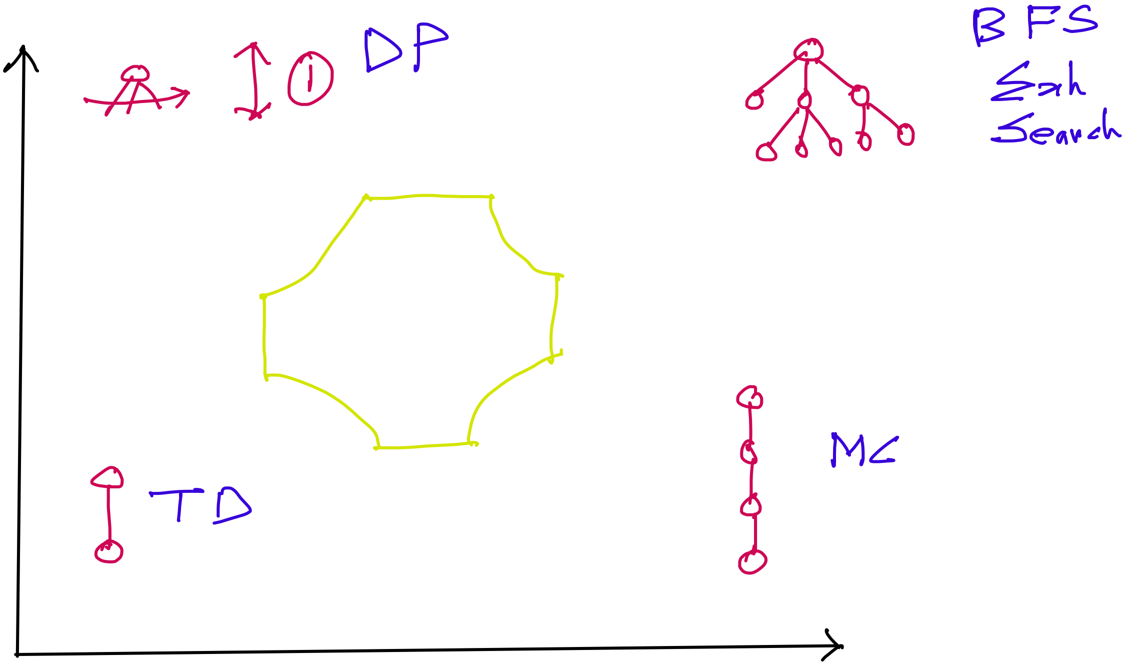
$$\boxed{\frac{\sum x_i}{N}}$$

$M_1 M_2 \dots M_K$

$$M_K = \frac{1}{K}\sum_{i=1}^{K} x_i = \frac{1}{K}\left(x_K + M_{K-1}(K-1)\right)$$

$$= \frac{M_{K-1}}{V} + \frac{1}{K}\left(M_K - M_{K-1}\right)$$

$\hat{V}_K(s) \Leftarrow \hat{V}_K(S-1) + \alpha(G_K - \hat{V}_{K-1}(s))$



$Q(S,A) + \alpha[R + \gamma Q(S',A')$

Policy Itr ← Eval ← Bellman Exp

Imp ← greedy

DP

Model Based → Policy Itr

Model Based → Value Itr

Model Free → Deep → MC

Model Free → One Step → TD

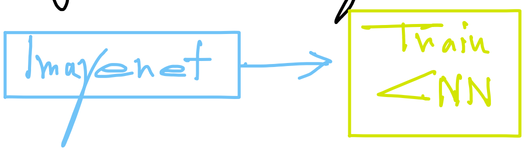
Deep ↕ One Step

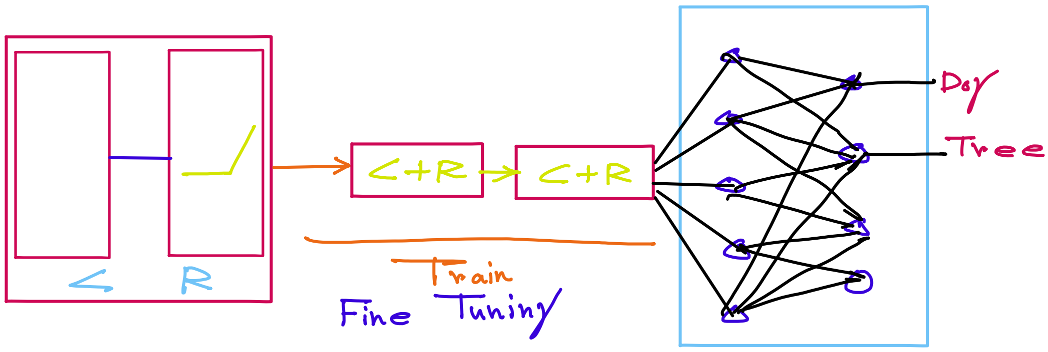
Control $Q(S,A)$ → SARSA (A')

Control $Q(S,A)$ → Q-Learning (max A')

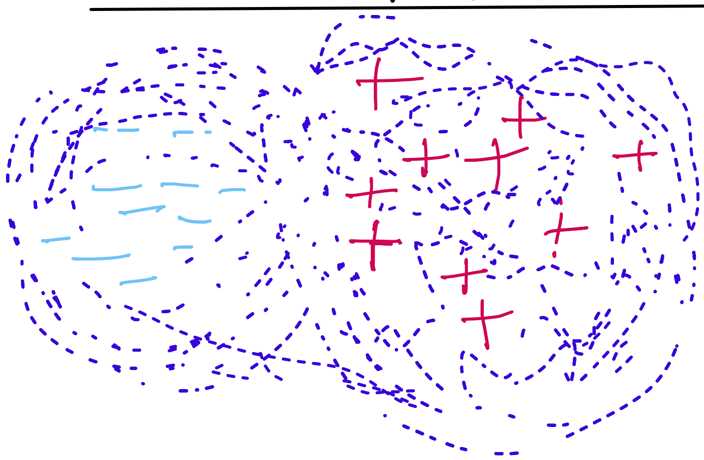
Few Shot Learning (Less labelled data)

(I) Transfer Learning





(II) Semi Supervised Learning



→ Graph Similarity 1983

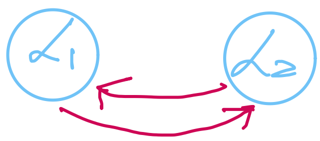
→ Transductive SVM

$$\max \frac{1}{2} w^T w$$

$$s.t.\ ①\ y_l (w^T x_l + b) \geq 1$$

$$②\ \hat{y}_u (w^T x_u + b) \geq 1$$

$$③\ \hat{y}_u \in \{+1, -1\}$$



(III) (Inter) Active Learning

$$\log_2 N \leftarrow \log_2 \left(\frac{1}{\epsilon}\right)$$

→ Density

→ Uncertainty

→ Ensemble

$$W_1 D + W_2 D$$

12 Meta Learning