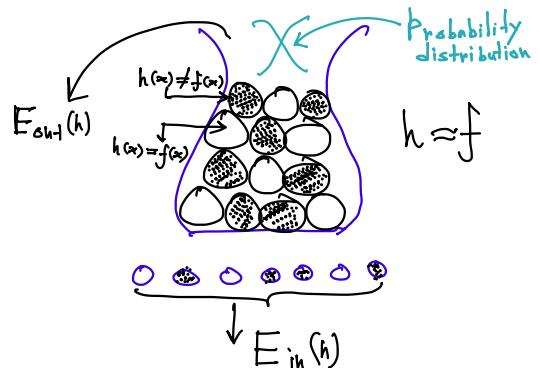
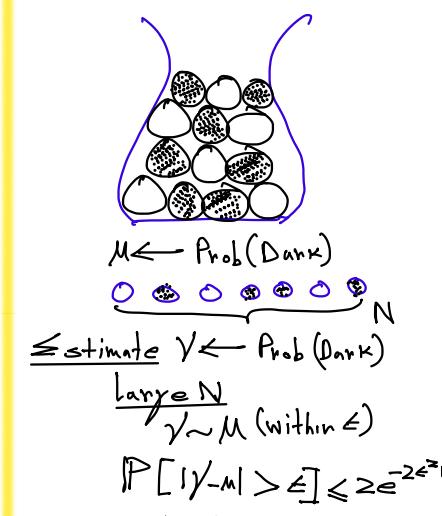


Possible — No

vs

Probable — Yes



$$P[|E_{out}(h) - E_{in}(h)| > \epsilon] \leq 2e^{-2\epsilon^2 N}$$

$$\sum_{h_1} \sum_{h_2} \sum_{h_3} M$$

$$P[|E_{out}(y) - E_{in}(y)|] \leq M e^{-2\epsilon^2 N}$$

P.A.-C

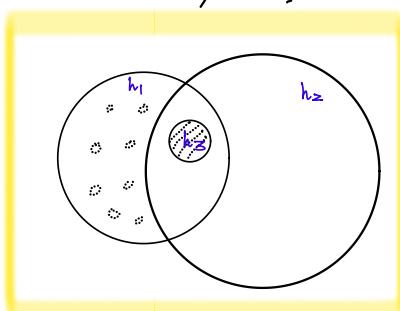
### Concept Learning

Boolean Formula  $\Rightarrow$  Anded

$$h_1: \langle \text{sunny?}, \text{strong?} \rangle \rightarrow (\text{sky = sunny}) \wedge (\text{wind = strong})$$

$$h_2: \langle \text{sunny?}, \text{?}, \text{?}, \text{?}, \text{same} \rangle \rightarrow \begin{cases} \text{S = sunny} \\ \wedge F = S \\ 1 \neq \end{cases} = F$$

$$h_3: \langle \text{sunny?}, \text{strong?}, \text{same} \rangle$$



$$\langle \phi, \phi, \phi, \phi, \phi, \phi \rangle$$

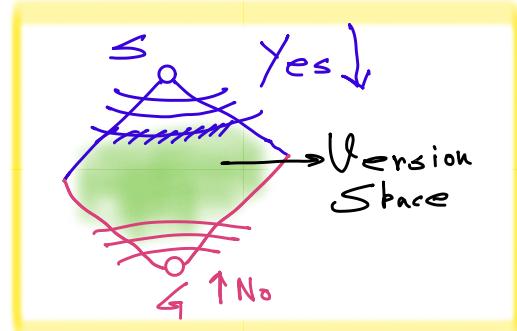
-1

$$\langle ?, ?, ?, ?, ?, ? \rangle$$

$$\# \text{sky} = 3(i)$$

$$\# \text{str} = 2(s)$$

$$M = 1 + \gamma \sqrt{3^5}$$



### Find S

$$S_0 \leftarrow \langle \phi, \phi, \phi, \phi, \phi, \phi \rangle$$

$$S_1 \leftarrow \langle \text{sunny}, \text{W}, \text{N}, \text{strong}, \text{W}, \text{same} \rangle$$

$$S_2 \leftarrow \langle \text{sun}, \text{W}, ?, \text{str}, \text{W}, \text{s} \rangle$$

$$S_3 \nearrow$$

$$S_4: \langle \text{sun}, \text{W}, ?, \text{str}, ?, ? \rangle$$

### Candidate Elimination

$$log \geq |Vs|$$

$S_0 \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$   
 $S_1 \leftarrow \langle S, W, N, S, W, S \rangle$   
 $S_2 \leftarrow \langle S, W, ?, S, W, S \rangle$   
 $S_3 \leftarrow \langle S, W, ?, ?, S, ? \rangle$   
 $S_4 \rightarrow \langle S, W, ?, S, ?, ? \rangle$

$G_0 \leftarrow \langle ?, ?, ?, ?, ?, ?, ? \rangle$   
 $\uparrow$   
 $G_1 = G_2$   
 $G_3 \leftarrow$

$\langle S, ?, ?, ?, ?, ? \rangle \langle ?, W, ?, ?, ?, ? \rangle \langle ?, ?, ?, ?, ?, ? \rangle C$   
 $G_4 \leftarrow \langle S, ?, ?, ?, ?, ? \rangle \langle ?, W, ?, ?, ?, ? \rangle$

$\langle S, W, ?, ?, ? \rangle \langle ?, W, ?, S, ? \rangle \langle S, ?, ?, S, ?, ? \rangle$

$S_{nn} \longrightarrow Y$   
 $\text{Cloudy} \longrightarrow Y$   
 $Rain \longrightarrow N$

Mitchell's Chapter 2

DR

	Season	Weather	Play Football
Train	Winter	Sunny	Y
	Summer	Rainy	Y
	Winter	Rainy	N
	Summer	Sunny	?

$S_0 \rightarrow \langle \emptyset, \emptyset \rangle$

$T_1: S_1 \rightarrow \langle W, S \rangle$

$T_2: S_2 \rightarrow \langle ?, ? \rangle$

$G_0 \rightarrow \langle ?, ? \rangle$

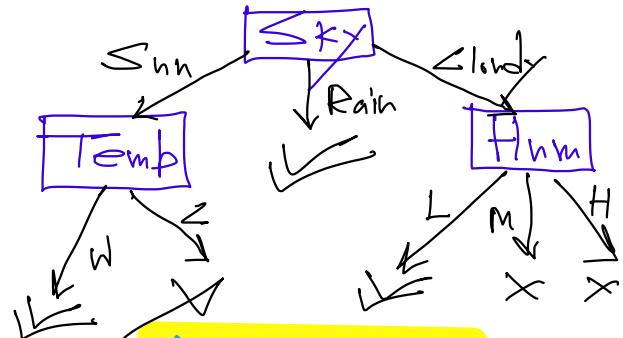
$T_3: \cancel{\langle ?, ? \rangle}$

$G_2 \rightarrow \langle ?, ? \rangle$

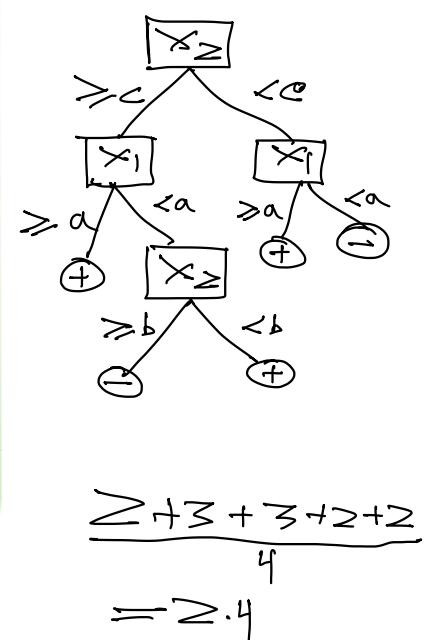
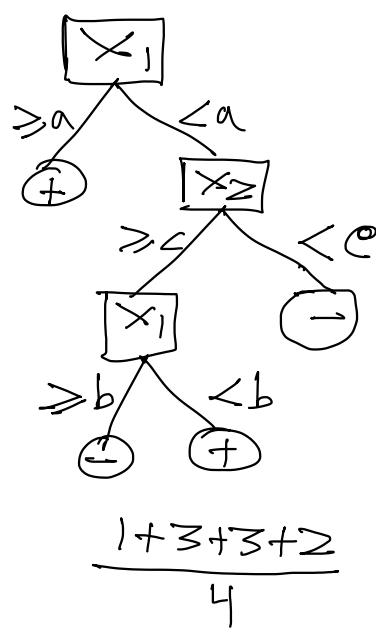
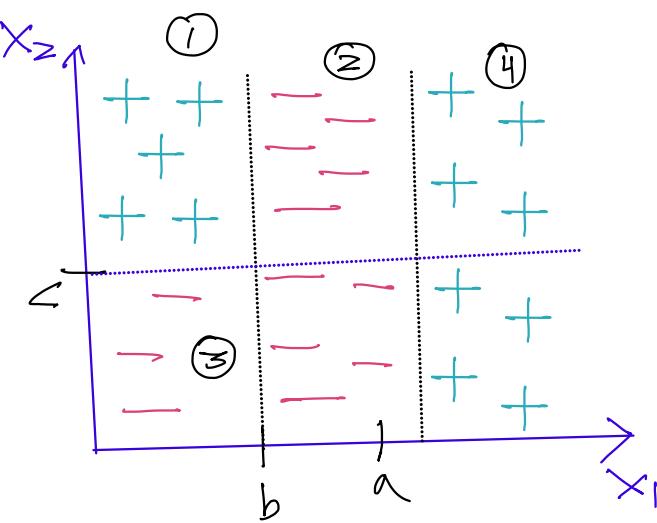
DR  $(\text{Sky}=\text{Sunny}) \wedge (\text{Temp}=\text{Warm})$

$\vee (\text{Sky}=\text{Rainy})$

$\vee (\text{Sky}=\text{Cloudy}) \wedge (\text{Hum}=\text{Low})$



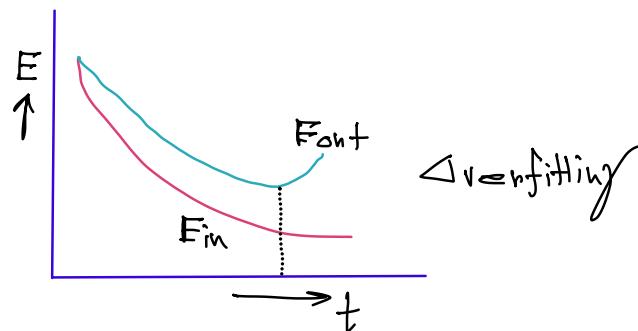
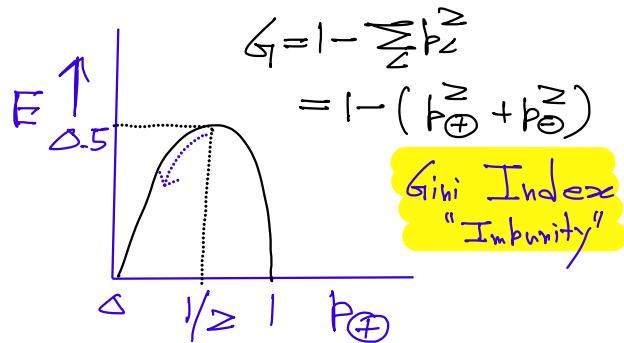
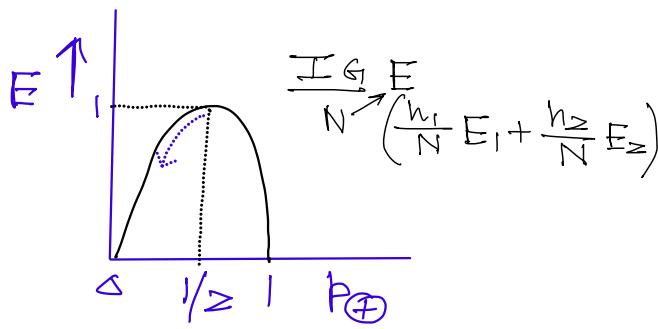
Decision Tree



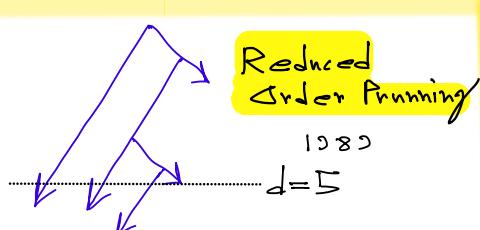
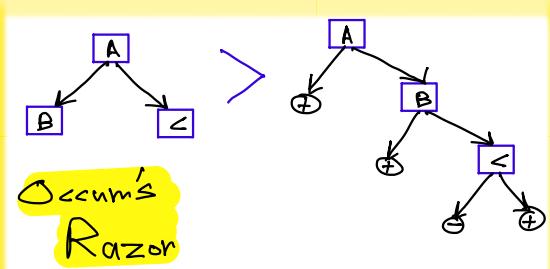
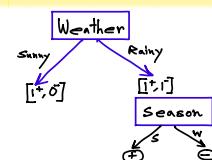
$$\text{Entropy} (p_+, p_-)$$

$$= -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$$

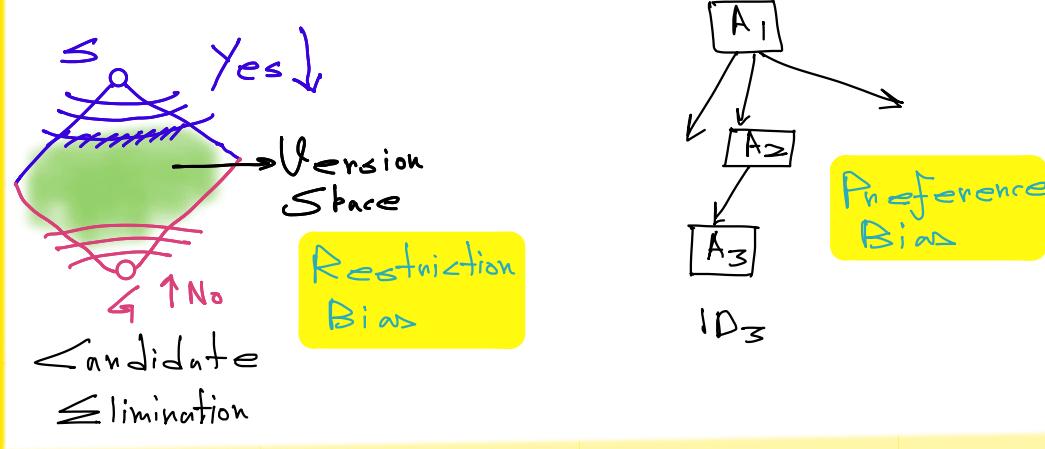
1D<sub>3</sub>  
1D87  
Quinlan



<u>Season</u>	<u>Weather</u>	<u>Play Football</u>
Winter	Sunny	Y
Summer	Rainy	Y
Winter	Rainy	N
Summer	Sunny	?



First construct  $(a \wedge b \wedge c \wedge d)$   
Full Tree and  $V(a \wedge c)$   
Then Eliminate  $V(a \wedge b \wedge c \wedge d)$



Temp	10	$1 \geq 14$	$\geq 22$	30	$3 \geq 38$	$4 \geq 48$
Play	N	N	N	N	Y	Y

$T \geq 26.5$        $T \leq 40$

A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	Out
a <sub>11</sub>	a <sub>21</sub>	a <sub>31</sub>	Y
a <sub>12</sub>	a <sub>22</sub>	a <sub>32</sub>	Y
a <sub>13</sub>	a <sub>23</sub>	a <sub>33</sub>	N
a <sub>14</sub>	a <sub>24</sub>	a <sub>34</sub>	N
a <sub>15</sub>	a <sub>25</sub>	a <sub>35</sub>	Y

$$f: x \rightarrow y \rightarrow \{+1, -1\}$$

$\nwarrow g(x) = ?$

$$P(y=1 | x_{new}) = ?$$

$$P(A=\text{male}) = \frac{|S_{\text{male}}|}{|S|}$$

$$0 \leq P(A) \leq 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

$$P(A|B) P(B) = P(A \wedge B)$$

$$= P(B|A) P(A)$$

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

Prior

Marginal

$$P(A_k \wedge B) = \frac{P(B|A_k) P(A_k)}{\sum_i P(B|A_i) P(A_i)}$$

$\geq N \xrightarrow{\text{Represent TPD}}$   
 $\xrightarrow{\geq 10^6 \text{ Rows} \approx 10^{30}}$  "Smart" Estimate  
 $\theta \leftarrow 0.22\% \cdot 10^3$  Probability

$$\textcircled{1} \quad 51 \text{ Heads} + 47 \text{ Tails}$$

$$\hat{\theta}_H = \hat{P}(x=1) = 0.51$$

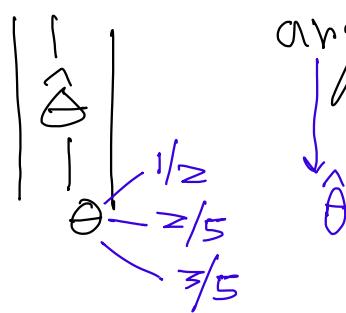
$$\textcircled{2} \quad \hat{\theta}_H \geq \text{Head} + 1 \text{ Tails}$$

$$\hat{\theta}_H = \hat{P}(x=1) = 0.67$$

$$\textcircled{3} \quad \alpha_H \text{ Heads} + \alpha_T \text{ Tails} \quad (\text{Online})$$

$$\frac{\alpha_H + 10}{(\alpha_H + 10) + (\alpha_T + 10)} \geq 0$$

H H T T H  
 $\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$



$\underset{\theta}{\operatorname{argmax}} \mathbb{P}[\text{Data} | \theta]$

MLE

$$\frac{\partial}{\partial \theta} [\mathbb{P}(D | \theta)] = 0$$

$$\frac{\partial}{\partial \theta} [\alpha_H \ln(\theta) + \alpha_T \ln(1-\theta)] = 0$$

$$\Rightarrow \frac{\alpha_H}{\theta} - \frac{\alpha_T}{1-\theta} = 0 \Rightarrow \theta = \frac{\alpha_H}{\alpha_H + \alpha_T}$$

MAP

$$\underset{\theta}{\operatorname{argmax}} [\mathbb{P}(D | \theta) \mathbb{P}(\theta)]$$

Marginal

"Independence"

$$P(x_1, x_2) = P(x_1) \cdot P(x_2)$$

$$x_1 \sim_{\text{ind}} x_2$$

Cond  
"Independence"

$$P(x_1 | x_2, y) = P(x_1 | y)$$

$$\text{or}, \quad P(x_1, x_2 | y) = P(x_1 | y) \cdot P(x_2 | y)$$

$$P(x_1, \dots, x_N | y) = \prod_i P(x_i | y)$$

$$\textcircled{1} \quad \frac{P(y=y_k) \prod_i P(x_i | y=y_k)}{\sum_j P(y=y_j) \prod_i P(x_i | y=y_j)} \geq N$$

Naive Bayes

$$\Rightarrow \frac{P[y=1 | x_1, \dots, x_N]}{P[y=0 | x_1, \dots, x_N]} \geq 1$$

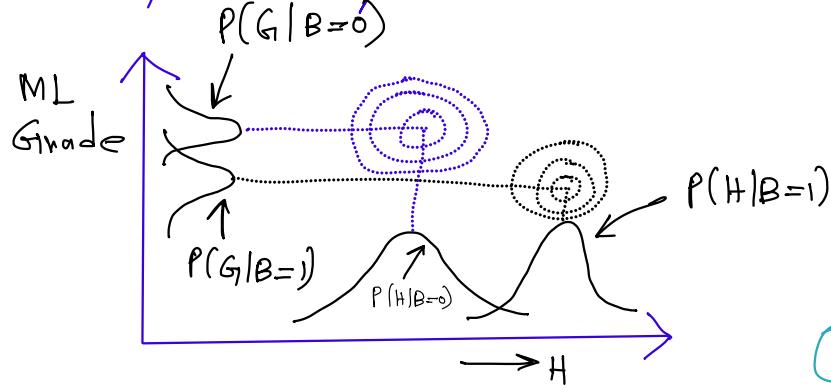
$$\Rightarrow \frac{P(y=1) \prod_i P(x_i | y=1)}{P(y=0) \prod_i P(x_i | y=0)} \geq 1$$

$$\Rightarrow \log\left(\frac{P(y=1)}{P(y=0)}\right) + \sum_i \log\left(\frac{P(x_i | y=1)}{P(x_i | y=0)}\right) \geq 0$$

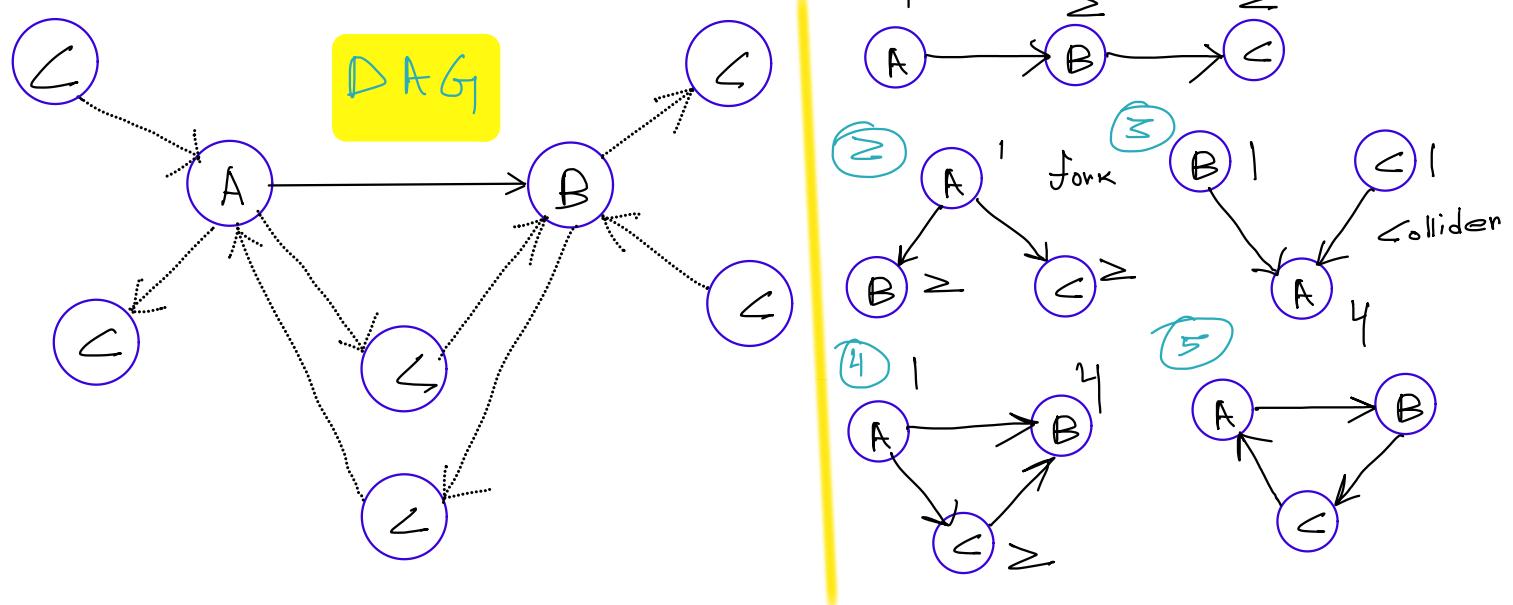
$$\Rightarrow \log\left(\frac{\hat{\theta}_1}{\hat{\theta}_0}\right) + \sum_i \left( x_i \log\left(\frac{\hat{\theta}_{i1}}{\hat{\theta}_{i0}}\right) + (1-x_i) \log\left(\frac{1-\hat{\theta}_{i1}}{1-\hat{\theta}_{i0}}\right) \right) \geq 0$$

Log  
Linear  
Model

$P[\text{Play} = B | \text{Height}, \text{ML Grade}]$

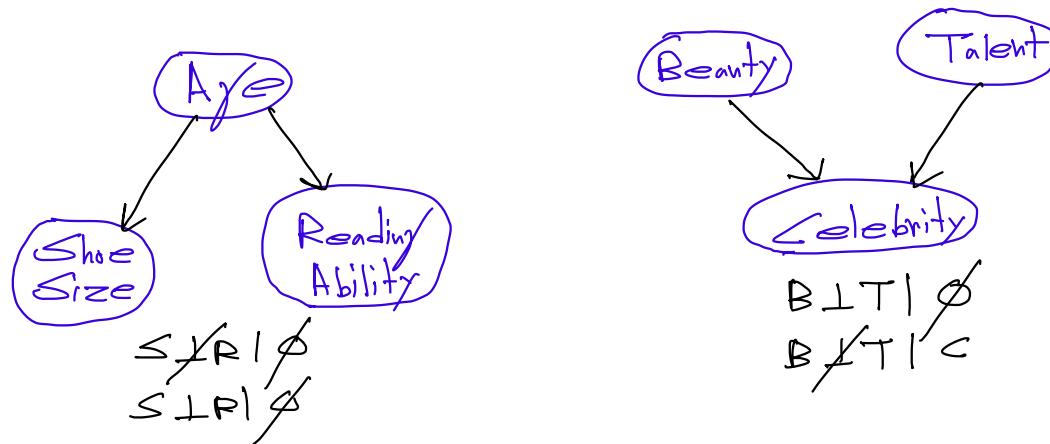


DAG



Fire  $\rightarrow$  Smoke  $\rightarrow$  Alarm  $F \perp A | S$

$$P(FSA) = \frac{P(A|SF) P(S|F) P(F)}{P(A|S)}$$



$$G = (V, E)$$

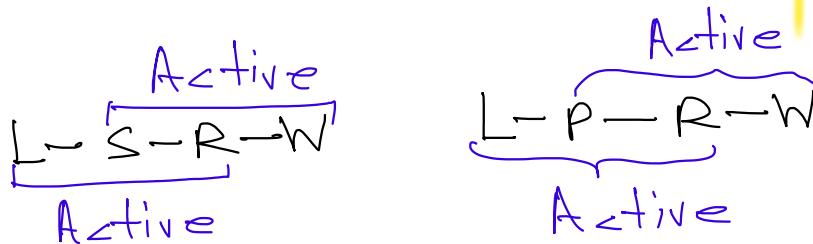
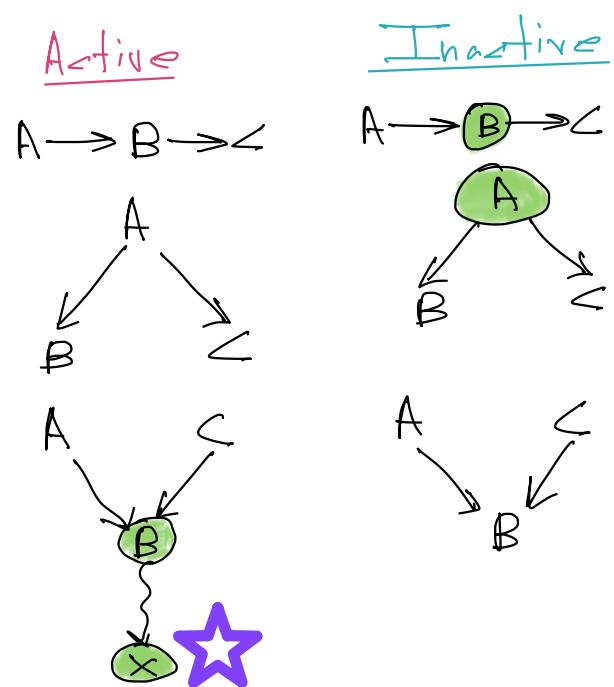
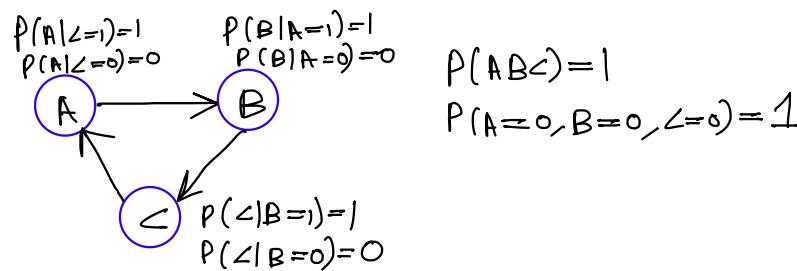
$$P(x_1, \dots, x_n) = \prod_i P(x_i | \text{Par}(x_i))$$

①

$$\text{Given: } P(ABC) = P(C|B) P(B|A) P(A)$$

$$P(A|BC) = \frac{P(ABC)}{P(BC)} = \frac{P(C|B) P(B|A)}{P(B)} \stackrel{(A)}{=} P(C|B) P(A|B)$$

$$\geq P(B|C|A) = \frac{P(ABC)}{P(A)} = \frac{P(B|A)P(C|A)P(A)}{P(A)}$$



Non-parametric Learning:  
 ↪ Instance / Case Based

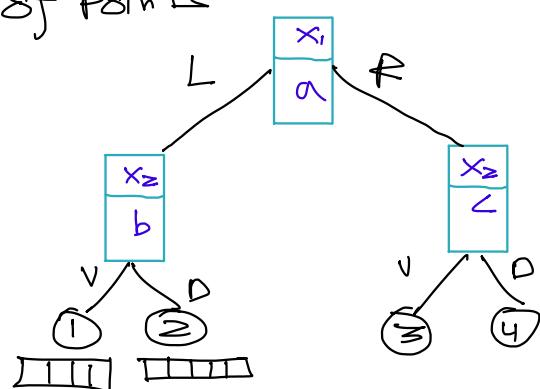
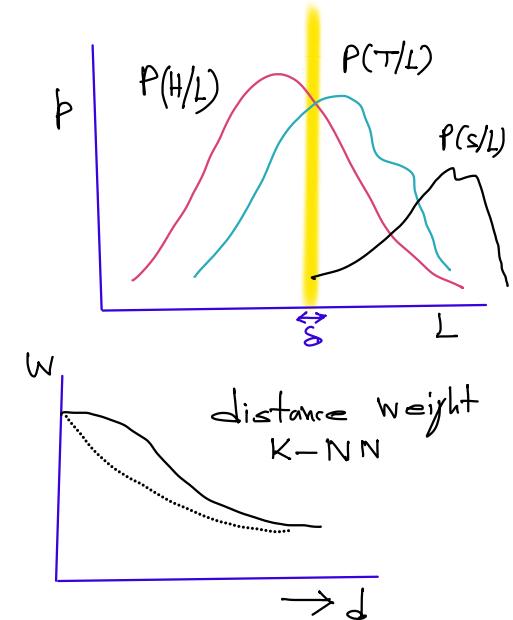
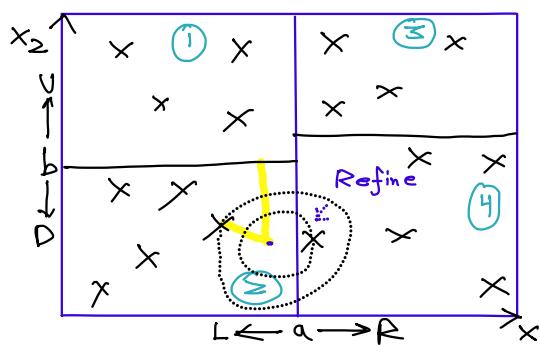
K-Nearest Neighbour:

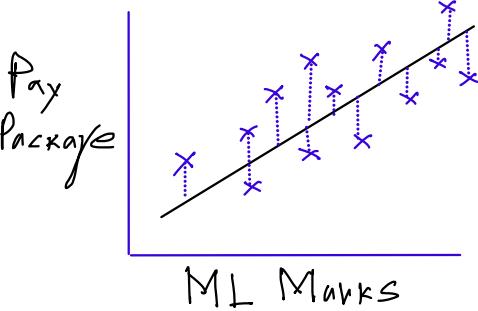
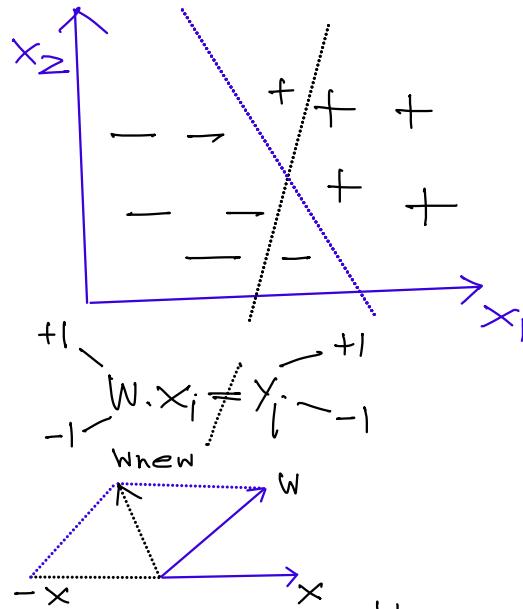
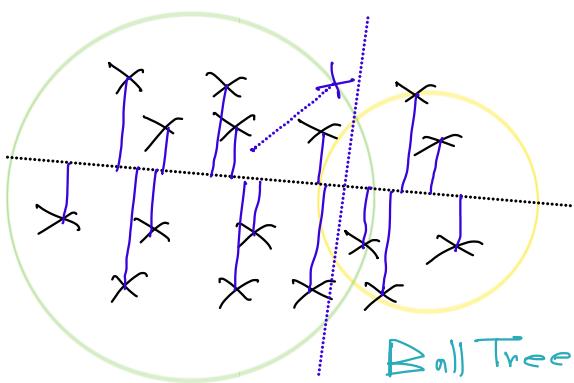
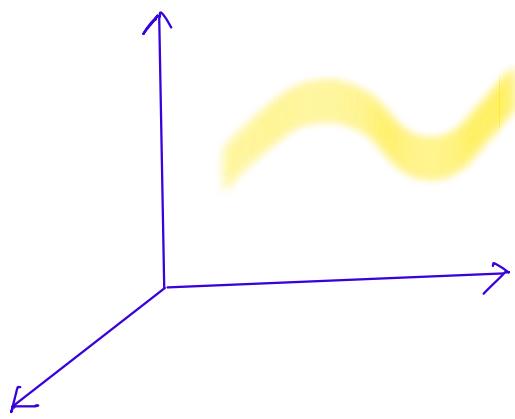
distance=?     $L = \{+, -, 0, *\}$   
 $T \rightarrow (x_1, l_1) \dots (x_N, l_N)$   
 $\textcircled{*} \rightarrow (x_{ij}, \dots, x_{ik})$   
 $\arg \max_l \sum_{j=1}^k \delta(l, f(x_{ij})) \cdot w_j(d(x, x_{ij}))$

- ①  $O(Nd) \rightarrow$  distance calc
- ②  $O(KN) / O(K + (N-K) \log K) \leftarrow$  Max Heap
- ③  $O(K) \leftarrow$  Bubble Sort

Condensing: Minimal consistent set of points

K-D Tree:





$$E_{in} = \frac{1}{N} \sum_{n=1}^N (w^T x_n - y_n)^2$$

$$\frac{\partial E_{in}}{\partial w} = 0 = \frac{1}{N} \|xw - y\|^2$$

$$X = \begin{bmatrix} x_1^T \\ \vdots \\ x_N^T \end{bmatrix}_{N \times (d+1)}$$

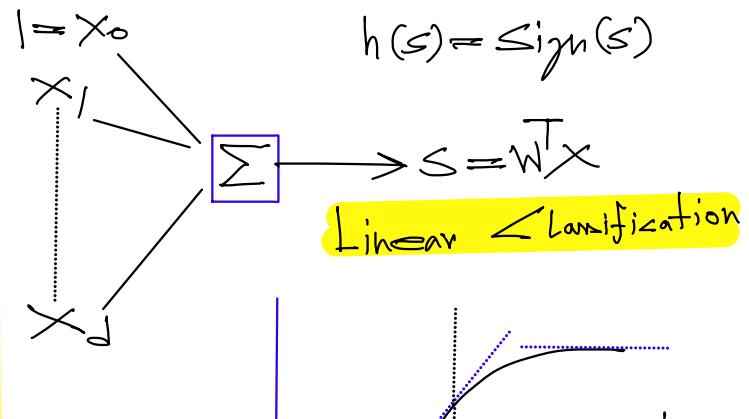
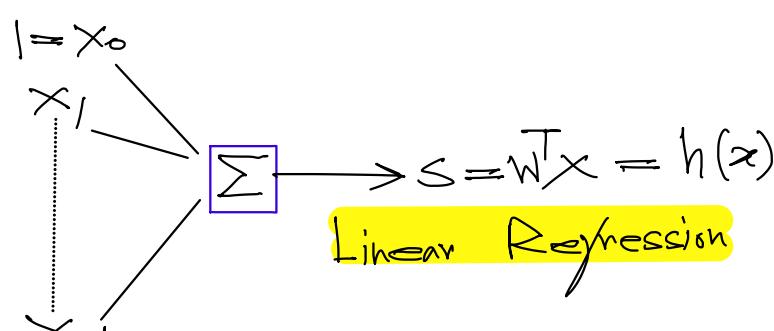
$$w = \begin{bmatrix} w_0 \\ \vdots \\ w_d \end{bmatrix}_{(d+1) \times 1}$$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}_{N \times 1}$$

$$\frac{\partial E_{in}}{\partial w} = \frac{1}{N} X^T (xw - y) = 0$$

$$w = \underbrace{(X^T X)^{-1}}_{\text{Pseudo-Inverse}} X^T Y$$

$$\underbrace{[(d+1) \times N]}_{[(d+1) \times (d+1)]} \underbrace{[N \times (d+1)]}_{[(d+1) \times N]} \Rightarrow \underbrace{[(d+1) \times N]}_{[(d+1) \times (d+1)]} \underbrace{[N \times 1]}_{[N \times 1]}$$



$$P[y|x] = \begin{cases} h(x) & x > 0 \\ 1 - h(x) & x \leq 0 \end{cases}$$

$$h(s) = h(y w^T x)$$

$$\arg \max_w \prod_{i=1}^N h(y_i w^T x_i)$$

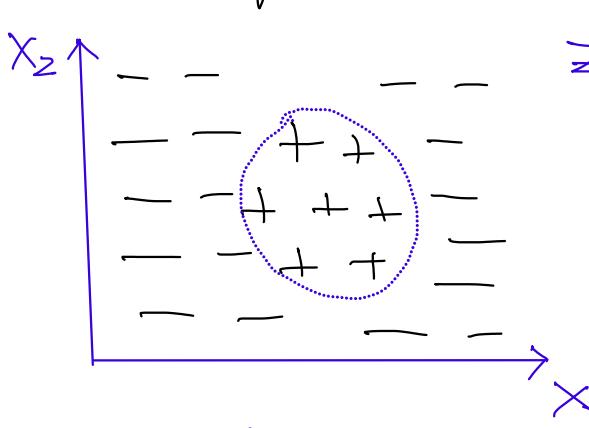
$$\text{Max} \frac{1}{\prod_{i=1}^N 1 + e^{-y_i w^T x_i}} \equiv \min \frac{1}{N} \sum_{i=1}^N \ln \left( 1 + e^{-y_i w^T x_i} \right)$$

*Cross Entropy*

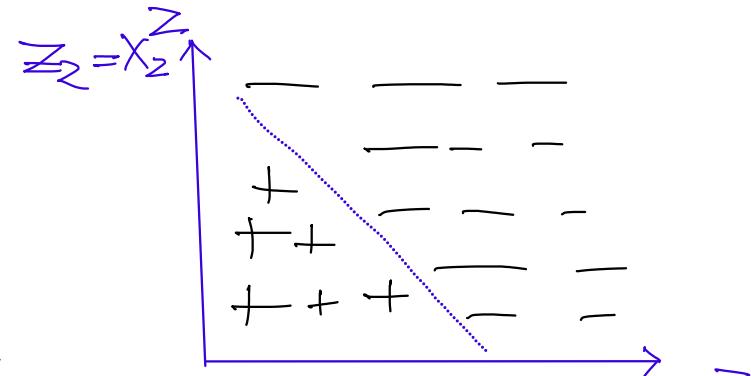
$$x_i x_i \frac{\partial E_{in}}{\partial w} = 0$$

$$\nabla E_{in} \frac{1}{N} \sum_{i=1}^N \frac{-y_i x_i}{1 + e^{y_i w^T x_i}} \quad w' \leftarrow w + \gamma$$

**Learning from Data**



$$x = (1, x_1, x_2)$$



$$z = \phi(x)$$

$$(1, x_1^z, x_2^z)$$

**Data Snooping**

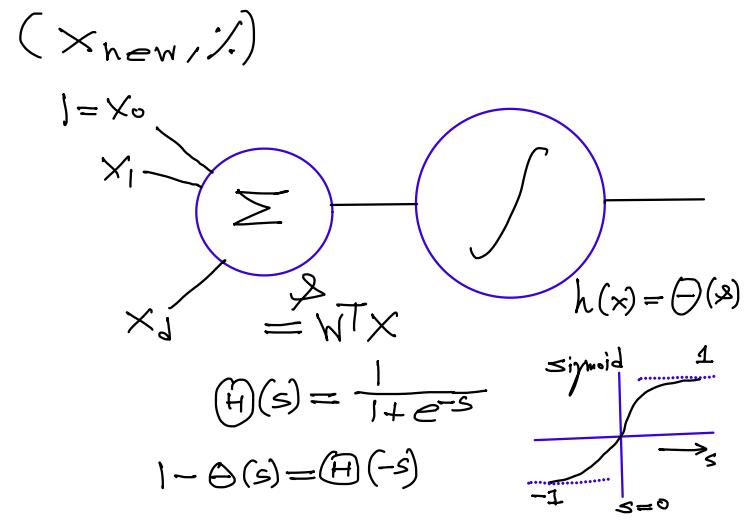
$$w = \phi^{-1}(\tilde{w})$$

$$(1, x_1, x_2, x_1^z, x_2^z, x_1 x_2) \Rightarrow (1, x_1^z + x_2^z)$$

$$(x_1, +1), (x_2, 0) \dots (x_N, +1)$$

$P[y|x] = h(x)$ , when  $y=1$   
 $= 1-h(x)$ , when  $y=0$

 $= \Theta(s) \quad y=+1$ 
 $\Theta(-s) \quad y=-1$ 
 $= \Theta(y_s)$ 
 $= \frac{1}{1+e^{-y_s w^T x}}$



$$\max \prod_{i=1}^N \hat{P}(y_i|x_i) = \prod_{i=1}^N \frac{1}{1+e^{-y_i w^T x_i}} \equiv \prod_{i=1}^N \ln \left( \frac{1}{1+e^{-y_i w^T x_i}} \right)$$

$$\min E = \frac{1}{N} \sum_{i=1}^N \underbrace{\ln(1+e^{-y_i w^T x_i})}_{\text{Cross Entropy Error}}$$

$$\frac{\partial E}{\partial w} = 0 \quad \frac{\partial E}{\partial w} = \frac{1}{N} \sum_{i=1}^N \frac{1}{1+e^{-y_i w^T x_i}} e^{-y_i w^T x_i} (-y_i x_i)$$

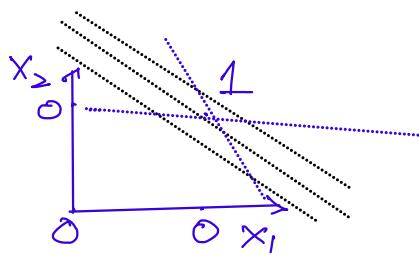
For each  $(x_i, y_i)$

$$\frac{\partial E}{\partial w} = \frac{-y_i x_i}{1+e^{-y_i w^T x_i}}$$

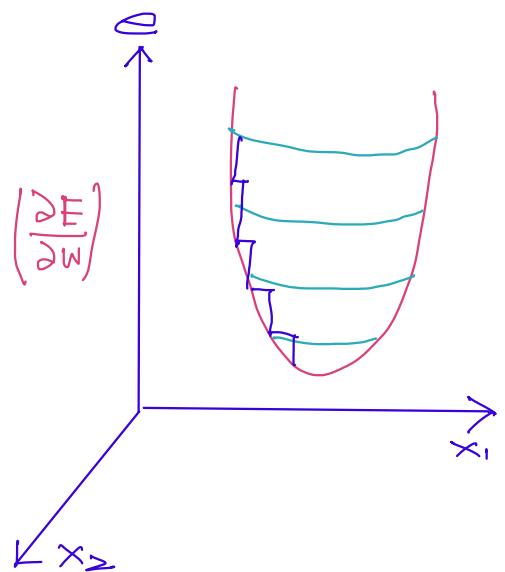
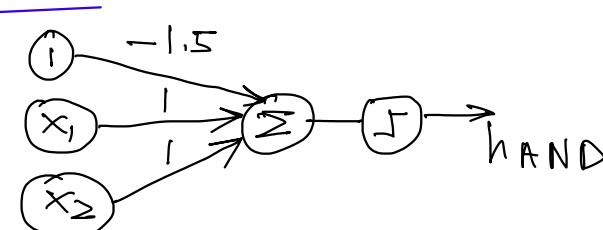
Stochastic Gradient Descent

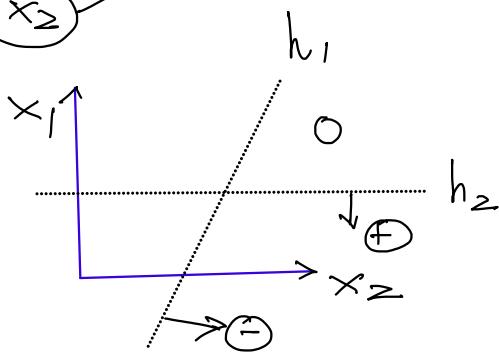
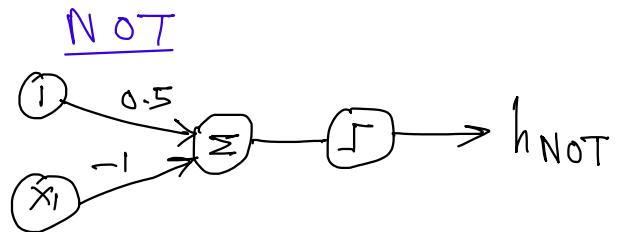
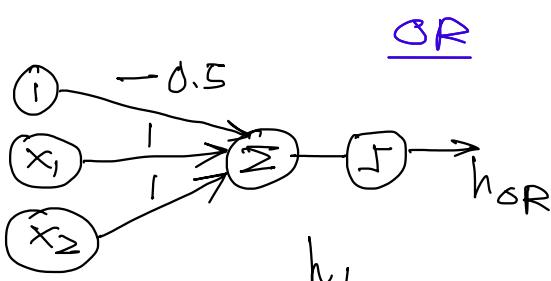
Logistic Regression

Ex:- Credit Repay%.  
Rainfall Chance%.

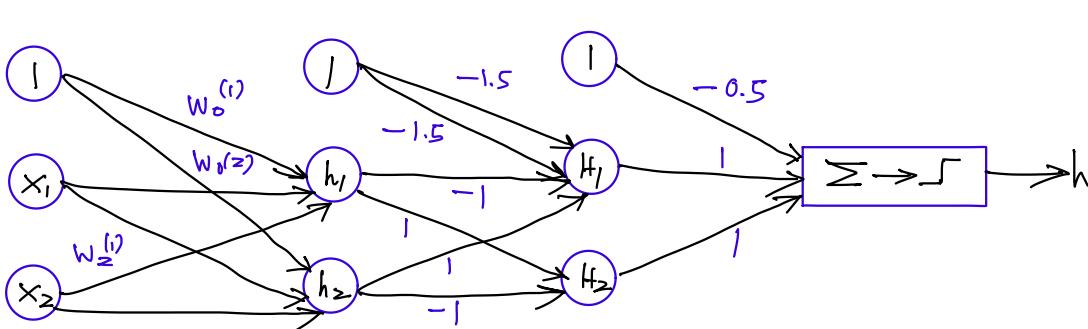
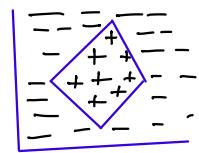


AND

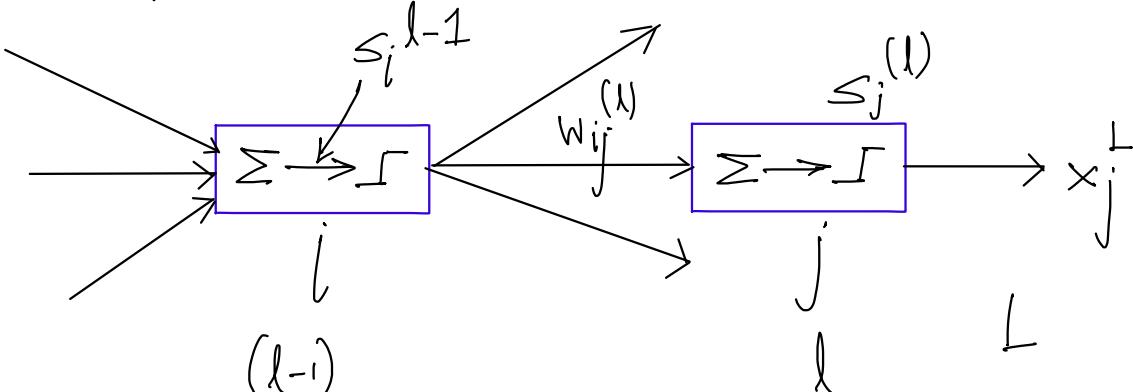
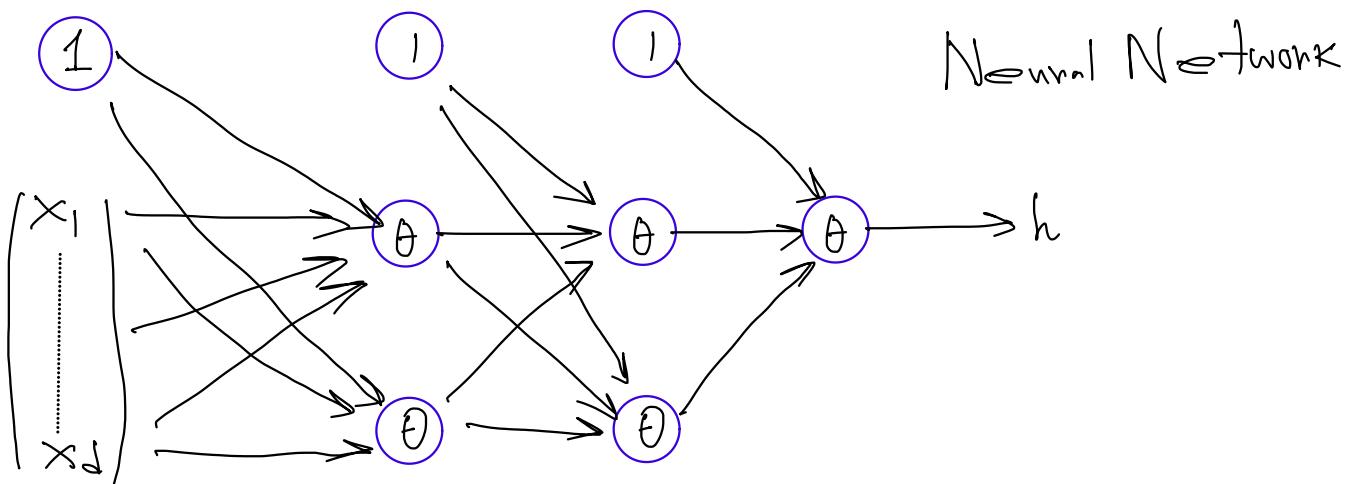




$$h_{XOR} = \underbrace{(h_1 \wedge h_2)}_{H_1} \vee \underbrace{(h_1 \wedge \neg h_2)}_{H_2}$$



ANN  
Multi Layer  
Perception  
Network  
(Feed-Fwd)



$$w_{ij}^{(l)} = \begin{cases} 1 & \text{if } 1 \leq l \leq L \text{ layers} \\ 0 & \text{if } 0 \leq i \leq d^{(l-1)} \text{ input} \\ 1 & \text{if } 1 \leq j \leq d^{(l)} \text{ output} \end{cases}$$

$$(x_0^{(0)} = 1, x_1^{(0)}, \dots, x_d^{(0)})$$

$$x_j^{(l)} = \Theta(\leq_j^{(l)}) = \Theta\left(\sum_{j=0}^l w_{ij}^{(l)} x_j^{(l-1)}\right)$$

$$x_0^{(0)} \rightarrow x^{(1)} \rightarrow x^{(2)} \dots \rightarrow x^{(l)}$$

$$\frac{\partial e}{\partial w_{ij}^{(l)}} = \frac{\partial e}{\partial s_j^{(l)}} * \frac{\partial s_j^{(l)}}{\partial w_{ij}^{(l)}} \\ = s_j^{(l)} * x_i^{(l-1)}$$

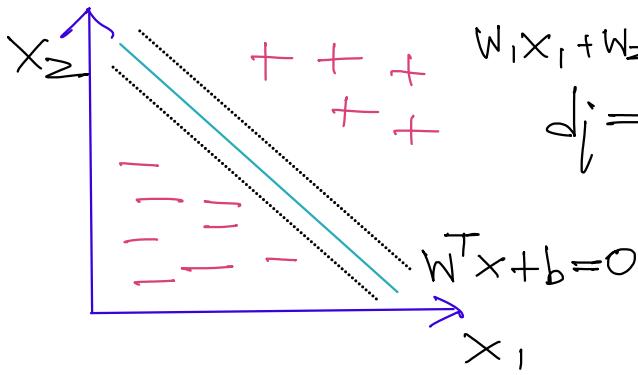
$$w_{ij}^{(l)} \leftarrow w_{ij}^{(l)} + \eta \frac{\partial e}{\partial w_{ij}^{(l)}}$$

$$s_1^{(l)} = \frac{\partial e}{\partial s_1^{(l)}} \\ = \frac{\partial}{\partial s_1^{(l)}} (\Theta(s_1^{(l)}) - y)^2$$

$$s_j^{(l-1)} = \frac{\partial e}{\partial s_j^{(l-1)}}$$

$$= \sum_{j=1}^{d-1} \frac{\partial e}{\partial s_j^{(l)}} * \frac{\partial s_j^{(l)}}{\partial x_j^{(l-1)}} * \frac{\partial x_j^{(l-1)}}{\partial s_j^{(l-1)}} \\ = s_j^{(l)} * w_{ij}^{(l)} * \Theta'(s_j^{(l)})$$

Back Propagation



$$w_1 x_1 + w_2 x_2 + b = 0$$

$$d_j = \frac{|w_1 x_1 + w_2 x_2 + b|}{\sqrt{w_1^2 + w_2^2}}$$

$$\text{constraint } y_i(w^T x_i + b) \geq 0 \quad \forall i$$

$$\underset{w,b}{\operatorname{Max}} \left[ \underset{j}{\operatorname{Min}} (d_j) = \frac{|w^T x + b|}{\|w\|} \right]$$

Constraint:  $y_i(w^T x_i + b) \geq 1$  for all  $i$  s.t.  $\operatorname{Min} \frac{1}{2} w^T w$  s.t. Primal Opt Problem

Dual OPT Prob: KKT

$$\underset{\alpha}{\operatorname{Min}} \frac{1}{2} w^T w + \sum_{i=1}^N \alpha_i (1 - y_i(w^T x_i + b)) \\ \text{s.t. } \alpha_i \geq 0$$

$$\alpha_i = 0 \Rightarrow y_i(w^T x_i + b) > 1$$

$$\alpha_i > 0 \Rightarrow y_i(w^T x_i + b) = 1$$

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w - \sum_{i=1}^n \alpha_i y_i x_i = 0 \Rightarrow w = \sum_{i=1}^n \alpha_i y_i x_i$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^n \alpha_i y_i = 0$$

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \alpha_i y_i y_j (x_i \cdot x_j) \alpha_j + \sum_{i=1}^N \alpha_i \rightarrow \sum_{i=1}^N \sum_{j=1}^N \alpha_i y_i y_j (x_i \cdot x_j) \alpha_j$$

$$= \sum_{i=1}^N \alpha_i - \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \alpha_i y_i y_j (x_i \cdot x_j) \alpha_j$$

$$= \Lambda^T V - \frac{1}{N} \Lambda^T H \Lambda$$

$$\Lambda = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} \quad W^T X_{SV} + b = 1$$

Support Vector Machine

$$W = \sum_{i \in SV} \alpha_i y_i x_i$$

Const

Hessian

$$H = \begin{bmatrix} & & & & \\ & x_i y_j x_i x_j & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

Issues:

$$\Rightarrow \text{Noise } ① \Rightarrow \text{Slack } \varepsilon_i \quad y_i (w^T x_i + b) \geq 1 - \varepsilon_i$$

$$M_i \rightarrow \varepsilon_i \geq 0 \quad (x_i)$$

$$\Delta b: \min_{w, b} \left( \frac{1}{2} w^T w + C \sum \varepsilon_i \right)$$

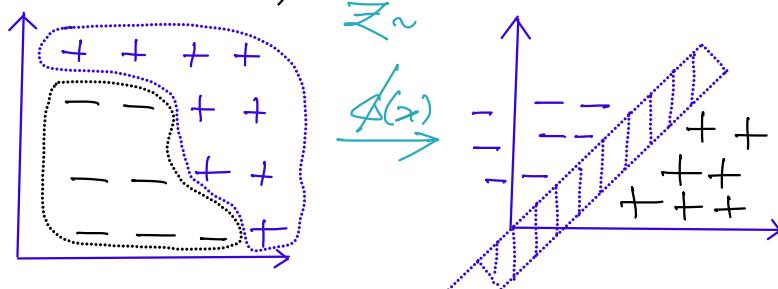
$$\text{Dual: } \min_{w, b} \frac{1}{2} w^T w + \sum_{i=1}^N \alpha_i (1 - \varepsilon_i - y_i (w^T x_i + b)) - \sum_{i=1}^N M_i \varepsilon_i$$

$$\text{s.t. } \alpha_i \geq 0, M_i \geq 0$$

$$\frac{\partial L}{\partial w} = 0 \quad \frac{\partial L}{\partial b} = 0 \quad \frac{\partial L}{\partial \varepsilon_i} = 0$$

$$W = \sum \alpha_i y_i x_i \quad \sum \alpha_i y_i = 0 \quad C = \sum M_i + \alpha_i$$

Non-Linearity:



$$x = (1, x_1, x_2) \xrightarrow{\phi} z = (1, x_1, x_2, x_1^2, x_2^2, x_1 x_2)$$

$$x' = (1, x'_1, x'_2) \xrightarrow{\phi} z' = (1, x'_1, x'_2, x'^2_1, x'^2_2, x'_1 x'_2)$$

$$K(x, x') = x \cdot x' \text{ linear}$$

$$\hookrightarrow = (1 + x \cdot x')^2$$

$$= 1 + \sum x_1 x'_1 + \sum x_2 x'_2 + x_1^2 x'_1^2 + x_2^2 x'_2^2 + \sum x_1 x'_1 x_2 x'_2$$

$$= (1, \sqrt{\sum x_1}, \sqrt{\sum x_2}, x_1^2, \sqrt{\sum x_1 x_2})^T$$

$$(1, \sqrt{\sum x'_1}, \sqrt{\sum x'_2}, x'_1^2, \sqrt{\sum x'_1 x'_2})^T$$

"Kernel"  
Gaussian Kernel

Ref - [2], [6]

Classifier Evaluation

Confusion Matrix

Prediction  $\leq$  true

Yes      No

Actual Class No

TP	FN
FP	TN

Accuracy ( $A$ ) =  $\frac{|TP| + |TN|}{|TP| + |TN| + |FP| + |FN|}$

Predicted  $\leq$  true

Yes      No

Actual Class No

TP	FN
FP	TN

Confusion Matrix

$$\text{Recall} = \frac{|TP|}{|TP| + |FN|}$$

$$\text{Precision} = \frac{|TP|}{|TP| + |FP|}$$

F-Score  $\frac{1}{F} = \frac{1}{2} \left( \frac{1}{P} + \frac{1}{R} \right) \Rightarrow F = \frac{2PR}{P+R}$

( $F_1$ -Measure)

$$W.A = \frac{w_1 |TP| + w_2 |TN|}{w_1 |TP| + w_2 |TN| + w_3 |FP| + w_4 |FN|}$$

Evaluation Method :- ① Holdout  $\frac{2}{3}$  Train +  $\frac{1}{3}$  Test  
 $\leq$  Train +  $(N-c)$  Test

② Random Subsampling → Random frac of  $\frac{K}{N}$

③ Cross Validation

↳ stratified Sampling

④

- $(x_1, y)$
- $(x_2, N)$
- $(x_3, X)$
- $(x_4, N)$
- $(x_5, Y)$
- $(x_6, Y)$

$x_1, x_3$   
 $x_5, x_6$

$y$

Bootstraping/  
(Replace)

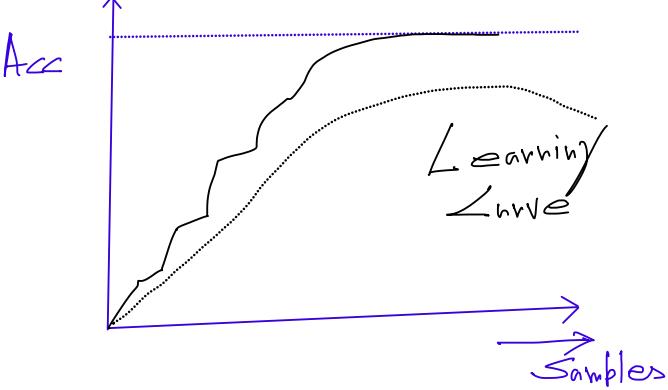
$x_2, x_4$

$N$

ROC (Receiver Operating  
Characteristics)

TPR

FPR



-ve → +ve

TP

FN

TN

FP

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