Assignment-2:-

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Notations:- Denote by $V_{ij}^{(m)}$ the value send by P_i to P_j in $OM(m)$ initiated by P_i. Also, denote Estimate of P_K for ~~value~~ $V_{ij}^{(m)}$ as $X_{ij(K)}^{(m)}$.

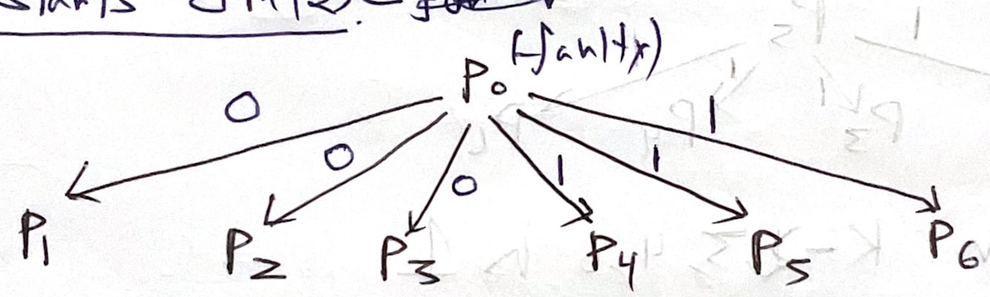
The executions are shown in a Depth first Search other. Which means for every $OM(1)$ we first show all $OM(0)$ initiated by it and then show next $OM(1)$.

~~Similarly for every OM(2), we first~~

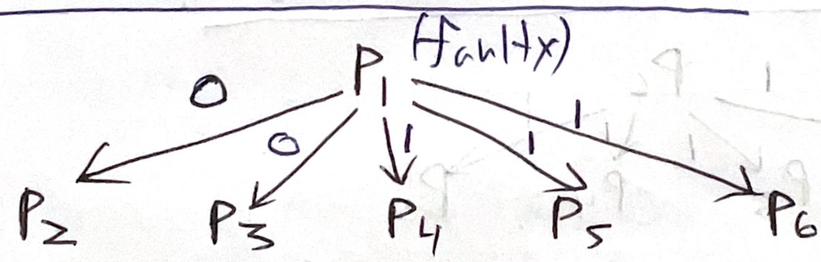
Case-1:-

P_0 (General) and P_1 is faulty:-

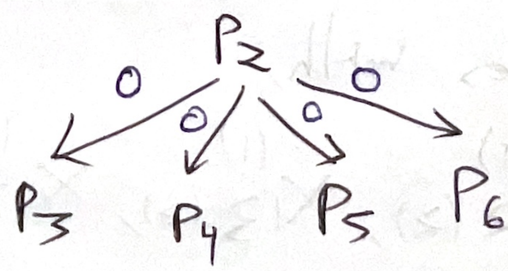
P_0 starts $OM(2)$:-



$\Rightarrow$ **P_1 starts $OM(1)$ with $V_{01}^{(2)}$:-**

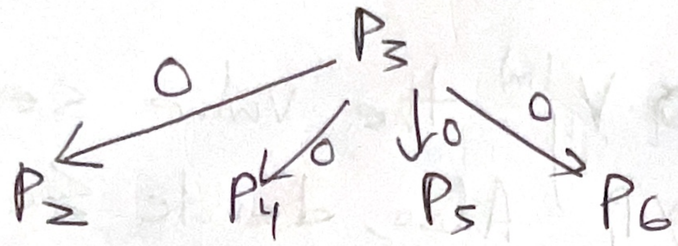


$\Rightarrow\Rightarrow$ **P_2 starts $OM(0)$ with $V_{12}^{(1)}$:-**



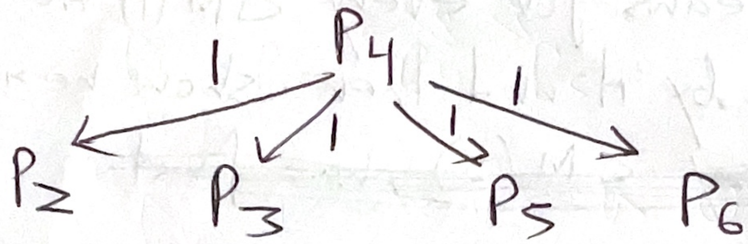
So, ~~$X_{ij(K)}^{(m)}$ for~~ $X_{12(K)}^{(1)}$ for $K=3,4,5,6$ is 0.

$\Longrightarrow\Longrightarrow$ <u>P_3 starts $\Delta M(0)$ with $V_{13}^{(1)}$:-</u>



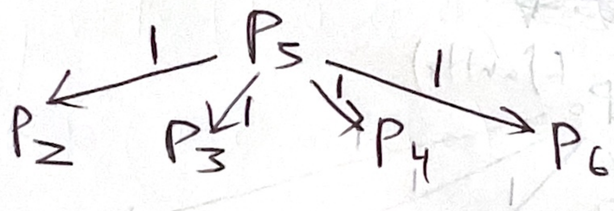
So, $X_{13(K)}^{(1)}$ for $K = \cancel{2}, 4, 5, 6$ is 0

$\Longrightarrow\Longrightarrow$ <u>P_4 starts $\Delta M(0)$ with $V_{14}^{(1)}$:-</u>



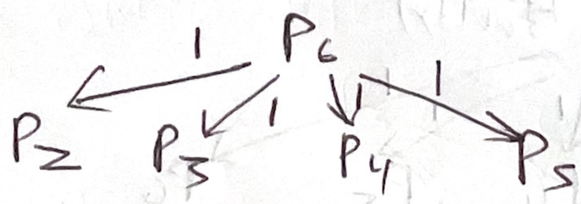
So, $X_{14(K)}^{(1)}$ for $K = 2, 3, 5, 6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_5 starts $\Delta M(0)$ with $V_{15}^{(1)}$:-</u>



So, $X_{15(K)}^{(1)}$ for $K = \cancel{2}, 3, 4, 6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_6 starts $\Delta M(0)$ with $V_{16}^{(1)}$:-</u>



So, $X_{16(K)}^{(1)}$ for $K = 2, 3, 4, 5$ is 1

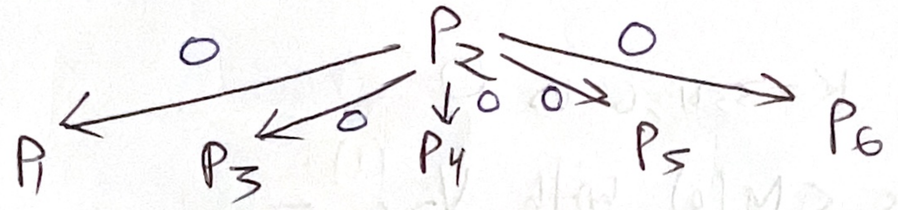
Now,
$\Delta M(1)$ by P_1 concludes with,

$$X_{01(2)}^{(2)} = \text{majority}\left(X_{12(2)}^{(1)}, X_{13(2)}^{(1)}, X_{14(2)}^{(1)}, X_{15(2)}^{(1)}, X_{16(2)}^{(1)}\right)$$
$$= \text{majority}(0, 0, 1, 1, 1) = \underline{1}$$

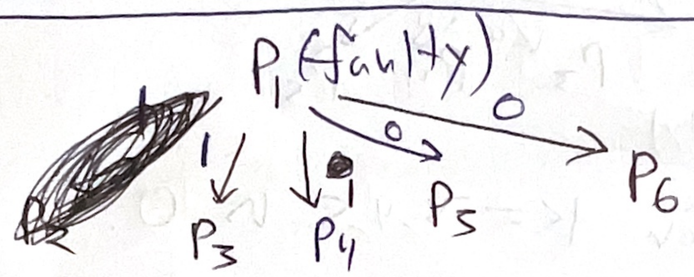
Similarly,

$$X^{(2)}_{01(3)} = X^{(2)}_{01(4)} = X^{(2)}_{01(5)} = X^{(2)}_{01(6)} = 1$$

$\Rightarrow$ <u>P_2 starts $\sigma M(1)$ with $V^{(2)}_{02}$:-</u>



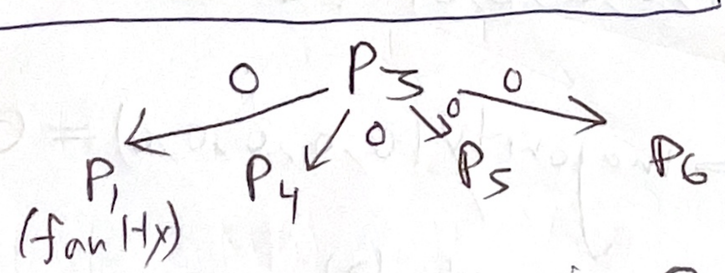
$\Rightarrow\Rightarrow$ <u>P_1 starts $\sigma M(0)$ with $V^{(1)}_{021}$:-</u>



So,

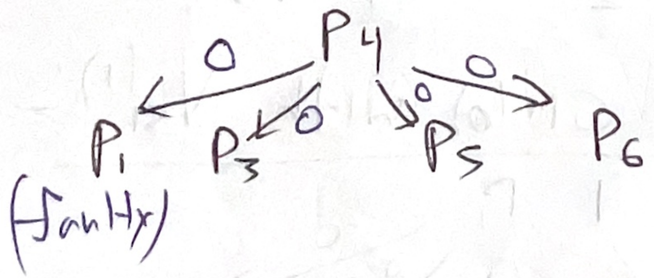
$$X^{(1)}_{21(3)} = X^{(1)}_{21(4)} = 1 \text{ and } X^{(1)}_{21(5)} = X^{(1)}_{21(6)} = 0$$

$\Rightarrow\Rightarrow$ <u>P_3 starts $\sigma M(0)$ with $V^{(1)}_{23}$:-</u>



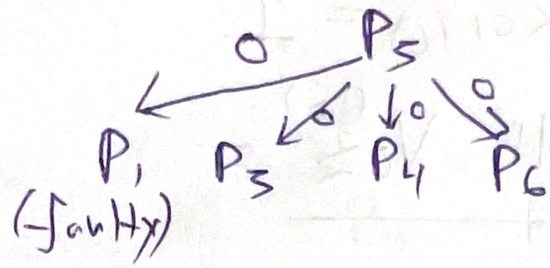
So, $X^{(1)}_{23(k)}$ for $k = 4, 5, 6$ is 0

$\Rightarrow\Rightarrow$ <u>P_4 starts $\sigma M(0)$ with $V^{(1)}_{23}$:-</u>



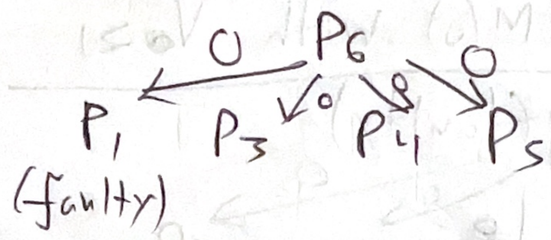
So, $X^{(1)}_{24(k)}$ for $k = 3, 5, 6$ is 0

$\Rightarrow\Rightarrow$ <u>P_5 starts OM(0) with $V_{25}^{(1)}$:-</u>



So, $X_{25}^{(1)}(k)$ for $k=3,4,6$ is 0

$\Rightarrow\Rightarrow$ <u>P_6 starts OM(0) with $V_{26}^{(1)}$:-</u>



So, $X_{26}^{(1)}(k)$ for $k=3,4,5$ is 0.

Now,

OM(1) by P_2 concludes. $X_{02}^{(2)}(2)=0$

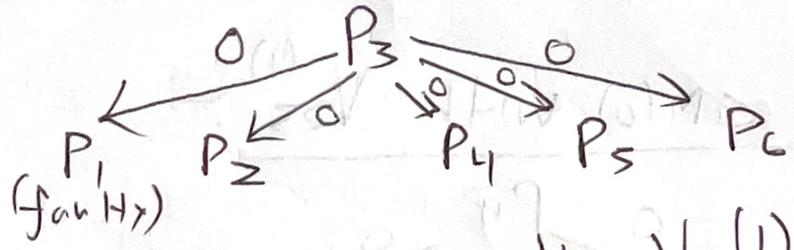
$X_{02}^{(2)}(3) = \text{majority}(1,0,0,0,0) = 0$

$X_{02}^{(2)}(4) = \text{majority}(1,0,0,0,0) = 0$

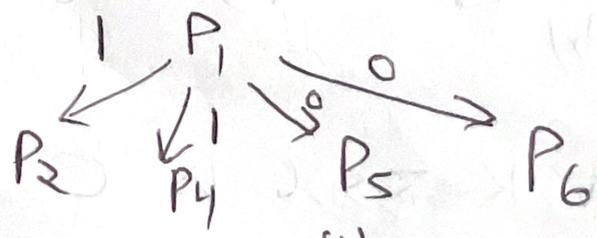
and

$X_{02}^{(2)}(5) = X_{02}^{(2)}(6) = \text{majority}(0,0,0,0,0) = 0$

$\Rightarrow$ <u>P_3 starts OM(1) with $V_{03}^{(2)}$:-</u>

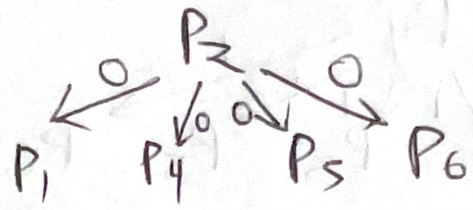


$\Rightarrow\Rightarrow$ <u>P_1 starts OM(0) with $V_{31}^{(1)}$:-</u>



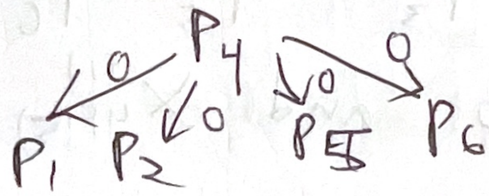
$X_{31}^{(1)}(2) = X_{31}^{(1)}(4) = 1$ and $X_{31}^{(1)}(5) = X_{31}^{(1)}(6) = 0$

$\Rightarrow\Rightarrow$ <u>P_2 starts $\Delta M(0)$ with $V_{32}^{(1)}$:-</u>



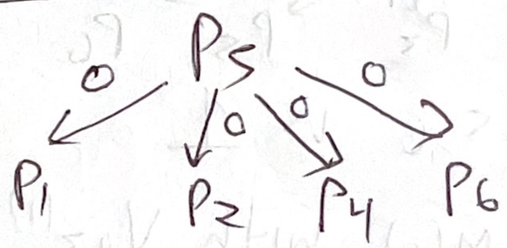
$X_{32}^{(1)}(K)$ for $K=4,5,6$ is 0

$\Rightarrow\Rightarrow$ <u>P_4 starts $\Delta M(0)$ with $V_{34}^{(1)}$:-</u>



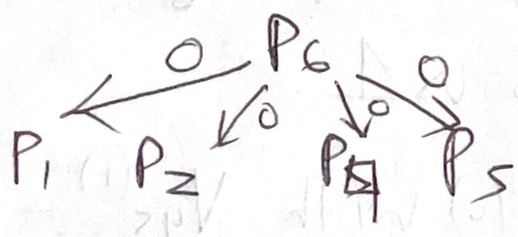
$X_{34}^{(1)}(K)$ for $K=2,5,6$ is 0

$\Rightarrow\Rightarrow$ <u>P_5 starts $\Delta M(0)$ with $V_{35}^{(1)}$:-</u>



$X_{35}^{(1)}(K)$ for $K=1,2,4,6$ is 0

$\Rightarrow\Rightarrow$ <u>P_6 starts with $V_{36}^{(1)}$, $\Delta M(0)$:-</u>



$X_{36}^{(1)}(K)$ for $K=2,4,5$ is 0

Now, $\Delta M(1)$ by P_3 concludes.

$X_{03}^{(2)}(3) = 0$

$X_{03}^{(2)}(2) = X_{03}^{(2)}(4) = \text{majority}(1,0,0,0,0) = 0$

$X_{03}^{(2)}(5) = X_{03}^{(2)}(6) = \text{majority}(0,0,0,0,0) = 0$

$\Rightarrow$ P_4 starts $\Delta M(1)$ with $V_{04}^{(2)}$:-

$$\begin{tikzcd}
 & P_4 \arrow[dl, "1"'] \arrow[dl, "1"] \arrow[dr, "1"] \arrow[dr, "1"] \arrow[drr, "1"] & & & \\
P_1 & P_2 & P_3 & P_5 & P_6
\end{tikzcd}$$

$\Rightarrow\Rightarrow$ P_1 starts $\Delta M(0)$ with $V_{41}^{(1)}$:-

$$\begin{tikzcd}
 & P_1 \arrow[dl, "0"'] \arrow[d, "0"] \arrow[dr, "1"] \arrow[drr, "1"] & & \\
P_2 & P_3 & P_5 & P_6
\end{tikzcd}$$

$X_{41}^{(1)}(2) = X_{41}^{(1)}(3) = 0$ and $X_{41}^{(1)}(5) = X_{41}^{(1)}(6) = 1$

$\Rightarrow\Rightarrow$ P_2 starts $\Delta M(0)$ with $V_{42}^{(1)}$:-

$$\begin{tikzcd}
 & P_2 \arrow[dl, "1"'] \arrow[d, "1"] \arrow[dr, "1"] \arrow[drr, "1"] & & \\
P_1 & P_3 & P_5 & P_6
\end{tikzcd}$$

$X_{42}^{(1)}(K)$ for $K = 3, 5, 6$ is 1

$\Rightarrow\Rightarrow$ P_3 starts $\Delta M(0)$ with $V_{43}^{(1)}$:-

$$\begin{tikzcd}
 & P_3 \arrow[dl, "1"'] \arrow[d, "1"] \arrow[dr, "1"] \arrow[drr, "1"] & & \\
P_1 & P_2 & P_5 & P_6
\end{tikzcd}$$

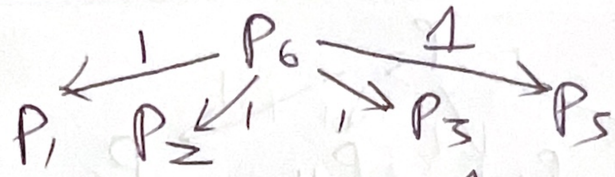
$X_{43}^{(1)}(K)$ for $K = 2, 5, 6$ is 1

$\Rightarrow\Rightarrow$ P_5 starts $\Delta M(0)$ with $V_{45}^{(1)}$:-

$$\begin{tikzcd}
 & P_5 \arrow[dl, "1"'] \arrow[d, "1"] \arrow[dr, "1"] \arrow[drr, "1"] & & \\
P_1 & P_2 & P_3 & P_6
\end{tikzcd}$$

$X_{45}^{(1)}(K)$ for $K = 2, 3, 6$ is 1

$\Longrightarrow\Rightarrow$ <u>P_6 starts $\Delta M(0)$ with $V_{46}^{(1)}$:</u>



$X_{46}^{(1)}(K)$ for $K = 2, 3, 5$ is 1

Now,

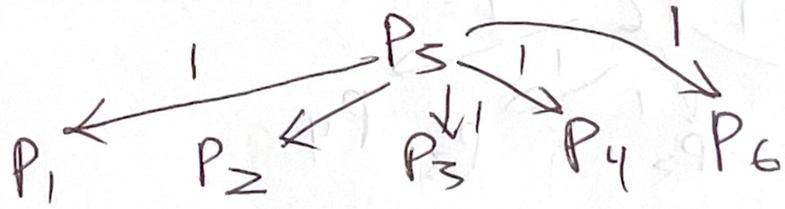
$\Delta M(1)$ by P_4 concludes,

$X_{04}^{(2)}(4) = 1$

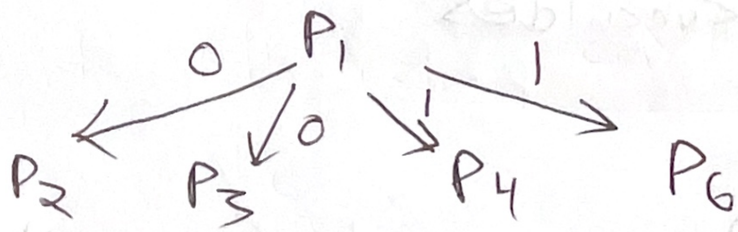
$X_{04}^{(2)}(2) = X_{04}^{(2)}(3) = \text{majority}(0,1,1,1,1) = 1$

$X_{04}^{(2)}(5) = X_{04}^{(2)}(6) = \text{majority}(1,1,1,1,1) = 1$

$\Longrightarrow$ <u>P_5 starts $\Delta M(1)$ with $V_{05}^{(2)}$:</u>

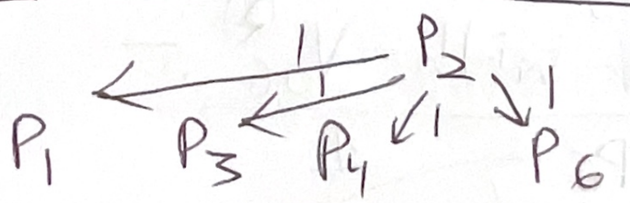


$\Longrightarrow\Rightarrow$ <u>P_1 starts $\Delta M(0)$ with $V_{51}^{(1)}$:</u>



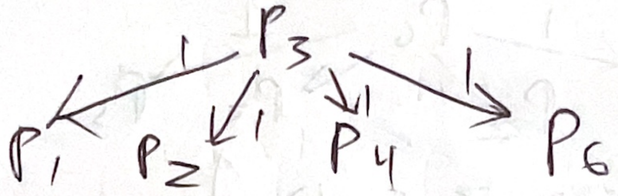
$X_{51}^{(1)}(2) = X_{51}^{(1)}(3) = 0$ and $X_{51}^{(1)}(4) = X_{51}^{(1)}(6) = 1$

$\Longrightarrow\Rightarrow$ <u>P_2 starts $\Delta M(0)$ with $V_{52}^{(1)}$:</u>



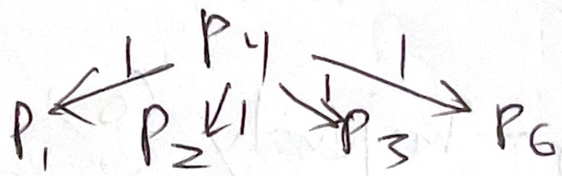
$X_{52}^{(1)}(K)$ for $K = 3, 4, 6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_3 starts OM(0) with $V_{53}^{(1)}$:-</u>



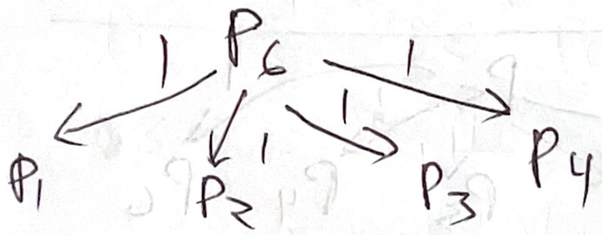
$X_{53}^{(1)}(k)$ for $k=2,4,6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_4 starts OM(0) with $V_{54}^{(1)}$:-</u>



$X_{54}^{(1)}(k)$ for $k=2,3,6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_6 starts OM(0) with $V_{56}^{(1)}$:-</u>



$X_{56}^{(1)}(k)$ for $k=2,3,4$ is 1

Now,

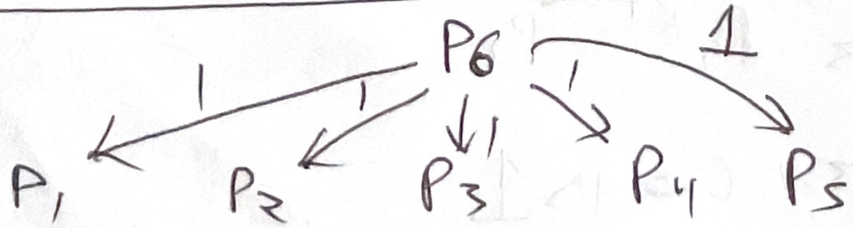
OM(1) by P_5 concludes

$X_{05}^{(2)}(5) = 1$

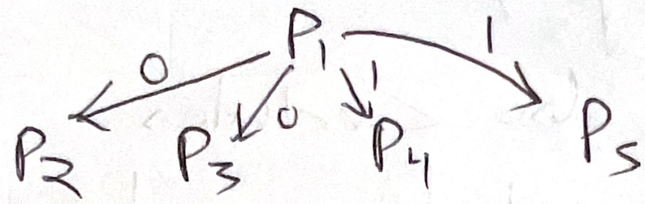
$X_{05}^{(2)}(2) = X_{05}^{(2)}(3) = \text{majority}(0,1,1,1,1) = 1$

$X_{05}^{(2)}(4) = X_{05}^{(2)}(6) = \text{majority}(1,1,1,1,1) = 1$

$\Longrightarrow$ <u>P_6 starts OM(1) with $V_{06}^{(2)}$:-</u>

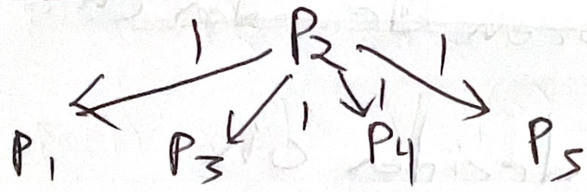


$\Longrightarrow\Longrightarrow$ P_1 starts $\triangleleft M(0)$ with $V_{61}^{(1)}$:-



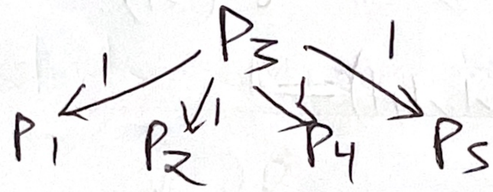
$X_{61}^{(1)}(2) = X_{61}^{(1)}(3) = 0$ and $X_{61}^{(1)}(4) = X_{61}^{(1)}(5) = 1$

$\Longrightarrow\Longrightarrow$ P_2 starts $\triangleleft M(0)$ with $V_{62}^{(2)}$:-



$X_{62}^{(2)}(k)$ for $k=3,4,5$ is 1

$\Longrightarrow\Longrightarrow$ P_3 starts ~~with~~ $\triangleleft M(0)$ with $V_{63}^{(1)}$:-

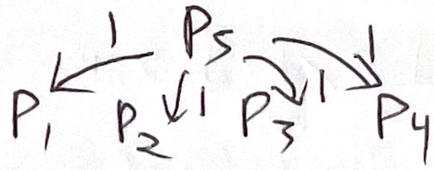


$X_{63}^{(1)}(k)$ for $k=2,4,5$ is 1

$\Longrightarrow\Longrightarrow$ P_4 starts $\triangleleft M(0)$ with $V_{64}^{(1)}$:-

$X_{64}^{(1)}(k)$ for $k=2,3,5$ is 1

$\Longrightarrow\Longrightarrow$ P_5 starts $\triangleleft M(0)$ with $V_{65}^{(1)}$:-



$X_{65}^{(1)}(k)$ for $k=2,3,4$ is 1

Now, $\triangleleft M(1)$ by P_6 concludes

$X_{06}^{(2)}(6) = 1$

$X_{06}^{(2)}(2) = X_{06}^{(2)}(3) = \text{majority}(0,1,1,1,1) = 1$

$X_{06}^{(2)}(4) = X_{06}^{(2)}(5) = \text{majority}(1,1,1,1,1) = 1$

Now, OM(2) by P_0 also concludes

Every correct Process has got same values of $X_{0j}^{(2)}(k)$ for all j and k ($k \neq 1, 0$) ($j \geq 1$)

~~Every correct process decides on~~

~~majority(X_{0j}~~ P_2 decides on

$\text{majority}(X_{01}^{(2)}(2), X_{02}^{(2)}(2), X_{03}^{(2)}(2), X_{04}^{(2)}(2), X_{05}^{(2)}(2), X_{06}^{(2)}(2))$

$= \text{majority}(0,0,0,1,1,1) = 1$

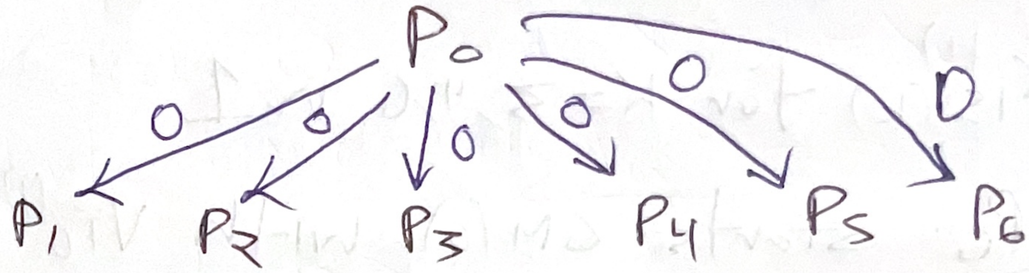
and similarly P_3, P_4, P_5, P_6 also

[Note: Although there is no majority but all correct process will then choose a default value which we take as 1]

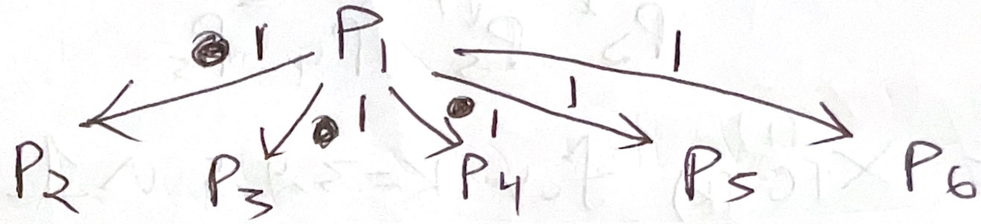
Case-2:-

P_1 and P_2 is faulty:-

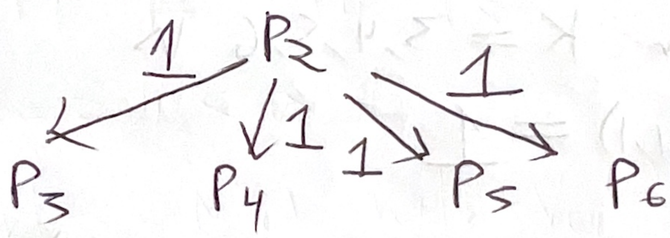
P_0 starts OM(2):-



⇒ **P_1 starts OM(1) with $V_{01}^{(2)}$:-**

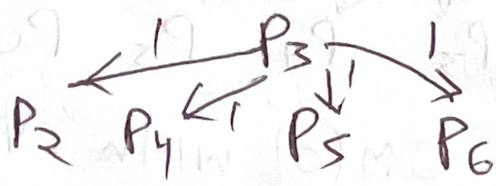


⇒⇒ **P_2 starts OM(0) with $V_{12}^{(1)}$:-**



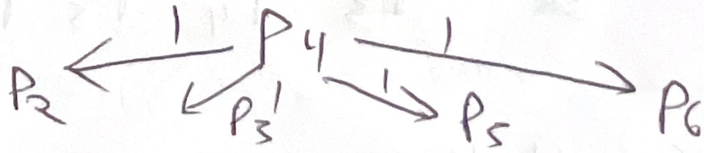
So, $X_{12}^{(1)}(K)$ for $K=3,4,5,6$ is 1

⇒⇒ **P_3 starts OM(0) with $V_{13}^{(1)}$:-**



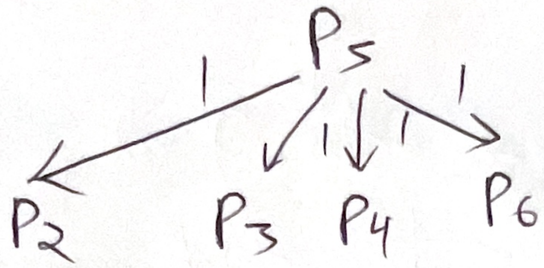
$X_{13}^{(1)}(K)$ for $K=4,5,6$ is 1

⇒⇒ **P_4 starts OM(0) with $V_{14}^{(1)}$:-**



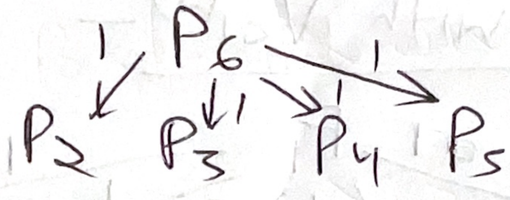
$X_{14}^{(1)}(K)$ for $K=3,5,6$ is 1

$\Rightarrow$ P_5 starts OM(0) with $V_{15}^{(1)}$:-



So $X_{15}^{(1)}(k)$ for $k=3,4,6$ is 1

$\Rightarrow\Rightarrow$ P_6 starts OM(0) with $V_{16}^{(1)}$:-



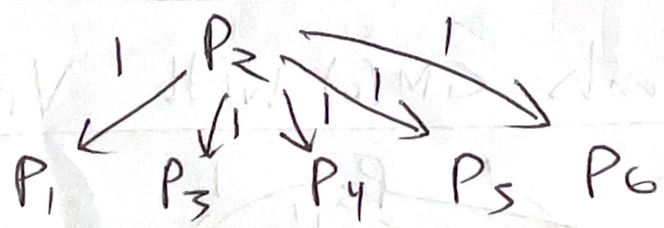
So, $X_{16}^{(1)}(k)$ for $k=3,4,5$ is 1

Now OM(1) by P_1 concludes,

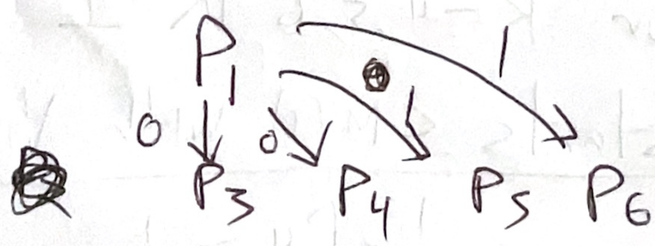
$X_{01}^{(2)}(k)$ for $k=3,4,5,6$ is

majority $(1,1,1,1,1) = 1$

$\Rightarrow$ P_2 starts OM(1) with $V_{02}^{(2)}$:-



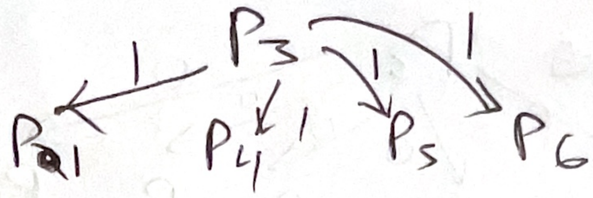
$\Rightarrow\Rightarrow$ P_1 starts OM(0) with $V_{21}^{(1)}$:-



So,
$X_{21}^{(1)}(k)$ for $k=3,4$ is 0

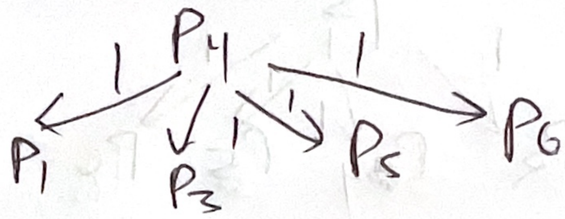
and $X_{21}^{(1)}(k)$ for $k=5,6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_3 starts OM(0) with $V_{23}^{(1)}$:-</u>



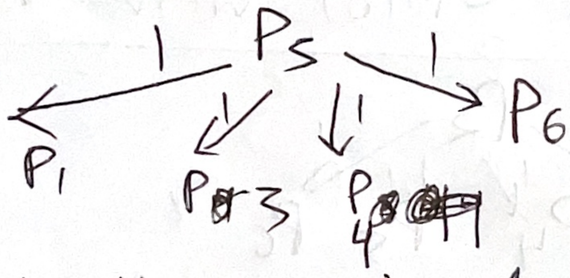
$X_{23}^{(1)}(k)$ for $k=4,5,6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_4 starts OM(0) with $V_{24}^{(1)}$:-</u>



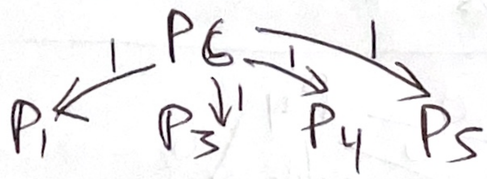
$X_{24}^{(1)}(k)$ for $k=3,5,6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_5 starts OM(0) with $V_{25}^{(1)}$:-</u>



$X_{25}^{(1)}(k)$ for $k=3,4,6$ is 1

$\Longrightarrow\Longrightarrow$ <u>P_6 starts OM(0) with $V_{26}^{(1)}$:-</u>



$X_{26}^{(1)}(k)$ for $k=3,4,5$ is 1

~~OM(1) by P2 concludes:-~~

Now, OM(1) by P_2 concludes:-

$X_{02}^{(2)}(k)$ for $k=3,4$ is majority $(0,1,1,1,1)=1$

$X_{02}^{(2)}(k)$ for $k=5,6$ is majority $(1,1,1,1,1)=1$

$\Longrightarrow$ P_3 starts $\Delta M(1)$ with $V_{03}^{(2)}$:-

$$\begin{tikzcd}
 & P_3 \arrow[dl, "0"'] \arrow[d, "0"] \arrow[dr, "0"] \arrow[drr, "0"] \arrow[drrr, "0"] & & & \\
P_1 & P_2 & P_4 & P_5 & P_6
\end{tikzcd}$$

$\Rightarrow\Rightarrow$ P_1 starts $\Delta M(0)$ with $V_{31}^{(1)}$:-

$$\begin{tikzcd}
 & P_1 \arrow[dl, "1"'] \arrow[d, "1"] \arrow[dr, "1"] \arrow[drr, "1"] & & \\
P_2 & P_4 & P_5 & P_6
\end{tikzcd}$$

So, $X_{31}^{(1)}(K)$ for $K=4,5,6$ is 1

$\Longrightarrow\Longrightarrow$ P_2 starts $\Delta M(0)$ with $V_{32}^{(1)}$:-

$$\begin{tikzcd}
 & P_2 \arrow[dl, "1"'] \arrow[d, "1"] \arrow[dr, "1"] \arrow[drr, "1"] & & \\
P_1 & P_4 & P_5 & P_6
\end{tikzcd}$$

So, $X_{32}^{(1)}(K)$ for $K=4,5,6$ is 1

$\Longrightarrow\Longrightarrow$ P_4 starts $\Delta M(0)$ with $V_{34}^{(1)}$:-

$$\begin{tikzcd}
 & P_4 \arrow[dl, "0"'] \arrow[dl, "0"] \arrow[d, "0"] \arrow[dr, "0"] & & \\
P_1 & P_2 & P_5 & P_6
\end{tikzcd}$$

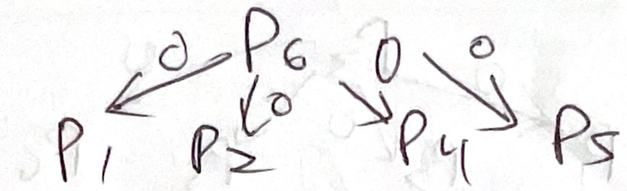
So, $X_{34}^{(1)}(K)$ for $K=5,6$ is 0

$\Longrightarrow\Longrightarrow$ P_5 starts $\Delta M(0)$ with $V_{35}^{(1)}$:-

$$\begin{tikzcd}
 & P_5 \arrow[dl, "0"'] \arrow[d, "0"] \arrow[dr, "0"] \arrow[drr, "0"] & & \\
P_1 & P_2 & P_4 & P_6
\end{tikzcd}$$

$X_{35}^{(1)}(K)$ for $K=4,6$ is 0

$\Longrightarrow\Longrightarrow$ <u>P_6 starts $\triangleleft M(0)$ with $V_{36}^{(1)}$:-</u>



So, $X_{36}^{(1)}(k)$ for $k=4,5$ is 0

Now,

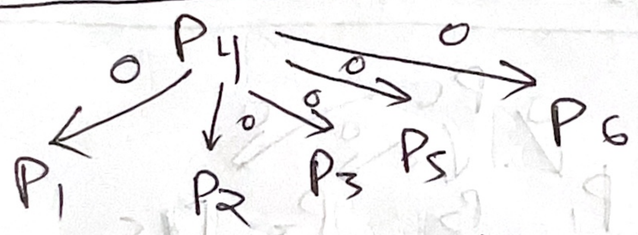
$\triangleleft M(1)$ P_3 concludes,

$X_{03}^{(2)}(3)=0$

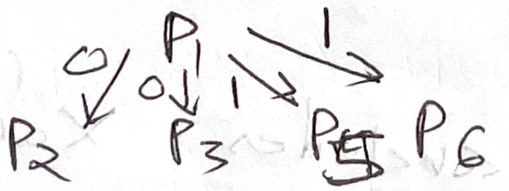
$X_{03}^{(2)}(k)$ for $k=4,5,6$ is

majority $(1,1,0,0,0)=0$

$\Longrightarrow$ <u>P_4 starts $\triangleleft M(1)$ with $V_{04}^{(2)}$:-</u>



$\Longrightarrow\Longrightarrow$ <u>P_1 starts $\triangleleft M(0)$ with $V_{41}^{(1)}$:-</u>

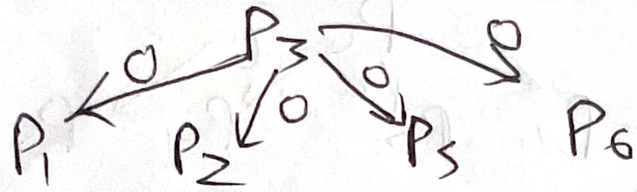


$X_{41}^{(1)}(3)=0$, $X_{41}^{(1)}(5)=X_{41}^{(1)}(6)=1$

$\Longrightarrow\Longrightarrow$ <u>P_2 starts $\triangleleft M(0)$ with $V_{42}^{(1)}$:-</u>

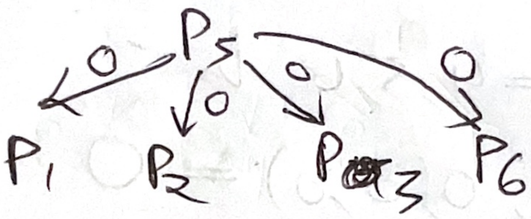
$X_{42}^{(1)}(3)=0$, $X_{42}^{(1)}(5)=X_{42}^{(1)}(6)=1$

$\Rightarrow\Rightarrow$ <u>P_3 starts OM(0) with $V_{43}^{(1)}$:-</u>



So, $X_{43}^{(1)}(k)$ for $k=5,6$ is 0

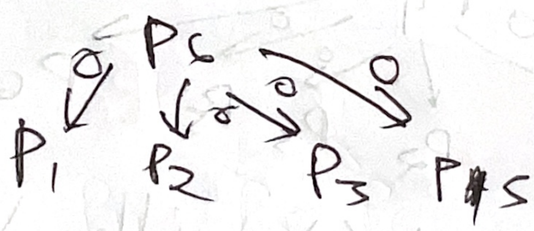
$\Rightarrow\Rightarrow$ <u>P_5 starts OM(0) with $V_{45}^{(1)}$:-</u>



So, $X_{45}^{(1)}(k)$ for $k=3,6$ is 0

$\Rightarrow\Rightarrow$ <u>P_6 starts OM(0) with $V_{46}^{(1)}$:-</u>

$\cancel{X_{46}^{(1)}(k)}$



So, $X_{46}^{(1)}(k)$ for $k=3,5$ is 0

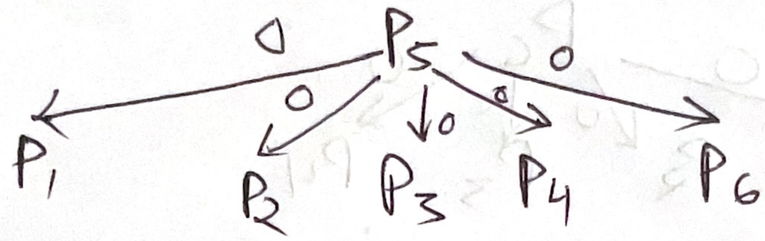
Now,

OM(1) by P_4 concludes $X_{04}^{(2)}(4)=4$

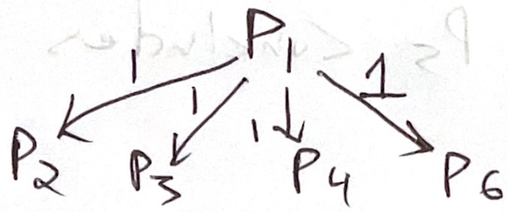
$X_{04}^{(2)}(3) = \text{majority}(0,0,0,0,0)=0$

$X_{04}^{(2)}(5) = X_{04}^{(2)}(6) = \text{majority}(1,1,0,0,0)=0$

$\Longrightarrow$ P_5 starts OM(1) with $V_{05}^{(2)}$:-

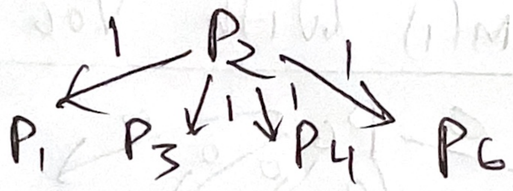


$\Longrightarrow\Longrightarrow$ P_1 starts OM(0) with $V_{51}^{(1)}$:-



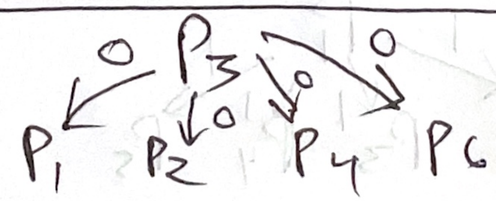
So, $X_{51(k)}^{(1)}$ for $k=3,4,6$ is 1

$\Longrightarrow\Longrightarrow$ P_2 starts OM(0) with $V_{52}^{(1)}$:-



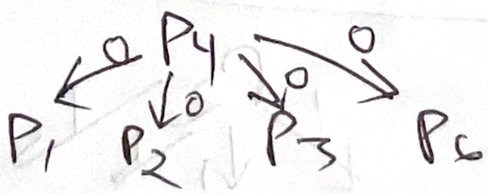
So, $X_{52(k)}^{(1)}$ for $k=3,4,6$ is 1

$\Longrightarrow\Longrightarrow$ P_3 starts OM(0) with $V_{53}^{(1)}$:-



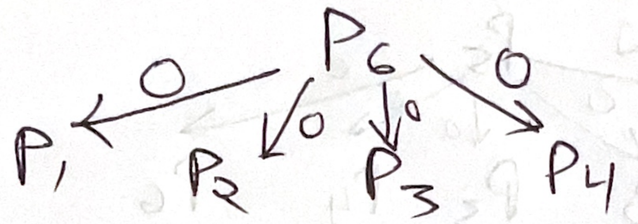
So, $X_{53(k)}^{(1)}$ for $k=4,6$ is 0

$\Longrightarrow\Longrightarrow$ P_4 starts OM(0) with $V_{54}^{(1)}$:-



So, $X_{54(k)}^{(1)}$ for $k=3,6$ is 0

$\Rightarrow\Rightarrow$ <u>P_6 starts $OM(0)$ with $V_{56}^{(1)}$:-</u>



So, $X_{56}^{(1)}(k)$ for $k=3,4$ is 0.

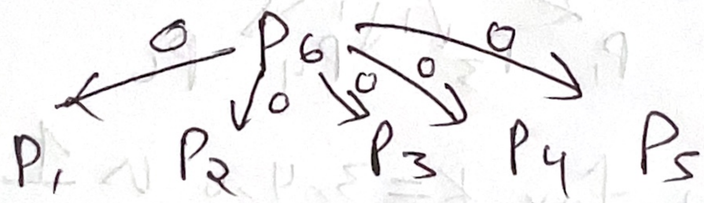
Now, $OM(1)$ by P_5 concludes

$X_{05}^{(2)}(5)=0$

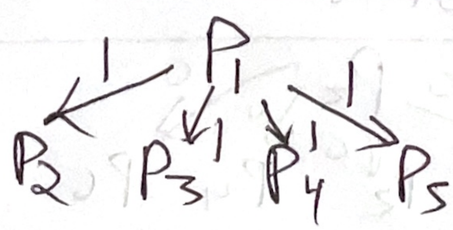
$X_{05}^{(2)}(k)$ for $k=3,4,6$ is

majority $(1,1,0,0,0)=0$

$\Rightarrow$ <u>P_6 starts $OM(1)$ with $V_{06}^{(2)}$:-</u>

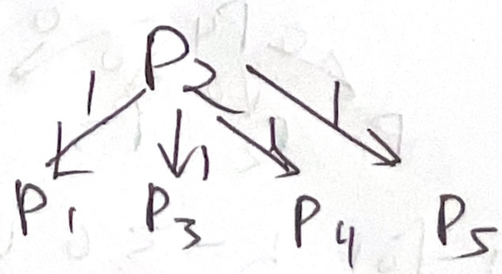


$\Rightarrow\Rightarrow$ <u>P_1 starts $OM(0)$ with $V_{61}^{(1)}$:-</u>



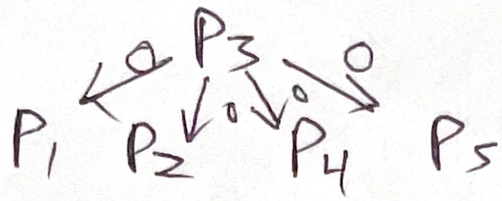
So, $X_{61}^{(1)}(k)$ for $k=3,4,5$ is 1

$\Rightarrow\Rightarrow$ <u>P_2 starts $OM(0)$ with $V_{62}^{(1)}$:-</u>



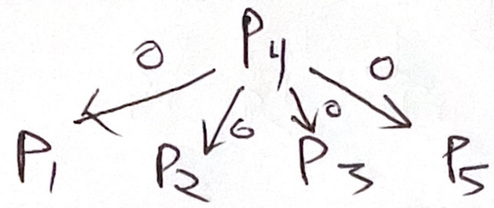
So, $X_{62}^{(1)}(k)$ for $k=3,4,5$ is $\underline{1}$

$\Rightarrow\Rightarrow$ <u>P_3 starts OM(0) with $V_{63}^{(1)}$:-</u>



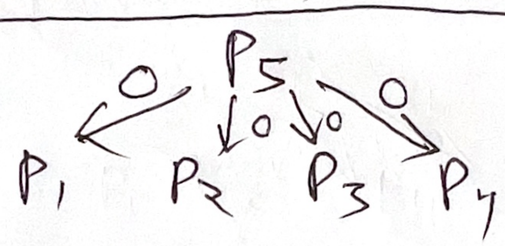
So, ~~X_{ij}~~ $X_{63(K)}^{(1)}$ for $K=4,5$ is 0

$\Rightarrow\Rightarrow$ <u>P_4 starts OM(0) with $V_{64}^{(1)}$:-</u>



So $X_{64(K)}^{(1)}$ for $K=3,5$ is 0

$\Rightarrow\Rightarrow$ <u>P_5 starts OM(0) with $V_{65}^{(1)}$:-</u>



So, $X_{65(K)}^{(1)}$ for $K=3,4$ is 0

Now, OM(1) by P_6 concludes

$X_{06}^{(2)}(6)=0$

$X_{06}^{(2)}(K)$ for $K=3,4,5$ is majority $(1,1,0,0,0)$
$=0$

Now, OM(2) by P_0 also concludes

Every correct process get the vector

$(1,1,0,0,0,0)$

And all decide on majority $(1,1,0,0,0,0)=0$