Mid-semester Examination (2023-24 Spring)

Computational Geometry (CS60064)

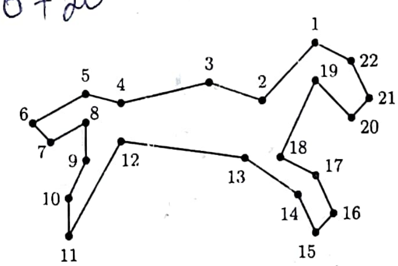
15-Feb-2024

Note: Answer all. Duration = 2 hours. Full marks = 60.

1. Prove or disprove: The intersection of two convex polygons is always a convex polygon. $\boxed{5}$

2. Classify the vertices of the following polygon required to subdivide it into y-monotone polygons. You need not show the subdivision. $\boxed{5}$

30 +50 +20



3. A convex polygon P is given as the counter-clockwise sequence of its n vertices. Assume that the vertices are in general positions, i.e., their x-coordinates are all distinct, and so also their y-coordinates. Given a query point p, the task is to determine whether P contains q. Suggest an $O(n)$-time algorithm for this, and another of $O(\log n)$ time complexity. Explain the time complexity only for the latter. The space complexity should be linear in either case. $\boxed{5 + 10}$

4. Recall the definition of isothetic frontier: Given a connected set S of integer points (pixels) and a square grid G, the isothetic frontier $F_G(S)$ is the smallest polygon containing S, without touching S, such that its vertices are all grid points and its edges lie on grid lines.
Given such a set S and a square grid G, suggest an algorithm to compute its isothetic frontier. Explain its time complexity. $\boxed{10 + 5}$

5. When is a set of integer points (pixels) said to be digitally convex? Give examples of two such sets—one being digitally convex and the other not. Suggest an algorithm to determine whether a set of integer points is digitally convex. Explain its time complexity. $\boxed{2 + 4 + 10 + 4}$

G(CH(S))

End-semester Examination (2023-24 Spring)

Computational Geometry (CS60064)

To answer all within 3 hours

Note: Unless mentioned, the following should be assumed about an input,

- Points are all distinct, no three being collinear, and no four being concyclic.
- A polygon is simple, i.e., edges intersect each other only at their endpoints, and it is described as clockwise or counter-clockwise sequence of its vertices, starting from an arbitrary vertex.

1. Prove or disprove: Graham Scan can be used to compute the convex hull of a polygon in linear time if the sequence of its vertices from the leftmost to the rightmost vertex is x-monotone, and so also the sequence from the rightmost to the leftmost vertex. $\boxed{5}$

2. Prove or disprove: An art gallery with n walls requires at most $\lfloor n/3 \rfloor$ guards, and this bound is tight. $\boxed{5}$

3. Prove or disprove: In Fortune's algorithm for Voronoi Diagram computation, the number of true circle events and that of false circle events are both linear in n. $\boxed{5}$

4. Argue why triangulating a monotone polygon is no harder than triangulating an arbitrary polygon. Justify this also with a reference to the time complexities of their algorithms discussed in the class. $\boxed{5}$

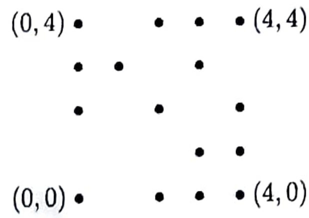
5. Consider the chain-code sequence: S = 001000001000001000001000000100000010000010000.
   Is it digitally straight? Provide reasoning.
   If S is digitally straight, identify a longest sub-sequence T of S that is not digitally straight; if not, determine a longest sub-sequence T of S that is digitally straight.
   In either case, you have to justify how you get T. $\boxed{5}$

6. Given n points on the plane, suggest an algorithm to find a pair having the largest slope. Explain its time complexity. $\boxed{7+3}$

7. What is meant by orthogonally convex polygon?
   Draw one example of orthogonally convex polygon and one of orthogonally non-convex polygon.
   Given an orthogonal polygon, suggest an algorithm to determine whether it is orthogonally convex.
   Explain its time complexity. $\boxed{1+2+5+2}$

8. Consider a set of n two-dimensional points $P = \{p_1, p_2, \ldots, p_n\}$ in general configuration. Assume that p_1 resides on the boundary of the minimum enclosing disk (MED) of P. Suggest an efficient randomized algorithm to find the MED of P. Deduce its expected time complexity. $\boxed{7+3}$

9. Draw the k-d tree containing the following point set. Notice that the points have all integer coordinates in $[0,4]$; and they have IDs 1 to 16 in the lexicographic order of non-decreasing (x,y)-coordinates. For example, $\mathrm{ID}(0,0)=1$, $\mathrm{ID}(0,4)=4$, $\mathrm{ID}(1,3)=5$, $\mathrm{ID}(4,0)=13$, $\mathrm{ID}(4,4)=16$.
You should label the partition lines as the algorithm progresses, and assign those labels to the corresponding nodes of the tree to create the correspondence. $\boxed{15}$



10. Let P be a set of 5 sites: $(0,0), (-2,0), (2,0), (0,-2), (0,2)$.
(a) Draw the Voronoi Diagram (VD) of P enclosed in an axis-parallel square S of length 8 and centered at $(0,0)$.
(b) Label its vertices, half-edges, and faces by integers IDs, and write their details in the corresponding Doubly Connected Edge List. You should disregard the infinite face lying outside S.
(c) How many vertices, edges, and faces the DCEL will have if $P = \{(i,j) : i,j \text{ are integers in } [-3,3]\}$? The VD is enclosed in the same square S. $\boxed{5+10+5}$