Spring 1996 **CMSC 754: Homework 1** Samir Khuller

Due in class: Feb 14.

Read Chapter 1 of the text (O'Rourke).

Problems 1–4 are due on Feb 14. Problem (5) can be turned in with HW 2.

(1) Let's consider the algorithm for the closest pair problem done in class. In class we observed that there are at most 8 points in the "window" of size $\delta \times 2\delta$. With the assumption that no two points have the same x co-ordinate, what is the best upper bound (smallest number) we can claim about the number of points in this "window"? (Don't just provide a number, try to give a proof.)

(2) Assume that $v_1 = x_1\vec{\imath} + y_1\vec{\jmath}$ and $v_2 = x_2\vec{\imath} + y_2\vec{\jmath}$ are two vectors in the xy plane. Let $v_1 \cdot v_2 = x_1x_2 + y_1y_2$.

(a) Prove that $v_1 \cdot v_2 = |v_1||v_2|\cos\theta$ where θ is the angle between the two vectors. (Hint: use elementary trigonometry to compute the length of the vector $v_1 - v_2$.)

(b) Use the definition of cross product given in O'Rourke's book (page 19) to prove that $|v_1 \times v_2|$ is the area of the parallelogram formed by the two vectors. (Recall that $\cos^2\theta + \sin^2\theta = 1$.)

(c) Prove that if $|v_1 \times v_2|$ is positive, then vector v_1 is clockwise from vector v_2 with respect to the origin.

(3) Problem 1 (page 10) in Exercises 1.1.4.

(4) Problem 2 (page 10) in Exercises 1.1.4.

(5) (Hard) Let $p_1p_2\ldots p_n$ be a polygonal path in the plane such that at every vertex the path "turns left" (i.e., p_{i+1} is to the left of the ray from p_{i-1} to p_i). Give a linear time algorithm to check the path for self intersections.