Computational Geometry (CS60064)
Spring 2024-25

Instructions

(a) The submission deadline is hard. There may be unforeseen glitches during submission. So, for safety, submit your files well ahead.

(b) All submissions should be on moodle only. No email submission will be accepted, excepting medical reasons.

(c) Do not forget to typeset your solutions. In particular, every mathematical expression must be properly typeset, e.g., the square of n must appear as n^2 and not as `n^2`. Improper typesetting may incur up to 25% deduction in marks.

You can use LaTeX for writing (that is what we recommend); else, typeset in Word and convert to pdf.

Handwritten text—converted to images or to pdf—will not be evaluated.

(d) You must submit all the source files and the final pdf as a single zip file. The name of the zip should be your roll number, followed by a hyphen, followed by the assignment number. For example, if your roll number is `XY190047`, then the zip file for the 1st assignment should be named as `XY190047-a1.zip`. For subsequent assignments, your zip files should be named as `XY190047-a2.zip`, `XY190047-a3.zip`, ...

If you typeset in LaTeX, then the zip should contain one tex file, image files if any, and the final pdf.

If you typeset in Word, then the zip should contain one odt/doc/docx file and the final pdf. Image files get embedded in a Word file, so image files are not needed.

Failing this, your assignment will not be evaluated.

Assignment 1

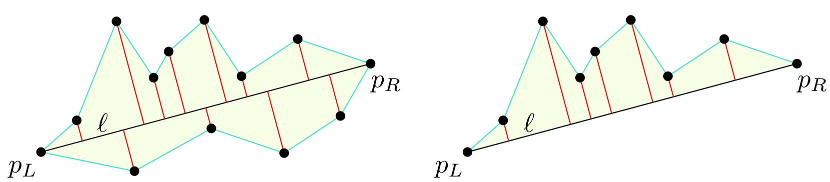
Submission deadline: 19-Jan-2025, 11:55 PM

1.1 Polygon Construction

Given n points on the xy-plane, design an algorithm to construct a simple polygon P such that all the given points serve as vertices of P, and no other points are included as vertices. Provide a proof of correctness for your algorithm and deduce its time complexity. (A *simple polygon* is defined as one in which no two edges intersect, except possibly at their endpoints.) $\boxed{4+3+3=10 \text{ marks}}$

Solution key:

Find the leftmost point p_L and the rightmost point p_R, and join them with a straight-line segment ℓ. Project all points that are above ℓ, on ℓ. Sort these footprints and connect the original points serially in the sorted order to form the upper chain of P; do the same for the lower chain. If one side of ℓ is empty, use ℓ as an edge of the polygon P (as shown in the figure on the right). This can be done in $O(n \log n)$ time.



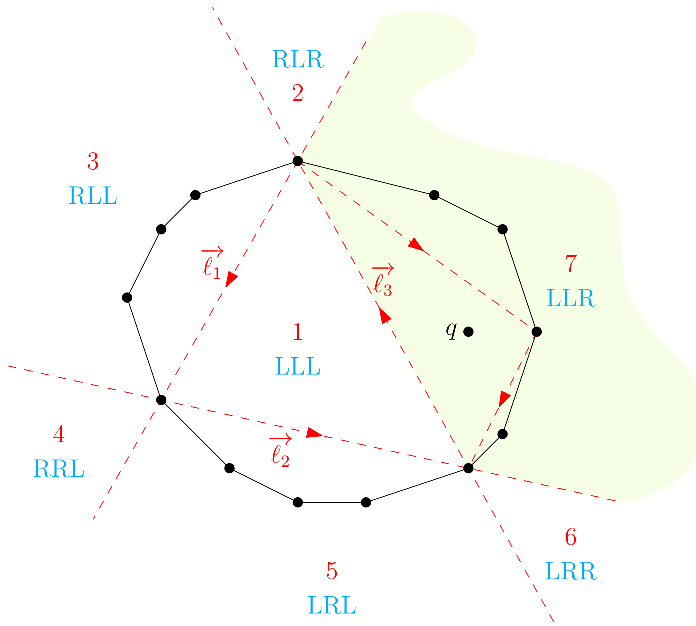
1.2 Point Location

A convex polygon P is provided as a counter-clockwise ordered sequence of n vertices, with their locations specified as (x, y) coordinates. Given a query point q, develop an algorithm to determine whether q lies inside P in $O(\log n)$ time, using $O(n)$ space, including any necessary preprocessing. Justify the time and space complexities of your algorithm. $\boxed{6+2+2=10 \text{ marks}}$

Solution key:

Choose three points on the boundary of P, which are almost equispaced—can be done in $O(1)$ time—via indexing. Construct three directed rays (cut-lines) $\overrightarrow{\ell_1}, \overrightarrow{\ell_2}, \overrightarrow{\ell_3}$ through these points—they partition the 2D-space into seven disjoint regions as shown. The location of the query point q w.r.t. these regions can be determined in $O(1)$ time via three orientation tests. Further refined partitioning can be done through $O(\log_3 n)$ steps, thus giving the precise location of q in $O(\log n)$ time, and in $O(n)$ space.

In the following example, q is initially identified to lie in Region 7, characterized by the unique 3-bit label LLR—indicating that q lies left of $\overrightarrow{\ell_1}$, left of $\overrightarrow{\ell_2}$, and right of $\overrightarrow{\ell_3}$. Subsequently, its position is evaluated with respect to $\overrightarrow{\ell_1}$ and two other rays, resulting in the label LLL, thereby confirming that q lies inside P.



Assignment 2

Submission deadline: 26-Jan-2025, 11:55 PM

2.1 Point location w.r.t. line

For some algorithm, we have to test whether a point r lies to the left or right of the directed line $\overrightarrow{pq}$ through two points p and q. Let $p = (p_x, p_y)$, $q = (q_x, q_y)$, and $r = (r_x, r_y)$.

(a) Show that the sign of the determinant

$$D = \begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines whether r lies to the left or right of the line.

(b) Show that $|D|$ is in fact twice the area of the triangle determined by p, q, and r.

(c) Why is this an attractive way to implement the basic test in any algorithm where the location of a point is determined w.r.t. a directed line? Provide arguments for both integer and floating-point coordinates. $\boxed{6 + 2 + 2 = 10 \text{ marks}}$

2.2 Point location in strip

Let S be a set of n disjoint line segments whose upper endpoints lie on the line $y = 1$ and whose lower endpoints lie on the line $y = 0$. These segments partition the horizontal strip $[-\infty : \infty] \times [0 : 1]$ into $n + 1$ regions: $R_1, \ldots, R_{n+1}$. Give an $O(n \log n)$-time algorithm to build a binary search tree on the segments in S such that the region containing a query point can be determined in $O(\log n)$ time. Also, describe the query algorithm in full detail. $\boxed{5 + 5 = 10 \text{ marks}}$

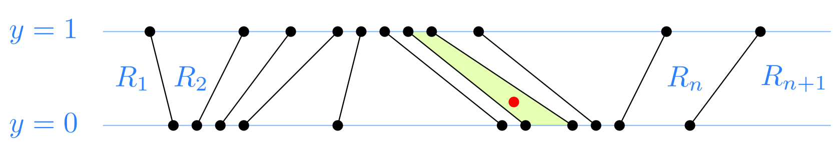


Figure 2.1: Determining the region of a query point in a strip.