Matrix dimensions and entries

A **matrix** is a rectangular array of values, where each value is an **entry** in both a row and a column.

We know how to solve systems of linear equations using the substitution, elimination, and graphing methods, but what we'll learn throughout this course is that solving linear systems can be made much easier when we use matrices instead of one of these other three methods.

This is especially true as our systems get larger. It's not too bad to use substitution, elimination, or graphing for a system of two equations with two unknowns, but what about a system of 20 equations with 20 unknowns? To tackle a system that large, we need to transition our understanding from these three methods, over to matrix methods.

But first, we need to learn understand the basics about matrices.

Matrix dimensions

A matrix is often described by the number of rows and columns that it has. For instance, a 3×4 matrix is a matrix with 3 rows and 4 columns. In the description "3×4," the number of rows always comes first, and the number of columns always comes second, so remember:

"rows $\times$ columns"

A matrix can be as small as 1×1, with one row and one column, in which case it looks like this:

$$[a]$$

Or it can have infinitely many rows and/or columns. It can have the same number of rows and columns, more rows than columns, or more columns than rows.

A 2×2 matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

A 2×3 matrix: $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

A 3×2 matrix: $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

Matrices are just a way of arranging data, especially large amounts of data. And in this section we'll learn all different ways to work with matrices, like how to add them or multiply them. And these kinds of matrix operations are useful in all kinds of fields, like statistics, economics, data analysis, and computer programming.

Example

Give the dimensions of each matrix.

$$A = \begin{bmatrix} 7 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 0 & 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We always give the dimensions of a matrix as rows $\times$ columns. Matrix A has 3 rows and 3 columns, so A is a 3×3 matrix. Matrix B has 1 row and 5 columns, so B is a 1×5 matrix. Matrix C has 3 rows and 1 column, so C is a 3×1 matrix.

Matrix entries

We call out a particular entry in a matrix using the name of the matrix and the row and column where the entry is sitting. So if the matrix is called K (uppercase letters are used to name matrices), and

$$K = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \end{bmatrix}$$

then if we want the entry in the first row, third column, we write that as $k_{1,3}$, since that's the entry in the first row, third column, of matrix K.

Example

Find $a_{2,3}$, $b_{1,4}$, and $c_{3,1}$.

$$A = \begin{bmatrix} 7 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 0 & 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The value of $a_{2,3}$ is the entry in the second row, third column of matrix A, which is 1, so $a_{2,3} = 1$. The value of $b_{1,4}$ is the entry in the first row, fourth column of matrix B, which is 0, so $b_{1,4} = 0$. The value of $c_{3,1}$ is the entry in the third row, first column of matrix C, so $c_{3,1} = 1$.
