Representing systems with matrices

We said that we wanted to start using matrices to solve systems, so we need to learn first how to represent a linear system in a matrix, instead of just as a list of separate equations, like this:

$$3x + 2y = 7$$

$$x - 6y = 0$$

Sometimes for systems like these, you'll use a matrix to represent only the left sides (when all the variable terms are consolidated on the left, and the constant terms are alone on the right sides). A matrix representing the left sides only would be

$$\begin{bmatrix} 3 & 2 \\ 1 & -6 \end{bmatrix}$$

Notice that we've just taken the coefficients on the x and y terms. The first column represents the coefficients on x, 3 and 1, and the second column represents the coefficients on y, 2 and -6. The first row represents the coefficients in the first equation, $3x + 2y = 7$, and the second row represents the coefficients in the second equation, $x - 6y = 0$.

Augmented matrices

But we can also bring the constants from the right side of these equations into the matrix. Whenever we add a column to a matrix that wasn't previously there, we say that we're **augmenting** the matrix, and we call the result an **augmented matrix**. So the augmented matrix for this system could look like this:

$$\begin{bmatrix}3 & 2 & 7\\1 & -6 & 0\end{bmatrix}$$

or like this:

$$\left[\begin{array}{cc|c}3 & 2 & 7\\1 & -6 & 0\end{array}\right]$$

You can use augmented matrices to represent systems of any size. If we added two more equations to the system, we'd simply add two more rows to the matrix. Or if we added another variable to the system, like z, we'd simply add one more column to the matrix.

Let's do an example with a few more variables.

Example

Represent the system with an augmented matrix called M.

$$-2x+y-t=7$$

$$x-y+z+4t=0$$

You always want to look at all the variables that are included in the system, not just the first equation, since the first equation may not include all the variables.

This particular system includes x, y, z, and t. Which means the augmented matrix will have four columns, one for each variable, plus a column for the constants, so five columns in total. Because there are two equations in the system, the matrix will have two rows. We could set up the matrix like this:

$$M = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & | & C_1 \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & | & C_2 \end{bmatrix}$$

Because there's no z-term in the first equation, the value of $m_{1,3}$ will be 0. If we fill in the matrix with that value and all the other coefficients and constants, we get

$$M = \begin{bmatrix} -2 & 1 & 0 & -1 & | & 7 \\ 1 & -1 & 1 & 4 & | & 0 \end{bmatrix}$$

Lining up the variables

Whenever you're building a matrix to represent a system, you want to be sure that you have all the variables in the same order, and all your constants grouped together on the same side of the equation.

That way, with everything lined up, it'll be easy to make sure that each entry in a column represents the same variable or constant, and that each row in the matrix captures the entire equation.

Let's do an example where the terms aren't already in order.

Example

Express the system of linear equations as a matrix called B.

$$2x + 3y - z = 11$$

$$7y = 6 - x - 4z$$

$$-8z + 3 = y$$

Before we do anything, we want to put each equation in order, with x, then y, then z on the left side, and the constant on the right side.

$$2x + 3y - z = 11$$

$$x + 7y + 4z = 6$$

$$-y - 8z = -3$$

We could also recognize that there is no x-term in the third equation, but we could add in a 0 "filler" term.

$$2x + 3y - z = 11$$

$$x + 7y + 4z = 6$$

$$0x - y - 8z = -3$$

Pulling all these values into a matrix gives

$$B=\begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & | & C_1 \\ b_{2,1} & b_{2,2} & b_{2,3} & | & C_2 \\ b_{3,1} & b_{3,2} & b_{3,3} & | & C_3 \end{bmatrix}$$

$$B=\begin{bmatrix} 2 & 3 & -1 & | & 11 \\ 1 & 7 & 4 & | & 6 \\ 0 & -1 & -8 & | & -3 \end{bmatrix}$$