SOLUTIONS (TEST #1, CSI 30, FAL 2014, OJAKIAN)

#1

(a) Proposition

(b) Proposition

(c) Not Proposition

(d) Proposition

#2

(a) $P(3) \equiv$ "$2(3)+10>3$" $\equiv$ "$16>3$" **True**

(b) $P(0) \equiv$ "$2(0)+10>0$" $\equiv$ "$10>0$" **True**

(c) $P(-10) \equiv$ "$2(-10)+10>-10$" $\equiv$ "$-10>-10$" **False**

#3

A	B	A→B	¬(A→B)	¬(A→B)→A
F	F	T	F	T
F	T	T	F	T
T	F	F	T	T
T	T	T	F	T

Tautology since this column shows it is always true

#4

For example

$P \wedge \neg P$

P	¬P	P∧¬P
F	T	F
T	F	F

Contradiction since this column shows it is always false

#5

(a) A rational number is a number that can be written as $\frac{p}{q}$, where $p, q \in \mathbb{Z}$ and $q \neq 0$

(b) A tautology is a propositional logic statement which is true for all settings of the variables.

#6

(a)

A	¬A	A↔¬A
F	T	F
T	F	F

All F → Contradiction

(b)

A	B	A∨B	A∧B	(A∨B)→(A∧B)
F	F	F	F	T
F	T	T	F	F
T	F	T	F	F
T	T	T	T	T

some T, some F → Contingency

(*7) (a) No Because for P = T, Q = F: $\begin{cases} P \to Q \text{ False} \\ Q \to P \text{ True} \end{cases}$

(b)

A	B	¬A	A∧B	A→(A∧B)	¬A ∨ (A∧B)
F	F	T	F	T	T
F	T	T	F	T	T
T	F	F	F	F	F
T	T	F	T	T	T

Some columns ⇒ **Equivalent**

(*8) $P \wedge \neg Q$

(*9) (a) $\neg(A \vee \neg B) \equiv \neg A \wedge \neg\neg B \equiv \boxed{\neg A \wedge B}$

(b) $\neg(\forall x \exists y\ x<y) \equiv \boxed{\exists x \forall y\ x \geq y}$

(*10) (a) True. Can take $x = 501$

(b) False. Can take $y = -1$ to make "$-7(-1) < 7(-1)$" false

(c) True. Can take $x = y = 1$

(d) False. Can take $x = y = -1$ to make "$2(-1)+3(-1) > (-1)+(-1)$" false

(*11) (a) True (b) False (c) True (d) False

(*12) Since n is even, $n = 2k$, for some integer k. Thus $21n+6 = 21(2k)+6 = 42k+6 = 2(21k+3)$, so (since $21k+3$ is an integer), $21n+6$ is even

(*13) Given $5n+2$ is odd. Assume for contradiction: n is even. Thus $n = 2k$, for some integer k. Thus $5n+2 = 5(2k)+2 = 2(5k+1)$, so $5n+2$ is even which contradicts $5n+2$ is odd.