Topic: Unit vectors and basis vectors

Question: Find the unit vector in the direction of $\vec{a} = (5, -9)$.

Answer choices:

A $\quad \vec{u} = \begin{bmatrix} \frac{5}{\sqrt{106}} \\ \frac{9}{\sqrt{106}} \end{bmatrix}$

B $\quad \vec{u} = \begin{bmatrix} -\frac{5}{\sqrt{106}} \\ -\frac{9}{\sqrt{106}} \end{bmatrix}$

C $\quad \vec{u} = \begin{bmatrix} \frac{5}{\sqrt{106}} \\ -\frac{9}{\sqrt{106}} \end{bmatrix}$

D $\quad \vec{u} = \begin{bmatrix} -\frac{5}{\sqrt{106}} \\ \frac{9}{\sqrt{106}} \end{bmatrix}$

Solution: C

First, find the length of $\vec{a}$.

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2}$$

$$||\vec{a}|| = \sqrt{5^2 + (-9)^2}$$

$$||\vec{a}|| = \sqrt{25 + 81}$$

$$||\vec{a}|| = \sqrt{106}$$

Then the unit vector in the direction of $\vec{a} = (5, -9)$ is

$$\vec{u} = \frac{1}{||\vec{a}||}\vec{a}$$

$$\vec{u} = \frac{1}{\sqrt{106}}\begin{bmatrix} 5 \\ -9 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} \frac{5}{\sqrt{106}} \\ -\frac{9}{\sqrt{106}} \end{bmatrix}$$

Topic: Unit vectors and basis vectors

Question: Find the unit vector in the direction of $\vec{v}=(2,-1,4)$.

Answer choices:

A $\quad \vec{u}=\begin{bmatrix} \frac{2}{\sqrt{21}} \\ \frac{1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}$

B $\quad \vec{u}=\begin{bmatrix} -\frac{2}{\sqrt{21}} \\ -\frac{1}{\sqrt{21}} \\ -\frac{4}{\sqrt{21}} \end{bmatrix}$

C $\quad \vec{u}=\begin{bmatrix} -\frac{2}{\sqrt{21}} \\ \frac{1}{\sqrt{21}} \\ -\frac{4}{\sqrt{21}} \end{bmatrix}$

D $\quad \vec{u}=\begin{bmatrix} \frac{2}{\sqrt{21}} \\ -\frac{1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}$

Solution: D

First, find the length of $\vec{v}$.

$$||\vec{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$||\vec{v}|| = \sqrt{2^2 + (-1)^2 + 4^2}$$

$$||\vec{v}|| = \sqrt{4 + 1 + 16}$$

$$||\vec{v}|| = \sqrt{21}$$

Then the unit vector in the direction of $\vec{v} = (2, -1, 4)$ is

$$\vec{u} = \frac{1}{||\vec{v}||}\vec{v}$$

$$\vec{u} = \frac{1}{\sqrt{21}}\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} \frac{2}{\sqrt{21}} \\ -\frac{1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}$$

Topic: Unit vectors and basis vectors

Question: Represent $\overrightarrow{x}=(-2,8,-4)$ with the standard basis vectors.

Answer choices:

A $\quad \overrightarrow{x}=2\hat{i}+8\hat{j}+4\hat{k}$

B $\quad \overrightarrow{x}=2\hat{i}-8\hat{j}+4\hat{k}$

C $\quad \overrightarrow{x}=-2\hat{i}+8\hat{j}-4\hat{k}$

D $\quad \overrightarrow{x}=-2\hat{i}-8\hat{j}-4\hat{k}$

Solution: C

The vector $\vec{x} = (-2,8,-4)$ is part of $\mathbb{R}^3$, which means we'll need to use the basis vectors for $\mathbb{R}^3$, which are $\hat{i} = (1,0,0)$, $\hat{j} = (0,1,0)$, and $\hat{k} = (0,0,1)$.

We're moving -2 units in the direction of the x-axis, 8 units in the direction of the y-axis, and -4 units in the direction of the z-axis.

$$\vec{x} = (-2,8,-4) = -2\begin{bmatrix}1\\0\\0\end{bmatrix} + 8\begin{bmatrix}0\\1\\0\end{bmatrix} - 4\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\vec{x} = (-2,8,-4) = \begin{bmatrix}-2\\0\\0\end{bmatrix} + \begin{bmatrix}0\\8\\0\end{bmatrix} + \begin{bmatrix}0\\0\\-4\end{bmatrix}$$

$$\vec{x} = (-2,8,-4) = \begin{bmatrix}-2+0+0\\0+8+0\\0+0-4\end{bmatrix}$$

$$\vec{x} = (-2,8,-4) = \begin{bmatrix}-2\\8\\-4\end{bmatrix}$$

So we can express $\vec{x} = (-2,8,-4)$ in terms of basis vectors as

$$\vec{x} = -2\hat{i} + 8\hat{j} - 4\hat{k}$$