proposition – is a sentence that declares a fact, i.e. declarative statement, that is either *true* or *false*, but not both.

examples:

"It is raining now"

$$1+6 = 10$$

Washington, D.C. is the capital of United States of America

not propositions:

"What time is it now?"

$$x+5=10$$

"Please, stand clear of the closing doors."

Do you see the difference between the two sets of examples? Why the last three sentences are not propositions?

proposition – is a sentence that declares a fact, i.e. *declarative statement*, that is either *true* or *false*, but not both.

we use letters to denote propositional variables (i.e. variables that represent propositions): *p, q, r, s*

examples:

p:1+7=10

q: "It is sunny outside"

If a proposition is true, its value can be denoted by T or True or 1

If a proposition is false, its value can be denoted by F or ⊥ (bottom) or False or 0

The area of logic that deals with propositions is called the *propositional logic* or *propositional calculus*.

It was first developed by the Greek philosopher Aristotle.

Can we build/construct new propositions?

New propositions can be constructed from existing propositions using *logical operators*, and are called compound propositions.

7	negation	¬p
٨	conjunction	p ^ q
V	disjunction	$p \vee q$

<u>other denotations</u> :	<u>meaning</u> :
p, not p	" <mark>not p"</mark>
<mark>p and q</mark>	<mark>"p and q'</mark>
p or q	"p or q"

example 1:

Let proposition p stand for "I will go to a movie theater", then $\neg p$ means "I will not go to a movie theater."

Truth table for negation operation:

p	¬р	
Т	F	
F	Т	or

p	¬р
1	0
0	1

example 2:

Let proposition p stand for "It is raining" and q stand for "I want to go to a movie theater", then $p \land q$ means "It is raining and I want to go to a movie theater."

Conjunction p ^ q is true when both p and q are true. Truth table for p ^ q:

p	q	p ^ q		p	q	p ^ q
Т	Т	Т		1	1	1
T	F	F		1	0	0
F	Т	F	or	0	1	0
F	F	F		0	0	0

example 3:

Let proposition p stand for "It is raining" and q stand for "I want to go to a movie theater", then $p \lor q$ means "It is raining or I want to go to a movie theater."

Disjunction p v q is true when at least one of p and q is true.

Truth table for p ∨ q:

p	q	pvq
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

or

 p
 q
 p ∨ q

 1
 1
 1

 1
 0
 1

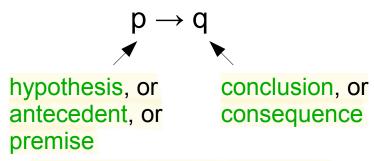
 0
 1
 1

 0
 0
 0

more logical operators:

- → implication p → q+ exclusive or p+q
- "if p then q", "p implies q" see page 6 for more "either p or q"

Implication:



if p is true then q holds

Implication is also called conditional statement.

Truth table for the implication $p \rightarrow q$:

p	q	$p \rightarrow q$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

example: Let p: "the weather is good", and q: "we'll go to the beach".

Then p→q stands for "If the weather is good we'll go to the beach"

more logical operators:

- $\begin{array}{ccc}
 & & \text{implication} & p \rightarrow q \\
 & & \text{exclusive or} & p \oplus q
 \end{array}$
- "if p then q", "p implies q"

 "either p or q"

Exclusive or:

p+q is true when exactly one of p and q is true

example: Let p: "the weather is good", and q: "we'll go to the beach".

Then p+q stands for "Either the weather is good or we'll go to the beach"

Truth table for the exclusive or p+q:

р	q	$p \oplus q$
Т	Т	F
Т	F	Т
F	Т	Т
F	F	F

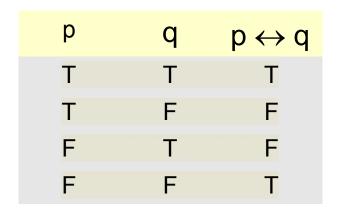
even more logical operators:

 \leftrightarrow biconditional statement $p \leftrightarrow q$ "p if and only if q", or "p iff q"

Biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

example: Let p: "we'll go to the beach", and q: "the weather is good".

Then p↔q stands for "we'll go to the beach if and only if the weather is good " Truth table for $p \leftrightarrow q$:



Biconditional statements are also called bi-implications.

List of considered logical operators:

Let's play with English:

Let p and q be propositions; p: "Swimming at the New Jersey shore is prohibited" and q: "Sharks have been spotted near the shore." Let's express the following propositions as English sentences.

- a) ¬p: Swimming at the New Jersey shore is allowed
- b) ¬p ^ ¬q : Swimming at the New Jersey shore is allowed and Sharks haven't been spotted near the shore
- c) $p \leftrightarrow q$: Swimming at the New Jersey shore is prohibited if and only if Sharks have been spotted near the shore.
- d) $\neg p \lor (p \land q)$ Swimming at the New Jersey shore is allowed or, it is prohibited and sharks have been spotted near the shore.

Converse, Contrapositive, and Inverse:

Let's start with an implication (conditional statement) $p \rightarrow q$

$$q \rightarrow p$$
 is the converse $\neg q \rightarrow \neg p$ is the contrapositive $\neg p \rightarrow \neg q$ is the inverse

! The contrapositive and the original statement have the same truth tables.

Example: what are the contrapositive, the converse and the inverse of the following conditional statement: "If you get 100% on the final, then you will get an A"?

contrapositive: If you won't get an A then you didn't get 100% on the final converse: If you will get an A then you get 100% on the final inverse: If you don't get 100% on the final, then you won't get an A

Precedence of Logical Operators:

$$\neg$$
, \wedge , \vee , \rightarrow , \leftrightarrow first last