# 2.1 Sets, power sets. Cartesian Products.



## Outline

What is a set?

Describing sets:
- Listing Elements
- Set-Builder Notation
- Venn Diagrams

Terminology:
- Subset
- Cardinality
- Cartesian Product

# *2.1 Sets, power sets. Cartesian Products.*

Set is an unordered collection of *distinct* objects.
- used to group objects together,
- often the objects with similar properties

This description of a set (without specification what an object is) was started by the German mathematician Georg Cantor, in 1895.

He is considered to be the founder of **set theory**.

We will use Cantor's original version of set theory (called **naive set theory**).

The objects in a set are called elements, or members, of the set. A set is said to contain its elements.

**Example** (set builder notation):
A = { x | x is even positive integer}

which means that A = { 2, 4, 6, 8, 10, ...}

-we can use this notation when it is not possible to list all the elements of the set.

**Example** (set builder notation):
T = { x | $x^2$=64} Can you tell what numbers set T consists of?

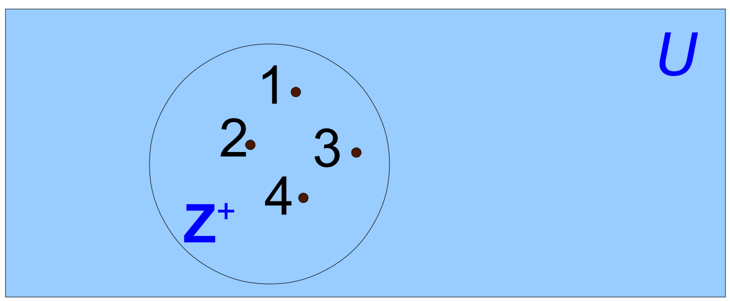
### Some of the well studied sets

**N** = { 0, 1, 2, 3, 4, ...} set of natural numbers
**Z** = {...,-3, -2, -1, 0, 1, 2, 3, ...} set of integers
$\mathbf{Z}^+$ = { 1, 2, 3, 4, ...} set of positive integers
**R** = the set of real numbers (rational and irrational numbers)

## Venn Diagrams

- is used to show relationships between sets.
- named after English mathematician Jogn Venn, who introduced their use in 1881.

In Venn diagram the universal set *U*, which contains <u>all the objects under consideration</u>, is represented by a rectangle.




*U* – universe

Let $U = \mathbf{Z}$ the set of integers

Inside the rectangle, circles and other geometrical figures are used to represent sets.

Sometimes points are used to represent particular elements of the set.

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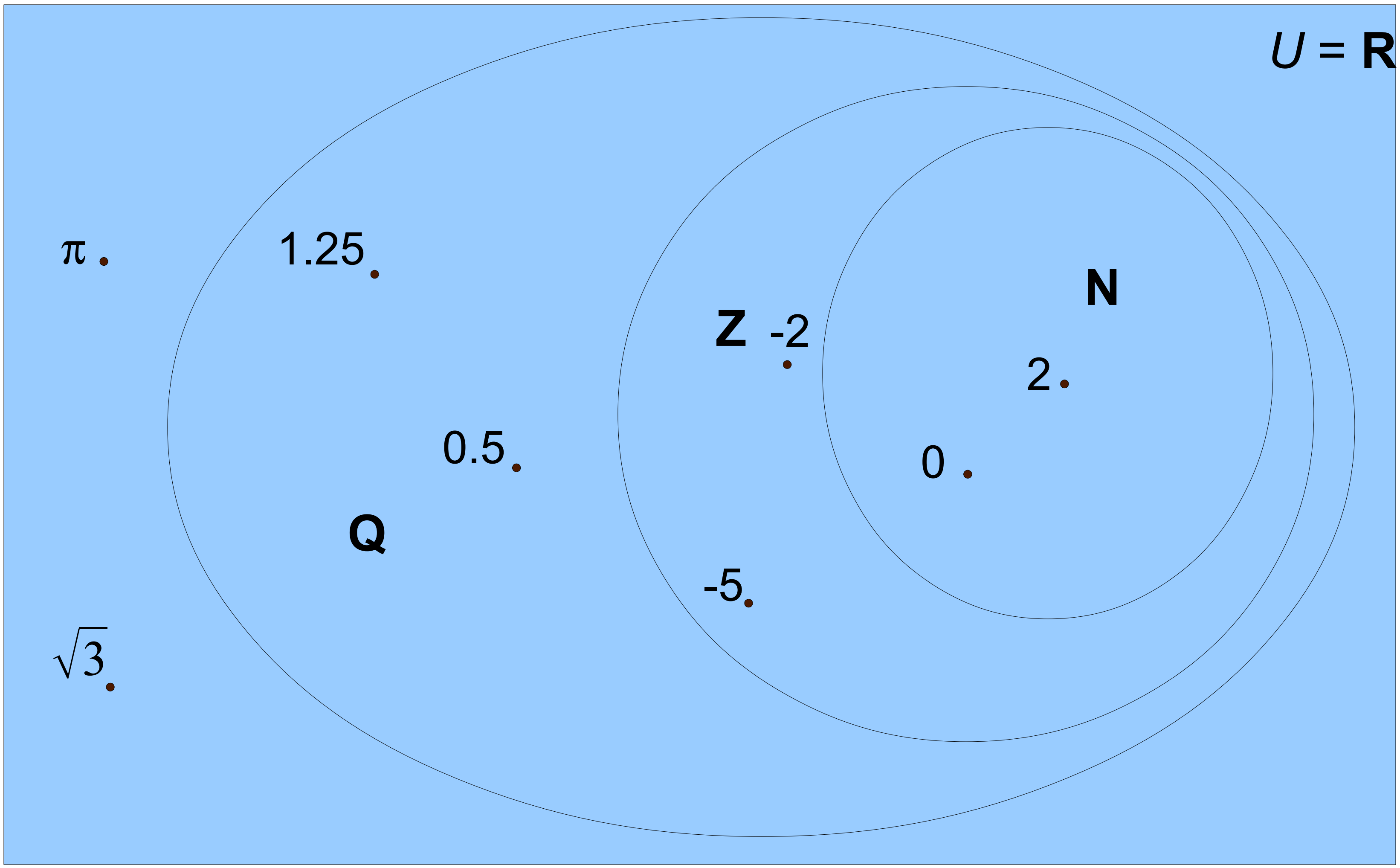
## Venn Diagrams

Venn diagrams allow us to visualize relationships between sets.



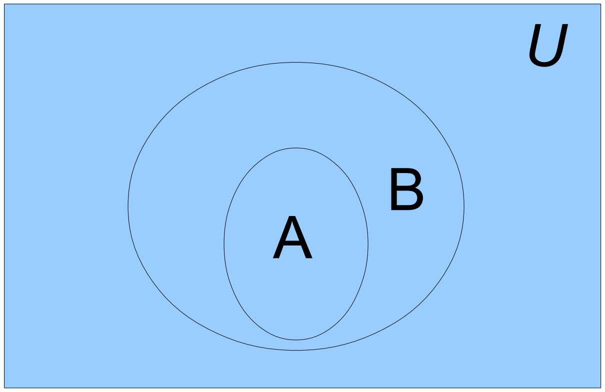
# 2.1 Sets, power sets. Cartesian Products.

Empty set is a set that has no elements; denotation: $\varnothing$

A is a subset of B iff every element of A is also an element of B.

denotation: $A \subseteq B$ or $B \supseteq A$

**Example**:
Let A={1,2,3} and B={0,1,2,5,3}.
Then $A \subseteq B$.




Venn diagram for $A \subseteq B$

# 2.1 Sets, power sets. Cartesian Products.

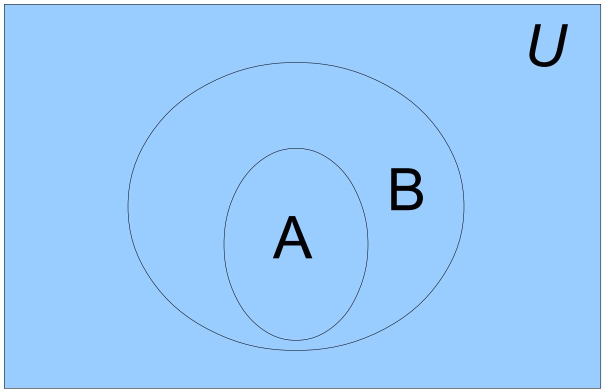
A is a proper subset of B iff A is a subset of B and set A and B are not equal.

denotation: $A \subset B$ or $B \supset A$

**Examples**:

**1)** Let A={1,2,3} and B={0,1,2,5,3}.
Then $A \subset B$.

**2)** Let A={1,2,3} and B={1,2,3}
Then $A \subseteq B$, but $A \not\subset B$




Venn diagram for $A \subseteq B$

Let S be a set. If there are exactly $n$ distinct elements in S, where $n$ is a nonnegative integer, we say that S is a finite set and that $n$ is the cardinality of S.

Denotation: $|S|$

**Examples**: find the cardinality of the given sets

**a)** The set of even positive integers less than 8.

**b)** The set of letters in English alphabet.

**c)** $\emptyset$

## Cartesian Products

The order of elements in the set is often important. Sets are unordered. Ordered n-tuples provide ordered collection.

An ordered n-tuple $(a_1, a_2, ..., a_n)$ is the ordered collection of the elements, where $a_1$ is the first element, $a_2$ is the second elements,..., and $a_n$ is the last, nth element.

n-tuples are equal iff each corresponding pair of their elements is equal, i.e. $(a_1, a_2, ..., a_n)=(b_1, b_2, ..., b_n)$ *iff* $a_i=b_i$, for $i=1,2, ..., n$

**Examples**:
1) (1,2,3,4)
2) (a,b)
3) (1,4,2,3)
4) (b,a)

## Cartesian Products

Cartesian product of two sets, A and B, is the set of all ordered pairs (a,b), where $a \in A$ and $b \in B$

<u>denotation</u>: $A \times B$

$A \times B = \{ (a,b) \mid a \in A \wedge b \in B\}$

**Example**: Find the Cartesian products $A \times B$ of the given sets
**a)** A = { 10, 23 }, B = { a, b, c }

**b)** A = { 1, 2, 3, 4}, B = { a, b }

Is $A \times B = B \times A$ for any two sets A and B?

## Cartesian Products

Cartesian product of the sets $A_1, A_2, ..., A_n$ is the set of all ordered n-tuples $(a_1, a_2, ..., a_n)$, where $a_i \in A_i$ for $i = 1, 2, ..., n$

<u>denotation</u>: $A_1 \times A_2 \times A_3 \times ... \times A_n$

$A_1 \times A_2 \times A_3 \times ... \times A_n = \{ (a_1, a_2, ..., a_n) \mid a_i \in A_i, \text{ for } i = 1, 2, ..., n \}$

**Example**: Find the Cartesian product $A \times B \times C$, where
A = { a, b, c }, B= { 0, 1 }, and C = { 2, 3 }.