Matrix addition and subtraction

In the same way that we can add and subtract real numbers, we can also add and subtract matrices. But matrices must have the same dimensions in order for us to be able to add or subtract them.

Dimensions must match

For instance, given a 2×3 matrix, you can only add it to another 2×3 matrix or subtract it from another 2×3 matrix. You couldn't add a 2×3 matrix to a 2×2 matrix, and you couldn't subtract a 3×3 matrix from a 2×4 matrix.

To add matrices, you simply add together entries from corresponding positions in each matrix. For instance, to add 2×2 matrices, you follow this pattern:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

For example,

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Or to add 2×4 matrices, you follow this pattern:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix} + \begin{bmatrix} 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1+9 & 2+10 & 3+11 & 4+12 \\ 5+13 & 6+14 & 7+15 & 8+16 \end{bmatrix} = \begin{bmatrix} 10 & 12 & 14 & 16 \\ 18 & 20 & 22 & 24 \end{bmatrix}$$

Subtracting matrices works the same way. You simply subtract corresponding entries. For instance, to subtract 2×2 matrices, you follow this pattern:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

For example,

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

Matrices can also be part of an equation. For instance, in the same way that $x+3=2$ gets solved as

$$x+3=2$$

$$x=2-3$$

$$x=-1$$

we can also solve an equation that contains matrices, like this:

$$X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$X = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 6-1 & 8-2 \\ 10-3 & 12-4 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

With the simple linear equation $x + 3 = 2$, we subtracted 3 from both sides to get x by itself, and then simplified $2 - 3$ on the right to find a value for x of $x = -1$. And we really did the same thing with the matrix equation. We subtracted the

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

matrix from both sides to get X by itself, and then simplified the difference of the matrices on the right to find a matrix value for X.

Let's do an example with matrix addition and subtraction.

Example

Solve for B.

$$\begin{bmatrix} 5 & -7 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ -1 & -6 \end{bmatrix} = B + \begin{bmatrix} 1 & 0 \\ 17 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 11 & -9 \end{bmatrix}$$

Let's start with the matrix addition on the left side of the equation.

$$\begin{bmatrix} 5+3 & -7+(-4) \\ -1+(-1) & 0+(-6) \end{bmatrix} = B + \begin{bmatrix} 1 & 0 \\ 17 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 11 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -11 \\ -2 & -6 \end{bmatrix} = B + \begin{bmatrix} 1 & 0 \\ 17 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 11 & -9 \end{bmatrix}$$

Subtract matrices on the right.

$$\begin{bmatrix} 8 & -11 \\ -2 & -6 \end{bmatrix} = B + \begin{bmatrix} 1-2 & 0-4 \\ 17-11 & 0-(-9) \end{bmatrix}$$

$$\begin{bmatrix} 8 & -11 \\ -2 & -6 \end{bmatrix} = B + \begin{bmatrix} -1 & -4 \\ 6 & 9 \end{bmatrix}$$

To isolate B, we'll subtract the matrix on the right from both sides in order to move it to the left.

$$\begin{bmatrix} 8 & -11 \\ -2 & -6 \end{bmatrix} - \begin{bmatrix} -1 & -4 \\ 6 & 9 \end{bmatrix} = B$$

$$\begin{bmatrix} 8-(-1) & -11-(-4) \\ -2-6 & -6-9 \end{bmatrix} = B$$

$$\begin{bmatrix} 9 & -7 \\ -8 & -15 \end{bmatrix} = B$$

The conclusion is that the value of B that makes the equation true is this matrix:

$$B = \begin{bmatrix} 9 & -7 \\ -8 & -15 \end{bmatrix}$$

Properties of matrix addition and subtraction

When it comes to addition and subtraction, matrices follow the same rules as real numbers.

Addition

Matrix addition **is commutative and associative**. The fact that it's commutative means that you can add two matrices together in either order, and still get the same answer.

$$A + B = B + A$$

The fact that matrix addition is associative means that you can group the addition in different ways (move the parentheses), and still get the same answer.

$$(A + B) + C = A + (B + C)$$

Subtraction

Matrix subtraction **is not commutative**, and it **is not associative**. The fact that it's not commutative means that you won't get the same result if you subtract matrices in different orders.

$$A - B \neq B - A$$

The fact that matrix subtraction is not associative means that you can't group the subtraction in different ways (move the parentheses) and still get the same answer.

$$(A - B) - C \neq A - (B - C)$$