1.1 Propositional Logic 1

proposition – is a sentence that declares a fact, i.e. *declarative statement*, that is either *true* or *false*, but not both.

<u>examples</u>:

"It is raining now"

$1+6 = 10$

Washington, D.C. is the capital of United States of America

<u>not propositions</u>:

"What time is it now?"

$x+5=10$

"Please, stand clear of the closing doors."

Do you see the difference between the two sets of examples?
Why the last three sentences are not propositions?

1.1 Propositional Logic 2

proposition – is a sentence that declares a fact, i.e. *declarative statement*, that is either *true* or *false*, but not both.

we use letters to denote propositional variables (i.e. variables that represent propositions): *p, q, r, s*

examples:
p : 1+7=10
q : “It is sunny outside”

If a proposition is true, its value can be denoted by T or True or 1
If a proposition is false, its value can be denoted by F or ⊥ (bottom) or False or 0

The area of logic that deals with propositions is called the *propositional logic* or *propositional calculus*.

It was first developed by the Greek philosopher Aristotle.

Can we build/construct new propositions?

1.1 Propositional Logic 3

New propositions can be constructed from existing propositions using *logical operators*, and are called compound propositions.

logical operators:

			other denotations:	meaning:
¬	negation	¬p	$\overline{p}$, not p	“not p”
∧	conjunction	p ∧ q	p and q	“p and q”
∨	disjunction	p ∨ q	p or q	“p or q”

example 1:
Let proposition *p* stand for “I will go to a movie theater”, then ¬*p* means “I will not go to a movie theater.”

Truth table for negation operation:

p	¬p
T	F
F	T

or

p	¬p
1	0
0	1

example 2:

Let proposition p stand for "It is raining" and q stand for "I want to go to a movie theater", then $p \wedge q$ means "It is raining and I want to go to a movie theater."

Conjunction $p \wedge q$ is true when both p and q are true. Truth table for $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

or

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

example 3:

Let proposition p stand for "It is raining" and q stand for "I want to go to a movie theater", then $p \vee q$ means "It is raining or I want to go to a movie theater."

Disjunction $p \vee q$ is true when at least one of p and q is true.

Truth table for $p \vee q$:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

or

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

more logical operators:

$\rightarrow$	implication	$p \rightarrow q$	" if p then q", "p implies q" *see page 6 for more*
$\oplus$	exclusive or	$p \oplus q$	"either p or q"

Implication:

$p \rightarrow q$

hypothesis, or antecedent, or premise

conclusion, or consequence

if p is true then q holds

Implication is also called conditional statement.

Truth table for the implication $p \rightarrow q$:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

example: Let p: "the weather is good", and q: "we'll go to the beach".

Then $p \rightarrow q$ stands for "If the weather is good we'll go to the beach"

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more logical operators:

$\rightarrow$	implication	$p \rightarrow q$	" if p then q", "p implies q"
$\oplus$	exclusive or	$p \oplus q$	"either p or q"

Exclusive or:

$p \oplus q$ is true when exactly one of p and q is true

Truth table for the exclusive or $p \oplus q$:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

example: Let p: "the weather is good", and q: "we'll go to the beach".

Then $p \oplus q$ stands for "Either the weather is good or we'll go to the beach" even more logical operators:

$\leftrightarrow$ biconditional statement $p \leftrightarrow q$ " p if and only if q", or "p iff q"

Biconditional statement $p \leftrightarrow q$ is true when *p* and *q* have the same truth values, and is false otherwise.

example: Let p: "we'll go to the beach", and q: "the weather is good".

Then $p \leftrightarrow q$ stands for "we'll go to the beach if and only if the weather is good "

Truth table for $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional statements are also called bi-implications.

List of considered logical operators:

¬	negation	¬p	“not p”
∧	conjunction	p ∧ q	“p and q”
∨	disjunction	p ∨ q	“p or q”
→	implication	p → q	“ if p then q”, “p implies q”
+	exclusive or	p + q	“either p or q”
↔	biconditional statement	p ↔ q	“ p if and only if q”

Let's play with English:

Let p and q be propositions; p: “Swimming at the New Jersey shore is prohibited” and q: “Sharks have been spotted near the shore.”

Let's express the following propositions as English sentences.

a) ¬p : Swimming at the New Jersey shore is allowed

b) ¬p ∧ ¬q : Swimming at the New Jersey shore is allowed and Sharks haven't been spotted near the shore

c) p ↔ q : Swimming at the New Jersey shore is prohibited if and only if Sharks have been spotted near the shore.

d) ¬p ∨ (p ∧ q) Swimming at the New Jersey shore is allowed or, it is prohibited and sharks have been spotted near the shore.

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Converse, Contrapositive, and Inverse:

Let's start with an implication (conditional statement) $p \to q$

$q \to p$ is the converse
$\neg q \to \neg p$ is the contrapositive
$\neg p \to \neg q$ is the inverse

! The contrapositive and the original statement have the same truth tables.

Example: what are the contrapositive, the converse and the inverse of the following conditional statement: "If you get 100% on the final, then you will get an A"?

contrapositive: If you won't get an A then you didn't get 100% on the final
converse: If you will get an A then you get 100% on the final
inverse: If you don't get 100% on the final, then you won't get an A

Precedence of Logical Operators:

$\neg$, $\wedge$, $\vee$, $\to$, $\leftrightarrow$

first *last*