Outline

- Representations of integers
    - Decimal
    - Binary
    - Hex
    - Other
- Conversions between representations
    - In General
    - Between Hex and Binary
- Arithmetic in different bases: Addition

Representation of Integers

Properties of decimal notation:
- Number of digits is 10 (it is a "base 10" representation).
- The digits are: 0,1,2,3,4,5,6,7,8,9
- Starting from right the positions represent: 10^0, 10^1, 10^2, 10^3, etc.

Some other notations: binary, HEX (hexadecimal), and octal.

Base b representation:
- The number of digits is b
- The digits are 0,1, …, "b-1"
- Starting from right the positions represent: b^0, b^1, b^2, b^3, etc.

Conversion: Base b to Base 10:
Just carry out definition.

[**Theorem behind base b numbers**]

Let $b \in \mathbf{Z}^+$ and $b > 1$. Then if n is a positive integer, it can be expressed uniquely in the form $n = a_k b^k + a_{k-1} b^{k-1} + \ldots + a_2 b^2 + a_1 b^1 + a_0$,
where k is a nonnegative integer, $a_0, a_1, \ldots, a_k$ are nonnegative integers less than b, and $a_k \neq 0$.

More informally: *For any base b, the base b numbers can represent all numbers, and if representations look different, they are different.*

A proof of this theorem can be constructed using mathematical induction.

Binary Expansion

Choosing the base *b* to be *2* gives binary expansions of integers.

digits: *0, 1*
binary expansions are used by computers to represent integers and do arithmetic with them.

Example: What is the decimal form (expansion) of the integer that has $(10\ 1101\ 1001)_2$ as its binary expansion?

$(10\ 1101\ 1001)_2$

$$= 1 + 0 \cdot 2 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 1 \cdot 2^6 + 1 \cdot 2^7 + 0 \cdot 2^8 + 1 \cdot 2^9$$
$$= 1 + 8 + 16 + 64 + 128 + 512$$
$$= 729_{10}$$

Answer: $(10\ 1101\ 1001)_2 = 729_{10}$

Hexadecimal Expansion (HEX)

Choosing the base b to be 16 gives hexadecimal expansions of integers.

digits: *0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F*
10 11 12 13 14 15

Example: What is the decimal expansion of the hexadecimal expansion of $(3DF0B)_{16}$?

Solution:

$(3DF0B)_{16}$

$= 11 + 0 \cdot 16 + 15 \cdot 16^2 + 13 \cdot 16^3 + 3 \cdot 16^4$

$= 11 + 3840 + 53248 + 196608$

$= 253707_{10}$

Conversion: Decimal to Another Base

Preliminary experiment: Successively divide a decimal number by 10

To convert N from Decimal to Base b:

- Divide N by b and record the remainder
- Make the quotient the "*new N*" and repeat the division by b
- Stop when the quotient = 0

Example 3: What is the hexadecimal expansion of the decimal number 4678?

<u>Solution</u>: *divide by 16, set aside the remainder; the quotient of division divide by 16 and set aside the remainder, and so on, till the quotient is 1 (or 0).*

$4678 \div 16 = 292\ R\ 6$

$292 \div 16 = 18\ R\ 4$

$18 \div 16 = 1\ R\ 2$

$1 \div 16 = 0\ R\ 1$

Now, starting from the end (from the last quotient): $(1\ 2\ 4\ 6)_{16}$

<u>Answer</u>: $(4678)_{10} = (1246)_{16}$

Hexadecimal Expansion (HEX)

Table1 on page 222, for speeding up HEX/Binary conversions.

TABLE 1 Hexadecimal, Octal, and Binary Representation of the Integers 0 through 15.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

Binary to HEX: Group the binary digits into groups of four (from the right), and add initial zeros at the start (if needed). Replace each group of 4 by its HEX digit.

HEX to Binary: Replace each HEX digit by the corresponding length 4 binary string.

Note: *Don't ignore zeros!*

Addition for Base b numbers

Could convert to decimal, do arithmetic, then convert back.

Or can think within that base, carrying out the steps analogous to base 10