MATH 313: LINEAR ALGEBRA
HOMEWORK 2

MELVYN B. NATHANSON

Use Gaussian elimination to solve the following systems of linear equations. Write the sets of solutions as affine subspaces.

(1)
$$\begin{aligned} 2x + y + z &= 0 \\ x + 2y + 3z &= 0. \end{aligned}$$

(2)
$$\begin{aligned} 2x + y + z &= -1 \\ x + 2y - z &= 5 \end{aligned}$$

(3)
$$\begin{aligned} 2x + y + z &= 4 \\ x + 2y - z &= -10 \end{aligned}$$

(4)
$$\begin{aligned} 3x - y + z + 2w &= 12 \\ -2x + 3y + 7z - 10w &= -15 \end{aligned}$$

(5)
$$\begin{aligned} x + 2y + 3z &= 0 \\ 4x + 5y + 6z &= 0 \\ 7x + 8y + 9z &= 0 \end{aligned}$$

(6)
$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 1 \\ x + 4y + 9z &= 2 \end{aligned}$$

(7)
$$\begin{aligned} x + 2y + 3z &= 7 \\ 3x - y - z &= 3 \end{aligned}$$

(8)
$$\begin{aligned} x + 2y &= 3 \\ 4x + 5y &= 6 \\ 7x + 8y &= 9 \end{aligned}$$

Date: February 8, 2019.

(9)
$$\begin{aligned} x + 4y + 3z &= 2 \\ 2x + y + z &= 3 \\ 3x - 2y - z &= 4 \end{aligned}$$

(10)
$$\begin{aligned} x + 4y + 3z &= 1 \\ 2x + y + z &= 3 \\ 3x - 2y - z &= 4 \end{aligned}$$

(11)
$$\begin{aligned} x + 2y + 3z &= 10 \\ 2x - 3y + 7z &= -9 \end{aligned}$$

(12)
$$\begin{aligned} x + 5y - z + 2w &= 17 \\ 3x + y + 5z - 4w &= -7 \end{aligned}$$

(13)
$$\begin{aligned} 5x + 15y + 13z - 6w &= 0 \\ 2x + 6y + 1z + 6w &= 0 \\ x + 3y + 2z &= 0 \end{aligned}$$

(14) Prove directly that a homogeneous system of 2 linear equations in 3 variables has a nonzero solution.
(15) Prove that if an inhomogeneous system of 2 linear equations in 3 variables has one solution, then it has at least two solutions.
(16) Let $m > n$. Prove that a homogeneous system of m linear equations in n variables has infinitely many solutions.