CMP 334 (2/27/19)

Quiz 2 (truth tables, formulas, circuits)

Signed binary arithmetic (review)

Ripple-carry adders

ALU building block circuits:

- Inverters, decoders, multiplexers

Condition flags and comparisons

- Comparisons and conditional branches

TOY assembly language

- HW 7 (`W ← X + Y + Z`)
- Relative conditional branch op: **`bc`**

Quiz 2 #1 solution

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Quiz 2 #1 solution

a	b	c	f	
0	0	0	0	$\overline{abc}$
0	0	1	0	$\overline{ab}c$
0	1	0	0	$\overline{a}b\overline{c}$
0	1	1	1	**$\overline{a}bc$**
1	0	0	1	**$a\overline{bc}$**
1	0	1	0	$a\overline{b}c$
1	1	0	0	$ab\overline{c}$
1	1	1	1	**abc**

Quiz 2 #1 solution

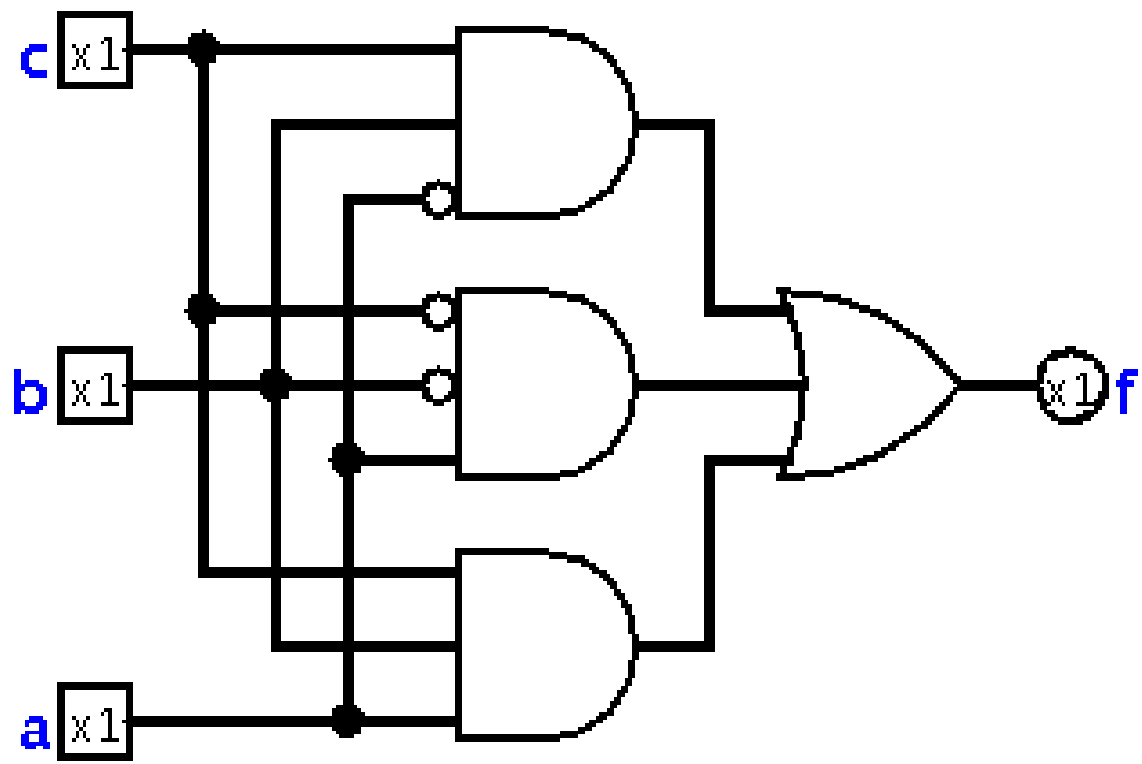
a	**b**	**c**	**f**	
0	0	0	0	$\overline{abc}$
0	0	1	0	$\overline{ab}c$
0	1	0	0	$\overline{a}b\overline{c}$
0	1	1	1	$\mathbf{\overline{a}bc}$
1	0	0	1	$\mathbf{a\overline{bc}}$
1	0	1	0	$a\overline{b}c$
1	1	0	0	$ab\overline{c}$
1	1	1	1	$\mathbf{abc}$

$$\mathbf{f} = \overline{a}bc + a\overline{bc} + abc$$

Quiz 2 #1 solution

a	b	c	f	
0	0	0	0	$\overline{a}\overline{b}\overline{c}$
0	0	1	0	$\overline{a}\overline{b}c$
0	1	0	0	$\overline{a}b\overline{c}$
0	1	1	1	**$\overline{a}bc$**
1	0	0	1	**$a\overline{b}\overline{c}$**
1	0	1	0	$a\overline{b}c$
1	1	0	0	$ab\overline{c}$
1	1	1	1	**abc**

$$\mathbf{f} = \overline{a}bc + a\overline{b}\overline{c} + abc$$



Quiz 2 #2 solution

n_3	n_2	n_1	n_0
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Quiz 2 #2 solution

n_3	n_2	n_1	n_0	**n**
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**2**
0	0	1	1	**3**
0	1	0	0	**4**
0	1	0	1	**5**
0	1	1	0	**6**
0	1	1	1	**7**
1	0	0	0	**8**
1	0	0	1	**9**
1	0	1	0	**A**
1	0	1	1	**B**
1	1	0	0	**C**
1	1	0	1	**D**
1	1	1	0	**E**
1	1	1	1	**F**

Quiz 2 #2 solution

n_3	n_2	n_1	n_0	n	n//5
0	0	0	0	**0**	0
0	0	0	1	**1**	0
0	0	1	0	**2**	0
0	0	1	1	**3**	0
0	1	0	0	**4**	0
0	1	0	1	**5**	1
0	1	1	0	**6**	1
0	1	1	1	**7**	1
1	0	0	0	**8**	1
1	0	0	1	**9**	1
1	0	1	0	**A**	2
1	0	1	1	**B**	2
1	1	0	0	**C**	2
1	1	0	1	**D**	2
1	1	1	0	**E**	2
1	1	1	1	**F**	3

Quiz 2 #2 solution

n_3	n_2	n_1	n_0	**n**	**n//5**	**n%5**
0	0	0	0	**0**	0	0
0	0	0	1	**1**	0	1
0	0	1	0	**2**	0	2
0	0	1	1	**3**	0	3
0	1	0	0	**4**	0	4
0	1	0	1	**5**	1	0
0	1	1	0	**6**	1	1
0	1	1	1	**7**	1	2
1	0	0	0	**8**	1	3
1	0	0	1	**9**	1	4
1	0	1	0	**A**	2	0
1	0	1	1	**B**	2	1
1	1	0	0	**C**	2	2
1	1	0	1	**D**	2	3
1	1	1	0	**E**	2	4
1	1	1	1	**F**	3	0

Quiz 2 #2 solution

n_3	n_2	n_1	n_0	**n**	**n//5**	**n**%5	**n%**5=4
0	0	0	0	**0**	0	0	0
0	0	0	1	**1**	0	1	0
0	0	1	0	**2**	0	2	0
0	0	1	1	**3**	0	3	0
0	1	0	0	**4**	0	4	**1**
0	1	0	1	**5**	1	0	0
0	1	1	0	**6**	1	1	0
0	1	1	1	**7**	1	2	0
1	0	0	0	**8**	1	3	0
1	0	0	1	**9**	1	4	**1**
1	0	1	0	**A**	2	0	0
1	0	1	1	**B**	2	1	0
1	1	0	0	**C**	2	2	0
1	1	0	1	**D**	2	3	0
1	1	1	0	**E**	2	4	**1**
1	1	1	1	**F**	3	0	0

Quiz 2 #2 solution

n_3	n_2	n_1	n_0	n	n//5	n%5	n%5=4	
0	0	0	0	**0**	0	0	0	$\overline{n}_3\overline{n}_2\overline{n}_1\overline{n}_0$
0	0	0	1	**1**	0	1	0	$\overline{n}_3\overline{n}_2\overline{n}_1 n_0$
0	0	1	0	**2**	0	2	0	$\overline{n}_3\overline{n}_2 n_1\overline{n}_0$
0	0	1	1	**3**	0	3	0	$\overline{n}_3\overline{n}_2 n_1 n_0$
0	1	0	0	**4**	0	4	**1**	$\overline{n}_3 n_2\overline{n}_1\overline{n}_0$
0	1	0	1	**5**	1	0	0	$\overline{n}_3 n_2\overline{n}_1 n_0$
0	1	1	0	**6**	1	1	0	$\overline{n}_3 n_2 n_1\overline{n}_0$
0	1	1	1	**7**	1	2	0	$\overline{n}_3 n_2 n_1 n_0$
1	0	0	0	**8**	1	3	0	$n_3\overline{n}_2\overline{n}_1\overline{n}_0$
1	0	0	1	**9**	1	4	**1**	$n_3\overline{n}_2\overline{n}_1 n_0$
1	0	1	0	**A**	2	0	0	$n_3\overline{n}_2 n_1\overline{n}_0$
1	0	1	1	**B**	2	1	0	$n_3\overline{n}_2 n_1 n_0$
1	1	0	0	**C**	2	2	0	$n_3 n_2\overline{n}_1\overline{n}_0$
1	1	0	1	**D**	2	3	0	$n_3 n_2\overline{n}_1 n_0$
1	1	1	0	**E**	2	4	**1**	$n_3 n_2 n_1\overline{n}_0$
1	1	1	1	**F**	3	0	0	$n_3 n_2 n_1 n_0$

Quiz 2 #2 solution

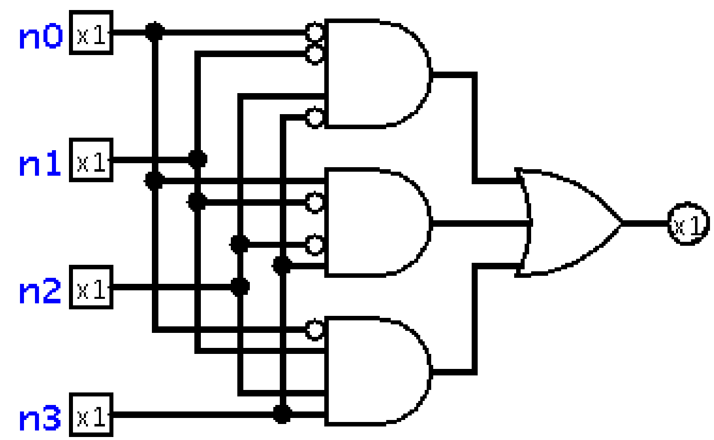
$$f = \overline{n}_3 n_2 \overline{n}_1 \overline{n}_0 + n_3 \overline{n}_2 \overline{n}_1 n_0 + n_3 n_2 n_1 \overline{n}_0$$

n_3	n_2	n_1	n_0	**n**	**n//5**	**n%5**	**n%5=4**	
0	0	0	0	**0**	0	0	0	$\overline{n}_3\overline{n}_2\overline{n}_1\overline{n}_0$
0	0	0	1	**1**	0	1	0	$\overline{n}_3\overline{n}_2\overline{n}_1 n_0$
0	0	1	0	**2**	0	2	0	$\overline{n}_3\overline{n}_2 n_1\overline{n}_0$
0	0	1	1	**3**	0	3	0	$\overline{n}_3\overline{n}_2 n_1 n_0$
0	1	0	0	**4**	0	4	**1**	$\overline{n}_3 n_2\overline{n}_1\overline{n}_0$
0	1	0	1	**5**	1	0	0	$\overline{n}_3 n_2\overline{n}_1 n_0$
0	1	1	0	**6**	1	1	0	$\overline{n}_3 n_2 n_1\overline{n}_0$
0	1	1	1	**7**	1	2	0	$\overline{n}_3 n_2 n_1 n_0$
1	0	0	0	**8**	1	3	0	$n_3\overline{n}_2\overline{n}_1\overline{n}_0$
1	0	0	1	**9**	1	4	**1**	$n_3\overline{n}_2\overline{n}_1 n_0$
1	0	1	0	**A**	2	0	0	$n_3\overline{n}_2 n_1\overline{n}_0$
1	0	1	1	**B**	2	1	0	$n_3\overline{n}_2 n_1 n_0$
1	1	0	0	**C**	2	2	0	$n_3 n_2\overline{n}_1\overline{n}_0$
1	1	0	1	**D**	2	3	0	$n_3 n_2\overline{n}_1 n_0$
1	1	1	0	**E**	2	4	**1**	$n_3 n_2 n_1\overline{n}_0$
1	1	1	1	**F**	3	0	0	$n_3 n_2 n_1 n_0$

Quiz 2 #2 solution

n_3	n_2	n_1	n_0	**n**	**n//5**	**n%5**	**n%5=4**	
0	0	0	0	**0**	0	0	0	$\overline{n}_3\overline{n}_2\overline{n}_1\overline{n}_0$
0	0	0	1	**1**	0	1	0	$\overline{n}_3\overline{n}_2\overline{n}_1 n_0$
0	0	1	0	**2**	0	2	0	$\overline{n}_3\overline{n}_2 n_1\overline{n}_0$
0	0	1	1	**3**	0	3	0	$\overline{n}_3\overline{n}_2 n_1 n_0$
0	1	0	0	**4**	0	4	**1**	$\overline{n}_3 n_2\overline{n}_1\overline{n}_0$
0	1	0	1	**5**	1	0	0	$\overline{n}_3 n_2\overline{n}_1 n_0$
0	1	1	0	**6**	1	1	0	$\overline{n}_3 n_2 n_1\overline{n}_0$
0	1	1	1	**7**	1	2	0	$\overline{n}_3 n_2 n_1 n_0$
1	0	0	0	**8**	1	3	0	$n_3\overline{n}_2\overline{n}_1\overline{n}_0$
1	0	0	1	**9**	1	4	**1**	$n_3\overline{n}_2\overline{n}_1 n_0$
1	0	1	0	**A**	2	0	0	$n_3\overline{n}_2 n_1\overline{n}_0$
1	0	1	1	**B**	2	1	0	$n_3\overline{n}_2 n_1 n_0$
1	1	0	0	**C**	2	2	0	$n_3 n_2\overline{n}_1\overline{n}_0$
1	1	0	1	**D**	2	3	0	$n_3 n_2\overline{n}_1 n_0$
1	1	1	0	**E**	2	4	**1**	$n_3 n_2 n_1\overline{n}_0$
1	1	1	1	**F**	3	0	0	$n_3 n_2 n_1 n_0$

$$f = \overline{n}_3 n_2 \overline{n}_1 \overline{n}_0 + n_3 \overline{n}_2 \overline{n}_1 n_0 + n_3 n_2 n_1 \overline{n}_0$$



Quiz 2 take aways

n_3	n_2	n_1	n_0
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Quiz 2 take aways

n_3	n_2	n_1	n_0	$n_3n_2n_1n_0$
0	0	0	0	$\overline{n}_3\overline{n}_2\overline{n}_1\overline{n}_0$
0	0	0	1	$\overline{n}_3\overline{n}_2\overline{n}_1n_0$
0	0	1	0	$\overline{n}_3\overline{n}_2n_1\overline{n}_0$
0	0	1	1	$\overline{n}_3\overline{n}_2n_1n_0$
0	1	0	0	$\overline{n}_3n_2\overline{n}_1\overline{n}_0$
0	1	0	1	$\overline{n}_3n_2\overline{n}_1n_0$
0	1	1	0	$\overline{n}_3n_2n_1\overline{n}_0$
0	1	1	1	$\overline{n}_3n_2n_1n_0$
1	0	0	0	$n_3\overline{n}_2\overline{n}_1\overline{n}_0$
1	0	0	1	$n_3\overline{n}_2\overline{n}_1n_0$
1	0	1	0	$n_3\overline{n}_2n_1\overline{n}_0$
1	0	1	1	$n_3\overline{n}_2n_1n_0$
1	1	0	0	$n_3n_2\overline{n}_1\overline{n}_0$
1	1	0	1	$n_3n_2\overline{n}_1n_0$
1	1	1	0	$n_3n_2n_1\overline{n}_0$
1	1	1	1	$n_3n_2n_1n_0$

Quiz 2 take aways

n_3	n_2	n_1	n_0	$n_3n_2n_1n_0$	***n***
0	0	0	0	$\bar{n}_3\bar{n}_2\bar{n}_1\bar{n}_0$	**0**
0	0	0	1	$\bar{n}_3\bar{n}_2\bar{n}_1n_0$	**1**
0	0	1	0	$\bar{n}_3\bar{n}_2n_1\bar{n}_0$	**2**
0	0	1	1	$\bar{n}_3\bar{n}_2n_1n_0$	**3**
0	1	0	0	$\bar{n}_3n_2\bar{n}_1\bar{n}_0$	**4**
0	1	0	1	$\bar{n}_3n_2\bar{n}_1n_0$	**5**
0	1	1	0	$\bar{n}_3n_2n_1\bar{n}_0$	**6**
0	1	1	1	$\bar{n}_3n_2n_1n_0$	**7**
1	0	0	0	$n_3\bar{n}_2\bar{n}_1\bar{n}_0$	**8**
1	0	0	1	$n_3\bar{n}_2\bar{n}_1n_0$	**9**
1	0	1	0	$n_3\bar{n}_2n_1\bar{n}_0$	**A**
1	0	1	1	$n_3\bar{n}_2n_1n_0$	**B**
1	1	0	0	$n_3n_2\bar{n}_1\bar{n}_0$	**C**
1	1	0	1	$n_3n_2\bar{n}_1n_0$	**D**
1	1	1	0	$n_3n_2n_1\bar{n}_0$	**E**
1	1	1	1	$n_3n_2n_1n_0$	**F**

Quiz 2 take aways

n_3	n_2	n_1	n_0	$n_3n_2n_1n_0$	***n***
0	0	0	0	$\overline{n_3}\,\overline{n_2}\,\overline{n_1}\,\overline{n_0}$	**0**
0	0	0	1	$\overline{n_3}\,\overline{n_2}\,\overline{n_1}\,n_0$	**1**
0	0	1	0	$\overline{n_3}\,\overline{n_2}\,n_1\overline{n_0}$	**2**
0	0	1	1	$\overline{n_3}\,\overline{n_2}\,n_1n_0$	**3**
0	1	0	0	$\overline{n_3}\,n_2\overline{n_1}\,\overline{n_0}$	**4**
0	1	0	1	$\overline{n_3}\,n_2\overline{n_1}\,n_0$	**5**
0	1	1	0	$\overline{n_3}\,n_2n_1\overline{n_0}$	**6**
0	1	1	1	$\overline{n_3}\,n_2n_1n_0$	**7**
1	0	0	0	$n_3\overline{n_2}\,\overline{n_1}\,\overline{n_0}$	**8**
1	0	0	1	$n_3\overline{n_2}\,\overline{n_1}\,n_0$	**9**
1	0	1	0	$n_3\overline{n_2}\,n_1\overline{n_0}$	**A**
1	0	1	1	$n_3\overline{n_2}\,n_1n_0$	**B**
1	1	0	0	$n_3n_2\overline{n_1}\,\overline{n_0}$	**C**
1	1	0	1	$n_3n_2\overline{n_1}\,n_0$	**D**
1	1	1	0	$n_3n_2n_1\overline{n_0}$	**E**
1	1	1	1	$n_3n_2n_1n_0$	**F**

$$n = n_3 \cdot 2^3 + n_2 \cdot 2^2 + n_1 \cdot 2^1 + n_0 \cdot 2^0$$

Quiz 2 take aways

n_3	n_2	n_1	n_0	$n_3n_2n_1n_0$	***n***	***s***
0	0	0	0	$\overline{n}_3\overline{n}_2\overline{n}_1\overline{n}_0$	**0**	**0**
0	0	0	1	$\overline{n}_3\overline{n}_2\overline{n}_1n_0$	**1**	**1**
0	0	1	0	$\overline{n}_3\overline{n}_2n_1\overline{n}_0$	**2**	**2**
0	0	1	1	$\overline{n}_3\overline{n}_2n_1n_0$	**3**	**3**
0	1	0	0	$\overline{n}_3n_2\overline{n}_1\overline{n}_0$	**4**	**4**
0	1	0	1	$\overline{n}_3n_2\overline{n}_1n_0$	**5**	**5**
0	1	1	0	$\overline{n}_3n_2n_1\overline{n}_0$	**6**	**6**
0	1	1	1	$\overline{n}_3n_2n_1n_0$	**7**	**7**
1	0	0	0	$n_3\overline{n}_2\overline{n}_1\overline{n}_0$	**8**	**–8**
1	0	0	1	$n_3\overline{n}_2\overline{n}_1n_0$	**9**	**–7**
1	0	1	0	$n_3\overline{n}_2n_1\overline{n}_0$	**A**	**–6**
1	0	1	1	$n_3\overline{n}_2n_1n_0$	**B**	**–5**
1	1	0	0	$n_3n_2\overline{n}_1\overline{n}_0$	**C**	**–4**
1	1	0	1	$n_3n_2\overline{n}_1n_0$	**D**	**–3**
1	1	1	0	$n_3n_2n_1\overline{n}_0$	**E**	**–2**
1	1	1	1	$n_3n_2n_1n_0$	**F**	**–1**

$$\boldsymbol{n} = \mathbf{n}_3 \cdot 2^3 + \mathbf{n}_2 \cdot 2^2 + \mathbf{n}_1 \cdot 2^1 + \mathbf{n}_0 \cdot 2^0$$

Quiz 2 take aways

n_3	n_2	n_1	n_0	$n_3n_2n_1n_0$	$\boldsymbol{n}$	$\boldsymbol{s}$
0	0	0	0	$\bar{n}_3\bar{n}_2\bar{n}_1\bar{n}_0$	**0**	**0**
0	0	0	1	$\bar{n}_3\bar{n}_2\bar{n}_1n_0$	**1**	**1**
0	0	1	0	$\bar{n}_3\bar{n}_2n_1\bar{n}_0$	**2**	**2**
0	0	1	1	$\bar{n}_3\bar{n}_2n_1n_0$	**3**	**3**
0	1	0	0	$\bar{n}_3n_2\bar{n}_1\bar{n}_0$	**4**	**4**
0	1	0	1	$\bar{n}_3n_2\bar{n}_1n_0$	**5**	**5**
0	1	1	0	$\bar{n}_3n_2n_1\bar{n}_0$	**6**	**6**
0	1	1	1	$\bar{n}_3n_2n_1n_0$	**7**	**7**
1	0	0	0	$n_3\bar{n}_2\bar{n}_1\bar{n}_0$	**8**	**–8**
1	0	0	1	$n_3\bar{n}_2\bar{n}_1n_0$	**9**	**–7**
1	0	1	0	$n_3\bar{n}_2n_1\bar{n}_0$	**A**	**–6**
1	0	1	1	$n_3\bar{n}_2n_1n_0$	**B**	**–5**
1	1	0	0	$n_3n_2\bar{n}_1\bar{n}_0$	**C**	**–4**
1	1	0	1	$n_3n_2\bar{n}_1n_0$	**D**	**–3**
1	1	1	0	$n_3n_2n_1\bar{n}_0$	**E**	**–2**
1	1	1	1	$n_3n_2n_1n_0$	**F**	**–1**

$$\boldsymbol{n} = n_3 \cdot 2^3 + n_2 \cdot 2^2 + n_1 \cdot 2^1 + n_0 \cdot 2^0$$

$$\boldsymbol{s} = -n_3 \cdot 2^3 + n_2 \cdot 2^2 + n_1 \cdot 2^1 + n_0 \cdot 2^0$$

HW 8: Two's Complement Circuit

Use the four step *Combinational Circuit Design Process* presented in class to design circuits that take a 3-bit unsigned binary integer **X** as input and produces as output a 3-bit unsigned binary integer **Y** that is the two's complement of **X**.

$$\mathbf{Y} = \mathbf{2}^{3} - \mathbf{X} = \overline{\mathbf{X}} + \mathbf{1}$$

Do not minimize the circuits for **Y** as a part of this assignments.

Extra credit: minimize the circuits for **Y**.

N–Bit Integers

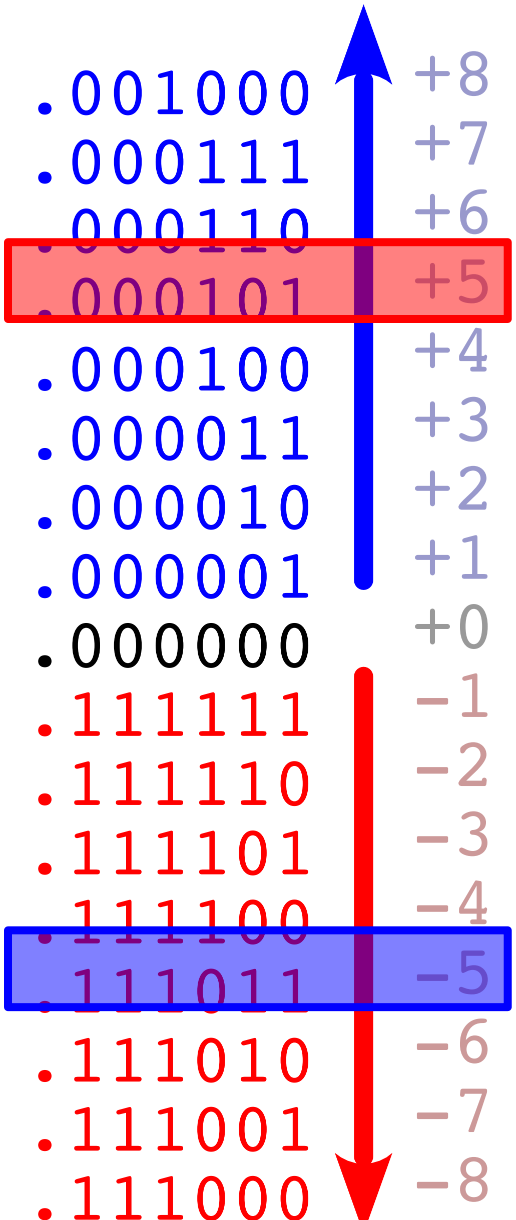
(N = 8)

Unsigned Integers

Signed Integers

. . . . . .	
. . . 0100000000	$= 2^N$
. . . 0011111111	$= 2^N - 1$
. . . . . .	
. . . 0010000000	$= 2^{N-1}$
. . . 0001111111	$= 2^{N-1} - 1$
. . . . . .	
. . . 0000000101	
. . . 0000000100	$= 4 = 2^2$
. . . 0000000011	$= 3 = 2^2 - 1$
. . . 0000000010	$= 2 = 2^1$
. . . 0000000001	$= 1 = 2^0 = 2^1 - 1$
. . . 0000000000	$= 0 = 2^0 - 1$
. . . 1111111111	$= -1 = -2^0$
. . . 1111111110	$= -2 = -2^1$
. . . 1111111101	$= -3 = -2^1 - 1$
. . . 1111111100	$= -4 = -2^2$
. . . 1111111011	$= -5 = -2^2 - 1$
. . . . . .	
. . . 1110000000	$= -2^{N-1}$
. . . 1101111111	$= -2^{N-1} - 1$
. . . . . .	
. . . 1100000000	$= -2^N$
. . . 1011111111	$= -2^N - 1$
. . . . . .	

Additive Inverse



THEROEM:

If **X** and **Y** are signed integers then

$$\mathbf{X - Y = X + \overline{Y} + 1}$$

Corollary:

$$\mathbf{-Y = \overline{Y} + 1}$$

$$-\ .0101 = \overline{.0101} + 1 = .1010 + 1 = .1011$$

$$-\ .0001 = \overline{.0001} + 1 = .1110 + 1 = .1111$$

$$-\ .0000 = \overline{.0000} + 1 = .1111 + 1 = .0000$$

$$-\ .1111 = \overline{.1111} + 1 = .0000 + 1 = .0001$$

$$-\ .1011 = \overline{.1011} + 1 = .0100 + 1 = .0101$$

n-bit Binary Numbers

Unsigned: $b_{n-1}b_{n-2} \dots b_1\ b_0$ $\quad$ ($b_i = 0$ or $b_i = 1$)

value: $b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \dots + b_1 2^1 + b_0 2^0$

range: $[0\ ..\ 2^n - 1]$

n-bit sum: $\mathbf{A} \overset{\cdots}{+} \mathbf{B} = \mathbf{A} + \mathbf{B} - \mathbf{c} \cdot \mathbf{2^n}$

n-bit diff: $\mathbf{A} \overset{\cdots}{-} \mathbf{B} \equiv \mathbf{A} \overset{\cdots}{+} (\mathbf{2}^n - \mathbf{B}) = \mathbf{A} \overset{\cdots}{+} \overline{\mathbf{B}} + 1 = \mathbf{A} - \mathbf{B} + \mathbf{2}^n - \mathbf{c} \cdot \mathbf{2}^n$

Signed: $b_{n-1}b_{n-2} \dots b_1\ b_0$ $\quad$ ($b_i = 0$ or $b_i = 1$)

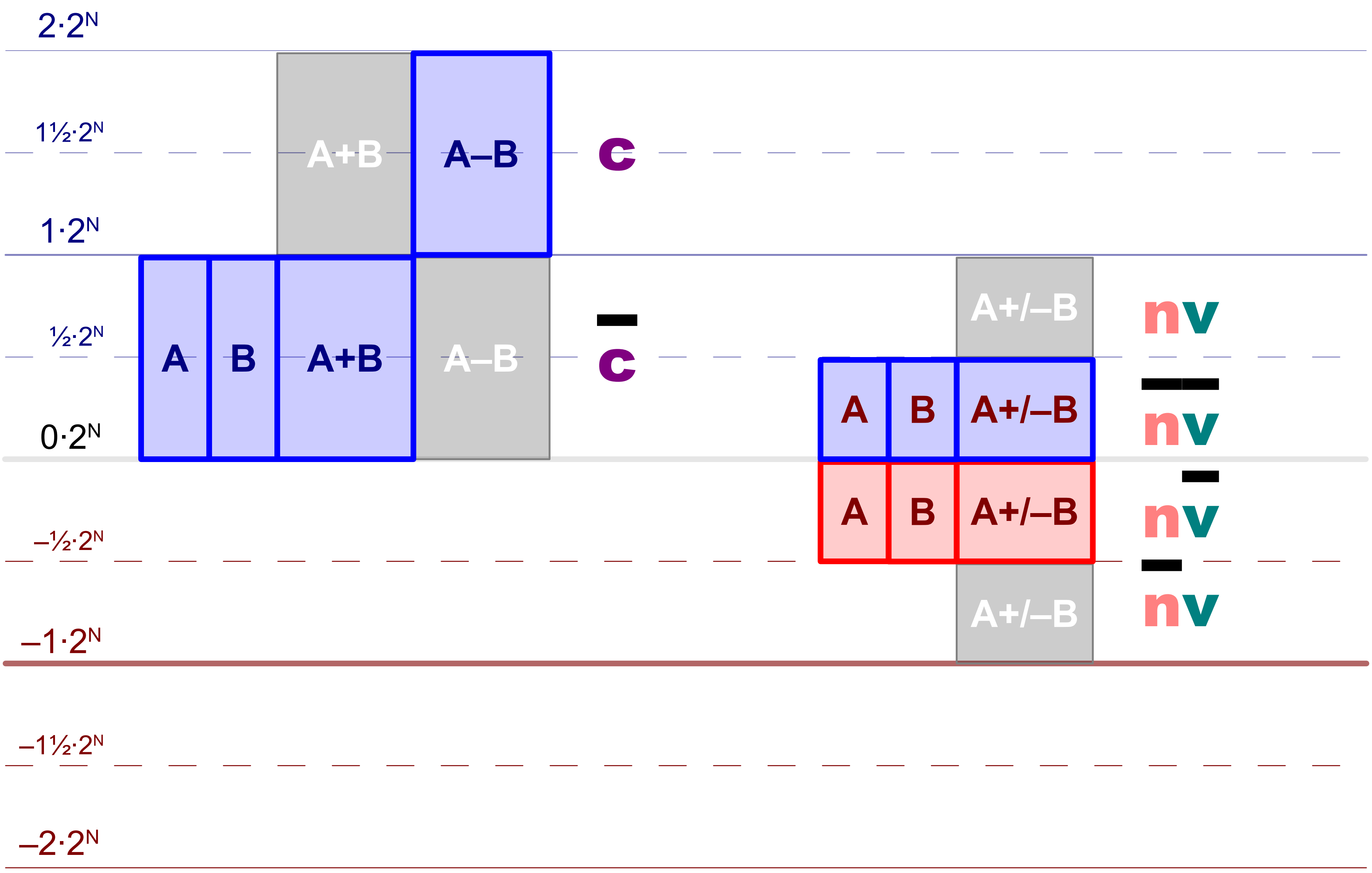
Value: $-b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \dots + b_1 2^1 + b_0 2^0$

Range: $[-2^{n-1}\ ..\ 2^{n-1} - 1]$

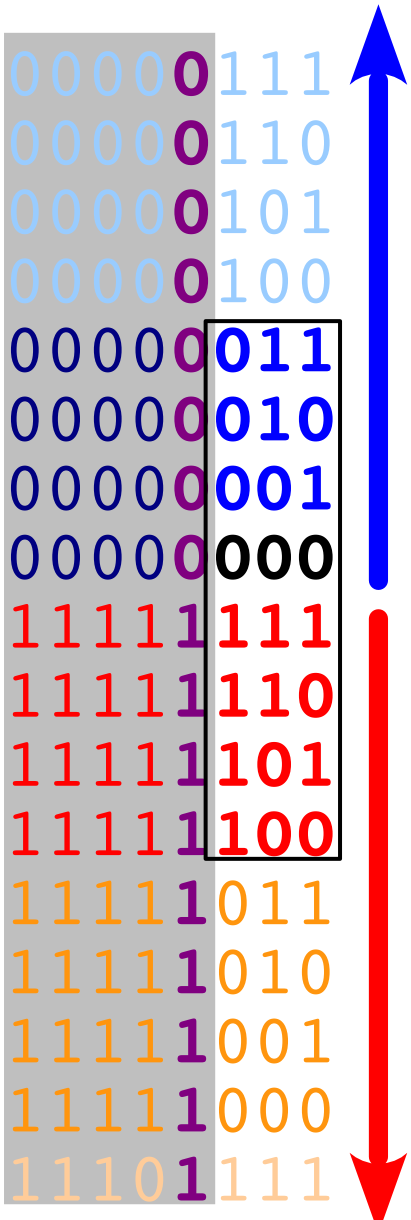
n-bit sum: $\mathbf{A} \overset{\cdots}{+} \mathbf{B} = \mathbf{A} + \mathbf{B}$ **iff** $\mathbf{v} = \mathbf{0}$

n-bit diff: $\mathbf{A} \overset{\cdots}{-} \mathbf{B} = \mathbf{A} - \mathbf{B}$ **iff** $\mathbf{v} = \mathbf{0}$

Condition Flags



3-Bit Signed Binary Integers



8 supported values: -4 .. 3

Sign: high bit: **0** positive, **1** negative

Not closed under addition

incorrect results: –8 .. –5, 4 .. 6

Not closed under subtraction

incorrect results: –7 .. –5, 4 .. 7

Correct: $X \overset{\cdots}{+} Y = Z$, $X \overset{\cdots}{+} Y = Z$

$X \overset{\cdots}{+} Y$, $X \overset{\cdots}{+} Y$ always correct. Why?

Incorrect: $X \overset{\cdots}{+} Y = Z$, $X \overset{\cdots}{+} Y = Z$

$X \overset{\cdots}{-} Y = Z$, $X \overset{\cdots}{-} Y = Z$

Honesty Criteria

The n-bit result **r** of a binary operation on n-bit values **a** and **b** is ***honest*** (***deceptive***) if it is *the same as* (*different from*) the whole number result of the same operation on the same values.

(n-bit) unsigned addition is *honest* iff (**c** = 0)

Carry flag is not set

(n-bit) unsigned subtraction is *honest* iff (**c** = 1)

Carry flag is set

(n-bit) signed addition is *honest* iff (**v** = 0)

a and **b** have different signs or **a**, **b**, and **r** have same sign

(n-bit) signed subtraction is *honest* iff (**v** = 0)

a and $\bar{\mathbf{b}}$ have different signs or **a**, $\bar{\mathbf{b}}$, and **r** have same sign

HW 9: Signed Binary Arithmetic

For each of the <**X**, **Y**> pairs in the table below:

a) Convert **X** and **Y** → binary
b) Compute $X \overset{\cdots}{+} Y$ (the 8-bit **sum**)
c) Compute $\overset{\cdots}{Y}$ (the 2's complement of **Y**)
d) Compute $X \overset{\cdots}{-} Y \equiv X \overset{\cdots}{+} \overset{\cdots}{Y}$ (the 8-bit **difference**)
e) Indicate the signs of **X**, **Y**, $X \overset{\cdots}{+} Y$, $\overset{\cdots}{Y}$, and $X \overset{\cdots}{-} Y$
f) Convert $X \overset{\cdots}{+} Y$, $\overset{\cdots}{Y}$, and $X \overset{\cdots}{-} Y$ → hexadecimal
g) Indicate condition flag (**z**, **n**, **c**, **v**) values for $X \overset{\cdots}{+} Y$, $X \overset{\cdots}{-} Y$
h) Is $X \overset{\cdots}{+} Y$ honest? is $X \overset{\cdots}{-} Y$ honest?

Where <**X**, **Y**> =

1) <`0x`**`4F`**, `0x`**`6D`**>
2) <`0x`**`B3`**, `0x`**`17`**>
3) <`0x`**`A3`**, `0x`**`95`**>
4) <`0x`**`6E`**, `0x`**`3A`**>

Signed Arithmetic: X1 i)

X	Y	X + Y	~Y	X – Y
0x8C	0x6F			
10001100	01101111	10001100	01101111	10001100
		01101111	10010000	10010001
		0 0 0 0 1 1 0 0	00000001	1 0 0 0 0 0 0 0
		011111011	10010001	100011101
		0xFB	0x91	0x1D
		$\overline{z}n\overline{c}\overline{v}$		$\overline{z}\overline{n}cv$
		(s) honest (u) honest		(s) deceptive (u) honest

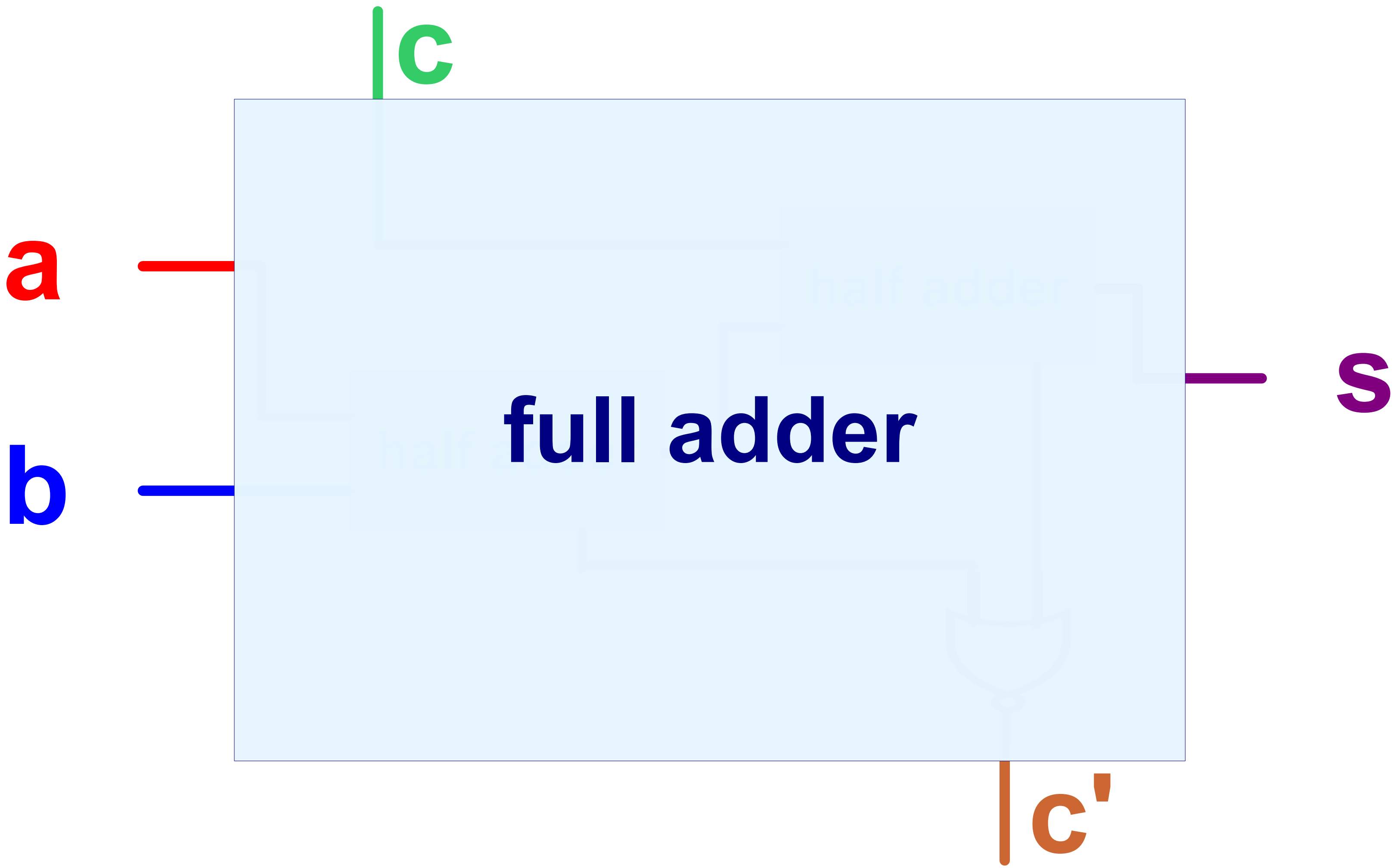
Signed Arithmetic: X2 i)

X	Y	X + Y	~Y	X – Y
0x54	0xF3			
01010100	11110011	01010100 11110011 1 1 1 1 0 0 0 0 101000111	11110011 00001100 00000001 00001101	01010100 00001101 0 0 0 1 1 1 0 0 001100001
		0x47	0x0D	0x61
		$\overline{z}\,\overline{n}\,c\,\overline{v}$ (s) honest (u) deceptive		$\overline{z}\,\overline{n}\,\overline{c}\,\overline{v}$ (s) honest (u) deceptive

Signed Arithmetic: X3 i)

X	Y	X + Y	~Y	X – Y
0x9E	0xCC			
10011110	11001100	10011110 11001100 1 0 0 1 1 1 0 0 101101010	$\overline{11001100}$ 00110011 00000001 00110100	10011110 00110100 0 0 1 1 1 1 0 0 011010010
		0x6A	0x34	0xD2
		$\overline{\text{zn}}$cv		$\overline{\text{z}}$n$\overline{\text{c}}$v
		(s) honest (u) deceptive		(s) honest (u) deceptive

Full Adder Black Box



Sum 3 1-bit inputs to give a 2-bit output

Full Adder Truth Table

$\begin{array}{cc} & 1 \\ + & 1 \\ \hline 1 & 0 \\ & 0 \end{array}$	$\begin{array}{cc} & 1 \\ + & 0 \\ \hline 0 & 0 \\ & 1 \end{array}$	$\begin{array}{cc} & 0 \\ + & 1 \\ \hline 0 & 0 \\ & 1 \end{array}$	$\begin{array}{cc} & 0 \\ + & 0 \\ \hline 0 & 0 \\ & 0 \end{array}$
$\begin{array}{cc} & 1 \\ + & 1 \\ \hline 1 & 1 \\ & 1 \end{array}$	$\begin{array}{cc} & 1 \\ + & 0 \\ \hline 1 & 1 \\ & 0 \end{array}$	$\begin{array}{cc} & 0 \\ + & 1 \\ \hline 1 & 1 \\ & 0 \end{array}$	$\begin{array}{cc} & 0 \\ + & 0 \\ \hline 0 & 1 \\ & 1 \end{array}$

carry out

carry in

Full Adder Truth Table

#	a b c	c'	s
0	0 0 0	0	0
1	0 0 1	0	1
2	0 1 0	0	1
3	0 1 1	1	0
4	1 0 0	0	1
5	1 0 1	1	0
6	1 1 0	1	0
7	1 1 1	1	1

Full Adder Boolean Formulas

#	a b c	c'	c'	s	s
0	0 0 0	0		0	
1	0 0 1	0		1	$\bar{a}\bar{b}c$
2	0 1 0	0		1	$\bar{a}b\bar{c}$
3	0 1 1	1	$\bar{a}bc$	0	
4	1 0 0	0		1	$a\bar{b}\bar{c}$
5	1 0 1	1	$a\bar{b}c$	0	
6	1 1 0	1	$ab\bar{c}$	0	
7	1 1 1	1	abc	1	abc

$$s = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$$

$$c' = \bar{a}bc + a\bar{b}c + ab\bar{c} + abc$$

Full Adder Boolean Formulas

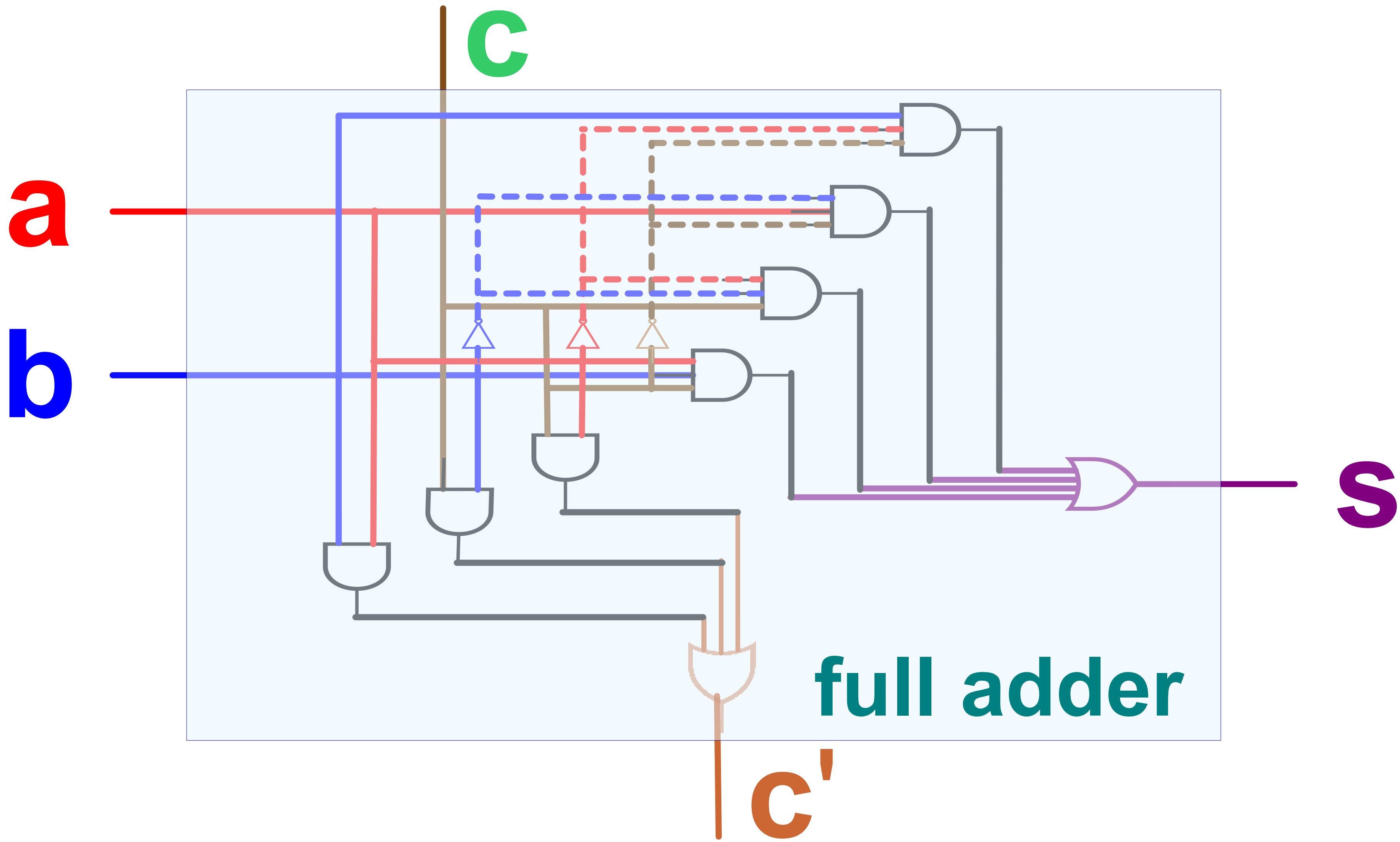
#	a	b	c	c'	s
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

$$s = \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$$

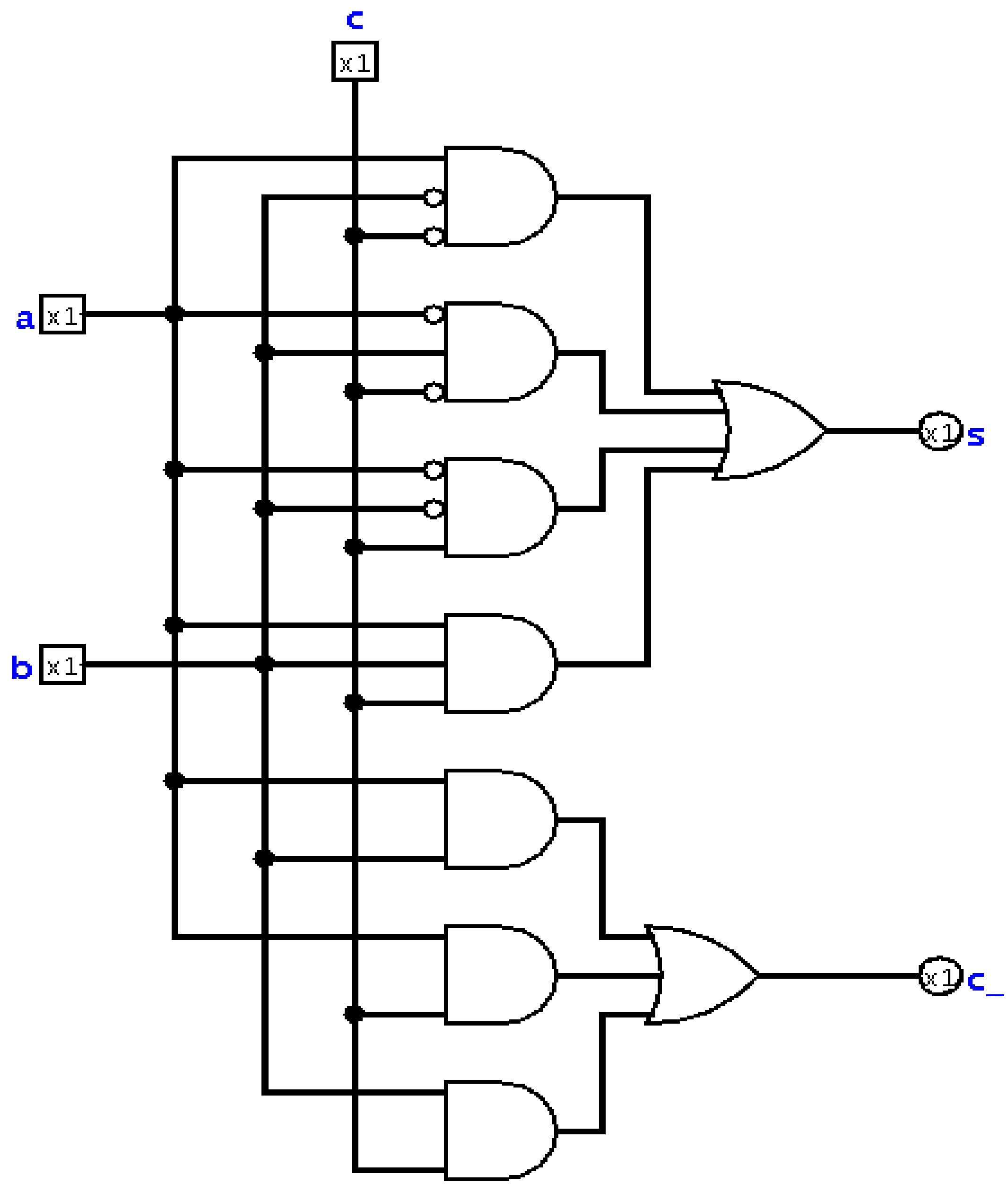
$$c' = bc + ac + ab$$

$$s = abc + \overline{ab}c + \bar{a}b\bar{c} + a\overline{bc}$$

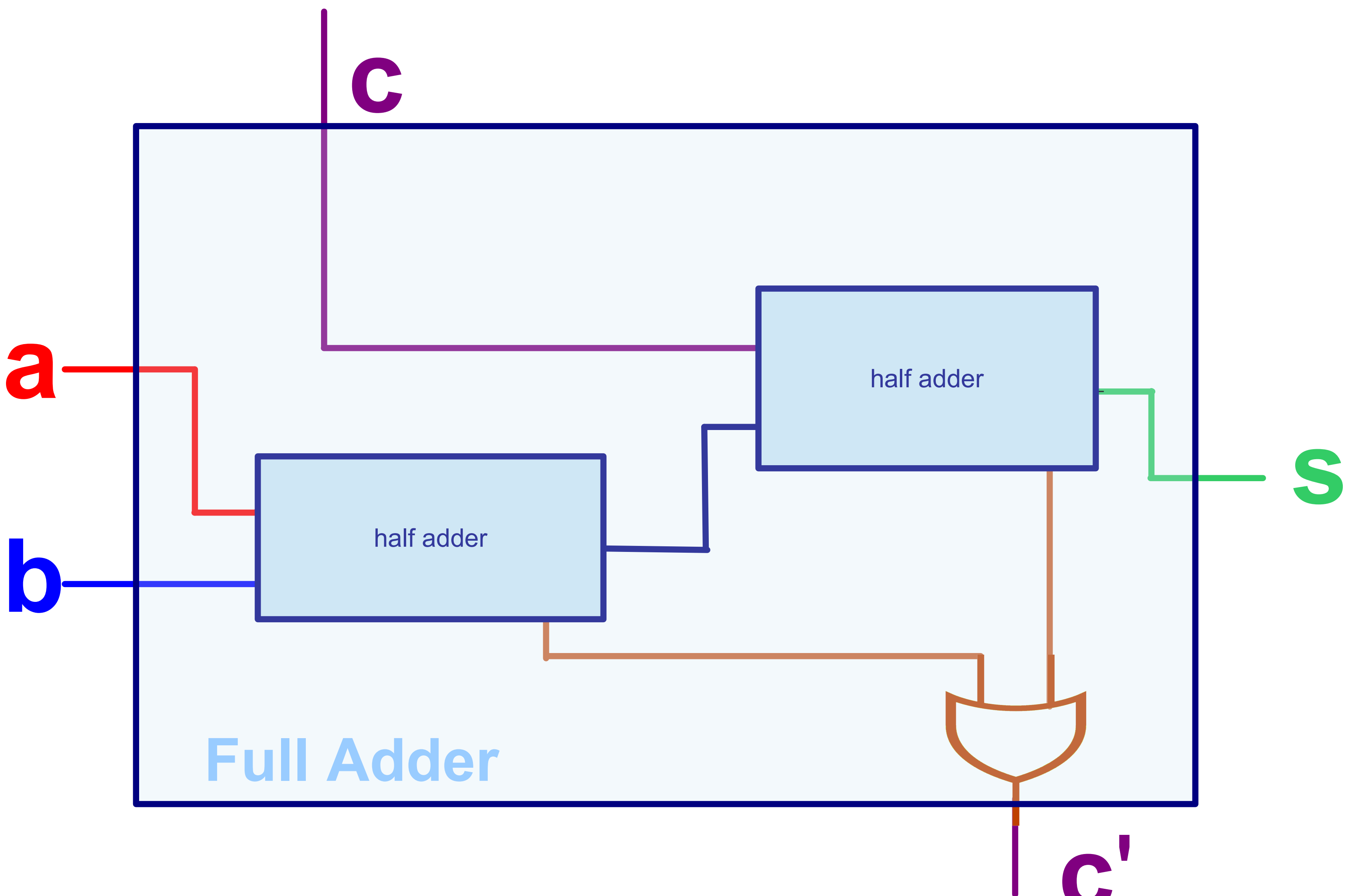
$$c' = ab + ac + bc$$



Full Adder Circuit

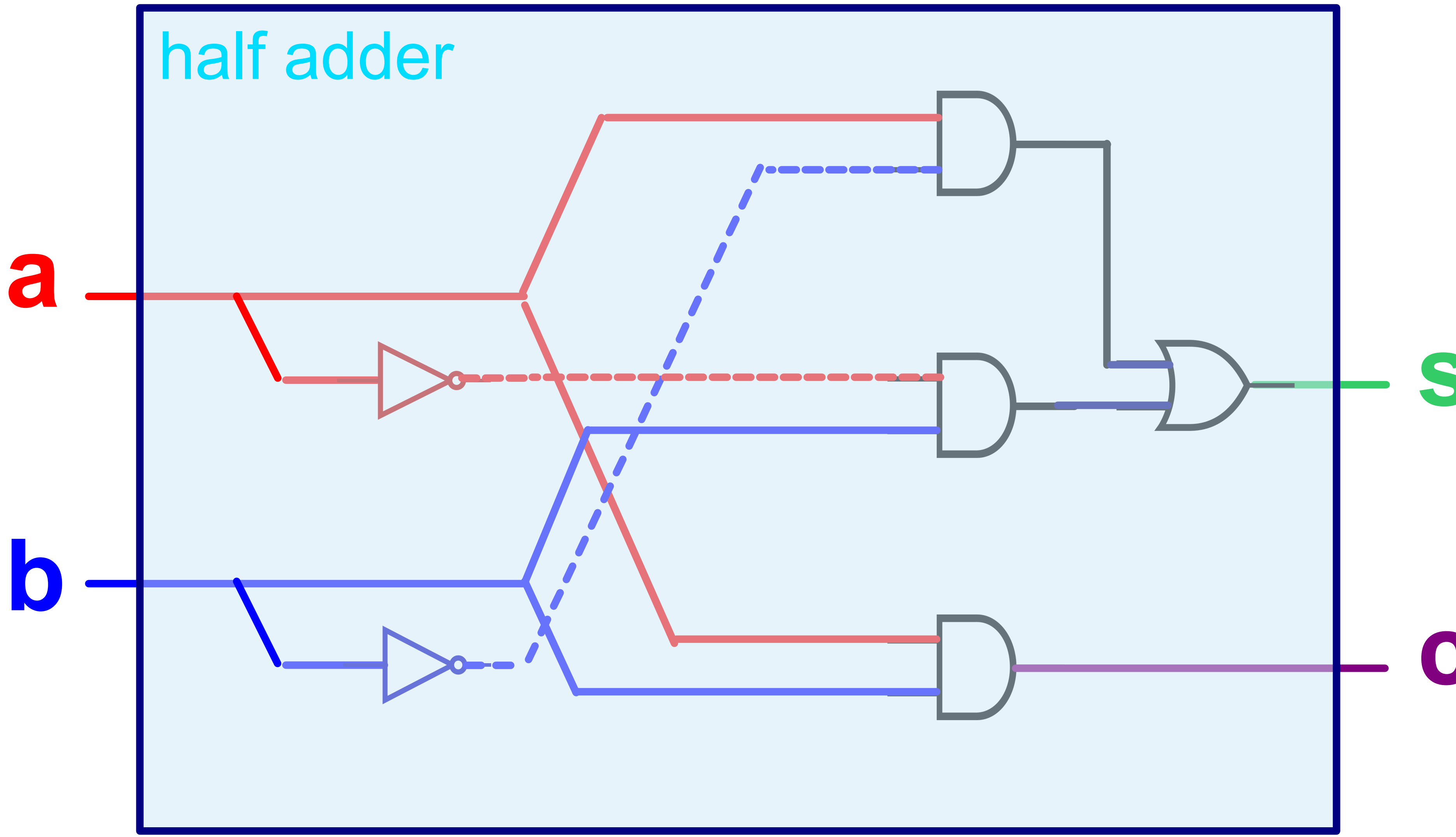


A **Full Adder** built of Half Adders

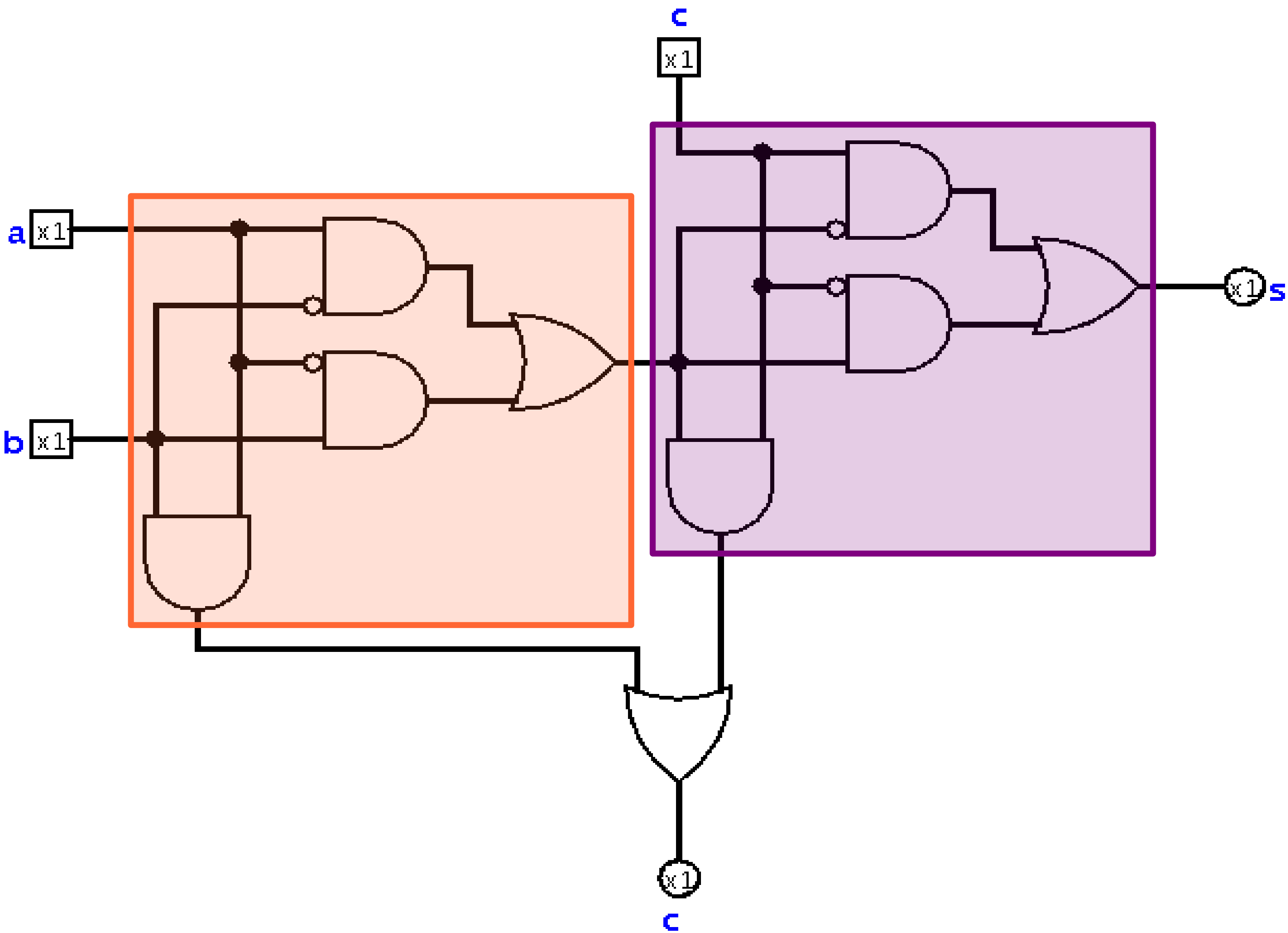


Half Adder Circuit

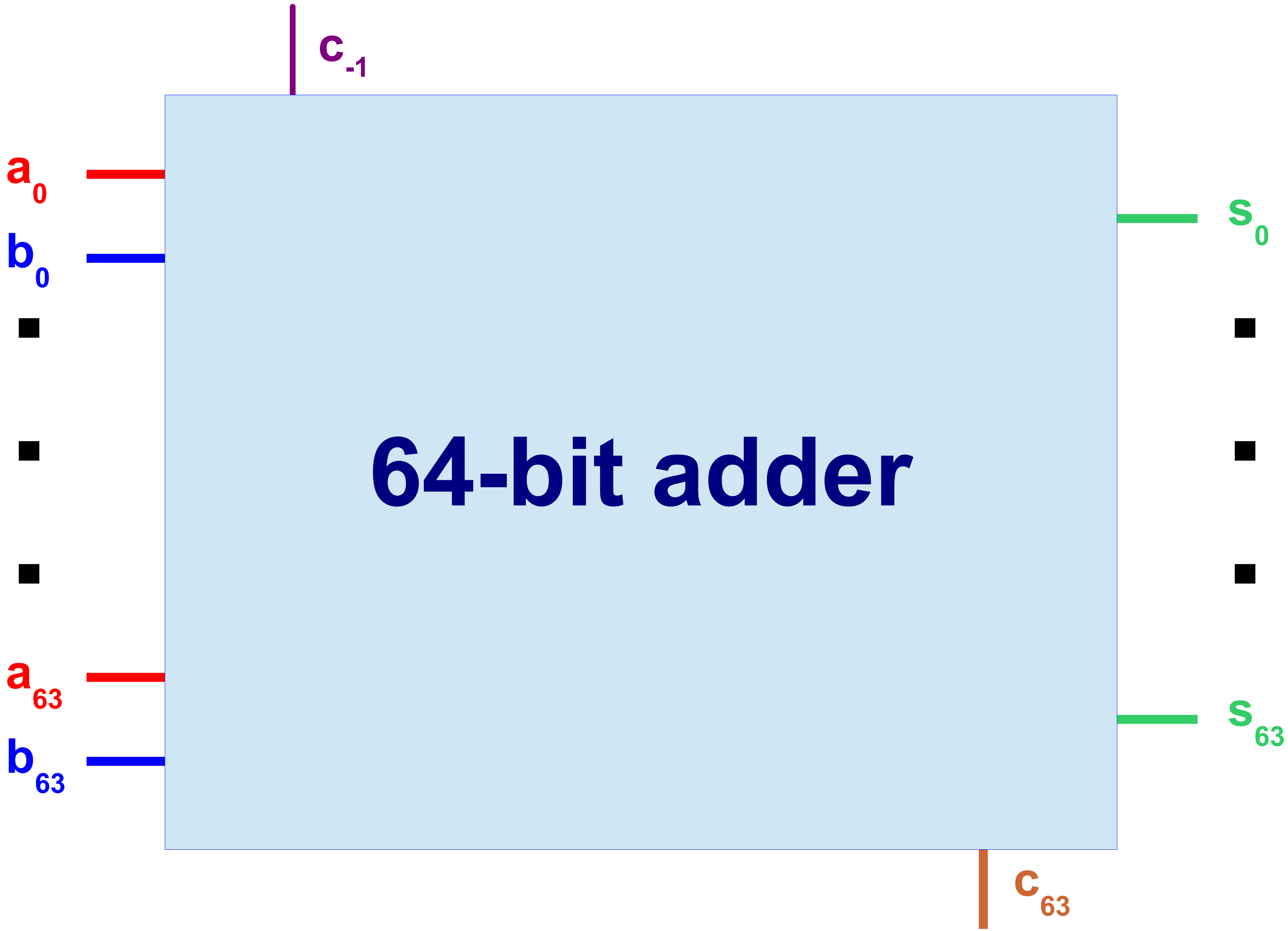
Half Adder Circuit



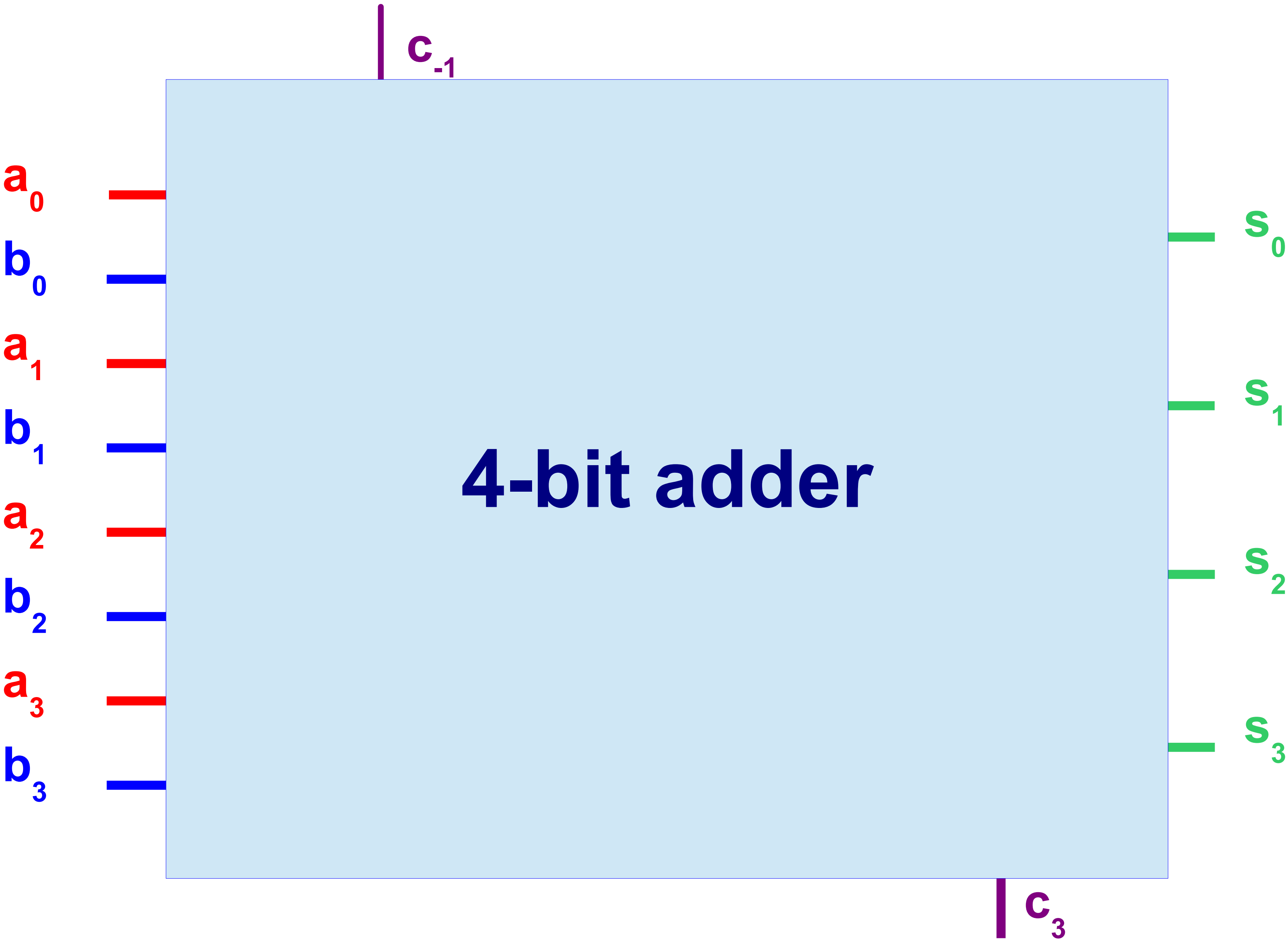
Full Adder built of Half Adders



64-bit Word Adder



4-bit Word Adder



4-Bit Ripple-Carry Addition

	1	1	1	0
+	0	1	1	1
				0

4-Bit Ripple-Carry Addition

	1	1	1	0
+	0	1	1	1
			0	0
				1

4-Bit Ripple-Carry Addition

	1	1	1	0
+	0	1	1	1
		1	0	0
			0	1

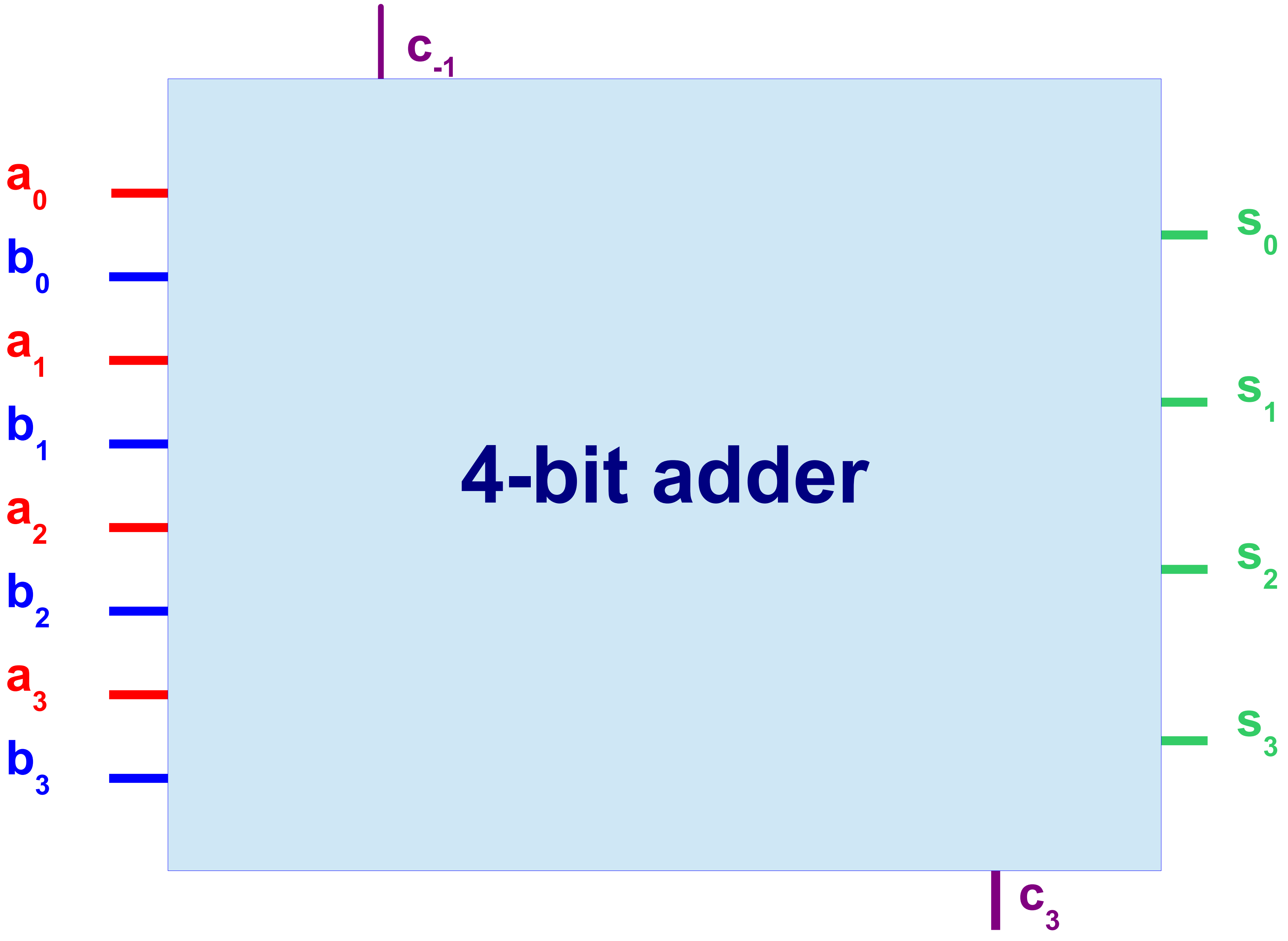
4-Bit Ripple-Carry Addition

	1	1	1	0
+	0	1	1	1
	1	1	0	0
		1	0	1

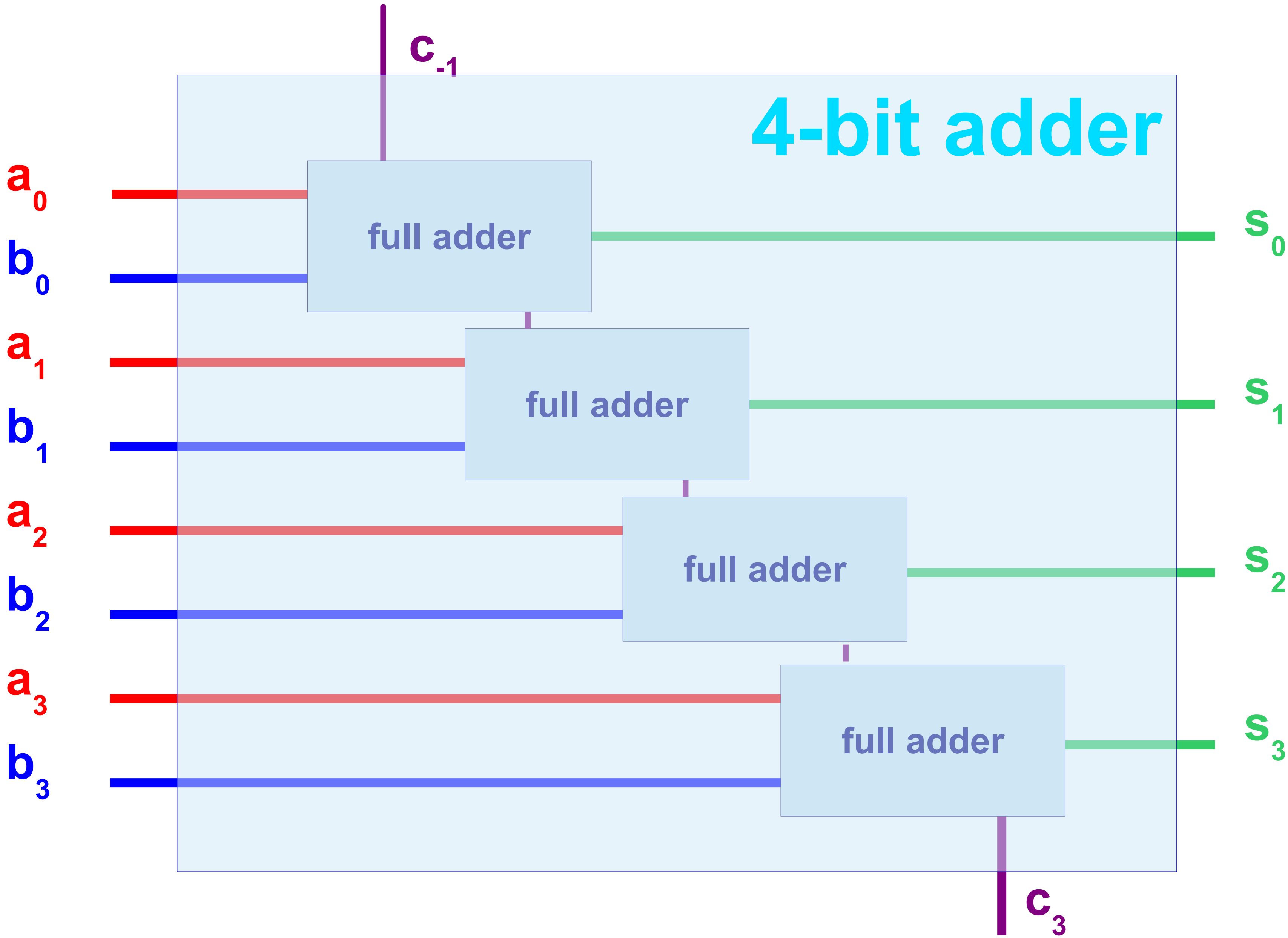
4-Bit Ripple-Carry Addition

		1	1	1	0
+		0	1	1	1
	1	1	1	0	0
		0	1	0	1

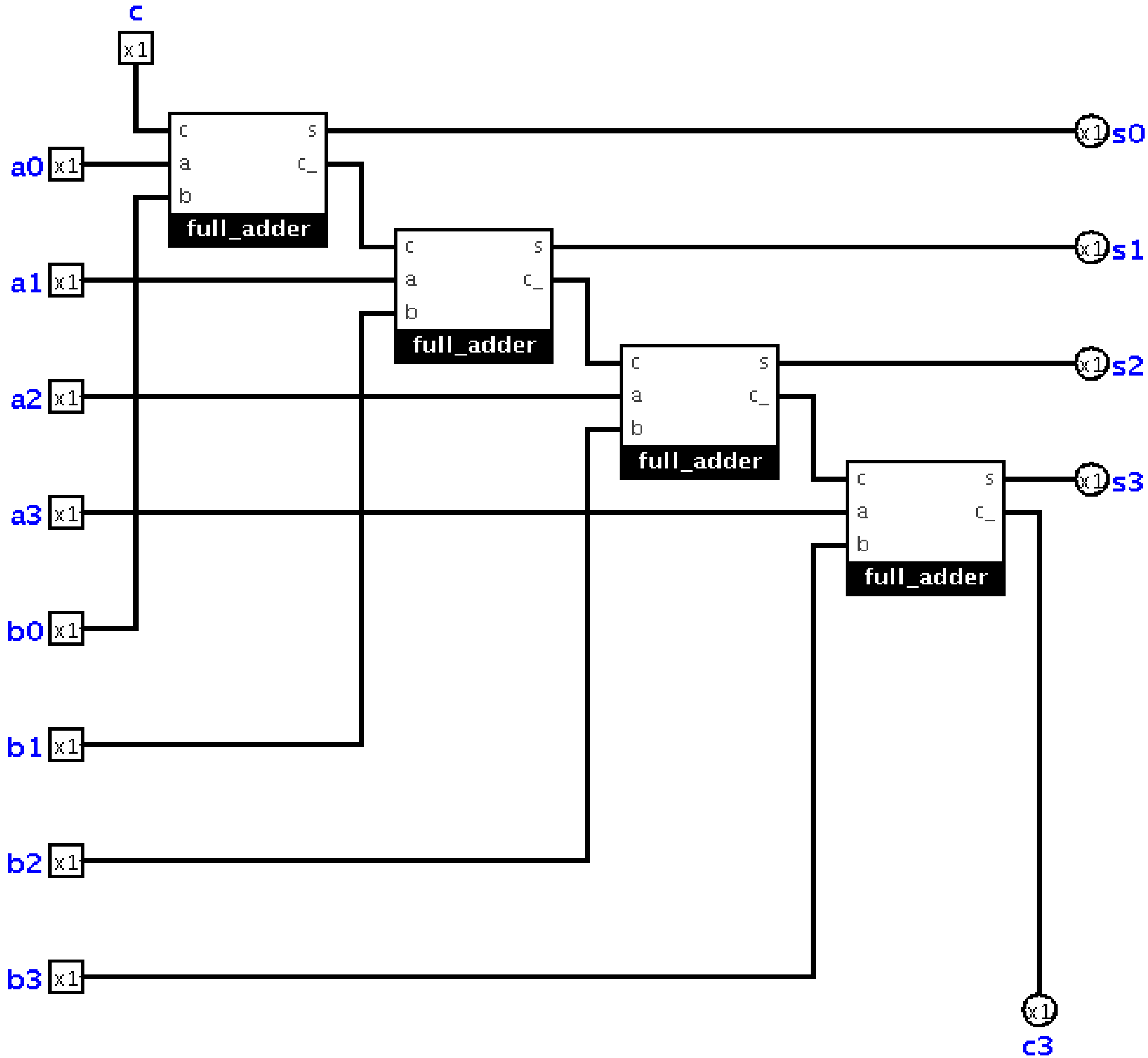
4-bit Ripple–Carry Adder



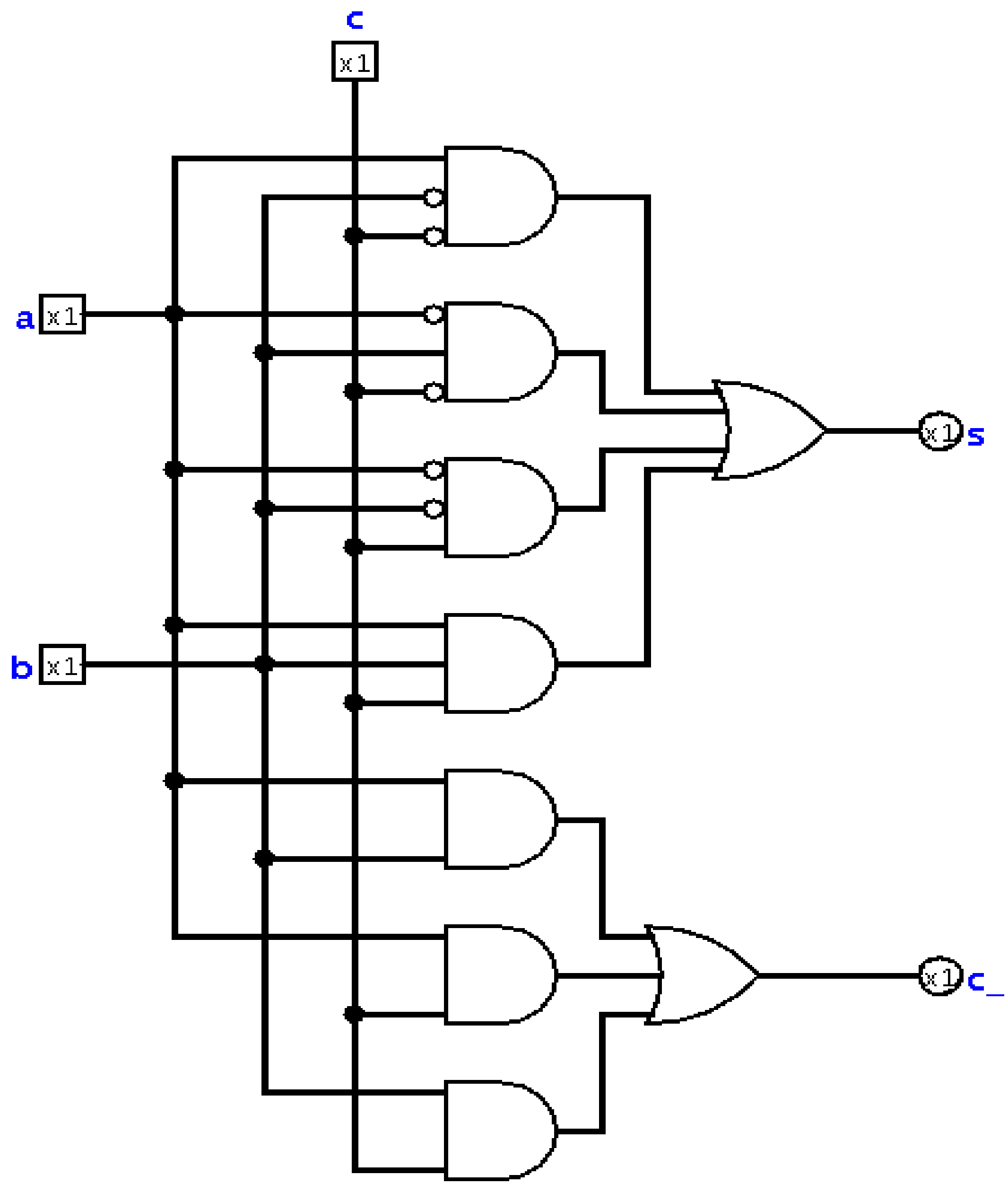
4-bit Ripple–Carry Adder



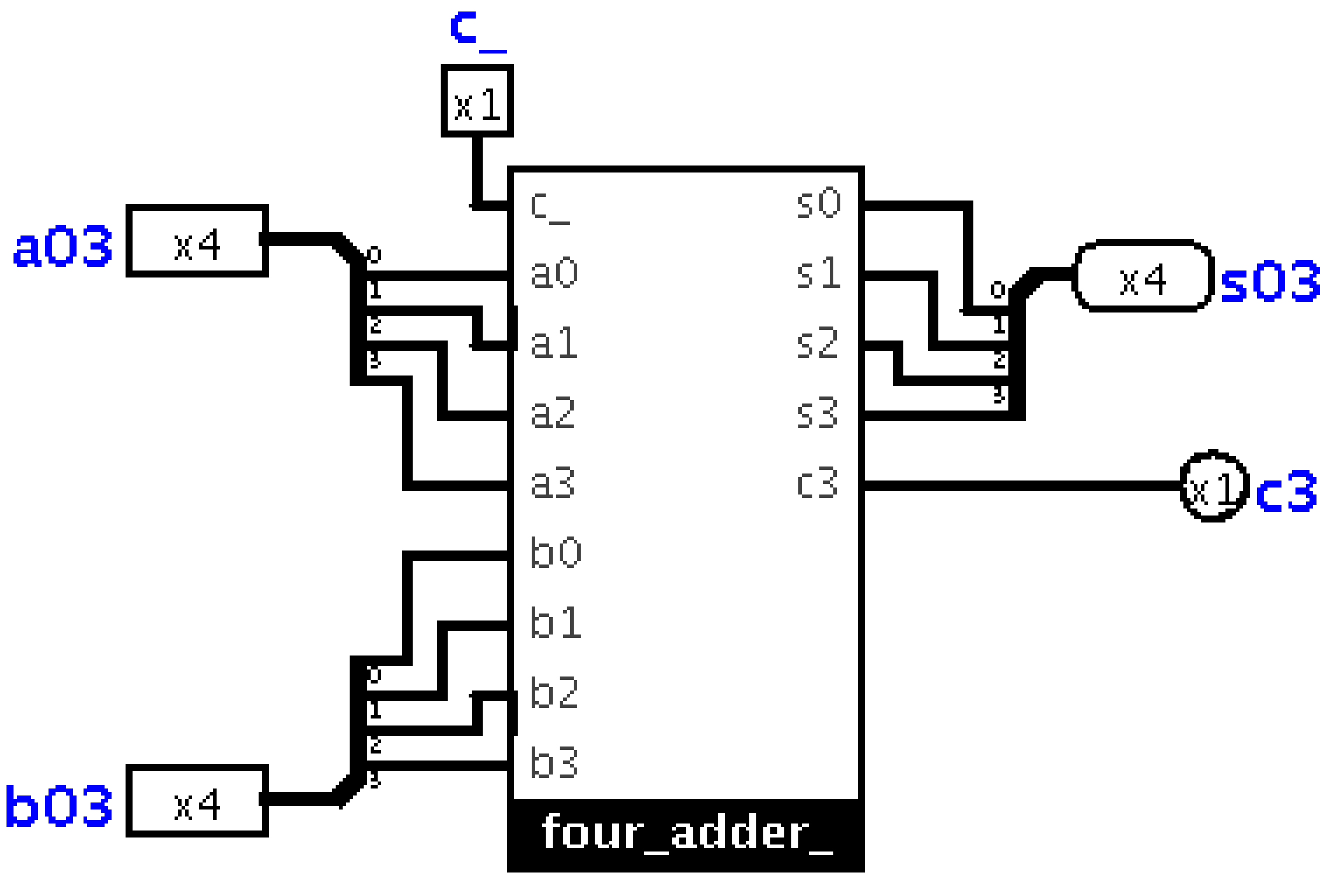
4-bit Ripple–Carry Adder



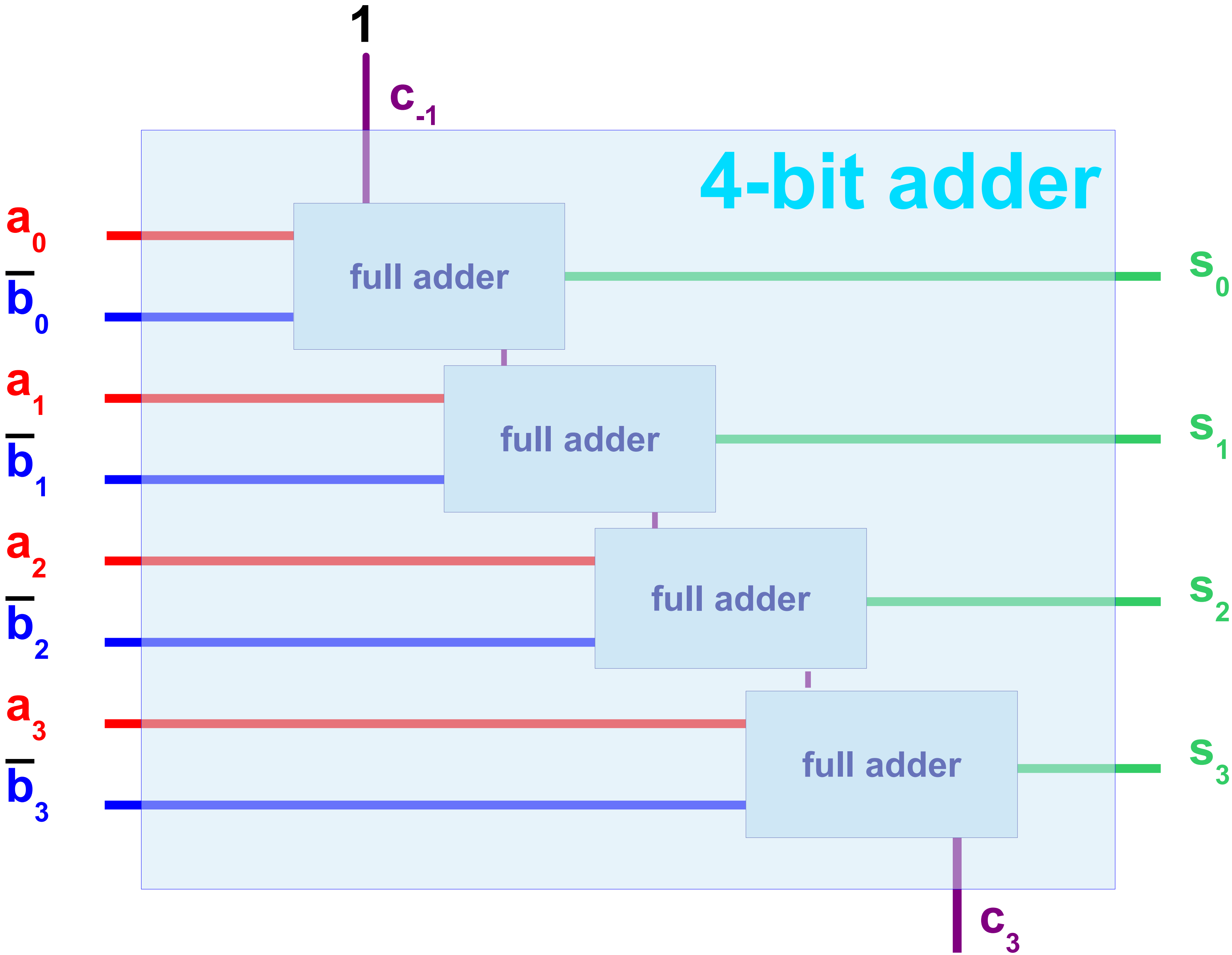
Full Adder Circuit



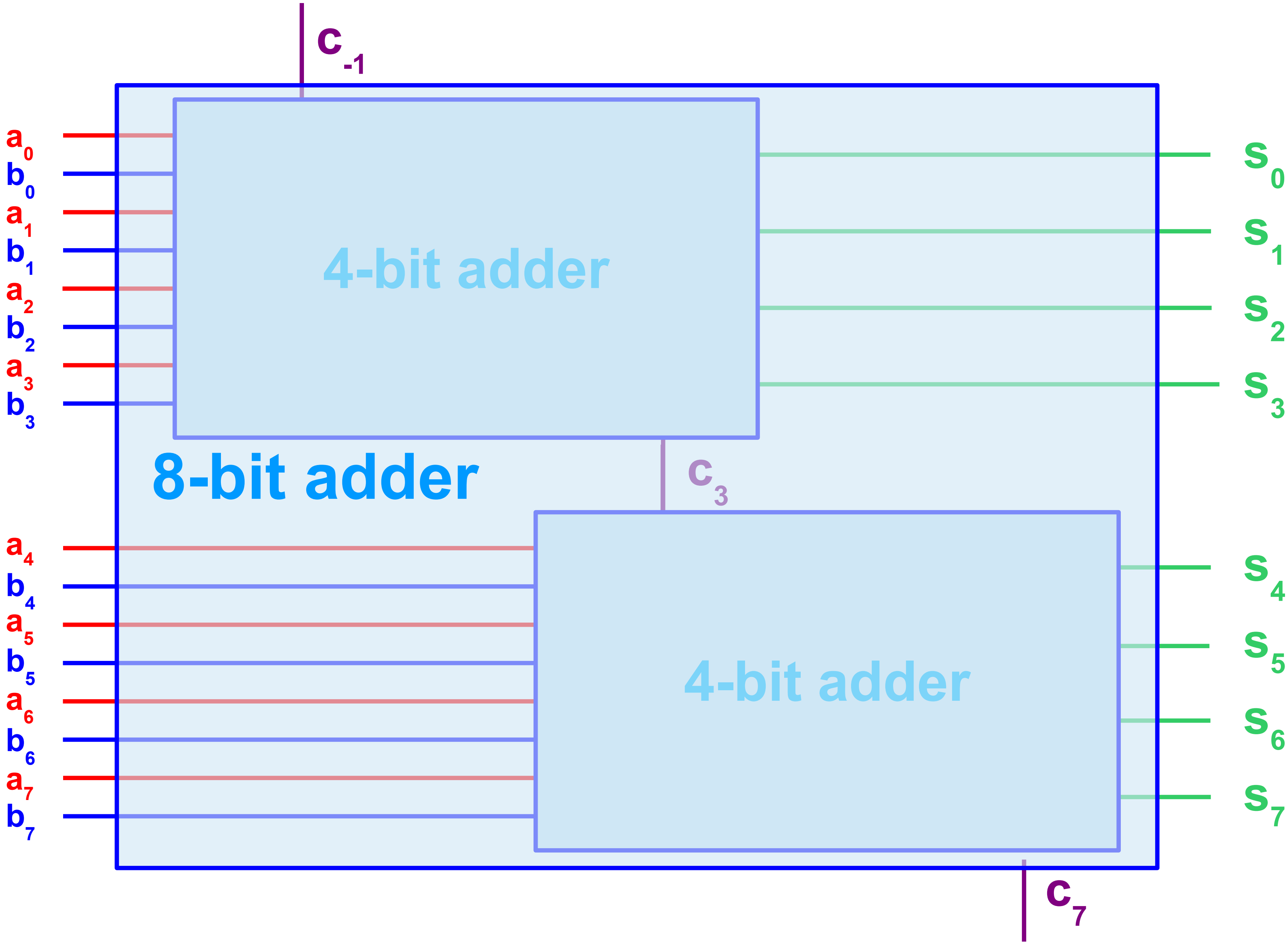
4-bit Ripple–Carry Adder



4-bit Subtraction



8-bit Ripple-Carry Adder



16-bit Ripple-Carry Adder