Topic: Linear systems in three unknowns

Question: Use any method to find the solution to the system of equations.

$$x + y - z = 4$$

$$x - y - z = 2$$

$$x + 2y + z = 1$$

Answer choices:

A $(x, y, z) = (-1,1,2)$

B $(x, y, z) = (1, -1,2)$

C $(x, y, z) = (1,1, -2)$

D $(x, y, z) = (1,1,2)$

Solution: C

So that we can stay organized, let's number the equations.

[1] $x + y - z = 4$

[2] $x - y - z = 2$

[3] $x + 2y + z = 1$

Since the coefficients of x and z in [1] are equal to the coefficients of x and z in [2], we'll be able to eliminate the variables x and z (and then solve for y) if we subtract [2] from [1].

$$x + y - z - (x - y - z) = (4) - (2)$$

$$x + y - z - x + y + z = 4 - 2$$

$$2y = 2$$

$$y = 1$$

Let's substitute 1 for y in [2] and [3], which will give us two equations in the variables x and z only. So

[2] $x - y - z = 2$

[3] $x + 2y + z = 1$

become

$$x - 1 - z = 2$$

$$x + 2(1) + z = 1$$

and then

[4] $x - z = 3$

[5] $x + z = -1$

Next, we'll add [4] and [5] in order to eliminate z and solve for x.

$$(x - z) + (x + z) = (3) + (-1)$$

$$x - z + x + z = 3 - 1$$

$$2x = 2$$

$$x = 1$$

Now that we know that $x = 1$ and $y = 1$, we'll substitute 1 for both x and y in [1], and then solve for z.

[1] $x + y - z = 4$

$$1 + 1 - z = 4$$

$$2 - z = 4$$

$$-z = 2$$

$$z = -2$$

We've found a possible solution, $(1,1,-2)$. Let's test it in the original system to make sure it satisfies the system.

[1] $x + y - z = 4$

$1 + 1 - (-2) = 4$

$1 + 1 + 2 = 4$

$4 = 4$

and

[2] $x - y - z = 2$

$1 - 1 - (-2) = 2$

$1 - 1 + 2 = 2$

$2 = 2$

and

[3] $x + 2y + z = 1$

$1 + 2(1) + (-2) = 1$

$1 + 2 - 2 = 1$

$1 = 1$

We've shown that $(1,1,-2)$ satisfies the system.

Topic: Linear systems in three unknowns

Question: Use any method to find the solution to the linear system.

$$x - y + z = -6$$

$$3x - 4y - z = -4$$

$$-2x + 3y + 4z = 14$$

Answer choices:

A $(x, y, z) = (-60, -46, -8)$

B $(x, y, z) = (60, -46, 8)$

C $(x, y, z) = (-60, 46, 8)$

D $(x, y, z) = (-60, -46, 8)$

Solution: D

So that we can stay organized, let's number the equations.

[1] $x - y + z = -6$

[2] $3x - 4y - z = -4$

[3] $-2x + 3y + 4z = 14$

We'll add [1] and [2] to eliminate z.

$$(x - y + z) + (3x - 4y - z) = (-6) + (-4)$$

$$x - y + z + 3x - 4y - z = -6 - 4$$

[4] $4x - 5y = -10$

Now we'll multiply [2] by 4, so that we can add the result to [3] and eliminate z.

[2] $3x - 4y - z = -4$

$$4(3x - 4y - z) = 4(-4)$$

[5] $12x - 16y - 4z = -16$

Adding [3] and [5], we get

$$(-2x + 3y + 4z) + (12x - 16y - 4z) = (14) + (-16)$$

$$-2x + 3y + 4z + 12x - 16y - 4z = 14 - 16$$

[6] $10x - 13y = -2$

With [4] and [6], we have a system of two equations in the variables x and y.

[4] $4x - 5y = -10$

[6] $10x - 13y = -2$

Let's solve [4] for y, and substitute the resulting expression for y into [6], and then solve for x.

[4] $4x - 5y = -10$

$$-5y = -4x - 10$$

$$5y = 4x + 10$$

[7] $y = \frac{4}{5}x + 2$

Now we'll plug this expression for y into [6].

[6] $10x - 13y = -2$

$$10x - 13\left(\frac{4}{5}x + 2\right) = -2$$

$$10x - \frac{52}{5}x - 26 = -2$$

$$5\left(10x - \frac{52}{5}x - 26\right) = 5(-2)$$

$$5(10x) + 5\left(-\frac{52}{5}x\right) + 5(-26) = 5(-2)$$

$$50x - 52x - 130 = -10$$

$$-2x = 120$$

$$x = -60$$

Next, we'll substitute -60 for x in [7], and then compute the value of y.

[7] $y = \frac{4}{5}x + 2$

$$y = \frac{4}{5}(-60) + 2$$

$$y = 4(-12) + 2$$

$$y = -48 + 2$$

$$y = -46$$

Now that we know that $x = -60$ and $y = -46$, we'll substitute -60 for x and -46 for y in [1], and then solve for z.

[1] $x - y + z = -6$

$$-60 - (-46) + z = -6$$

$$-60 + 46 + z = -6$$

$$-14 + z = -6$$

$$z = 8$$

We've found a possible solution, $(-60,-46,8)$. Let's test it in the original system to make sure it satisfies the system.

[1] $x-y+z=-6$

$-60-(-46)+8=-6$

$-60+46+8=-6$

$-14+8=-6$

$-6=-6$

and

[2] $3x-4y-z=-4$

$3(-60)-4(-46)-8=-4$

$-180+184-8=-4$

$4-8=-4$

$-4=-4$

and

[3] $-2x+3y+4z=14$

$-2(-60)+3(-46)+4(8)=14$

$120-138+32=14$

$-18+32=14$

$$14 = 14$$

We've shown that $(-60, -46, 8)$ satisfies the system.

Topic: Linear systems in three unknowns

Question: Solve the system for x, y, and z.

$$x + 2z = 3$$

$$3x - 2y + z = -11$$

$$2x + y + 3z = 9$$

Answer choices:

A $(x, y, z) = (3, -1, 5)$

B $(x, y, z) = (-1, 3, 2)$

C $(x, y, z) = (2, 5, -1)$

D $(x, y, z) = (-1, 5, 2)$

Solution: D

So that we can stay organized, let's number the equations.

[1] $x + 2z = 3$

[2] $3x - 2y + z = -11$

[3] $2x + y + 3z = 9$

Solve [1] for x.

$$x + 2z = 3$$

$$x = 3 - 2z$$

Substitute this expression for x into [2] and [3], and then simplify.

[2] $3x - 2y + z = -11$

$$3(3 - 2z) - 2y + z = -11$$

$$9 - 6z - 2y + z = -11$$

$$-2y - 5z = -20$$

and

[3] $2x + y + 3z = 9$

$$2(3 - 2z) + y + 3z = 9$$

$$6 - 4z + y + 3z = 9$$

$$y - z = 3$$

Now we can solve the resulting equations as a system of two equations in the variables y and z.

$$-2y - 5z = -20$$

$$y - z = 3$$

We'll multiply the second equation by 2.

$$y - z = 3$$

$$2(y - z) = 2(3)$$

$$2y - 2z = 6$$

To eliminate y, we can add this equation to the equation $-2y - 5z = -20$ that we found earlier.

$$(-2y - 5z) + (2y - 2z) = (-20) + (6)$$

$$-2y - 5z + 2y - 2z = -20 + 6$$

$$-7z = -14$$

$$z = 2$$

Now we'll substitute 2 for z in the equation $y - z = 3$ that we found earlier, and then solve for y.

$$y - z = 3$$

$$y - 2 = 3$$

$$y = 5$$

Finally, we'll substitute 2 for z in the equation $x = 3 - 2z$ that we found much earlier, and then compute the value of x.

$$x = 3 - 2z$$

$$x = 3 - 2(2)$$

$$x = 3 - 4$$

$$x = -1$$

The solution is $(-1,5,2)$. If we plug the coordinates of this point into all three of the original equations, we'll see that it satisfies all of them.