MATH 313: LINEAR ALGEBRA
HOMEWORK 1

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(1) Write the solutions of the following linear equations as vectors in $\mathbf{R}^2$:
(a)
$$x + y = 3.$$
(b)
$$x + 3y = 1$$
(2) Write the solutions of the following linear equations as vectors in $\mathbf{R}^3$:
(a)
$$x + y + z = 0.$$
(b)
$$x + y + z = -7.$$
(3) Write the solutions of the following linear equations as vectors in $\mathbf{R}^4$:
(a)
$$x + y + z + w = 0$$
(b)
$$x + 3y - 7z + 2w = 1$$
(4) Compute the following linear combinations of vectors:
(a)
$$3\begin{pmatrix}7\\2\end{pmatrix} - 8\begin{pmatrix}9\\-1\end{pmatrix}$$
(b)
$$\begin{pmatrix}1\\2\\3\end{pmatrix} + 2\begin{pmatrix}4\\5\\6\end{pmatrix} + 3\begin{pmatrix}7\\8\\9\end{pmatrix}$$
(c)
$$8\begin{pmatrix}-1\\0\\1\\2\\3\end{pmatrix} + 3\begin{pmatrix}4\\-7\\-5\\1\\1\end{pmatrix} - 2\begin{pmatrix}7\\-8\\0\\1\\9\end{pmatrix}$$
(5) In the vector space $\mathbf{R}^3$, let
$$\mathbf{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad \mathbf{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \qquad \mathbf{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

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Prove that every vector $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbf{R}^3$ has a unique representation as a linear combination of the vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$.

(6) Prove that if

$$x \begin{pmatrix} 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

then $x = y = 0$.

(7) (a) Let W be the set of solutions of the homogeneous linear equation

$$5x - 8y = 0.$$

Compute a set S of vectors in $\mathbf{R}^2$ such that W is the set of linear combinations of vectors in S.

(b) Let L be the set of solutions of the inhomogeneous linear equation

$$5x - 8y = 1.$$

Compute a vector $\mathbf{v}$ such that $L = \mathbf{v} + W$.

(8) (a) Let W be the set of solutions of the homogeneous linear equation

$$5x - 8y - 2z = 0.$$

Compute a set S of vectors in $\mathbf{R}^3$ such that W is the set of linear combinations of vectors in S.

(b) Let L be the set of solutions of the inhomogeneous linear equation

$$5x - 8y - 2z = 3.$$

Compute a vector $\mathbf{v}$ such that $L = \mathbf{v} + W$.

(9) In the vector space $\mathbf{R}^2$, draw the vector subspace

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R}^2 : x + y = 0 \right\}$$

and the affine subspaces

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R}^2 : x + y = c \right\}$$

for $c = -2, -1, 1, 2$.

(10) Consider the affine subspace

$$L = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R}^2 : x + y = 1 \right\}$$

Prove that

$$\begin{pmatrix} x \\ y \end{pmatrix} \in L$$

if and only if

$$\begin{pmatrix} x \\ y \end{pmatrix} = (1 - t) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

for some $t \in \mathbf{R}$.

(11) In the vector space $\mathbf{R}^2$, draw the vector subspace

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R}^2 : 5x - 2y = 0 \right\}$$

and the affine subspaces

$$\left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R}^2 : 5x - 2y = c \right\}$$

for $c = -2, -1, 1, 2$.

(12) In the vector space $\mathbf{R}^3$, draw the vector subspace

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbf{R}^3 : x + y + z = 0 \right\}$$

and the affine subspaces

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbf{R}^3 : x + y + z = c \right\}$$

for $c = -2, -1, 1, 2$.

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