Topic: Linear independence in two dimensions

Question: Which vector set might be linearly independent?

Answer choices:

A A set of 2 two-dimensional vectors

B A set of 3 two-dimensional vectors

C A set of 4 two-dimensional vectors

D A set of 4 three-dimensional vectors

Solution: A

Any n n-dimensional linearly independent vectors can span $\mathbb{R}^n$. Which means that any $n+1$ or greater set of vectors in $\mathbb{R}^n$ will be linearly dependent.

So given two-dimensional vectors, only a set of two or fewer can be linearly independent, or given three-dimensional vectors, only a set of three or fewer can be linearly independent.

Answer choices B, C, and D all have too many vectors for the dimension in which they're defined, so only answer choice A can be a linearly independent set.

Topic: Linear independence in two dimensions

Question: Which vector set is linearly independent?

Answer choices:

A $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

B $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$

C $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

D $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Solution: B

In $\mathbb{R}^n$ space, only vector sets with n or fewer vectors can be linearly independent. For instance, in $\mathbb{R}^2$, only vector sets with one or two vectors can be a linearly independent set, and any set with three or more vectors will be linearly dependent.

Because all of the vectors in these answer choices are in $\mathbb{R}^2$, a linearly independent set will include two or fewer vectors, which leaves only answer choices A and B as possibilities.

Let's test answer choice A by setting up the vector equation.

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Change the equation into an augmented matrix, then put the matrix into reduced row-echelon form.

$$\begin{bmatrix} 1 & 3 & | & 0 \\ 2 & 6 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

The rref form of the matrix gives the equation

$$c_1 + 3c_2 = 0$$

$$c_1 = -3c_2$$

This equation tells us that there are an endless number of solutions to the system. We can choose any value for c_2, and we'll get a different value for c_1, and all of those combinations will give us the zero vector. Because $(c_1, c_2) = (0,0)$ isn't the only solution, that tells us that the vectors in answer choice A are linearly dependent.

Which means answer choice B must be the correct choice, but let's verify that those vectors are, in fact, linearly independent. We'll test answer choice B by setting up the vector equation.

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Change the equation into an augmented matrix, then put the matrix into reduced row-echelon form.

$$\begin{bmatrix} 1 & 3 & | & 0 \\ 2 & -6 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & | & 0 \\ 0 & -12 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & | & 0 \\ 0 & 1 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \end{bmatrix}$$

The rref form of the matrix gives the equations

$$c_1 = 0$$

$$c_2 = 0$$

These equations tell us that $(c_1, c_2) = (0,0)$ is the only solution, which means the vectors in answer choice B are linearly independent.

Topic: Linear independence in two dimensions

Question: Which vector set is linearly independent?

Answer choices:

A $\quad \vec{a}=\begin{bmatrix}-3\\5\end{bmatrix}, \vec{b}=\begin{bmatrix}9\\-15\end{bmatrix}, \vec{c}=\begin{bmatrix}-1\\6\end{bmatrix}$

B $\quad \vec{a}=\begin{bmatrix}-3\\5\end{bmatrix}, \vec{b}=\begin{bmatrix}9\\15\end{bmatrix}, \vec{c}=\begin{bmatrix}1\\-6\end{bmatrix}$

C $\quad \vec{a}=\begin{bmatrix}-3\\5\end{bmatrix}, \vec{b}=\begin{bmatrix}9\\-15\end{bmatrix}$

D $\quad \vec{a}=\begin{bmatrix}-3\\5\end{bmatrix}, \vec{b}=\begin{bmatrix}9\\15\end{bmatrix}$

Solution: D

In $\mathbb{R}^n$ space, only vector sets with n or fewer vectors can be linearly independent. For instance, in $\mathbb{R}^2$, only vector sets with one or two vectors can be a linearly independent set, and any set with three or more vectors will be linearly dependent.

Because all of the vectors in these answer choices are in $\mathbb{R}^2$, a linearly independent set will include two or fewer vectors, which leaves only answer choices C and D as possibilities.

Let's test answer choice C by setting up the vector equation.

$$c_1 \begin{bmatrix} -3 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} 9 \\ -15 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Change the equation into an augmented matrix, then put the matrix into reduced row-echelon form.

$$\begin{bmatrix} -3 & 9 & | & 0 \\ 5 & -15 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & | & 0 \\ 5 & -15 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

The rref form of the matrix gives the equation

$$c_1 - 3c_2 = 0$$

$$c_1 = 3c_2$$

This equation tells us that there are an endless number of solutions to the system. We can choose any value for c_2, and we'll get a different value for c_1, and all of those combinations will give us the zero vector. Because $(c_1, c_2) = (0,0)$ isn't the only solution, that tells us that the vectors in answer choice C are linearly dependent.

Which means answer choice D must be the correct choice, but let's verify that those vectors are, in fact, linearly independent. We'll test answer choice D by setting up the vector equation.

$$c_1 \begin{bmatrix} -3 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} 9 \\ 15 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Change the equation into an augmented matrix, then put the matrix into reduced row-echelon form.

$$\begin{bmatrix} -3 & 9 & | & 0 \\ 5 & 15 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & | & 0 \\ 5 & 15 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & | & 0 \\ 0 & 30 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & | & 0 \\ 0 & 1 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \end{bmatrix}$$

The rref form of the matrix gives the equations

$$c_1 = 0$$

$$c_2 = 0$$

These equations tell us that $(c_1, c_2) = (0,0)$ is the only solution, which means the vectors in answer choice D are linearly independent.