Topic: Linear combinations and span

Question: How many linearly independent vectors are needed to span $\mathbb{R}^4$?

Answer choices:

A 1

B 2

C 3

D 4

Solution: D

Any n n-dimensional linearly independent vectors will span $\mathbb{R}^n$. So in order to span $\mathbb{R}^4$, we'll need 4 linearly independent vectors.

Topic: Linear combinations and span

Question: Will the vectors span $\mathbb{R}^2$?

$$\vec{u} = (3,1)$$

$$\vec{v} = (6,2)$$

Answer choices:

A Yes, because the vectors aren't parallel

B Yes, because the vectors are parallel

C No, because the vectors aren't parallel

D No, because the vectors are parallel

Solution: D

The vectors $\vec{u}=(3,1)$ and $\vec{v}=(6,2)$ are parallel. We can tell this by sketching them, or by looking at the slope of each vector, where the slope of $\vec{u}$ is $3/1=3$, and the slope of $\vec{v}$ is $6/2=3$.

Parallel vectors can never span their space. So $\vec{u}=(3,1)$ and $\vec{v}=(6,2)$ can't span $\mathbb{R}^2$ because of the fact that they're parallel.

Topic: Linear combinations and span

Question: Can the standard basis vectors $\mathbf{i}=(1,0,0)$, $\mathbf{j}=(0,1,0)$ and $\mathbf{k}=(0,0,1)$ span $\mathbb{R}^3$?

Answer choices:

A Yes, because three linearly independent vectors in $\mathbb{R}^3$ will span $\mathbb{R}^3$

B Yes, because three linearly dependent vectors in $\mathbb{R}^3$ will span $\mathbb{R}^3$

C No, because three linearly independent vectors in $\mathbb{R}^3$ won't span $\mathbb{R}^3$

D No, because three linearly dependent vectors in $\mathbb{R}^3$ won't span $\mathbb{R}^3$

Solution: A

Any 3 three-dimensional linearly independent vectors will span $\mathbb{R}^3$. The three-dimensional basis vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ are linearly independent, which is why they span $\mathbb{R}^3$.