Topic: Identity matrices

Question: Which identity matrix is I_3?

Answer choices:

A $\quad I_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B $\quad I_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C $\quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

D $\quad I_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Solution: C

We always call the identity matrix I, and it's always a square matrix, like 2×2, 3×3, 4×4, etc. For that reason, it's common to abbreviate I_{2x2} as just I_2, or I_{3x3} as just I_3, etc. So I_3 is the 3×3 identity matrix.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Topic: Identity matrices

Question: Which identity matrix can be multiplied by A (in other words, IA), if A is a 2×4 matrix?

Answer choices:

A $\quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B $\quad I_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

C $\quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D $\quad I_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Solution: A

Start by setting up the equation $IA = A$. Next, substitute the dimensions for A into the equation.

$$I \cdot A = A$$

$$I \cdot 2 \times 4 = 2 \times 4$$

Break down the dimensions of the identity matrix as rows $\times$ columns.

$$R \times C \cdot 2 \times 4 = 2 \times 4$$

In order to be able to multiply matrices, we need the same number of columns in the first matrix as we have rows in the second matrix. So the identity matrix must have 2 columns.

$$R \times 2 \cdot 2 \times 4 = 2 \times 4$$

And the dimensions of the resulting matrix come from the rows of the first matrix and the columns of the second matrix. So the identity matrix must have 2 rows.

$$2 \times 2 \cdot 2 \times 4 = 2 \times 4$$

Therefore, the identity matrix we need is I_2.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Topic: Identity matrices

Question: If we want to find IA, which identity matrix should we use, and what is the product?

$$A = \begin{bmatrix} -3 & 7 & 1 \\ 2 & 8 & 4 \end{bmatrix}$$

Answer choices:

A Use $I_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and the product is $IA = \begin{bmatrix} 2 & 8 & 4 \\ -3 & 7 & 1 \end{bmatrix}$

B Use $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and the product is $IA = \begin{bmatrix} -3 & 7 & 1 \\ 2 & 8 & 4 \end{bmatrix}$

C Use $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and the product is $IA = \begin{bmatrix} -3 & 7 & 1 \\ 2 & 8 & 4 \end{bmatrix}$

D Use $I_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, and the product is $IA = \begin{bmatrix} 2 & 8 & 4 \\ -3 & 7 & 1 \end{bmatrix}$

Solution: B

Matrix A is a 2×3 matrix, and we need to find IA. We also know that IA will be 2×3. So we'll set up an equation of dimensions.

$$I \cdot A = A$$

$$I \cdot 2 \times 3 = 2 \times 3$$

$$R \times C \cdot 2 \times 3 = 2 \times 3$$

For matrix multiplication to be valid, we need the same number of columns in the first matrix as we have rows in the second matrix.

$$R \times 2 \cdot 2 \times 3 = 2 \times 3$$

The dimensions of the result are given by the rows from the first matrix, and columns from the second matrix.

$$2 \times 2 \cdot 2 \times 3 = 2 \times 3$$

So the identity matrix is 2×2, which means it's I_2.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Then the product of I_2 and matrix A is

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 7 & 1 \\ 2 & 8 & 4 \end{bmatrix}$$

$$IA = \begin{bmatrix} 1(-3) + 0(2) & 1(7) + 0(8) & 1(1) + 0(4) \\ 0(-3) + 1(2) & 0(7) + 1(8) & 0(1) + 1(4) \end{bmatrix}$$

$$IA = \begin{bmatrix} -3 & 7 & 1 \\ 2 & 8 & 4 \end{bmatrix}$$