Topic: Linear subspaces

Question: Which part of the definition of a subspace is redundant, because it's already contained in the other parts of the definition?

Answer choices:

A The set includes the zero vector.

B The set is closed under scalar multiplication.

C The set is closed under addition.

D The set is defined in $\mathbb{R}^2$.

Solution: A

The requirement that the set contains the zero vector is redundant. If the set is closed under scalar multiplication, that means any vector in the set can be multiplied by any scalar, and the resulting vector will still be in the set.

By definition, that means we can multiply by the zero vector, and the zero vector must still be in the set. So it's redundant to say that the zero vector must be included in the set. If the set is closed under scalar multiplication, then the zero vector would already be included.

So really only answer choices B and C are needed to form the definition of a subspace. Answer choice D isn't part of the definition of a subspace at all. A subspace can be defined in any $\mathbb{R}^n$ space, not just in $\mathbb{R}^2$.

Topic: Linear subspaces

Question: Which of these are possible subspaces of $\mathbb{R}^2$?

Answer choices:

A $\mathbb{R}^2$

B $\overrightarrow{O} = (0,0)$

C A line through $(0,0)$

D All of these

Solution: D

All of these are subspaces within $\mathbb{R}^2$. The space $\mathbb{R}^2$ itself is a subspace of $\mathbb{R}^2$, the zero vector $\vec{O} = (0,0)$ is a subspace of $\mathbb{R}^2$, and any line in $\mathbb{R}^2$ that runs through the origin is also a subspace of $\mathbb{R}^2$.

Topic: Linear subspaces

Question: Is V a subspace of $\mathbb{R}^2$?

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; x = 1 \right\}$$

Answer choices:

A Yes

B No, because it’s not closed under addition

C No, because it’s not closed under scalar multiplication

D No, because it’s not closed under addition or scalar multiplication

Solution: D

In order for V to be a subspace, it must be closed under addition and closed under scalar multiplication. Let's pick a couple of vectors in V to see what they do when we add them and scale them. From

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \ \middle| \ x = 1 \right\}$$

we can choose any vectors where $x = 1$. Let's pick

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

If we add $\vec{v}_1$ and $\vec{v}_2$, we get

$$\vec{v}_1 + \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+1 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

The result isn't in V since its x component isn't $x = 1$, so V isn't closed under addition. If we multiply $\vec{v}_1$ by a scalar, let's choose 3, we get

$$3\vec{v}_1 = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

The result isn't in V since its x component isn't $x = 1$, so V isn't closed under scalar multiplication.