Topic: The elimination matrix

Question: Which elimination matrix accomplishes the row operation?

$$-2R_3 + R_1 \to R_1$$

Answer choices:

A $\quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

B $\quad E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C $\quad E = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

D $\quad E = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution: C

The row operation $-2R_3 + R_1 \to R_1$ means we're leaving the second and third rows alone, but replacing the first row with 1 of the first row and -2 of the third row.

So to get the elimination matrix that accomplishes the row operation, we'll put a 1 in $E_{1,1}$ and a -2 in $E_{1,3}$.

$$E = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Topic: The elimination matrix

Question: Which elimination matrix accomplishes the row operations?

$$(1/2)R_2 \to R_2$$

$$-R_2 + R_1 \to R_1$$

Answer choices:

A $E = \begin{bmatrix} 1 & -1 \\ 0 & \frac{1}{2} \end{bmatrix}$

B $E = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$

C $E = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

D $E = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 \end{bmatrix}$

Solution: B

The row operation $(1/2)R_2 \to R_2$ means we're leaving the first row alone, but multiplying the second row by a scalar of $1/2$, so we'll put a $1/2$ in $E_{2,2}$.

$$E_1 = \begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$$

The row operation $-R_2 + R_1 \to R_1$ means we're leaving the second row alone, but replacing the first row with 1 of the first row and -1 of the second row, so we'll put a 1 in $E_{1,1}$, and a -1 in $E_{1,2}$.

$$E_2 = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

We can consolidate these two row operations into one elimination matrix, simply by multiplying E_2 by E_1.

$$E = E_2E_1$$

$$E = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$$

$$E = \begin{bmatrix}1(1)+(-1)(0) & 1(0)+(-1)\left(\frac{1}{2}\right)\\ 0(1)+1(0) & 0(0)+1\left(\frac{1}{2}\right)\end{bmatrix}$$

$$E = \begin{bmatrix}1+0 & 0-\frac{1}{2}\\ 0+0 & 0+\frac{1}{2}\end{bmatrix}$$

$$E = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$$

Topic: The elimination matrix

Question: Which elimination matrix puts A

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

into reduced row-echelon form, where E accounts for this set of row operations?

1. $-R_1 \to R_1$

2. $-3R_3 + R_2 \to R_2$

3. $2R_2 + R_1 \to R_1$

Answer choices:

A $\quad E = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

B $\quad E = \begin{bmatrix} -1 & 2 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

C $\quad E = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

D $\quad E = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

Solution: B

The row operation $-R_1 \to R_1$ means we're leaving the second and third rows alone, but multiplying the first row by a scalar of -1, so we'll put a -1 into $E_{1,1}$.

$$E_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The row operation $-3R_3 + R_2 \to R_2$ means we're leaving the first and third rows alone, but replacing the second row with 1 of the second row and -3 of the third row, so we'll put a 1 in $E_{2,2}$ and a -3 in $E_{2,3}$.

$$E_2E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The row operation $2R_2 + R_1 \to R_1$ means we're leaving the second and third rows alone, but replacing the first row with 1 of the first row and a 2 of the second row, so we'll put a 1 in $E_{1,1}$ and a 2 in $E_{1,2}$.

$$E_3E_2E_1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now we can find the consolidated elimination matrix E by finding the product $E_3E_2E_1$.

$$E = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1(-1)+0(0)+0(0) & 1(0)+0(1)+0(0) & 1(0)+0(0)+0(1) \\ 0(-1)+1(0)-3(0) & 0(0)+1(1)-3(0) & 0(0)+1(0)-3(1) \\ 0(-1)+0(0)+1(0) & 0(0)+0(1)+1(0) & 0(0)+0(0)+1(1) \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1(-1)+2(0)+0(0) & 1(0)+2(1)+0(0) & 1(0)+2(-3)+0(1) \\ 0(-1)+1(0)+0(0) & 0(0)+1(1)+0(0) & 0(0)+1(-3)+0(1) \\ 0(-1)+0(0)+1(0) & 0(0)+0(1)+1(0) & 0(0)+0(-3)+1(1) \end{bmatrix}$$

$$E = \begin{bmatrix} -1 & 2 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

We’ve found the elimination matrix, and we can check to make sure that it reduces A to the identity matrix.

$$EA = \begin{bmatrix} -1 & 2 & -6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} -1(-1)+2(0)-6(0) & -1(2)+2(1)-6(0) & -1(0)+2(3)-6(1) \\ 0(-1)+1(0)-3(0) & 0(2)+1(1)-3(0) & 0(0)+1(3)-3(1) \\ 0(-1)+0(0)+1(0) & 0(2)+0(1)+1(0) & 0(0)+0(3)+1(1) \end{bmatrix}$$

$$EA = \begin{bmatrix} 1+0+0 & -2+2+0 & 0+6-6 \\ 0+0+0 & 0+1+0 & 0+3-3 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Because multiplying the elimination matrix by A gives us the identity matrix, we know that we got the correct elimination matrix.