Given two questions I was sent today, limits vs. continuity at end points needs clarification. Let me try again.

def: $f(x)$ is <u>continuous at a</u> if $\lim\limits_{x \to a} f(x) = f(a)$

This means ① $\lim\limits_{x \to a} f(x)$ is a real number

② $f(a)$ exists (a is in domain of f)

③ ① = ②

But what happens if a is the end point of a domain? For example, $f(x) = \sqrt{x}$ has a domain which is $[0, \infty)$. Is $f(x)$ continuous at $x = 0$? Yes! Continuity at end points of intervals only needs a one-sided limit -- see Definition 3 in 1.8 notes. So in this case; since 0 is an end point, $f(x) = \sqrt{x}$ is continuous at $x = 0$ because $\lim\limits_{x \to 0^+} \sqrt{x} = f(0) = 0$

↖ one-sided limit!

<u>However</u> if we were just asked to evaluate $\lim\limits_{x \to 0} \sqrt{x}$, we would in this case say DNE.

✱ (This is where I may have confused some of you!)

$\lim\limits_{x \to 0^+} \sqrt{x} = \sqrt{0} = 0$ because when approaching from the right, all x are positive so can take $\sqrt{x}$

$\lim\limits_{x \to 0^-} \sqrt{x}$ DNE   because when approaching from left, x are negative, $\sqrt{x}$ is undefined -- we cant approach from left.

<u>So</u> $0 \neq$ DNE therefore $\lim\limits_{x \to 0} \sqrt{x}$ DNE, as one sided limits aren't same real number.