Scalar multiplication

We've learned about matrix addition and subtraction, and in this lesson we want to start looking at matrix multiplication.

Before we work through matrix multiplication, let's first walk through what it looks like to multiply an entire matrix by a scalar.

Scalar multiplication

We talked earlier about how to multiply a single row by a scalar, but you can also multiply an entire matrix by a scalar.

If we want to multiply a matrix by 3, then 3 is the called the **scalar**, we distribute the scalar across every entry in the matrix, and the result of the scalar multiplication looks like this:

$$3\begin{bmatrix} 6 & 2 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} 3(6) & 3(2) \\ 3(-1) & 3(-4) \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ -3 & -12 \end{bmatrix}$$

Notice that this also translates to dividing through a matrix by a scalar. If you want to divide through a matrix by 6, that's the same as multiplying through the matrix by $1/6$, and we're right back to the same kind of scalar multiplication.

$$\frac{1}{6}\begin{bmatrix} 6 & 2 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{6}(6) & \frac{1}{6}(2) \\ \frac{1}{6}(-1) & \frac{1}{6}(-4) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2}{3} \end{bmatrix}$$

This translates as well to the kinds of matrix equations we solved in the last lesson.

Example

Solve the equation for K.

$$2\begin{bmatrix} 5 & -7 \\ -1 & 0 \end{bmatrix} + K = -3\begin{bmatrix} 2 & 4 \\ 11 & -9 \end{bmatrix}$$

Apply the scalars to the matrices.

$$\begin{bmatrix} 2(5) & 2(-7) \\ 2(-1) & 2(0) \end{bmatrix} + K = \begin{bmatrix} -3(2) & -3(4) \\ -3(11) & -3(-9) \end{bmatrix}$$

$$\begin{bmatrix} 10 & -14 \\ -2 & 0 \end{bmatrix} + K = \begin{bmatrix} -6 & -12 \\ -33 & 27 \end{bmatrix}$$

Subtract the matrix on the left from both sides of the equation in order to isolate K.

$$K = \begin{bmatrix} -6 & -12 \\ -33 & 27 \end{bmatrix} - \begin{bmatrix} 10 & -14 \\ -2 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} -6-10 & -12-(-14) \\ -33-(-2) & 27-0 \end{bmatrix}$$

$$K = \begin{bmatrix} -16 & 2 \\ -31 & 27 \end{bmatrix}$$

Multiplying by a zero scalar

What happens when we multiply a matrix by a zero scalar? Well, like any other scalar, we distribute the 0 across every entry in the matrix,

$$(0)\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 9(0) & -6(0) & 2(0) \\ 1(0) & 0(0) & -7(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and we just end up with a matrix full of zeros. Any matrix that's full of only zeros, regardless of the dimensions of the matrix, we call a "zero matrix."