Topic: Spans as subspaces

Question: Is V a subspace?

$$V = \text{Span}\left(\begin{bmatrix} -3 \\ -2 \end{bmatrix}\right)$$

Answer choices:

A Yes

B No, because it's not closed under addition

C No, because it's not closed under scalar multiplication

D No, because it's not closed under addition or scalar multiplication

Solution: A

The set V is a subspace, because V is a span of a vector, and a span is always a subspace.

The span of a vector is all the possible linear combinations of that vector. For instance, we could create the linear combination

$$c_1\begin{bmatrix}-3\\-2\end{bmatrix}+c_2\begin{bmatrix}-3\\-2\end{bmatrix}+c_3\begin{bmatrix}-3\\-2\end{bmatrix}$$

and then factor out the vector.

$$(c_1+c_2+c_3)\begin{bmatrix}-3\\-2\end{bmatrix}$$

The constant $(c_1+c_2+c_3)$ is still just a constant, which means we could rewrite the linear combination as

$$c_4\begin{bmatrix}-3\\-2\end{bmatrix}$$

This result is still just a linear combination of the vector in the set, which means it's still contained within the span. Therefore, the set is closed under addition.

And because multiplying a linear combination of the vector by a scalar still just gives a linear combination of the vector, the set is also closed under scalar multiplication.

Topic: Spans as subspaces

Question: Is V a subspace?

$$V = \text{Span}\left(\begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}\right)$$

Answer choices:

A Yes

B No, because it's not closed under addition

C No, because it's not closed under scalar multiplication

D No, because it's not closed under addition or scalar multiplication

Solution: A

The set V is a subspace, because V is a span of vectors, and a span is always a subspace.

The span of vectors is all the possible linear combinations of those vectors. For instance, we could create the linear combination

$$c_1\begin{bmatrix}1\\-4\end{bmatrix}+c_2\begin{bmatrix}1\\-4\end{bmatrix}+c_3\begin{bmatrix}0\\2\end{bmatrix}+c_4\begin{bmatrix}0\\2\end{bmatrix}$$

and then factor out each vector.

$$(c_1+c_2)\begin{bmatrix}1\\-4\end{bmatrix}+(c_3+c_4)\begin{bmatrix}0\\2\end{bmatrix}$$

The constants (c_1+c_2) and (c_3+c_4) are still just constants, which means we could rewrite the linear combination as

$$c_5\begin{bmatrix}1\\-4\end{bmatrix}+c_6\begin{bmatrix}0\\2\end{bmatrix}$$

This result is still just a linear combination of the vectors in the set, which means it's still contained within the span. Therefore, the set is closed under addition.

And because multiplying a linear combination of the vectors by a scalar still just gives a linear combination of the vectors, the set is also closed under scalar multiplication.

Topic: Spans as subspaces

Question: Is V a subspace?

$$V = \text{Span}\left(\begin{bmatrix}1\\0\\-6\end{bmatrix}, \begin{bmatrix}-2\\1\\3\end{bmatrix}\right)$$

Answer choices:

A Yes

B No, because it's not closed under addition

C No, because it's not closed under scalar multiplication

D No, because it's not closed under addition or scalar multiplication

Solution: A

The set V is a subspace, because V is a span of vectors, and a span is always a subspace.

The span of vectors is all the possible linear combinations of those vectors. For instance, we could create the linear combination

$$c_1\begin{bmatrix}1\\0\\-6\end{bmatrix}+c_2\begin{bmatrix}1\\0\\-6\end{bmatrix}+c_3\begin{bmatrix}-2\\1\\3\end{bmatrix}+c_4\begin{bmatrix}-2\\1\\3\end{bmatrix}$$

and then factor out each vector.

$$(c_1+c_2)\begin{bmatrix}1\\0\\-6\end{bmatrix}+(c_3+c_4)\begin{bmatrix}-2\\1\\3\end{bmatrix}$$

The constants (c_1+c_2) and (c_3+c_4) are still just constants, which means we could rewrite the linear combination as

$$c_5\begin{bmatrix}1\\0\\-6\end{bmatrix}+c_6\begin{bmatrix}-2\\1\\3\end{bmatrix}$$

This result is still just a linear combination of the vectors in the set, which means it's still contained within the span. Therefore, the set is closed under addition.

And because multiplying a linear combination of the vectors by a scalar still just gives a linear combination of the vectors, the set is also closed under scalar multiplication.