CMP 334 (3/6/19)

HW 8 (two's complement circuit)

Combinational circuit design

- Inverters, decoders, multiplexers

Signed Arithmetic (review)

Condition flags and comparisons

Arithmetic / **L**ogical **U**nits

- HW 7 (`W ← X + Y + Z`)
- ALU operations (explicit and implicit)
- Condition flags and conditional branches
- The **TOY** ALU

HW 8: Two's Complement Circuit

Use the four step *Combinational Circuit Design Process* presented in class to design circuits that take a 3-bit unsigned binary integer **X** as input and produces as output a 3-bit unsigned binary integer **Y** that is the two's complement of **X**.

$$\mathbf{Y = 2^3 - X = \overline{X} + 1}$$

Do not minimize the circuits for **Y** as a part of this assignments.

Extra credit: minimize the circuits for **Y**.

Combinational Circuit Design

Combinational circuit

Output determined by input

Design process

1. Specify semantics
   - Black Box: *input* and *output* (informal semantics)
   - Truth Table (formal semantics)
2. Truth table → Boolean formula
3. Minimize boolean formula (optional)
   - Boolean algebra
   - Karnaugh maps
4. Boolean formula → combinational circuit

Combinational Circuit Design

Combinational circuit

Output determined by input

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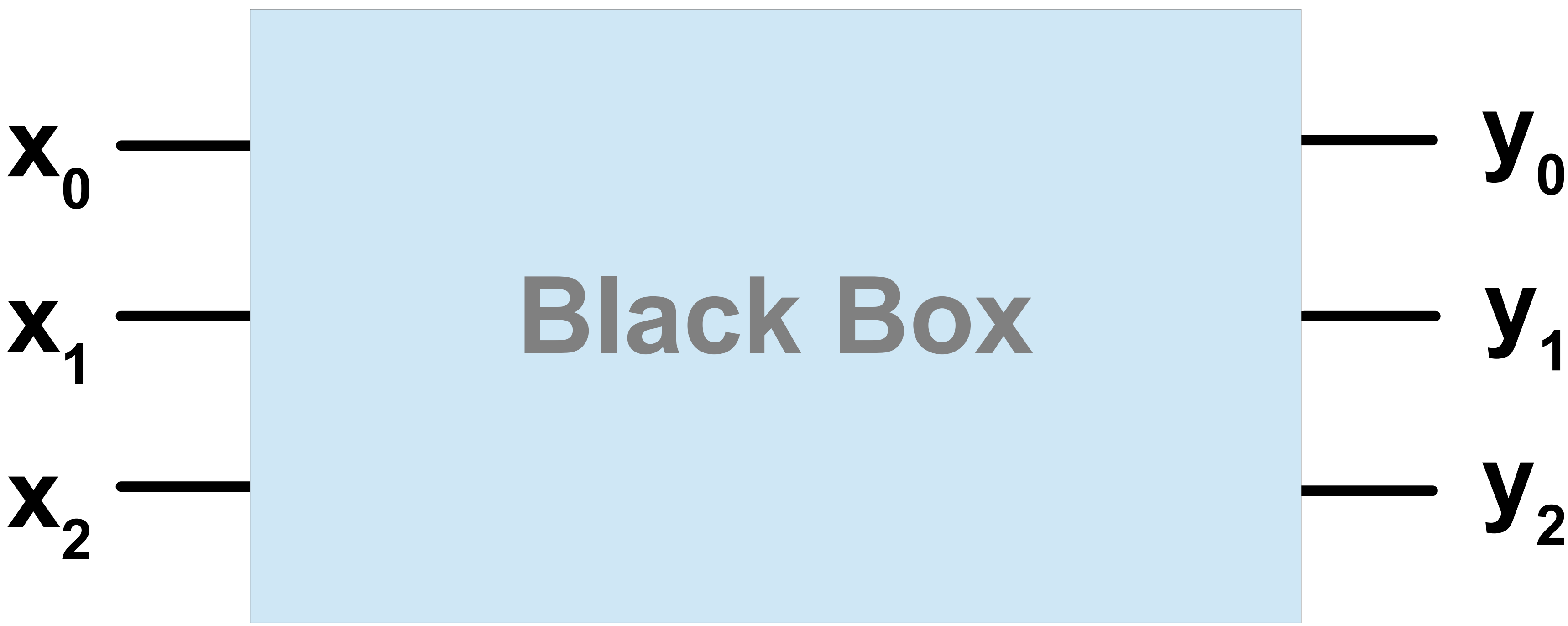
HW 8: Two's Complement



Informal semantics:

$$\mathbf{X} = 4 \cdot x_2 + 2 \cdot x_1 + x_0$$

$$\mathbf{Y} = 4 \cdot y_2 + 2 \cdot y_1 + y_0 = \ddddot{\mathbf{X}} = 8 - \mathbf{X} = \overline{\mathbf{X}} + 1$$

Two's Complement Truth Table

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
	0	0	0				
	0	0	1				
	0	1	0				
	0	1	1				
	1	0	0				
	1	0	1				
	1	1	0				
	1	1	1				

Two's Complement Truth Table

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

Two's Complement Truth Table

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
0	0	0	0	8			
1	0	0	1	7			
2	0	1	0	6			
3	0	1	1	5			
4	1	0	0	4			
5	1	0	1	3			
6	1	1	0	2			
7	1	1	1	1			

Two's Complement Truth Table

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
0	0	0	0	8	0	0	0
1	0	0	1	7	1	1	1
2	0	1	0	6	1	1	0
3	0	1	1	5	1	0	1
4	1	0	0	4	1	0	0
5	1	0	1	3	0	1	1
6	1	1	0	2	0	1	0
7	1	1	1	1	0	0	1

Combinational Circuit Design

Combinational circuit

Output determined by input

Design process

1. Specify semantics

   Black Box: *input* and *output* (informal semantics)

   Truth Table (formal semantics)

2. Truth table → Boolean formula

3. Minimize boolean formula (optional)

   Boolean algebra

   Karnaugh maps

4. Boolean formula → combinational circuit

Boolean Formula for $\mathbf{y}_0$

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
0	0	0	0	8	0	0	**0**
1	0	0	1	7	1	1	**1**
2	0	1	0	6	1	1	**0**
3	0	1	1	5	1	0	**1**
4	1	0	0	4	1	0	**0**
5	1	0	1	3	0	1	**1**
6	1	1	0	2	0	1	**0**
7	1	1	1	1	0	0	**1**

Boolean Formula for $\mathbf{y}_0$

X	x_2	x_1	x_0	Y	y_0
0	0	0	0	8	0
1	0	0	1	7	1
2	0	1	0	6	0
3	0	1	1	5	1
4	1	0	0	4	0
5	1	0	1	3	1
6	1	1	0	2	0
7	1	1	1	1	1

Boolean Formula for $\mathbf{y}_0$

X	x_2	x_1	x_0	Y	y_0	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{x}_2\overline{x}_1x_0$
2	0	1	0	6	0	
3	0	1	1	5	1	
4	1	0	0	4	0	
5	1	0	1	3	1	
6	1	1	0	2	0	
7	1	1	1	1	1	

Boolean Formula for $\mathbf{y}_0$

X	x_2	x_1	x_0	Y	y_0	
0	0	0	0	8	0	
1	0	0	1	7	1	$\bar{x}_2\bar{x}_1x_0$
2	0	1	0	6	0	
3	0	1	1	5	1	$\bar{x}_2x_1x_0$
4	1	0	0	4	0	
5	1	0	1	3	1	
6	1	1	0	2	0	
7	1	1	1	1	1	

Boolean Formula for $\mathbf{y}_0$

X	x_2	x_1	x_0	Y	y_0	
0	0	0	0	8	0	
1	0	0	1	7	1	$\bar{x}_2\bar{x}_1x_0$
2	0	1	0	6	0	
3	0	1	1	5	1	$\bar{x}_2x_1x_0$
4	1	0	0	4	0	
5	1	0	1	3	1	$x_2\bar{x}_1x_0$
6	1	1	0	2	0	
7	1	1	1	1	1	

Boolean Formula for $\mathbf{y}_0$

X	x_2	x_1	x_0	Y	y_0	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{x}_2\overline{x}_1x_0$
2	0	1	0	6	0	
3	0	1	1	5	1	$\overline{x}_2x_1x_0$
4	1	0	0	4	0	
5	1	0	1	3	1	$x_2\overline{x}_1x_0$
6	1	1	0	2	0	
7	1	1	1	1	1	$x_2x_1x_0$

$$y_0 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

X	x_2	x_1	x_0	Y	y_0	
0	0	0	0	8	0	
1	0	0	1	7	1	$\bar{x}_2\bar{x}_1x_0$
2	0	1	0	6	0	
3	0	1	1	5	1	$\bar{x}_2x_1x_0$
4	1	0	0	4	0	
5	1	0	1	3	1	$x_2\bar{x}_1x_0$
6	1	1	0	2	0	
7	1	1	1	1	1	$x_2x_1x_0$

Boolean Formula for $\mathbf{y}_1$

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
0	0	0	0	8	0	**0**	0
1	0	0	1	7	1	**1**	1
2	0	1	0	6	1	**1**	0
3	0	1	1	5	1	**0**	1
4	1	0	0	4	1	**0**	0
5	1	0	1	3	0	**1**	1
6	1	1	0	2	0	**1**	0
7	1	1	1	1	0	**0**	1

Boolean Formula for y_1

X	x_2	x_1	x_0	Y	y_1
0	0	0	0	8	0
1	0	0	1	7	1
2	0	1	0	6	1
3	0	1	1	5	0
4	1	0	0	4	0
5	1	0	1	3	1
6	1	1	0	2	1
7	1	1	1	1	0

Boolean Formula for $\mathbf{y}_1$

X	x_2	x_1	x_0	Y	y_1
0	0	0	0	8	0
1	0	0	1	7	1
2	0	1	0	6	1
3	0	1	1	5	0
4	1	0	0	4	0
5	1	0	1	3	1
6	1	1	0	2	1
7	1	1	1	1	0

$\overline{x}_2\overline{x}_1x_0$

Boolean Formula for $\mathbf{y}_1$

X	x_2	x_1	x_0	Y	y_1	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{x}_2\overline{x}_1x_0$
2	0	1	0	6	1	$\overline{x}_2x_1\overline{x}_0$
3	0	1	1	5	0	
4	1	0	0	4	0	
5	1	0	1	3	1	
6	1	1	0	2	1	
7	1	1	1	1	0	

Boolean Formula for $\mathbf{y}_1$

$\mathbf{X}$	$\mathbf{x_2}$	$\mathbf{x_1}$	$\mathbf{x_0}$	$\mathbf{Y}$	$\mathbf{y_1}$	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0$
2	0	1	0	6	1	$\overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0$
3	0	1	1	5	0	
4	1	0	0	4	0	
5	1	0	1	3	1	$\mathbf{x}_2\overline{\mathbf{x}}_1\mathbf{x}_0$
6	1	1	0	2	1	
7	1	1	1	1	0	

Boolean Formula for $\mathbf{y}_1$

X	x_2	x_1	x_0	Y	y_1	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{x}_2\overline{x}_1x_0$
2	0	1	0	6	1	$\overline{x}_2x_1\overline{x}_0$
3	0	1	1	5	0	
4	1	0	0	4	0	
5	1	0	1	3	1	$x_2\overline{x}_1x_0$
6	1	1	0	2	1	$x_2x_1\overline{x}_0$
7	1	1	1	1	0	

$$y_1 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1\bar{x}_0 + x_2\bar{x}_1x_0 + x_2x_1\bar{x}_0$$

X	x_2	x_1	x_0	Y	y_1	
0	0	0	0	8	0	
1	0	0	1	7	1	$\bar{x}_2\bar{x}_1x_0$
2	0	1	0	6	1	$\bar{x}_2x_1\bar{x}_0$
3	0	1	1	5	0	
4	1	0	0	4	0	
5	1	0	1	3	1	$x_2\bar{x}_1x_0$
6	1	1	0	2	1	$x_2x_1\bar{x}_0$
7	1	1	1	1	0	

Boolean Formula for $\mathbf{y}_2$

X	x_2	x_1	x_0	Y	y_2	y_1	y_0
0	0	0	0	8	**0**	0	0
1	0	0	1	7	**1**	1	1
2	0	1	0	6	**1**	1	0
3	0	1	1	5	**1**	0	1
4	1	0	0	4	**1**	0	0
5	1	0	1	3	**0**	1	1
6	1	1	0	2	**0**	1	0
7	1	1	1	1	**0**	0	1

Boolean Formula for $\mathbf{y}_2$

X	x_2	x_1	x_0	Y	y_2
0	0	0	0	8	0
1	0	0	1	7	1
2	0	1	0	6	1
3	0	1	1	5	1
4	1	0	0	4	1
5	1	0	1	3	0
6	1	1	0	2	0
7	1	1	1	1	0

Boolean Formula for $\mathbf{y}_2$

X	x_2	x_1	x_0	Y	y_2
0	0	0	0	8	0
1	0	0	1	7	1
2	0	1	0	6	1
3	0	1	1	5	1
4	1	0	0	4	1
5	1	0	1	3	0
6	1	1	0	2	0
7	1	1	1	1	0

$\bar{x}_2\bar{x}_1x_0$

Boolean Formula for $\mathbf{y}_2$

$\mathbf{X}$	$\mathbf{x_2}$	$\mathbf{x_1}$	$\mathbf{x_0}$	$\mathbf{Y}$	$\mathbf{y_2}$	
0	0	0	0	8	0	
1	0	0	1	7	1	$\bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0$
2	0	1	0	6	1	$\bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0$
3	0	1	1	5	1	
4	1	0	0	4	1	
5	1	0	1	3	0	
6	1	1	0	2	0	
7	1	1	1	1	0	

Boolean Formula for $\mathbf{y}_2$

X	x_2	x_1	x_0	Y	y_2	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{x}_2\overline{x}_1x_0$
2	0	1	0	6	1	$\overline{x}_2x_1\overline{x}_0$
3	0	1	1	5	1	$\overline{x}_2x_1x_0$
4	1	0	0	4	1	
5	1	0	1	3	0	
6	1	1	0	2	0	
7	1	1	1	1	0	

Boolean Formula for $\mathbf{y}_2$

X	x_2	x_1	x_0	Y	y_2	
0	0	0	0	8	0	
1	0	0	1	7	1	$\bar{x}_2\bar{x}_1x_0$
2	0	1	0	6	1	$\bar{x}_2x_1\bar{x}_0$
3	0	1	1	5	1	$\bar{x}_2x_1x_0$
4	1	0	0	4	1	$x_2\bar{x}_1\bar{x}_0$
5	1	0	1	3	0	
6	1	1	0	2	0	
7	1	1	1	1	0	

$$y_2 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1\overline{x}_0 + \overline{x}_2x_1x_0 + x_2\overline{x}_1\overline{x}_0$$

X	x_2	x_1	x_0	Y	y_2	
0	0	0	0	8	0	
1	0	0	1	7	1	$\overline{x}_2\overline{x}_1x_0$
2	0	1	0	6	1	$\overline{x}_2x_1\overline{x}_0$
3	0	1	1	5	1	$\overline{x}_2x_1x_0$
4	1	0	0	4	1	$x_2\overline{x}_1\overline{x}_0$
5	1	0	1	3	0	
6	1	1	0	2	0	
7	1	1	1	1	0	

Two's Complement Formulas

$$\mathbf{y}_0 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\mathbf{x}_0$$

$$\mathbf{y}_1 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\overline{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Combinational Circuit Design

Combinational circuit

Output determined by input

Design process

1. Specify semantics
   - Black Box input and output (informal semantics)
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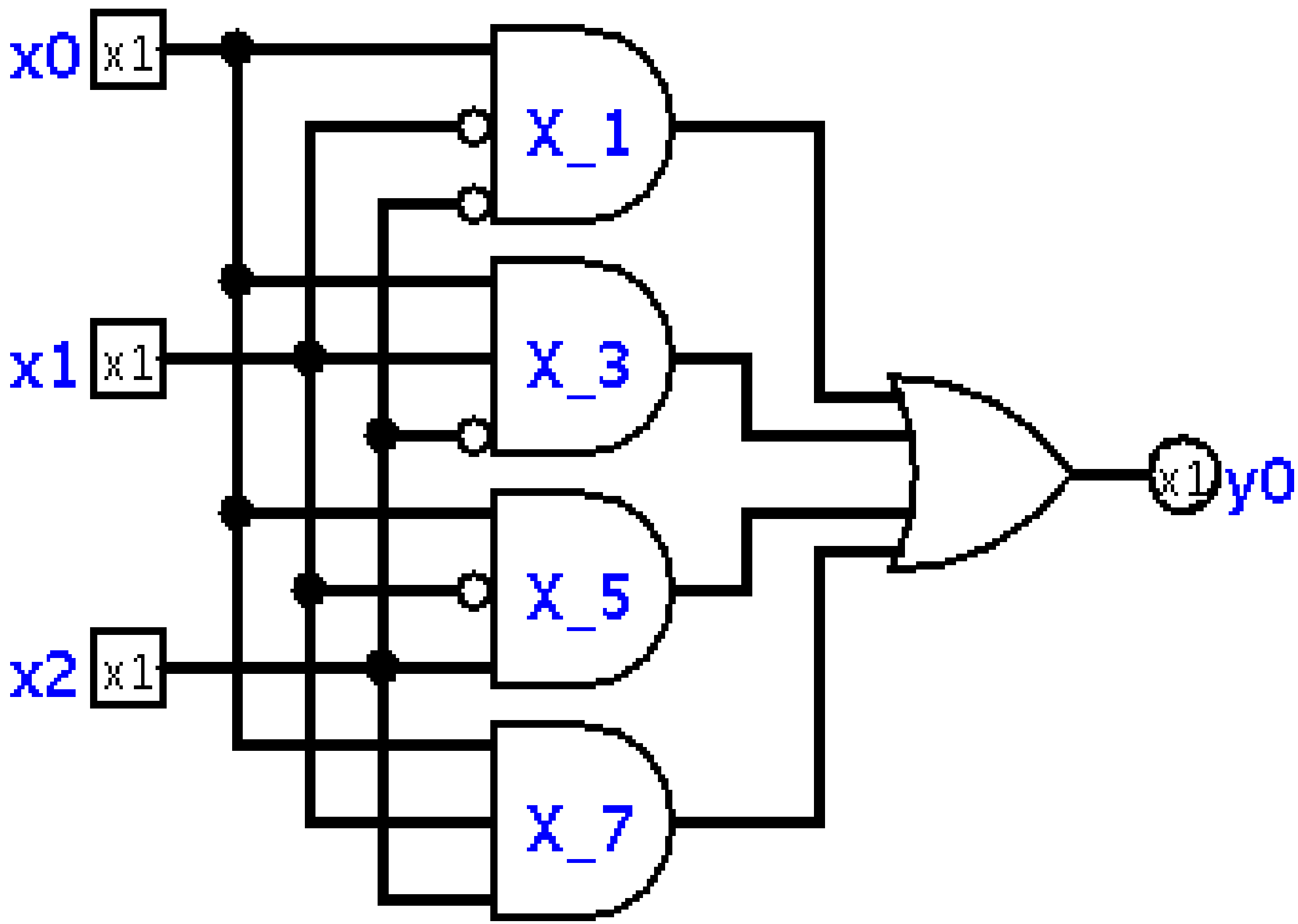
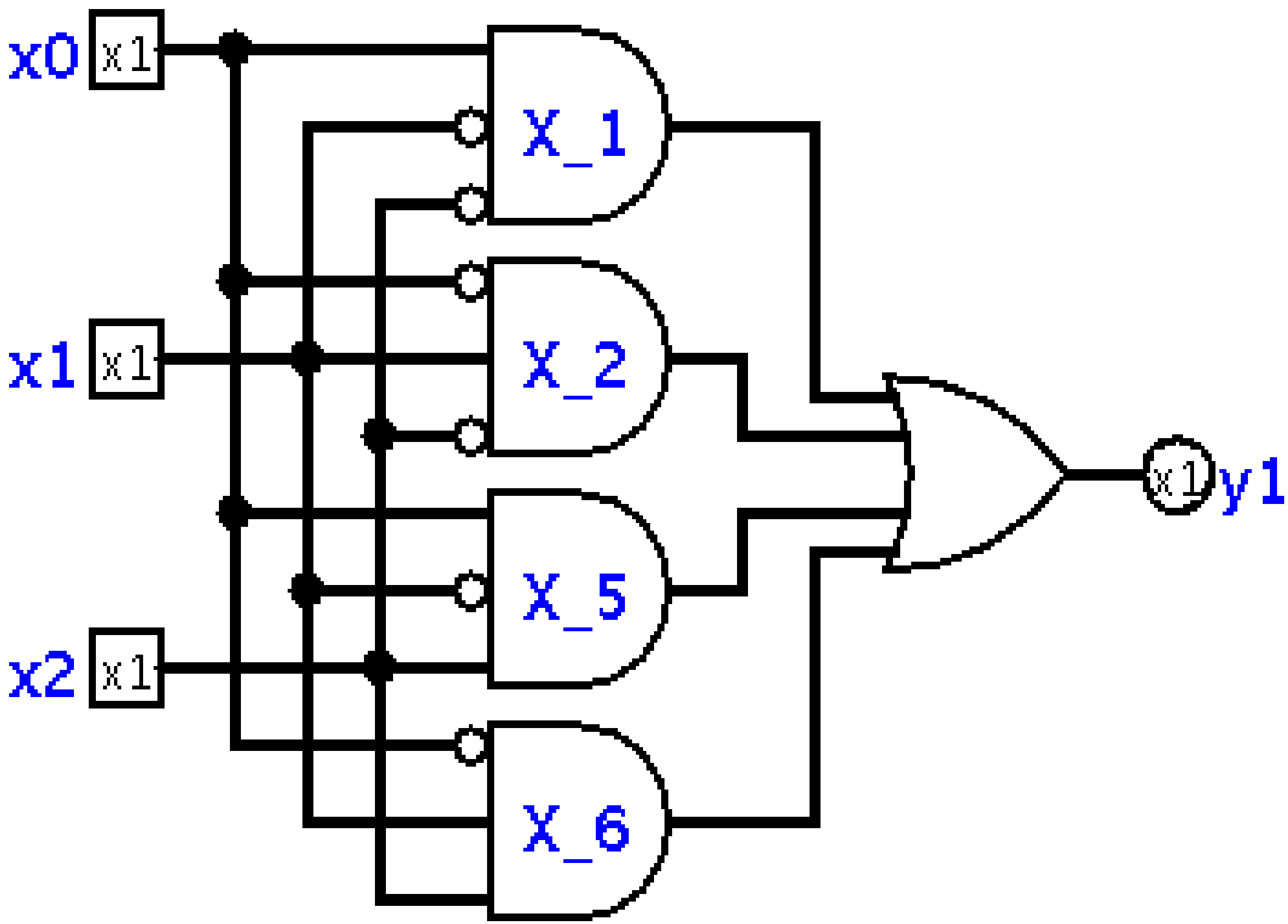
2. Truth table → Boolean formula
3. Minimize boolean formula (optional)
   - Boolean algebra
   - Karnaugh maps
4. Boolean formula → combinational circuit

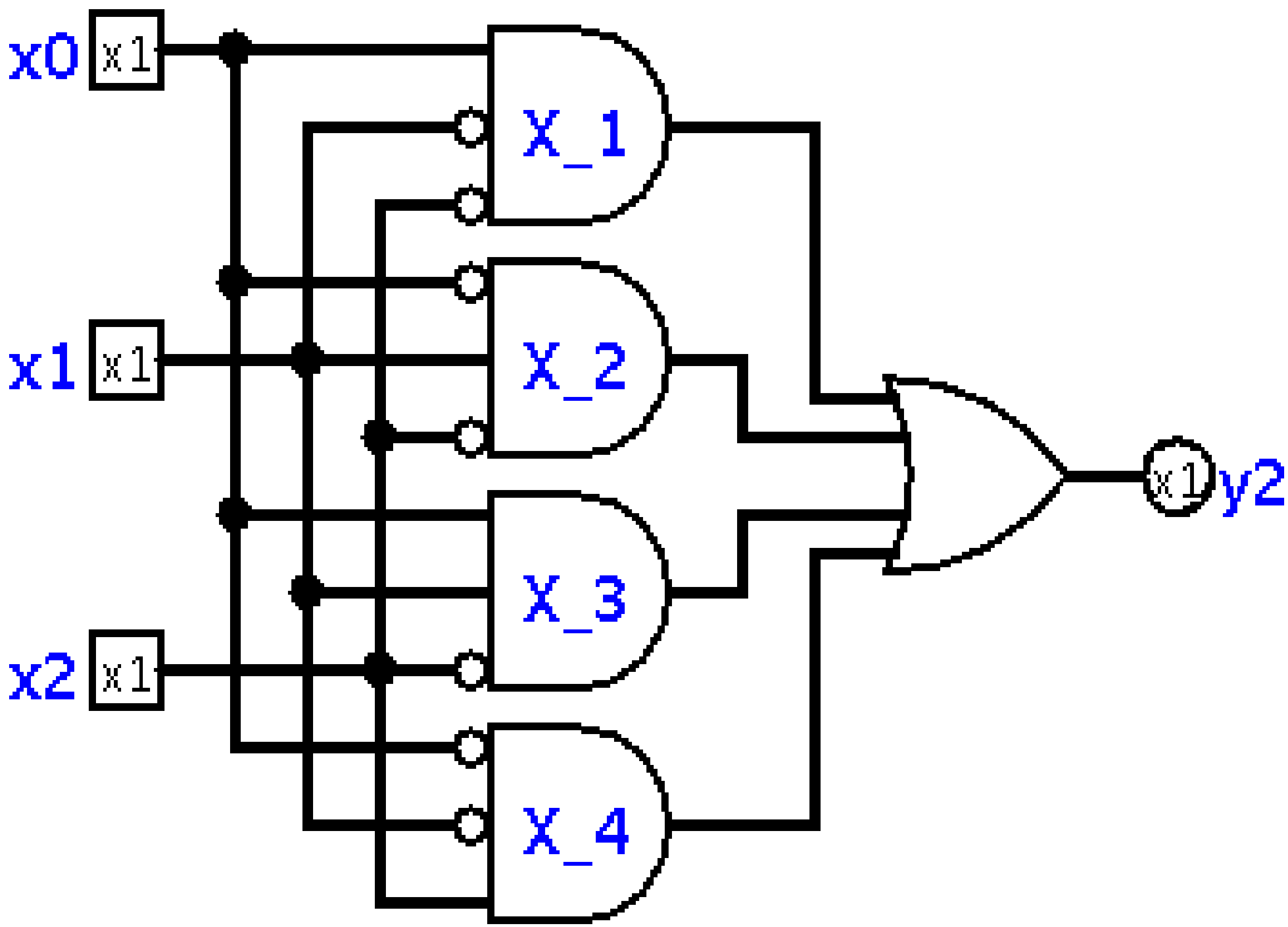
$$y_0 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$



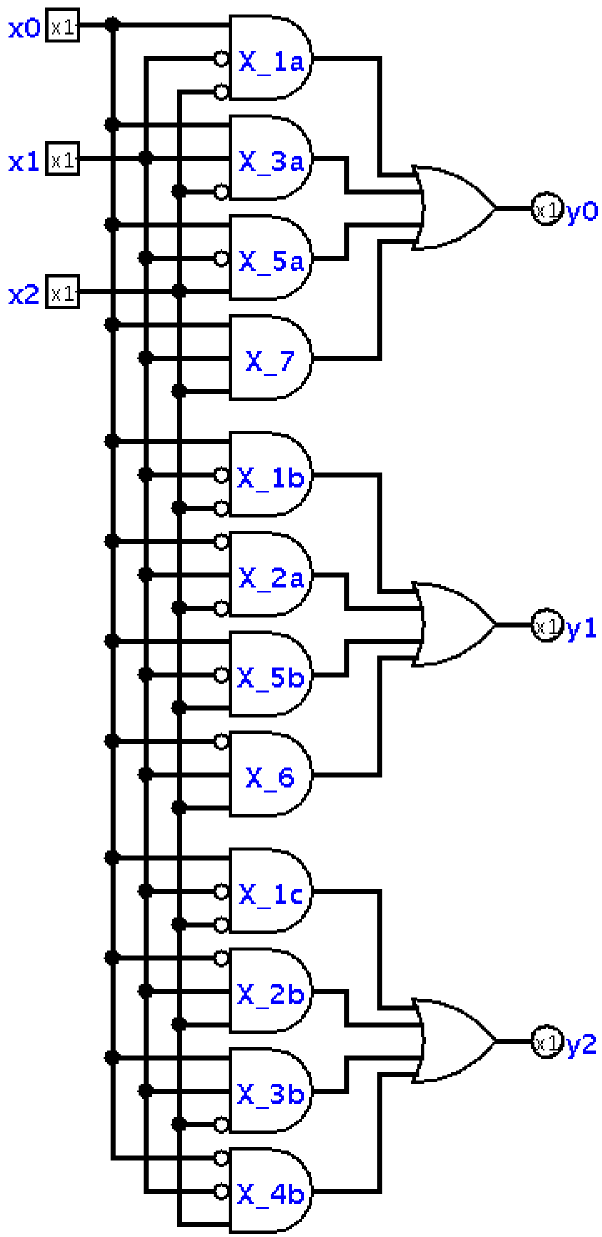
$$y_1 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1\bar{x}_0 + x_2\bar{x}_1x_0 + x_2x_1\bar{x}_0$$



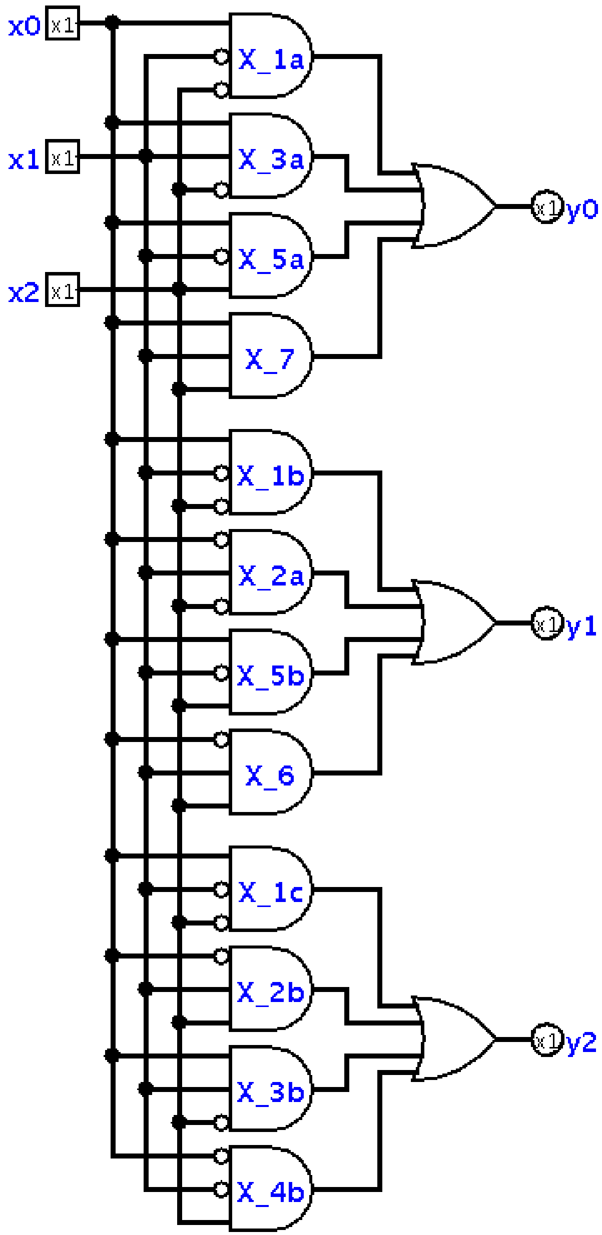
$$y_2 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1\bar{x}_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1\bar{x}_0$$

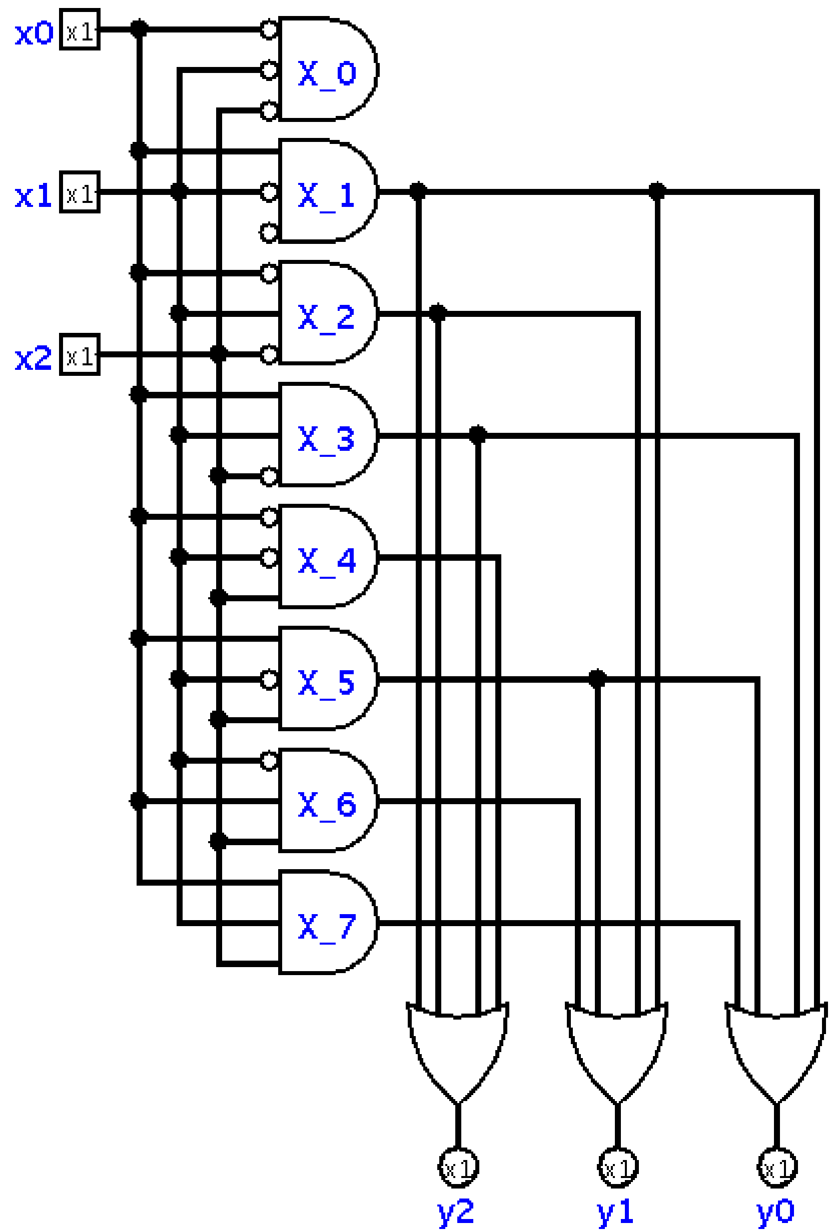


Two's Complement Circuits



Two's Complement Gate Array





Combinational Circuit Design

Combinational circuit

Output determined by input

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1. Specify semantics

   Black Box: *input* and *output* (informal semantics)

   Truth Table (formal semantics)

2. Truth table → Boolean formula
3. Minimize boolean formula (optional)

   Boolean algebra

   Karnaugh maps

4. Boolean formula → combinational circuit

Simplification of Boolean Formulas

$$\mathbf{y_0} = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\mathbf{x}_0$$

$$\mathbf{y}_1 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\overline{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_0$ Formula

$$\mathbf{y_0 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1x_0 + x_2\overline{x}_1x_0 + x_2x_1x_0}$$

$$y_1 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1\overline{x}_0 + x_2\overline{x}_1x_0 + x_2x_1\overline{x}_0$$

$$y_2 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1\overline{x}_0 + \overline{x}_2x_1x_0 + x_2\overline{x}_1\overline{x}_0$$

Simplification of $\mathbf{y}_0$ Formula

$$\mathbf{y_0 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1x_0 + x_2\overline{x}_1x_0 + x_2x_1x_0}$$

Simplification of $\mathbf{y}_0$ Formula

$$\mathbf{y_0 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1x_0 + x_2\overline{x}_1x_0 + x_2x_1x_0}$$

Simplification of $\mathbf{y}_0$ Formula

$$\mathbf{y_0 = \overline{x}_2\overline{x}_1x_0 + \overline{x}_2x_1x_0 + x_2\overline{x}_1x_0 + x_2x_1x_0}$$

$$\mathbf{y_0 = \overline{x}_2x_0 + x_2\overline{x}_1x_0 + x_2x_1x_0}$$

Simplification of $\mathbf{y}_0$ Formula

$$\mathbf{y_0 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0}$$

$$\mathbf{y_0 = \bar{x}_2x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0}$$

Simplification of y_0 Formula

$$y_0 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

$$y_0 = \bar{x}_2x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

$$y_0 = \bar{x}_2x_0 + x_2x_0$$

Simplification of y_0 Formula

$$y_0 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

$$y_0 = \bar{x}_2x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

$$y_0 = \bar{x}_2x_0 + x_2x_0$$

Simplification of $\mathbf{y}_0$ Formula

$$y_0 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

$$y_0 = \bar{x}_2x_0 + x_2\bar{x}_1x_0 + x_2x_1x_0$$

$$y_0 = \bar{x}_2x_0 + x_2x_0$$

$$y_0 = x_0$$

Simplification of $\mathbf{y}_1$ Formula

$$\mathbf{y}_0 = \mathbf{x}_0$$

$$\mathbf{y}_1 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_1$ Formula

$$\mathbf{y}_1 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\overline{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_1$ Formula

$$\mathbf{y}_1 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_1$ Formula

$$\mathbf{y}_1 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_1 = \bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_1$ Formula

$$\mathbf{y}_1 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_1 = \bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_1$ Formula

$$\mathbf{y}_1 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_1 = \bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \mathbf{x}_2\mathbf{x}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_1 = \bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\bar{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_2$ Formula

$$\mathbf{y}_0 = \mathbf{x}_0$$

$$\mathbf{y}_1 = \overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\overline{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_2$ Formula

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_2$ Formula

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_2$ Formula

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Simplification of $\mathbf{y}_2$ Formula

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\overline{\mathbf{x}}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \overline{\mathbf{x}}_2\overline{\mathbf{x}}_1\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1 + \overline{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$$

Simplification of y_2 Formula

$$y_2 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1\bar{x}_0 + \bar{x}_2x_1x_0 + x_2\bar{x}_1\bar{x}_0$$

$$y_2 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1 \qquad\qquad\qquad + x_2\bar{x}_1\bar{x}_0$$

$$y_2 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1 + \bar{x}_2x_1x_0 + x_2\bar{x}_1\bar{x}_0$$

$$y_2 = \bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1 + \bar{x}_2x_1x_0 + x_2\bar{x}_1\bar{x}_0$$

Simplification of $\mathbf{y}_2$ Formula

$$\mathbf{y}_2 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\bar{\mathbf{x}}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1 + \bar{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \bar{\mathbf{x}}_2\bar{\mathbf{x}}_1\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1 + \bar{\mathbf{x}}_2\mathbf{x}_1\mathbf{x}_0 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$$

$$\mathbf{y}_2 = \bar{\mathbf{x}}_2\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$$

$$y_2 = \overline{x}_2 x_0 + \overline{x}_2 x_1 + x_2 \overline{x}_1 \overline{x}_0$$

$$y_2 = \overline{x}_2 \overline{x}_1 x_0 + \overline{x}_2 x_1 \overline{x}_0 + \overline{x}_2 x_1 x_0 + x_2 \overline{x}_1 \overline{x}_0$$

$$y_2 = \overline{x}_2 \overline{x}_1 x_0 + \overline{x}_2 x_1 + x_2 \overline{x}_1 \overline{x}_0$$

$$y_2 = \overline{x}_2 \overline{x}_1 x_0 + \overline{x}_2 x_1 + \overline{x}_2 x_1 x_0 + x_2 \overline{x}_1 \overline{x}_0$$

$$y_2 = \overline{x}_2 \overline{x}_1 x_0 + \overline{x}_2 x_1 + \overline{x}_2 x_1 x_0 + x_2 \overline{x}_1 \overline{x}_0$$

$$y_2 = \overline{x}_2 x_0 + \overline{x}_2 x_1 + x_2 \overline{x}_1 \overline{x}_0$$

Simplified 2's Complement Formulas

$$y_0 = x_0$$

$$y_1 = \overline{x}_1 x_0 + x_1 \overline{x}_0$$

$$y_2 = \overline{x}_2 x_0 + \overline{x}_2 x_1 + x_2 \overline{x}_1 \overline{x}_0$$

Combinational Circuit Design

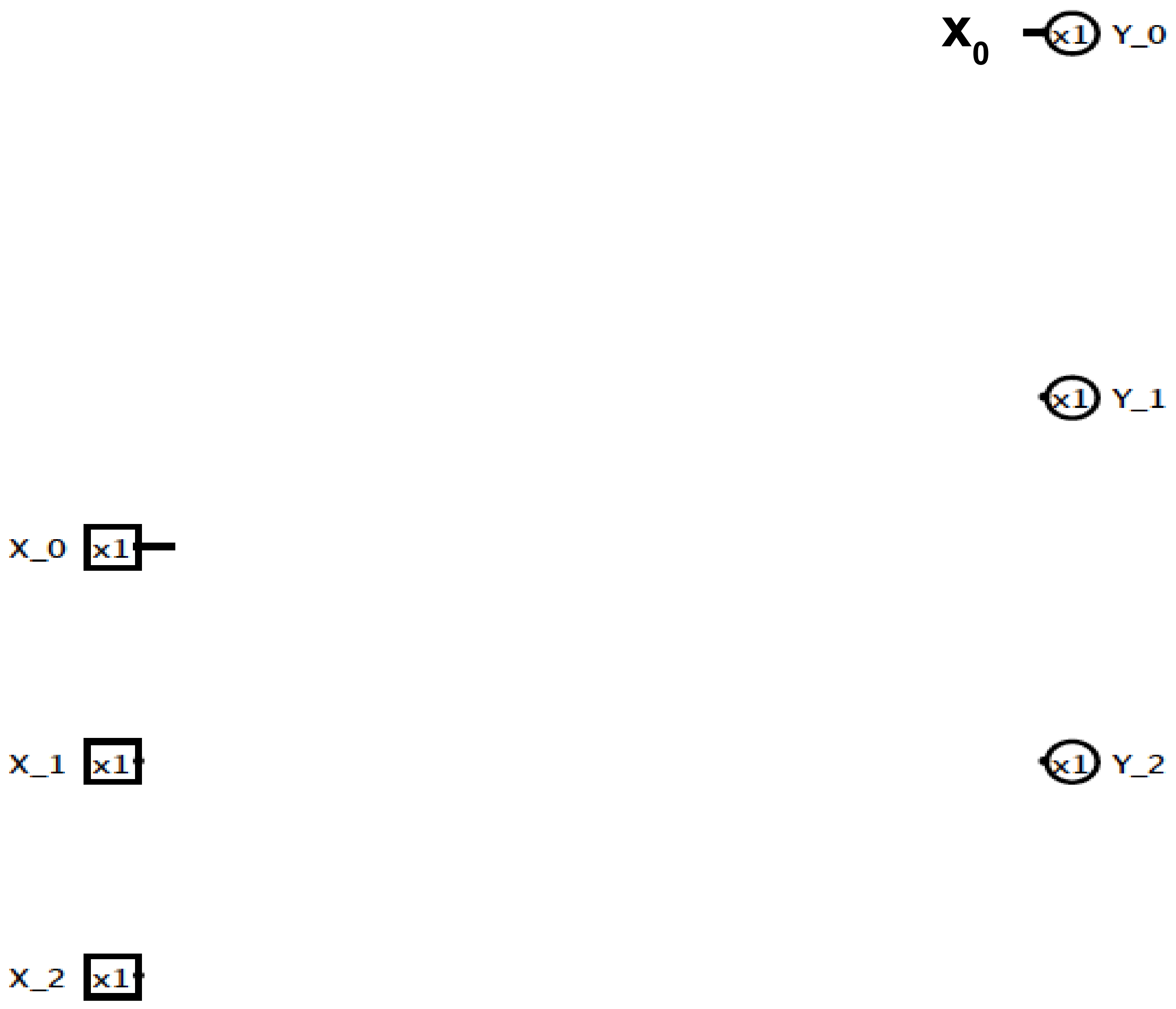
Combinational circuit

Output determined by input

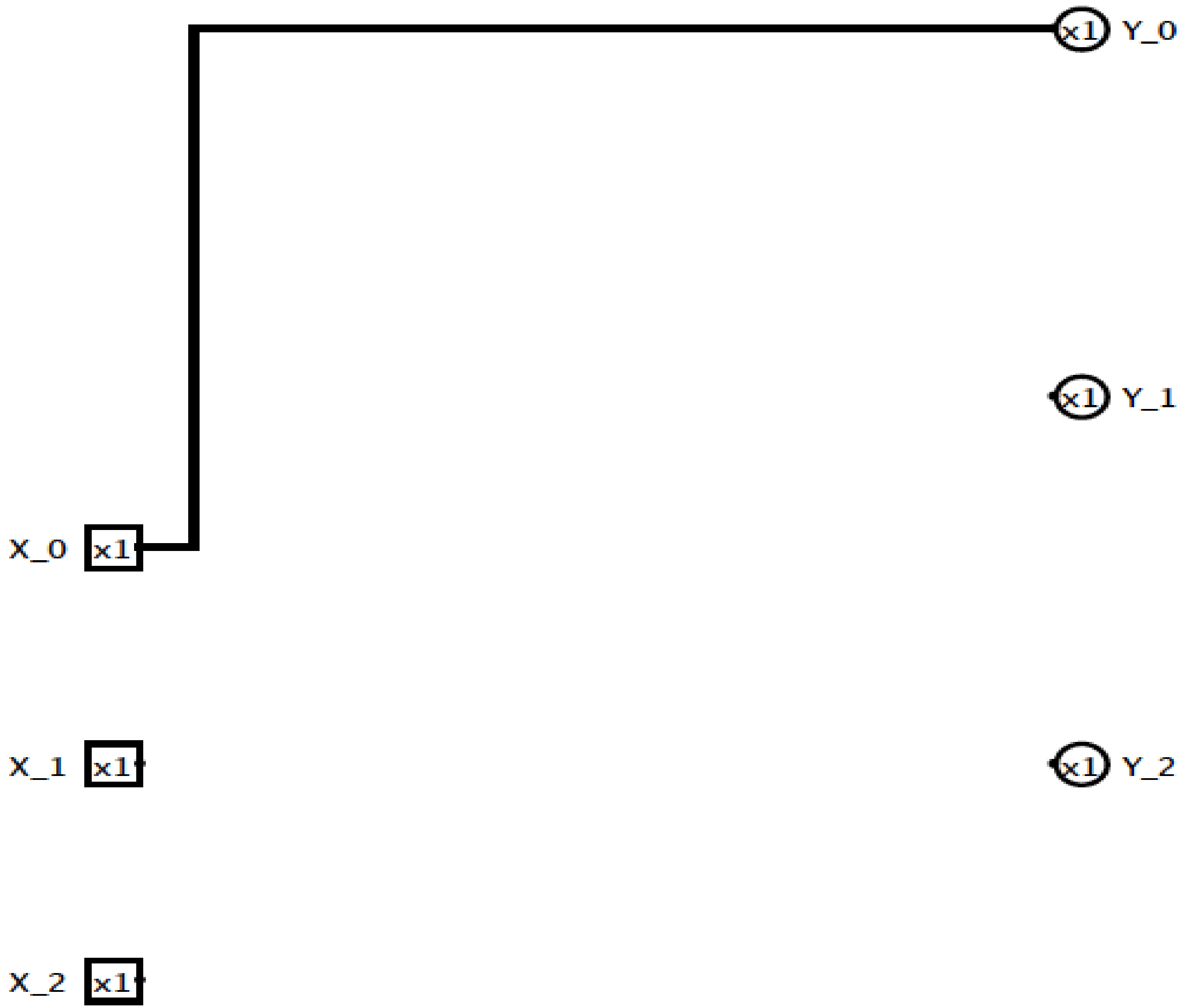
Design process

1. Specify semantics
   - Black Box input and output (informal semantics)
   - Truth Table (formal semantics)
2. Truth table → Boolean formula
3. Minimize boolean formula (optional)
   - Boolean algebra
   - Karnaugh maps
4. Boolean formula → combinational circuit

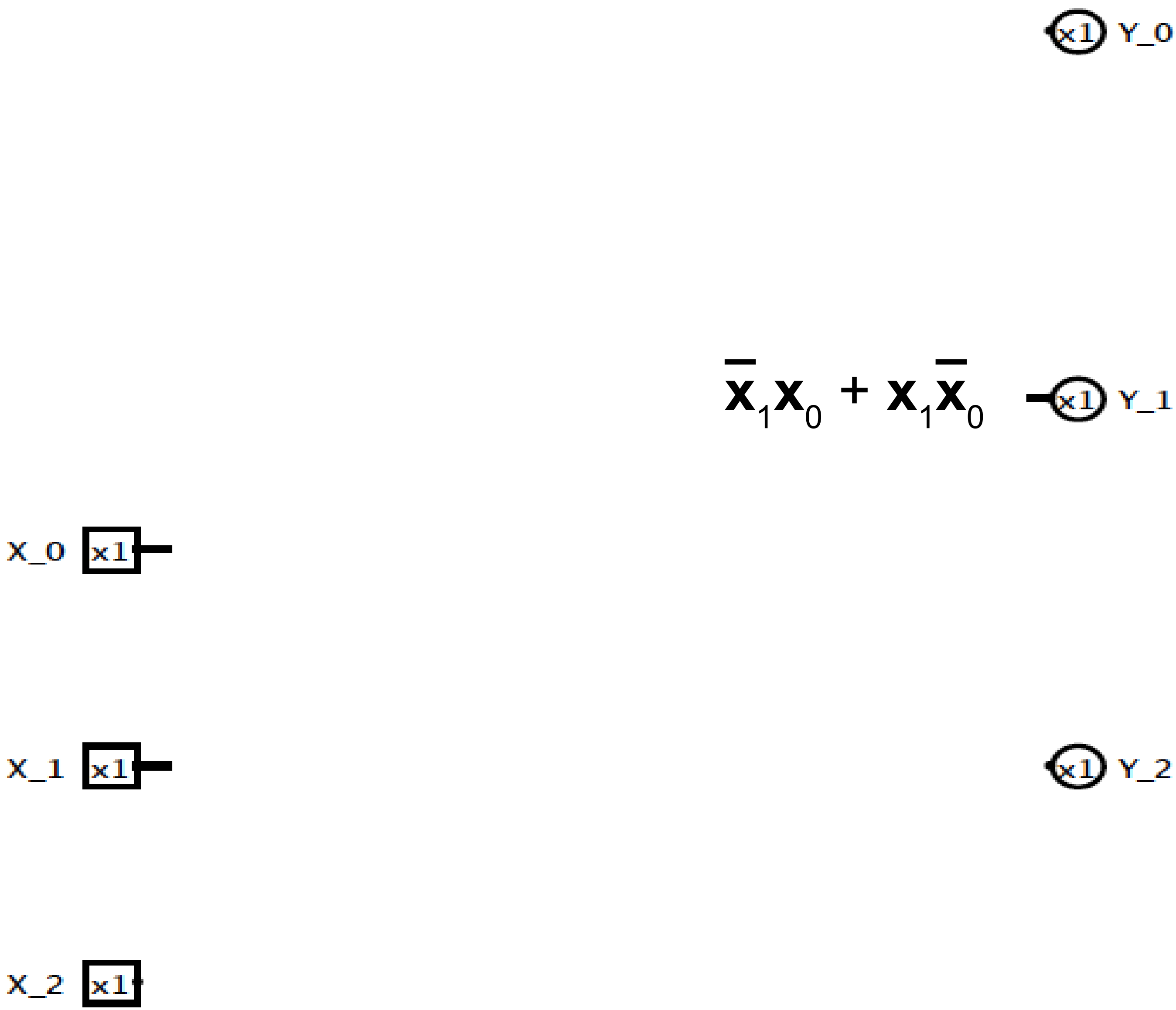
Circuit: $y_0 = x_0$



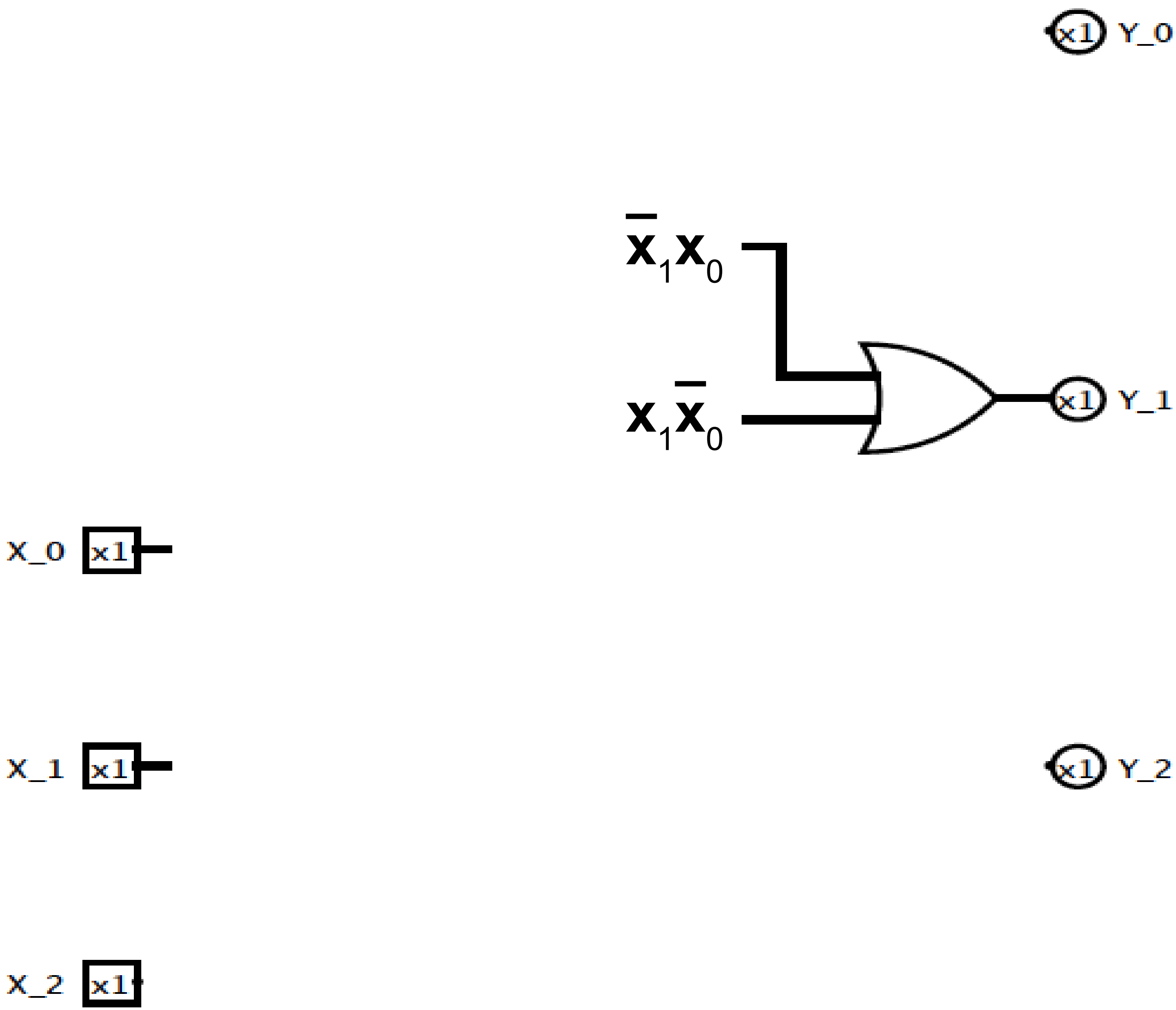
Circuit: $\mathbf{y_0 = x_0}$



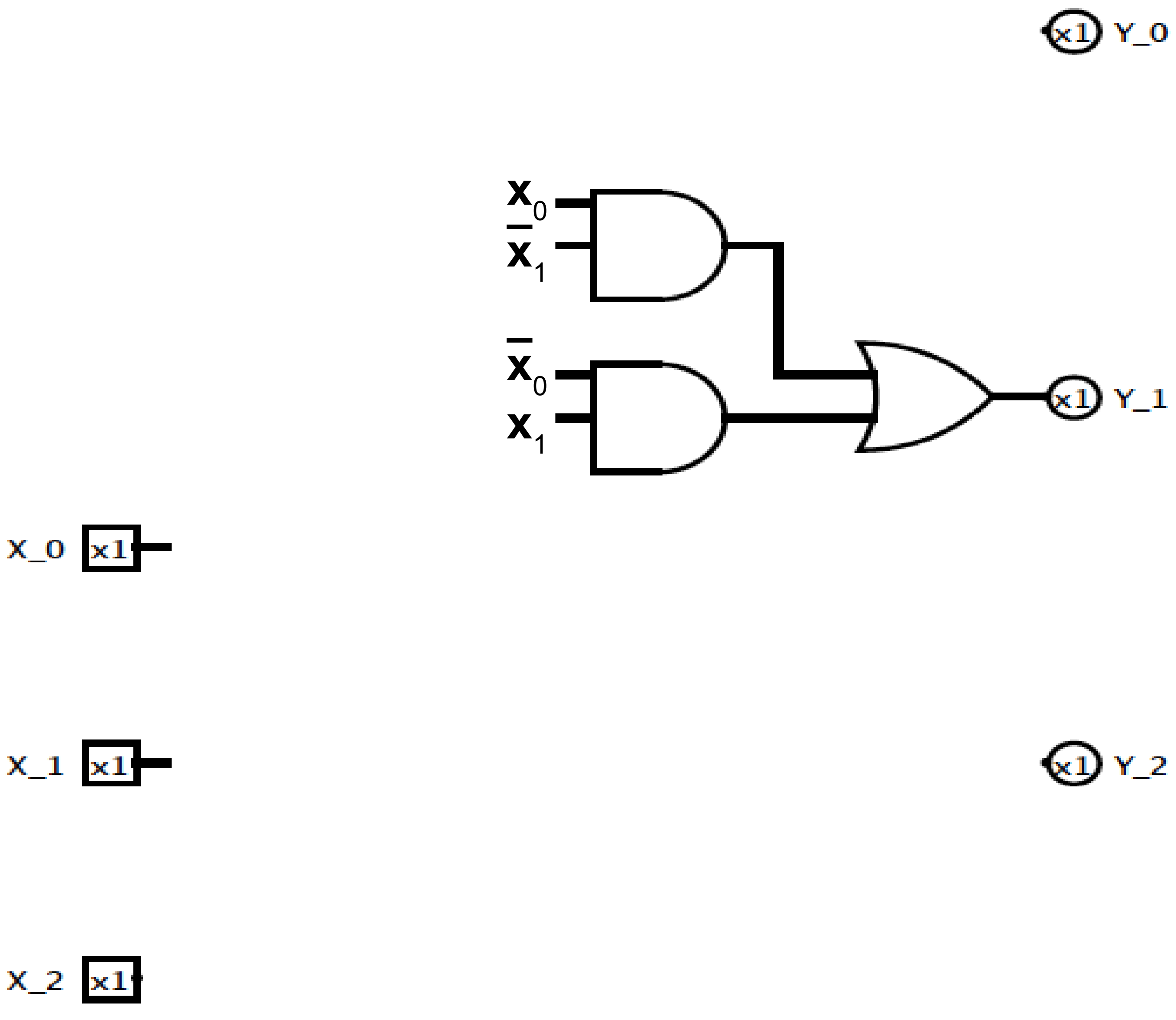
Circuit: $\mathbf{y}_1 = \overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\overline{\mathbf{x}}_0$



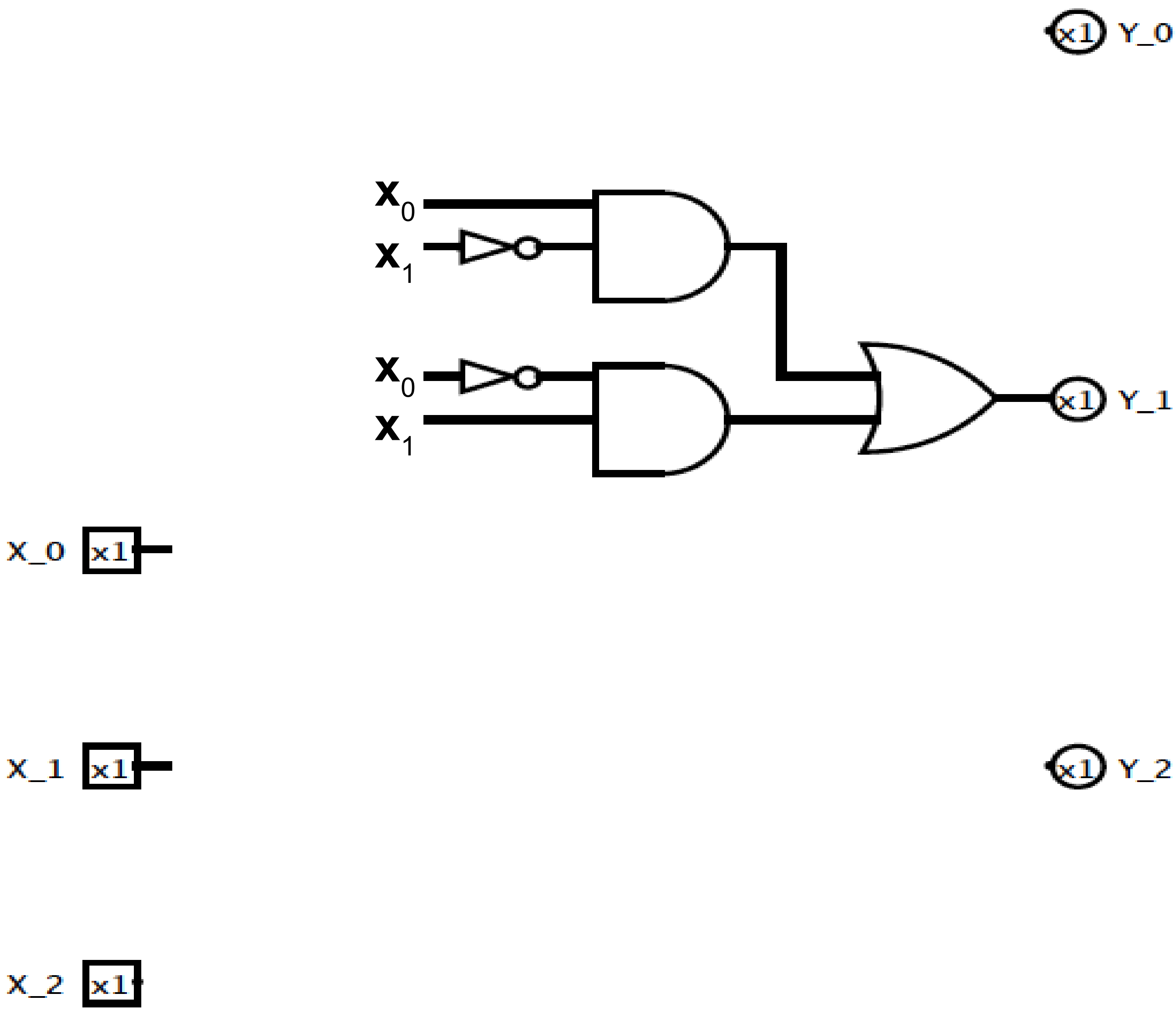
Circuit: $\mathbf{y}_1 = \overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\overline{\mathbf{x}}_0$



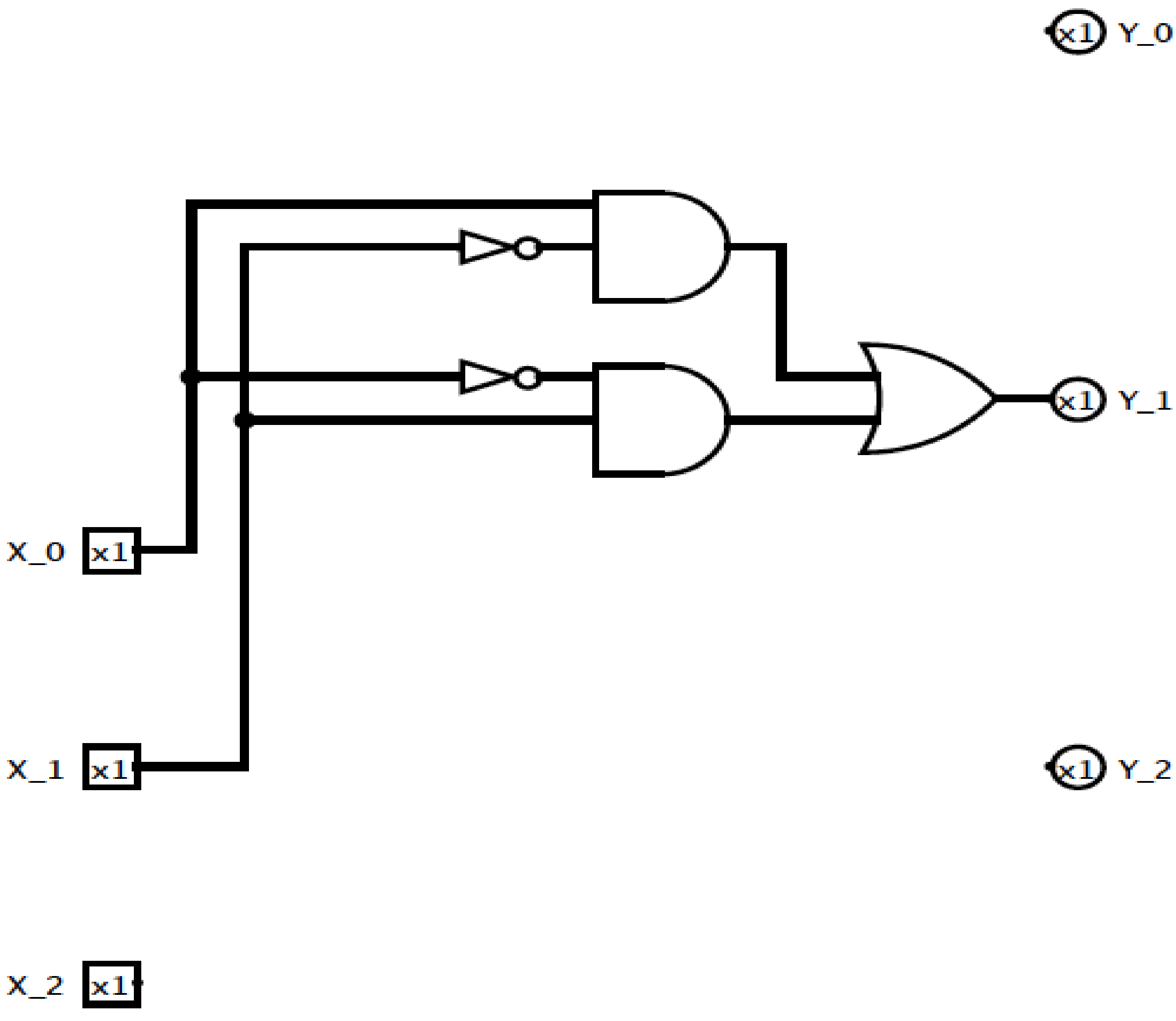
Circuit: $\mathbf{y}_1 = \bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\bar{\mathbf{x}}_0$



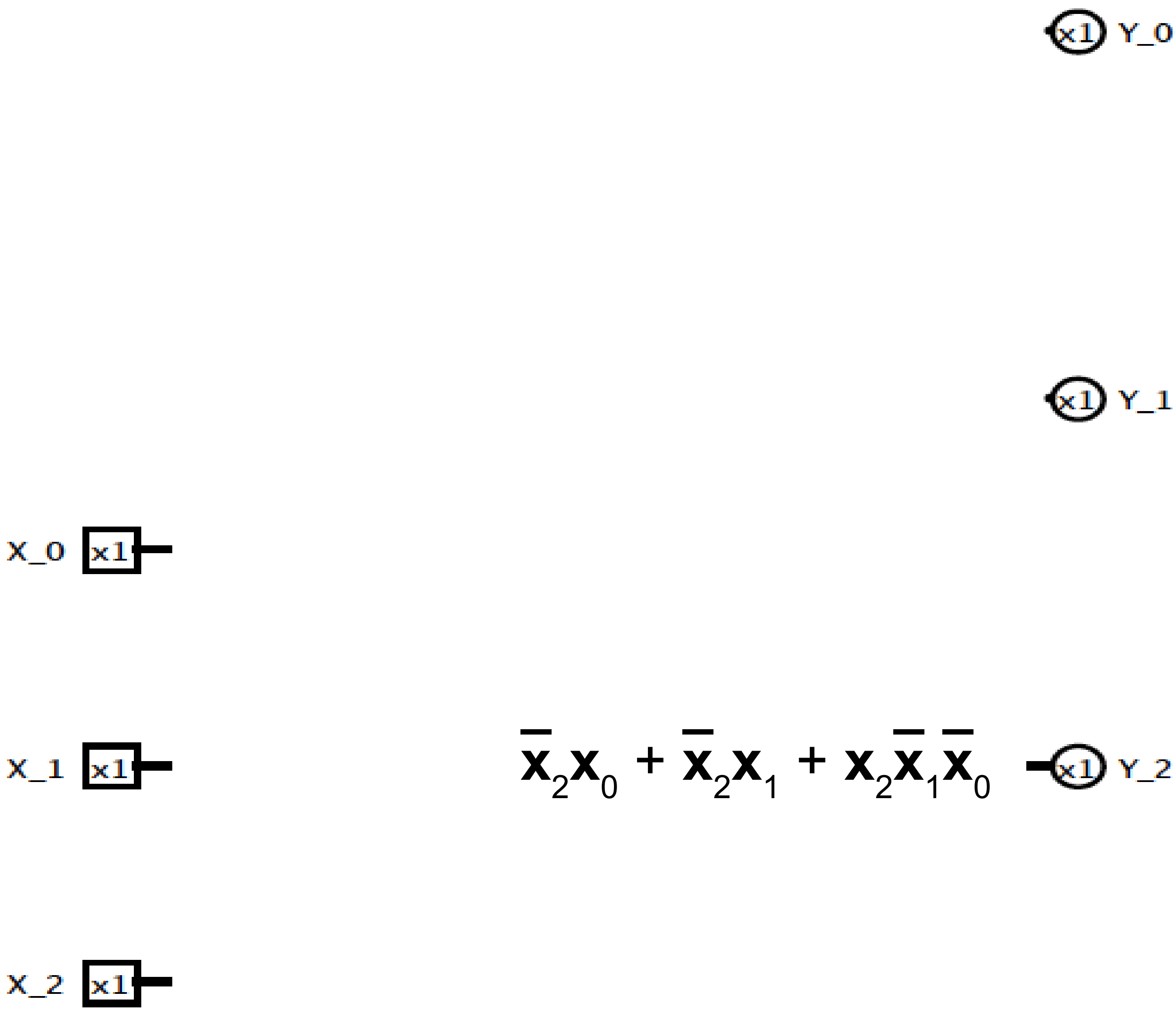
Circuit: $\mathbf{y}_1 = \overline{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\overline{\mathbf{x}}_0$



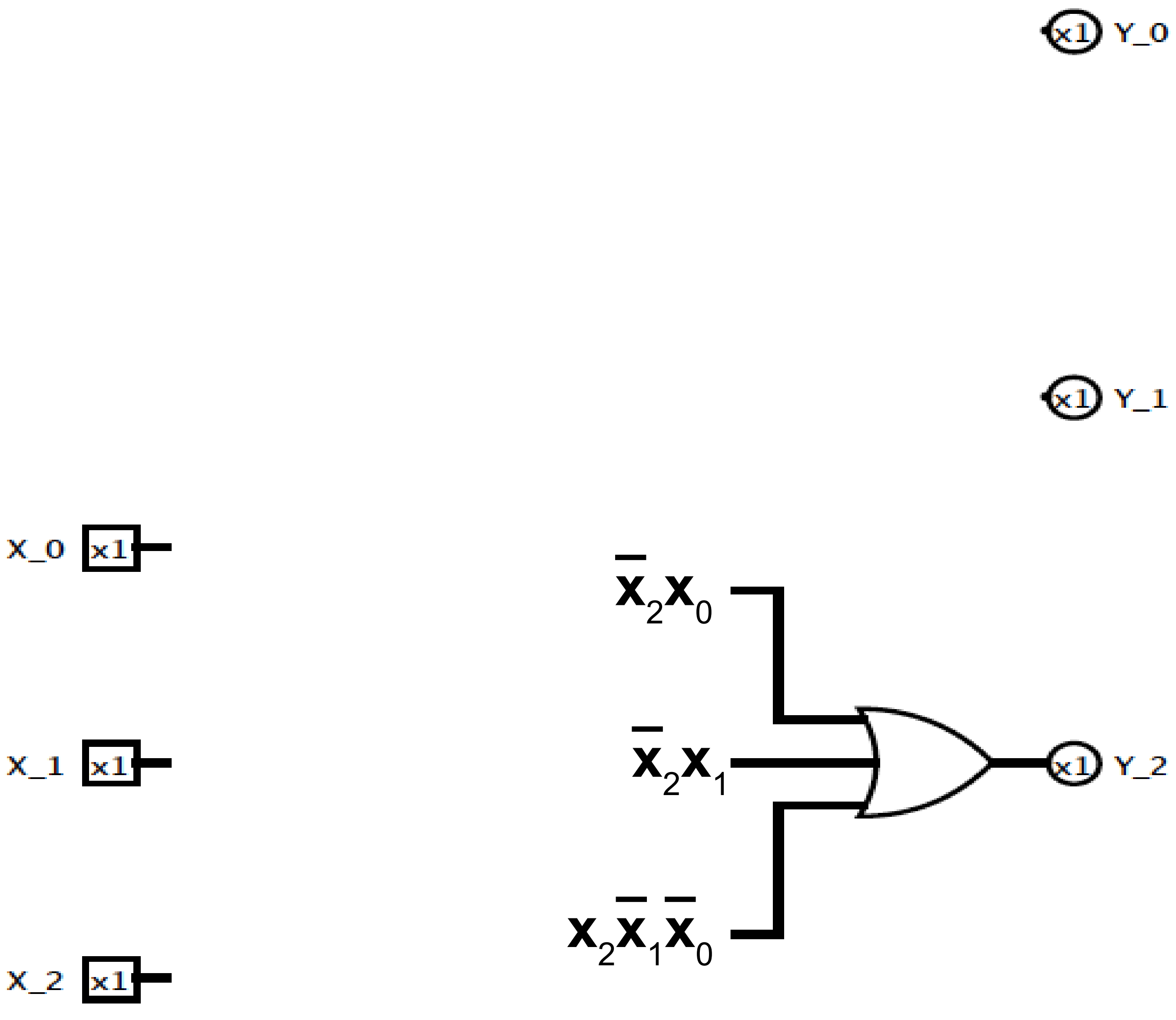
Circuit: $\mathbf{y}_1 = \bar{\mathbf{x}}_1\mathbf{x}_0 + \mathbf{x}_1\bar{\mathbf{x}}_0$



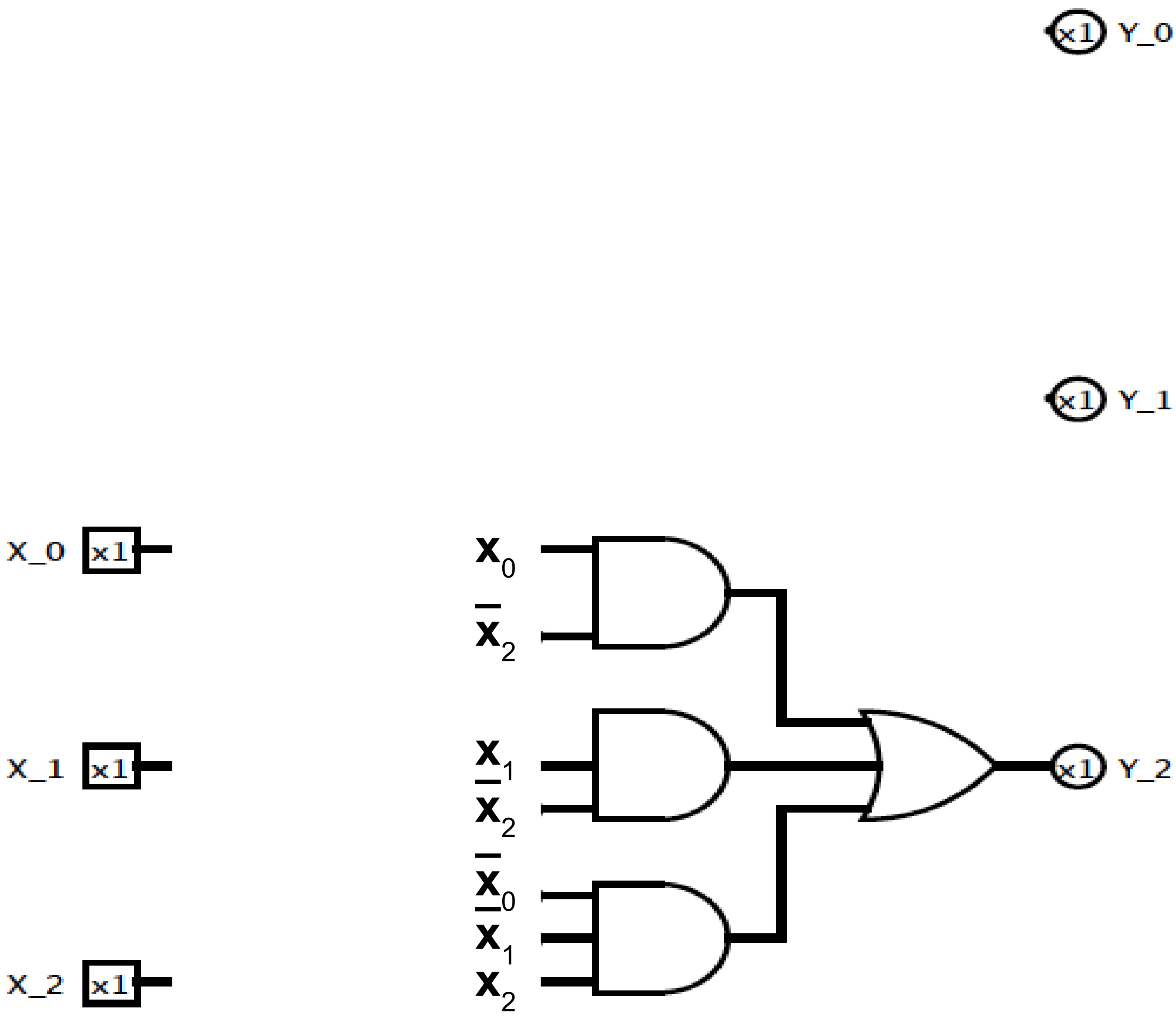
Circuit: $y_2 = \bar{x}_2 x_0 + \bar{x}_2 x_1 + x_2 \bar{x}_1 \bar{x}_0$



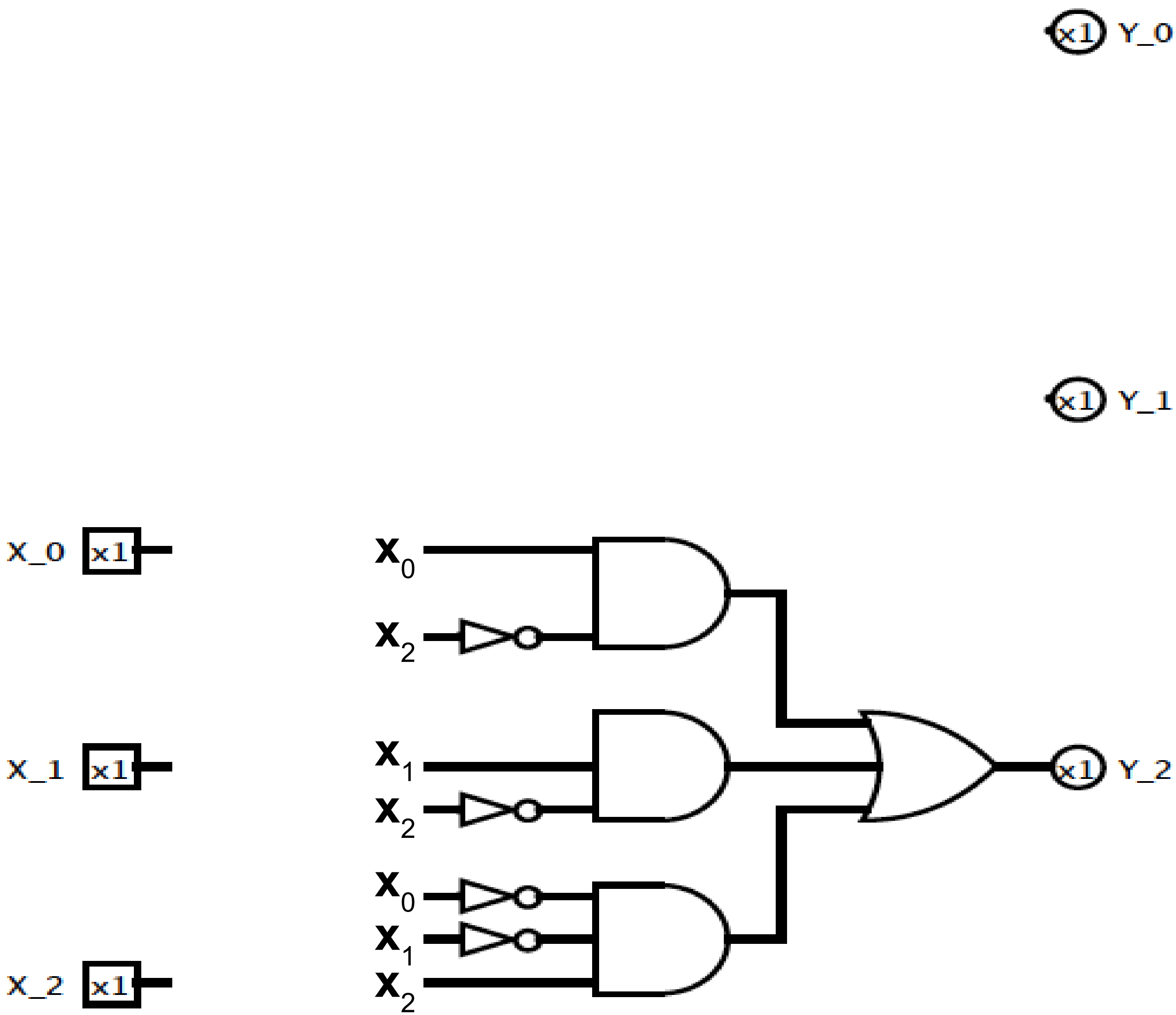
Circuit: $y_2 = \bar{x}_2 x_0 + \bar{x}_2 x_1 + x_2 \bar{x}_1 \bar{x}_0$



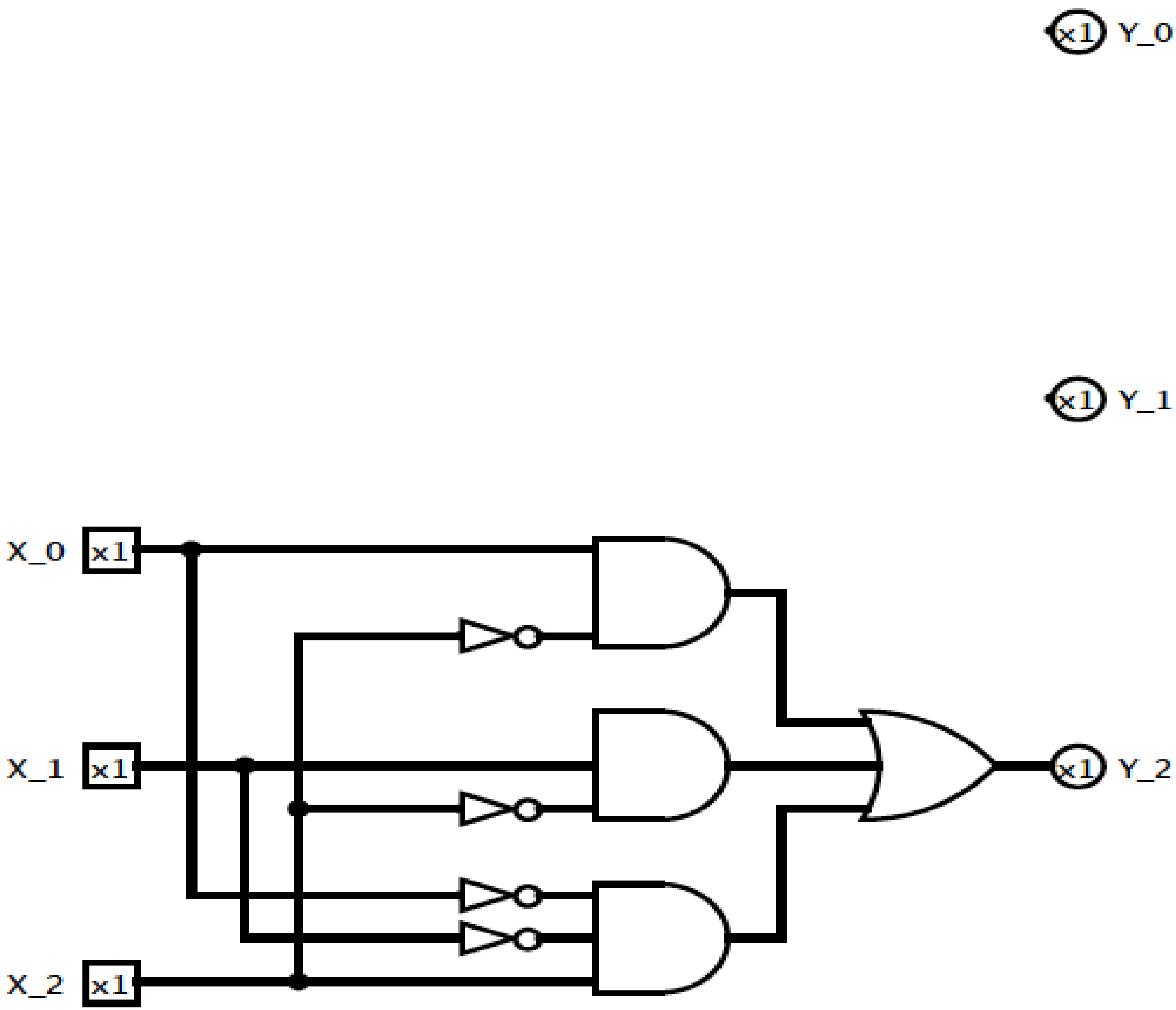
Circuit: $\mathbf{y}_2 = \bar{\mathbf{x}}_2\mathbf{x}_0 + \bar{\mathbf{x}}_2\mathbf{x}_1 + \mathbf{x}_2\bar{\mathbf{x}}_1\bar{\mathbf{x}}_0$



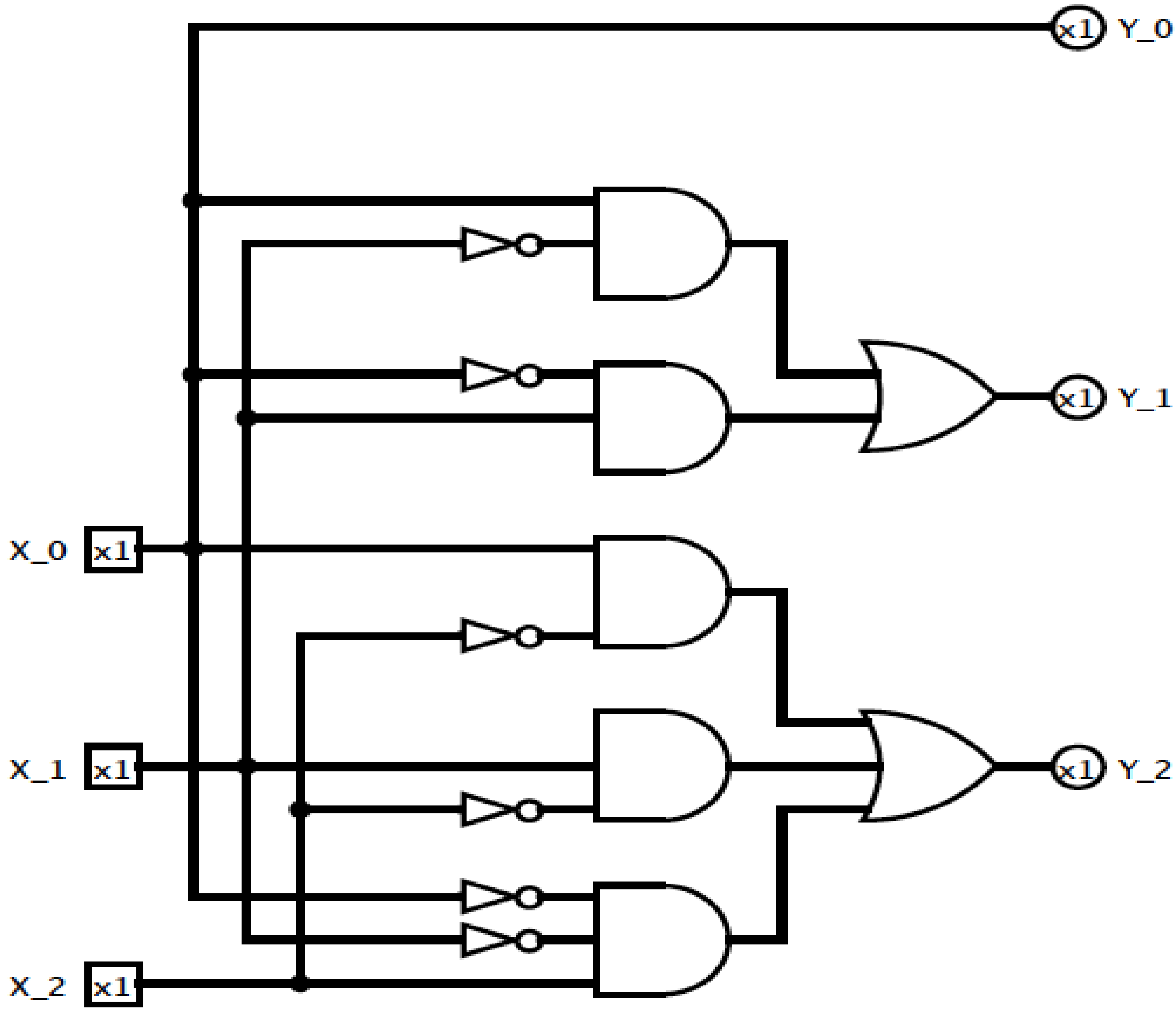
Circuit: $y_2 = \bar{x}_2 x_0 + \bar{x}_2 x_1 + x_2 \bar{x}_1 \bar{x}_0$



Circuit: $\mathbf{y}_2 = \overline{\mathbf{x}}_2\mathbf{x}_0 + \overline{\mathbf{x}}_2\mathbf{x}_1 + \mathbf{x}_2\overline{\mathbf{x}}_1\overline{\mathbf{x}}_0$



Two's Complement Circuits



Combinational Circuit Design

Combinational circuit

Output determined by input

Design process

1. Specify semantics

   Black Box: *input* and *output* (informal semantics)

   Truth Table (formal semantics)

2. Truth table → Boolean formula

3. Minimize boolean formula (optional)

   Boolean algebra

   Karnaugh maps

4. Boolean formula → combinational circuit

Inverters, Decoders, Multiplexer

Inverter: *select* data input or its negation

1 data input

1 selector input

1 output

Decoder: *select* unique output to be 1 (true)

N selector inputs

2^N outputs

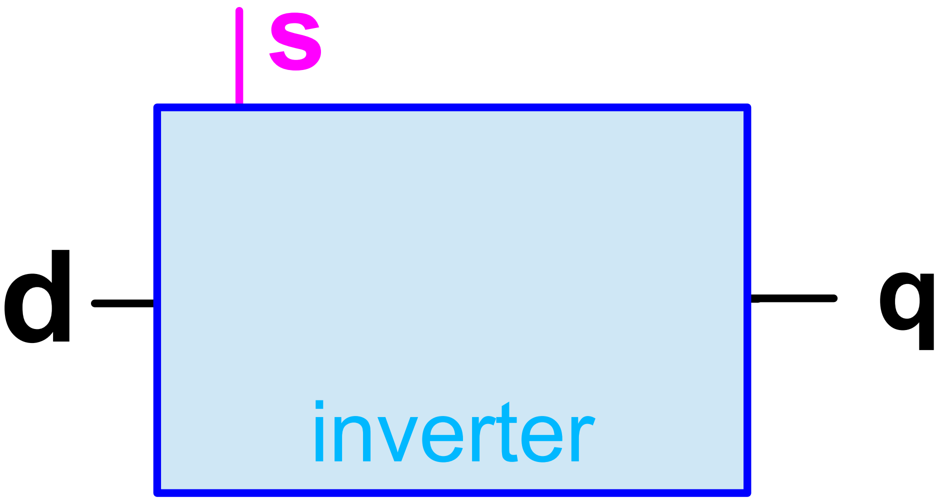
Multiplexer: *select* unique data input to be output

2^N data inputs

N selector inputs

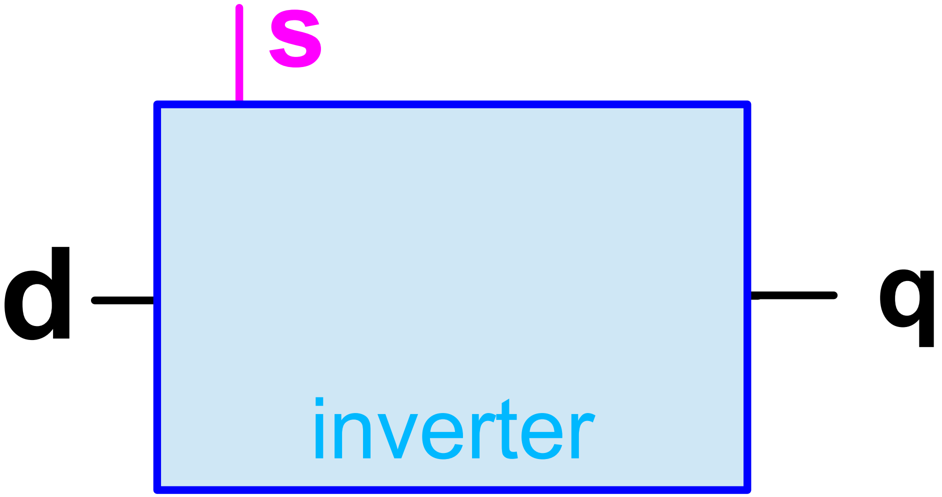
1 output

Inverter Black Box



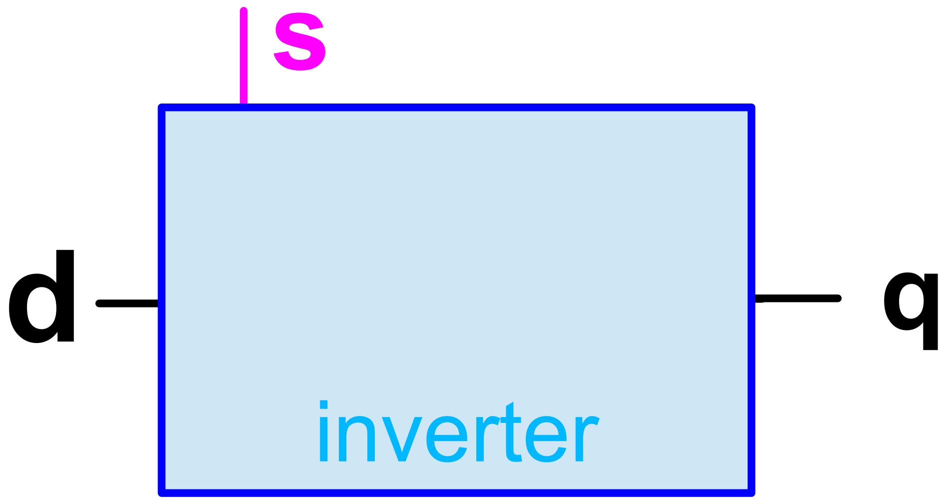
Inverter Truth Table

s	d	q
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**



Inverter Formula

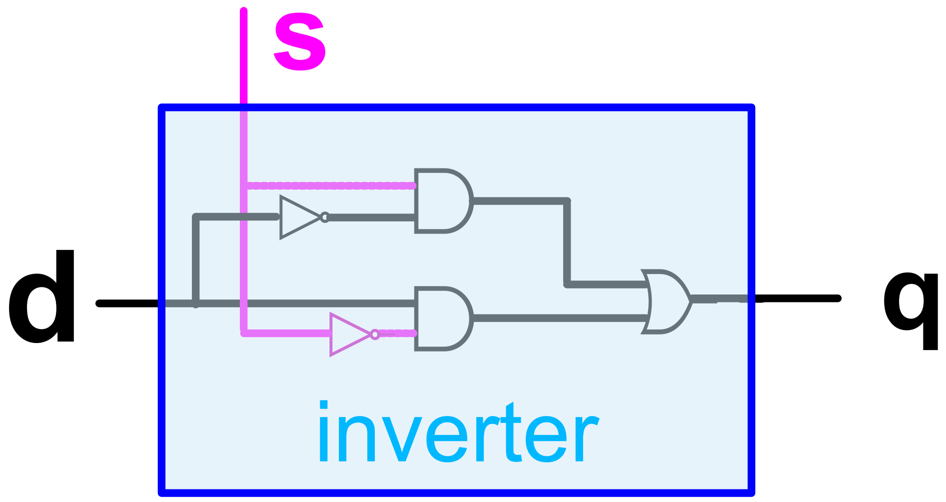
s	**d**	**q**
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**



$$q = \overline{s}d + s\overline{d}$$

Inverter Circuit

s	d	q
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

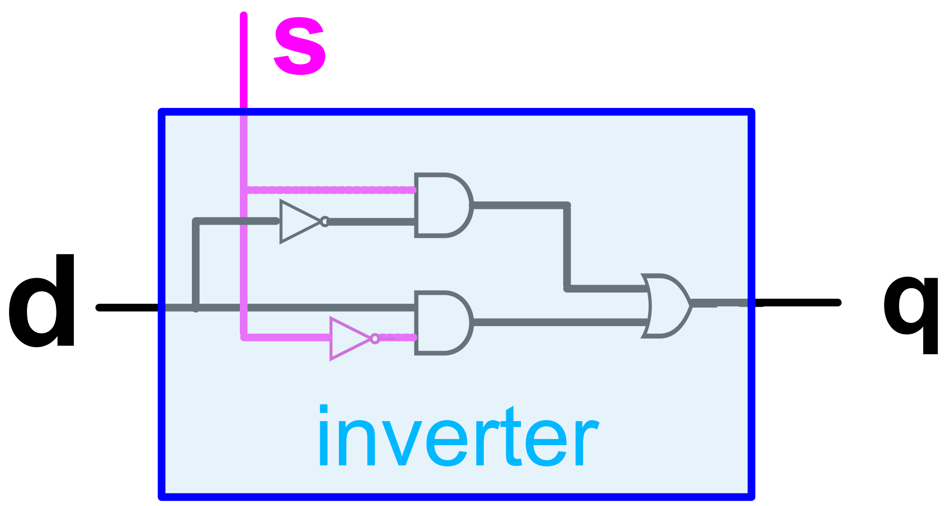


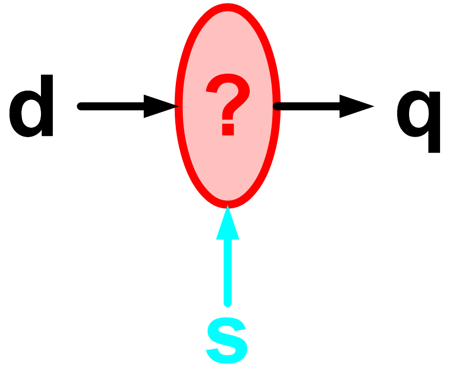
$$q = \overline{s}d + s\overline{d}$$

Inverter Component Icon

s	d	q
0	0	0
0	1	1
1	0	1
1	1	0

$$q = \overline{s}d + s\overline{d}$$

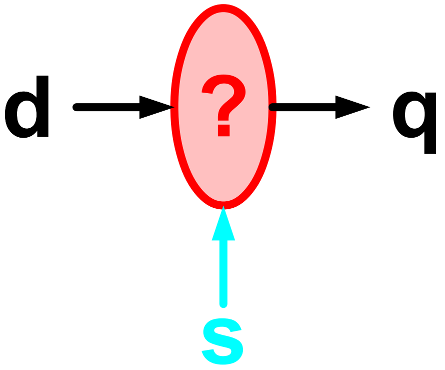


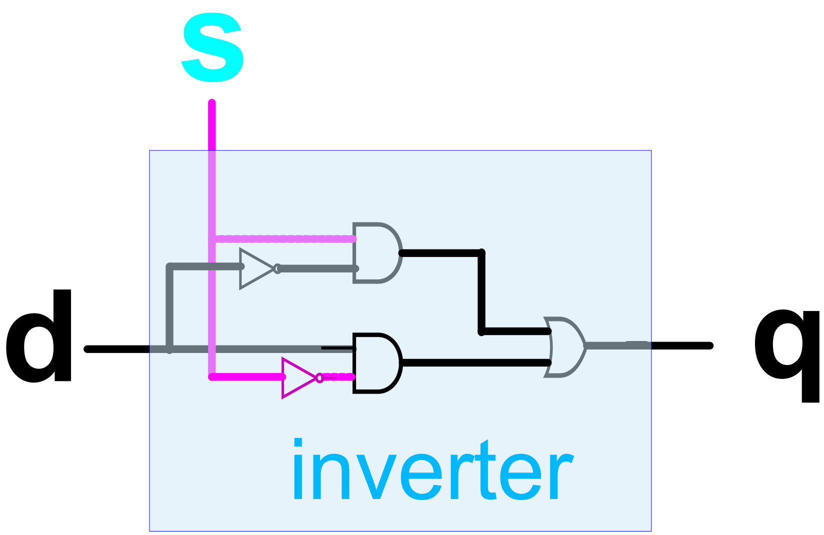


Inverter Summary

Inverter: *select* data input or its negation

- 1 data input
- 1 selector input
- 1 output





N-Bit Decoder

Each different combination of N input bits uniquely specifies one of 2^N outputs. An output is **1** if and only if the corresponding input combination is active (true). For any input, exactly one output is **1**.

N-Bit Decoder Truth Table:

- N input (selector) columns
- 2^N output columns
- 2^N rows

Exactly one 1 in each output column

Exactly one 1 in each output row

1-Bit Decoder Black Box

1-Bit Decoder Truth Table

s	w_0	w_1
0	1	0
1	0	1

1-Bit Decoder Formulas

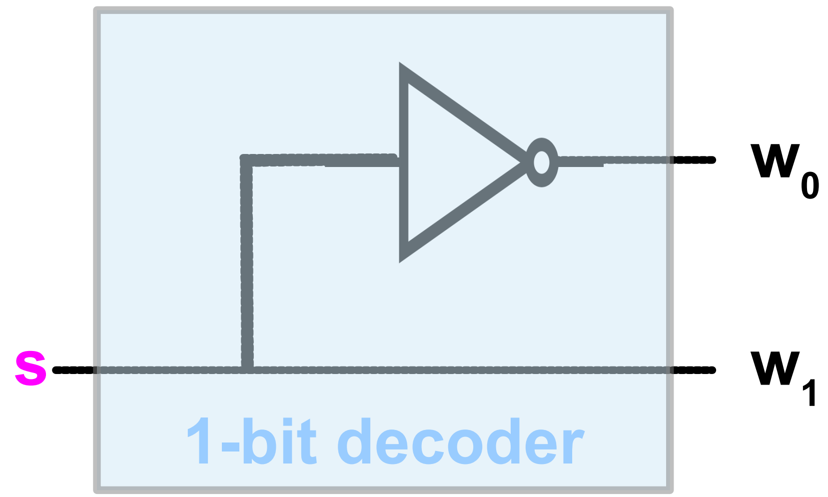
s	w_0	w_1
0	1	0
1	0	1

$$w_0 = \overline{s}$$

$$w_1 = s$$

1-Bit Decoder Circuit

s	w_0	w_1
0	1	0
1	0	1

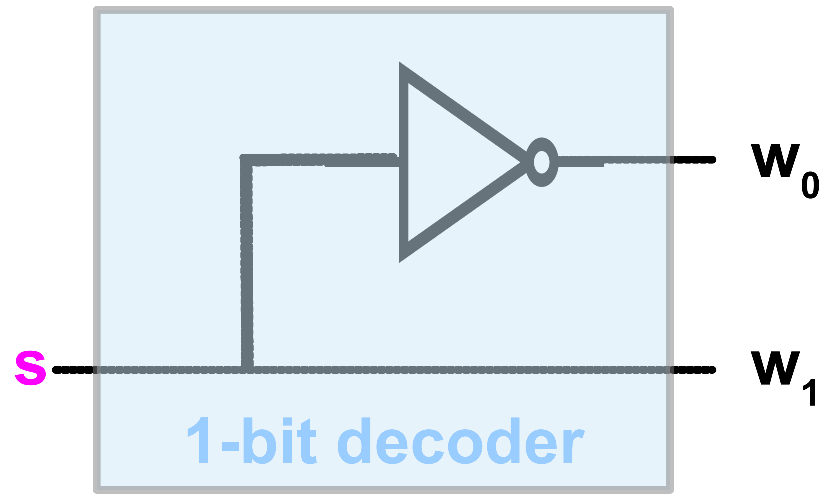


$$w_0 = \overline{s}$$

$$w_1 = s$$

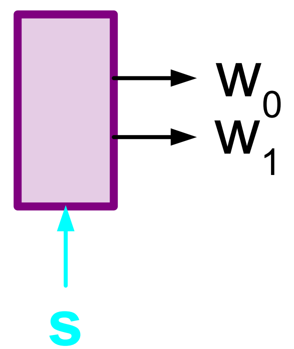
1-Bit Decoder Component Icon

s	w_0	w_1
0	1	0
1	0	1



$$w_0 = \overline{s}$$

$$w_1 = s$$



2-Bit Decoder

2-bit Decoder Truth Table

#	s_1	s_0	w_{00}	w_{01}	w_{10}	w_{11}
0	0	0	**1**	0	0	0
1	0	1	0	**1**	0	0
2	1	0	0	0	**1**	0
3	1	1	0	0	0	**1**

2-bit Decoder Formulas

#	s_1	s_0	w_{00}	w_{01}	w_{10}	w_{11}
0	0	0	**1**	0	0	0
1	0	1	0	**1**	0	0
2	1	0	0	0	**1**	0
3	1	1	0	0	0	**1**

$$w_{11} = s_1 s_0$$

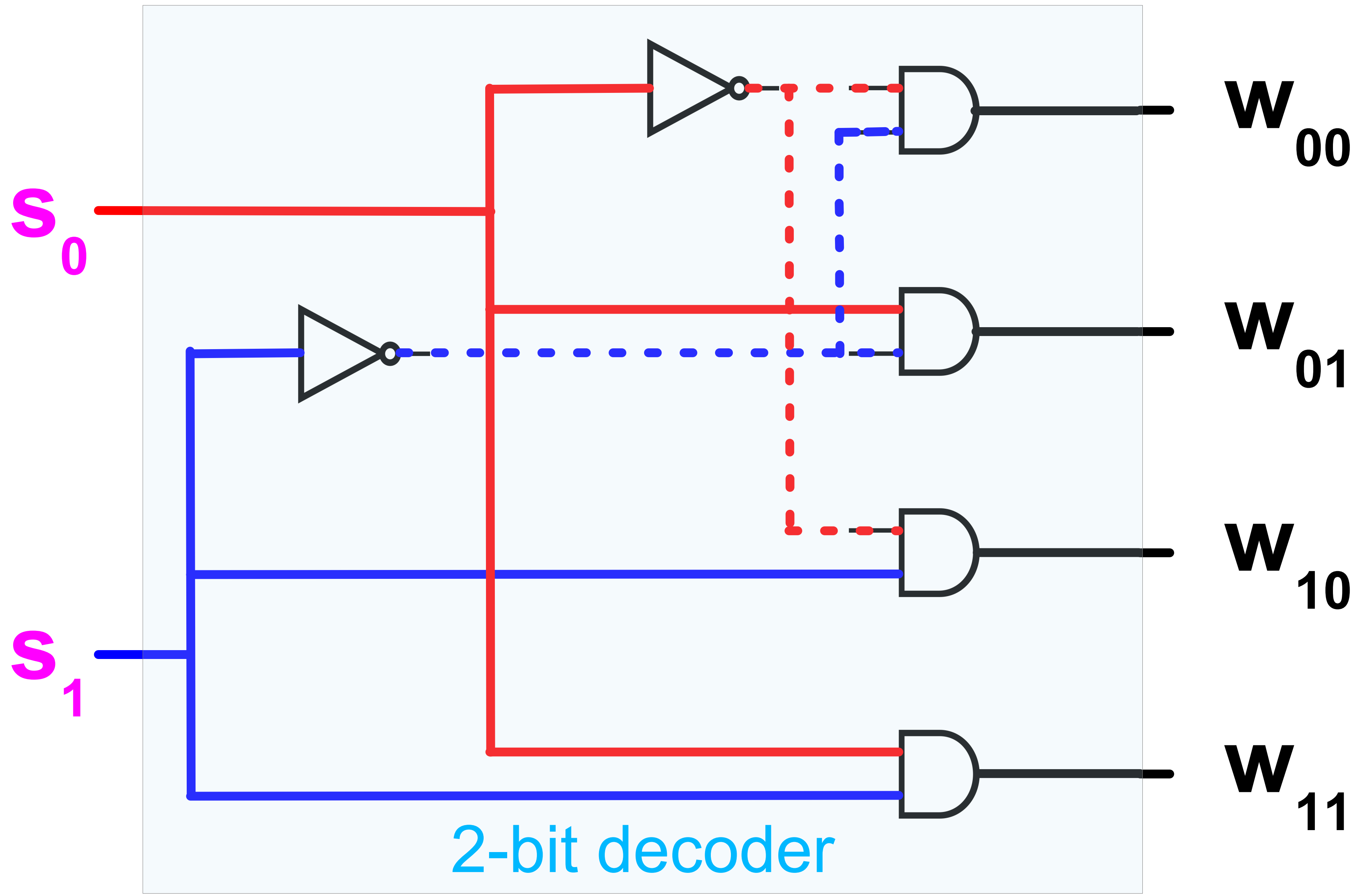
$$w_{10} = s_1 \overline{s_0}$$

$$w_{01} = \overline{s_1} s_0$$

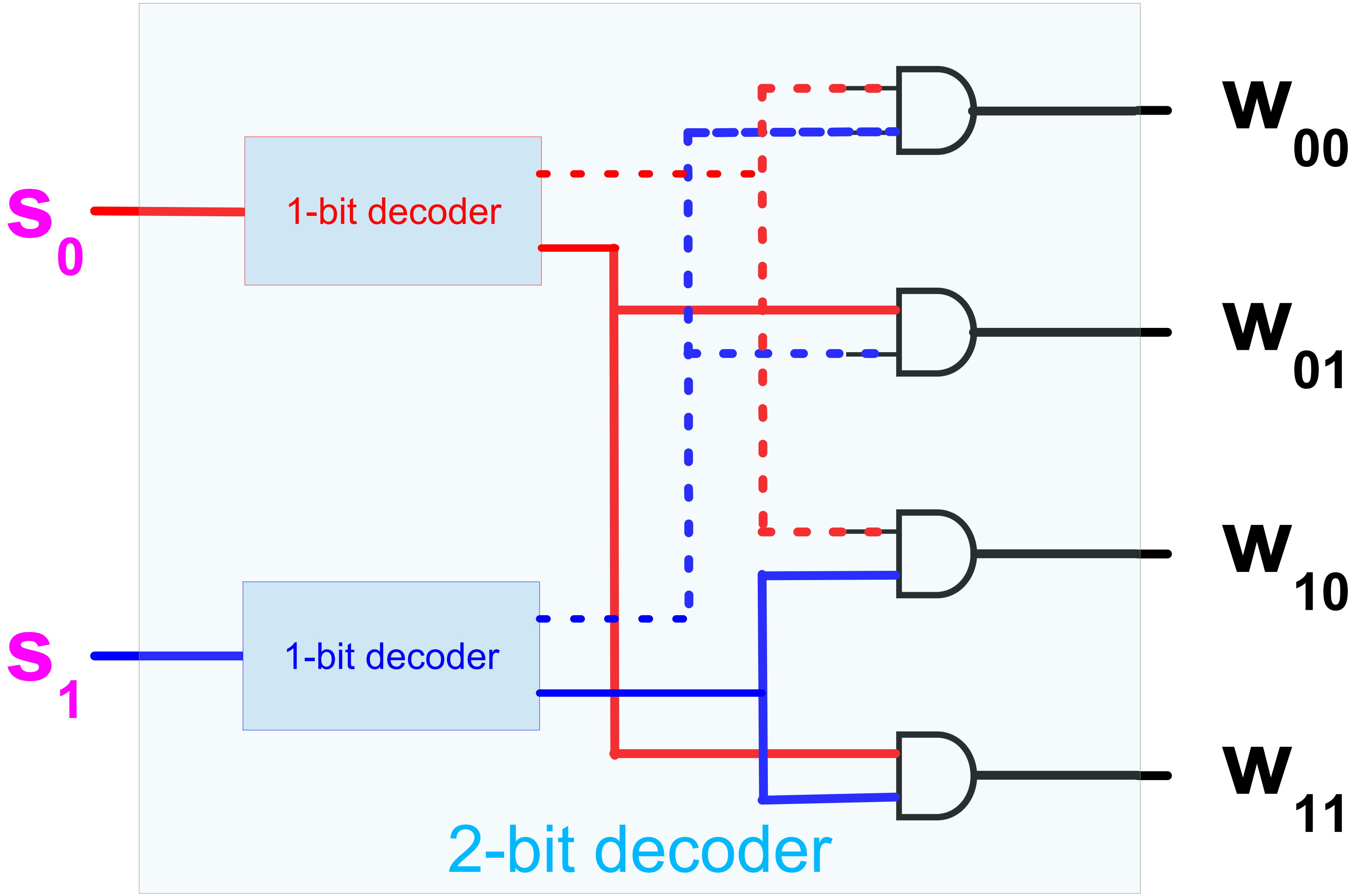
$$w_{00} = \overline{s_1}\,\overline{s_0}$$

2-Bit Decoder Circuit

2-Bit Decoder Circuit



2-Bit Decoder Circuit

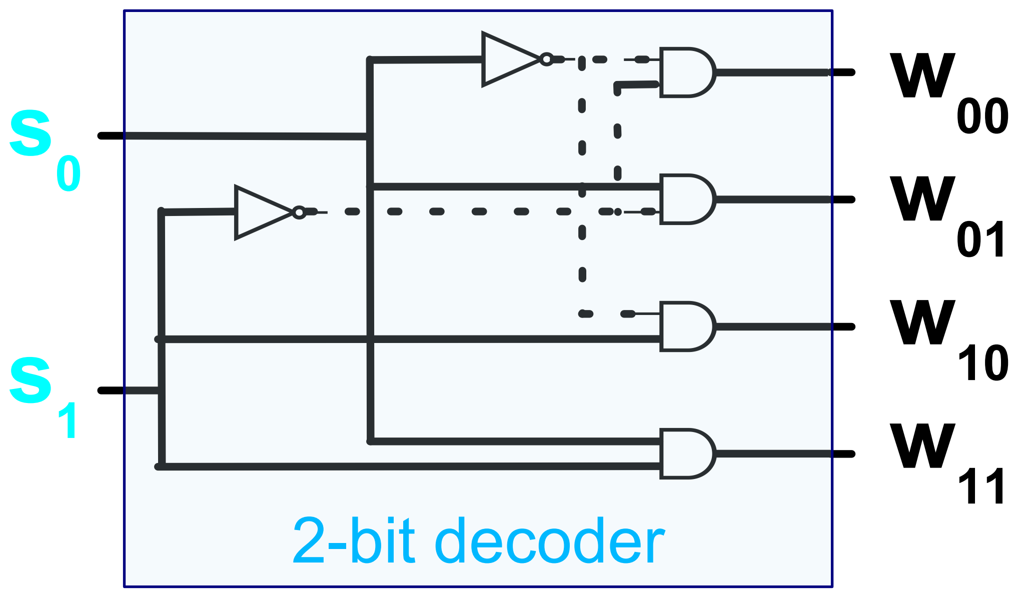


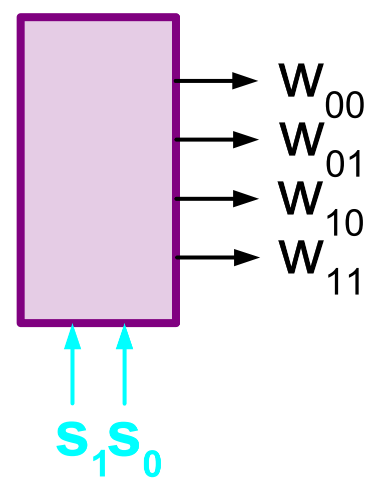
Decoder Summary

Decoder: *select* unique output to be 1 (true)

N selector inputs

2^N outputs





M-way Multiplexer ($M \equiv 2^N$)

N (lg M) *selector* input bits

choose one of 2^N (M) *data* input bitss to output

2^N-way multiplexer truth table:

2^N data input columns	($\mathbf{d}_i$ $0 \le \mathbf{i} < 2^N$)
N selector input columns	($\mathbf{s}_j$ $0 \le \mathbf{j} < N$, $0 \le \mathbf{s} < 2^N$)
1 output column	($\mathbf{x} = \mathbf{d}_s$)

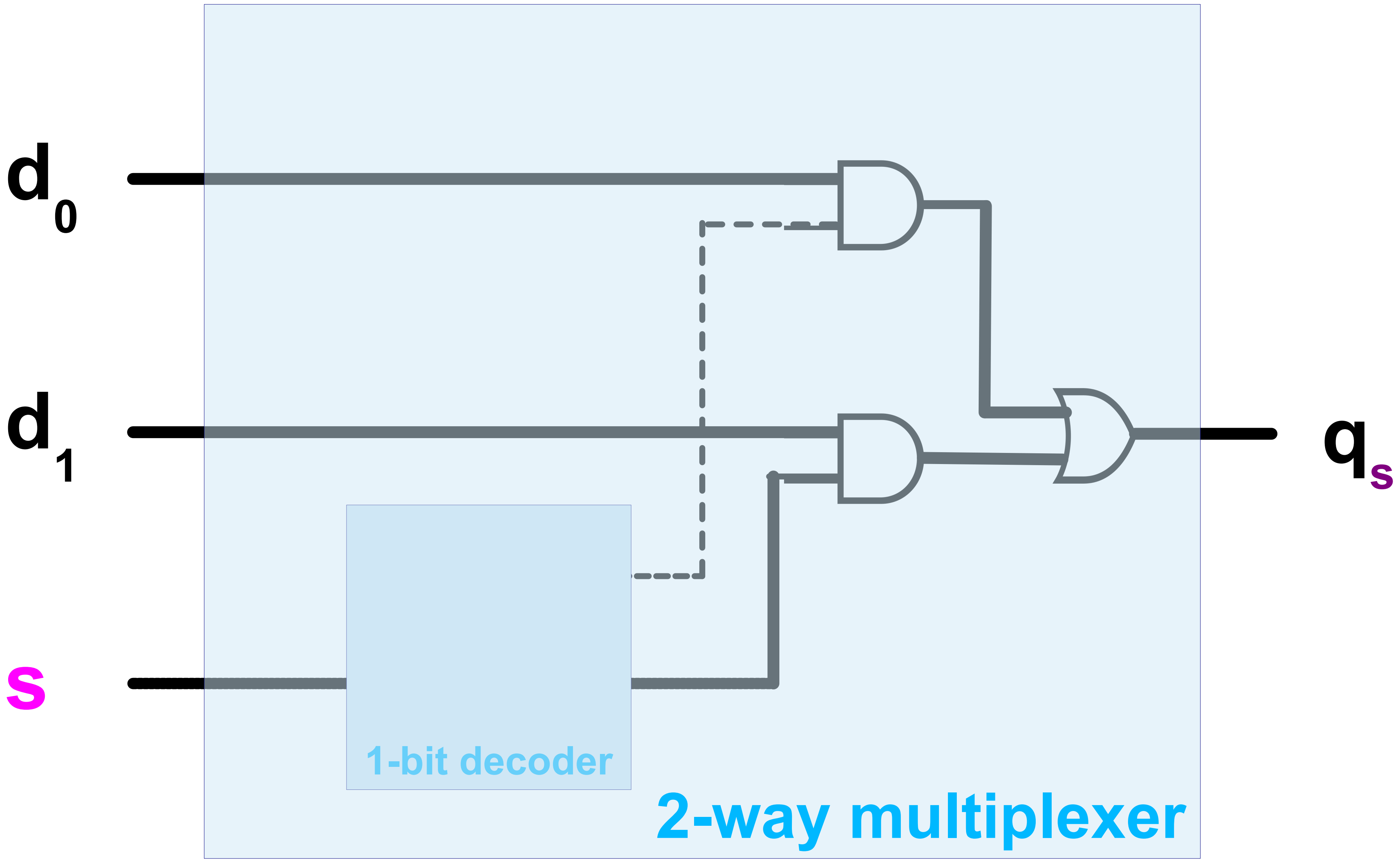
2-Way **Multiplexer**

2-Way **Multiplexer**

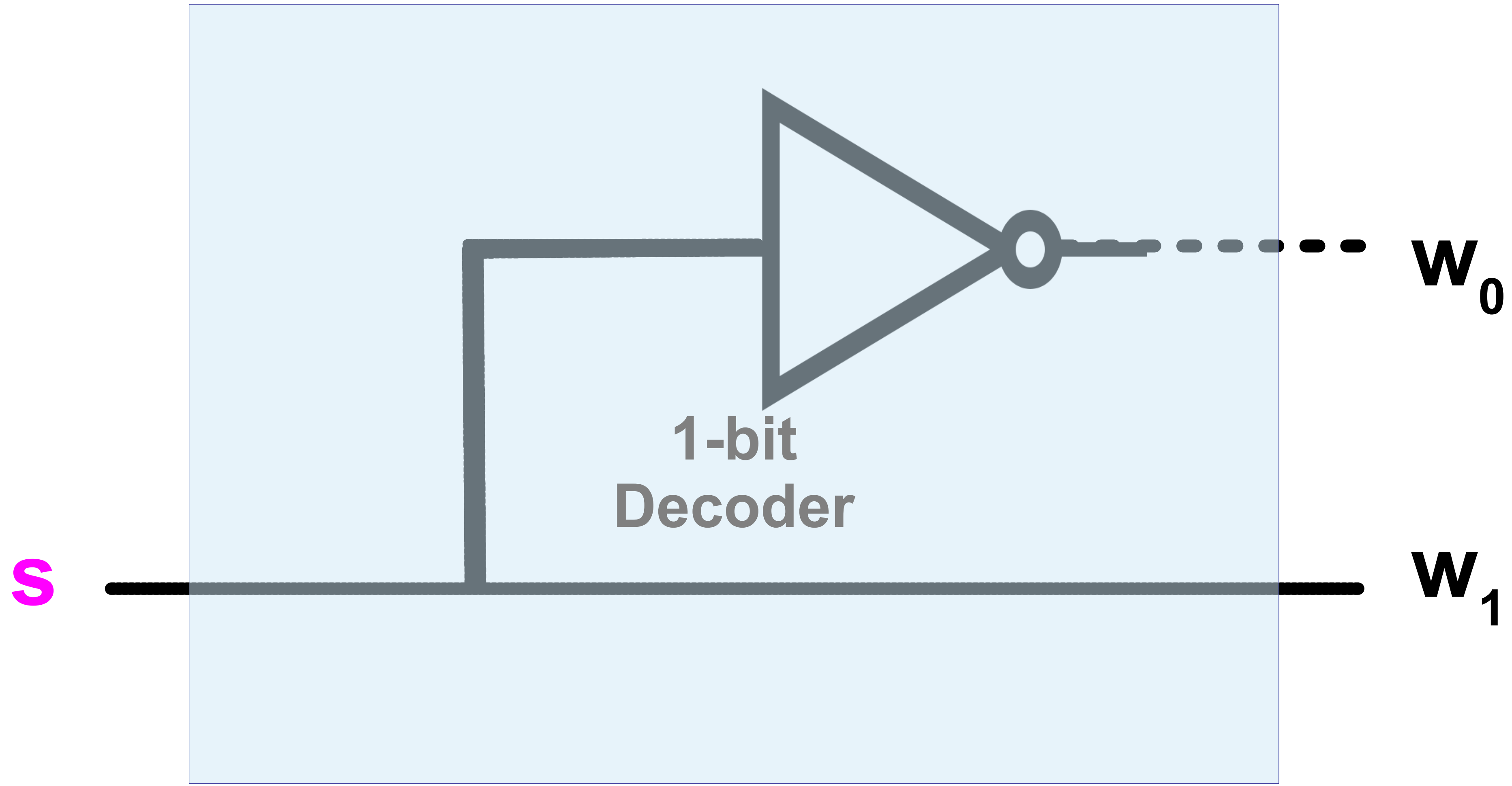
#	s	d_1	d_0	q
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

2-Way **Multiplexer**

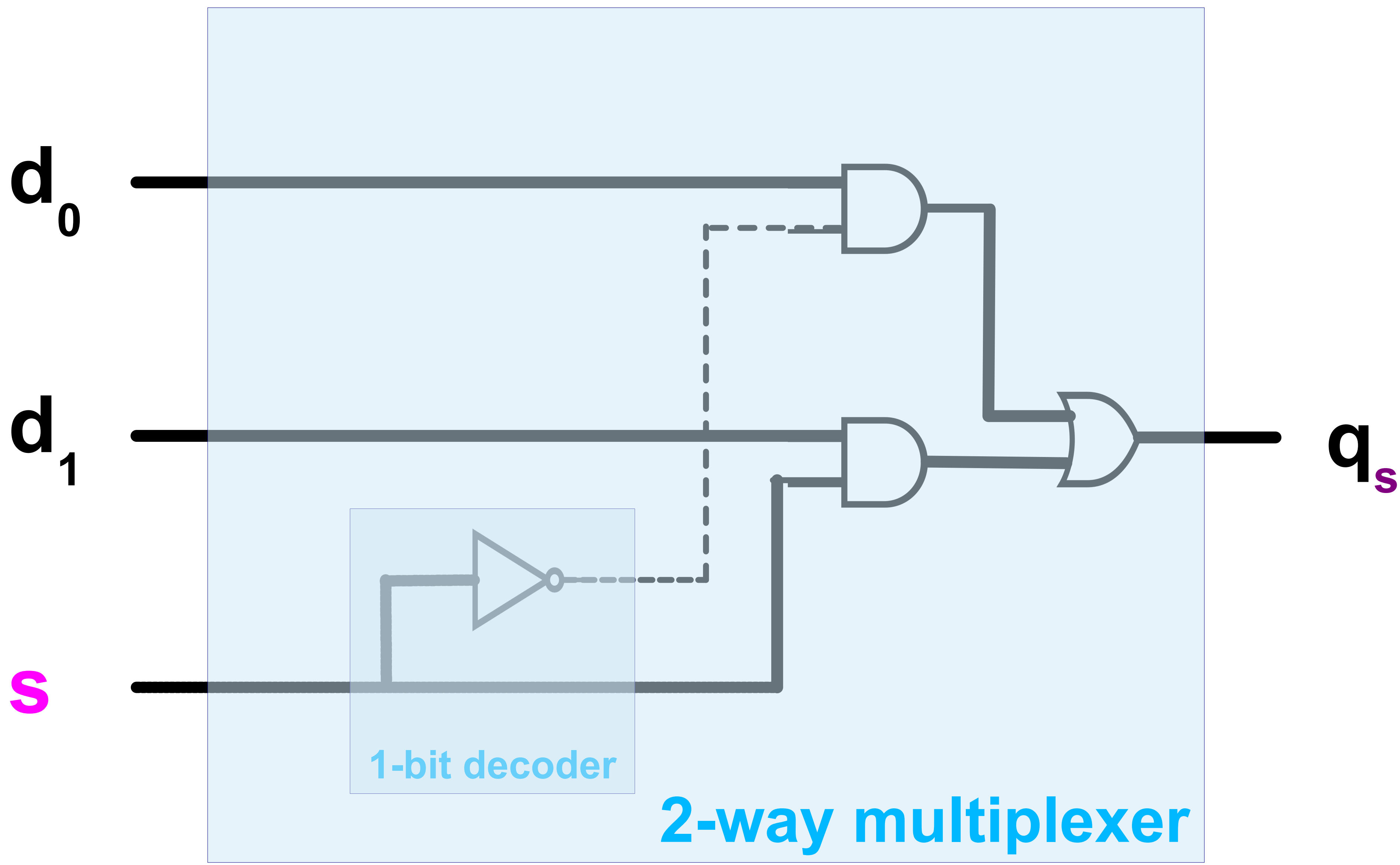
2-Way **Multiplexer**



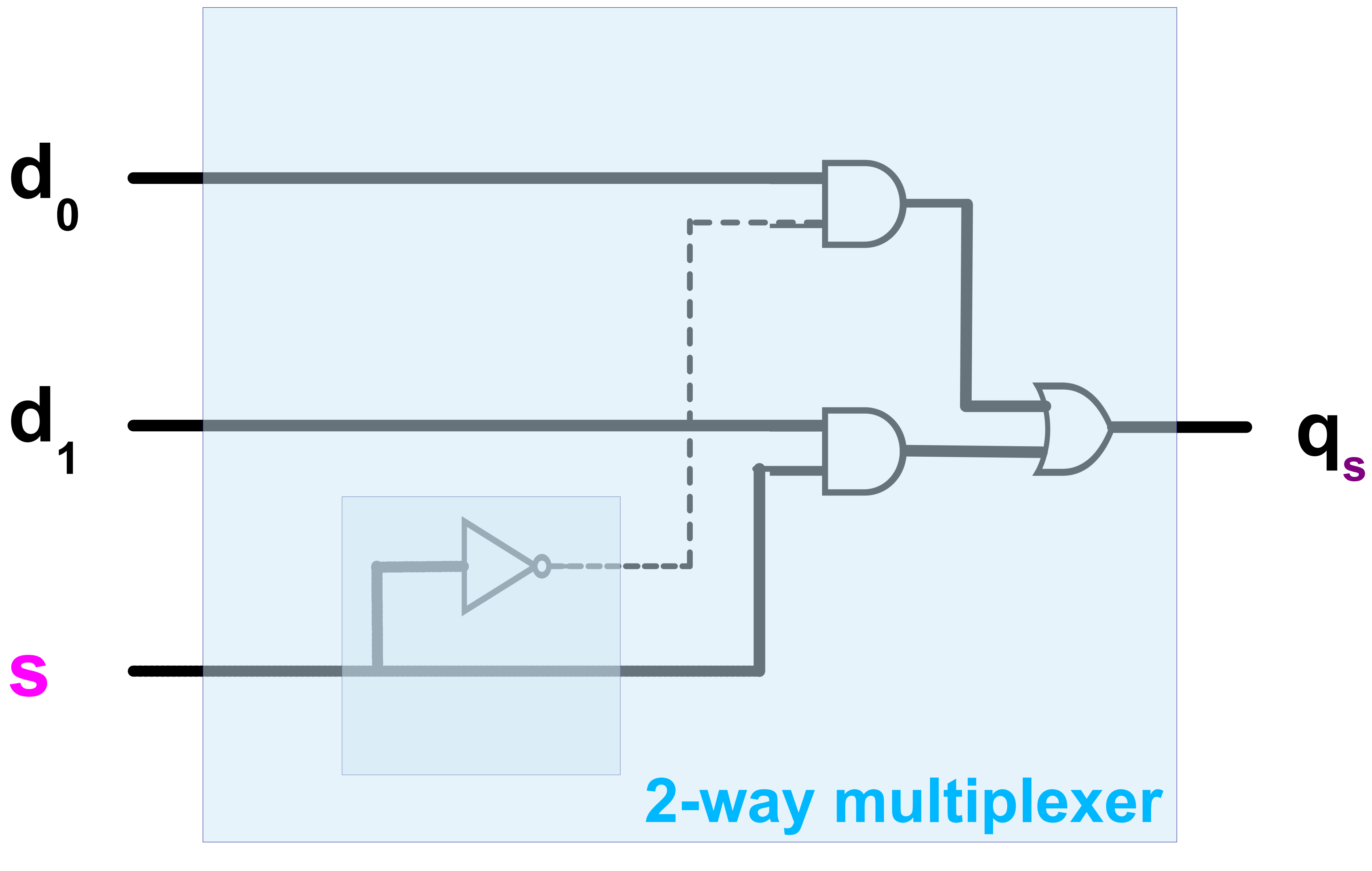
1-Bit Decoder



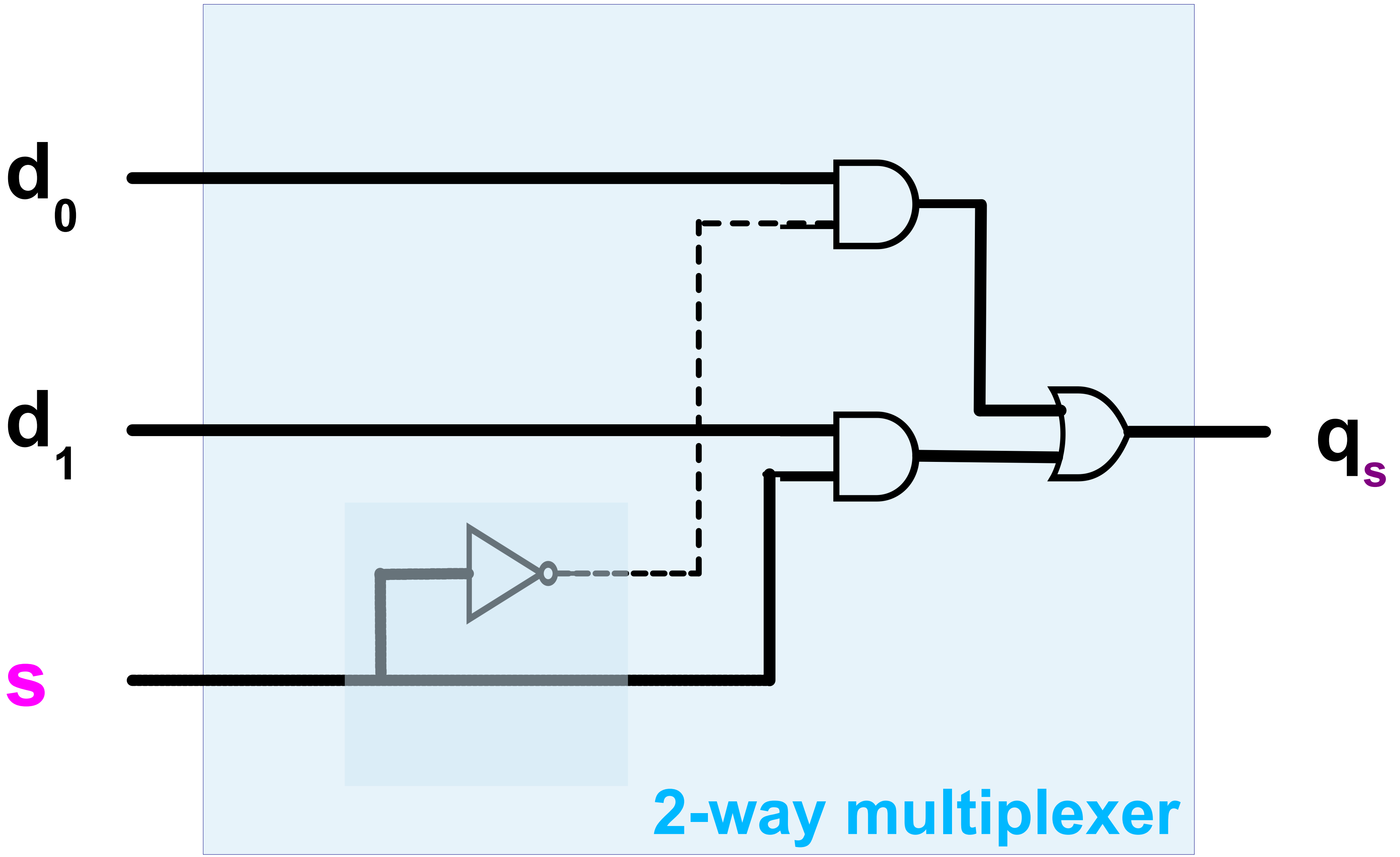
2-Way **Multiplexer**



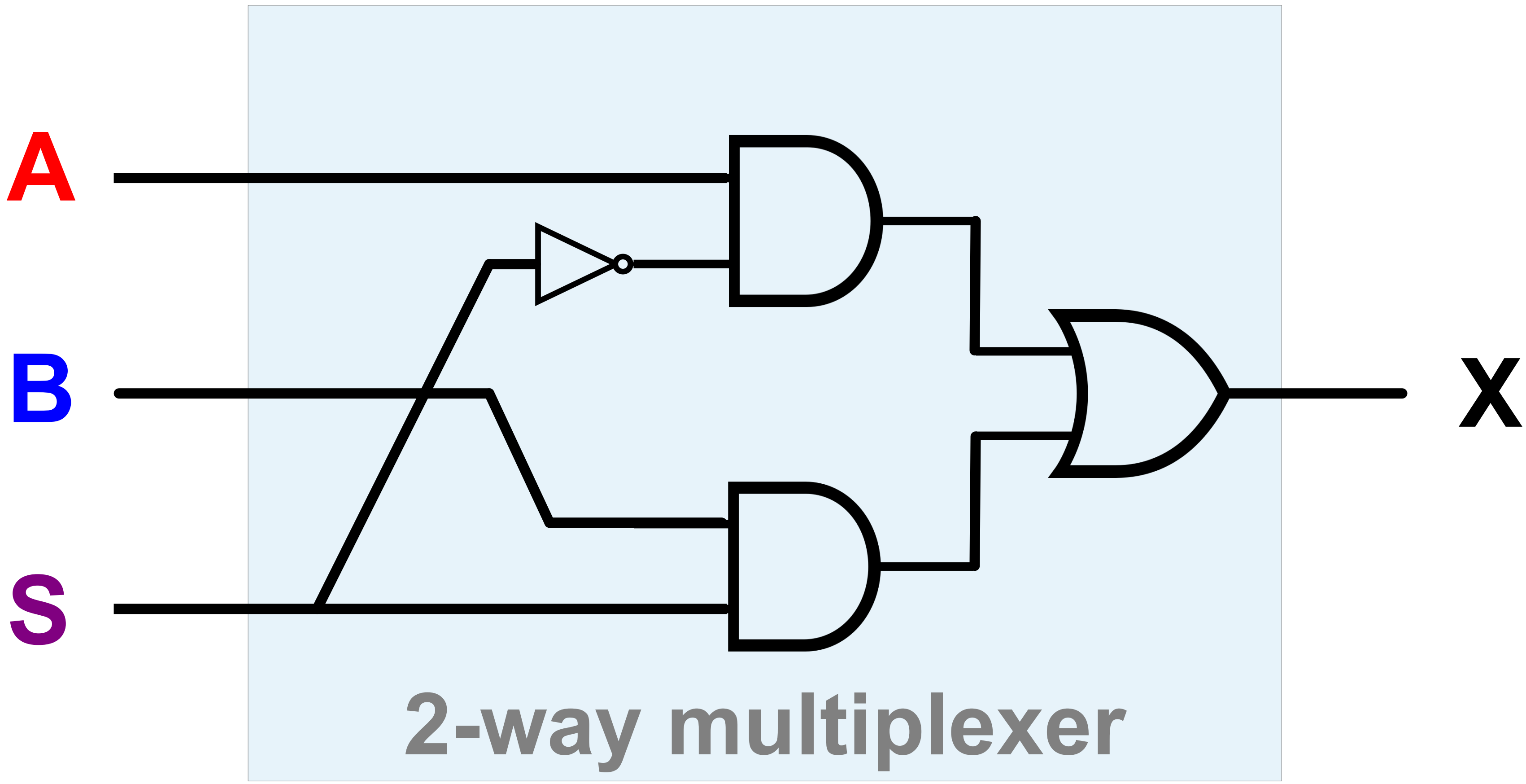
2-Way **Multiplexer**



2-Way **Multiplexer**



Two Way Multiplexer Circuit



$$X = \overline{S}A + SB$$

4-Way Multiplexer Black Box

4-Way Multiplexer

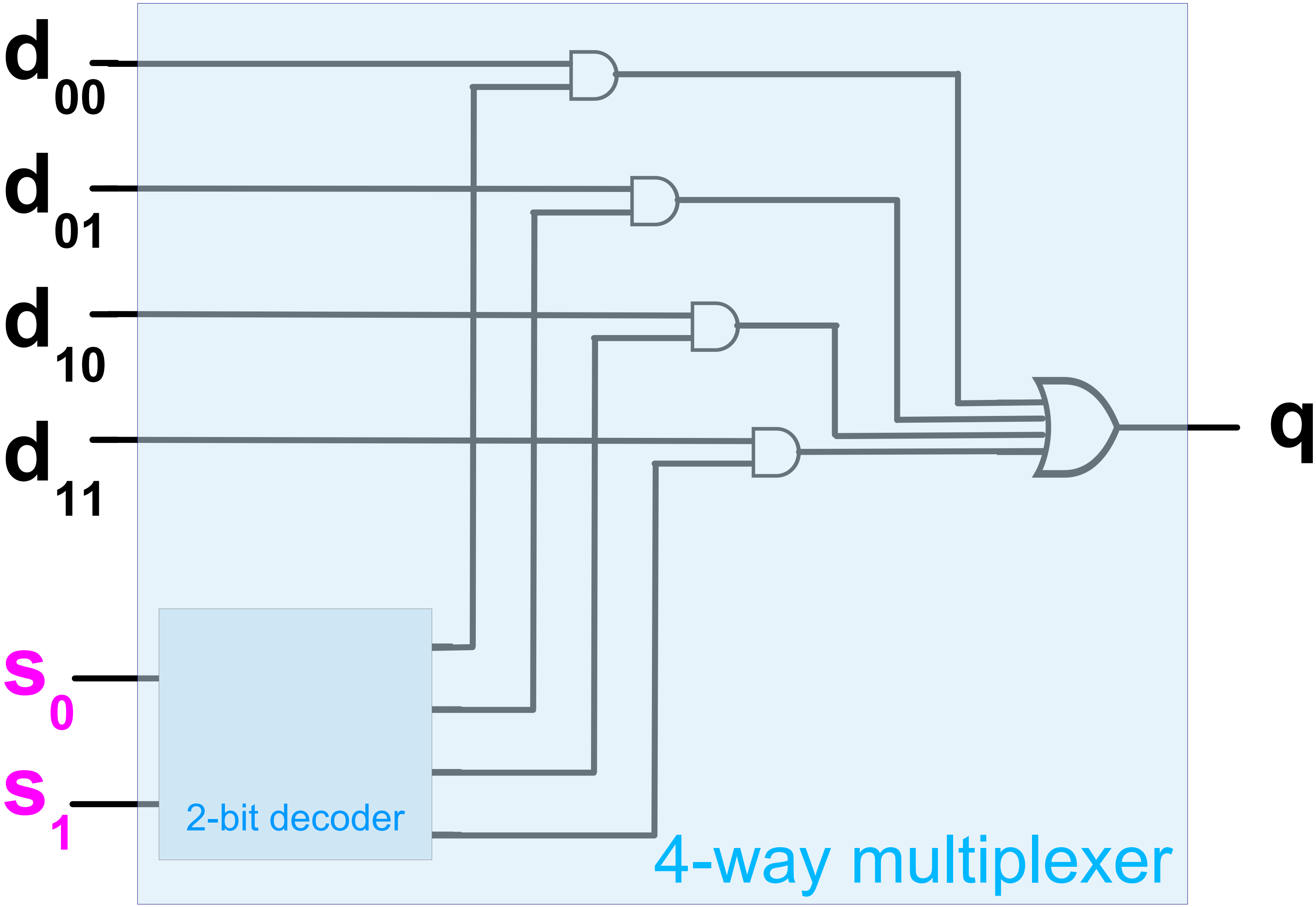
s_1	s_0	d_{11}	d_{10}	d_{01}	d_{00}	q
0	0	√	√	√	0	0
0	0	√	√	√	1	1
0	1	√	√	0	√	0
0	1	√	√	1	√	1
1	0	√	0	√	√	0
1	0	√	1	√	√	1
1	1	0	√	√	√	0
1	1	1	√	√	√	1

4-Way Multiplexer

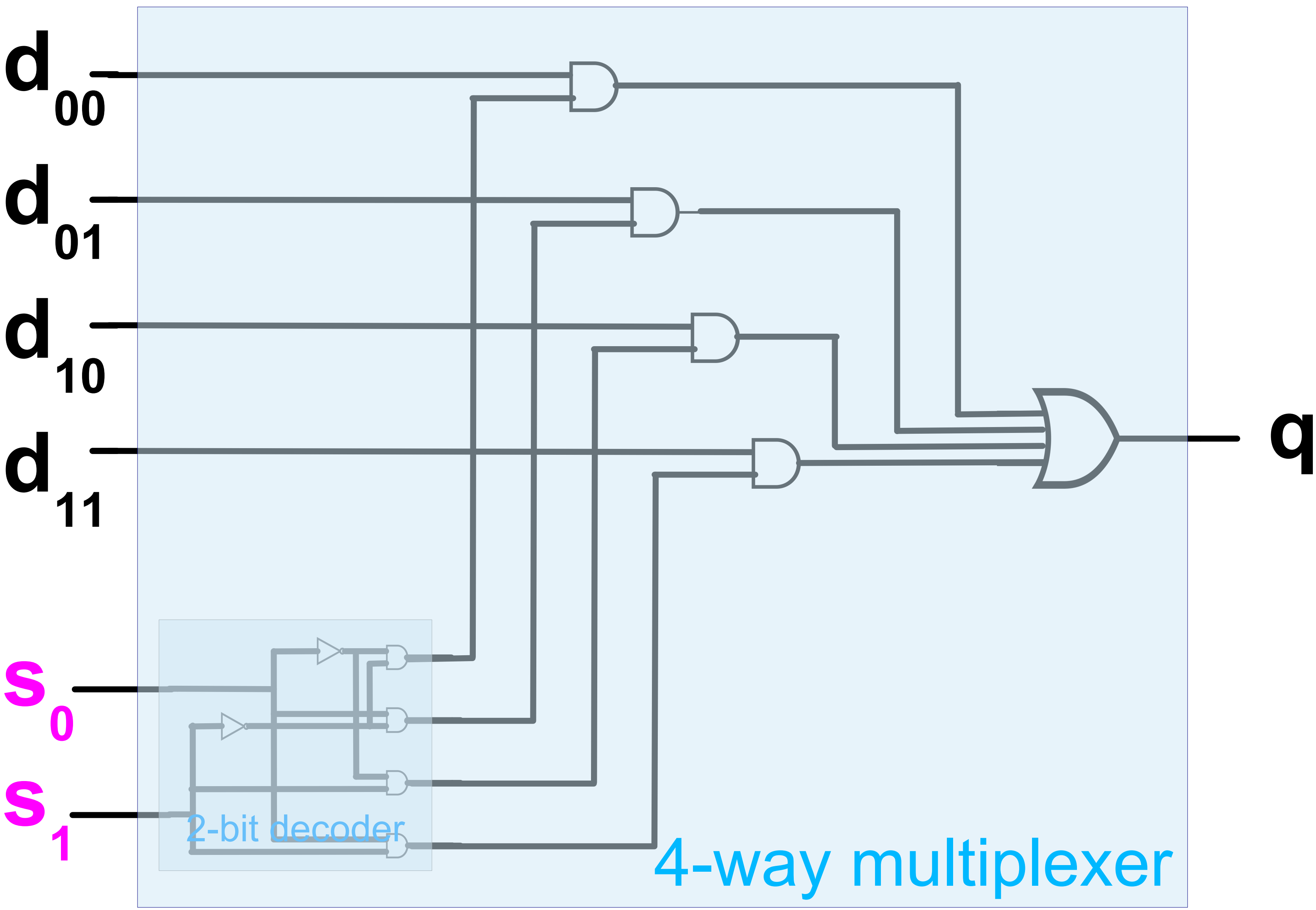
s_1	s_0	d_{11}	d_{10}	d_{01}	d_{00}	q	
0	0	√	√	√	0	0	
0	0	√	√	√	1	1	$\overline{s}_1\overline{s}_0 d_{00}$
0	1	√	√	0	√	0	
0	1	√	√	1	√	1	$+ \overline{s}_1 s_0 d_{01}$
1	0	√	0	√	√	0	
1	0	√	1	√	√	1	$+ s_1\overline{s}_0 d_{10}$
1	1	0	√	√	√	0	
1	1	1	√	√	√	1	$+ s_1 s_0 d_{11}$

4-Way Multiplexer

4-Way Multiplexer

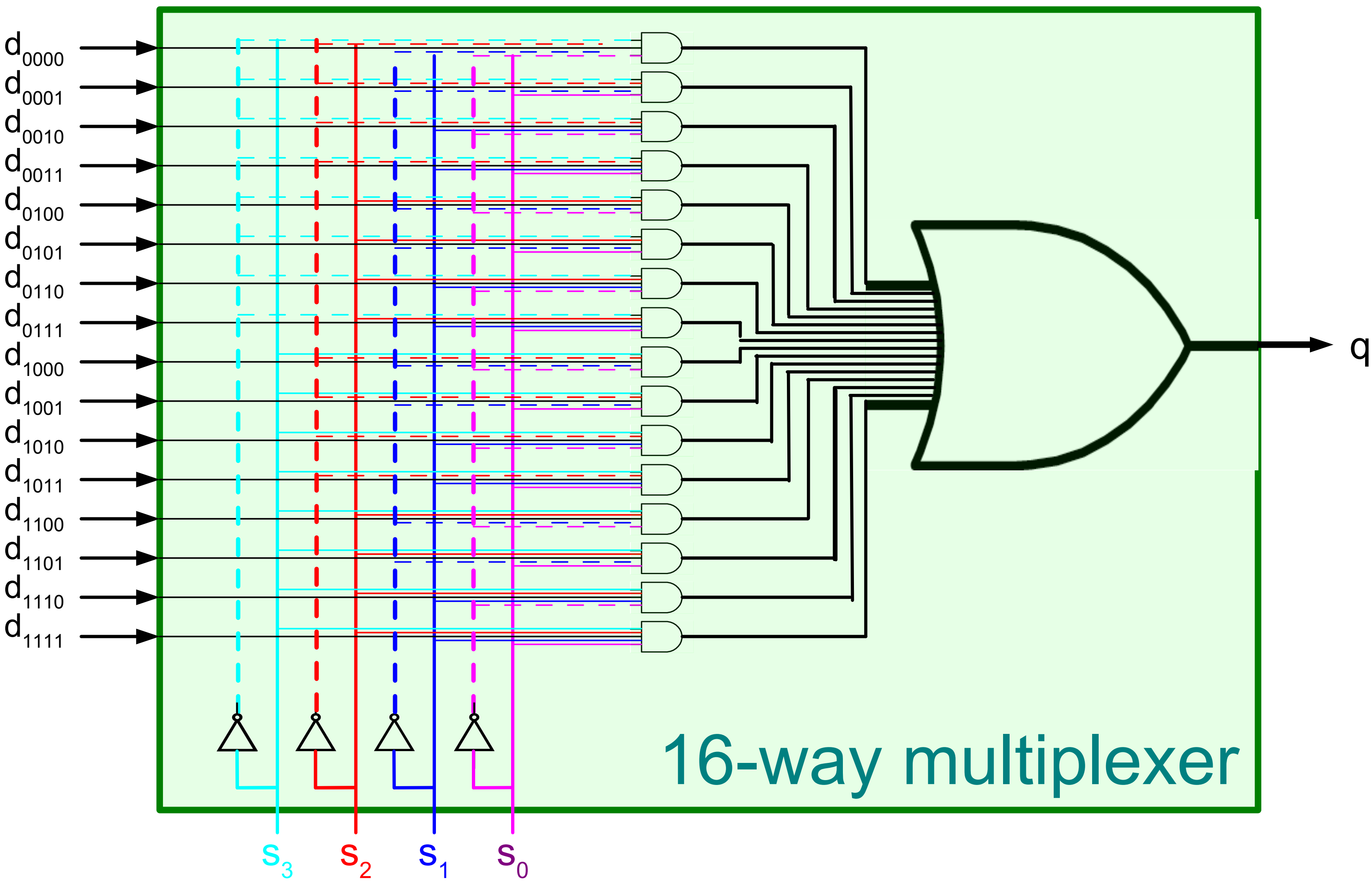


4-Way Multiplexer

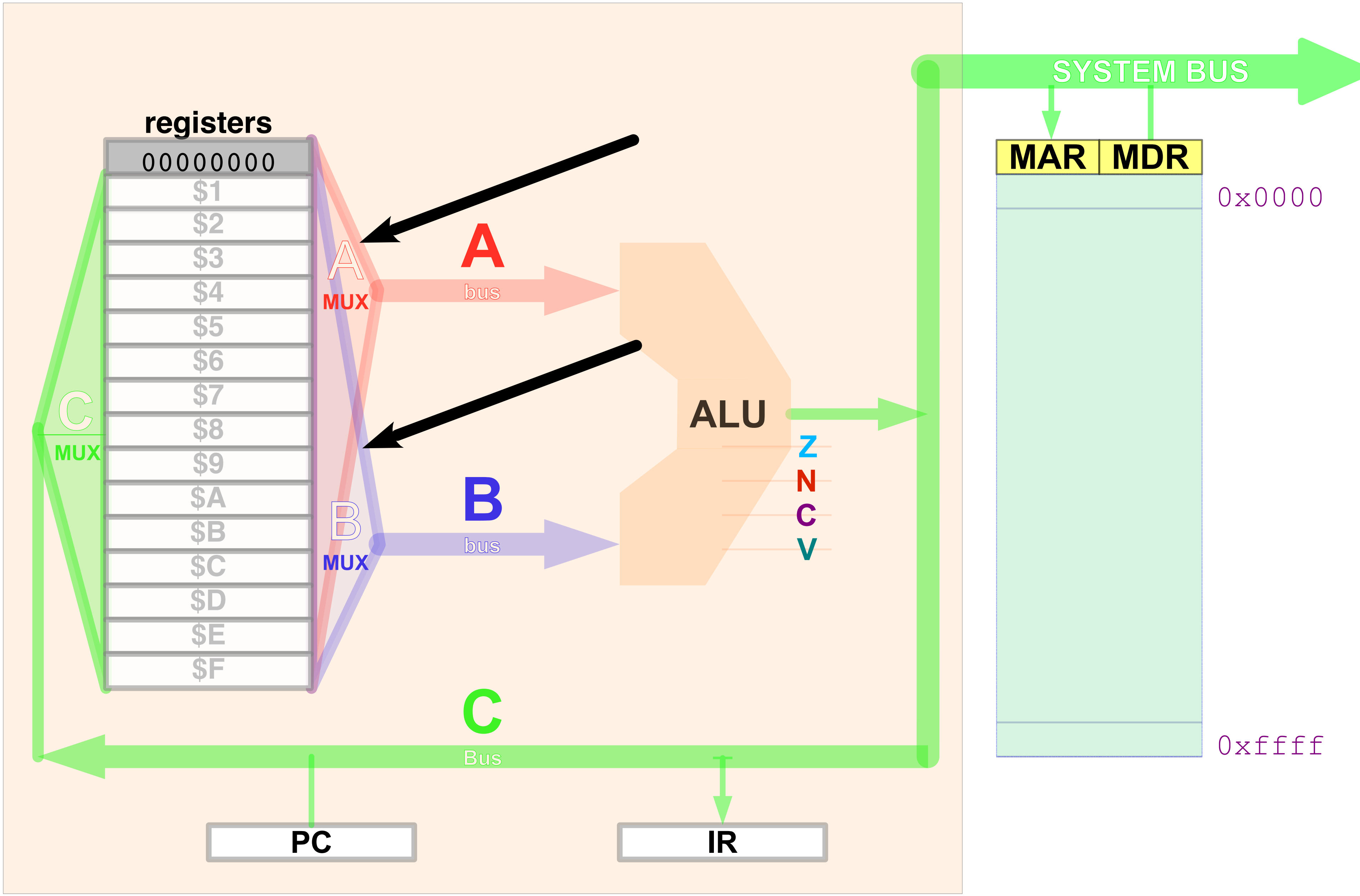


16-Way Multiplexer

16-Way Multiplexer

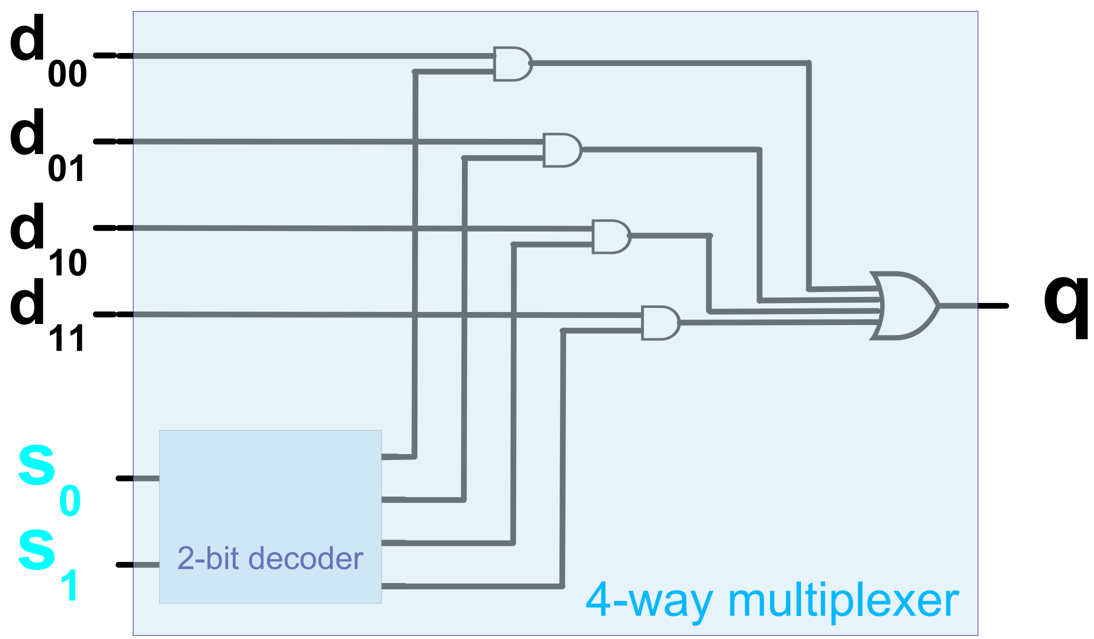


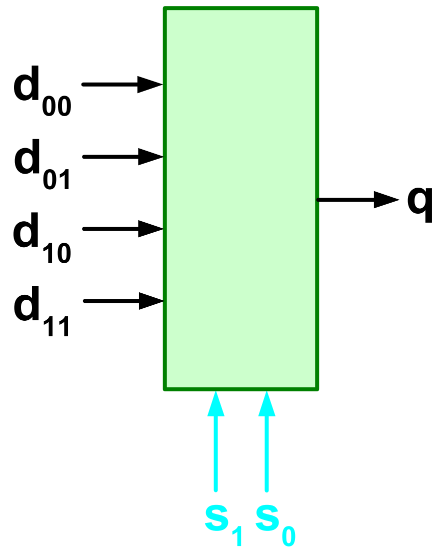
A and B Multiplexers



2^N-Way Multiplexer Summary

N selector inputs specify one of 2^N data inputs to output





n-bit Binary Numbers

Unsigned: $b_{n-1}b_{n-2} \ldots b_1\ b_0$ $(b_i = 0 \text{ or } b_i = 1)$

value: $b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \ldots + b_1 2^1 + b_0 2^0$

range: $[0\ ..\ 2^n - 1]$

n-bit sum: $\mathbf{A} \overset{\cdots}{+} \mathbf{B} = \mathbf{A} + \mathbf{B} - c \cdot 2^n$

n-bit diff: $\mathbf{A} \overset{\cdots}{-} \mathbf{B} \equiv \mathbf{A} \overset{\cdots}{+} (2^n - \mathbf{B}) = \mathbf{A} \overset{\cdots}{+} \overline{\mathbf{B}} + 1 = \mathbf{A} - \mathbf{B} + 2^n - c \cdot 2^n$

Signed: $b_{n-1}b_{n-2} \ldots b_1\ b_0$ $(b_i = 0 \text{ or } b_i = 1)$

Value: $-b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \ldots + b_1 2^1 + b_0 2^0$

Range: $[-2^{n-1}\ ..\ 2^{n-1} - 1]$

n-bit sum: $\mathbf{A} \overset{\cdots}{+} \mathbf{B} = \mathbf{A} + \mathbf{B}$ **iff** $v=0$

n-bit diff: $\mathbf{A} \overset{\cdots}{-} \mathbf{B} = \mathbf{A} - \mathbf{B}$ **iff** $v=0$

Honesty Criteria

The n-bit result **r** of a binary operation on n-bit values **a** and **b** is ***honest*** (***deceptive***) if it is *the same as* (*different from*) the whole number result of the same operation on the same values.

(n-bit) unsigned addition is *honest* iff (**c** = 0)

Carry flag is not set

(n-bit) unsigned subtraction is *honest* iff (**c** = 1)

Carry flag is set

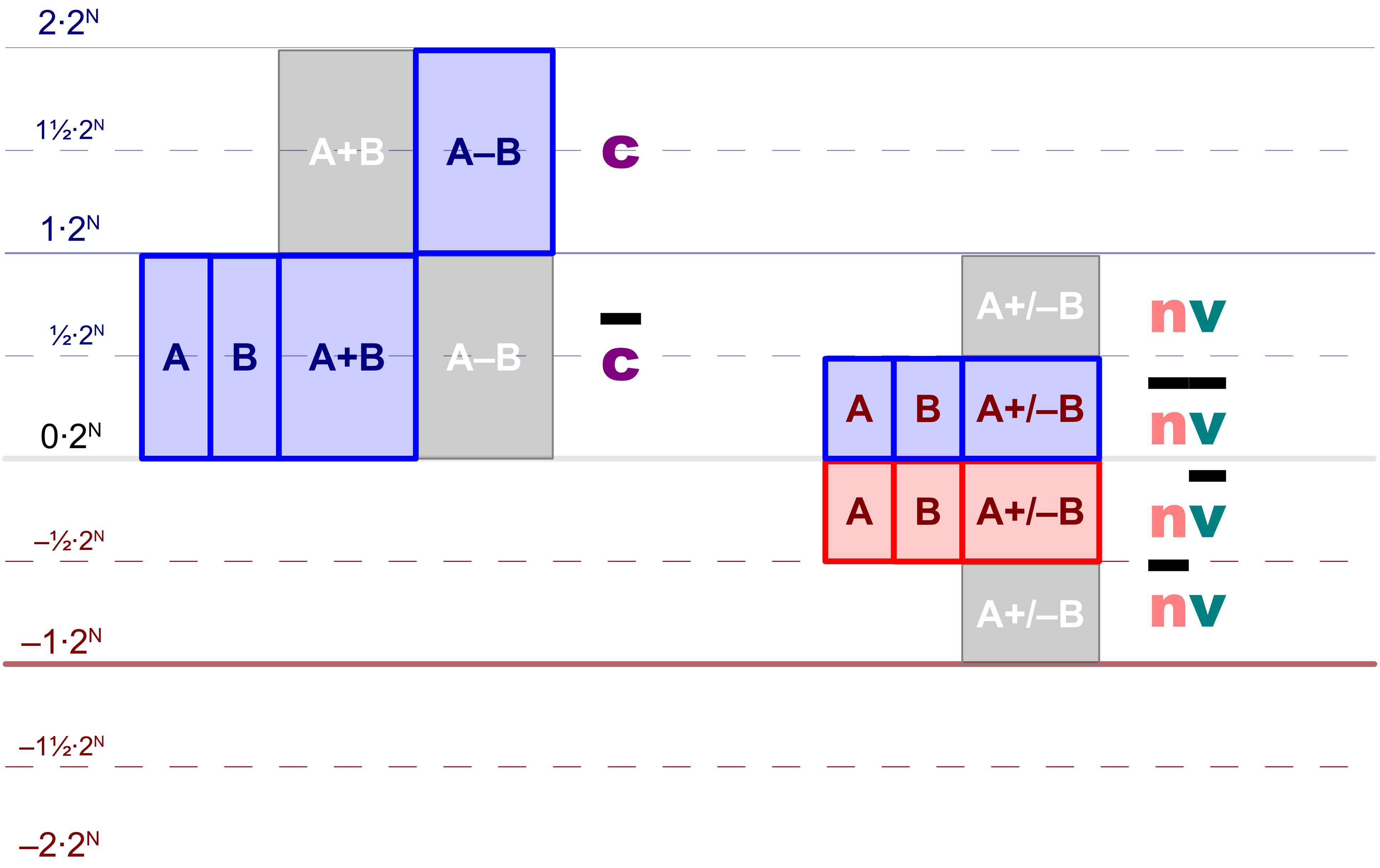
(n-bit) signed addition is *honest* iff (**v** = 0)

a and **b** have different signs or **a**, **b**, and **r** have same sign

(n-bit) signed subtraction is *honest* iff (**v** = 0)

a and **b** have same sign or **a** and **r** have same sign

Condition Flags



Comparisons and Condition Flags

A = B ??

A ≥ B ??

A ≥ B

unsigned

A = B

both

signed

Comparisons and Condition Flags

$A = B$ if and only if $A - B = 0$

$A \geq B$ if and only if $A - B \geq 0$

$A - B \geq 0$

unsigned

both

$A - B = 0$

signed

Comparisons and Condition Flags

$A = B$ if and only if $A - B = 0$

$A \geq B$ if and only if $A - B \geq 0$

$A–B \geq 0$

unsigned

both

$A–B = 0$ z

signed

Comparisons and Condition Flags

$A = B$ if and only if $A - B = 0$

$A \geq B$ if and only if $A - B \geq 0$

$A–B \geq 0$ c

unsigned

$A–B = 0$ z

both

signed

Comparisons and Condition Flags

$A \neq B$ if and only if $A - B \neq 0$

$A < B$ if and only if $A - B < 0$

$A–B \geq 0$ c $A–B < 0$

unsigned

$A–B = 0$ z $A–B \neq 0$

both

signed

Comparisons and Condition Flags

$A \neq B$ if and only if $A - B \neq 0$

$A < B$ if and only if $A - B < 0$

unsigned

$A - B \geq 0$ c

$A - B < 0$ $\overline{c}$

both

$A - B = 0$ z

$A - B \neq 0$ $\overline{z}$

signed

Comparisons and Condition Flags

$A \leq B$ if and only if $A - B \leq 0$

$A > B$ if and only if $A - B > 0$

unsigned	$A–B \geq 0$	c	$A–B < 0$	$\overline{c}$
	$A–B \leq 0$		$A–B > 0$	
both	$A–B = 0$	z	$A–B \neq 0$	$\overline{z}$
signed				

Comparisons and Condition Flags

$A \leq B$ if and only if $A - B \leq 0$

$A > B$ if and only if $A - B > 0$

unsigned	$A-B \geq 0$	c	$A-B < 0$	$\overline{c}$
	$A-B \leq 0$	$z+\overline{c}$	$A-B > 0$	
both	$A-B = 0$	z	$A-B \neq 0$	$\overline{z}$
signed				

Comparisons and Condition Flags

$A \le B$ if and only if $A - B \le 0$

$A > B$ if and only if $A - B > 0$

unsigned	$A\!-\!B \ge 0 \quad c$	$A\!-\!B < 0 \quad \bar{c}$
	$A\!-\!B \le 0 \quad z+\bar{c}$	$A\!-\!B > 0 \quad \bar{z}c$
both	$A\!-\!B = 0 \quad z$	$A\!-\!B \ne 0 \quad \bar{z}$
signed		

Comparisons and Condition Flags

$A \geq B$ if and only if $A - B \geq 0$

$A < B$ if and only if $A - B < 0$

unsigned

$A–B \geq 0$ c

$A–B < 0$ $\overline{c}$

$A–B \leq 0$ $z+\overline{c}$

$A–B > 0$ $\overline{z}c$

both

$A–B = 0$ z

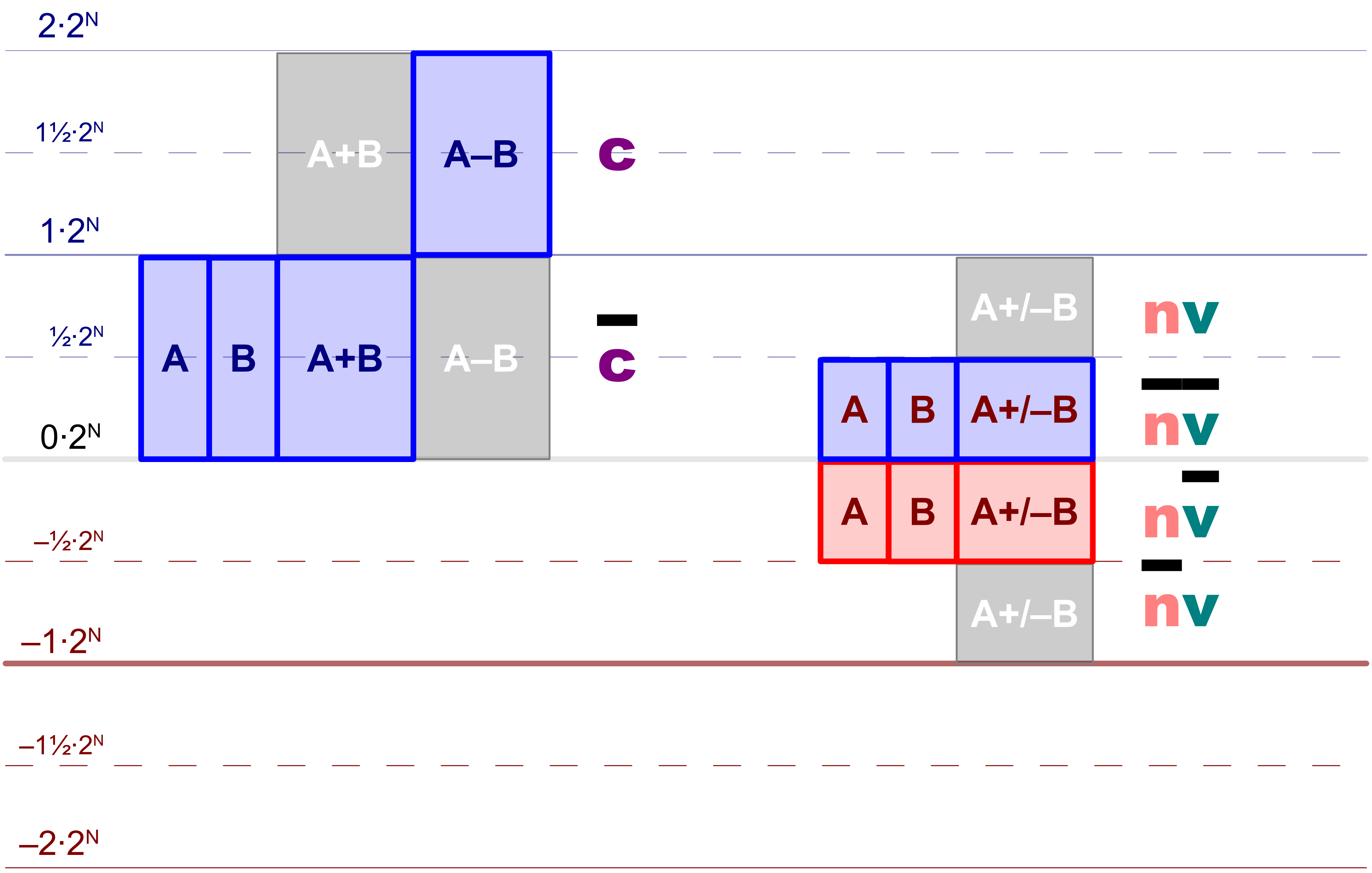
$A–B \neq 0$ $\overline{z}$

signed

$A–B \geq 0$

$A–B < 0$

Condition Flags



Comparisons and Condition Flags

$A \geq B$ if and only if $A - B \geq 0$

$A < B$ if and only if $A - B < 0$

unsigned	$A-B \geq 0$	c	$A-B < 0$	$\bar{c}$
	$A-B \leq 0$	$z+\bar{c}$	$A-B > 0$	$\bar{z}c$
both	$A-B = 0$	z	$A-B \neq 0$	$\bar{z}$
signed	$A-B \geq 0$	$\overline{nv}+nv$	$A-B < 0$	

Comparisons and Condition Flags

$A \geq B$ if and only if $A - B \geq 0$

$A < B$ if and only if $A - B < 0$

unsigned	$A\!-\!B \geq 0$	c	$A\!-\!B < 0$	$\bar{c}$
	$A\!-\!B \leq 0$	$z + \bar{c}$	$A\!-\!B > 0$	$\bar{z}c$
both	$A\!-\!B = 0$	z	$A\!-\!B \neq 0$	$\bar{z}$
signed	$A\!-\!B \geq 0$	$\bar{n}\bar{v} + nv$	$A\!-\!B < 0$	$n\bar{v} + \bar{n}v$

Comparisons and Condition Flags

$A \geq B$ if and only if $A - B \geq 0$

$A < B$ if and only if $A - B < 0$

unsigned	$A-B \geq 0$	c	$A-B < 0$	$\bar{c}$
	$A-B \leq 0$	$z+\bar{c}$	$A-B > 0$	$\bar{z}c$
both	$A-B = 0$	z	$A-B \neq 0$	$\bar{z}$
signed	$A-B \geq 0$	$\bar{n}\bar{v}+nv$	$A-B < 0$	$n\bar{v}+\bar{n}v$
	$A-B \leq 0$		$A-B > 0$	

Comparisons and Condition Flags

$A \geq B$ if and only if $A - B \geq 0$

$A < B$ if and only if $A - B < 0$

unsigned	$A-B \geq 0$	c	$A-B < 0$	$\bar{c}$
	$A-B \leq 0$	$z+\bar{c}$	$A-B > 0$	$\bar{z}c$
both	$A-B = 0$	z	$A-B \neq 0$	$\bar{z}$
signed	$A-B \geq 0$	$\bar{n}\bar{v}+nv$	$A-B < 0$	$n\bar{v}+\bar{n}v$
	$A-B \leq 0$	$z+n\bar{v}+\bar{n}v$	$A-B > 0$	

Comparisons and Condition Flags

$A \geq B$ if and only if $A - B \geq 0$

$A < B$ if and only if $A - B < 0$

unsigned	$A{-}B \geq 0$	c	$A{-}B < 0$	$\bar{c}$
	$A{-}B \leq 0$	$z+\bar{c}$	$A{-}B > 0$	$\bar{z}c$
both	$A{-}B = 0$	z	$A{-}B \neq 0$	$\bar{z}$
signed	$A{-}B \geq 0$	$\overline{nv}+nv$	$A{-}B < 0$	$n\bar{v}+\bar{n}v$
	$A{-}B \leq 0$	$z+n\bar{v}+\bar{n}v$	$A{-}B > 0$	$\overline{znv}+nv$

Comparisons and Condition Flags

unsigned	$A \geq B$	c	$A < B$	$\bar{c}$
	$A \leq B$	$z+\bar{c}$	$A > B$	$\bar{z}c$
both	$A = B$	z	$A \neq B$	$\bar{z}$
signed	$A \geq B$	$\overline{nv}+nv$	$A < B$	$n\bar{v}+\bar{n}v$
	$A \leq B$	$z+n\bar{v}+\bar{n}v$	$A > B$	$\overline{znv}+nv$

HW 10: Condition Flags

For each line in the table on the following page indicate whether the assertion would be true (T), false (?), or unknown (?) if the codition flags obtained their indicated values after the ALU performed the indicated operation.

operation		flags	assertion	T / F / ?
$A \overset{\cdots}{-} B$	unsigned	$\overline{Z}N\overline{C}V$	result is honest	
$A \overset{\cdots}{+} B$	signed	$\overline{Z}\overline{N}\overline{C}\overline{V}$	result is honest	
$A \overset{\cdots}{+} B$	unsigned	$\overline{Z}N\overline{C}V$	result is honest	
$A \overset{\cdots}{-} B$	signed	$Z\overline{N}\overline{C}\overline{V}$	result is honest	
$A \overset{\cdots}{+} B$	unsigned	$\overline{Z}\overline{N}CV$	result is honest	
$A \overset{\cdots}{+} B$	signed	$\overline{Z}NC\overline{V}$	result is honest	
$A \overset{\cdots}{-} B$	unsigned	$\overline{Z}\overline{N}CV$	$A > B$	
$A \overset{\cdots}{-} B$	signed	$\overline{Z}N\overline{C}V$	$A = B$	
$A \overset{\cdots}{-} B$	unsigned	$\overline{Z}N\overline{C}V$	$A < B$	
$A \overset{\cdots}{+} B$	signed	$Z\overline{N}CV$	$A \leq B$	
$A \overset{\cdots}{-} B$	unsigned	$\overline{Z}NC\overline{V}$	$A > B$	
$A \overset{\cdots}{-} B$	signed	$\overline{Z}\overline{N}\overline{C}\overline{V}$	$A \leq B$	
$A \overset{\cdots}{+} B$	unsigned	$Z\overline{N}\overline{C}\overline{V}$	$A < B$	
$A \overset{\cdots}{-} B$	signed	$Z\overline{N}CV$	$A \geq B$	
$A \overset{\cdots}{-} B$	unsigned	$\overline{Z}NC\overline{V}$	$A \leq B$	
$A \overset{\cdots}{-} B$	signed	$Z\overline{N}\overline{C}\overline{V}$	$A \geq B$	

Honesty Criteria

The n-bit result **r** of a binary operation on n-bit values **a** and **b** is ***honest*** (***deceptive***) if it is *the same as* (*different from*) the whole number result of the same operation on the same values.

(n-bit) unsigned addition is *honest* iff (**c** = 0)

Carry flag is not set

(n-bit) unsigned subtraction is *honest* iff (**c** = 1)

Carry flag is set

(n-bit) signed addition is *honest* iff (**v** = 0)

a and **b** have different signs or **a**, **b**, and **r** have same sign

(n-bit) signed subtraction is *honest* iff (**v** = 0)

a and **b** have same sign or **a** and **r** have same sign

HW 9: Signed Binary Arithmetic

For each of the <X, Y> pairs in the table below:

a) Convert X and Y → binary

b) Compute $X \overset{\cdots}{+} Y$ (the 8-bit **sum**)

c) Compute $\overset{\cdots}{Y}$ (the 2's complement of Y)

d) Compute $X \overset{\cdots}{-} Y \equiv X \overset{\cdots}{+} \overset{\cdots}{Y}$ (the 8-bit **difference**)

e) Indicate the signs of X, Y, $X \overset{\cdots}{+} Y$, $\overset{\cdots}{Y}$, and $X \overset{\cdots}{-} Y$

f) Convert $X \overset{\cdots}{+} Y$, $\overset{\cdots}{Y}$, and $X \overset{\cdots}{-} Y$ → hexadecimal

g) Indicate condition flag (**z**, **n**, **c**, **v**) values for $X \overset{\cdots}{+} Y$, $X \overset{\cdots}{-} Y$

h) Is $X \overset{\cdots}{+} Y$ honest? is $X \overset{\cdots}{-} Y$ honest?

Where <X, Y> =

1) <`0x`**`4F`**, `0x`**`6D`**>
2) <`0x`**`B3`**, `0x`**`17`**>
3) <`0x`**`A3`**, `0x`**`95`**>
4) <`0x`**`6E`**, `0x`**`3A`**>

Signed Arithmetic Example: X4

X	Y	X + Y	~Y	X – Y
`0x5A`	`0x5A`			

Signed Arithmetic: X4 a)

X	Y	X + Y	~Y	X – Y
`0x5A`	`0x5A`			
`01011010`	`01011010`			

Signed Arithmetic: X4 b)

X	Y	X + Y	~Y	X – Y
0x5A	**0x5A**			
01011010	01011010	01011010		
		01011010		
		0 1 0 1 1 0 1 0		
		010110100		

Signed Arithmetic: X4 c)

X	Y	X + Y	~Y	X – Y
0x5A	**0x5A**			
01011010	01011010	01011010	$\overline{01011010}$	
		$\underline{01011010}$	10100101	
		0 1 0 1 1 0 1 0	$\underline{00000001}$	
		010110100	10100110	

Signed Arithmetic: X4 d)

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010	$\overline{01011010}$	01011010
		01011010	10100101	10100110
		0 1 0 1 1 0 1 0	00000001	1 1 1 1 1 1 1 0
		010110100	10100110	100000000

Signed Arithmetic: X4 e)

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010	$\overline{01011010}$	01011010
		01011010	10100101	10100110
		0 1 0 1 1 0 1 0	00000001	1 1 1 1 1 1 1 0
		010110100	10100110	100000000

Signed Arithmetic: X4 f)

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010	$\overline{01011010}$	01011010
		01011010	10100101	10100110
		0 1 0 1 1 0 1 0	00000001	1 1 1 1 1 1 1 0
		010110100	10100110	100000000
		0xB4	0xA6	0x00

Signed Arithmetic: X4

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010	$\overline{01011010}$	01011010
		01011010	10100101	10100110
		0 1 0 1 1 0 1 0	00000001	1 1 1 1 1 1 1 0
		010110100	10100110	100000000
		0xB4	0xA6	0x00
		oVerflow		no oVerflow

Signed Arithmetic: X4

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010 01011010	$\overline{01011010}$ 10100101 00000001	01011010 10100110
		0 1 0 1 1 0 1 0		1 1 1 1 1 1 1 0
		010110100	10100110	100000000
		0xB4	0xA6	0x00
		oVerflow		no oVerflow

Signed Arithmetic: X4

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010 01011010 0 1 0 1 1 0 1 0 010110100	01011010 10100101 00000001 10100110	01011010 10100110 1 1 1 1 1 1 1 0 100000000
		0xB4	0xA6	0x00
		oVerflow		no oVerflow

Signed Arithmetic: X4 g)

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010	$\overline{01011010}$	01011010
		01011010	10100101	10100110
		0 1 0 1 1 0 1 0	00000001	1 1 1 1 1 1 1 0
		010110100	10100110	100000000
		0xB4	0xA6	0x00
		$\bar{z}n\bar{c}v$		$z\bar{n}c\bar{v}$

Signed Arithmetic: X4 h)

X	Y	X + Y	~Y	X – Y
0x5A	0x5A			
01011010	01011010	01011010 01011010 0 1 0 1 1 0 1 0 010110100	$\overline{01011010}$ 10100101 00000001 10100110	01011010 10100110 1 1 1 1 1 1 1 0 100000000
		0xB4	0xA6	0x00
		$\bar{z}n\bar{c}v$		$z\bar{n}c\bar{v}$
		(s) deceptive		(s) honest