

1.3 Propositional Equivalences 1

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An important step used in a mathematical argument is the replacement of a statement with another statement with the same truth value.

tautology is a compound proposition that is always true, no matter what the truth values of the propositions that occur in it.

example: $\neg p \vee p$

p	$\neg p$	$\neg p \vee p$
0	1	1
1	0	1

contradiction is a compound proposition that always false.

example: $\neg p \wedge p$

p	$\neg p$	$\neg p \wedge p$
0	1	0
1	0	0

contingency is a compound statement that is neither a tautology nor a contradiction.

example: $p \rightarrow q \vee \neg p$

Two compound propositions, **p** and **q**, are *logically equivalent* if $p \leftrightarrow q$ is a tautology.

We'll write $p \equiv q$ or $p \leftrightarrow q$

Ways to determine whether two compound proposition are equivalent:

- truth tables (columns giving their truth values agree)
- use laws

Example 1: proof by truth tables that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Reading of both compound propositions:

Let **p**: “the weather is good” and **q**: “we'll go swimming” Then,

$p \rightarrow q$: “If the weather is good, we'll go swimming”; and

$\neg p \vee q$: “the weather is not good or we'll go swimming”