Zero matrices

A **zero matrix** is a matrix with all zero values, like these:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We always name the zero matrix with a capital O. And optionally, you can add a subscript with the dimensions of the zero matrix. Since the values in a zero matrix are all zeros, just having the dimensions of the zero matrix tells you what the entire matrix looks like. As an example, $O_{2\times3}$ is

$$O_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Adding and subtracting the zero matrix

Adding the zero matrix to any other matrix doesn't change the matrix's value. And subtracting the zero matrix from any other matrix doesn't change that matrix's value.

Just like with non-zero matrices, matrix dimensions have to be the same in order to be able to add or subtract them.

Adding the zero matrix:

$$\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix}$$

Subtracting the zero matrix:

$$\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix}$$

Adding opposite matrices

Adding opposite matrices always results in the zero matrix. Matrices K and $-K$ are **opposite matrices**. So are A and $-A$, and so are X and $-X$. In other words, to get the opposite of a matrix, multiply it by a scalar of -1. So if

$$K = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix}$$

then the opposite of K is

$$-K = (-1)\begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} = \begin{bmatrix} (-1)9 & (-1)(-6) & (-1)2 \\ (-1)1 & (-1)0 & (-1)(-7) \end{bmatrix} = \begin{bmatrix} -9 & 6 & -2 \\ -1 & 0 & 7 \end{bmatrix}$$

If we now add K and $-K$, we'll end up with the zero matrix that has the same dimensions as K and $-K$. Since K and $-K$ are both 2×3, we should get $O_{2\times3}$.

$$K + (-K) = \begin{bmatrix} 9 & -6 & 2 \\ 1 & 0 & -7 \end{bmatrix} + \begin{bmatrix} -9 & 6 & -2 \\ -1 & 0 & 7 \end{bmatrix}$$

$$K + (-K) = \begin{bmatrix} 9+(-9) & -6+6 & 2+(-2) \\ 1+(-1) & 0+0 & -7+7 \end{bmatrix}$$

$$K + (-K) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O_{2\times3}$$

Let's do an example with zero matrices.

Example

Find k and x.

$$\begin{bmatrix} 7 & -1 \\ 4 & 0 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -7 & 1 \\ k & 0 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 9 \\ 0 & -6 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -9 \\ 0 & 6 \\ -1 & x \end{bmatrix}$$

Adding the zero vector won't change the value of a matrix, and neither will subtracting out the zero vector, so we can simplify the equation to

$$\begin{bmatrix} 7 & -1 \\ 4 & 0 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -7 & 1 \\ k & 0 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 9 \\ 0 & -6 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -9 \\ 0 & 6 \\ -1 & x \end{bmatrix}$$

Let's add the matrices on the left together, and separately add the matrices on the right together.

$$\begin{bmatrix} 7+(-7) & -1+1 \\ 4+k & 0+0 \\ -2+2 & 3+(-3) \end{bmatrix} = \begin{bmatrix} 2+(-2) & 9+(-9) \\ 0+0 & -6+6 \\ 1+(-1) & 1+x \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 4+k & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1+x \end{bmatrix}$$

The matrices are equal when entries from corresponding positions in each matrix are equal. So we get

$$4 + k = 0$$

$$k = -4$$

and

$$0 = 1 + x$$

$$x = -1$$

Therefore, we can say that $k = -4$ and $x = -1$ are the values that make the equation true.