Polynomial Chaos Expansions for Wind Energy

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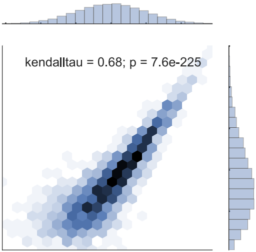
Outline

- (A very short) Introduction to PCE
- Case A: A simple model
    - A1. 2D Not correlated Normal
    - A2. 2D Normal
    - A3. 2D Conditionally correlated
- Case A: PCE for Aero-elastic model

Uncertainty propagation problem

Input variables $\mathbf{x} \in \mathbb{R}^M$ → Model $\mathcal{M}(\mathbf{x})$ → Output variables $\mathbf{y} = \mathcal{M}(\mathbf{x}) \in \mathbb{R}^L$

Random inputs $\mathrm{PDF}(\mathbf{x})$ → Model $\mathcal{M}(\mathbf{x})$ → **Random** outputs $\mathrm{PDF}(\mathbf{y})$



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(A very short) Introduction to PCE

Monte-Carlo simulation

- Obtain a response sample by evaluating the model in each input realization: $\mathbf{y}_i = \mathcal{M}(\mathbf{x}_i)$
- Input sample, $\mathbf{x}_i$, can be generated using advanced sampling methods such as: Latin Hypercube (LHS), Halton or Hammersley sampling.
- **Pros:** Very robust and easy to implement and parallelize
- **Cons:** Convergence is slow ($\propto N^{-1/2}$)

Polynomial Chaos expansion

- Build a polynomial surrogate of the model that captures the central trend: mean $\mathbb{E}(\mathbf{y})$, variance $\mathbb{V}(\mathbf{y})$ and sensitivity index for each input $S_i = \mathbb{V}\left(\mathbb{E}_{\forall k \neq i}\left(\mathbf{y}(\mathbf{x}|x_i)\right)\right)/\mathbb{V}(\mathbf{y})$.
- A polynomial basis is built with respect to $\mathrm{PDF}(\mathbf{x})$.
- The model is evaluated, $\mathbf{y}_i = \mathcal{M}(\mathbf{x}_i)$, and *projected/fitted* to the polynomial basis.
- **Pros:** Convergence is fast ($\propto N^{-l}$, $l > 1$, l is problem dependent)
- **Cons:** The surrogate does not capture the actual $\mathrm{PDF}(\mathbf{y})$

(A very short) Introduction to PCE

Single uncertain variable x

$$y \approx \sum_{l=0}^{P} y_l \, \pi_l(x)$$

Define an inner product using $\mathrm{PDF}(x)$:

$$\langle f, g \rangle = \int f(x)\, g(x)\, \mathrm{PDF}(x)\, dx$$

The polynomial basis is constructed such that $\pi_0 = 1$ and:

$$\langle \pi_l, \pi_k \rangle = \delta_{lk}$$

$$\langle 1, \pi_l \rangle = 0 \quad \forall l > 0 \qquad \Longleftrightarrow \qquad \int \pi_l(x)\, \mathrm{PDF}(x)\, dx = 0 \quad \forall l > 0$$

Methods to find the coefficients y_l

Semi-Spectral projection

- Use a quadrature rule to approximate the integrals (nodes, x_i and weights ω_i). Gaussian quadrature is widely used.

$$y_l = \langle y, \pi_l \rangle = \int y(x)\, \pi_l(x)\, \mathrm{PDF}(x)\, dx \approx \sum_{i=0}^{N} \omega_i\, y(x_i)\, \pi_l(x_i)$$

- **Pros:** Very good for low number of dimensions
- **Cons:** Unstable for heavy tailed PDFs. Quadrature rules fail with most correlated variables

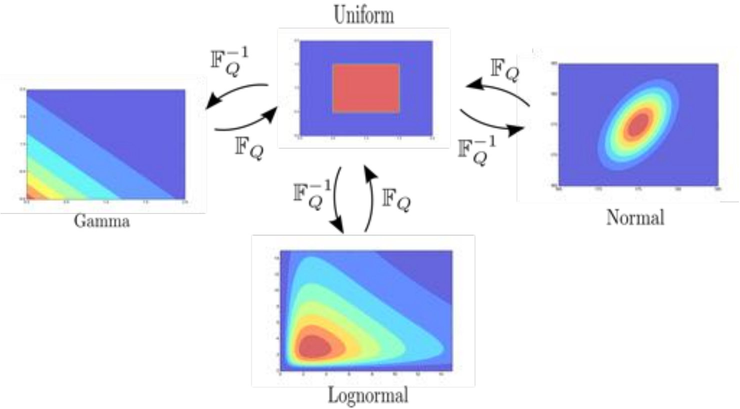
Point collocation

- Generate a small sample and fit the polynomial basis using Least squares or some other optimization method (e.g. LAR, LASSO).
- **Pros:** Very robust. Optimization algorithms are design to handle large number of dimension (sparsity) and correlated inputs.
- **Cons:** Not as efficient as semi-spectral collocation.

How to deal with correlated inputs

Rosenblatt Transformation [Rosenblatt 1952]

- Transforms the correlated input variable space into a multi-dimensional uncorrelated uniform space. Solve the propagation problem in the uniform space.
- The transformation consists in using the inverse of the CDF of each variable in a sequence. Chaospy includes this transformation [Feinberg 2015]. Graph reproduced from Chaospy tutorials.



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Case A: A simple model
Model Description

Input variables $\mathbf{x} = (I; a) \in \mathbb{R}^2$ → Model $\mathcal{M}(\mathbf{x}; t) = I\, e^{-a\, t}$ → Output variables $\mathbf{y} = \mathcal{M}(\mathbf{x}; t) \in \mathbb{R}^L$

Variables without uncertainty

- t: Location of the evaluation.

Uncertain variables

- I: Initial condition.
- a: Rate of dissipation.

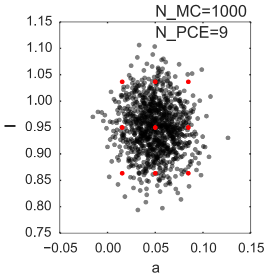
Case A1: A simple model

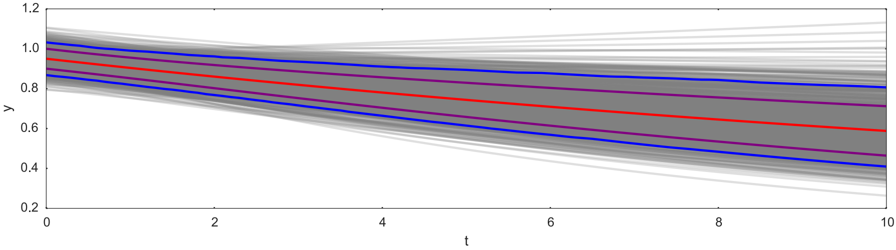
Inputs: 2D Not correlated Normal

$$y = I\, e^{-a\, t}$$

$$I \sim Normal(\mu_I = 0.95, \sigma_I = 0.05)$$

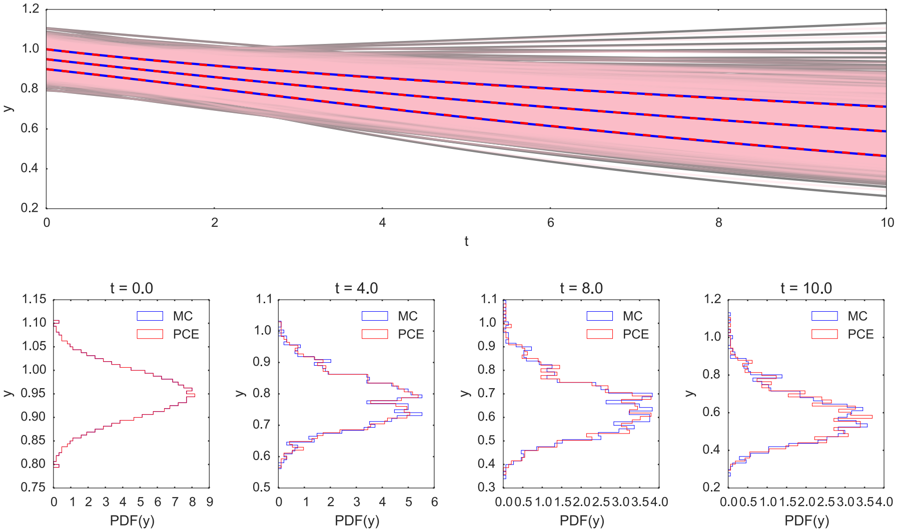
$$a \sim Normal(\mu_a = 0.05, \sigma_a = 0.02)$$





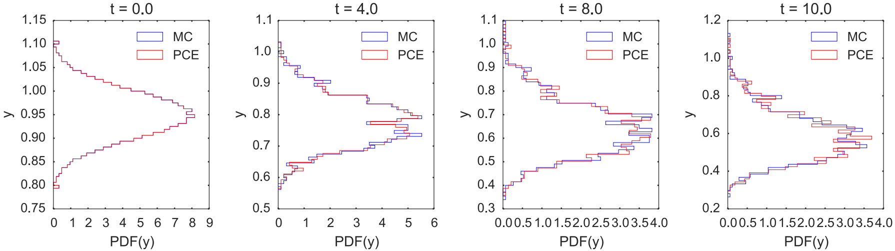
Case A1: A simple model

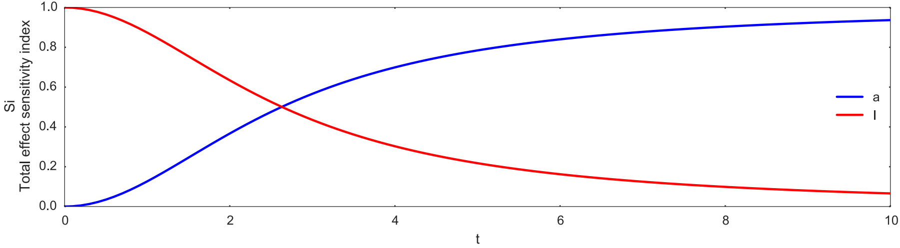
Inputs: 2D Not correlated Normal



Case A1: A simple model

Inputs: 2D Not correlated Normal





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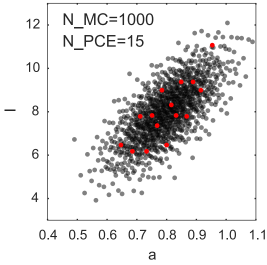
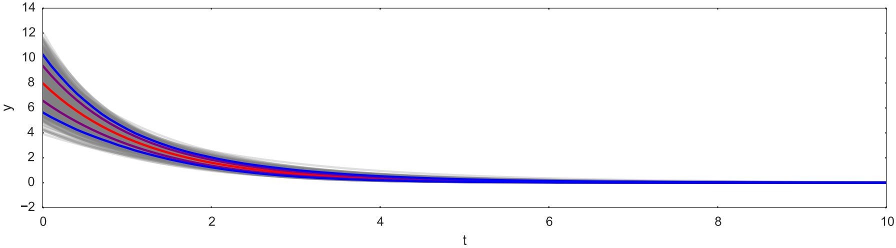
Case A2: A simple model
Inputs: 2D Correlated Normal

$$y = I\, e^{-a\, t}$$

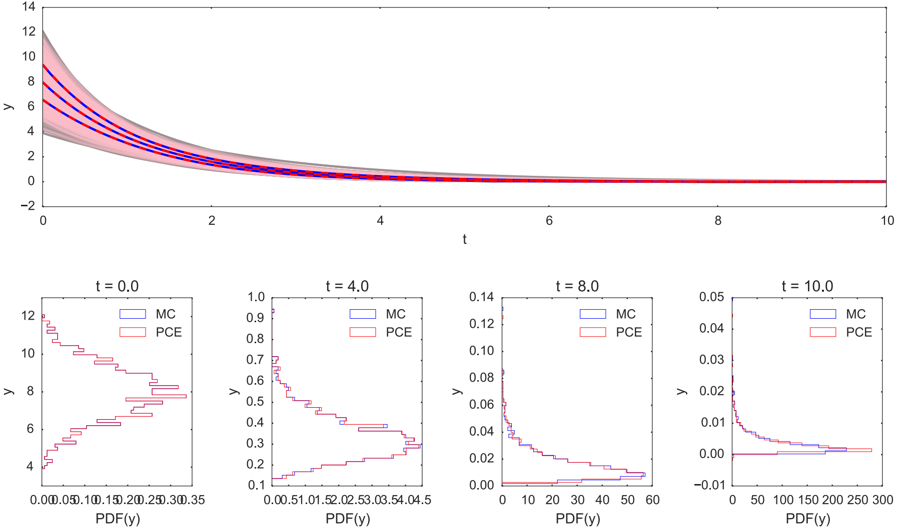
$$\begin{bmatrix} a \\ I \end{bmatrix} \sim Normal(\mu, \mathbb{C})$$

$$\mu = \begin{bmatrix} 0.8 \\ 8. \end{bmatrix} \quad \mathbb{C} = \begin{bmatrix} 0.01 & 0.1 \\ 0.1 & 2. \end{bmatrix}$$



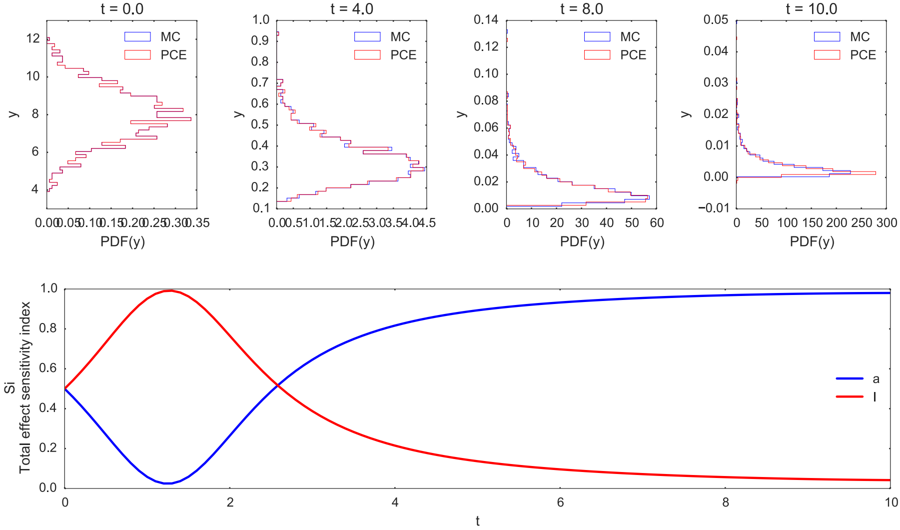


Case A2: A simple model
Inputs: 2D Correlated Normal



Case A2: A simple model

Inputs: 2D Correlated Normal



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Case A3: A simple model

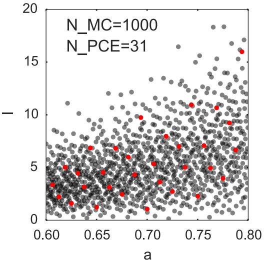
Inputs: 2D Conditionally Correlated

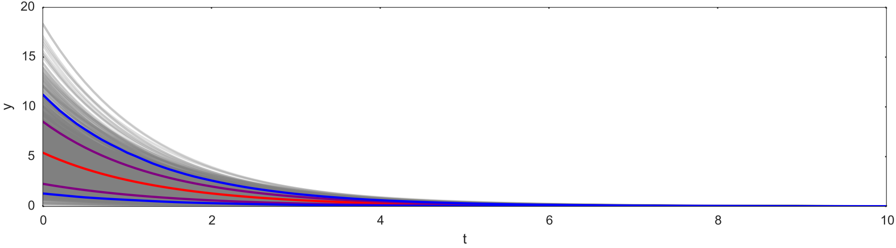
$$y = I\, e^{-a\, t}$$

$$a \sim Uniform(0.6, 0.8)$$

$$I \sim Weibull(k = 2.,$$

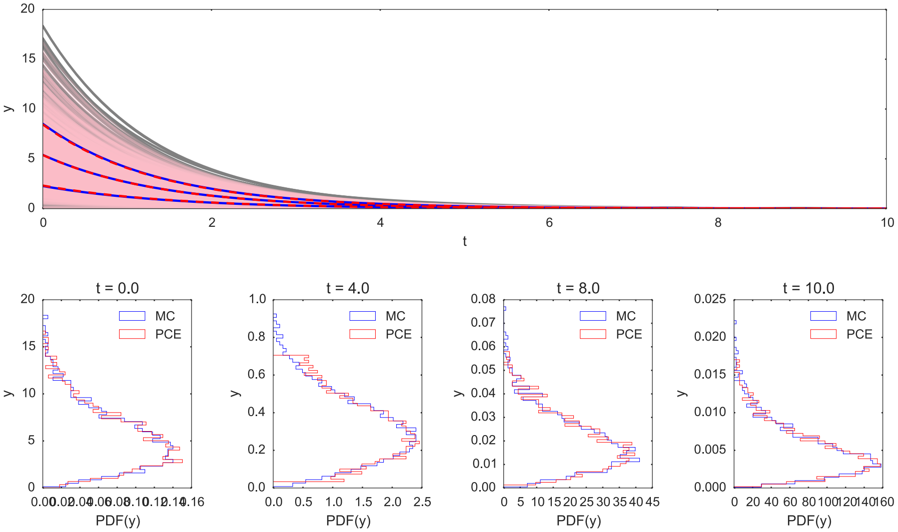
$$A = 6(0.3 + a)^4)$$





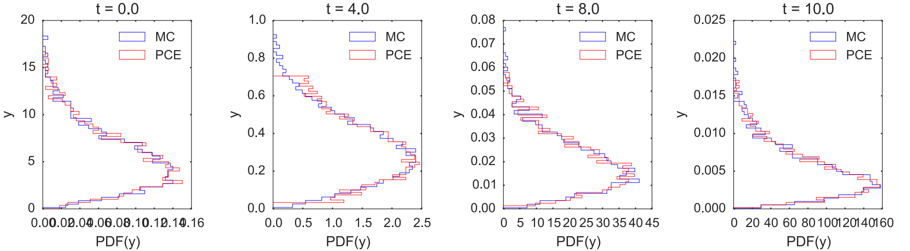
Case A3: A simple model

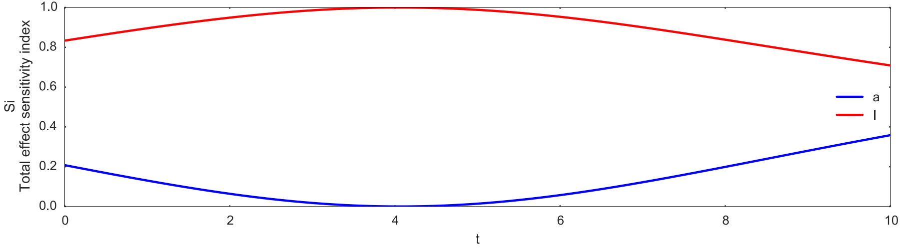
Inputs: 2D Conditionally Correlated



Case A3: A simple model

Inputs: 2D Conditionally Correlated





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DTU

Wind turbine model

Inputs: 4D Conditionally Correlated. [Dimitrov 2015]

Input variables

$$\mathbf{x} \in \mathbb{R}^4$$
$$\mathbf{x} = (TSeed; \mathsf{WS}; \sigma_{\mathsf{WS}}; \alpha)$$

→ Model $\mathcal{M}(\mathbf{x})$ →

Output variables

$$\mathbf{y} = \mathcal{M}(\mathbf{x}) \in \mathbb{R}$$

Model

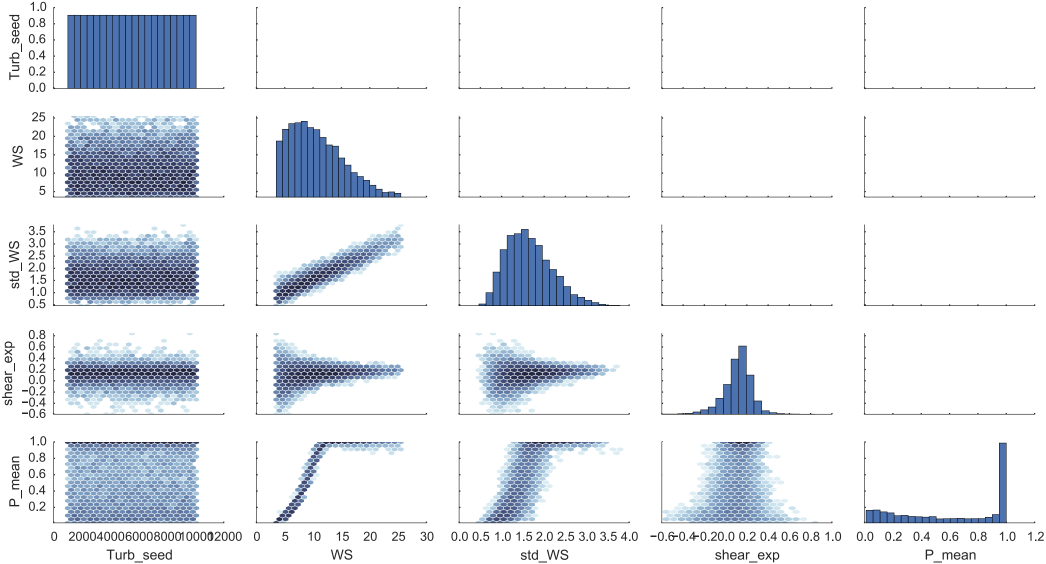
- HAWC2 DTU 10MW

Uncertain variables [Dimitrov 2015]

- TSeed $= Uniform(0, N)$
- WS $= Weibull_{\mathsf{truncated}}(k = 2, A = 10, \mathsf{low} = 3.5, \mathsf{high} = 25.5)$
- $\sigma_{\mathsf{WS}} = Lognormal(\mu = \mu(\mathsf{WS}), \sigma = \sigma(\mathsf{WS}))$
- $\alpha = Normal(\mu = 0.088(\log(\mathsf{WS}) - 1), \sigma = 1/\mathsf{WS})$

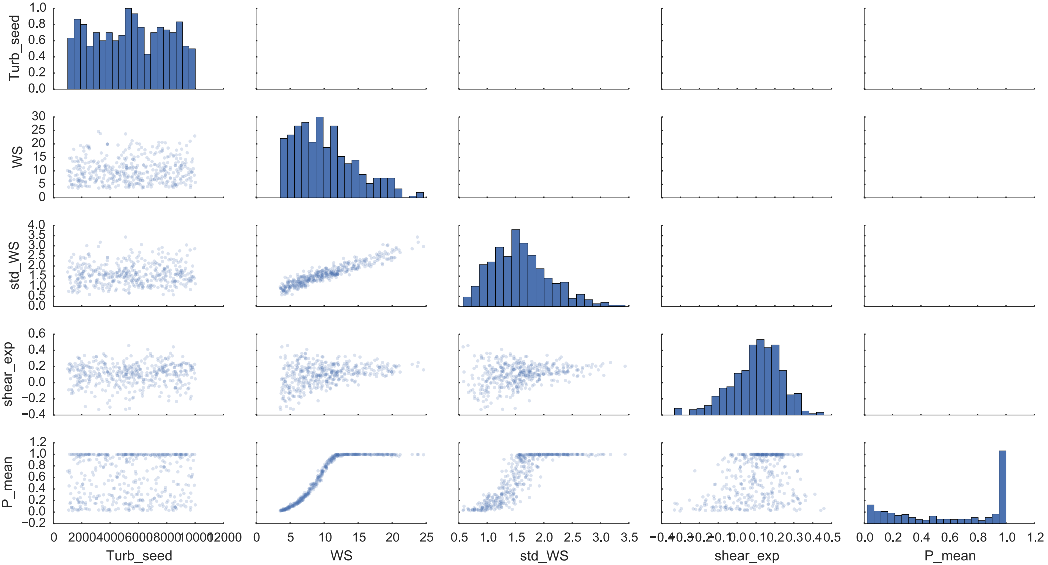
Wind turbine model

Full MC sample: 10000 HAWC2 simulations



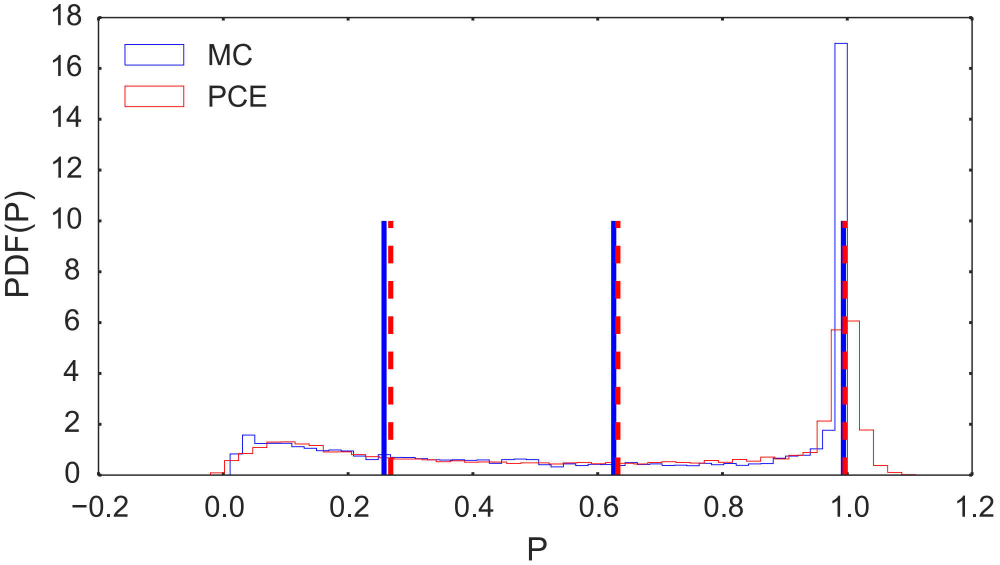
Wind turbine model

PCE point collocation: randomly chosen 100 HAWC2 simulations

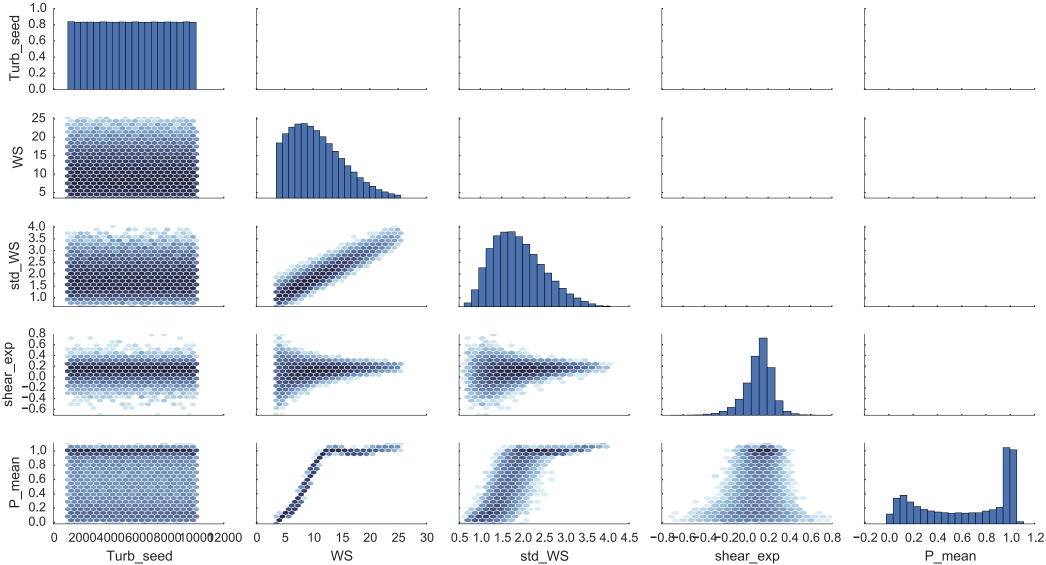


Wind turbine model

Output $\mathrm{PDF(y)}$ comparison



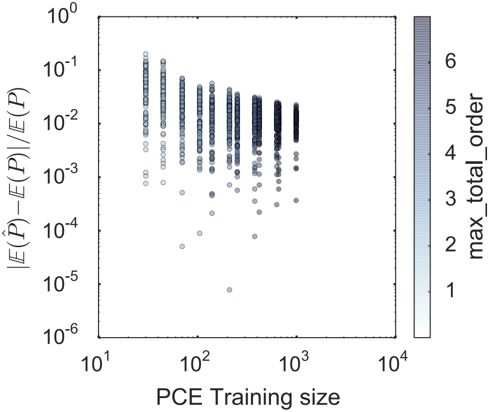
Wind turbine model
Polynomial surrogate

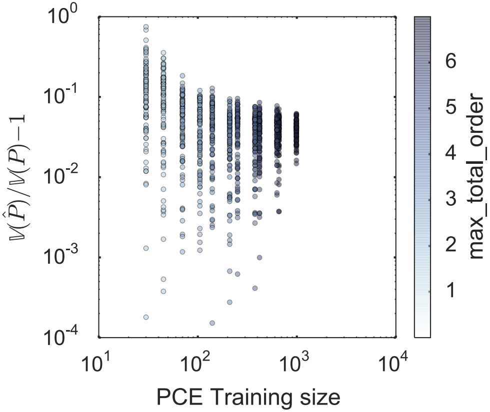


Wind turbine model

Convergence of 10-min mean power statistics

- PCE training size: Size of the subset of the MC sample used to train the PCE. 100 random subset were selected for each training size.
- Max total order: Term with the highest total degree (sum of the degrees in each variable).

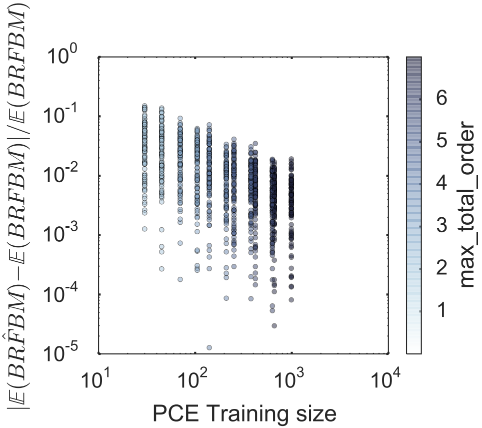


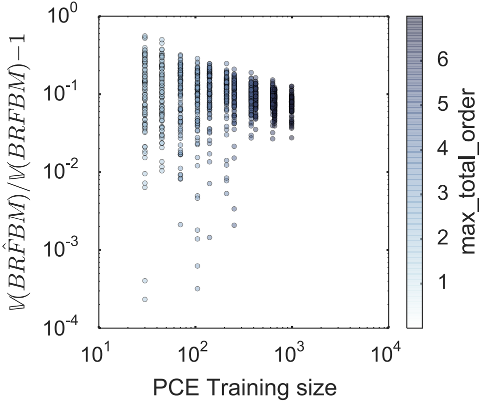


Wind turbine model

Convergence of 10-min mean blade root flap-wise bending moment statistics

- PCE training size: Size of the subset of the MC sample used to train the PCE. 100 random subset were selected for each training size.
- Max total order: Term with the highest total degree (sum of the degrees in each variable).





Conclusions

PCE:

- Efficient uncertainty propagation that enables to compute the statistics of the output such as: mean, standard deviation and sensitivity analysis.
- The polynomial surrogate will not capture all the details of a complex model, but it does predicts the main behavior.

PCE as Aero-ealastic model surrogate:

- Using the turbulent seed as an uncertain variable DOES NOT capture the behavior caused by the turbulent field. It translate the stochasticity into the selection of the training points.
- Estimate the mean, variance of the effect of the turbulent seed for every simulation using sample of turbulent seeds in advance.

References I

[Rosenblatt 1952] Rosenblatt, M 1952 Remarks on a multivariate transformation. Annals of Mathematical Statistics Vol. 23, pp 470-472.]

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[Dimitrov 2015] Dimitrov, N., Natarajan, A., & Kelly, M. (2015). Model of wind shear conditional on turbulence and its impact on wind turbine loads. Wind Energy, 18(11), 1917-1931.

Questions?

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