Should the Fed Target The Output Gap or Output Growth? A Bayesian Econometric Investigation Into Indeterminacy*

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February 9, 2021

Abstract

Should the Fed target output gap or output growth to improve stabilization? Using a Bayesian procedure allowing for trend inflation and indeterminacy, we show a rule targeting output growth achieves determinacy during the pre-Volcker and Great Moderation periods. A rule aiming at output gap implies indeterminacy during both, an outcome hardly reconcilable with stability experienced after 1984. We show that the average frequency of price changes is very sensitive to small variations in the mean prior of the price stickiness parameter under a rule aiming at output gap, but not with a rule targeting output growth.

JEL classification: E31, E32, E37.

Keywords: Bayesian estimation; Indeterminacy; Taylor rules; Trend inflation; Output gap; Output growth.

^{*}We are grateful to Guido Ascari, Yasuo Hirose, Hashmat Khan, and Carl Walsh for useful comments and suggestions. Brault acknowledges financial support from the Social Sciences and Humanities Research Council for doctoral scholarship 752-2020-2683. Phaneuf acknowledges financial support from the Social Sciences and Humanities Research Council for project 435-2020-1061-Achieving Macroeconomic Stability in Economies with Positive Long Run (Trend) Inflation.

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1 Introduction

What measure of economic activity should the Fed target to improve stabilization? The output gap or output growth? This is a timely question addressed by macroeconomists. For instance, Walsh (2003) showed that a social loss function in which the policy objectives are inflation and output growth improves stabilization over one where the objectives are inflation and the output gap. Using a calibrated New Keynesian model with positive trend inflation, Coibion and Gorodnichenko (2011) showed that targeting output growth requires a much smaller policy response to inflation to ensure determinacy than aiming at the output gap. Khan et al. (2020) demonstrated that with a policy rule including policy responses to both the output gap and output growth, the influence of the output gap is disproportionately important and threatens the prospect of determinacy even at low rates of trend inflation like 2% or 3%.

Missing from the literature, however, is some evidence based on econometric estimation that targeting output growth indeed improves stabilization. This is what our paper does. We estimate a New Keynesian model in which monetary policy is assumed to target either the output gap or output growth using a Bayesian estimation method that formally accounts for positive trend inflation and indeterminacy along the lines of Lubik and Schorfheide (2004), Hirose et al. (2020) and Bianchi and Nicoló (2020). We estimate the model using data covering for the pre-Volcker era and Great Moderation.

We begin by providing a simple example illustrating why with positive trend inflation aiming at the output growth achieves determinacy with a lower policy response to inflation than reacting to the output gap. This is followed by the estimation of models allowing policy to react to output gap or output growth with data covering the pre-Volcker and Great Moderation years. A key difference with previous econometric works that have looked at whether the US economy experienced indeterminacy and when, is that we investigate the determinacy properties of our models over a plausible range of mean priors of the Calvo price stickiness parameter.

Specifically, we choose mean priors that are broadly consistent with microeconomic price

adjustment evidence and ranging from 0.5 to 0.65 (Bils and Klenow (2004); Nakamura and Steinsson (2008)). We intend to show that when monetary policy targets the output gap, the estimated frequency of price adjustment is quite sensitive to modest variations in the mean prior of the price stickiness parameter. We show this has a significant impact on the determinacy outcome, especially during the Great Moderation period. By contrast, when the Fed targets output growth, estimates of the price stickiness parameter are much more stable upon varying the mean prior of the price stickiness parameter.

Our first set of substantive findings pertains to the pre-Volcker period. Based on point estimates of either the output gap or the output growth policy rule, we find evidence that monetary policy was active during the pre-Volcker period. That is, monetary policy did more than complying with the Taylor principle requiring a policy response to inflation greater than one. Interestingly, based on 90% confidence intervals, we report several cases for which there is no uncertainty that the Fed's policy was active during the pre-Volcker era. In many cases we find that the policy responses to inflation are close to Taylor (1993)'s prescription of 1.5 and even exceed it. This finding contrasts with most of the previous literature suggesting that indeterminacy mainly resulted from the Fed's response to inflation that was not strong enough (Clarida et al. 2000, Lubik and Schorfheide 2004).

The estimated probability of indeterminacy implied by the model where the Fed targets the output gap is always 100% for the pre-Volcker period. We obtain estimates of the policy responses to the output gap that were significantly higher for the pre-Volcker period than for the Great Moderation, and this by a factor of 2 to 4. Interestingly, our estimates of trend inflation are broadly in-line with the average annualized rate of inflation observed during the pre-Volcker years. A Bayesian method not allowing explicitly for indeterminacy significantly underestimates trend inflation during the Great Inflation years as noted by Smets and Wouters (2007), which induces some bias in favor of the determinacy outcome.

We obtain very different results when monetary policy targets output growth. We find that the probability of determinacy varies between nearly 63% and 99% depending on the mean prior for the price stickiness parameter during the pre-Volcker years, and this despite the fact the estimated level of trend inflation is broadly consistent with actual average infla-

tion. The determinacy outcome results mostly from a moderately active policy prior to 1980, and because the frequency of price adjustment was relatively high and in-line with microeconomic evidence on price changes. It is worth emphasizing that there is no need to resort to indexation for this finding.

Another striking result is that for the Great Moderation, we obtain indeterminacy with certainty when assuming a policy rule reacting to the output gap, except for the lowest mean prior of the price stickiness parameter.¹ Our estimates under the output gap-rule suggest monetary policy was significantly more aggressive in fighting inflation after 1982. However, we also find that the estimated degree of price stickiness rises significantly with higher mean priors of the price stickiness parameter. The average frequency of a price change implied by the estimated output gap-rule model—once every 12 months or more—is inconsistent with the evidence based on microeconomic data. Combined with an estimated level of trend inflation which is 3% or more, the higher degree of price rigidity poses a threat to determinacy even after 1983 despite policy becoming more active.

By contrast, a policy rule targeting output growth ensures determinacy with certainty during the Great Moderation, and this even if we increase the mean prior of the price stickiness parameter to 0.75. One reason is that monetary policy was more active during that period. But also, the estimated price stickiness parameter is much lower and more stable upon varying the mean prior of the price stickiness parameter. The estimated frequency of price adjustment is still consistent with evidence of price changes based on microeconomic data. Furthermore the estimated trend inflation is lower than that under the gap-rule. Finally, the estimated policy response to output growth is much stronger during the Great Moderation.

We conclude from our empirical investigation that a policy rule targeting output growth achieves determinacy with higher certainty. It ensures determinacy with a high probability during the pre-Volcker period and certainty during the Great Moderation. With a rule targeting the output gap, the probability of indeterminacy is nearly 100% in both periods. Since indeterminacy is hard to reconcile with the increased macroeconomic stability experienced

¹We find similar results for a rule targeting both the output gap and output growth. Khan et al. (2020) show that the effect of the output gap on the prospect of indeterminacy is disproportionately important compared to that of output growth.

after 1982, it is hard to believe that Fed targeted the output gap, at least after 1982.

The rest of the paper is organized as follows. Section 2 presents a simple illustrative example showing why targeting output growth helps in achieving determinacy compared to targeting the output gap. Section 3 presents the model to be estimated. Section 4 discusses the data and estimation strategy. Section 5 analyzes the estimation results and their implication for the prospect of indeterminacy during the pre-Volcker era. Section 6 does the same for the Great Moderation. Section 7 contains concluding remarks.

2 Targeting Output Gap Versus Output Growth

Coibion and Gorodnichenko (2011) and Khan et al. (2020) argue that a policy rule targeting output growth ensures determinacy with much smaller policy responses to inflation than one aiming at the output gap.

We offer a simple example illustrating the implications for determinacy of a rule targeting the output gap or output growth.² Suppose a Taylor rule wherein the central bank targets both the output gap and its rate of change:

$$r_t = \alpha_\pi \pi_t + \alpha_x x_t + \alpha_\Delta (x_t - x_{t-1}), \tag{1}$$

with π denoting inflation and x the output gap; π is expressed as the deviation of inflation from its steady-state value.³ As shown by Bullard and Mitra (2002), with a rule targeting only the output gap ($\alpha_{\Delta} = 0$) and zero trend inflation, the condition for determinacy is

$$\alpha_{\pi} + \alpha_{x} \left(\frac{1-\beta}{\kappa} \right) > 1$$
,

where β is the discount factor, and κ represents the elasticity of inflation with respect to the output gap. This can be expressed as

$$\alpha_{\pi} + \alpha_{x} \frac{\partial x}{\partial \pi} \mid_{LR} > 1,$$

²We are thankful to Carl Walsh for suggesting this example.

³Note that the change in the output gap is equivalent to the deviation of output growth from trend growth.

where $\frac{\partial x}{\partial \pi}$ | LR represents the long-run multiplier of inflation to the output gap. It also represents the long-run effect of a permanent rise in inflation on the nominal interest. Therefore, with zero trend inflation, determinacy can be achieved insofar as the rise in the nominal interest rate after a permanent increase in inflation is more than one-for-one.

Under positive trend inflation, the condition $\alpha_{\pi} + \alpha_{x} \frac{\partial x}{\partial \pi} \mid_{LR} > 1$ still holds but with a long-run multiplier term depending on positive trend inflation. Ascari and Ropele (2009) show that if $\overline{\pi}$ is non-zero trend inflation, then

$$\frac{\partial x}{\partial \pi} \mid_{LR} = \delta(\overline{\pi}).$$

For trend inflation greater than some value $\overline{\pi}^* > 0$ up to an upper limit, $\delta(\overline{\pi}) < 0$. Thus, if trend inflation rises above $\overline{\pi}^*$, the minimum value of α_{π} necessary to ensure determinacy rises and exceeds 1.

With the policy rule (1), the effect of a permanent rise on the nominal interest rate is

$$\alpha_{\pi} + \alpha_{x} \frac{\partial x}{\partial \pi} \mid_{LR} + \alpha_{\Delta} \frac{\partial (x - x)}{\partial \pi} \mid_{LR} = \alpha_{\pi} + \alpha_{x} \frac{\partial x}{\partial \pi}.$$

With a Taylor rule targeting output growth but not the output gap, $\alpha_x = 0$. Then the previous expression resumes to

$$\alpha_{\pi} > 1$$
,

since $\alpha_{\Delta} \frac{\partial (x-x)}{\partial \pi} \mid_{LR} = 0$. Thus, with a policy rule targeting only output growth, the condition for determinacy is independent of the level of trend inflation.

While our example is designed to convey some intuition as to why a policy rule aiming at output growth improves stabilization, Khan et al. (2020) show this intuition remains valid for more complex models with positive trend inflation, nominal rigidities, and economic growth.

3 The Model

Our framework includes firm-specific labour, positive trend inflation and consumer habit formation. There is no capital accumulation. There is Calvo price stickiness. Aggregate fluctuations are driven by shocks to the discount rate, TFP, and monetary policy, and if in a state of indeterminacy, sunspot shocks.

We abstract from backward-looking price-setting mechanisms like rule-of-thumb behavior of price setters and indexation to past inflation and/or steady-state inflation. Both assumptions lack microfoundations. Furthermore, in the case of indexation it has been criticized because it conterfactually implies that all prices change every 3 months (Chari et al. 2009, Cogley and Sbordone 2008, Woodford 2007).⁴

3.1 Representative Consumer

The representative consumer maximizes expected utility over final consumption goods *C* and differentiated labour *L*

$$\max_{C_t, L_t(i), B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t b_t \left[\ln(C_t - h\bar{C}_{t-1}) - \frac{1}{1 + \eta^{-1}} \int_0^1 L_t(i)^{1 + \eta^{-1}} di \right], \tag{2}$$

where β is the subjective discount factor, h the degree of external habit formation, $L_t(i)$ is labour specific to intermediate-good firm $i \in [0,1]$, η is the elasticity of labour supply, and b_t is an intertemporal preference shock which follows an AR(1) process given by

$$\ln b_t = (1 - \rho_b) \ln b + \rho_b \ln b_{t-1} + \epsilon_t^b,$$

where ϵ_t^b is i.i.d. $N(0, \sigma_h^2)$.

The representative consumer is subject to the following budget constraint

$$P_t C_t + B_{t+1} = \int_0^1 W_t(i) L_t(i) di + B_t R_t + T_t, \tag{3}$$

where B_t is the stock of nominal bonds that the household enters the period with, $W_t(i)$ is the nominal wage page by sector i, P_t is the price of the final consumption good, R_t is the (gross) nominal interest rate, and T_t is profits from ownership of the firms.

The necessary first-order conditions for the representative consumer are given by

⁴Ascari et al. (2018) offer a survey of the empirical irrelevance of the forms of indexation used in DSGE models. Phaneuf et al. (2018) emphasize that price indexation also has the property that it delivers inertial inflation responses not only to monetary policy shocks, but also to TFP shocks, the latter being inconsistent with available VAR evidence.

$$1 = \beta E_t \left[\frac{b_{t+1}}{b_t} \left(\frac{C_t - h\bar{C}_{t-1}}{C_{t+1} - h\bar{C}_t} \right) \left(\frac{R_t}{\pi_{t+1}} \right) \right], \tag{4}$$

$$L_t(i)^{\eta^{-1}} = \frac{w_t(i)}{C_t - h\bar{C}_{t-1}},\tag{5}$$

where $\pi_t \equiv P_t/P_{t-1}$ and real wage is defined as $w_t = W_t/P_t$.

3.2 Final Goods Firms

Final goods firms combine intermediate inputs into a single aggregate output good using a CES aggregator. Aggregate consumption is equal to aggregate output

$$C_t = Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

 θ denoting the elasticity of substitution between intermediate inputs. Final goods firms maximize profits subject to intermediate input costs

$$\max_{Y(i)} P_{t} Y_{t} - \int_{0}^{1} P_{t}(i) Y_{t}(i) di.$$
 (6)

The first order condition for final goods firms yields the standard downward sloping demand curve for each intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t,\tag{7}$$

and the aggregate price index given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}.$$
 (8)

3.3 Intermediate Goods Firms

Intermediate goods are produced by a continuum of monopolistically competitive firms with a constant returns to scale production function given by

$$Y_t(i) = A_t L_t(i), (9)$$

where A_t is the aggregate productivity level governed by an AR(1) process given by

$$\ln A_t = \ln \bar{A} + \ln A_{t-1} + \epsilon_t^A,$$

where ϵ_t^A is i.i.d. $N(0, \sigma_A^2)$.

Intermediate goods firms minimize costs subject to meeting demand. The cost minimization problem for firm i is given by

$$\underset{L_t(i)}{\text{Min}} W_t L_t(i) \tag{10}$$

subject to

$$A_t L_t(i) \ge \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t. \tag{11}$$

The first-order condition for the cost minimization problem yields firm i's nominal marginal cost

$$MC_t(i) = \frac{W_t(i)}{A_t}. (12)$$

Firms are subject to Calvo pricing. Each period firms face a probability of reoptimizing their price given by $1 - \xi_p$. A firm setting its price optimally in period t maximizes the following discounted expected flow of profits

$$\operatorname{Max} E_{t} \sum_{\tau=0}^{\infty} \xi_{p}^{\tau} \Lambda_{t,t+\tau} Y_{t+\tau}(i) \left[P_{t}(i) - M C_{t+\tau}(i) \right], \tag{13}$$

where $\Lambda_{t,t+\tau} \equiv \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t}$ and Λ_t is the marginal utility of nominal income to the representative consumer in period t.

Taking the first-order condition for optimal pricing that we combine with equations (5), (7), (9), and (12), it is straightforward to show that the optimal price reset equation is given by

$$\left(\frac{P_t^{\star}}{P_t}\right)^{1+\frac{\theta}{\eta}} = \left(\frac{\theta}{\theta-1}\right) E_t \frac{\sum_{\tau=0}^{\infty} \xi_p^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_{t+\tau}}{P_t}\right)^{1+\theta(1+\eta^{-1})} \left(\frac{Y_{t+\tau}}{A_{t+\tau}}\right)^{1+\eta^{-1}} \left(Y_{t+\tau} - hY_{t+\tau-1}\right)}{\sum_{\tau=0}^{\infty} \xi_p^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_{t+\tau}}{P_t}\right)^{\theta} Y_{t+\tau}}, \quad (14)$$

and output under flexible prices by

$$\left(\frac{Y_t^F}{A_t}\right)^{1+\eta^{-1}} = \frac{\theta - 1}{\theta} + h\left(\frac{Y_t^F}{A_t}\right)^{\eta^{-1}} \left(\frac{Y_{t-1}^F}{A_t}\right). \tag{15}$$

3.4 Monetary Policy

We consider two Taylor rule specifications, each characterized by an interest rate smoothing effect whose importance is governed by ρ_R . The gap-rule states the Fed reacts to deviations of inflation from a fixed target, π_t/π , and to the output gap, $X_t \equiv Y_t/Y_t^F$, where Y_t^F is the level of output when assuming perfectly flexible prices. The growth-rule states that the Fed reacts to deviations of inflation from a fixed target and to deviations of current output growth from trend output growth, $\frac{Y_t}{Y_{t-1}}g_Y^{-1}$, where g_Y denotes output growth which stems from neutral technological progress. The general form of the rule is

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_{\pi}} \left(X_t\right)^{\alpha_{\chi}} \left(\frac{Y_t}{Y_{t-1}} g_Y^{-1}\right)^{\alpha_{\Delta}} \right]^{1-\rho_R} \epsilon_t^R.$$

The gap-rule model assumes $\alpha_{\Delta} = 0$, and the growth-rule model sets $\alpha_x = 0$. ϵ_t^R is a monetary policy shock which is i.i.d. $N(0, \sigma_R^2)$.

3.5 Log-Linearization

Solving the model requires detrending output, which is done by removing trend growth and taking a log-linear approximation of the stationary model around the non-stochastic steady state.

4 Model Solution, Estimation and Data

4.1 Rational Expectations Solution Under Indeterminacy

To solve the model with potentially multiple equilibria, we follow the solution method proposed by Lubik and Schorfheide (2003). The linear rational expectations (LRE) model can be represented in its canonical form by

$$\Gamma_0(\vartheta)\mathbf{s}_t = \Gamma_1(\vartheta)\mathbf{s}_{t-1} + \Psi(\vartheta)\epsilon_t + \Pi(\vartheta)\eta_t, \tag{16}$$

where \mathbf{s}_t is a vector of endogenous variables, ϵ_t is a vector of exogenous structural disturbances, and η_t is a vector of one-step ahead forecast errors for the expectational variables in the model. $\Gamma_0(\vartheta)$, $\Gamma_1(\vartheta)$, $\Psi(\vartheta)$, and $\Pi(\vartheta)$ are matrices containing potentially non-linear combinations of the structural parameters. Following Lubik and Schorfheide (2003), the full set of LRE solutions can be characterized by,

$$\mathbf{s}_{t} = \mathbf{\Phi}(\boldsymbol{\vartheta})\mathbf{s}_{t-1} + \mathbf{\Phi}_{\boldsymbol{\epsilon}}(\boldsymbol{\vartheta}, \tilde{\boldsymbol{M}})\boldsymbol{\epsilon}_{t} + \mathbf{\Phi}_{\boldsymbol{\zeta}}(\boldsymbol{\vartheta}, \boldsymbol{M}_{\boldsymbol{\zeta}})\boldsymbol{\zeta}_{t}, \tag{17}$$

where $\Phi(\vartheta)$, $\Phi_{\epsilon}(\vartheta, \tilde{M})$, and $\Phi_{\zeta}(\vartheta, M_{\zeta})$ are matrices of the coefficients. Following Lubik and Schorfheide (2004) we impose that $M_{\zeta} = 1$, implying that ζ_t is no longer a vector of sunspot shocks, but instead a reduced form sunspot shock (now given by ζ_t) which captures a sunspot shock to all of the expectational variables in the system. ζ_t is assumed to follow a process given by $\zeta_t \sim \text{i.i.d. } N(0, \sigma_{\zeta}^2)$.

Under this setup, indeterminacy can impact endogenous variables in two different ways. First, the non-fundamental expectation errors, ζ_t , can impact model dynamics. Second, be-

cause the model features multiplicity of equilibria, the propagation structural shocks, ϵ_t , is not unique and is in part determined by the arbitrary matrix \tilde{M} .

We replace \tilde{M} with $M^*(\theta) + M$. $M^*(\theta)$ is found by minimizing the difference between the impact responses of endogenous variables to structural disturbances at the boundary between determinacy and indeterminacy. With the gap rule and positive trend inflation this boundary is unknown since satisfying the Taylor principle does not guarantee determinacy. We follow Hirose et al. (2020), who proposed perturbing the Taylor Rule response parameter to inflation, α_{π} , to find this boundary. The remaining matrix M is estimated from the data.

4.2 Econometric Strategy

We use a full information Bayesian estimation strategy to characterize the posterior distributions of the structural parameters and shocks across the determinacy and indeterminacy regions of the model. The main challenge is that the associated posterior distributions are potentially multi-modal and difficult to estimate using standard methods. We follow Hirose et al. (2020) and use the sequential Monte Carlo (SMC) algorithm proposed by Herbst and Schorfheide (2016, 2014), which allows approximating the posteriors in both the determinacy and indeterminacy regions in a single estimation.

In this case the likelihood function is given by

$$p(\mathbf{X}^T|\boldsymbol{\vartheta}_S,S) = 1\{\boldsymbol{\vartheta}_S \in \mathbf{\Theta}^D\} p^D(\mathbf{X}^T|\boldsymbol{\vartheta}_D,D) + 1\{\boldsymbol{\vartheta}_S \in \mathbf{\Theta}^I\} p^I(\mathbf{X}^T|\boldsymbol{\vartheta}_I,I),$$
(18)

where $\mathbf{\Theta}^D$ and $\mathbf{\Theta}^I$ are the determinacy and indeterminacy regions of the parameter space, $p^D(\mathbf{X}^T|\boldsymbol{\vartheta}_D,D)$ and $p^I(\mathbf{X}^T|\boldsymbol{\vartheta}_I,I)$ the likelihood functions under determinacy and indeterminacy, and finally $1\{\boldsymbol{\vartheta}_S\in\mathbf{\Theta}^D\}$ and $1\{\boldsymbol{\vartheta}_S\in\mathbf{\Theta}^I\}$ are indicator functions which equal 1 if the parameters are respectively in the determinacy or indeterminacy region.

The SMC algorithm constructs a sequence of tempered posteriors given by

$$\pi_n(\boldsymbol{\vartheta}) = \frac{[p(\boldsymbol{X}^T | \boldsymbol{\vartheta}_S, S)]^{\phi_n} p(\boldsymbol{\vartheta}_S | S)}{\int_{\boldsymbol{\vartheta}_S} [p(\boldsymbol{X}^T | \boldsymbol{\vartheta}_S, S)]^{\phi_n} p(\boldsymbol{\vartheta}_S | S) d\boldsymbol{\vartheta}_S'}$$
(19)

where ϕ_n for $n=1,\ldots,N_\phi$ is a sequence that slowly increases from zero to one. As noted in (Herbst and Schorfheide 2016, p. 76), the algorithm consists of three primary steps: 1) correction, which reweights particles to reflect the density in iteration n; 2) selection, which eliminates particle degeneracy by resampling; and 3) mutation, which propagates the particles forward using a Markov transition kernel to adapt to the current bridge density. Our estimation uses N=10,000, $N_\phi=200$, and $\lambda=2.5$

After obtaining approximations to the posterior distributions, we compute the posterior probability of determinacy as

$$P(\boldsymbol{\vartheta} \in \boldsymbol{\Theta}^{D} | \boldsymbol{X}^{T}) = \frac{1}{N} \sum_{i=1}^{N} 1\{\boldsymbol{\vartheta}_{N_{\boldsymbol{\phi}}} \in \boldsymbol{\Theta}^{D}\}, \tag{20}$$

effectively counting the number of draws which yield determinacy in the final stage of the SMC algorithm.

4.3 Data

To estimate a subset of the structural parameters of the model we use three U.S. quarterly time series: per capita real GDP growth, GDP deflator based inflation, and the Federal Funds rate. Data sources are reported in an Online Appendix.

We estimate the model's parameters over two different samples. The first sample corresponds to the pre-Volcker era and runs from 1960Q1 to 1979Q2. The second sample corresponds to the Great Moderation period and spans from 1982Q4 to 2008Q4.⁶

The observables are mapped into the model in the following manner,

$$\begin{bmatrix}
100 \log \Delta Y_t \\
100 \log \Delta P_t \\
100 \log R_t
\end{bmatrix} = \begin{bmatrix} \bar{A} \\ \bar{\pi} \\ \bar{R} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + \epsilon_t^A \\ \tilde{\pi}_t \\ \tilde{R}_t \end{bmatrix},$$
(21)

⁵This imposes a tempering schedule according to $\phi_n = \left(\frac{n-1}{N_{\phi}-1}\right)^{\lambda}$.

⁶The exclusion of the periods from late 1979 to early 1982 is standard (e.g., Clarida et al. (1999)) since during this period the Federal Reserve was not following an interest rate target.

where \bar{A} , $\bar{\pi}$, and \bar{R} are the steady state values of the growth rate of output, inflation, and the nominal interest rate, respectively. These values are expressed in net terms given by $100(\bar{A}-1)$, $100(\bar{\pi}-1)$, and $100(\bar{R}-1)$.

4.4 Calibration and Prior Distributions

All parameters are estimated except two which are calibrated. We set the elasticity of substitution between differentiated goods to 9, implying a 12.5% price markup with zero trend inflation. The Frisch elasticity of labour supply is set to 1. These values are standard in the literature.

The remaining structural parameters are estimated. The parameter governing habit consumer formation has a mean prior of 0.7, with a prior standard deviation of 0.1. We explore a range of plausible priors for the Calvo price stickiness parameter given by a beta distribution with mean priors of 0.5, 0.55, and 0.65 for the pre-Volcker years, and means 0.5, 0.55, 0.65 and 0.75 for the Great Moderation. The mean priors ranging from 0.5 to 0.65 are consistent with the micro-level evidence in Bils and Klenow (2004) and Nakamura and Steinsson (2008). In each case, the prior standard deviation is set to 0.1.

Priors for the average rate of inflation, growth rate of output, and nominal interest rate are set at their average sample values. We use relatively diffuse priors around these means. The average rate of inflation and output growth have normal distribution priors with means of 0.985 and 0.370, and standard deviations of 0.75 and 0.15. The average nominal interest rate has gamma distribution prior with mean equal to 1.597 and standard deviation equal to 0.25.

The parameters describing monetary policy are standard. The interest rate smoothing coefficient has a beta prior with mean equal to 0.6 and a standard deviation of 0.2. The policy response to inflation is characterized by a gamma distribution with prior mean equal to 1.5 and standard deviation equal to 0.3. Both the policy response to output growth and output gap have gamma priors with means of 0.125 and standard deviations of 0.1.

The exogenous disturbances have standard priors. Shocks have inverse gamma priors with means of 0.5 and standard deviations of 4. Persistence parameters have beta distri-

butions with means of 0.5 and standard deviations of 0.2. Lastly, for the correlation between sunspot shocks and structural shocks, we use normal distributions with mean 0 and standard deviation equal to 1.

5 Estimation Results: The Pre-Volcker Period

This section presents and analyzes our results for the pre-Volcker period (1960:Q1 to 1979:Q2). We report estimates for the output-gap and output-growth rule models. We also provide estimates of structural parameters and shocks for mean priors assigned to the Calvo price stickiness parameter ξ_p of 0.5, 0.55 and 0.65, with 90% confidence intervals for the estimated parameters. The log $p(X^T)$ represents the marginal data density of a model, while Prob(det) is the posterior probability of equilibrium determinacy implied by the model.

5.1 The Output Gap-Rule Model

Table 1 reports the results for the output gap-rule model. We find indeterminacy with certainty (less than one percent probability of determinacy) for the three mean priors of ξ_p . A first factor leading to indeterminacy under the output gap-rule is the estimated trend inflation. Our estimates imply an average (annualized) rate of inflation between 4.64% and 4.85% depending on the prior assigned to ξ_p , compared to an average rate of inflation of 4.33% in the data. The output gap-rule model hence overestimates trend inflation. Based on model simulations in Khan et al. (2020), such levels of trend inflation represent a threat to determinacy unless the policy response to inflation deviates sharply from the original Taylor principle.

A second factor leading to indeterminacy under the output gap-rule is the estimated average frequency of price adjustment. We find it varies from once every 7.25 months to 8.43 months upon chaging the prior on ξ_p from 0.5 to 0.65. These estimates are more in-line with the microeconomic evidence reported by Nakamura and Steinsson (2008) than by Bils and Klenow (2004). More rigid prices favor indeterminacy.

A third factor is that the estimates of policy responses to inflation are greater than 1,

but do not widely deviate from 1. In fact, they exceed Taylor (1993)'s original prescription of 1.5, but are not strong enough to ensure determinacy. Interestingly, based on the 90% confidence intervals for the policy responses to inflation, we find that α_{π} is higher than 1. That is, monetary policy under the output gap-rule was moderately active during the pre-Volcker period.

A fourth key factor causing indeterminacy under the output gap-rule is the policy response to the output gap. We find estimates lying between 0.29 and 0.35 depending on the prior assigned to ξ_p . Based on calibrated New Keynesian models, Coibion and Gorodnichenko (2011) and Khan et al. (2020) showed that policy responses of this magnitude require very strong policy responses to inflation to achieve determinacy, in the range of 3 to 6 for trend inflation between 4% and 6%. Our findings confirm that targeting the output gap during the pre-Volcker years generates indeterminacy.⁷

5.2 The Output Growth-Rule Model

Table 2 reports the results for the output growth-rule model. We find a probability of determinacy ranging from 99% to 62.4% depending on the mean prior assigned to ξ_p . Trend inflation is estimated to be about 4.6%. A key factor explaining determinacy under the output growth-rule is the average waiting time between price adjustment. We find it is 5.25 months to 6.4 months depending on the ξ_p prior. Price changes are more frequent when assuming a rule targeting output growth. These estimates are more consistent with the evidence in Bils and Klenow (2004). A higher frequency of price adjustment increases the prospect of determinacy.

The output growth-rule model implies that monetary policy was weakly active, with estimates of the policy response to inflation ranging from 1.27 to 1.37. This is lower than under the output gap-rule. But recall, as we have seen in Section 2, that under the output growth-rule the response to inflation does not need to deviate very much from 1 to achieve deter-

⁷We find similar evidence for a policy rule targeting the output gap and output growth (not reported). Our evidence suggests the economy was in a state of indeterminacy with certainty (less than one percent probability of determinacy) or near-certainty (less than five percent probability of determinacy) during the pre-Volcker period depending on the prior assigned to ξ_p .

minacy. Furthermore, the output growth-rule precludes a response to the output gap, which helps achieving determinacy. The estimated policy response to output growth is nearly 0.2, which also favors determinacy.

When looking at the marginal data density statistics $\log P(X^T)$ implied by the output gap-rule and output growth-rule models, we find that the output gap-rule model seems to provide a better fit for the pre-Volcker period. This finding is perhaps not surprising, since the pre-Volcker period was one of high macroeconomic instability and that the output gap-rule is associated with more unstable outcomes than the output growth-rule.

What do we learn from our findings for the pre-Volcker period? The existing literature on the prospect of determinacy in economies with positive trend inflation has established that policy rules targeting the output gap are more destabilizing than rules targeting output growth. Our estimation results to confirm this. But our evidence also confirms the potential stabilizing effectiveness of targeting output growth.

6 Estimation Results: The Great Moderation

This section discusses our results for the Great Moderation (1982:Q4 to 2008:Q4).

6.1 The Outout Gap-Rule Model

The estimation results for the output gap-rule model are presented in Table 3. The estimated average rate of inflation drops significantly. Estimates of average inflation lie between 2.82% and 3.64%. Average inflation was 2.55% during that period. So, while predicting a decline in trend inflation, the output gap-rule model significantly overestimates trend inflation during the Great Moderation period. At the same time, we find much stronger policy responses to inflation, which now range from 2.31 to 2.6 depending on the mean prior ξ_p . Therefore, the output gap-rule model predicts monetary policy fought inflation more strongly after 1982.

When looking at the posterior probability of equilibrium determinacy, we find it is very sensitive to modest variations in the mean prior ξ_p . With a mean prior of 0.5, the estimated probability of determinacy is 87%. However, this probability drops to nearly 0% when the

mean prior is set to either 0.55 or 0.65.

The factors explaining this drastic change in the determinacy outcome are the following. First, estimated trend inflation is higher than 3% when the mean prior on ξ_p is 0.55 or 0.65. Khan et al. (2020) have shown that with levels of trend inflation like 3% and 4%, the parameter governing the policy response to inflation consistent with determinacy may well exceed 3 or 4.

Also, the estimated average waiting time between a price change increases significantly with higher priors of ξ_p . It increases from once every 5.3 months on average with a prior 0.5 to once every 12 months for the slightly higher prior 0.55, and once every 12.8 months with prior 0.65. The evidence on the frequency of price changes with mean priors 0.55 and 0.65 is thus inconsistent with the microeconomic evidence in Bils and Klenow (2004) and Nakamura and Steinsson (2008). Note also that the policy response to the output gap is much weaker during the Great Moderation than the pre-Volcker period, meaning that it is not a key factor explaining indeterminacy during the Great Moderation.⁸

6.2 The Output Growth-Rule Model

Table 4 reports the estimation results for the output growth-rule model. In this case, we report estimates for priors of the Calvo price stickiness parameter set at 0.5, 0.65 and 0.75. By doing this, we wish to show that estimates of the output growth-rule model are more stable than those of the output gap rule model.

A first main difference between the two models is the overall stability of estimates from the output growth-rule model upon varying the mean prior on ξ_p . For instance, the average rate of inflation varies from 2.8% with prior 0.5 to 2.87% with prior 0.75. Estimated trend inflation is closer to the actual average rate of inflation with a policy rule targeting output growth than the output gap. The estimated average frequency of price adjustment is also much more stable with the output growth-rule. It lies between once every 5.66 months with prior 0.5 and once every 6.4 months with prior 0.75. These estimates are broadly consistent

⁸The estimation results for a model where policy aims at the output gap and output growth are broadly the same as those reported for the gap-rule model. In this model too, we find the estimated Calvo probability of price non-reoptimization is highly sensitive to the choice of a mean prior on ξ_p .

with the evidence on price changes in Bils and Klenow (2004), as were those for the pre-Volcker period.

When looking at estimates of the policy rule, we find that the policy response to inflation varies between 2.2 with prior 0.5 and 2.06 with prior 0.75. Therefore, while our estimates suggest the Fed adopted a more aggressive stance against inflation during the Great Moderation, they indicate monetary policy was somewhat less accommodative after 1982 under the output gap-rule than the output growth-rule.

The estimated policy response to output growth is significantly higher during the Great Moderation than the pre-Volcker period, between 0.46 and 0.481 depending on the mean prior on ξ_p . A higher policy response to output growth favors the determinacy outcome.

Based on the estimation results from the output growth-rule model, we find a probability of determinacy of 100% for the Great Moderation, and this for mean priors on ξ_p ranging from 0.5 to 0.75.

We compare the marginal data density statistics, $\log P(X^T)$, predicted by the output growth-rule and output gap-rule models. This comparison is based on mean priors on ξ_p of 0.5, 0.55 and 0.65 in both models. This comparison suggests the following ranking of models in terms of model fit: output growth-rule model ($-56.36 \mid \xi_p = 0.5$; $-57.32 \mid \xi_p = 0.55$; $-57.63 \mid \xi_p = 0.65$) > output gap-rule model ($-68.93 \mid \xi_p = 0.5$; $-63.44 \mid \xi_p = 0.55$; $-62.5 \mid \xi_p = 0.65$).

These findings tell us that the output growth-rule model provides a better fit of the data than the output gap-rule model during the Great Moderation. The fact that the output gap-rule model delivers indeterminacy with certainty for the post-1982 period for plausible priors of ξ_p casts serious doubts as to whether the Fed targeted the output gap at all during the Great Moderation.

7 Conclusion

Using a Bayesian method allowing for positive trend inflation and indeterminacy, we have shown that a policy rule targeting output growth improves stabilization prospects. For example, such a rule is consistent with determinacy during both the pre-Volcker era and Great Moderation. By sharp contrast, a rule targeting the output gap predicts indeterminacy in both periods. The idea that the Fed targeted the output gap after 1982 is therefore hard to reconcile with the greater macroeconomic stability experienced during the Great Moderation.

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Table 1: 1960I:1979II POSTERIOR ESTIMATES GAP RULE

	Prio	r		Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.524	0.514	0.520
				[0.405,0.651]	[0.383,0.635]	[0.401,0.637]
ξ_p	Beta		0.1	0.586	0.614	0.644
,				[0.483,0.688]	[0.521,0.718]	[0.547,0.743]
α_{π}	Gamma	1.5	0.3	1.674	1.877	1.651
				[1.189,2.191]	[1.375,2.408]	[1.138,2.158]
α_{x}	Gamma	0.125	0.1	0.289	0.296	0.348
				[0.122,0.419]	[0.150,0.438]	[0.159,0.575]
$lpha_{\Delta}$	Gamma	0.125	0.1	_	_	_
				[—,—]	[—,—]	[—,—]
$ ho_R$	Beta	0.6	0.2	0.565	0.547	0.564
				[0.399,0.721]	[0.408,0.708]	[0.400,0.742]
$ar{A}$	Normal	0.370	0.15	0.405	0.439	0.431
				[0.178,0.619]	[0.223,0.663]	[0.231,0.632]
$\bar{\pi}$	Normal	0.985	0.75	1.212	1.161	1.162
				[0.943,1.482]	[0.905,1.433]	[0.891,1.431]
$ar{R}$	Gamma	1.597	0.25	1.486	1.448	1.479
				[1.239,1.736]	[1.204,1.707]	[1.232,1.714]
$ ho_b$	Beta	0.5	0.2	0.465	0.462	0.475
				[0.149,763]	[0.178,0.729]	[0.173,0.760]
$ ho_A$	Beta	0.5	0.2	0.754	0.798	0.683
				[0.587,0.886]	[0.693,0.912]	[0.356,0.882]
$ ho_R$	Beta	0.5	0.2	0.509	0.564	0.532
				[0.323,0.699]	[0.394,0.719]	[0.309,0.711]
σ_b	Inverse Gamma	0.5	4	0.932	0.954	1.327
				[0.271,1.873]	[0.267,1.755]	[0.281,2.393]
σ_A	Inverse Gamma	0.5	4	0.584	0.523	0.563
				[0.338,0.816]	[0.290,0.734]	[0.340,0.783]
σ_R	Inverse Gamma	0.5	4	0.286	0.316	0.289
				[0.209,0.362]	[0.218,0.402]	[0.208,0.373]
σ_{ζ}	Inverse Gamma	0.5	4	0.339	0.336	0.323
				[0.249,0.425]	[0.242,0.423]	[0.250,0.400]
M_b	Normal	0	1	-0.039	-0.058	-0.002
				[-0.414,0.248]	[-0.435,0.207]	[-0.249,0.228]
M_A	Normal	0	1	0.372	0.839	0.240
				[-0.661,1.294]	[-0.560,2.162]	[-0.718,1.126]
M_R	Normal	0	1	0.372	0.499	0.223
				[-0.474,1.013]	[-0.205,1.127]	[-0.621,0.982]
$\log p(X^T)$				-141.9454	-141.5080	-140.8163
Prob(det)				0.0004	0.0000	0.0000

Table 2: 1960I:1979II POSTERIOR ESTIMATES GROWTH RULE

	Prio	r			Posterior	
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.553	0.577	0.586
				[0.453,0.648]	[0.477,0.671]	[0.482,0.695]
ξ_p	Beta		0.1	0.429	0.486	0.531
,				[0.342,0.513]	[0.365,0.634]	[0.394,0.691]
$lpha_{\pi}$	Gamma	1.5	0.3	1.372	1.308	1.273
				[1.088,1.678]	[0.908,1.671]	[0.881,1.662]
α_{x}	Gamma	0.125	0.1	_	_	_
				[—,—]	[—,—]	[—,—]
$lpha_{\Delta}$	Gamma	0.125	0.1	0.171	0.203	0.216
				[0.047,0.282]	[0.057,0.360]	[0.057,0.354]
$ ho_R$	Beta	0.6	0.2	0.428	0.507	0.534
				[0.275,0.586]	[0.328,0.710]	[0.357,0.726]
$ar{A}$	Normal	0.370	0.15	0.412	0.411	0.400
				[0.196,0.614]	[0.212,0.621]	[0.182,0.644]
$\bar{\pi}$	Normal	0.985	0.75	1.118	1.152	1.144
				[0.890,1.338]	[0.826,1.457]	[0.875,1.406]
$ar{R}$	Gamma	1.597	0.25	1.456	1.492	1.487
				[1.217,1.721]	[1.162,1.829]	[1.179,1.746]
$ ho_b$	Beta	0.5	0.2	0.865	0.740	0.715
				[0.806,0.932]	[0.355,0.947]	[0.345,0.946]
$ ho_A$	Beta	0.5	0.2	0.243	0.372	0.378
				[0.069,0.385]	[0.078,0.752]	[0.086,0.765]
$ ho_R$	Beta	0.5	0.2	0.498	0.491	0.492
				[0.390,0.624]	[0.345,0.640]	[0.343,0.646]
σ_b	Inverse Gamma	0.5	4	0.900	0.889	0.897
				[0.502,1.315]	[0.342,1.346]	[0.327,1.393]
σ_A	Inverse Gamma	0.5	4	1.769	1.563	1.672
				[1.391,2.137]	[0.649,2.185]	[0.532,2.408]
σ_R	Inverse Gamma	0.5	4	0.314	0.295	0.285
				[0.241,0.384]	[0.220,0.373]	[0.212,0.357]
σ_{ζ}	Inverse Gamma	0.5	4	0.596	0.470	0.445
				[0.257,0.924]	[0.236,0.704]	[0.222,0.707]
M_b	Normal	0	1	-0.003	0.012	-0.109
				[-1.609,1.572]	[-1.295,1.502]	[-1.574,1.462]
M_A	Normal	0	1	-0.057	0.208	-0.143
				[-1.674,1.561]	[-1.041,1.682]	[-1.386,0.963]
M_R	Normal	0	1	-0.060	0.125	-0.035
				[-1.695,1.509]	[-1.351,1.517]	[-1.325,0.998]
$\log p(X^T)$				-144.4772	-146.7588	-146.8734
Prob(det)				0.9881	0.7079	0.6239

Table 3: 1982IV:2008IV POSTERIOR ESTIMATES GAP RULE

	Prio	r			Posterior	
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.584	0.536	0.640
				[0.474,0.727]	[0.425,0.652]	[0.538,0.748]
ξ_p	Beta	_	0.1	0.433	0.749	0.766
				[0.284,0.735]	[0.694,0.804]	[0.707,0.828]
α_{π}	Gamma	1.5	0.3	2.316	2.510	2.588
				[1.941,2.770]	[2.149,2.882]	[2.156,3.001]
α_x	Gamma	0.125	0.1	0.146	0.064	0.066
				[0.004,0.304]	[0.023,0.107]	[0.019,0.116]
$lpha_{\Delta}$	Gamma	0.125	0.1	_		_
				[—,—]	[—,—]	[—,—]
$ ho_R$	Beta	0.6	0.2	0.614	0.591	0.615
				[0.514,0.708]	[0.504,0.677]	[0.529,0.705]
$ar{A}$	Normal	0.370	0.15	0.389	0.379	0.365
				[0.226,0.554]	[0.220,0.543]	[0.172,0.550]
$\bar{\pi}$	Normal	0.985	0.75	0.705	0.909	0.840
				[0.528,0.873]	[0.750,1.067]	[0.617,1.059]
R	Gamma	1.597	0.25	1.407	1.627	1.526
				[1.163,1.640]	[1.435,1.843]	[1.240,1.807]
$ ho_b$	Beta	0.5	0.2	0.879	0.583	0.591
				[0.841,0.952]	[0.310,0.858]	[0.271,0.909]
$ ho_A$	Beta	0.5	0.2	0.269	0.735	0.718
				[0.066,0.700]	[0.629,0.838]	[0.427,0.903]
$ ho_R$	Beta	0.5	0.2	0.639	0.758	0.771
				[0.517,0.742]	[0.705,0.807]	[0.711,0.831]
σ_b	Inverse Gamma	0.5	4	1.635	0.644	0.720
				[1.040,2.553]	[0.320,0.989]	[0.210,1.466]
σ_A	Inverse Gamma	0.5	4	1.107	0.487	0.527
				[0.491,1.571]	[0.298,0.672]	[0.291,0.771]
σ_R	Inverse Gamma	0.5	4	0.243	0.259	0.261
				[0.188,0.296]	[0.214,0.303]	[0.202,0.315]
σ_{ζ}	Inverse Gamma	0.5	4	0.664	0.380	0.404
				[0.225,1.240]	[0.272,0.474]	[0.313,0.498]
M_b	Normal	0	1	0.118	-0.478	-0.126
				[-1.323,1.607]	[-0.972,-0.040]	[-0.636,0.297]
M_A	Normal	0	1	-0.115	-0.317	-0.184
				[-1.651,1.570]	[-0.698,0.075]	[-0.584,0.297]
M_R	Normal	0	1	0.393	2.146	1.987
				[-1.058,2.256]	[1.499,2.767]	[1.367,2.679]
$\log p(X^T)$				-68.9284	-63.4437	-62.4963
Prob(det)				0.8696	0.0017	0.0001

Table 4: 1982IV:2008IV POSTERIOR ESTIMATES GROWTH RULE

	Prio	r			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.65$	$\xi_p = 0.75$	
h	Beta	0.7	0.1	0.569	0.608	0.606	
				[0.474,0.659]	[0.518,0.695]	[0.516,0.701]	
ξ_p	Beta	_	0.1	0.470	0.528	0.532	
				[0.362,0.576]	[0.415,0.637]	[0.419,0.651]	
α_{π}	Gamma	1.5	0.3	2.198	2.080	2.057	
				[1.739,2.670]	[1.598,2.549]	[1.633,2.504]	
α_{x}	Gamma	0.125	0.1	_	_	_	
				[—,—]	[—,—]	[—,—]	
$lpha_{\Delta}$	Gamma	0.125	0.1	0.460	0.481	0.480	
				[0.291,0.649]	[0.299,0.670]	[0.287,0.660]	
$ ho_R$	Beta	0.6	0.2	0.601	0.601	0.599	
				[0.489,0.703]	[0.497,0.707]	[0.500,0.720]	
Ā	Normal	0.370	0.15	0.410	0.391	0.396	
				[0.243,0.588]	[0.206,0.572]	[0.206,0.571]	
$\bar{\pi}$	Normal	0.985	0.75	0.702	0.715	0.717	
_				[0.543,0.862]	[0.546,0.884]	[0.537,0.881]	
R	Gamma	1.597	0.25	1.435	1.445	1.439	
				[1.171,1.698]	[1.206,1.714]	[1.197,1.687]	
$ ho_b$	Beta	0.5	0.2	0.922	0.919	0.921	
	_			[0.892,0.955]	[0.886,0.950]	[0.889,0.955]	
$ ho_A$	Beta	0.5	0.2	0.163	0.188	0.201	
	_			[0.031,0.289]	[0.031,0.338]	[0.034,0.356]	
$ ho_R$	Beta	0.5	0.2	0.573	0.601	0.599	
				[0.480,0.670]	[0.503,0.696]	[0.499,0.698]	
σ_b	Inverse Gamma	0.5	4	1.703	1.640	1.677	
		o -		[1.111,2.237]	[1.089,2.180]	[1.095,2.290]	
σ_A	Inverse Gamma	0.5	4	1.293	1.423	1.393	
		0.5		[0.983,1.579]	[1.064,1.747]	[1.079,1.731]	
σ_R	Inverse Gamma	0.5	4	0.232	0.227	0.229	
		0.5	4	[0.182,0.280]	[0.177,0.273]	[0.177,0.279]	
σ_{ζ}	Inverse Gamma	0.5	4	0.602	0.615	0.537	
λ./ι	Nouse - 1	0	1	[0.276,0.951]	[0.263,0.961]	[0.293,0.794]	
M_b	Normal	0	1	0.026	0.034	-0.041	
λΛ	Normal	0	1	[-1.644,1.659]	[-1.486,1.680]	[-1.680,1.573]	
M_A	Normal	0	1	-0.011	0.005	-0.039	
λA	Normal	0	1	[-1.676,1.551]	[-1.603,1.735]	[-1.595,1.373]	
M_R	Normal	0	1	0.004	0.033	-0.025	
100 m (VT)				[-1.545,1.660]	[-1.606,1.646]	[-1.580,1.484]	
$\log p(X^T)$				-56.36 1.0000	-57.6253 1.0000	-61.1323	
Prob(det)				1.0000	1.0000	0.9982	