Higher Order Interest-Smoothing, Time-Varying Inflation Target and the Prospect of Indeterminacy*

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Abstract

Single equation estimation highlights the importance of higher order interest rate smoothing in explaining interest rate inertia. We provide evidence conditioned on a Bayesian model consistent approach showing that higher order interest rate smoothing is empirically relevant and has important implications for the prospect of determinacy. Based on an estimated New Keynesian model with positive trend inflation allowing the joint possibility of determinacy and indeterminacy, we find the preferred interest rate rule characterizing the Fed's behavior includes second order interest-smoothing, a time-varying inflation target, a response to output growth, and a persistent policy shock. This is true for the pre-Volcker era and Great Moderation. Importantly, our evidence suggests this rule avoided self-fulfilling revisions in inflationary expectations and indeterminacy during the pre-Volcker years. Including an observable for the inflation target in the estimation is a key factor leading to these findings.

JEL classification: E31, E32, E37.

Keywords: Taylor rules; higher order interest-smoothing; time-varying target inflation; Bayesian estimation; indeterminacy; positive trend inflation.

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1 Introduction

Following Taylor (1993)'s influential contribution to the monetary policy debate and the empirical work by Clarida et al. (2000) on monetary policy rules, a consensus in the macroeconomic literature holds that US monetary policy ought to be described by a rule wherein the Fed adjusts the nominal interest rate in response to deviations of inflation and output from target levels, while smoothing short-term variations in the nominal interest rate, first order interest-smoothing being the key mechanism leading to interest rate inertia.

Rudebusch (2002, 2006) subsequently questioned the source of inertia: If the Fed reacts to persistent variables omitted from the policy rule, persistent policy shocks might explain interest rate inertia without any smoothing intention from the Fed. Coibion and Gorodnichenko (2011, 2012) (CG) challenge this interpretation. Referring to Woodford (2003), CG argue that theoretical models of policy inertia point to higher order interest-smoothing as optimal. Using a variety of methods and data for the period 1983:I-2006:IV, they report evidence of statistically significant smoothing parameters of order two, and of autoregressive parameters in the error terms of the policy rules which are either negative or statistically insignificant.

Until now the debate about the rule that best describes the Fed's policy has been conducted mostly within single-equation approaches. The present paper offers new evidence from a Bayesian estimated model consistent approach. Using data from 1964:I to 1979:II and 1983:I to 2005:I, and a procedure permitting the joint possibility of determinacy and indeterminacy, we investigate if higher order interest-smoothing receives empirical support and whether it matters for the prospect of indeterminacy. We show that second order interest-smoothing is a robust empirical fact which has major implications for the determinacy outcome.

Our evidence is based on a comparison of various estimated models with policy rules embedding first and second order interest-smoothing. Policy rules also include responses to deviations of inflation from a time-varying inflation target as in Del Negro and Eusepi (2011) and Del Negro et al. (2015), and adjustments to the level of the output gap and output growth, or to output growth only. Monetary policy shocks are either persistent or white-noise. The trend inflation rate is positive.

A novel aspect of our paper is that it combines the solution method to models of indeterminacy proposed by Bianchi and Nicoló (2020) (BN) with the Sequential Monte Carlo (SMC) algorithm proposed by Herbst and Schorfheide (2014). This permits the joint estimation of determinacy and indeterminacy regions of the model in a single estimation, even when the region between the two is unknown. The advantage of the BN approach over others currently used is that it does not require finding the boundary region for each likelihood evaluation which is more computationally costly than our procedure.[1]

To estimate our models, we use for the first time in the literature on indeterminacy an observable providing information about movements in the inflation target by Aruoba and Schorfheide (2011), who estimate the common factor from two series capturing inflation expectations and actual inflation. Having a series measuring time-varying inflation target helps identify the parameters of the policy rule, in particular those of interest-smoothing versus persistent policy shocks, during both the so-called Great Inflation and Great Moderation.[2]

We consider Taylor rules with policy responses to different measures of economic activity for the following reasons. While Clarida et al. (2000) used of various measures of the output gap, Smets and Wouters (2007) assumed mixed policy responses to the level of the output gap and output growth, where the output gap is defined as in the Textbook New Keynesian (NK) model as the short-run deviations of output from its level at flexible prices (and nominal wages) (Galí 2003). Still, Orphanides (2002), Walsh (2003), Sims (2013) and Khan et al. (2020) put forward arguments favouring policy rules targeting output growth only.

Our substantive findings are summarized as follows. Since previous Bayesian studies on

[1]Estimating the same model with identical SMC algorithm parameters using the approach in Hirose et al. (2020), we find that our approach is nearly four times faster (90 minutes compared to 5.5 hours).

[2]The inflation target series is strongly correlated with inflation expectations from the Survey of Professional Forecasters. For periods where the target series and data on inflation expectations overlap, the correlation coefficient with one and ten year ahead inflation expectations are 0.94 and 0.96, respectively.

policy rules and indeterminacy have typically relied on first order interest-smoothing and policy responses to deviations of inflation and output from targets, we first provide simulation results about the exact boundaries of indeterminacy with second order smoothing and policy responses to the output gap, the output growth, or both. We search for the minimum policy response to inflation consistent with determinacy depending on interest-smoothing being "high" or "mild" in a sense to be defined below. We show that insofar as the policy rule minimally responds to the output gap, interest-smoothing does not have any effect on determinacy regions. However, the policy response to inflation required for determinacy can depart very significantly from the original Taylor Principle for an inflation trend between 0% and 8%. When the policy rule targets output growth only, we show that it makes a difference on the determinacy regions whether smoothing is high or mild. That is, with high smoothing, determinacy is achieved for a policy response to inflation which is about 1. With mild smoothing, we find that the response to inflation consistent with determinacy will depend very much on the degree of price flexibility.

Next, we report simulation results from an exercise where data are generated from our model when it is characterized by indeterminacy and determinacy, and then estimate our model on this data. We show that with sample sizes close to our empirical samples our estimation method correctly identifies data generated from a model with determinacy or indeterminacy, with HPD intervals containing the true parameters.

In contrast to the previous literature, we show that the model delivering the largest estimated marginal data densities for the pre-Volcker period is that with a policy rule including two lags of smoothing, a time-varying inflation target, a response to output growth, and a persistent policy shock. We find it has larger data densities than the same model but a white-noise policy shock, or a similar model where policy responds to the output gap and output growth rather than to output growth only. This model is also preferred for the period 1983:I to 2005:I, meaning that according to our evidence the Fed adhered to the same rule during the postwar period prior to the Great Recession.

Our Taylor rule estimates have important implications for the determinacy outcome. We find that for the pre-Volcker era, the estimated model with second order policy smoothing, a time-varying inflation target, and a policy response to output growth delivers determinacy with probability 1. Unlike Coibion and Gorodnichenko (2011, 2012), our evidence points to both second order interest-smoothing and mildly persistent policy shocks. We also find that with a policy rule targeting output growth, determinacy is obtained for the pre-Volcker era with a policy shock which is either persistent or white-noise. However, with a policy rule targeting output gap and output growth, we obtain determinacy with high probability only if the policy shock is persistent.

If we follow the previous literature on indeterminacy, and replace second order interest-smoothing by first order smoothing, we find lower marginal data densities and a near-zero probability of determinacy with a rule responding to the output gap and output growth (see also Lubik and Schorfheide (2004); Hirose et al. (2020)), but with probability 1 if the rule responds to output growth only.

Adding an observable for the time-varying inflation target plays a key role in our estimations. For if we include a time-varying inflation target, but without the additional observable on the inflation target, then we find a probability of determinacy of 0.5 or lower for a rule which targets output gap and growth, or output growth only.

The rest of the paper is organized as follows. Section 2 describes our economic model. Section 3 describes the BN solution method to models with indeterminacy, the SMC estimation algorithm, and the data and priors used in the estimation. Section 4 discusses simulation evidence from our model about the determinacy regions and suitability of our estimation strategy. Section 5 analyses our results. Section 6 puts our new findings into perspective in the broader literature. Section 7 contains concluding remarks.

2 The Model

Our framework includes positive trend inflation and a real adjustment friction in the form of external consumer habit formation. There is no capital accumulation. There is Calvo price stickiness. Aggregate fluctuations are driven by shocks to the discount rate, TFP, to the time-varying inflation target, to the policy rule, and if in a state of indeterminacy, by sunspot shocks.

2.1 Households

The representative consumer maximizes expected utility over final consumption goods C and labour supply L

$$\underset{C_t, L_t, B_t}{\text{Max}} \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t b_t \left[\log(C_t - h\bar{C}_{t-1}) - v \frac{L_t^{1+\eta}}{1+\eta} \right], \tag{1}$$

where β is the subjective discount factor, h the degree of external habit formation, η is the inverse elasticity of labour supply, and b_t is an intertemporal preference shock which follows an AR(1) process given by

$$\log b_t = (1-\rho_b)\log b + \rho_b \log b_{t-1} + \epsilon_t^b, \tag{2}$$

where ϵ_t^b is i.i.d. $N(0, \sigma_b^2)$. The representative consumer is subject to the following budget constraint

$$B_t + P_t C_t = R_{t-1} B_{t-1} + W_t L_t + \Pi_t, \tag{3}$$

where B_{t-1} is the stock of nominal bonds that the household enters period t with, W_t is the nominal wage rate, P_t is the price of the final consumption good, R_t is the gross nominal interest rate, and Π_t is profits from ownership of the firms.

2.2 Final goods firms

Final goods firms operate in a perfectly competitive environment and package intermediate goods into a final aggregate good, Y_t, sold at price P_t. Their maximization problem is given by

$$\underset{Y_t(i)}{\text{Max}} \ P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \tag{4}$$

where

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{5}$$

$P_t(i)$ and $Y_t(i)$ are prices and quantities of intermediate goods, and ϵ is the elasticity of substitution between intermediate goods. The maximization problem yields the standard downward sloping demand function for intermediate firm i's input, which is a function of its relative price and the price elasticity of demand

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{6}$$

and the aggregate price index is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{7}$$

The market clearing condition is given by

$$Y_t = C_t. \tag{8}$$

2.3 Intermediate producers

Intermediate goods are produced by a continuum of monopolistically competitive firms with a constant returns to scale production function given by

$$Y_t(i) = A_t L_t(i), \tag{9}$$

where A_t is a technology shock common to all firms. Technology evolves according to

$$\log g_{A,t} = (1 - \rho_z) \log g_A + \rho_z \log g_{A,t-1} + \epsilon_t^z, \tag{10}$$

where $g_{A,t} \equiv A_t / A_{t-1}$ and ϵ_t^z is i.i.d. $N(0, \sigma_z^2)$.

Intermediate producers minimize total costs each period subject to meeting demand

$$\underset{L_t(i)}{\text{Min}}\, W_t L_t(i), \tag{11}$$

and

$$A_t L_t(i) \geq \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t. \tag{12}$$

The minimization problem yields the following first-order condition,

$$MC_t = \frac{W_t}{A_t}, \tag{13}$$

where MC_t is the nominal marginal cost in period t, and since all firms are subject to the same technology shock and nominal wages, marginal costs are the same across all firms. Firms are subject to Calvo pricing. Each period firms face a probability of reoptimizing their price given by $1 - \xi_p$. A firm setting its price optimally in period t maximizes the following discounted expected flow of profits

$$\underset{P_t(i)}{\text{Max}}\, \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{P_t(i)}{P_{t+\tau}} \left(\frac{P_t(i)}{P_{t+\tau}}\right)^{-\epsilon} Y_{t+\tau} - mc_{t+\tau} \left(\frac{P_t(i)}{P_{t+\tau}}\right)^{-\epsilon} Y_{t+\tau} \right), \tag{14}$$

where mc_t is the real marginal cost in in period t and λ_t is the marginal utility of nominal income to the representative consumer in period t. Lastly we denote price dispersion in period t by

$$v_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di. \tag{15}$$

2.4 Monetary policy

Monetary policy is set according to an endogenous feedback rule which takes the form

$$\left(\frac{R_t}{R}\right) = \left(\frac{R_{t-1}}{R}\right)^{\rho_{R,1}} \left(\frac{R_{t-2}}{R}\right)^{\rho_{R,2}} \left[\left(\frac{\pi_t}{\pi_t^\star}\right)^{\alpha_\pi} \left(X_t\right)^{\alpha_x} \left(\frac{Y_t}{Y_{t-1}}\right)^{\alpha_{\Delta y}}\right]^{1-\rho_{R,1}-\rho_{R,2}} v_t^r, \tag{16}$$

where R is the gross nominal interest rate, $\pi_t^\star$ is a time-varying inflation target, X_t is the output gap defined as $\frac{Y_t}{Y_t^n}$, and $\frac{Y_t}{Y_{t-1}}$ is the gross growth rate of output. The natural rate of output, Y_t^n, is given by

$$v\left(\frac{Y_t^n}{A_t}\right)^{1+\eta} = \left(\frac{\epsilon-1}{\epsilon}\right) + vh\left(\frac{Y_t^n}{A_t}\right)\left(\frac{Y_{t-1}^n}{A_t}\right). \tag{17}$$

v_t^r and $\pi_t^\star$ are exogenous processes given by

$$\log v_t^r = \rho_r \log v_{t-1}^r + \epsilon_t^r, \tag{18}$$

$$\log \pi_t^\star = (1-\rho_\pi)\pi + \rho_\pi \log \pi_{t-1}^\star + \epsilon_t^\pi, \tag{19}$$

where ϵ_t^r is i.i.d. $N(0, \sigma_r^2)$ and ϵ_t^π is i.i.d. $N(0, \sigma_\pi^2)$.

2.5 Log-Linearization

Solving the model requires detrending output, which is done by removing trend growth and taking a log-linear approximation of the stationary model around the non-stochastic steady state. The full set of non-linear equations which characterize the equilibrium of the model and the log-linearized model are reported in the Appendix of the paper.

3 Model Solution, Estimation, and Data

3.1 Rational Expectations Solution Under Indeterminacy

To solve the Linear Rational Expectations (LRE) model allowing for the possibility of indeterminacy, we use the solution method proposed by Bianchi and Nicoló (2020) (henceforth BN). A standard LRE system can be cast in its canonical form given by

$$\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)\epsilon_t + \Pi(\theta)\eta_t, \tag{20}$$

where s_t is a vector of endogenous variables, ϵ_t a vector of exogenous disturbances, and η_t a vector of one-step ahead forecast errors for the expectational variables in the model. The solution method proposed by BN is to augment the canonical form with additional autoregressive equations, with the number of additional equations being equal to the degree of indeterminacy. These additional equations can be used to provide the "missing" explosive roots. This requires that when the model is characterized by indeterminacy of order p, then p of the auxiliary equations must be explosive. When the model is determinant, the auxiliary equations are not explosive and do not influence endogenous variables in the model. In our case we permit one degree of indeterminacy and hence augment the LRE system in (20) with one additional equation which is given by

$$\omega_t = \left(\frac{1}{\alpha_{BN}}\right)\omega_{t-1} - \zeta_t + \eta_{f,t}, \tag{21}$$

where ζ_t is a sunspot shock which follows $\zeta_t \sim$ i.i.d. $N(0, \sigma_\zeta^2)$. $\eta_{f,t}$ is an expectational error. BN show that the choice of expectational error does not affect the solution of the model when the correlations between the sunspot shock and fundamental shocks are left unrestricted. We assume that this expecational error is associated with inflation, that is, $\eta_{f,t} = \eta_{\pi,t} = \pi_t - E_{t-1}\pi_t$. Additionally, since the model contains trend inflation, the exact boundary of indeterminacy is unknown. As such we treat α_{BN} as a parameter to be estimated alongside the other structural parameters of the model.

Expanding the state space to include the additional auxiliary equation, the LRE model takes the form

$$\hat{\Gamma}_0(\theta)\hat{s}_t = \hat{\Gamma}_1(\theta)\hat{s}_{t-1} + \hat{\Psi}(\theta)\hat{\epsilon}_t + \hat{\Pi}(\theta)\eta_t, \tag{22}$$

where $\hat{s}_t \equiv (s_t, \omega_t)'$ and $\hat{\epsilon}_t \equiv (\epsilon_t, \zeta_t)'$. The matrices $\hat{\Gamma}_0$, $\hat{\Gamma}_1$, $\hat{\Psi}$, $\hat{\Pi}$ are redefined to include the auxiliary equation. (22) can now be solved using standard methods and the augmented representation contains solutions for the model in both the determinacy and indeterminacy regions (given the parameter requirements discussed above). The BN characterization of equilibrium indeterminacy is equivalent to the characterizations one would get using the methodology of Lubik and Schorfheide (2003, 2004) or Farmer et al. (2015).

3.2 Econometric Strategy

To estimate the posterior distributions of the structural parameters and shocks we use Bayesian estimation. Because the model features regions of determinacy and indeterminacy, posterior densities are potentially multi-modal and standard posterior approximation methods such as the random walk Metropolis Hastings (RWMH) algorithm can often get stuck at local modes and fail to explore the entire posterior distribution. This is driven by the construction of the algorithm, which relies on highly correlated draws. Instead we employ the Sequential Monte Carlo (SMC) method proposed in Herbst and Schorfheide (2014) and discussed in Herbst and Schorfheide (2016). SMC is an importance sampling algorithm but overcomes the main challenge associated with importance sampling, which is finding good proposal densities, by recursively constructing a sequence of distributions which begins at some easy-to-sample initial distribution (in our case, the prior distributions) and using these distributions as proposal densities in the subsequent stages.

The sequence of distributions are given by

$$\pi_n(\theta) = \frac{[p(Y|\theta)]^{\phi_n} p(\theta)}{\int [p(Y|\theta)]^{\phi_n} p(\theta) d\theta}, \tag{23}$$

where ϕ_n increases from 0 to 1 for $n = 1, \dots, N^{\phi}$.[3] The sequence of distributions for $\phi_n \in (0, 1)$ are referred to as *tempered posteriors* or *bridge* distributions, and the distribution associated with $\phi_n = 1$ is the approximated posterior distribution. The parameter ϕ_n is referred to as the *tempering schedule* and is determined by

[3] Because the SMC is based on recursively computing the bridge densities, the posterior density typically denoted $p(\theta|Y)$ is abbreviated by $\pi_n(\theta)$, where n is the bridge density in iteration n.

$$\phi_n = \left(\frac{n-1}{N_\phi - 1}\right)^\lambda, \tag{24}$$

where n is the current stage and N_ϕ is the total number of stages. λ determines the shape of the tempering schedule. A value of $\lambda = 1$ implies a linear tempering schedule. For high values of λ, initial bridge distributions will be quite similar to the prior distributions, and bridge distributions will be quite different in the final stages of the algorithm. We use a value of $\lambda = 2$, which is the suggested value by Herbst and Schorfheide (2016). The remaining choices to be made are the number of stages and the number of particles in each stage. We follow Herbst and Schorfheide (2016) and use 200 stages ($N_\phi = 200$). However we opt for a larger number of particles than typically recommended and use 25,000 ($N = 25,000$). The rationale for this is that with the Bianchi and Nicoló (2020) solution approach, likelihood evaluation requires a solution to exist and be unique (including the appended auxiliary equation). However, many of the draws at each stage may be discarded due to: (1) the model being determinant and the auxiliary equation being explosive; or (2) the model being indeterminant and the auxiliary equation being non-explosive. This potentially leads to a non-negligible decline in the number of particles at each stage, which we counteract by increasing the total number of particles. After initializing the algorithm (i.e., drawing initial particles from the prior distributions) and equalizing the initial weights, the algorithm proceeds in three steps:

1. **Correction**: The correction step reweights particles from the previous stage to areas of parameter space with higher likelihoods. Reweighting occurs according to incremental and normalized weights given by

$$\tilde{w}_n^i = [p(Y|\theta_{n-1}^i)]^{\phi_n - \phi_{n-1}}, \qquad \tilde{W}_n^i = \frac{\tilde{w}_n^i W_{n-1}^i}{\frac{1}{N}\sum_{i=1}^N \tilde{w}_n^i W_{n-1}^i},$$

   for all N particles.

2. **Selection**: The selection step computes the effective sample size (ESS) which is given by $ESS_n = \frac{N}{\frac{1}{N}\sum_{i=1}^N (\tilde{W}_i^n)^2}$. If ESS falls below a threshold use multinomial resampling to

resample the particles from support points and weights $\{\theta^i_{n-1}, \tilde{W}^i_n\}$ and equalize the weights such that $W^i_n = 1$. We use the threshold suggested by Herbst and Schorfheide (2016), which resamples when $ESS_n < N/2$. In effect, this step ensures that particles do not become too concentrated, and if a function of the variance of the weights falls below a threshold, then resample.

3. **Mutation**: The mutation steps propagates particles $\{\hat{\theta}_i, W^i_n\}$ using a single step of the RWMH algorithm with appropriate tuning parameters to ensure a reasonable acceptance rate.

The final sampling approximation, $\pi_{N_\phi}(\theta)$, yields the estimated posterior density for the parameters.

Model fit. As noted by Herbst and Schorfheide (2016), the correction step approximates the marginal data density as a by-product without having to compute any additional likelihood evaluations. We use this approximation to rank our models. The approximation is given by

$$\hat{p}_{SMC}(Y) = \prod_{n=1}^{N_\phi} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}^i_n W^i_{n-1} \right). \tag{25}$$

Probability of determinacy. To assess the probability of determinacy our analysis examines the posterior distribution of the parameter α_{BN}. As discussed in the previous section, when the model is characterized by determinacy, the parameter α_{BN} must be strictly greater than 1 such that the appended equation has no impact on the dynamics of the model. Thus our probability of determinacy is computed as

$$\mathbb{P}(\text{Determinacy}) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\{\alpha^i_{BN,N_\phi} > 1\}, \tag{26}$$

where $\mathbf{1}$ is an indicator function which equals 1 if $\alpha_{BN} > 1$.

3.3 Data

To estimate the parameters of the model we use four U.S. quarterly time series: per capita real GDP growth, GDP deflator based inflation, the Federal Funds rate, and a measure of target inflation from Aruoba and Schorfheide (2011). Our sample periods are dictated by the availability of data on target inflation. The construction of the estimation data is described in detail in the Appendix.

We estimate the model's parameters over two different samples. The first sample corresponds to the pre-Volcker era and spans the periods from 1964:I to 1979:II. The second sample corresponds to the Great Moderation and spans from 1983:I to 2005:I.

The mapping of observables to model variables is given by

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100\Delta \log P_t \\ 100 \log R_t \\ 100\Delta \log P_t^{\star} \end{bmatrix} = \begin{bmatrix} \bar{g_A} \\ \bar{\pi} \\ \bar{r} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + \epsilon_t^z \\ \tilde{\pi}_t \\ \tilde{r}_t \\ \tilde{\pi}_t^{\star} \end{bmatrix}, \tag{27}$$

where $\bar{g_A}$, $\bar{\pi}$, and $\bar{r}$ are the steady state values of the growth rate of output, inflation, and the nominal interest rate, respectively. These values are expressed in net terms given by $100(\bar{g_A} - 1)$, $100(\bar{\pi} - 1)$, and $100(\bar{r} - 1)$.

3.4 Calibration and Prior Distributions

All parameters are estimated except two which are calibrated. We set the elasticity of substitution between differentiated goods to 9, implying a 12.5% price markup with zero trend inflation. The Frisch elasticity of labour supply is set to 1. These values are standard in the literature. The remaining structural parameters are estimated. Mean priors and prior standard deviations for these parameter values are presented in Table 1.

The parameter governing consumer habit formation has a beta distribution prior with a mean of 0.7 and standard deviation of 0.1. As in Justiniano et al. (2011), the prior for the Calvo price stickiness parameter is given by a beta distribution with a mean of 0.66 and standard deviation of 0.1.

Priors for the average rate of inflation, growth rate of output, and nominal interest rate are set at roughly their average sample values for the post-WWII period. We use relatively diffuse priors around these means. The average rate of inflation, output growth, and nominal interest rate have normal distribution priors with means of 0.8, 0.4, and 1.2, with standard deviations of 0.75, 0.2, and 0.4, respectively.

For the response parameters governing monetary policy, we generally follow those specified in Smets and Wouters (2007). The response parameters of inflation, the output gap, and output growth have normal distribution priors with means of 1.5, 0.125, and 0.125 with prior standard deviations of 0.3, 0.05, and 0.05. Our monetary policy rule features interest rate smoothing of order two. For these two parameters we rely closely on the empirical estimates in Coibion and Gorodnichenko (2011). For the order one interest rate smoothing parameter, we specify a normal distribution prior with a mean of 1.1 and standard deviation of 0.25. For interest rate smoothing of order two, we specify a normal prior with mean of 0 and standard deviation of 0.25.[4] When monetary policy is characterized by smoothing of order one, we specify a beta prior with a mean of 0.6 and standard deviation of 0.2.

The exogenous disturbances have standard priors. Shocks have Inverse Gamma priors with means of about 0.6 and standard deviations of 0.3. Persistence parameters of shocks have beta distributions with means of 0.5 and standard deviations of 0.2. An identical prior was used for the time-varying inflation target persistence in Del Negro et al. (2015).

For the parameter α_{BN}, which determines whether the model is characterized by indeterminacy, we use a Uniform prior over the range $[0.5, 1.5]$ as suggested by Bianchi and Nicoló (2020). Lastly, for the correlation between sunspot shocks and fundamental shocks, we use Uniform priors over the range $[-1, 1]$.

4 Determinacy Regions and Simulated Data

This Section identifies the determinacy regions under the assumption of first and second order interest-smoothing for a level of annualized trend inflation between 0% to 8%. Coibion and Gorodnichenko (2011) show that targeting inflation and the output gap increases the prospect of indeterminacy as trend inflation rises relative to targeting inflation and output growth. However, when examining these two cases, they do not combine output targeting

[4] It is worth noting that with these priors the sum of autoregressive coefficients could yield a rule which is super-inertial. However, we discard draws where the sum of coefficients is greater than one.

with interest-smoothing. Furthermore, when they look at the impact of interest-smoothing on the determinacy outcome, it is restricted to first order smoothing. Here, we perform a similar exercise, but combining inflation and output targeting with second order interest-smoothing.

4.1 Determinacy Regions

Our key findings can be summarized as follows. We identify through numerical simulations the minimum policy response to inflation required for determinacy when assuming second order smoothing. We consider "high" smoothing which is broadly consistent with CG' estimates (first order smoothing degree of 1.35 and second order degree of -0.4), and "mild " smoothing (first order degree of 0.75 and second order degree of -0.4).

With a policy rule targeting both the level of the output gap and output growth, we find that interest-smoothing is basically irrelevant for the determinacy outcome which is driven primarily by the policy response to the output gap. With a rule responding only to output growth, interest-smoothing matters for determinacy. If smoothing is "high", determinacy is achieved with a policy reaction to inflation slightly above 1, and this up to an annualized inflation trend of 8% and different degrees of price rigidity. If smoothing is "mild ", determinacy is somewhat sensitive to trend inflation between 3% and 5% if the Calvo stickiness parameter is 0.66 or 0.75. With relatively flexible prices (Calvo stickiness parameter of 0.55), determinacy is achieved with a policy response to inflation slightly higher than 1.

Figure 1 looks at the policy response to inflation consistent with determinacy when the policy rule targets the output gap with a response parameter of 0.2. The determinacy regions are traced for a first order smoothing parameter which is either 1.35 (high) or 0.75 (mild), and a second order smoothing parameter of -0.4. The high positive first order and negative second order smoothing parameter values are broadly consistent with the estimates reported by Coibion and Gorodnichenko (2011). The mild positive first order smoothing parameter more or less corresponds to some estimates we report below. The Calvo price stickiness parameter is set at 0.55, 0.66 and 0.75, respectively. Other parameters take their mean prior values. Figure 2 displays the determinacy regions with a policy rule reacting to output growth with a response coefficient of 0.2.

With flexible prices (Calvo stickiness parameter of 0.55) and a rule targeting the output gap, Figure 1 reveals that determinacy is achieved with a policy reaction to inflation that lies between 1 and 1.22 for an inflation trend varying between 0% and 8%. With a price stickiness parameter of 0.66, a 5% trend inflation requires a policy response to inflation of about 1.25 to be consistent with determinacy. When the stickiness parameter is 0.75, there is a huge increase in the minimum response to inflation consistent with determinacy to 1.75 with a 5% trend inflation.

If the policy rule targets output growth, trend inflation does not matter for the prospect of determinacy insofar as first order interest-smoothing is high. That is, determinacy is safely achieved with a policy response to inflation that complies with the original Taylor principle. With mild first order smoothing, the same conclusion applies insofar as prices are relatively flexible (Calvo stickiness parameter of 0.55). With a stickiness parameter of 0.66, a policy response to inflation of 1.01 is consistent with determinacy up to 5% trend inflation. With a stickiness parameter of 0.75, the minimum policy response to inflation needed to ensure determinacy departs from 1.01 when trend inflation exceeds 3% and reaches 1.38 at a level of trend inflation of 5%.

Therefore, a policy rule targeting output growth in an economy with positive trend inflation is generally consistent with determinacy with a lower policy reaction to inflation than a rule aiming at the output gap. In fact, when policy reactions to both the output gap and output growth are combined, the influence of output gap on the determinacy outcome is disproportionately important relative to that of output growth. To see this, Figure 3 combines policy responses to both the output gap and output growth, with response parameters of 0.2. There is almost no difference between this figure and 1 where the nominal interest rate responds only to the output gap.

4.2 Simulated Data

To ensure that our estimation strategy is capable of delivering accurate structural parameter estimates and posterior probabilities of determinacy, we conduct a simulation exercise in the following manner. First, we calibrate the structural parameters of the model such that in one case the model is characterized by determinacy and in the other the model is characterized by indeterminacy. We generate a sequence of 5,000 normally distributed shocks, with standard deviations given by the calibrated values. We then iterate the shocks through the model, keeping the final 75 observations for output growth, inflation, the nominal interest rate, and the inflation target.[5] In both cases we iterate the sunspot shocks through the model, but in the case of determinacy these shocks have no impact on other endogenous variables of the model.

Using the simulated data for the 4 observables, we then estimate the model parameters and probability of determinacy using this data. The estimation results along with the calibrated parameter values used in generating the data are reported in Table 2. The prior distributions for structural parameters are the same as those described in Section 3.4.

Two points are worth emphasizing. First, our simulation evidence suggests that even with only 75 observations, the structural parameters are estimated quite well. In almost all cases the 90% HPDI interval includes the true parameter value. Second, the simulation reveals that our estimation accurately identifies data generated from indeterminacy and determinacy. That is, α_{BN} converges to the correct region. While the simulation exercise benefits from no model uncertainty (the data is drawn from the correct model), it does give some validation to our results in the rest of the paper.

5 Estimation Results

This Section presents our estimation results. We begin by presenting estimates of a model with two interest-smoothing lags, a time-varying inflation target, with and without a persistent policy shock, for the period 1964:I-1979:II. This is followed by estimates of models with only one lag of interest-smoothing. Finally, we report estimates for the period 1983:I-2005:I. We provide estimates of the structural parameters and shocks with their 90% confidence intervals. The log $p(X^T)$ represents the marginal data density of a model, while *Prob(det)* is the

[5]Our choice of 75 observations is motivated by our empirical sample sizes which are 62 for the pre-1979 period and 89 for the post-1983 period.

posterior probability of equilibrium determinacy implied by the model estimates.

5.1 Second Order Smoothing: 1964:I-1979:II

Table 3 presents four sets of results. The first column contains estimates of a model that includes a policy rule with second order smoothing, a time-varying inflation target, policy responses to both the level of the output gap and output growth (hereafter mixed output rule or MO-rule), and an AR(1) policy shock. The second column presents estimates with a policy rule that aims at output growth only (hereafter output growth rule or OG-rule). Columns 3 and 4 provide estimates for the same two models, except that the monetary policy shocks are white-noise shocks.

Table 3 unveils the following main findings. The log marginal data densities $\log p(X^T)$ reported in the second to last row of the table reveal that the preferred model by this criterion is the OG-rule model, followed by the MO-model. This finding, which is new to the literature, is of interest in light of previous contributions by Walsh (2003), Sims (2013) and Khan et al. (2020) that put forth arguments in favour of adopting policy rules targeting output growth. Compared to previous works that favoured MO-rule over OG-rule models, and which we discuss in Section 7, our results are generated with a different solution method and estimation procedure, second order interest rate smoothing, a time-varying inflation target and a additional observable for the inflation target that stands as a proxy for inflation expectations and which dates back to the 1960s.

Models with second order smoothing and a persistent policy shock, whether the policy rule includes a response to the output gap and output growth or to output growth only, predict determinacy with a high probability, that is probability .85 for the MO-rule and probability .99 for the OG-rule. The estimated policy response to inflation is 2.0 for the MO-rule and 2.13 for the OG-rule.

The estimated first order smoothing parameters are respectively 0.83 and 0.81 for the MO-rule and OG-rule models, while estimates of the second order smoothing parameters are -0.32 and -0.34. The first order smoothing parameters are thus smaller than those reported by Coibion and Gorodnichenko (2011, 2012), while the second order smoothing parameters are consistent with CG' estimates.

The estimated AR(1) parameter of the policy shock process is .38 in the MO-rule model and .37 in the OG-rule model. While moderate, the estimated HPD interval for AR(1) parameters of the policy shock do not contain zero. Therefore, unlike the evidence in CG, ours confirms both the presence of second order interest-smoothing and persistent policy shocks in the MO-rule and OG-rule models.

The estimated price stickiness parameters are .626 and .622, respectively. The estimated average annualized rate of inflation is 4.27 percent for the MO-rule model and 4.17 percent for the OG-rule model. Therefore, based on these estimates and the simulated determinacy regions identified in Section 4.1, determinacy is safely achieved under both the mixed rule and the growth-rule for our pre-1980 sample.

How important is it to have persistent monetary policy shocks in addition to second order smoothing and a time-varying inflation target for the determinacy outcome? The third and fourth columns of Table 3 presents estimates with white-noise policy shocks. Having a white-noise policy shock has important consequences for the determinacy outcome under the MO-rule, for then the probability of determinacy is down from .85 to only .33. Note in this particular case that the Calvo price stickiness parameter is about .7 and that the policy response to inflation is 1.4. Furthermore, the average annualized of inflation is about 4.6 percent. Therefore, given these estimated parameter values and based on our simulation of determinacy regions in Section 4.1, determinacy would have required a stronger policy response to inflation.

By contrast, under the output growth policy rule, the probability of determinacy is 1. The policy response to inflation at 2.09 is significantly higher than in the MO-rule model. Note also that with white-noise policy shocks, the estimated first order interest-smoothing parameters are significantly higher compared to estimates with persistent policy shocks. Therefore, there seems to be some tradeoff between the degree of interest-smoothing and the degree of persistence in the policy shock.

5.2 First Order Interest-Smoothing: 1964:I-1979:II

How important is it to have second order interest-smoothing for our results? The previous literature using a Bayesian model consistent approach to monetary policy and indeterminacy has generally focused on first order interest-smoothing models. Table 4 reports estimation results with one lag of smoothing combined with a time-varying inflation target and persistent policy shock, and this for the sample 1964:I-1979:II, using our four observables in the estimation.[6]

The results are striking. The probability of determinacy is now down to .01 with first order smoothing when policy responds both to the output gap and output growth. The policy response to inflation drops to 1.39. Furthermore, the Calvo price stickiness parameter rises to .815. The estimated average annualized rate of inflation is about 4.8 percent. Together, these factors contribute to predicting indeterminacy when the model features first order smoothing.

By stark contrast, the estimated probability of determinacy is 1 for the OG-rule model. The estimated policy reaction to inflation remains high at 2.13, while the probability of price non-reoptimization at .64 is much lower than in MO-rule model. The estimated average inflation rate is 4.23 percent. These estimates concur in making of determinacy an outcome with certainty under the OG-rule. Note also that based on the log marginal data densities $\log p(X^T)$, the OG-rule model with first order smoothing and a persistent policy shock is preferred to the MO-rule model. This finding also contrasts with the rest of the literature which generally holds that a rule specification with responses to the output gap and output growth is generally better for the pre-Volcker period.

5.3 Second Order-Smoothing: 1983:I-2005:I

Table 5 presents our estimation results from 1983:I to 2005:I. Again, we report estimates for the MO-rule and OG-rule models, with and without a persistent policy shock. Based on the log marginal data densities $\log p(X^T)$ reported in the second to last row of the table, the

[6]We do not report the results with a white-noise policy shock because they are significantly worse than those with a persistent policy shock.

model preferred by this criterion is the OG-rule model with a persistent policy shock. This model also predicts an estimated probability of determinacy of 1. This is also the case for the MO-rule model with a persistent policy shock.

The estimated models with two smoothing lags and a persistent policy shock are strongly preferred to models with second order smoothing and a white-noise policy shock. In the case of the MO-rule model with a white-noise policy shock, the estimated probability of determinacy drops to a low .06. Still, the OG-rule model delivers determinacy with probability 1.

Therefore, our evidence points to the statistical superiority of models with two lags of smoothing and a persistent policy shock prior to 1980 and after 1982. Our evidence hence makes a strong case in favour of a policy rule with two smoothing lags and a persistent policy shock.

5.4 Inflation Target Observable

A novel aspect of our work is the use of a series for the inflation target that serves as an observable in the estimation of our models, and which helps identify the parameters of the policy rule and of the inflation target shock process. Haque (forthcoming) assumes a time-varying inflation target in studying indeterminacy, but without the use of a series for the inflation target as an observable.

Table 6 report estimates of our model with two interest-smoothing lags, a time-varying inflation target, policy responses to output gap gap and output growth, and to output growth only, with a persistent policy shock, using three observables instead of four in the estimation. Without the observable on the inflation target, the probability of determinacy is down to .51 for the MO-rule model and to .42 for the OG-model. These results confirm that having a series for the inflation target makes an important difference for the determinacy outcome.

6 Related Literature

This Section explains how our findings relate to the broader literature on monetary policy rules and indeterminacy. Most of the previous literature has concluded that the US economy experienced indeterminacy during the 1960 & 1970s and determinacy after 1982. These studies have typically assumed policy rules with one interest-smoothing lag and responses to the output gap, or to both the output gap and output growth. These rules have also included policy responses to deviations of inflation from a fixed target or from a target which is time-varying. Policy shocks were either white-noise or persistent.

Clarida et al. (2000) explain periods of indeterminacy and determinacy using estimates of policy rules with first order smoothing where the inflation response parameters are lower than 1 during the pre-Volcker period and hence leading to indeterminacy, while higher than 2 after 1982 and resulting into determinacy.

Using a prototypical NK price setting model estimated with a Bayesian method that permits the possibility of determinacy or indeterminacy, Lubik and Schorfheide (2004) corroborate CGG's findings that monetary policy was highly accommodative during the pre-Volcker period and conclude that the US economy was in a state of indeterminacy. Compared to our preferred policy rule, theirs includes first order interest-smoothing, responses to deviations of inflation and output from target levels, and a white-noise policy shock. Trend inflation is zero. The model is estimated conditional on information from three observables which are HP detrended real GDP, CPI-U inflation and the average federal funds rate.

Hirose et al. (2020) extend the work of Lubik and Schorfheide by estimating different versions of a NK price setting model with positive trend inflation. They use a full-information Bayesian method and a SMC algorithm such that they can assess regions of determinacy and indeterminacy in a single estimation. Their model includes a policy rule with first order smoothing, policy responses to deviations of inflation from a fixed target, to level of the output gap, and to deviations of output growth from trend. The policy shock follows a AR(1) process. The estimation uses three observables: real GDP growth, inflation, and the federal funds rate. They conclude that the pre-Volcker years were characterized by indeterminacy with certainty or a high probability, while the economy experienced determinacy after 1982. They argue, in line with Coibion and Gorodnichenko (2011), that a more active response to inflation was not sufficient to explain U.S. macroeconomic stability after 1982. A lower level of trend inflation, and/or a weaker response to the output gap and a stronger response to output growth are also required to achieve determinacy.

Unlike Lubik and Schorfheide (2004) and Hirose et al. (2020), our estimations reveal that the model preferred by the criterion of marginal data densities is one with a policy rule including second order smoothing, a time-varying inflation target and a reaction to output growth only. Determinacy rather than indeterminacy is then reached with near-certainty. Our rule specification plays an important role driving our new results. That is, consistent with the evidence in Lubik and Schorfheide and Hirose et al., we obtain indeterminacy with near-certainty if the policy rule assumes first order smoothing, a time-varying inflation target and responses to both the output gap and output growth.

Bilbiie and Straub (2013) augment the standard NK price setting model with limited asset market participation. They argue that the pre-1980s were characterized by low asset market participation implying that lower interest rates has contractionary rather than expansionary effects on the economy, and thus led to determinacy during the pre-Volcker years. Compared to our policy rules, theirs features one lag of interest-smoothing, responses to one-period ahead expected inflation and actual output, and a white-noise policy shock. Trend inflation is zero.[7]

Coibion and Gorodnichenko (2011) import single-equation estimates of policy rules featuring two interest-smoothing lags in a calibrated NK model with sticky prices. They conclude that high positive trend inflation and a "dovish" monetary policy led to indeterminacy prior to 1980, while determinacy was achieved during the Great Moderation by setting a lower level of trend inflation and adopting a "hawkish" policy stance against inflation.[8]

[7] Ascari et al. (2017) show that a small amount of nominal wage stickiness will normally prevent inversion of the slope of the IS curve when accounting for limited asset market participation, and hence will invalidate the Inverted Taylor Principle.

[8] Ascari and Ropele (2009) show that with positive trend inflation, determinacy will require policy responses to inflation stronger than dictated by the original Taylor Principle. Khan et al. (2020) show that a policy rule targeting the output gap makes determinacy very unlikely if average (trend) annualized inflation reaches 4

While our evidence is consistent with second order interest-smoothing, it also confirms the significance of mildly persistent policy shocks. Furthermore, CG conclude to indeterminacy during the pre-Volcker period, our estimation results point to determinacy.

Haque et al. (2021) estimate a NK price setting model with positive trend inflation, commodity price shocks and a real wage rigidity. Their policy rule includes first order smoothing, responses to the level of the output gap and output growth, and a white-noise policy shock. They find when combining these ingredients that the estimated policy response to the output gap is nearly zero, so that determinacy is achieved. By contrast, our main new findings hinge primarily on our different Taylor rule.

Haque (forthcoming) argues that adding a time-varying inflation target to an otherwise standard NK sticky price model with first order interest-smoothing, policy responses to the output gap and output growth, and a white-noise policy shock is sufficient to restore determinacy prior to 1980. Unlike us, he does not use an observable for target inflation. When dropping the inflation target series as an observable, the probability of determinacy is down to .42 with a rule responding to output growth and to .51 with a rule reacting to output gap and output growth. Therefore, it is not accounting for time-varying inflation target per se that drives our determinacy results prior to 1980, but the fact that we combine second order smoothing, a time-varying inflation, and a persistent policy shock in a model estimated with an observable conveying information about movements in the inflation target.

7 Conclusion

Previous work based on a single equation estimation approach highlighted the importance of higher order interest rate smoothing in explaining interest rate inertia (Coibion and Gorodnichenko 2011, 2012). Using a Bayesian model consistent approach to monetary policy rules that allows the possibility of both determinacy and indeterminacy, we have offered new evidence confirming the empirical relevance of higher order interest rate smoothing and its implications for the prospect of determinacy.

percent.

While our evidence pointed to both second order interest-smoothing and mildly persistent policy shocks, we have shown that our determinacy findings for the pre-Volcker era do not the depend on the policy shock being persistent or white-noise insofar as the policy rule responds to deviations of inflation from a time-varying target and output growth.

On the whole, our findings suggest the Fed followed a consistent policy rule throughout the postwar period prior to the Great Recession, and one which avoided self-fulfilling inflation expectations prior to 1980. This rule differs however from those which have been used so far in the Bayesian macroeconomic literature.

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A Data construction

To estimate the structural parameters of our model, we use four observables. These observables correspond to per capita output growth, inflation, the federal funds rate, and a measure of target inflation. Our first three observables were downloaded from the Federal Reserve Bank of St. Louis on September 28th, 2021. The exact variables and corresponding FRED codes are

– Gross Domestic Product (`GDP`)

– Gross Domestic Product Implicit Price Deflator (`GDPDEF`)

– Effective Federal Funds rate (`FEDFUNDS`)

– Population Level (`CNP16OV`)

Prior to defining per capita GDP, we fit the population level with a Hodrick Prescott filter with a smoothing parameter of 10,000 and use the trend from this series as our measure of population. The rationale for this, as noted by Pfeifer (2020), is that population levels are periodically updated due to censuses or benchmarking in the Current Population Survey. These updates cause spikes in population growth rates not related to changes in the actual population.

Our measure of target inflation is from Aruoba and Schorfheide (2011). The authors estimate this measure by combining three inflation expectation measures in a small state space model and extracting the common factor using the Kalman filter. The series is available for download at Frank Schorfheide's website `https://web.sas.upenn.edu/schorf/publications/` from the paper *Sticky Prices versus Monetary Frictions: An Estimation of Policy Trade-offs*. The data file is titled "inflation-target" and we use the data under the column heading *filtered f0*.

Our observables are then defined as

$$100 \times \Delta \log Y_t = 100 \times \Delta \log \left(\frac{GDP}{GDPDEF \times P\hat{O}P} \right), \tag{28}$$

$$100 \times \Delta \log P_t = 100 \times \Delta \log \left(\frac{GDPDEF}{GDPDEF_{-1}} \right), \tag{29}$$

$$100 \times \log R_t = 100 \times \log \left(1 + \frac{FEDFUNDS}{400} \right), \tag{30}$$

$$100 \times \Delta \log P_t^{\star} = \left(\frac{\text{filtered f0}}{4} \right), \tag{31}$$

where $P\hat{O}P$ is the filtered population level.

B Full Set of Non-linear Equilibrium Conditions

Below we describe the full set of equations which characterize the equilibrium of the model. There are 17 equations and 17 endogenous variables.

$$\lambda_t P_t = \frac{b_t}{C_t - hC_{t-1}} \tag{32}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) R_t \right] \tag{33}$$

$$b_t v L_t^{\eta} = \lambda_t W_t \tag{34}$$

$$mc_t = \frac{w_t}{A_t} \tag{35}$$

$$C_t = Y_t \tag{36}$$

$$Y_t = \frac{A_t L_t}{v_t^p} \tag{37}$$

$$v_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di \tag{38}$$

$$P_t^{1-\epsilon} = \int_0^1 P_t(i)^{1-\epsilon} di \tag{39}$$

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}}{X_{2,t}} \tag{40}$$

$$X_{1,t} = \lambda_t mc_t P_t^{\epsilon} Y_t + \beta \xi_p \mathbb{E}_t X_{1,t+1} \tag{41}$$

$$X_{2,t} = \lambda_t P_t^{\epsilon-1} Y_t + \beta \xi_p \mathbb{E}_t X_{2,t+1} \tag{42}$$

$$\left(\frac{R_t}{R} \right) = \left(\frac{R_{t-1}}{R} \right)^{\rho_{R,1}} \left(\frac{R_{t-2}}{R} \right)^{\rho_{R,2}} \left[\left(\frac{\pi_t}{\pi_t^{\star}} \right)^{\alpha_\pi} \left(X_t \right)^{\alpha_x} \left(\frac{Y_t}{Y_{t-1}} \right)^{\alpha_{\Delta y}} \right]^{1-\rho_{R,1}-\rho_{R,2}} v_t^r \tag{43}$$

$$X_t = \frac{Y_t}{Y_t^n} \tag{44}$$

$$v \left(\frac{Y_t^n}{A_t} \right)^{1+\eta} = \left(\frac{\epsilon - 1}{\epsilon} \right) + vh \left(\frac{Y_t^n}{A_t} \right) \left(\frac{Y_{t-1}^n}{A_t} \right) \tag{45}$$

$$\log g_{A,t} = (1 - \rho_z) \log g_A + \rho_z \log g_{A,t-1} + \epsilon_t^z \tag{46}$$

$$\log b_t = (1 - \rho_b) \log b + \rho_b \log b_{t-1} + \epsilon_t^b \tag{47}$$

$$\log v_t^r = \rho_r \log v_{t-1}^r + \epsilon_t^r \tag{48}$$

$$\log \pi_t^{\star} = (1 - \rho_\pi) \pi + \rho_\pi \log \pi_{t-1}^{\star} + \epsilon_t^{\pi} \tag{49}$$

C Log-linearized model

After detrending the model by removing trend growth, the log-linearized model can be characterized by 11 endogenous variables and 11 equations which are described below.

$$\begin{aligned}\tilde{y}_t =& \frac{h}{h+g_A}\left(\tilde{y}_{t-1} - \tilde{g}_{A,t}\right) + \frac{g_A}{h+g_A}\mathbb{E}_t\left(\tilde{y}_{t+1} + \tilde{g}_{A,t+1}\right) \\ & - \frac{g_A - h}{h+g_A}\left(\tilde{r}_t - \mathbb{E}_t\tilde{\pi}_{t+1} - \tilde{b}_t + \mathbb{E}_t\tilde{b}_{t+1}\right)\end{aligned} \tag{50}$$

$$\begin{aligned}\tilde{\pi}_t =& \beta[1+\epsilon(1-\xi_p\pi^{\epsilon-1})(\pi-1)]\mathbb{E}_t\tilde{\pi}_{t+1} + \beta(1-\xi_p\pi^{\epsilon-1})(\pi-1)\mathbb{E}_t\tilde{x}_{1,t+1} \\ & + \left(\frac{(1-\xi_p\pi^{\epsilon-1})(1-\beta\xi_p\pi^{\epsilon})}{\xi_p\pi^{\epsilon-1}}\right)((1+\eta)\tilde{y}_t + \eta\tilde{v}_t^p) + \beta(1-\pi)(1-\xi_p\pi^{\epsilon-1})\tilde{b}_t \\ & + \left(\frac{(1-\beta\xi_p\pi^{\epsilon-1})(1-\xi_p\pi^{\epsilon-1})}{\xi_p\pi^{\epsilon-1}}\right)\left(\frac{h}{g_A-h}\right)(\tilde{y}_t - \tilde{y}_{t-1} + \tilde{g}_{A,t})\end{aligned} \tag{51}$$

$$\tilde{x}_{1,t} = (1-\beta\xi_p\pi^{\epsilon})(\tilde{b}_t + (1+\eta)\tilde{y}_t + \eta\tilde{v}_t^p) + \beta\xi_p\pi^{\epsilon}\mathbb{E}_t[\tilde{x}_{1,t+1} + \epsilon\tilde{\pi}_{t+1}] \tag{52}$$

$$\tilde{v}_t^p = \frac{\epsilon\xi_p\pi^{\epsilon-1}(\pi-1)}{1-\xi_p\pi^{\epsilon-1}}\tilde{\pi}_t + \pi^{\epsilon}\xi_p\tilde{v}_{t-1}^p \tag{53}$$

$$\tilde{y}_t^n = \frac{h}{(1+\eta)g_A - h\eta}\left(\tilde{y}_{t-1}^n - \tilde{g}_{A,t}\right) \tag{54}$$

$$\begin{aligned}\tilde{r}_t =& \rho_{R,1}\tilde{r}_{t-1} + \rho_{R,2}\tilde{r}_{t-2} \\ & + (1-\rho_{R,1}-\rho_{R,2})\left(\alpha_\pi(\tilde{\pi}_t - \tilde{\pi}_t^\star) + \alpha_x\tilde{x}_t + \alpha_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1} + \tilde{g}_{A,t})\right) + \tilde{v}_t^r\end{aligned} \tag{55}$$

$$\hat{x}_t = \hat{y}_t - \hat{y}_t^n \tag{56}$$

$$\tilde{g}_{A,t} = \rho_z\tilde{g}_{A,t-1} + \epsilon_t^z \tag{57}$$

$$\tilde{b}_t = \rho_b\tilde{b}_{t-1} + \epsilon_t^b \tag{58}$$

$$\tilde{v}_t^r = \rho_r\tilde{v}_{t-1}^r + \epsilon_t^r \tag{59}$$

$$\tilde{\pi}_t^\star = \rho_\pi\tilde{\pi}_{t-1}^\star + \epsilon_t^\pi \tag{60}$$

Table 1: Prior distributions

Parameter	Domain	Density	Para(1)	Para(2)
h	[0,1)	Beta	0.7	0.1
ξ_p	[0,1)	Beta	0.66	0.1
α_π	$\mathbb{R}^+$	Normal	1.5	0.3
α_x	$\mathbb{R}^+$	Normal	0.125	0.05
$\alpha_{\Delta y}$	$\mathbb{R}^+$	Normal	0.125	0.05
$\rho_{R,1}$	$\mathbb{R}^+$	Normal	1.1	0.25
$\rho_{R,2}$	$\mathbb{R}$	Normal	0	0.25
$\bar{A}$	$\mathbb{R}$	Normal	0.4	0.2
$\bar{\pi}$	$\mathbb{R}$	Normal	0.8	0.75
$\bar{r}$	$\mathbb{R}^+$	Normal	1.2	0.4
ρ_b	[0,1)	Beta	0.5	0.2
ρ_z	[0,1)	Beta	0.5	0.2
ρ_r	[0,1)	Beta	0.5	0.2
ρ_π	[0,1)	Beta	0.5	0.2
σ_b	$\mathbb{R}^+$	InvGamma	0.5	4
σ_z	$\mathbb{R}^+$	InvGamma	0.5	4
σ_r	$\mathbb{R}^+$	InvGamma	0.5	4
σ_π	$\mathbb{R}^+$	InvGamma	0.5	4
σ_s	$\mathbb{R}^+$	InvGamma	0.5	4
α_{BN}	[0.5,1.5]	Uniform	0.5	1.5
$\rho_{b,s}$	[-1,1]	Uniform	-1	1
$\rho_{z,s}$	[-1,1]	Uniform	-1	1
$\rho_{r,s}$	[-1,1]	Uniform	-1	1
$\rho_{\pi,s}$	[-1,1]	Uniform	-1	1

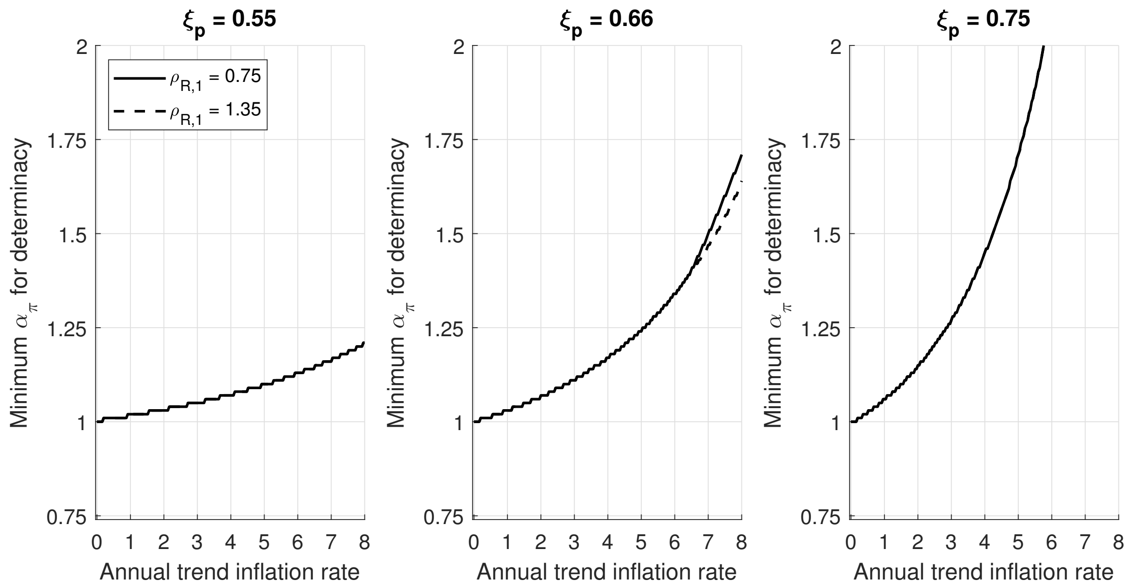
Notes: For the beta and normal densities, Para(1) and Para(2) refer to the means and standard deviations of the prior. For the Uniform densities, Para(1) and Para(2) refer to the lower and upper bounds. For the Inverse Gamma distribution, Para(1) and Para(2) refer to s and v where $p_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$.

Table 2: ESTIMATION OF SIMULATED DATA

	Indeterminacy		Determinacy	
	True	**75 observations**	**True**	**75 observations**
h	0.85	0.771 [0.707,0.838]	0.85	0.820 [0.778,0.863]
ξ_p	0.75	0.719 [0.627,0.819]	0.75	0.735 [0.684,0.781]
α_π	0.8	0.943 [0.767,1.121]	1.9	1.702 [1.462,1.945]
α_x	0.12	0.144 [0.065,0.218]	0.12	0.115 [0.035,0.184]
$\alpha_{\Delta y}$	0.18	0.127 [0.045,0.202]	0.18	0.121 [0.050,0.198]
$\rho_{R,1}$	0.90	0.902 [0.690,1.103]	0.90	0.942 [0.793,1.094]
$\rho_{R,2}$	-0.25	-0.353 [-0.488,-0.218]	-0.25	-0.376 [-0.491,-0.258]
$\bar{A}$	0.50	0.564 [0.388,0.762]	0.50	0.430 [0.225,0.624]
$\bar{\pi}$	1.00	0.740 [0.533,0.955]	1.00	0.989 [0.846,1.124]
$\bar{r}$	1.40	1.015 [0.790,1.264]	1.40	1.264 [0.990,1.539]
ρ_b	0.80	0.552 [0.239,0.862]	0.80	0.753 [0.656,0.870]
ρ_z	0.25	0.399 [0.165,0.647]	0.25	0.247 [0.100,0.388]
ρ_r	0.50	0.551 [0.236,0.623]	0.50	0.514 [0.389,0.652]
ρ_π	0.99	0.887 [0.805,0.980]	0.99	0.953 [0.920,0.988]
σ_b	1.00	0.522 [0.284,0.764]	1.00	0.829 [0.584,1.046]
σ_z	1.20	0.760 [0.474,1.090]	1.20	1.091 [0.856,1.303]
σ_r	0.30	0.311 [0.260,0.358]	0.30	0.358 [0.288,0.425]
σ_π	0.05	0.126 [0.109,0.142]	0.05	0.125 [0.109,0.141]
σ_s	0.50	0.564 [0.484,0.650]	0.50	0.579 [0.279,0.887]
α_{BN}	0.50	0.762 [0.557,0.997]	1.50	1.240 [1.003,1.443]
$\rho_{b,s}$	0.00	-0.079 [-0.811,0.613]	0.00	-0.003 [-0.647,0.667]
$\rho_{z,s}$	0.00	-0.146 [-0.439,0.133]	0.00	-0.070 [-0.719,0.583]
$\rho_{r,s}$	0.00	-0.110 [-0.490,0.285]	0.00	-0.004 [-0.688, 0.651]
$\rho_{\pi,s}$	0.00	-0.125 [-0.465,0.269]	0.00	0.042 [-0.600, 0.718]
Prob(det)		0.0000		1.0000

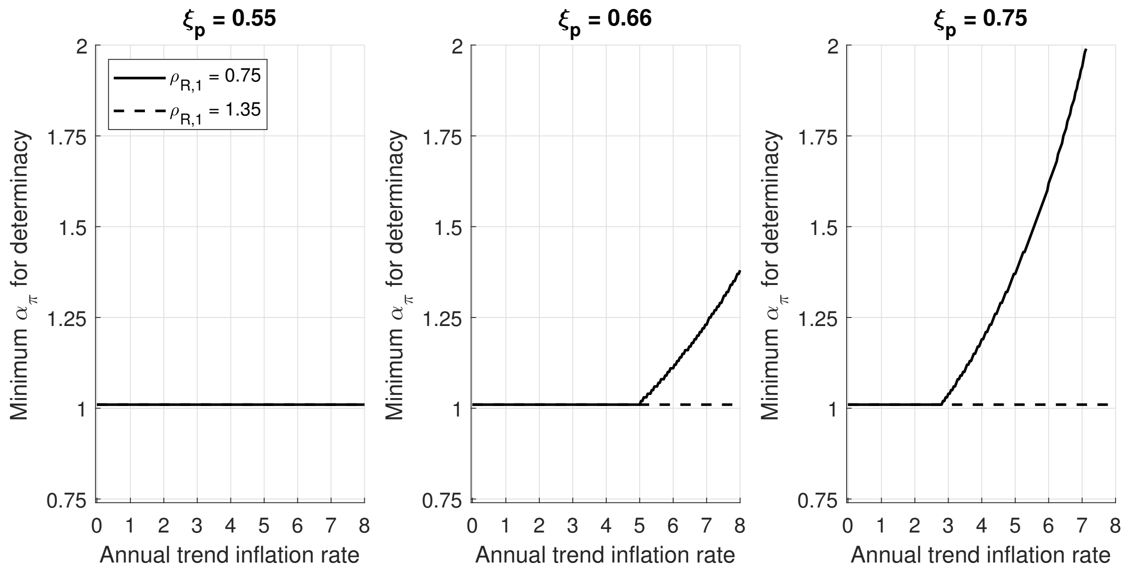
Notes: In the estimates columns, the numbers in brackets are 90% HPDI intervals. The priors used in the estimation of the simulated data are the same as the priors listed in Table 1.

Figure 1: Indeterminacy region with output gap response



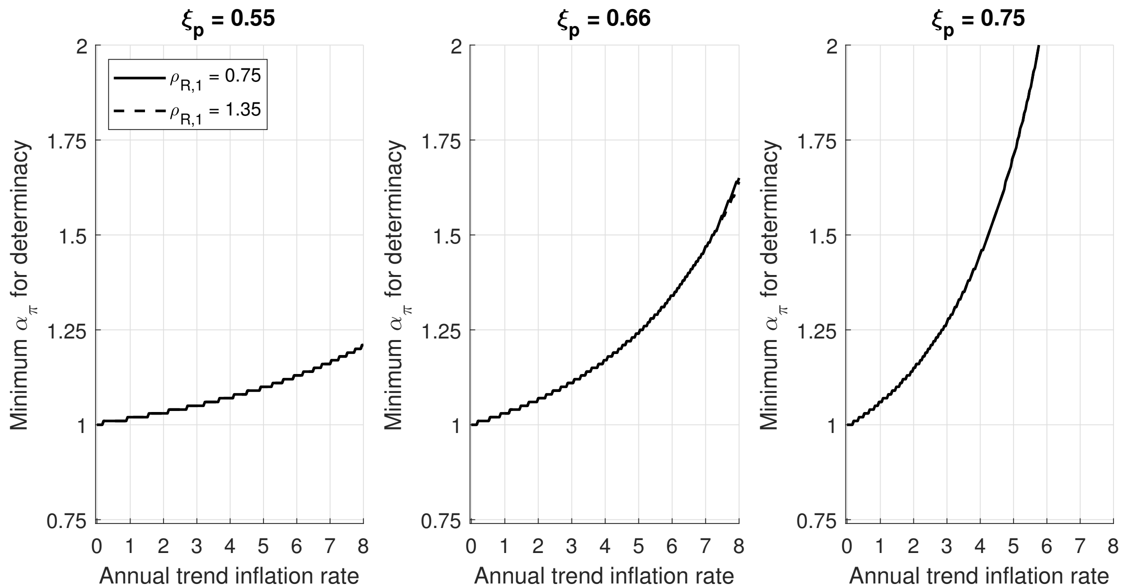
Notes: The output gap response, α_x, is 0.2 and the output growth response, $\alpha_{\Delta y}$, is 0.

Figure 2: Indeterminacy region with output growth response



Notes: The output gap response, α_x, is 0 and the output growth response, $\alpha_{\Delta y}$, is 0.2.

Figure 3: Indeterminacy region with output gap and growth response



Notes: The output gap response, α_x, is 0.2 and the output growth response, $\alpha_{\Delta y}$, is 0.2.

Table 3: AR(2) Policy Rules 1964Q1:1979Q2

	Persistent policy shocks		**White noise policy shocks**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
h	0.500	0.469	0.542	0.472
	[0.370,0.641]	[0.358,0.573]	[0.419,0.661]	[0.359,0.585]
ξ_p	0.626	0.622	0.696	0.577
	[0.521,0.711]	[0.540,0.713]	[0.526,0.852]	[0.469,0.683]
α_π	1.999	2.125	1.399	2.092
	[0.790,2.596]	[1.808,2.476]	[0.734,2.254]	[1.757,2.405]
α_x	0.119	—	0.158	—
	[0.044,0.195]	[—,—]	[0.077,0.236]	[—,—]
$\alpha_{\Delta y}$	0.148	0.135	0.136	0.130
	[0.079,0.223]	[0.064,0.208]	[0.066,0.209]	[0.053,0.202]
$\rho_{R,1}$	0.834	0.807	1.051	0.971
	[0.616,1.069]	[0.600,1.026]	[0.814,1.257]	[0.782,1.169]
$\rho_{R,2}$	-0.317	-0.338	-0.313	-0.358
	[-0.502,-0.130]	[-0.525,-0.130]	[-0.473,-0.141]	[-0.526,-0.163]
$\bar{A}$	0.442	0.451	0.417	0.445
	[0.195,0.694]	[0.203,0.692]	[0.190,0.663]	[0.189,0.680]
$\bar{\pi}$	1.068	1.043	1.148	1.049
	[0.861,1.277]	[0.832,1.259]	[0.921,1.390]	[0.851,1.253]
$\bar{r}$	1.359	1.324	1.453	1.339
	[1.117,1.596]	[1.099,1.542]	[1.260,1.654]	[1.165,1.502]
ρ_b	0.728	0.793	0.561	0.799
	[0.424,0.896]	[0.710,0.883]	[0.243,0.895]	[0.724,0.895]
ρ_z	0.185	0.160	0.402	0.171
	[0.052,0.272]	[0.062,0.249]	[0.073,0.766]	[0.077,0.265]
ρ_r	0.380	0.373	—	—
	[0.204,0.545]	[0.220,0.528]	[—,—]	[—,—]
ρ_π	0.916	0.932	0.930	0.933
	[0.865,0.971]	[0.887,0.983]	[0.890,0.974]	[0.888,0.977]
σ_b	1.226	1.310	1.303	1.473
	[0.680,1.834]	[0.922,1.695]	[0.380,1.984]	[1.070,1.924]
σ_z	1.596	1.520	1.257	1.521
	[1.083,2.177]	[1.212,1.833]	[0.508,1.807]	[1.162,1.833]
σ_r	0.366	0.369	0.269	0.314
	[0.245,0.480]	[0.273,0.474]	[0.215,0.330]	[0.249,0.381]
σ_π	0.151	0.148	0.144	0.147
	[0.127,0.173]	[0.126,0.171]	[0.122,0.164]	[0.124,0.167]
σ_s	0.513	0.571	0.458	0.604
	[0.288,0.741]	[0.275,0.865]	[0.343,0.572]	[0.280,0.926]
α_{BN}	1.195	1.262	0.936	1.251
	[0.804,1.500]	[1.052,1.495]	[0.514,1.303]	[1.028,1.470]
$\rho_{b,s}$	-0.005	-0.027	0.077	-0.014
	[-0.679,0.676]	[-0.697,0.621]	[-0.381,0.598]	[-0.695,0.633]
$\rho_{z,s}$	-0.006	0.014	-0.126	-0.011
	[-0.594,0.601]	[-0.645,0.684]	[-0.478,0.270]	[-0.719,0.622]
$\rho_{r,s}$	0.016	-0.040	-0.088	-0.006
	[-0.580,0.672]	[-0.715,0.610]	[-0.627,0.269]	[-0.633,0.685]
$\rho_{\pi,s}$	0.130	-0.013	0.402	0.007
	[-0.471,0.878]	[-0.698,0.655]	[-0.454,0.897]	[-0.697,0.651]
log p(X^T)	-93.1928	-91.7187	-95.5699	-95.5688
Prob(det)	0.8489	0.9928	0.3271	1.0000

Table 4: AR(1) Policy Rules 1964Q1:1979Q2

	Mixed rule	**Growth rule**
h	0.602	0.491
	[0.504,0.703]	[0.382,0.598]
ξ_p	0.815	0.642
	[0.739,0.902]	[0.563,0.724]
α_π	1.388	2.129
	[0.818,1.893]	[1.793,2.451]
α_x	0.162	—
	[0.104,0.228]	[—,—]
$\alpha_{\Delta y}$	0.120	0.133
	[0.050,0.191]	[0.063,0.206]
$\rho_{R,1}$	0.639	0.486
	[0.505,0.793]	[0.334,0.626]
$\rho_{R,2}$	—	—
	[—,—]	[—,—]
$\bar{A}$	0.368	0.449
	[0.115,0.636]	[0.205,0.686]
$\bar{\pi}$	1.195	1.058
	[0.932,1.487]	[0.845,1.247]
$\bar{r}$	1.410	1.354
	[1.140,1.682]	[1.141,1.560]
ρ_b	0.520	0.817
	[0.214,0.806]	[0.745,0.894]
ρ_z	0.661	0.167
	[0.267,0.946]	[0.063,0.262]
ρ_r	0.466	0.396
	[0.277,0.671]	[0.250,0.542]
ρ_π	0.945	0.929
	[0.914,0.981]	[0.882,0.980]
σ_b	1.174	1.342
	[0.238,2.637]	[0.962,1.717]
σ_z	0.874	1.600
	[0.321,1.628]	[1.248,1.927]
σ_r	0.278	0.356
	[0.224,0.333]	[0.278,0.437]
σ_π	0.145	0.148
	[0.126,0.167]	[0.126,0.171]
σ_s	0.471	0.644
	[0.359,0.559]	[0.278,1.033]
α_{BN}	0.760	1.248
	[0.535,0.972]	[1.045,1.492]
$\rho_{b,s}$	-0.009	-0.026
	[-0.444,0.498]	[-0.703,0.646]
$\rho_{z,s}$	-0.021	0.031
	[-0.330,0.288]	[-0.646,0.705]
$\rho_{r,s}$	-0.225	0.013
	[-0.465,0.018]	[-0.652,0.688]
$\rho_{\pi,s}$	0.707	0.006
	[0.557,0.868]	[-0.642,0.671]
log p(X^T)	-94.0730	-93.6336
Prob(det)	0.0101	1.0000

Table 5: AR(2) Policy Rules 1983Q1:2005Q1

	Persistent policy shocks		**White noise policy shocks**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
h	0.635	0.659	0.784	0.710
	[0.546,0.739]	[0.571,0.749]	[0.682,0.880]	[0.609,0.809]
ξ_p	0.806	0.817	0.863	0.803
	[0.769,0.843]	[0.785,0.848]	[0.835,0.902]	[0.754,0.851]
α_π	2.414	2.372	1.241	1.984
	[2.049,2.750]	[2.018,2.746]	[0.835,1.607]	[1.537,2.420]
α_x	0.085	—	0.144	—
	[0.006,0.150]	[—,—]	[0.105,0.191]	[—,—]
$\alpha_{\Delta y}$	0.166	0.173	0.158	0.203
	[0.099,0.232]	[0.106,0.240]	[0.092,0.220]	[0.130,0.281]
$\rho_{R,1}$	0.687	0.683	1.275	1.262
	[0.525,0.860]	[0.512,0.836]	[1.148,1.427]	[1.112,1.435]
$\rho_{R,2}$	-0.254	-0.263	-0.353	-0.416
	[-0.418,-0.099]	[-0.423,-0.105]	[-0.521,-0.208]	[-0.582,-0.268]
$\bar{A}$	0.525	0.482	0.269	0.480
	[0.349,0.708]	[0.283,0.677]	[-0.004,0.498]	[0.278,0.687]
$\bar{\pi}$	0.784	0.855	0.801	0.816
	[0.617,0.948]	[0.682,1.032]	[0.695,0.923]	[0.676,0.947]
$\bar{r}$	1.508	1.568	1.283	1.476
	[1.313,1.732]	[1.305,1.812]	[1.067,1.490]	[1.284,1.661]
ρ_b	0.848	0.838	0.437	0.794
	[0.784,0.916]	[0.775,0.906]	[0.270,0.567]	[0.686,0.894]
ρ_z	0.177	0.186	0.711	0.229
	[0.047,0.300]	[0.044,0.343]	[0.458,0.962]	[0.048,0.417]
ρ_r	0.767	0.771	—	—
	[0.703,0.834]	[0.708,0.835]	[—,—]	[—,—]
ρ_π	0.930	0.956	0.958	0.920
	[0.893,0.979]	[0.922,0.990]	[0.930,0.992]	[0.870,0.970]
σ_b	1.227	1.189	1.603	1.592
	[0.875,1.533]	[0.876,1.520]	[1.029,2.186]	[1.132,2.015]
σ_z	1.112	1.174	0.863	1.264
	[0.879,1.355]	[0.929,1.432]	[0.274,1.420]	[0.930,1.586]
σ_r	0.256	0.246	0.163	0.185
	[0.203,0.306]	[0.195,0.298]	[0.137,0.185]	[0.159,0.214]
σ_π	0.113	0.111	0.134	0.116
	[0.100,0.126]	[0.097,0.124]	[0.118,0.154]	[0.100,0.129]
σ_s	0.550	0.695	0.308	0.563
	[0.271,0.818]	[0.283,1.136]	[0.191,0.379]	[0.288,0.856]
α_{BN}	1.281	1.209	0.750	1.230
	[1.081,1.500]	[1.000,1.411]	[0.517,0.899]	[1.001,1.436]
$\rho_{b,s}$	0.126	-0.132	-0.055	-0.018
	[-0.485,0.746]	[-0.694,0.503]	[-0.419,0.352]	[-0.671,0.665]
$\rho_{z,s}$	-0.156	0.044	-0.027	0.089
	[-0.787,0.457]	[-0.558,0.753]	[-0.306,0.241]	[-0.578,0.713]
$\rho_{r,s}$	0.083	0.011	0.138	-0.064
	[-0.538,0.718]	[-0.670,0.671]	[-0.078,0.372]	[-0.737,0.559]
$\rho_{\pi,s}$	0.065	-0.036	0.743	0.056
	[-0.604,0.690]	[-0.669,0.632]	[0.545,0.950]	[-0.601,0.702]
log p(X^T)	61.5134	66.3211	36.2191	38.5321
Prob(det)	1.000	1.000	0.0626	1.0000

Table 6: Additional Results: 3 observables 1964Q1:1979Q2

	Mixed rule	**Growth rule**
log p(X^T)	-130.4705	-132.0124
Prob(det)	0.5089	0.4243