Monetary policy uncertainty and the cyclicality of interest rates*

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Joshua Brault†

Carleton University

& Ottawa-Carleton GSE

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Abstract

Since the onset of the Great Moderation US nominal and real interest rates underwent a significant shift in their cyclicality from inverted leading indicators to positive lagging indicators over the business cycle. In this paper I provide a structural interpretation of these facts using a New Keynesian model featuring imperfect information about the current state of the economy. Agents and the central bank solve a signal extraction problem to infer the current state from noisy signals. The model's ability to explain the changes in cyclicality rests on the fact that information frictions, which hinder accurate estimates about the economy, have become less severe since the onset of the Great Moderation. Estimates of the structural model, real-time measurement data, and Federal Reserve Greenbook forecasts support this interpretation.

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†Department of Economics, D886 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada. *E-mail:* joshua.brault@carleton.ca. Tel: +1.613.520.2600 (ext 2053).

1 Introduction

One of the central aims of business cycle analysis is to understand the comovement between macroeconomic aggregates. An important feature of comovement is leading indicators, which are variables that tend to precede movements in output over the business cycle. These indicators are an important source of information for policymakers, practitioners, and market participants since they yield information about future economic conditions without having to formulate a structural theory for why these relationships exists.

Among these indicators, a large literature has emphasized that short term nominal and real interest rates are leading indicators of the US business cycle.[1] Lower current nominal and real interest rates are strongly correlated with higher future output, a property that King and Watson (1996) refer to as the "*inverted leading indicator*" property of nominal and real interest rates.

But recently Brault and Khan (2020) have documented significant shifts in the cyclicality of US short term nominal and real interest rates since the onset of the Great Moderation. First, the inverted leading indicator property highlighted by King and Watson (1996) has completely disappeared. Instead both short term nominal and real interest rates positively lag movements in output. Second, the contemporaneous correlation between output and the real interest rate switched from mildly countercyclical to procyclical, and the contemporaneous correlation between output and the nominal interest rate also experienced a dramatic rise in its procyclicality. These changes are depicted in Figure 1, which plots the cross-correlogram between output, nominal and real interest rates.[2]

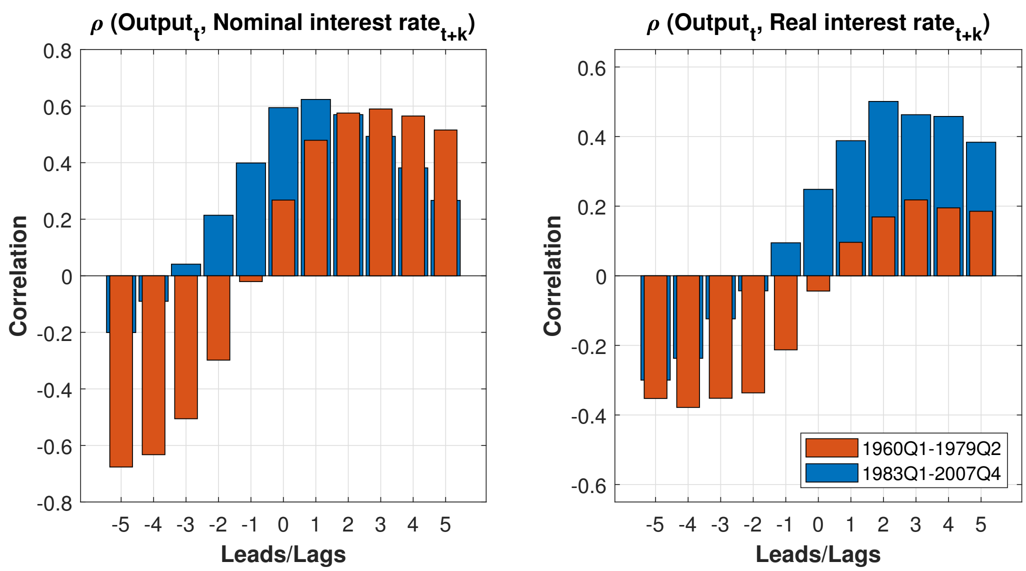
In this paper I provide a structural interpretation of these changes using a New-Keynesian model with imperfect information. In the model agents and the central bank are subject to symmetric imperfect information about the current state of the economy and must solve a signal extraction problem in the presence of noisy signals about macroeconomic aggregates. The model's ability to explain the changes in the cyclicality of nominal and real interest rates rests on the fact that uncertainty about the current state of the economy has become less severe since the onset of the Great Moderation.

Uncertainty about the current state of the economy can arise from several sources. One prominent source emphasized in a series of articles by Athanasios Orphanides (e.g., Or-

[1] I use short term to refer to the three month treasury bill rate.

[2] In this paper I use a strict statistical definition of leading and lagging indicators. A variable is considered is leading indicator if the largest absolute correlation in the cross-correlogram, $\rho(y_t, x_{t+k})$, belongs to the variable at any time for $k = [-5, -1]$.

Figure 1: CROSS-CORRELOGRAM BETWEEN OUTPUT, NOMINAL AND REAL INTEREST RATES



Notes: The real interest rate is an ex-post measure defined as the federal funds rate at time t less realized inflation (annualized) between t and $t + 1$. Output is real GDP detrended using the Hodrick-Prescott filter with a smoothing parameter, $\lambda = 1600$.

phanides (2001, 2003a,b)) is that monetary policymakers must make decisions based on real-time data, even though these data are often substantially revised in the future as more comprehensive information becomes available and methodologies improve. Real-time policy recommendations can then differ quite dramatically from what would have been recommended if policymakers had access to ex-post revised data.

A second source of uncertainty is that there is often a publication lag between the period when monetary authorities set their policy rate and the data which are available. Policymakers then need to rely on a combination of staff projections using historical data and advanced estimates from agencies such as the Bureau of Economic Analysis (BEA). For example, when FOMC members meet this year on November 4-5, only advanced estimates of third quarter GDP will be released by the BEA.

Both of the above examples suggest that in reality central banks are faced with the challenge of setting their policy rate while being uncertain about the true state of macroeconomic aggregates such as output, the output gap, and inflation. These information challenges are emphasized by former Federal Reserve's Chair Ben Bernanke,

> The measurement issues I just raised point to another important concern of policymakers, namely, the necessity of making decisions in "real time," under

conditions of great uncertainty — including uncertainty about the underlying state of the economy — and without the benefit of hindsight.[3]

But have these information challenges remained relatively constant throughout postwar US business cycles? The evidence put forth in this paper suggests that they have not and information-related issues have declined since the onset of the Great Moderation.

I begin by investigating two potential sources of information improvements. The first is evidence on real-time data from the Federal Reserve Bank of Philadelphia. This evidence suggests that real-time data has become more accurate, providing monetary authorities with better information about the current state of the economy. I construct real-time measurement errors in output growth and GDP deflator-based inflation by comparing initial releases to most recent values. I find that the standard deviation of output growth and inflation real-time measurement errors has fallen by 45% and 16% since the onset of the Great Moderation.

The second source of information improvement I investigate comes from the Federal Reserve's Greenbook. The Greenbook is produced before each FOMC meeting and contains staff projections about current and future states of the economy. The rationale for studying these projections is that even with the same real-time data, Federal Reserve employees may have improved techniques and understanding about the macroeconomy, allowing them to produce more accurate forecasts. I construct forecast errors in output growth and inflation by comparing Greenbook forecasts to the most recent values for these variables. I find that forecast errors have become significantly smaller since the onset of the Great Moderation, with output growth forecast errors falling by 45% and inflation errors by 14%.

The empirical evidence points to improvements in both real-time data and Federal Reserve forecasts. While I remain agnostic about the source of the decline in real-time measurement and forecast errors, the important point for this paper is that both sources support the interpretation that information challenges about the current state of the US economy have declined since the onset of the Great Moderation.

To understand how a reduction in information frictions matters for interest rate cyclicality, I construct a small scale New-Keynesian model where agents and the central bank have symmetric imperfect information about current state of the economy.[4] Agents and the cen-

[3]"Outstanding Issues in the Analysis of Inflation", a speech made by Ben Bernanke at the Federal Reserve Bank of Boston's 53rd Annual Economic Conference (2008). A transcript of the speech is available at https://www.federalreserve.gov/newsevents/speech/bernanke20080609a.htm.

[4]The assumption of symmetric information between agents and the central bank may appear strong given recent work by Nakamura and Steinsson (2018). The authors document a *Fed information effect* where private agent forecasts about future output growth increase in response to unexpected increases in interest rates, the

tral bank receive noisy signals about output and inflation, and must solve a signal extraction problem to formulate optimal estimates about the current state of the economy. These optimal estimates are formulated through a Kalman filter updating equation as in Svensson and Woodford (2003). After obtaining estimates about the current state of the economy, the central bank conducts monetary policy according to a Taylor rule which responds to deviations of expected inflation and the output gap. The model features six sources of exogenous variation, including: neutral technology shocks, cost-push shocks, preference (demand) shocks, monetary policy shocks, and shocks to the signals about output and inflation (which I refer to as *noise shocks*). Larger noise shocks generate more uncertainty about the current economic state.

I estimate the structural parameters of the model for two different subsamples using Bayesian maximum likelihood methods. The first subsample (1960Q1:1979Q2) corresponds to the Great Inflation period where interest rate dynamics were characterized by an inverted leading indicator property, and the second subsample corresponds to the Great Moderation period (1983Q1:2007Q4) where interest rate dynamics displayed a positive lagging property. Estimates of the structural parameters indicate a substantial fall in noise shocks. From the Great Inflation to the Great Moderation, output and inflation noise shocks fell by approximately 68% and 51%, respectively.

I emphasize three novel channels in which uncertainty matters for the cyclicality of nominal and real interest rates. Two channels which were prominent during the Great Inflation period produce inverted leading comovements between output, nominal, and real interest rates. The third channel, which was muted during the Great Inflation period and more prominent during the Great Moderation, produces positive lagging comovements between output, nominal, and real interest rates.

In the first channel I emphasize that because the central bank relies on optimal estimates about the current state of the economy to set its policy rate, a reduction in uncertainty leads policy rate responses which are closer to the policy responses implied by the Taylor rule under perfect information. That is, a reduction in uncertainty reduces undesirable movements in the policy rate. Interactions between these undesirable movements and real rigidities generate inverted leading comovements in the nominal and real interest rate, similar to monetary policy shocks in a standard macroeconomic model.

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opposite of what would be predicted by a symmetric information model (i.e., the Fed has superior information about the economy). But recently, Bauer and Swanson (2020) challenge this interpretation by showing that both the Fed and private agents are responding to news and there is little role of a Fed information effect. The latter work is supportive of a symmetric information assumption.

In the second channel I emphasize the role that a relative reduction in inflation noise shocks has played. A positive inflation noise shock leads the central bank to believe both output and inflation have risen and consequently the central bank incorrectly tightens monetary policy by raising the nominal interest rate and due to sticky prices, the real interest rate. Since actual inflation and output were initially unchanged, the rise in the real interest rate causes households to reduce consumption and output through intertemporal substitution. In the presence of real rigidities, this produces a negative leading comovement between output, nominal, and real interest rates.

In the third channel I show that a reduction in uncertainty fundamentally alters the propagation of preference shocks. A fall in uncertainty allows the central bank to more easily distinguish preference shocks from statistical noise, which leads to more aggressive policy rate responses. More aggressive policy rate responses produce stronger positive lagging comovements between output, nominal, and real interest rates.

To emphasize the strength of the proposed channels I conduct a simple counterfactual experiment where I change only the amount of uncertainty and examine the implications for the cyclicality of nominal and real interest rates. I find that by changing the amount of uncertainty, the change in the cross-correlograms closely resembles the changes in the unconditional data presented in Figure 1. In particular, both nominal and real interest rates experience a dramatic rise in their procyclicality and the real interest rate switches from an inverted leading indicator to a positive lagging indicator. The magnitude of the cross-correlations is comparable with those in the unconditional data.

In terms of practical relevance, my paper clarifies that short term nominal and real interest rates should not be considered leading indicators of future economic conditions when policy is conducted in close accordance with a Taylor rule based on ex-post revised data.[5] Additionally, the paper provides a structural rationale for why nominal and real interest rates displayed an inverted leading indicator property during the Great Inflation period. This was primarily because policy implementation was challenging due to significant misunderstandings about the current state of the economy. The necessity of setting the policy rate in real time suggests that information frictions will never completely disappear, and by extension, even in periods of good monetary policy short term nominal and real interest rates can be expected to positively lag output over the business cycle.

[5] For a recent example emphasizing real interest rates as an inverted leading indicator, see the following video from Vox's Centre for Economic Policy Research (CEPR): https://voxeu.org/content/predicting-real-interest-rate.

Shocks versus structure: One alternative explanation that readers may find compelling is that this change is driven primarily by a shift in the relative importance of technology, aggregate demand, and monetary policy shocks. For reasons which I discuss below this does not seem to be the entire story.

First, a large literature has documented a rise in the relative importance of neutral technology shocks since the onset of the Great Moderation (e.g., Galí and Gambetti (2009), Barnichon (2010)); A result that is consistent with my structural estimates. Mertens (2010) argues that changes in the cyclicality of the real interest rate between the Great Inflation and the Great Moderation can mostly be attributed to this fact.

But this interpretation is problematic when one also considers the changes in the cyclicality of the nominal interest rate. Neutral technology shocks generate a negative comovement between output and the nominal interest rate (both empirically and in a New-Keynesian model). Then a rise in the relative importance of technology shocks would suggest an increasingly negative comovement between output and the nominal interest rate. However this is exactly opposite to the observed rise in procyclicality in the data. The mechanism in the present paper can account for the observed rise in procyclicality of both the nominal and real interest rate.

Second, Brault and Khan (2020) take the Smets and Wouters (2007) (SW) model and estimate the model on pre-1984 and post-1984 data. This model features a wide array of structural shocks, real and nominal frictions intended to provide an accurate representation of the data. But the model is unable to generate the change in comovement between output, nominal and real interest rates exhibited by the data even though the relative importance of exogenous shocks changes substantially. Since model features in SW are embedded in many contemporary models, this suggests that standard models will be unable to capture this change.

Related literature: This paper is broadly related to two strands of literature. One strand seeks to explain why nominal (or real) interest rates lead the business cycle. King and Watson (1996) document the inverted leading indicator property of nominal and real interest rates in the unconditional data.[6] Additionally they investigate the ability of a real business

[6]Other works documenting this property for either real or nominal interest rates include Zarnowitz (1988), Bernanke and Blinder (1992), Fiorito and Kollintzas (1994), Chari, Christiano and Eichenbaum (1995), Beaudry and Guay (1996), Stock and Watson (1999a), Boldrin, Christiano and Fisher (2001), Dotsey, Lantz and Scholl (2003), and Mertens (2010).

cycle model, a sticky price model, and a liquidity effect model to account for these facts. They show that while each model can capture one aspect, none can account for all of the facts they document. Beaudry and Guay (1996) show that a modified RBC model with habit formation and capital adjustment costs significantly improves the model's ability to explain the comovement between output and real interest rates. However this improvement depends largely on comparing with the comovement induced by identified technology shocks and not the unconditional data. Boldrin, Christiano and Fisher (2001) present a two-sector RBC model with habit formation and limited factor mobility and show that real interest rates are inversely correlated with future output. Lastly, Pintus, Wen and Xing (2017) show that a model featuring collateral constraints in the style of Kiyotaki-Moore with redistribution shocks can account for the inverted leading indicator property of real interest rates.

Relative to this strand of literature this paper is the first, to my knowledge, to provide a comprehensive explanation of why nominal and real interest rates featured an inverted leading indicator property during the Great Inflation and a positive lagging property during the Great Moderation.

A second related strand of literature explores imperfect information as a propagation mechanism and source of endogenous persistence in DSGE models. Lippi and Neri (2007) estimate a small scale DSGE model with imperfect information and discretionary monetary policy for the euro area. They find that under information frictions the weight on output gap stabilization is small and that observations on unit labour costs contain important information on potential output. Collard, Dellas and Smets (2009) estimate a small scale New-Keynesian DSGE model with a variety of imperfect information structures and find that information frictions provide a plausible mechanism for inertia in variables such as inflation and increase the model's fit relative to a full information structure. Collard and Dellas (2010) consider imperfect information about monetary shocks and show that the model can exhibit inflation inertia without ad hoc model features such as price indexation. Neri and Ropele (2011) estimate a small scale NK model using real-time data in the euro area and compare to a model with perfect information. They find that the estimated policy rule is more inertial and less aggressive towards inflation. Additionally, they assess the output gap inflation tradeoff facing the ECB. Givens and Salemi (2015) estimate an NK model with optimal discretionary policy using real-time and ex-post US data for the 1965-2010 period. They find that the Federal Reserve increased its concern for stabilizing the output gap after 1979 and that the tension between optimal and observed policy is smaller during this period.

Relative to this literature, I focus on a particular set of moments — nominal and real interest rate cyclicality — and whether changes in the degree of uncertainty can explain the changes in cyclicality since the onset of the Great Moderation.

The rest of the paper is organized as follows. Section 2 explores two plausible sources of information improvements and shows that both point to improvements in understanding about real-time macroeconomic conditions. Section 3 describes the DSGE model, including how the model is solved and estimated under the assumption of symmetric imperfect information. Section 4 describes the three channels in which a reduction in uncertainty can help to explain the change in the cyclicality of the nominal and real interest rates. Section 5 contains concluding remarks.

2 Empirical investigation into noise

In the following section I explore two potential sources in which monetary policy authorities may have better information about the current state of the economy: Better real-time information and improved forecasting/signal extraction methods from the Fed.

2.1 Real-time measurement error

One source of uncertainty in the implementation of monetary policy is that the information available (i.e., *real-time data*) is often substantially revised in the future as more comprehensive information becomes available. Then a naive policymaker who sets the policy rate based on this information may do so incorrectly (according to a Taylor rule) due to the presence of measurement error in real-time data.

But given the advances in technology and data processing over the past several decades it is reasonable to believe that such advances could lead to more accurate real-time data, and by extension a reduction in uncertainty on behalf of policymakers. However, as Arouba (2008) notes, it is also quite possible that the rise in technology has made real-time data collection more challenging since there has been a large increase in the variety of goods.

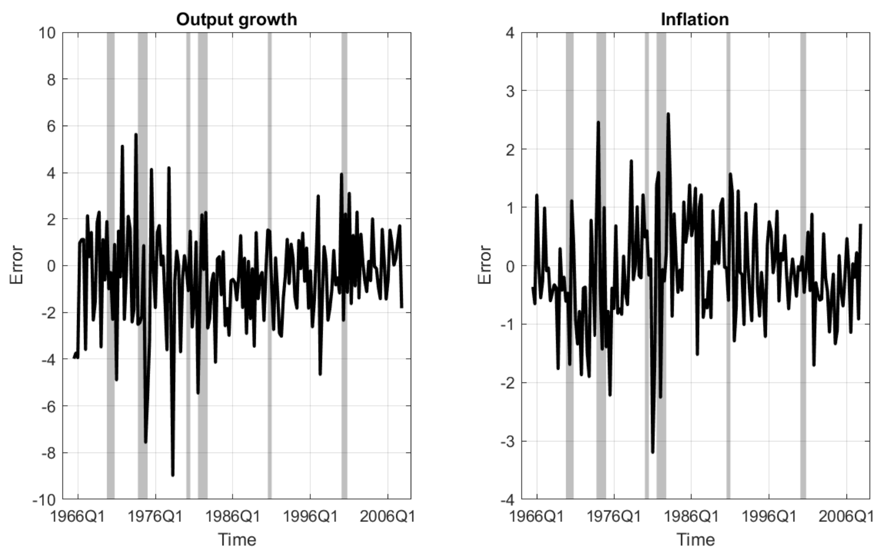
To investigate if real-time data measurement has become more accurate since the onset of the Great Moderation, I construct a series of real-time measurement errors for output growth and inflation using data from the Federal Reserve Bank of Philadelphia's Real-Time Data Set for Macroeconomists (more information on the data is available in Appendix D). Measurement errors are constructed by comparing first releases of output growth and inflation to their most recent values.[7] Specifically, measurement errors are defined as,

$$\text{Measurement Error}_{i,t} = \text{First Release}_{i,t} - \text{Most Recent}_{i,t}, \tag{2.1}$$

for $i \in \{\text{Output growth}, \text{Inflation}\}$. Equation (2.1) says that positive errors correspond to periods where initial estimates of output growth and inflation overstate the true values. In this case positive errors could lead the central bank to increase their policy rate more than would be required by their Taylor rule.

Figure 2 plots real-time measurement errors for quarter-over-quarter output growth and inflation from 1965Q3:2007Q4 (in annualized percentage points).[8]

Figure 2: REAL-TIME MEASUREMENT ERROR IN OUTPUT GROWTH AND INFLATION



Notes: Output growth and inflation errors are reported in quarter over quarter rates in annualized percentage points. The inflation measure corresponds to prices in the GDP deflator. Gray bands indicate recessions as defined by the NBER Business Cycle Dating Committee.

It is worth remarking on two facts about Figure 2. First, the size of measurement errors in output growth and inflation are quantitatively large. The figure implies that at points in the sample, initial reports of output growth differed from the true value by as much as

[7] Since I am only examining data up to 2007Q4, treating the most recent observations as the true value seems reasonable given that most of the revisions have already taken place.

[8] The measurement error series starts in 1965Q3 due to data limitations.

8% in annualized terms (approximately 2% in quarterly growth rates). Inflation errors are smaller, but still quantitatively large with errors being as large as 0.75% in quarterly terms. To put these errors in perspective, average output growth and inflation in the ex-post revised data over this period was 3.27% and 4.01% in annualized terms. This implies that there were periods where initial releases of output growth and inflation were substantially different than their final values.

Second, it is apparent from the figure that there is a substantial difference in the magnitude of measurement errors when comparing the Great Inflation period to the Great Moderation. Large errors are visually noticeable in both output growth and inflation during the Great Inflation period, but have since declined quite dramatically. How large the declines? Table 1 reports the standard deviation of measurement errors for output growth and inflation in the pre-1979 and post-1983 periods.

Table 1: STANDARD DEVIATION OF OUTPUT GROWTH AND INFLATION REAL-TIME MEASUREMENT ERRORS

Moment $\downarrow$ / **Time** $\rightarrow$	1965Q3:1979Q2	1983Q1:2007Q4
$\sigma(\Delta y^{ME})$	2.89	1.58
$\sigma(\pi^{ME})$	0.93	0.78

Notes: Output growth and inflation are quarter-over-quarter growth rates in annualized terms.

It is evident from the table that measurement errors in output growth are largest, and have also exhibited the largest decline. The measurement errors for output growth have declined by about 45%, while measurement errors for inflation have declined by about 16%. This suggests that real-time data has become more accurate and points to one potential source of reduction in uncertainty for policymakers.

2.2 Fed Greenbook forecasts

A second distinct source of reduction in uncertainty may come from the Federal Reserve itself through a variety of channels. First, Assumption 1 in the structural model assumes that agents and the central bank are aware of the structural equations, parameters values, and shock processes governing the economy. This is clearly a strong assumption, and the more likely case is that both agents and the central bank have learned more about structure and dynamics over time.

A second channel in which the Fed could have reduced its uncertainty is through its information set itself. In the structural model which will presented in this paper it is assumed that only output and inflation are observable with some error. In reality there is a wide array of information available, some of which may provide more accurate information on variables such as output and inflation. An example showing information set expansion as a useful tool in reducing uncertainty is presented in Nimark (2008), who adds a bond market with different maturity bonds into a small scale New-Keynesian model. Since bond yields are observed at a much higher frequency, if the bond market is not too noisy, then bond returns can provide important real-time information to the central bank about macroeconomic variables and structural shocks.

Lastly, it is quite possible that the Fed has a better understanding about what drives measurement errors in the first place. For example, Arouba (2008) shows that real-time measurement errors can be forecast using other real-time variables. Amir-Ahmadi, Matthes and Wang (2017) use a Bayesian VAR with sign restrictions to examine the impact of monetary policy shocks on real-time and ex-post measures of real activity. They find that impulse responses to a monetary shock can be significantly different when one compares IRFs based on real-time versus ex-post data, something that policymakers should be aware of. Both of the above suggest that the Fed may have a better understanding of where measurement errors comes from and how to extract information from noisy signals.

To consider the above possibilities I examine forecasts about the current state of the economy from the Federal Reserve Greenbook.[9] The Greenbook contains internal forecasts from Federal Reserve staff about past, current, and future macroeconomic variables. I focus on forecasts about the current state of output growth and inflation using the last forecast in a given quarter about that specific quarter (e.g., the forecast of output growth in September for the third quarter of that year) and compare those forecasts to most recent values.[10]

Similar to the construction of real-time measurement errors, I define forecast errors by the following equation,

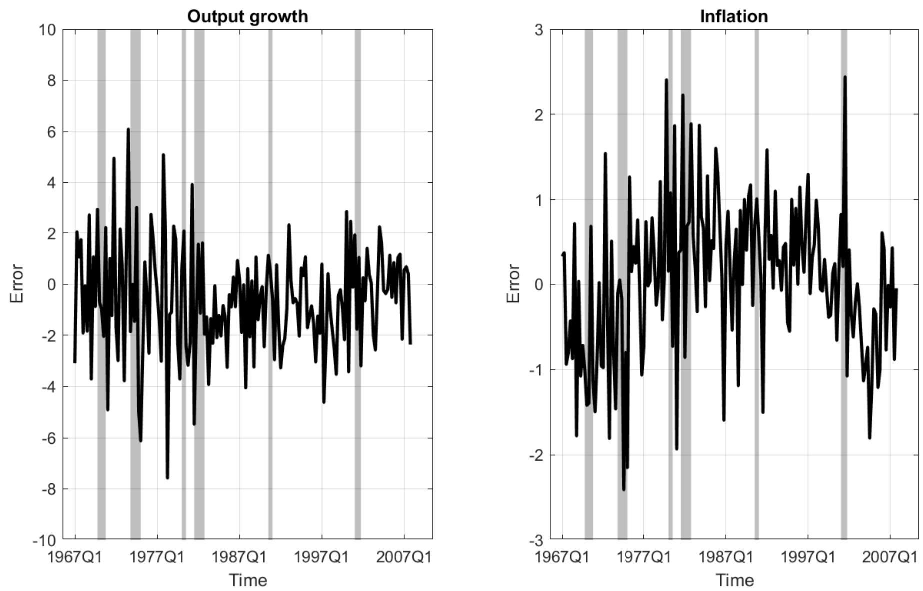
$$\text{Forecast Error}_{i,t} = \text{Greenbook forecast}_{i,t} - \text{Most Recent}_{i,t}, \tag{2.2}$$

[9]Due to data limitations, I only examine forecasts from 1967Q1 onwards.

[10]I also examined forecasts from the beginning and middle of each quarter. There is no significant difference between the timing of the forecast within a quarter and the conclusions I draw about the quality of forecasts before and after the Great Moderation.

for $i \in \{\text{Output growth}, \text{Inflation}\}$. Equation (2.2) states that when the Fed projections are overly optimistic about output growth and inflation, errors will be positive and potentially lead to unnecessary tightening of monetary policy. These forecast errors are pictured in Figure 3.

Figure 3: GREENBOOK FORECAST ERRORS IN OUTPUT GROWTH AND INFLATION



Notes: Output growth and inflation errors are reported in quarter over quarter rates in annualized percentage points. The inflation measure corresponds to prices in the GDP deflator. Gray bands indicate recessions as defined by the NBER Business Cycle Dating Committee.

Similar to the measurement errors in the previous section, forecasts errors from the Federal Reserve's Greenbook are quantitatively large and appear to decline since the onset of the Great Moderation, substantially so in the case of output growth. I report the standard deviation of these forecast errors for output growth and inflation in Table 2.

Table 2: STANDARD DEVIATION OF GREENBOOK FORECAST ERRORS IN OUTPUT GROWTH AND INFLATION

Moment $\downarrow$ / **Time** $\rightarrow$	1967Q1:1979Q2	1983Q1:2007Q4
$\sigma(\Delta y^{FE})$	2.85	1.57
$\sigma(\pi^{FE})$	0.90	0.77

Notes: Output growth and inflation are quarter-over-quarter growth rates in annualized terms.

Interestingly, the level and decline in Fed forecasts errors almost identically matches the level and decline from the real-time data. Output growth forecast errors have declined by 45% and inflation forecast errors by 14% since the onset of the Great Moderation.

The above suggests that both real-time data and Fed forecasts have become more accurate since the onset of the Great Moderation period, and by extension, monetary policymakers should face lower levels of uncertainty about the current state of the economy. In the following section I estimate a structural model where agents and the central bank face imperfect information due to noise shocks. I estimate the magnitude of noise in output and inflation in the model, but the magnitudes of measurement errors and forecast errors in this section serve as useful comparisons.

3 Model, Solution, and Estimation

3.1 Model

The model in this section is a small scale New-Keynesian model comprised of a continuum of intermediate goods firms who operate in a monopolistically competitive environment. Intermediate goods firms produce output with labour as the only input and are subject to sticky prices via the Calvo mechanism. Final goods firms package intermediate goods into a final aggregate good which are sold in a perfectly competitive environment. Households optimize over consumption, labour supply, and one period riskless bonds. The model is closed by a monetary policy authority who sets nominal interest rates in response to deviations of inflation and the output gap with interest rate smoothing.

Since many features of this model are relatively standard (e.g., Galí (2008, Ch. 3)), I simply report the log-linearized equations of the model and leave the derivation to Appendix B. I use $\tilde{x}$ to denote a variable x that has been log-linearized around its steady state. The model structure is given by,

$$\tilde{y}_t = \frac{h}{1+h}\tilde{y}_{t-1} + \frac{1}{1+h}\tilde{y}_{t+1|t} - \frac{1-h}{1+h}(\tilde{i}_t - \tilde{\pi}_{t+1|t}) + \frac{1-h}{1+h}\tilde{v}^y_t(1-\rho_y), \tag{3.1}$$

$$\tilde{\pi}_t = \kappa_1(\tilde{y}_t - \tilde{y}^F_t) + \kappa_2(\tilde{y}_{t-1} - \tilde{y}^F_{t-1}) + \beta E_t\tilde{\pi}_{t+1} + \tilde{v}^\pi_t, \tag{3.2}$$

$$\tilde{i}_t = \rho_i\tilde{i}_{t-1} + (1-\rho_i)\{\alpha_\pi\tilde{\pi}_{t|t} + \alpha_y(\tilde{y}_{t|t} - \tilde{y}^F_{t|t})\} + \tilde{v}^m_t, \tag{3.3}$$

where $\tilde{y}$ is output, $\tilde{i}$ is the nominal interest rate, and $\tilde{\pi}$ is the inflation rate. $\tilde{v}^y$, $\tilde{v}^\pi$, and $\tilde{v}^m$

are structural shocks to demand, inflation, and the monetary policy rule. Equation (3.1) is the dynamic IS equation, (3.2) is the New Keynesian Phillips curve, and (3.3) is a Taylor-type rule which allows for interest rate smoothing. The nominal interest rate and inflation are reported in percentage point deviation from steady state (i.e., $x_t - \bar{x}$). All remaining variables are reported in percentage deviation from steady state (i.e., $\frac{x_t - \bar{x}}{\bar{x}}$).

The reduced form parameters are given by: $\kappa_1 = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{1+\eta(1-\varphi)(1-h)}{1-h}$ and $\kappa_2 = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{h}{1-h}$. β is the household discount factor, θ is the Calvo probability of a firm being able to reset its price in a given period, h is the external habit formation parameter, η is the inverse Frisch elasticity of labour supply, and φ is the fraction of fixed costs in total production. The natural rate of output, the level of output that would prevail in the absence of sticky prices, is given by,

$$\tilde{y}_t^F = \frac{(1-h)(1+\eta)}{1+\eta(1-\varphi)(1-h)}\tilde{a}_t + \frac{h}{1+\eta(1-\varphi)(1-h)}\tilde{y}_{t-1}^F, \tag{3.4}$$

where $\tilde{y}_t^F$ denotes output in the flexible price environment and a_t is a neutral technology shock. This shock, along with the structural shocks in (3.1), (3.2), and (3.3), are governed by the following processes,

$$\log a_t = \rho_a \log a_{t-1} + (1-\rho_a)\log(\bar{a}) + \epsilon_t^a \qquad \epsilon_t^a \sim N(0, \sigma_a^2), \tag{3.5}$$

$$\log v_t^y = \rho_y \log v_{t-1}^y + \epsilon_t^y \qquad \epsilon_t^y \sim N(0, \sigma_y^2), \tag{3.6}$$

$$\log v_t^\pi = \rho_\pi \log v_{t-1}^\pi + \epsilon_t^\pi \qquad \epsilon_t^\pi \sim N(0, \sigma_\pi^2), \tag{3.7}$$

$$\log v_t^m = \epsilon_t^m \qquad \epsilon_t^m \sim N(0, \sigma_m^2). \tag{3.8}$$

There are two points worth mentioning about the model setup above. First, expectations about future output and inflation are consistent with rational expectations in the sense that forecast errors are unpredictable. However the expectations will differ from a standard full information rational expectations model since agents imperfectly observe the true state of the economy and use these imperfect observations in their formation of expectations about future output and inflation. In an attempt to differentiate this from standard notation, the expectations are denoted $x_{t+1|t}$ for $x \in \{\tilde{y}, \tilde{\pi}\}$.

Second, since estimates of the current state are an input variable into the central bank's feedback rule, these estimates will be affected by the imperfect information. I have denoted limited information of these variables by $x_{t|t}$ for $x \in \{\tilde{y}, \tilde{\pi}, \tilde{y}^F\}$. In the following subsection I discuss how these estimates of the state are formed and what information is assumed to be known.

3.2 Solution

In the following subsection I describe the solution the model in the presence of imperfect information. This solution method closely follows Svensson and Woodford (2003). The log-linearized model can be represented in compact matrix form by,

$$\begin{bmatrix} \mathbf{X}_{1,t+1} \\ E_t\mathbf{X}_{2,t+1} \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix} + \mathbf{A}_2 \begin{bmatrix} \mathbf{X}_{1,t|t} \\ \mathbf{X}_{2,t|t} \end{bmatrix} + \mathbf{C}\boldsymbol{\epsilon}_t, \tag{3.9}$$

where $\mathbf{X}_1$ is comprised of predetermined and exogenous variables, $\mathbf{X}_2$ contains the forward looking variables (output and inflation), and $\boldsymbol{\epsilon}$ is a vector of structural shocks. The matrices $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{C}$ contain non-linear functions of the structural parameters.

The primary difference between a standard full information New-Keynesian model and the model presented here is that agents and the central bank cannot perfectly observe all variables in the model. Instead they only observe a subset of these variables with noise and lag, and must solve a signal extraction problem to infer their true values. Agent's and the central bank's observation equation is given by,

$$\mathbf{Z}_t = \begin{bmatrix} \tilde{\pi}_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{v}_t^{\eta_\pi} \\ \tilde{v}_t^{\eta_y} \end{bmatrix}, \tag{3.10}$$

where $\tilde{v}_t^{\eta_\pi}$ and $\tilde{v}_t^{\eta_y}$ are noise shocks preventing the central bank from perfectly observing lags of output and inflation.[11] Equation (3.10) implies that agents and the central bank do not observe structural shocks or the natural rate of output. However since there is symmetric information, agents are aware of the central bank's interest rate, and by extension any monetary policy shocks. The noise shocks are allowed to be potentially serially correlated and their process is given by,

[11] A compromise is made in terms of inflation observation timing. The model is based on a quarterly frequency, but inflation data are reported at a monthly frequency and thus a one quarter lag is too long.

$$\ln v_t^{\eta\pi} = \rho_{\eta\pi} \ln v_{t-1}^{\eta\pi} + \epsilon_t^{\eta\pi} \qquad \epsilon_t^{\eta\pi} \sim N(0, \sigma^2_{\eta\pi}), \tag{3.11}$$

$$\ln v_t^{\eta y} = \rho_{\eta_y} \ln v_{t-1}^{\eta y} + \epsilon_t^{\eta y} \qquad \epsilon_t^{\eta y} \sim N(0, \sigma^2_{\eta_y}). \tag{3.12}$$

It is assumed that there is no correlation between the noise shocks and underlying structural shocks. This structure gives rise to Assumption 1.

Assumption (A1). The information set of agents and the central bank at any time t is composed of $I_t \equiv \{\mathbf{A}_1, \mathbf{A}_2, \mathbf{C}, \mathbf{D}_1, \mathbf{\Sigma}_{\epsilon\epsilon}, \mathbf{\Sigma}_{vv}, \mathbf{Z}_{t-s} | s \geq 0\}$.

Assumption 1 states that agents and the central bank know the structural equations of the model and the corresponding values of the structural parameters. Additionally, it is assumed that they know the distributions of the exogenous shocks (both structural shocks and noise shocks). Lastly, it is assumed that they have access to the full history of observables, $\mathbf{Z}_{t-s}$.

Under this setup it is well known that the solution to the rational expectations equilibrium is independent of the computation of the state (i.e., the signal extraction problem). This is commonly referred to as the *separation principle* (e.g., Pearlman (1992), Svensson and Woodford (2003)). Then one can solve for the linear mapping between the predetermined/exogenous variables and the forward looking variables using standard techniques.[12] This mapping is given by,

$$\mathbf{X}_{2,t|t} = \mathbf{G}^\star \mathbf{X}_{1,t|t}. \tag{3.13}$$

Given the linear mapping in (3.13), it is straightforward to cast the model in state space form and use the one-sided Kalman filter to compute the expected values of the state variables. The dynamics of the model are then represented by the following set of equations,

[12] In this paper I use a simple iterative fixed point algorithm. Alternatively, one could also solve for the matrix $\mathbf{G}^\star$ using the Blanchard-Kahn method.

$$\mathbf{X}_{t+1} = \mathbf{H}\mathbf{X}_t + \mathbf{J}\mathbf{X}_{t|t} + \mathbf{B}\epsilon_{t+1}, \tag{3.14}$$

$$\mathbf{X}_{t|t} = \mathbf{X}_{t|t-1} + \mathbf{K}(\mathbf{Z}_t - \mathbf{Z}_{t|t-1}), \tag{3.15}$$

$$\mathbf{Z}_t = \mathbf{L}\mathbf{X}_t + \mathbf{M}\mathbf{X}_{t|t} + \mathbf{v}_t, \tag{3.16}$$

where,

$$\mathbf{H} = \mathbf{A}_{11}^1 - \mathbf{A}_{12}^1(\mathbf{A}_{22}^1)^{-1}\mathbf{A}_{21}^1, \tag{3.17}$$

$$\mathbf{J} = \mathbf{A}_{12}^1\Big[(\mathbf{A}_{22}^1)^{-1}\mathbf{A}_{21}^1 + \mathbf{G}^\star\Big] + \mathbf{A}_{11}^2 + \mathbf{A}_{12}^2\mathbf{G}^\star, \tag{3.18}$$

$$\mathbf{L} = \mathbf{D}_1^1 - \mathbf{D}_2^1(\mathbf{A}_{22}^1)^{-1}\mathbf{A}_{21}^1, \tag{3.19}$$

$$\mathbf{M} = \mathbf{D}_2^1\Big[(\mathbf{A}_{22}^1)^{-1}\mathbf{A}_{21}^1 + \mathbf{G}^\star\Big], \tag{3.20}$$

$$\mathbf{K} = \mathbf{P}\mathbf{L}^{'}\left(\mathbf{L}\mathbf{P}\mathbf{L}^{'} + \mathbf{\Sigma}_v^2\right)^{-1}, \tag{3.21}$$

where the subscripts on the matrix $\mathbf{A}$ refer to the number of rows or columns associated with the predetermined or forward looking variables (e.g., $\mathbf{A}_{11}$ is the first n_1 rows and columns of $\mathbf{A}$, where $n_1 = 11$, the number of predetermined and exogenous variables) and $\mathbf{P}$ is a covariance matrix of prediction errors. In Appendix C I report the matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{C}$ which are used in the solution, as well as additional details on the solution of the model.

There are a few points worth emphasizing about the above model dynamics relative to a standard full information rational expectations DSGE model. First, Equation (3.14) highlights that the signal extraction problem endogenously feeds back into the dynamics of the model through estimates of the current state of the economy. Second, these estimates will also impact the expectations channels in the dynamic IS and NKPC equations through Equation (3.13). As the literature on imperfect information has documented (e.g., Collard et al. (2009)), these properties generate rich dynamics between the signal extraction problem and the impact of exogenous shocks.

One convenient feature of this model setup is that it nests the perfect information case when $\mathbf{X}_{t|t} = \mathbf{X}_t$. In this case the standard state space representation can be obtained by adding matrices $\mathbf{H}$ and $\mathbf{J}$ together, and adding $\mathbf{L}$ and $\mathbf{M}$ together.

3.3 Estimation

The model in Section 3.1 is estimated using US quarterly data on output, inflation, and the nominal interest rate. I estimate the model for two different periods. The first is a subsample corresponding to the period 1960Q1:1979Q2. The rationale for estimating the model on this subsample is that nominal and real interest rates displayed an inverted leading indicator property during this period. The second subsample corresponds to the period 1983Q1:2007Q4. A similar rationale exists for this sample, nominal and real interest rates displayed a positive lagging property during this period.

Output is measured as real per capita GDP, inflation as the log difference in the GDP deflator, and the nominal interest rate is the average quarterly federal funds rate. Since I do not model any non-stationary factors in the structural model, I detrend real GDP using a one-sided Hodrick-Prescott filter. Inflation and the federal funds rate are converted to quarterly units. I provide more information on the construction of the data in Appendix D.

The vector of observables corresponds to the following in the model,

$$\begin{bmatrix} 100 \times \pi_t \\ 100 \times Y_t^{1hp} \\ 100 \times i_t \end{bmatrix} = \begin{bmatrix} \bar{\pi} \\ 0 \\ \bar{i} \end{bmatrix} + \begin{bmatrix} \pi_t \\ y_t \\ i_t \end{bmatrix}, \tag{3.22}$$

where $\bar{\pi} = 100(\pi - 1)$ and $\bar{i} = 100(\frac{1}{\beta} - 1)$.

I use the Kalman filter to evaluate the log-likelihood of the model and use the first 4 observations to initialize the Kalman filter. After obtaining a posterior mode using a simulated annealing algorithm, I estimate the posterior density functions of the structural parameters using the random walk Metropolis-Hastings algorithm with 5,000,000 draws. I drop the first 10% of draws to eliminate any issues associated with initial conditions.

Prior to estimation I fixed the inverse Frisch elasticity of labour supply parameter, η, to 1. My rationale for fixing this parameter was motivated by previous estimation attempts in which I found the parameter to be poorly identified. Posterior estimates had large standard deviations and no clear peak. The lack of identification likely stems from no inclusion of a wage observable, which is a standard observable used in medium scale DSGE models (e.g., Smets and Wouters (2007)).

The remaining structural parameters are estimated. The vector of estimated structural parameters is given by $\Theta = \{\beta, h, \pi, \varphi, \theta, \alpha_\pi, \alpha_y, \rho_a, \rho_y, \rho_v, \rho_i, \rho_{\eta_\pi}, \rho_{\eta_y}, \sigma_a, \sigma_v, \sigma_y, \sigma_m, \sigma_{\eta_\pi}, \sigma_{\eta_y}\}$.

Prior distributions can be found in Table 3.

Table 3: PRIOR AND POSTERIOR DISTRIBUTIONS OF STRUCTURAL PARAMETERS AND SHOCK PROCESSES

	Prior			**Posterior**			
				1960Q1:1979Q2		**1983Q1:2007Q4**	
	Distribution	Mean	SD	Mean	SD	Mean	SD
β	Beta	0.99	0.01	0.9859	0.0014	0.9879	0.0014
h	Beta	0.20	0.10	0.1858	0.0908	0.2615	0.0754
π	Normal	0.90	0.25	1.0023	0.1809	0.6799	0.1480
φ	Beta	0.25	0.125	0.3108	0.1454	0.3411	0.1498
θ	Beta	0.66	0.05	0.7940	0.0257	0.8303	0.0225
α_π	Normal	1.50	0.25	1.4976	0.2066	1.8484	0.1920
α_y	Normal	0.125	0.05	0.1388	0.0459	0.1400	0.0423
ρ_a	Beta	0.60	0.20	0.6123	0.1890	0.6011	0.2118
ρ_π	Beta	0.60	0.20	0.8054	0.0571	0.8666	0.0495
ρ_y	Beta	0.60	0.20	0.6020	0.0855	0.8215	0.0354
ρ_i	Beta	0.60	0.20	0.5301	0.0905	0.6661	0.0572
ρ_{η_π}	Beta	0.60	0.20	0.6962	0.1798	0.7421	0.1621
ρ_{η_y}	Beta	0.60	0.20	0.4102	0.1724	0.4128	0.1726
σ_a	IG	0.50	4.00	0.1544	0.1593	0.3923	0.3810
σ_π	IG	0.50	4.00	0.1127	0.0229	0.0261	0.0053
σ_y	IG	0.50	4.00	6.7836	2.4763	3.9642	1.1019
σ_m	IG	0.50	4.00	0.0302	0.0100	0.0103	0.0022
σ_{η_π}	IG	0.50	4.00	1.6691	0.9060	0.8125	0.4405
σ_{η_y}	IG	0.50	4.00	0.8163	0.5329	0.1699	0.1101

Notes: Posterior means and standard deviations are computed using the random walk Metropolis-Hastings algorithm with 5 million draws. IG in the prior distribution column represents an inverse gamma distribution. The acceptance ratio for the RWMH algorithm in the pre-1979 and post-1983 estimations was 0.2017 and 0.1554.

I assign prior distributions which are commonly used in the literature. For the household discount factor I use a beta distribution with a mean of 0.99 and standard deviation of 0.01. This prior can be justified based on the average real interest rate throughout the postwar period. For fixed costs, I use a beta distribution with a mean of 0.25 and standard deviation of 0.125. For the Calvo parameter I use a beta distribution with a mean of 0.66 and a standard deviation of 0.05 which is consistent with the microeconomic evidence on price adjustments. For the Taylor rule response parameters, I use a normal distributions with a mean of 1.5 and standard deviation of 0.25 for the inflation response parameter and a mean of 0.125 and standard deviation of 0.05 for the output gap response parameter. Since I have relatively little information on size and persistence of the noise shock parameters, and how this propagation mechanism will impact other exogenous disturbances, I use relatively neutral priors for persistence parameters with a mean of 0.6 and a standard deviation of 0.2. For the shock processes I assign inverse gamma distributions with a mean of 0.5 and standard deviation of 4.

Table 3 reports the estimated posterior means and standard deviations of the structural parameters for the two samples. Most of the parameter estimates fall within the range of those found in other studies. However it is worth emphasizing a few parameter estimates which are relevant for this study.

First, estimates of the amount of noise in output and inflation are sizable, and consistent with the interpretation that uncertainty about the state of the economy has become less severe since the onset of the Great Moderation. Both output and inflation noise shocks have declined significantly. The estimated standard deviation of inflation noise shocks declines by roughly 51% and output noise shocks by roughly 79%. As I discuss in the next section, the decline in noise shocks matters not only for the distribution of structural shocks, but also for the propagation of other shocks in the model.

Second, evidence from papers such as Clarida, Galí and Gertler (2000) and Lubik and Schorfheide (2004) suggests that the Federal Reserve likely did not satisfy a Taylor principle in the Great Inflation period which opened the possibility of a role for self-fulfilling expectations. In this paper I find an estimate for α_π in the Great Inflation period which is significantly above one (1.49) and rules out any possibility of indeterminacy.

The rationale for this result is that the necessary condition for determinacy under imperfect information is much weaker than the perfect information case. Determinacy in a perfect information setup requires the monetary policy authority to adjust the nominal interest rate greater than one for one to movements in inflation (that is, $\alpha_\pi > 1$ with respect to π_t) in a zero steady state inflation environment. But under an imperfect information setup, determinacy only requires that the monetary policy authority adjust the nominal interest rate greater than one for one to movements in its expectation of current inflation. Since inflation is noisy, the central bank's expectation of current inflation is much less responsive than actual inflation since it cannot perfectly distinguish inflation from noise. Then movements in the nominal interest rate appear much more sensitive inflation since it is based on expected inflation and this leads to a much larger Taylor rule coefficient (that is, $\alpha_\pi > 1$ with respect to $\pi_{t|t}$).

4 Results and discussion

In the following section I focus on three channels in which changes in the amount of uncertainty about the state of the economy impacts the cyclicality of nominal and real interest rates. The first result is derived analytically while the second and third are shown through a series of impulse response functions.

Additionally, I discuss about the role (or lack thereof) of technology shocks in this setup. Typically in estimated small scale New-Keynesian models, technology shocks play a significant role in output fluctuations. For example, Ireland (2004) finds that technology shocks account for roughly one quarter of output fluctuations in a post-WWII sample and about 40% in a post-1980 sample. However in this setup technology shocks are second-order in terms of driving movements in output, inflation, and the nominal interest rate. I discuss why this is the case in a model with imperfect information.

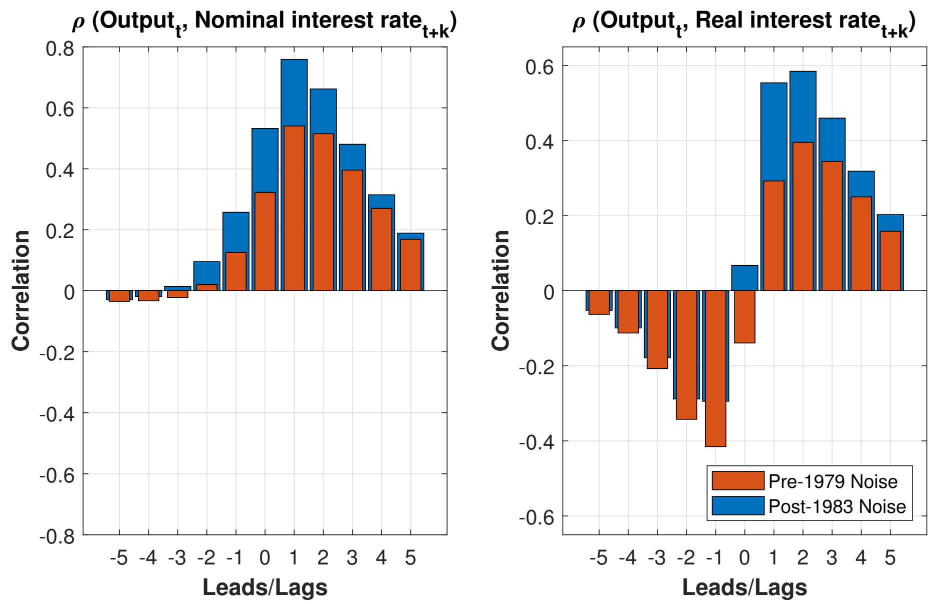
But prior to discussing the channels in which uncertainty impacts the cyclicality of nominal and real interest rates, a natural litmus test is to see whether the change in uncertainty about the current state can explain a substantial share of the observed changes in the cyclicality of nominal and real interest rates.

To address this question I run the following counterfactual scenario. First, I simulate the model for a large number of periods ($t = 20,000$) using the estimated structural parameters and shock processes from the pre-1979 sample estimation with the magnitude of uncertainty (σ_{η_π} and σ_{η_y}) equal to the pre-1979 levels. I compute the correlograms between output, nominal and real interest rates. Then in a counterfactual experiment I again simulate the model using the same structural parameters and shock processes, but changing only the degree of uncertainty to the estimated values in the post-1983 sample. That is, only changing σ_{η_π} and σ_{η_y}. Again I compute the correlograms between output, nominal and real interest rates. The results of this experiment are depicted in Figure 4.

Figure 4 highlights that the degree of uncertainty plays a significant role in the cyclicality of both the nominal and real interest rate. When the degree of uncertainty equals the amount in the pre-1979 sample, real interest rates are an inverted leading indicator of output with the largest correlation being $\rho(y_t, r_{t-1}) = -0.41$. When the degree of uncertainty is reduced, real interest rates become a positive lagging indicator with the largest correlation becoming $\rho(y_t, r_{t+2}) = 0.58$. The model also matches a mildly countercyclical real interest rate in the pre-1979 period and a modestly procyclical real rate in the counterfactual.

The nominal interest rate also displays changes consistent with the observed changes

Figure 4: UNCERTAINTY AND THE CYCLICALITY OF INTEREST RATES COUNTERFACTUAL



Notes: The counterfactual uses structural parameter estimates from the pre-1979 estimation. I then change only σ_{η_π} and σ_{η_y} to match the estimates in post-1983 period.

in its unconditional cyclicality in the data. While the model misses the inverted leading comovement in the pre-1979 period (likely to the almost zero contribution of technology shocks), a change in uncertainty produces a stronger positive lagging indicator in the correlogram and a rise in the procyclicality of the nominal interest rate — both of which are features of the change in unconditional cyclicality.

In the following sections I take a closer inspection of exactly how changes in uncertainty alter the comovements of nominal and real interest rates.

4.1 Uncertainty and the cyclicality of the nominal and real interest rate

4.1.1 Undesirable policy rate movements

The first channel in which uncertainty impacts the cyclicality of nominal and real interest rates is through the monetary policy feedback rule. Since the rule relies directly on estimates about the current state of the economy, when the level of uncertainty is high, inferring the true state of the economy becomes more difficult. To see the implications of this, note that the interest rate rule can be written as a function of the expected predetermined and forward looking variables in the following way,

$$i_t = \mathbf{F}_1 \mathbf{X}_{1,t|t} + \mathbf{F}_2 \mathbf{X}_{2,t|t}, \tag{4.1}$$

where I have, for the moment, abstracted from monetary policy shocks since they do not alter this illustration. The matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ are comprised of the monetary policy structural parameters ρ_i, α_π and α_y. Using (3.13) and (3.15), the feedback rule can be rewritten in the following manner,

$$i_t = (\mathbf{F}_1 + \mathbf{F}_2 \mathbf{G}^\star)\mathbf{X}_{1,t|t-1} + (\mathbf{F}_1 + \mathbf{F}_2 \mathbf{G}^\star)\mathbf{K}(\mathbf{Z}_t - \mathbf{Z}_{t|t-1}). \tag{4.2}$$

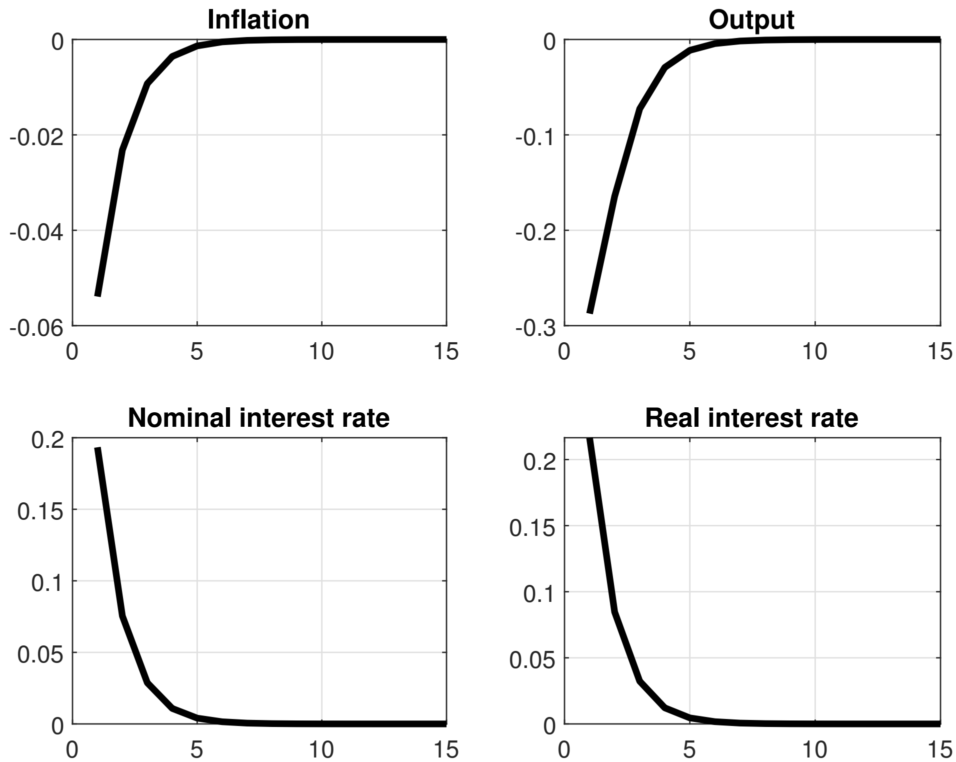
The first part of (4.2) states that the monetary policy authority sets the nominal interest rate as a function of the expected value of predetermined and exogenous variables conditional on information in $t-1$. But the more relevant part of the equation with respect to this paper is the second term. This implies that the policy rate is impacted by the difference between observables (output and inflation) at any time t and their expected values conditional on information in the previous period, $t-1$.

As emphasized by Neri and Ropele (2011), in essence, this generates undesirable movements in the policy rate and by extension fluctuations in the economy. When uncertainty is higher these conditional expectations will increasingly be at odds with actual output and inflation and fluctuations due to undesirable movements in the policy rate will be more prominent.

What are the properties of these undesirable movements and what do they imply for the cyclicality of nominal and real interest rates in the model? They propagate similar to how exogenous movements in the policy rate would in a perfect information model. An exogenous rise in the policy rate in the presence of sticky prices leads to a rise in the real interest rate, though less than one for one. Households, through the intertemporal substitution channel, reduce consumption and by definition a fall in output occurs. Additionally, the interaction of the shock and real rigidities in the form of habit formation produce movements in nominal and real interest rates that are inverted and leading (the largest correlation is with nominal and real rates at $t-1$, albeit this is hard to see in Figure 5).

Since errors in the feedback rule due to imperfect information do not directly correspond to an exogenous shock, I simulate this type of error using a standard monetary policy shock. Figure 5 plots the IRFs of inflation, output, the nominal and ex-post real interest rate to an exogenous increase in the policy rate by a magnitude of 0.25%.

Figure 5: MONETARY POLICY SHOCK



Notes: The IRF in this figure uses the estimated structural parameters from the pre-1979 estimation. The real interest rate reported here is the ex-post real interest rate, consistent with the focus in the empirical section of the paper.

Since the Great Inflation period is characterized by a higher degree of uncertainty (based on higher estimated values of σ_{η_π} and σ_{η_y}), these types of errors are more prominent. All else equal, an increase in these types of errors has two implications for the cyclicality of nominal and real interest rates: First, since these undesirable policy rate movements generate negative comovement between output, nominal and real interest rates, nominal and real interest rates should appear less procyclical/ more countercyclical over the business cycle; Second, due to the interaction of the undesirable policy rate movements and habit formation, nominal and real interest rates should appear to more often negatively lead output movements.

Both of the above features are consistent with the unconditional data in the pre-1979 period. During this period, nominal and real interest rates were more countercyclical and featured inverted leading properties.

4.1.2 Inflation noise shocks

The second channel in which uncertainty impacts the cyclicality of nominal and real interest rates is through shocks to the signals about inflation (i.e., an inflation *noise shock*).[13] Even though agents and the central bank are cautious in response to signals they receive about output and inflation, noise shocks still generate movements in their beliefs about the current state of the economy. The extent of this movement is determined by the level of uncertainty.

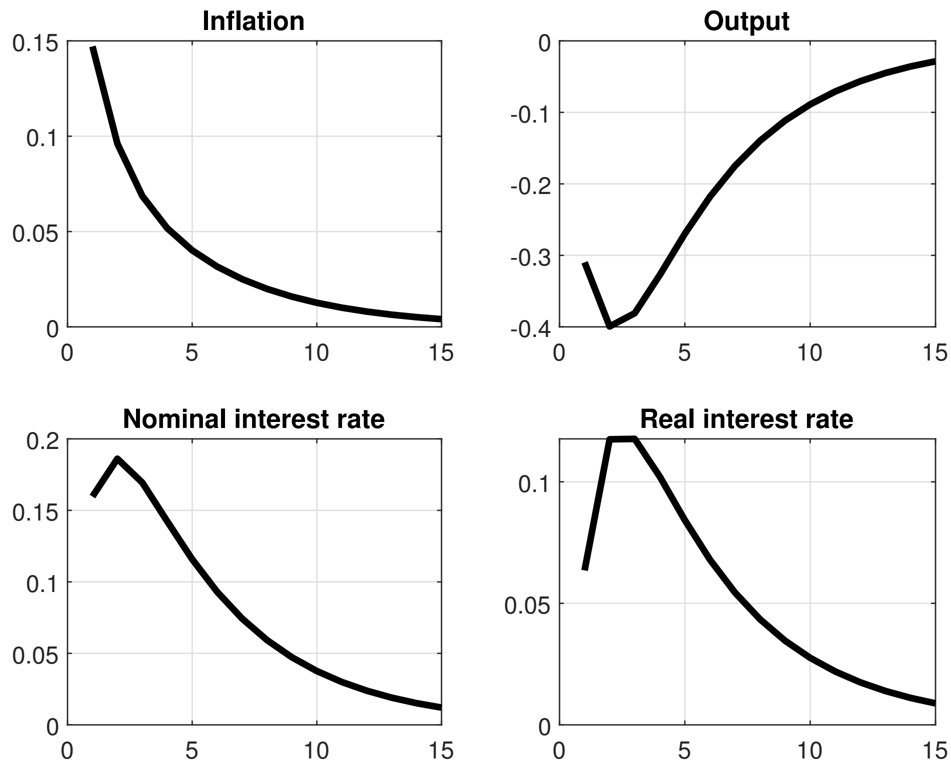
Consider a positive noise shock to inflation, that is, agents and the central bank receive a signal that output is higher than it actually is. Naturally, agents and the central bank will interpret at least part of the signal as reflecting that inflation is actually higher. Additionally, since the structure of the economy is known (Assumption 1), a perceived rise in inflation also leads to a perceived rise in the output gap. The rise in expected inflation and the output gap leads the central bank to raise the policy rate in an effort to lean against the perceived rise in inflation and the output gap. But since actual inflation and output were unchanged, this rise in the nominal interest rate produces a rise in the real rate and a contraction in output. Figure 6 plots the IRF for inflation, output, nominal and ex-post real interest rate to an inflation noise shock.

With respect to the cyclicality of nominal and real interest rates, inflation noise shocks produce countercyclical movements and because of habit formation, these countercyclical movements lead movements in output. This implies that the larger the magnitude of inflation noise shocks, the more nominal and real interest rates should exhibit cyclicality consistent with inverted leading indicators. Inflation noise shocks were especially prominent during the Great Inflation period, with the estimated standard deviation being more than twice as large as the estimated standard deviation during the Great Moderation period. This is consistent with the unconditional data presented in the introduction.

It is worth emphasizing a key difference between the IRF generated by an inflation noise shock and those generated by errors in the policy rate discussed in 4.1.1. This difference comes from the response of inflation itself. While there is a contraction in output induced by the monetary policy response, inflation actually rises, in contrast to the positive comovement

[13] The section omits a discussion of output noise shocks for two reasons: First, inflation noise shocks are much more prominent in terms of driving output and nominal interest rate fluctuations compared to output noise shocks; Second, output noise shocks generate positive comovement between output, nominal, and real interest rates, suggesting that they cannot explain inverted leading properties of nominal and real interest rates in the pre-1979 period.

Figure 6: INFLATION NOISE SHOCK



Notes: The IRF in this figure uses the estimated structural parameters from the pre-1979 estimation. The real interest rate reported here is the ex-post real interest rate, consistent with the focus in the empirical section of the paper. The impulse response function is computed using a one standard deviation inflation noise shock.

between output and inflation in the policy rate errors channel. The rationale for this difference is due to the impact of noise on expectations about future inflation, which rise enough to more than offset the downward pressure on inflation through the NKPC.

Another important feature to highlight with respect to reduced uncertainty is that conditional on the same magnitude of inflation noise shock, a reduction in uncertainty will lead to a larger impact on inflation, output, nominal and real interest rates. The rationale for this result is that as uncertainty falls, agents and the central bank perceive movements in inflation as increasingly reflecting fundamentals and not noise, leading to larger changes in expected values of inflation, output, and the output gap.

4.1.3 Demand shocks

The third channel in which uncertainty impacts the cyclicality of nominal and real interest rates is through the propagation of preference shocks (i.e, demand shocks).

In a full information setup a positive preference shock yields an increase in demand for current consumption (and by definition current output). Since the flexible price level of output remains unchanged, an output increase generates a positive output gap and a rise in inflation. In response to a rise in inflation and a positive output gap, the Taylor rule implies a rise in the nominal interest rate and, because of sticky prices, a less than one for one rise in the real interest rate. In terms of comovement, a preference shock generates strong positive comovement between output, nominal and real interest rates.

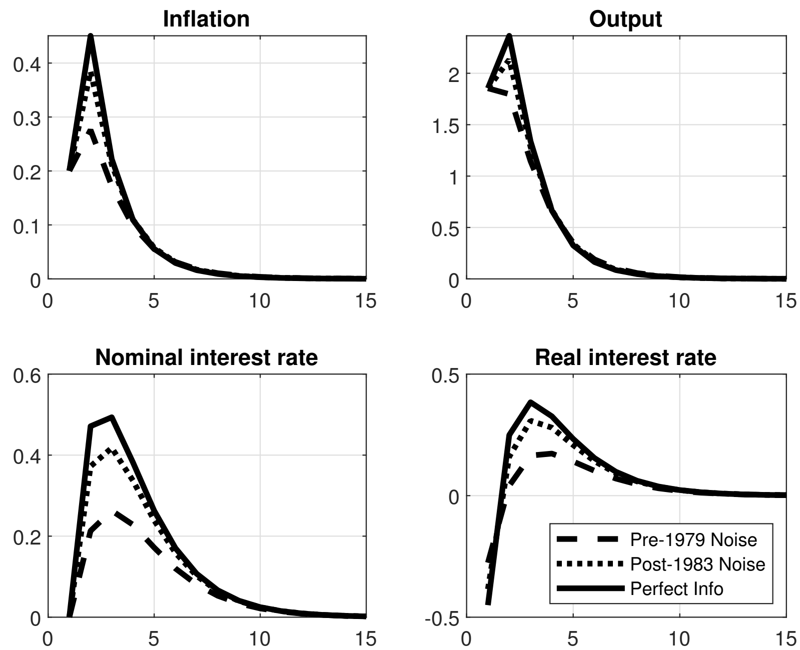
Under the imperfect information setup, the level of uncertainty fundamentally alters the propagation of preference shocks. Specifically, conditional on a positive preference shock nominal and real interest rates rise less than they would under a full information scenario. The rationale for this change is that when uncertainty about the current state of the economy is higher, agents and the central bank have more difficulty distinguishing the preference shock from statistical noise. As a result, expectations about the current state of the economy react much more cautiously, leading the central bank to raise the policy rate less than they would if they could perfectly observe the economy. This cautious response decreases the procyclicality of both the nominal and real interest rates and due to the presence of noise, these rates lag output (i.e., increased uncertainty pushes down positive lagging comovements of both rates).

To illustrate this channel, Figure 7 reports IRFs of inflation, output, nominal and ex-post real interest rates in response to a one standard deviation positive preference shock for three levels of uncertainty. The first level corresponds to the level of uncertainty in the pre-1979 sample, the second to the level of uncertainty in the post-1983 sample and the last to the full information case, where the central bank can perfectly observe output and inflation.[14]

The figure highlights that the response of nominal and real interest rates is disproportionately pushed downwards as the level of uncertainty rises. Consequently, the magnitude of the positive comovement between output, nominal and real interest rates is reduced. For example, in the case of perfect information the nominal interest rate peak response occurs three quarters after the impact of the shock with a rise in the nominal rate of 0.4935. Compared to the case of post-1983 noise, where the peak response occurs three periods after the shock

[14] It is important to note that even in the perfect information case I maintain a one period lag on observables which explains why the nominal interest rate does not respond until the second period in the IRF.

Figure 7: UNCERTAINTY AND DEMAND SHOCKS



Notes: Impulse response functions are generated using the structural parameters from the 1965Q3:1979Q2 sample. The real interest rate measure is the ex-post real interest rate, consistent with the focus in the empirical section of the paper. The IRF is in response to a one standard deviation preference shock.

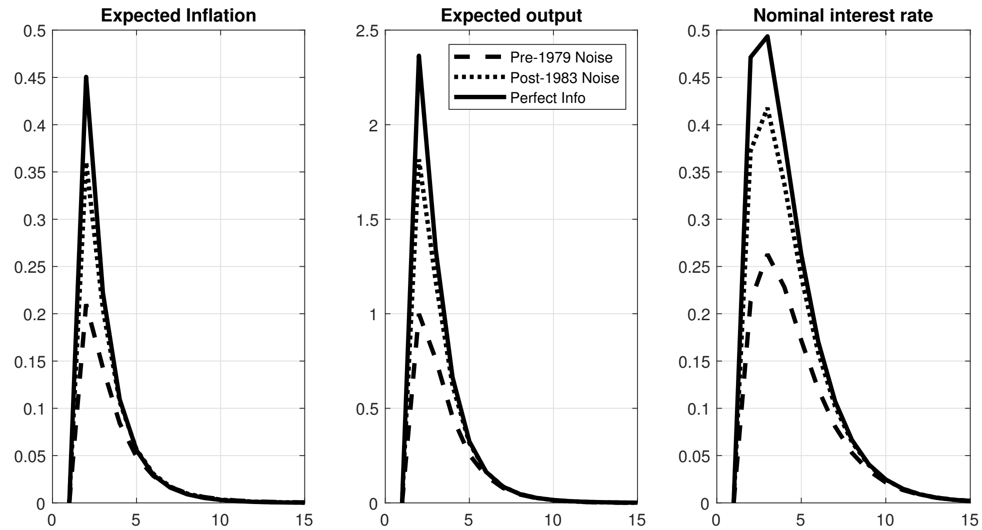
with a rise in the nominal rate equal to 0.4178. Finally, in the case of pre-1979 noise the peak response again occurs three periods after the shock with a response equal to 0.2630.

To illustrate why the peak response of the nominal interest rate is much lower when uncertainty rises, Figure 8 plots the perceived state (i.e., expectations) of current output and inflation and the associated policy rate response.

The left and middle subfigures depict the central banks expectations about the current state, that is $\pi_{t|t}$ and $y_{t|t}$. The figure highlights that even a moderate amount of uncertainty reduces the perceived boom in output and inflation substantially. Comparing the case of perfect information to the amount of uncertainty in the Great Inflation period, expectations of the peak inflation and output response fall by 53% and 57%, respectively.

To quantify the impact of varying levels of uncertainty on the contemporaneous comovement between output, nominal, and ex-post real interest rates, Table 4 reports the contemporaneous correlation between these variables conditioned on the preference shock in the figure. That is, $\rho(y_t, i_t|\epsilon_t^y)$ and $\rho(y_t, r_t|\epsilon_t^y)$.

Figure 8: UNCERTAINTY AND EXPECTATIONS OF THE STATE OF THE ECONOMY



Notes: Impulse response functions are generated using the structural parameters from the 1965Q3:1979Q2 sample. The real interest rate measure is the ex-post real interest rate, consistent with the focus in the empirical section of the paper. The IRF is in response to a one standard deviation preference shock.

Table 4: CONTEMPORANEOUS COMOVEMENTS CONDITIONAL ON A PREFERENCE SHOCK

Moment	**Pre-1979 Noise**	**Post-1983 Noise**	**Perfect Information**
Corr(y_t, i_t)	0.58	0.65	0.69
Corr(y_t, r_t)	-0.18	0.04	0.12

Notes: y_t is output, i_t is the nominal interest rate, and r_t is the ex-post real interest rate.

The conditional correlations align with the intuition in the provided in the IRFs. When uncertainty is higher, the procyclicality of nominal and real interest rates is suppressed. It is important to highlight that if uncertainty is sufficiently high than it is possible for the conditional comovement between output and the ex-post real interest rate to be negative since this comovement is dominated by movements in actual inflation, as opposed to movements in the policy rate.

The above discussion leads to the following proposition.

Proposition 1. *Let $\sigma_{\eta_\pi} \to \infty$ and $\sigma_{\eta_y} \to \infty$. Conditional on a demand shock output and nominal interest rates are uncorrelated. That is, $Corr(Y_t, i_t) \to 0$ and $Corr(Y_t, r_t)$ is entirely determined by the comovement between output and the negative of one period ahead inflation.*

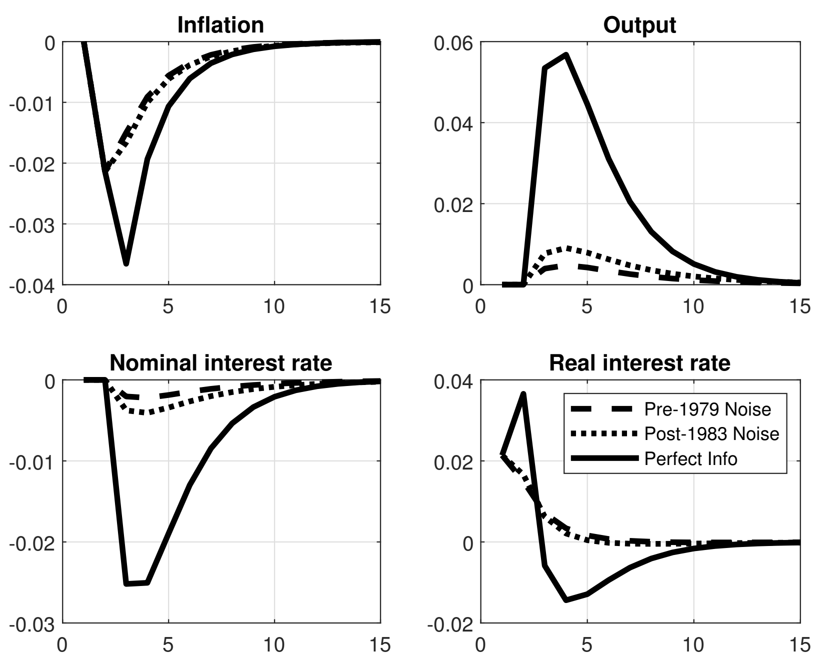
Proof is in Appendix A.

Proposition 1 establishes that the level of uncertainty alters the contemporaneous co-movement output, nominal, and real interest rates in response to preference shocks. Additionally, since all of the peak responses in nominal and real rates occur after the peak response in output, uncertainty also depresses positive lagging correlations of nominal and real interest rates. Since the Great Inflation period is characterized by a higher degree of uncertainty, this channel implies that nominal and real interest rates should appear less procyclical/ more countercyclical and less procyclical for positive lags of these variables. Both of which are features consistent with the correlogram in Figure 1.

4.2 Technology shocks

The channels in which uncertainty impacted the cyclicality of nominal and real interest rates was void of any discussion about technology shocks, I discuss the reasons for this here.

Figure 9: UNCERTAINTY AND TECHNOLOGY SHOCKS



Notes: IRF is in response to a two standard deviation positive technology shock. I use the estimated structural parameters from the 1965Q3:1979Q2 sample.

Typically technology shocks play a non-negligible role in generating fluctuations in estimated small scale New-Keynesian models. But in the current setup, estimation results suggest technology shocks play almost no role. The rationale for this result is due to a breakdown in the usual propagation mechanism of technology shocks when imperfect information is introduced.

In a full information model, a positive shock to technology reduces firm's marginal costs and raises the natural rate of output. The consequences of this is a fall in inflation due to negative output gap. In an attempt to stabilize inflation the central bank lowers nominal interest rates greater than one for one to inflation, ultimately causing a decline in the real interest rate. Through the intertemporal substitution channel, households increase current consumption and output.

Under imperfect information this channel is disrupted by the fact that inflation is only imperfectly observed. Then a positive technology shock which leads to a decline in inflation only generates a small decline in the nominal interest rate as the central bank cannot distinguish the technology shock from noise. Since the nominal interest rate (and real rate) exhibit a smaller decline, the intertemporal substitution channel is largely muted and the associated output response is small. To illustrate this, Figure 9 displays the response of inflation, output, nominal and real interest rates to a two standard deviation technology shock under different levels of uncertainty.

The quantitatively small effects of technology shocks I find lines up well with others estimating this class of models (see, e.g., (Collard, Dellas and Smets 2009, Figure 5) or (Givens and Salemi 2015, Table 5)).

5 Conclusion

In this paper I have proposed a structural interpretation to explain changes in the cyclicality of short term nominal and real interest rates since the onset of the Great Moderation period. This structural explanation hinges on uncertainty about the current state of the economy becoming less severe since the onset of the Great Moderation.

When uncertainty declines, the cyclicality of nominal and real interest rates is impacted by three primary channels. First, a decline in uncertainty leads to a better understanding about the current state of the economy and allows monetary policy authorities to set their policy rate in much closer accordance with rule outcomes based on ex-post revised data. When real-time policy rates differ from what ex-post outcomes would suggest, the difference results in macroeconomic fluctuations similar to monetary policy shocks. These fluctuations feature inverted leading properties for nominal and real interest rates.

Second, shocks to signals about inflation (what I refer to in the paper as *noise shocks*), lead to policy responses which attempt to lean against perceived inflation booms. But since the origin of the boom is purely noise, policy actions generate macroeconomic fluctuations characterized where nominal and real interest rates are characterized by inverted leading properties. These fluctuations are distinct from the first channel since inflation noise generates strong impacts on expectations about future inflation. This leads to positive comovements between nominal interest rates and inflation (contrary to the negative comovement between these variables in the first channel).

Third, uncertainty leads the central bank to react increasingly cautious to demand shocks since monetary policy authorities cannot distinguish the demand shock from statistical noise. This depresses the procyclicality of current and future nominal and real interest rates (i.e., the lagging cross-correlations). This effect can be sufficiently strong that real interest rates are negatively correlated with output in response to a demand shock.

Taken together, these channels imply that the Great Inflation, which was characterized by a larger degree of uncertainty, should exhibit comovements in nominal and real interest rates most consistent with channels one and two. That is, rates should appear more countercylcical and leading. The shift in the Great Moderation should place more emphasis on channel three, that is, rates should positively lag output. This is precisely what the unconditional data show, suggesting that the above mechanism is a promising explanation for the change in the cyclicality of nominal and real interest rates.

Empirical evidence on real-time macroeconomic data releases and Federal Reserve forecasts are both consistent with the notion that monetary policy authorities have a better understanding of current macroeconomic conditions than in the past. Real-time data errors and Federal Reserve forecast errors of output growth and inflation have fallen substantially since the onset of the Great Moderation, supporting the proposed explanation herein.

The implications of the paper are to reaffirm that nominal and real interest rates are not leading indicators of the business cycle and should not be treated as such. The Great Inflation period in the US featured a leading indicator property because implementation of monetary policy was challenging due to difficultly understanding current macro aggregates. Well functioning monetary policy (that is, in close accordance with a rule used here under perfect information) will lead to nominal and real interest rates lagging the business cycle.

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A Proofs of Propositions

Proposition 1

Let $\mathbf{K}^{'}$ be a submatrix of the Kalman gain associated with predetermined variables and $\mathbf{K}^{''}$ a submatrix of the Kalmain gain associated with exogenous variables (i.e., shocks). As $\sigma_{\eta_\pi} \rightarrow \infty$ and $\sigma_{\eta_y} \rightarrow \infty$, $\mathbf{K}^{'} \rightarrow 0$. Let $\mathbf{X}^{'}_{t|t}$ be the time t expectation of predetermined variables. Then in response to a demand shock from the steady state (i.e., $\mathbf{X}_0 = \mathbf{0}$), it is straightforward to show that predetermined variables evolve according to,

$$\mathbf{X}^{'}_{t|t} = \mathbf{X}^{'}_{t|t-1} + \mathbf{K}^{'}(\mathbf{Z}_t - \mathbf{Z}_{t|t-1}). \tag{A.1}$$

Since $\mathbf{X}_{1|0} = \mathbf{0}$ and $\mathbf{K}^{'} = \mathbf{0}$, it is straightforward to iterate equation (A.1) to show that $\mathbf{X}^{'}_{t|t} = 0$ for all t. Next, note that the rational expectations equilibria (REE) solution is independent of the noise shocks. Then it can be shown that,

$$\mathbf{X}_{2,t|t} = \mathbf{G}^{\star}\mathbf{X}_{1,t|t} = \mathbf{G}^{'\star}\mathbf{X}^{'}_{1,t|t}. \tag{A.2}$$

Then it is straightforward to show by substituting (A.1) into (A.2) that $\mathbf{X}_{2,t|t} = \mathbf{0}$ for all t. The monetary policy rule can be represented as a function of expected predetermined variables and current values of output and inflation (this representation was used in (4.1)),

$$i_t = \mathbf{F}_1\mathbf{X}_{1,t|t} + \mathbf{F}_2\mathbf{X}_{2,t|t}. \tag{A.3}$$

It has been shown above that both $\mathbf{X}_{1,t|t}$ and $\mathbf{X}_{2,t|t}$ are null matrices for all t, then the nominal interest rate is equal to zero for all t in response to a demand shock. Since the nominal rate is zero for all t, the $Cov(y_t, i_t) = 0$ and the associated conditional correlation $Corr(y_t, i_t) = 0$. Additionally, since the ex-post real interest rate is given by $r_t = i_t - \pi_{t+1}$, a fixed nominal interest rate implies that real interest rate is entirely determined by $-\pi_{t+1}$. A positive demand shock introduces a positive output gap and increases inflation. Then the conditional correlation between output and the real interest rate is determined entirely by $Corr(y_t, -\pi_{t+1})$ which belongs to interval $(0, -1]$. $\square$

B Derivation of the economic model

The model is a relatively standard 3 equation New Keynesian model. I describe the derivation of the log linearized model below.

B.1 Households

There are an infinite number of identical households distributed over the unit interval. This representative household has the following maximization problem,

$$\underset{c_{t+\tau}, l_{t+\tau}, b_{t+\tau}}{\text{Max}} \quad E_t \sum_{\tau=0}^{\infty} \beta^{\tau} b_{t+\tau} \left[\log(c_{t+\tau} - h\bar{c}_{t+\tau-1}) - v \frac{l_{t+\tau}^{1+\eta}}{1+\eta} \right] \tag{B.1}$$

where $\beta \in (0, 1)$ is the discount factor, c_t is consumption in period t, h is the external habit formation parameter which is proportional to past aggregate consumption, l_t is labour supplied in period t, and η governs the elasticity of labour supplied. The representative household faces the following budget constraint,

$$b_t + c_t = \left(\frac{1 + i_{t-1}}{1 + \pi_t} \right) b_{t-1} + w_t l_t + \Pi_t. \tag{B.2}$$

The maximization problem yields the following first order conditions,

$$\lambda_t = \frac{b_t}{c_t - hc_{t-1}} \tag{B.3}$$

$$1 = \beta E_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right] \tag{B.4}$$

$$b_t v l_t^{\eta} = \lambda_t w_t \tag{B.5}$$

B.2 Final goods firms

Final goods firms operate in a perfectly competitive environment and package intermediate goods into a final aggregate good, y_t, sold at price p_t. Their maximization problem is given by,

$$\underset{y_t(i)}{\text{Max}} \; p_t y_t - \int_0^1 p_t(i) y_t(i) di \tag{B.6}$$

where,

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{B.7}$$

where $p_t(i)$ and $y_t(i)$ are prices and quantities of intermediate goods, and ϵ_t is the elasticity of substitution between intermediate goods. The maximization problem yields the standard downward sloping demand function for intermediate firm i's input , which is a function of its relative price and the price elasticity of demand,

$$y_t(i) = \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon} y_t \tag{B.8}$$

and implies an aggregate price index given by,

$$p_t = \left(\int_0^1 p_t(i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{B.9}$$

B.3 Intermediate goods firms

There is a continuum of intermediate goods producer on the unit interval $i \in [0,1]$ who produce goods using a constant returns to scale technology and minimize costs subject to meeting demand. In this setup, wages are common to all firms. The minimization problem is given by,

$$\underset{l_t(i)}{\text{Min}} \; w_t l_t(i) - \Phi \quad \text{subject to} \tag{B.10}$$

$$a_t l_t(i) \geq \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon} y_t \tag{B.11}$$

which yields a nominal marginal cost equation given by,

$$\chi_t = \frac{w_t}{a_t} \tag{B.12}$$

Intermediate goods producers operate in a monopolistically competitive environment and set prices for the good they produce. Prices are fixed for a stochastic number of periods following Calvo (1983). In each period intermediate goods firms have a probability to adjust their price given by θ. Thus optimal price setting takes into account not only the current optimal price, but the possibility that the firm may not be able to adjust its price for a long time. A firm setting its price optimally in period t then maximizes real profits given by the following,

$$\underset{p_t(i)}{\text{Max}} \; E_t \sum_{\tau=0}^{\infty} \theta^{\tau} \left\{ \Lambda_{t,t+\tau} \left(\frac{p_t}{p_{t+\tau}} \right) \left(p_t(i) \left(\frac{p_t(i)}{p_{t+\tau}} \right)^{-\epsilon} y_{t+\tau} - \chi_{t+\tau} \left(\frac{p_t(i)}{p_{t+\tau}} \right)^{-\epsilon} y_{t+\tau} \right) \right\} \tag{B.13}$$

where $\Lambda_{t,t+\tau}$ is a stochastic discount factor given by $\Lambda_{t,t+\tau} = \beta^{\tau} \left(\frac{u'(c_t)}{u'(c_{t+\tau})} \right)$. The firms optimal price is given by,

$$p_t(i) = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{E_t \sum_{\tau=0}^{\infty} \theta^{\tau} \Lambda_{t,t+\tau} \chi^R_{t+\tau} p^{\epsilon}_{t+\tau} y_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} \theta^{\tau} \Lambda_{t,t+\tau} p^{\epsilon-1}_{t+\tau} y_{t+\tau}} \tag{B.14}$$

where $\chi^R_{t+\tau} \equiv \frac{\chi_{t+\tau}}{p_{t+\tau}}$ is the real marginal cost in each period $t + \tau$.

B.4 Monetary policy

Monetary policy is conducted according to the following Taylor type rule,

$$\frac{i_t}{i^{\star}} = \left(\frac{i_{t-1}}{i^{\star}} \right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi^{\star}} \right)^{\alpha_{\pi}} \left(\frac{Y_t}{Y^F_t} \right)^{\alpha_y} \right]^{1-\rho_i} \epsilon^m_t \tag{B.15}$$

where ρ_i is the degree of interest rate smoothing, $i^{\star}$ is the steady state nominal interest rate, and ϵ^m is a monetary policy shock.

B.5 Equilibrium and Log-linearization

In equilibrium, goods market clearing imposes that all output is consumed, $y_t = c_t$. Then the intertemporal IS equation becomes,

$$\widetilde{y}_t = \frac{h}{1+h}\widetilde{y}_{t-1} + \frac{1}{1+h}E_t\widetilde{y}_{t+1} - \frac{1-h}{1+h}(\widetilde{i}_t - E_t\widetilde{\pi}_{t+1}) + \frac{1-h}{1+h}\widetilde{v}_t^y(1-\rho_y) \tag{B.16}$$

where $\frac{1-h}{1+h}\widetilde{v}_t^y(1-\rho_y) \equiv \frac{1-h}{1+h}(\widetilde{b}_t - E_t\widetilde{b}_{t+1})$. Output fluctuations are represented in percentage deviations from steady state (i.e., $\frac{x_t - \bar{x}}{x}$), while fluctuations in interest rates and inflation are represented in percentage point deviation from steady state (i.e., $x_t - \bar{x}$). The optimal pricing equation given in (B.14) can be rewritten as in the typical NKPC form. First let,

$$X_{1,t} = E_t \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} c_{t+\tau}^{-1} \chi_{t+\tau}^{R} p_{t+\tau}^{\epsilon} y_{t+\tau}, \tag{B.17}$$

and rewrite in the following manner,

$$X_{1,t} = c_t^{-1}\chi_t^R p_t^{\epsilon} y_t + \beta\theta X_{1,t+1}. \tag{B.18}$$

Similarly, the denominator of the optimal price setting equation can be written as,

$$X_{2,t} = c_t^{-1} p_t^{\epsilon-1} y_t + \beta\theta X_{2,t+1}. \tag{B.19}$$

The optimal price reset equation can then be written as,

$$p_t(i) = \left(\frac{\epsilon}{\epsilon-1}\right) p_t \frac{x_{1,t}}{x_{2,t}}, \tag{B.20}$$

where $x_{1,t} = \frac{X_{1,t}}{p_t^{\epsilon}}$ and $x_{2,t} = \frac{X_{2,t}}{p_t^{\epsilon-1}}$. The optimal price reset equation can be written in terms of inflation by dividing both sides by p_{t-1},

$$1 + \pi_t^{\#} = \left(\frac{\epsilon}{\epsilon - 1} \right) (1 + \pi_t) \frac{x_{1,t}}{x_{2,t}}, \tag{B.21}$$

where $\pi^{\#}$ is the inflation based on the optimal price change. Log-linearizing the above equation, it is straightforward to show that,

$$\widetilde{\pi}_t^{\#} = \widetilde{\pi}_t + \widetilde{x}_{1,t} - \widetilde{x}_{2,t}, \tag{B.22}$$

where, in a zero inflation steady state, $\widetilde{x}_{1,t}$ and $\widetilde{x}_{2,t}$ are given by,

$$\widetilde{x}_{1,t} = (1 - \beta\theta)\widetilde{\chi}_t^R + \beta\theta\widetilde{x}_{1,t+1} + \beta\theta\epsilon\widetilde{\pi}_{t+1}, \tag{B.23}$$

$$\widetilde{x}_{2,t} = \beta\theta\widetilde{x}_{2,t+1} + \beta\theta(\epsilon - 1)\widetilde{\pi}_{t+1}. \tag{B.24}$$

Substituting (B.23) and (B.24) into (B.22) and using the identity that $\widetilde{\pi}_t^{\#} = \frac{1}{1-\theta}\widetilde{\pi}_t$, we arrive at the following,

$$\frac{\theta}{1 - \theta}\pi_t = (1 - \beta\theta)\widetilde{\chi}_t^R + \beta\theta \underbrace{(\widetilde{x}_{1,t+1} - \widetilde{x}_{2,t+1})}_{\frac{\theta}{1-\theta}E_t\widetilde{\pi}_{t+1}} + \beta\theta\widetilde{\pi}_{t+1}, \tag{B.25}$$

$$\pi_t = \frac{(1 - \beta\theta)(1 - \theta)}{\theta}\widetilde{\chi}_t^R + \beta E_t\widetilde{\pi}_{t+1}. \tag{B.26}$$

As a final step I rewrite the NKPC in terms of deviations of output its flexible price level. This can be done by log linearizing the intratemporal condition given in (B.5) and the production function given in (B.11) and substituting these two into the log linearized marginal cost condition given in (B.12). After making these substitutions, one can arrive at the following,

$$\widetilde{\chi}_t = \frac{\eta(1 - \varphi)(1 - h) + 1}{(1 - h)}\widetilde{y}_t - \frac{h}{1 - h}\widetilde{y}_{t-1} - (1 + \eta)\widetilde{a}_t, \tag{B.27}$$

and in the scenario where prices are completely flexible, marginal costs are constant. This yields the following,

$$\tilde{a}_t = \frac{\eta(1-\varphi)(1-h)+1}{(1-h)(1+\eta)}\tilde{y}_t^F - \frac{h}{(1-h)(1+\eta)}\tilde{y}_{t-1}^F. \tag{B.28}$$

Substituting (B.27) and (B.28) into (B.26) yields a standard New Keynesian Phillips Curve relating inflation to the current and past output gap and expected future inflation,

$$\pi_t = \kappa_1(\tilde{y}_t - \tilde{y}_t^F) - \kappa_2(\tilde{y}_{t-1} - \tilde{y}_{t-1}^F) + \beta E_t \pi_{t+1}, \tag{B.29}$$

where,

$$\kappa_1 = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{\eta(1-\varphi)(1-h)+1}{1-h} \tag{B.30}$$

$$\kappa_2 = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{h}{1-h} \tag{B.31}$$

Lastly, log linearizing the Taylor rule yields the following,

$$\widetilde{i}_t = \rho_i \widetilde{i}_{t-1} + (1-\rho_i)\{\alpha_\pi \widetilde{\pi}_t + \alpha_y(\widetilde{y}_t - \widetilde{y}_t^F)\} + v_t^R \tag{B.32}$$

In summary, the model dynamics are represented by the following equations,

$$\widetilde{y}_t = \frac{h}{1+h}\widetilde{y}_{t-1} + \frac{1}{1+h}E_t\widetilde{y}_{t+1} - \frac{1-h}{1+h}(\widetilde{i}_t - E_t\widetilde{\pi}_{t+1}) + \frac{1-h}{1+h}\widetilde{v}_t^y(1-\rho_y) \tag{B.33}$$

$$\pi_t = \kappa_1(\tilde{y}_t - \tilde{y}_t^F) - \kappa_2(\tilde{y}_{t-1} - \tilde{y}_{t-1}^F) + \beta E_t \pi_{t+1} + v_t^\pi \tag{B.34}$$

$$\widetilde{i}_t = \rho_i \widetilde{i}_{t-1} + (1-\rho_i)\{\alpha_\pi \widetilde{\pi}_t + \alpha_y(\widetilde{y}_t - \widetilde{y}_t^F)\} + v_t^m \tag{B.35}$$

where I also admit for potential supply disturbances in the NKPC given by v_t^π.

C Model setup and solution

In the following I describe the matrices that make up the following matrix form representation of the model and are used to solve the filtering problem of the central bank. The matrix form of the model is given by,

$$\mathbf{A}_0 \begin{bmatrix} \mathbf{X}_{1,t+1} \\ E_t \mathbf{X}_{2,t+1} \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix} + \mathbf{A}_2 \begin{bmatrix} \mathbf{X}_{1,t|t} \\ \mathbf{X}_{2,t|t} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_1 \\ 0 \end{bmatrix} \boldsymbol{\epsilon}_{t+1}, \tag{C.1}$$

where $\mathbf{X}_{1,t}$ is the vector of predetermined variables, $\mathbf{X}_{2,t}$ is the vector of forward looking (or jump) variables, and $\boldsymbol{\epsilon}_{t+1}$ is the vector of structural shocks and measurement errors. $\mathbf{X}_{1,t|t}$ and $\mathbf{X}_{2,t|t}$ are the expected values of the vectors of predetermined variables and jump variables conditional on information at time t. To keep notation compact I include the measurement errors in the vector $\mathbf{X}_{1,t}$. These vectors are given by the following,

$$\mathbf{X}'_{1,t} = \begin{bmatrix} \pi_{t-1} & y^F_{t-1} & y_{t-1} & i_{t-1} & y^F_t & a_t & v^\pi_t & v^y_t & v^m_t & \eta^\pi_t & \eta^y_t \end{bmatrix}, \tag{C.2}$$

$$\mathbf{X}'_{2,t} = \begin{bmatrix} y_t & \pi_t \end{bmatrix}, \tag{C.3}$$

$$\boldsymbol{\epsilon}'_{t+1} = \begin{bmatrix} \epsilon^a_{t+1} & \epsilon^\pi_{t+1} & \epsilon^y_{t+1} & \epsilon^m_{t+1} & \eta^\pi_{t+1} & \eta^y_{t+1} \end{bmatrix}. \tag{C.4}$$

Then the matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{C}_1$ are given by the following,

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -OG_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & -\frac{1-h}{1+h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1-h}{1+h} & \frac{1}{1+h} \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \rho_i & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & OG_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_v & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_y & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\eta_\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\eta_y} & 0 & 0 \\ 0 & -\kappa_2 & \kappa_2 & 0 & \kappa_1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & -\kappa_1 \\ 0 & 0 & -\frac{h}{1+h} & 0 & 0 & 0 & 0 & -\frac{1-h}{1+h} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\mathbf{A}_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(1-\rho_i)\alpha_y & 0 & 0 & 0 & 0 & 0 & 0 & (1-\rho_i)\alpha_\pi & (1-\rho_i)\alpha_y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\mathbf{C}_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

In the matrices I use the following reduced form notation,

$$
\begin{aligned}
OG_1 &= \frac{(1-h)(1+\eta)}{1+\eta(1-\varphi)(1-h)} \\
OG_2 &= \frac{h}{1+\eta(1-\varphi)(1-h)} \\
\kappa_1 &= \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{(1+\eta(1-\varphi)(1-h))}{1-h} \\
\kappa_2 &= \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{h}{1-h}
\end{aligned}
$$

Based on the above the model can be written in a more compact notation given by,

$$
\begin{bmatrix} \mathbf{X}_{1,t+1} \\ E_t\mathbf{X}_{2,t+1} \end{bmatrix} = \mathbf{A}^1 \begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{2,t} \end{bmatrix} + \mathbf{A}^2 \begin{bmatrix} \mathbf{X}_{1,t|t} \\ \mathbf{X}_{2,t|t} \end{bmatrix} + \mathbf{C}\boldsymbol{\epsilon}_{t+1}, \tag{C.5}
$$

where $\mathbf{A}^1 = \mathbf{A}_0^{-1}\mathbf{A}_1$, $\mathbf{A}^2 = \mathbf{A}_0^{-1}\mathbf{A}_2$, and $\mathbf{C} = \mathbf{A}_0^{-1}\begin{bmatrix}\mathbf{C}_1 & \mathbf{0}\end{bmatrix}'$. The solution to the model is then a simple application of the methods shown in Svensson and Woodford (2003). As they show, under the assumption of symmetric partial information the estimation of the partially observed state and the computation of the rational expectations equilibrium can be separated. Thus a linear mapping exists relating the predetermined state variables to the forward looking variables and one can use standard methods of eliminating bubbles to find this solution given by,

$$
\mathbf{X}_{2,t|t} = \mathbf{G}^{\star}\mathbf{X}_{1,t|t}. \tag{C.6}
$$

In this paper I use a simple fixed point iteration to solve for the matrix $\mathbf{G}^{\star}$ which satisfies the following

$$
\mathbf{G} = (\mathbf{G}\mathbf{A}_{12} - \mathbf{A}_{22})^{-1}(\mathbf{A}_{21} - \mathbf{G}\mathbf{A}_{11}), \tag{C.7}
$$

where $\mathbf{A} = \mathbf{A}^1 + \mathbf{A}^2$ and subscripts indicate rows and columns of the predetermined variables and jump variables (e.g., A_{11} are the first n_1 rows and n_1 columns of matrix A and n_1 is equal to the number of predetermined variables). The model dynamics can then be described by the following system of equations,[15]

$$\mathbf{X}_{1,t+1} = \mathbf{H}\mathbf{X}_{1,t} + \mathbf{J}\mathbf{X}_{1,t|t} + \mathbf{C}_1\boldsymbol{\epsilon}_{t+1} \tag{C.8}$$

$$\mathbf{X}_{1,t|t} = \mathbf{X}_{1,t|t-1} + \mathbf{K}(\mathbf{Z}_t - \mathbf{Z}_{t|t-1}) \tag{C.9}$$

$$\mathbf{Z}_t = \mathbf{L}\mathbf{X}_t + \mathbf{M}\mathbf{X}_{t|t} \tag{C.10}$$

where

$$\mathbf{H} = \mathbf{A}^1_{11} - \mathbf{A}^1_{12}(\mathbf{A}^1_{22})^{-1}\mathbf{A}^1_{21} \tag{C.11}$$

$$\mathbf{J} = \mathbf{A}^1_{12}\left[(\mathbf{A}^1_{22})^{-1}\mathbf{A}^1_{21} + \mathbf{G}^\star\right] + \mathbf{A}^2_{11} + \mathbf{A}^2_{12}\mathbf{G}^\star \tag{C.12}$$

$$\mathbf{L} = \mathbf{D}^1_1 - \mathbf{D}^1_2(\mathbf{A}^1_{22})^{-1}\mathbf{A}^1_{21} \tag{C.13}$$

$$\mathbf{M} = \mathbf{D}^1_2\left[(\mathbf{A}^1_{22})^{-1}\mathbf{A}^1_{21} + \mathbf{G}^\star\right] \tag{C.14}$$

$$\mathbf{K} = \mathbf{P}\mathbf{L}^{'}(\mathbf{L}\mathbf{P}\mathbf{L}^{'} + \sigma^2_{vv})^{-1} \tag{C.15}$$

The matrix **P** is the covariance matrix for the prediction errors which can be solved as the solution to a Riccati equation (see equation (25) in Svensson and Woodford (2003) for this formula). These equilibrium dynamics can be represented in a more compact fashion (which I use in the code) given by the following,

$$\underbrace{\begin{bmatrix} \mathbf{X}_{1,t+1} \\ \mathbf{X}_{1,t+1|t+1} \end{bmatrix}}_{\tilde{\mathbf{X}}_{t+1}} = \underbrace{\begin{bmatrix} \mathbf{H} & \mathbf{J} \\ \mathbf{KLH} & (\mathbf{H}+\mathbf{J}) - \mathbf{KLH} \end{bmatrix}}_{\tilde{\mathbf{A}}} \underbrace{\begin{bmatrix} \mathbf{X}_{1,t} \\ \mathbf{X}_{1,t|t} \end{bmatrix}}_{\tilde{\mathbf{X}}_t} + \underbrace{\begin{bmatrix} \mathbf{C}_1 \\ \mathbf{KLC}_1 \end{bmatrix}}_{\tilde{\mathbf{C}}} \boldsymbol{\epsilon}_{t+1} \tag{C.16}$$

and the forward looking variables can be obtained using,

$$\mathbf{X}_{2,t} = \mathbf{G}^1\mathbf{X}_{1,t} + \mathbf{G}^2\mathbf{X}_{1,t|t} \tag{C.17}$$

where,

[15]Note that since I carry the measurement error shocks in the state vector, one could alternatively depict the observation equation with an error term. Instead, in this case the matrix L picks the observable and its measurement error from the state vector.

$$\mathbf{G}^1 = -(\mathbf{A}^1_{22})^{-1}\mathbf{A}^1_{21} \tag{C.18}$$

$$\mathbf{G}^2 = \mathbf{G}^\star - \mathbf{G}^1 \tag{C.19}$$

and from the definition of the rational expectations equilibrium, the expected value of the forward looking variables is given by,

$$\mathbf{X}_{2,t|t} = \mathbf{G}^\star \mathbf{X}_{1,t|t} \tag{C.20}$$

C.1 Computing impulse response functions

Based on the notation given in equation (C.16), it is straightforward to compute impulse response functions by iterating the state space forward. For example, the state space at a given time t for a structural shock at time t_0 can be computed from

$$\tilde{\mathbf{X}}_t = \tilde{\mathbf{A}}^{t-1}\tilde{\mathbf{C}}\boldsymbol{\epsilon}_{t_0} \tag{C.21}$$

and the corresponding values for the forward looking variables and expected forward looking variables can be found using equations (C.17) and (C.20).

D Data and Estimation

D.1 Data

To estimate the structural parameters of the model I use data from the Federal Reserve Bank of St. Louis database. The exact FRED codes are below in parentheses. These variables include:

– Gross Domestic Product (GDP).

– Gross Domestic Product Implicit Price Deflator (GDPDEF).

– Effective Federal Funds rate (FEDFUNDS). This measure is converted to quarterly frequency by taking averages.

– Population Level (CNP16OV).

It is well known that the population series used is problematic due to irregular updating from census population measures. This causes spikes in the series unrelated to business cycles. To avoid introducing spurious dynamics into my per capita real GDP measure, I use a smoothed value of this population series which is obtained by fitting an HP-filtered trend with a smoothing parameter, $\lambda = 10,000$, as suggested by Pfeifer (2020). I use the following variable construction,

$$Y_t = log\left(\frac{GDP}{GDPDEF * POP^{HP}}\right) * 100, \tag{D.1}$$

$$\pi_t = log\left(\frac{GDPDEF_t}{GDPDEF_{t-1}}\right) * 100, \tag{D.2}$$

$$i_t = \frac{FEDFUNDS}{4}. \tag{D.3}$$

Since I am not explicitly modeling any non-stationary factors I detrend output using a one-sided HP filter with a smoothing parameter, $\lambda = 1600$ (Stock and Watson 1999b, pg. 301).

In the empirical section of the paper I make use of data from the Real-Time Data Set for Macroeconomists from the Federal Reserve Bank of Philadelphia. I use first release data for real GDP/GNP under the heading NIPA Product Side - Real and inflation data from the Price Index for GNP/GDP under the Price Level Indices heading. Both of these variables can be obtained in Excel format from https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/first-second-third.

The second source of data used in the empirical investigation of noise uses data from the Federal Reserve's Greenbook. This data is also available from the Federal Reserve Bank of Philadelphia at https://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data/. Again, I use measures for Real GDP and GDP Price inflation.

D.2 Estimation

To estimate the posterior mode of the model I use a simulated annealing algorithm as in Goffe et al. (1994). The algorithm combines grid search with random movements in the parameter space which allows for movements parameter vector which are not always improvements in the likelihood. I initialize the temperature to 5 and set $N_t = 5$ and $N_s = 20$. The temperature convergence criterion is set to 1e-8. The algorithm is described below.

Algorithm 1

procedure SIMULATED ANNEALING
while $(T > T^\star)$
  for $k = 1 : N_T$
    for $i = 1 : N_s$
      Draw $\theta_j^{'} = \theta_j^0 + r \cdot v_j$ where $r \sim U[-1, 1]$ and v_j is an element of the step size
      vector V.
      Evaluate $f(\Theta^{'})$
      if $f(\Theta^{'}) > f(\Theta^0)$
        $\Theta^{(1)} = \Theta^{'}$
      elseif $f(\Theta^{'}) < f(\Theta^0)$
        $\Theta^1 = \begin{cases} \Theta^{(1)} = \Theta^{'} & \text{with probability } e^{(f(\Theta^{'}) - f(\Theta^0))/T} \\ \Theta^{(1)} = \Theta^0 & \text{with probability } 1 - e^{(f(\Theta^{'}) - f(\Theta^0))/T} \end{cases}$
      end
    end
    Set $V = V'$ such that roughly 50% of all moves are accepted
  end
  Reduce temperature such that $T' = r_T \cdot T$ so fewer downhill steps accepted
end
return $\{\Theta^\star\}$

After obtaining posterior mode estimates of the structural parameters, I simulate the posterior distributions of the parameters using the Random-Walk Metropolis Hastings algorithm with 2 million replications (S in algorithm below). The algorithm is described below.

Algorithm 2

procedure RANDOM-WALK METROPOLIS HASTINGS
Initialize Θ^0 to posterior mode estimates computed in Algorithm 1
for $i = 1 : S$
    Generate a candidate draw, $\Theta^\star \sim q(\Theta^\star|\Theta^{(s-1)})$, where
    $q(\Theta^\star|\Theta^{(s-1)}) = N(\Theta^{s-1}, \Sigma)$ and Σ is a multivariate normal distribution
    if $U(0,1) \leq \alpha_s$
        Set $\Theta^s = \Theta^\star$
    else
        $\Theta^s = \Theta^{s-1}$
    end
    Where α_s is an acceptance probability given by $\alpha_s = \min\left(1, \frac{p(\Theta^\star|y)}{p(\Theta^{(s-1)}|y)} \frac{q(\Theta^{(s-1)}|\Theta^\star)}{q(\Theta^\star|\Theta^{(s-1)})}\right)$
end
return $\{\Theta^s\}$ for $s = 1 \dots S$

In Table 5 I report the model predicted standard deviations of inflation, output, and the nominal interest rate in the pre-1979 and post-1983 periods.

Table 5: DATA AND MODEL STANDARD DEVIATIONS

	Output	**Inflation**	**Nominal rate**
Pre-1979			
Data	1.7258	0.6755	0.6062
Model	3.0081	0.4881	0.6109
Post-1983			
Data	1.3406	0.2211	0.6131
Model	1.4114	0.2540	0.4714

Notes: Output in the data is real GDP detrended using a one-sided Hodrick-Prescott filter.

In Table 6 I report the unconditional variance decomposition of output, inflation, and the nominal interest rate in the pre-1979 and post-1983 periods.

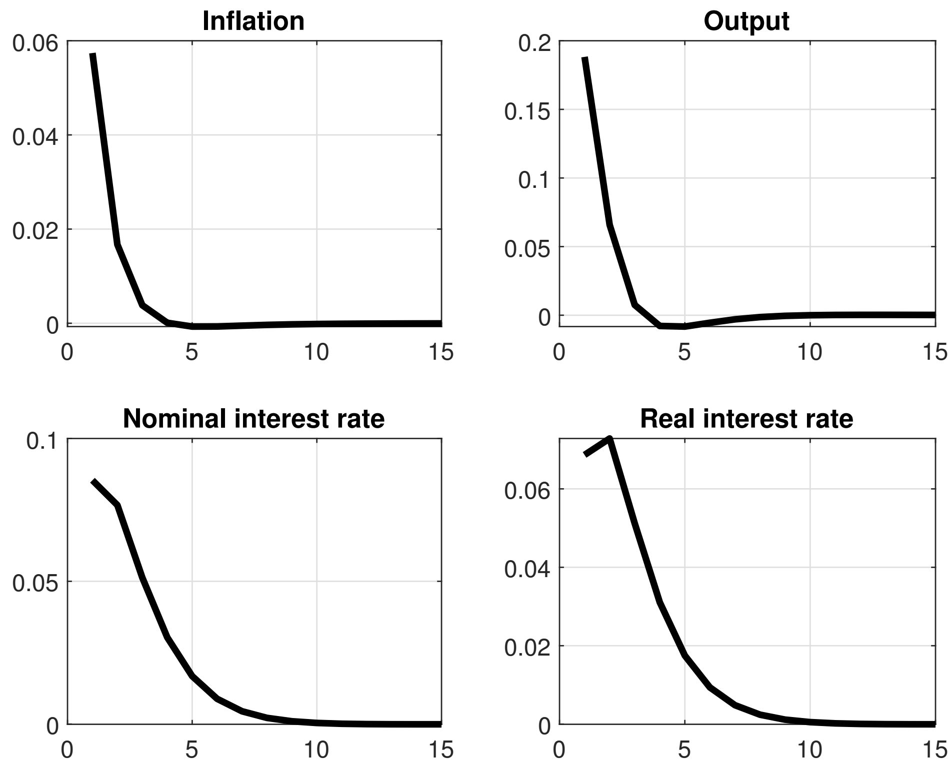
Table 6: UNCONDITIONAL VARIANCE DECOMPOSITION

	Pre-1979					
Moment ↓ / Shock →	Tech.	Cost push	Preference	MP	Inf. noise	Output noise
Output	0.0001	0.0277	0.6842	0.0303	0.2311	0.0265
Inflation	0.0031	0.8675	0.0553	0.0040	0.0598	0.0103
Nominal rate	0.0002	0.0599	0.1890	0.1231	0.5067	0.1211
	Post-1983					
Moment ↓ / Shock →	Tech.	Cost push	Preference	MP	Inf. noise	Output noise
Output	0.0002	0.0316	0.4603	0.0375	0.3940	0.0765
Inflation	0.0086	0.7870	0.0888	0.0048	0.0623	0.0485
Nominal rate	0.0001	0.0218	0.4287	0.0754	0.2732	0.2008

Notes: Output in the data is real GDP detrended using a one-sided Hodrick-Prescott filter.

E Additional results

Figure 10: OUTPUT NOISE SHOCK



Notes: The IRF in this figure uses the estimated structural parameters from the pre-1979 estimation. The real interest rate reported here is the ex-post real interest rate, consistent with the focus in the empirical section of the paper. The impulse response function is computed using a one standard deviation output noise shock.