Capitalist Spirit Model

A representative household has a utility function given by

$$\underset{c_T, w_{T+1}}{\text{Max}} \quad u(c_T) + v(w_{T+1}), \tag{0.1}$$

subject to

$$w_{T+1} = w_T - c_T. \tag{0.2}$$

The optimization problem (my preferred interpretation) is that the household is deciding how to allocate lifetime resources, w_T, between lifetime consumption, c_T, and end of life wealth, which acts as a bequest to children. Using Carroll (2000)'s utility function specifications

$$u(c_T) = \frac{c_T^{1-\rho}}{1-\rho}, \tag{0.3}$$

$$v(w_{T+1}) = \frac{(w_{T+1} + \gamma)^{1-\alpha}}{1-\alpha}, \tag{0.4}$$

where $\rho = 2$, $\alpha = 1$, and $\gamma = 1$. In this case the first order condition is given by

$$c_T^{-2} = (w_{T+1} + 1)^{-1}, \tag{0.5}$$

and substituting in the budget constraint, we get

$$c_T^{-2} = (w_T - c_T + 1)^{-1}. \tag{0.6}$$

A little bit of algebra yields

$$c_T^2 + c_T - (w_T + 1) = 0. \tag{0.7}$$

The solution to the optimal consumption choice is quadratic in c_T, and therefore has two solutions given by

$$c_{T,1}, c_{T,2} = \frac{-1 \pm \sqrt{(1)^2 - 4(-1)(w_T + 1)}}{2}. \tag{0.8}$$

We can rule out one of these solutions by imposing that the household cannot end with positive debt (if $c_T < 0$ then $w_{T+1} > w_T$ and the household has debt). Then the solution to the problem is given by

$$c_{T,1} = \frac{-1 + \sqrt{1 + 4(w_T + 1)}}{2}. \tag{0.9}$$

However, there exists a region in which this is not the optimal choice of consumption. Consider a very small amount of lifetime resources (w_T is very small), in this case lifetime consumption would be negative. For this region the optimal choice of consumption will be to simply consume its entire lifetime resources and not leave a bequest. That is,

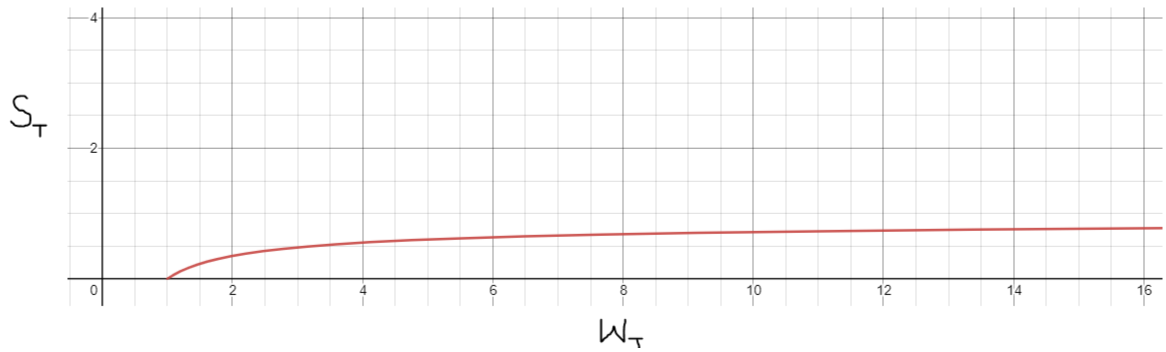
$$c_T = \text{Min}[c_{T,1}, w_T] \tag{0.10}$$

[Why does $\rho > \alpha$ matter?]

Consider the Saving rate (defined as w_{T+1}/w_T) for a case where $\rho > \alpha$

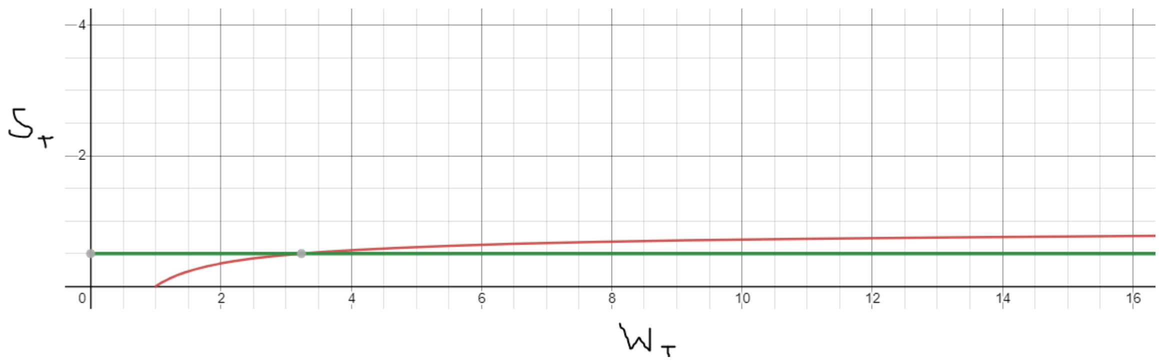
$$s_T = \begin{cases} c_T = w_T \quad \text{if } \; -1 + \sqrt{1 + 4(w_T + 1)} < 0, \\ c_T = \frac{-1+\sqrt{1+4(w_T+1)}}{2} \quad \text{if } \; -1 + \sqrt{1 + 4(w_T + 1)} > 0. \end{cases} \tag{0.11}$$

This saving rate is depicted in the following graph. What it shows is that the saving rate is zero up until a sufficient level of wealth. After that the saving rate rises, but is concave, and



eventually converges to a saving rate of one in the infinite limit.

Now consider the case where $\rho = \alpha = 1$ and $\gamma = 0$ (i.e., log utility arguments). In this case since the utility arguments are the same it is optimal for the household to simply divide resources evenly between lifetime consumption and the bequest ($c_T = \frac{1}{2}w_T$). Again using the budget constraint and defining the saving rate as w_{T+1}/w_T, the saving rate in this case is depicted in green, with the previous case in red.

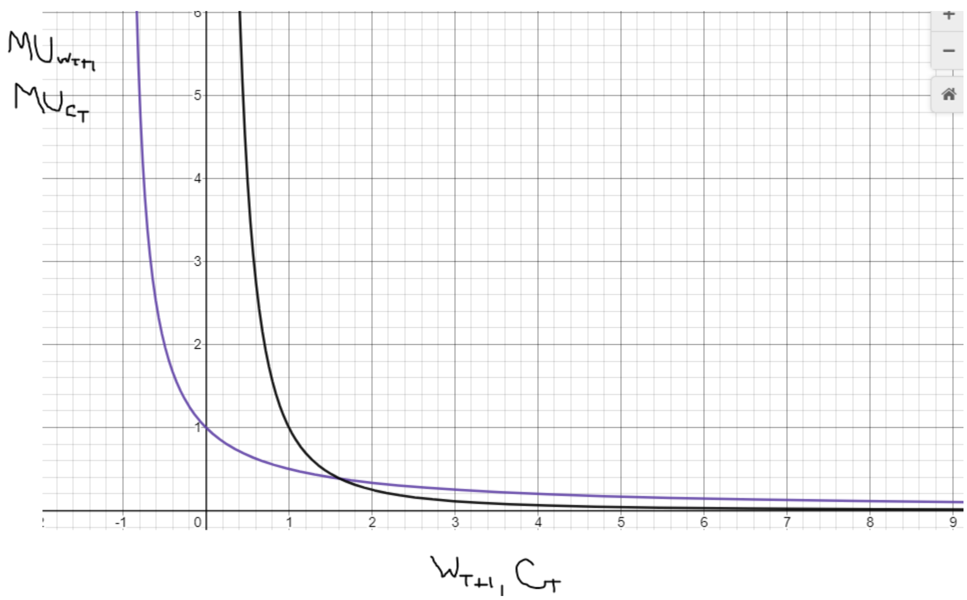


What it shows is that in the log utility case, the saving rate out of wealth is constant. Compared to the case with $\rho > \alpha$, where the saving rate is concave in lifetime wealth. This is the key feature, alongside $\gamma > 0$, which Carroll argues is a better fit to the micro data on consumption-saving behaviour. Note that in the limit as wealth tends to infinity, the green line stays constant at $1/2$, whereas the red line converges to 1.

[What is the role played by γ?]

γ controls the level of wealth required before a household will begin having a positive saving rate. For a higher γ, the household will require more lifetime wealth to begin saving.

Why is this the case? Consider the case where $\rho = 2$, $\alpha = 1$, and $\gamma = 1$. The figure below shows the marginal utility of bequest and lifetime consumption on the y-axis, and the corresponding level of consumption and bequest on the x-axis. From above, we impose that the household cannot leave a negative bequest. When lifetime wealth is below 1, the marginal utility of consumption (black line) is always greater than the marginal utility of bequest (purple line). When consumption equals exactly 1, the marginal utility of consumption is equal to the marginal utility of the first unit of bequest, this is the point where the saving rate becomes positive.



Since the marginal utility of consumption declines faster than the marginal utility of bequest, as lifetime wealth rises, households put more and more resources into bequests (as a fraction of lifetime wealth) and hence drive up the saving rate. γ determines where the marginal utilities are equal to each other, and as such, where the saving rate becomes positive. For example, if $\gamma = 2$, then it wouldn't be until $c_T = 2$ that households would find it optimal to begin giving bequests and start saving (holding our calibrations of ρ and α constant).