1. $T(1) = 1, \quad T(n) = T\left(\frac{n}{2}\right) + 1$ mit $a = 1$, $b = 2$ und $f(n) = 1$

$\Rightarrow n^{\log_2 1} = n^0 = 1$

$\Rightarrow$ 2. Fall: $f(n) = \Theta\left(n^{\log_2 1}\right) = \Theta(1) = 1$

$\Rightarrow \underline{\underline{T(n)}} = \Theta\left(n^{\log_2 1} \log n\right) = \Theta\left(n^0 \log n\right) = \underline{\underline{\Theta(\log n)}}$

2. $T(1) = 1, \quad T(n) = 3T\left(\frac{n}{4}\right) + n\log n$ mit $a = 3$, $b = 4$ und $f(n) = n\log n$

$f(n) = n\log(n) \geq n^{\log_4 3} \quad \wedge \quad \cancel{3n\log}\; 3\frac{n}{4}\log\left(\frac{n}{4}\right) \leq c\, n\log(n)$

$\Leftrightarrow \frac{3}{4}n\left(\log(n) - \log(4)\right) \leq c\,n\log(n)$

$\Leftrightarrow \frac{3}{4}n\left(\log(n) - 2\right) \leq c\,n\log(n)$

$\Leftrightarrow \frac{3}{4}n\log(n) - \frac{6}{4}n \leq c\,n\log(n)$

$\Leftrightarrow \frac{3}{4}\cdot\frac{\cancel{n\log(n)}}{\cancel{n\log(n)}} - \frac{\cancel{6}^{3}}{\cancel{4}_{2}}\cdot\frac{n}{n\log n} \leq c \Leftrightarrow \frac{3}{4} - \frac{3}{2\log(n)} \leq c$ Wahr!

$\Rightarrow$ 3. Fall: $\underline{\underline{T(n) = \Theta(n\log n)}}$

3. $T(1) = 1, \quad T(n) = 7T\left(\frac{n}{2}\right) + n^2$ mit $a = 7$, $b = 2$ und $f(n) = n^2$

$\log_2 7 < \log_2 4 \Leftrightarrow \log_2 7 < 2$

$\Rightarrow n^{\log_2 7} < n^2 \quad \Rightarrow$ 1. Fall: $n^2 = O\left(n^{\log_2 7}\right)$

$\Rightarrow \underline{\underline{T(n) = \Theta\left(n^{\log_2 7}\right)}}$