Beh.: Siehe Angabe

IA.: $f(1) = \frac{\phi^n - \hat{\phi}^n}{\cancel{2}\sqrt{5}} = \frac{\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}}{\sqrt{5}} = \frac{2\sqrt{5}}{2\sqrt{5}} = 1$

IV.: Beh. gilt für $f(n)$

IS.: $f(n) \to f(n+1)$

$f(n+1) = f(n) + f(n-1) \iff f(n) = f(n+1) - f(n-1)$

$$f(n) = \frac{\phi^{n+1} - \hat{\phi}^{n+1}}{\sqrt{5}} - \frac{\phi^{n-1} - \hat{\phi}^{n-1}}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \left(\phi^2 \cdot \phi^{n-1} - \phi^{n-1} - \hat{\phi}^2 \hat{\phi}^{n-1} + \hat{\phi}^{n-1}\right)$$

$$= \frac{1}{\sqrt{5}} \cdot \left(\phi^{n-1} \cdot (\phi^2 - 1) - \hat{\phi}^{n-1} \cdot (\hat{\phi}^2 - 1)\right)$$

Nebenrechnungen:

$$\phi^2 - 1 = \left(\frac{1+\sqrt{5}}{2}\right)^2 - 1 = \frac{1 + 2\sqrt{5} + 5}{4} - 1 = \frac{1+\sqrt{5}}{2} = \phi$$

$$\hat{\phi}^2 - 1 = \left(\frac{1-\sqrt{5}}{2}\right)^2 - 1 = \frac{1 - 2\sqrt{5} + 5}{4} - 1 = \frac{1-\sqrt{5}}{2} = \hat{\phi}$$

Fortsetzung:

$$f(n) = \frac{1}{\sqrt{5}} \cdot \left(\phi^{n-1} \cdot \phi - \hat{\phi}^{n-1} \cdot \hat{\phi}\right) = \frac{1}{\sqrt{5}} \cdot \left(\phi^n - \hat{\phi}^n\right) = \underbrace{\frac{\phi^n - \hat{\phi}^n}{\sqrt{5}}}_{IV}$$

Somit gezeigt.