3.7

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March 12, 2017

(a) A *biparte graph* is a graph G=(V, E) whose vertices can be partitioned into two sets (V=$V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$) such that there are no edges between vertices in the same set (for instance, if $u, v \in V_1$, then there is no edge between$u$ and v).

An algorithm that can determine wheather an undirected graph is bipartite is one that uses DFS to seek out if there are odd number cycles in the graph. The reason why this algorithm will work is because every point must be in one of the two sets $V_1 or V_2$. Now being that no vertex in the subset can share an edge with another vertex, it follows that the only shared edges must then be between vertices of the two submatrices.

Then, let's suppose we have V_1 and V_2. Then, let there be an odd cycle between the two sets.
Then, starting at u from $V_1, therewillbetwopossibilities.$
1) u will connect one or more vertices $\in V_1$ or
2) u will share two or more edges with vertices in V_2.

Case 1 immediately fails the definition for biparte graph while Case 2 will also inevitably fail since in order to complete the odd circuit, at least two vertices from the other graph will need to share an edge in order to prevent a crossover in V_1.

Even circuits however do not affect this tracing back and forth from V_1 to V_2 back to V_1
Will always be done in moves of two, which means if there are only even circuits, it is possible to divide V into V_1, V_2that fit the definition.

The algorithm's time complexity is $T(n) = O(N)$ because using DFS, we will look at at most N edges, with a small constant factor for checking for odd circuits.

(b)
Look at **(a)**

(c)
To rephrase the question, when we get we have V_1 with $2k_1$ cycles and V_2 with $2k_2 + 1$ cycles where $V_1 and V_2$ are sets that come from V and k_1 and k_2 are someintegers which create even numbers when multiplied by 2 and odd when multiplied by 2 and added a 1. Then the question becomes how many more V_P do we have to add to ensure no two vertices from the same subset share vertices. Well, since the only thing we need to do to is to break the 1 odd cycle, making $V_2 = 2k_2$, which only requires moving one number into a new subsetV_3 which will only take 1 more color.