

Two paths in a graph are called *edge-disjoint* if they have no edges in common.

Show that in any undirected graph, it is possible to pair up the vertices of odd degree and find paths between each such pair so that all these paths are edge-disjoint.

Let us pick a number of odd number vertices, and quantify it as  $2k$  vertices. Then pair up these vertices, and separate them using set of even degree vertices that connect all the odd points.

Since every time you cross an even node, you use up an even number of paths to go into the vertex, and then out of the vertex, any even node will stay even. Since all the odd vertices are connected by an even degree vertex, then there must a unique path between them and hence they are edge-disjoint.