B1

Andrew Lee

March 7, 2017

If f(n) = O(g(n)), and g(n) = O(f(n)) then f(n) = $\theta(g(n))$

If $f(n) = O(g(n)),$, then by the definition of Big-O, $f(n) \leq c * g(n) \forall c > 0$

Furthermore, if $g(n) = O(f(n))$, then by the definition of Big-O,
$g(n) \leq c * f(n) \forall c > 0$

Then $\exists$ constants $c_1, c_2,$ such that $c_1 * f(n) = c_2 * g(n)$

Hence, $f(n) = \frac{c_2 * g(n)}{c_1}$, and $g(n) = \frac{c_1 * f(n)}{c_2}$

Hence, $f(n) \geq c_3 * g(n)$ and $g(n) \geq c_4 * f(n)$ for some constants c_3 *and* c_4
Therefore, by the definition of Big-Ω, $f(n) = \Omega(g(n))$ and $g(n) = \Omega(f(n))$.

Because $f(n) = O(g(n))$ and $f(n) = \Omega(g(n)), f(n) = \theta(g(n))$ and
because $g(n) = O(f(n))$ and $g(n) = \Omega(f(n)), g(n) = \theta(f(n))$.