3.12

Andrew Lee

March 12, 2017

Either prove or give a counterexample: if u, v is an edge in an undirected graph, and during depth-first search post(u) $<$ post(v), then v is an ancestor of u in the DFS tree.

The only case when a vertex ucan have a smaller post value than another vertex v is when there is a braching in in the graph between say, vertex 1 and between vertex 2 and vertex 3. Since DFS will always pick the smallest value, and continue until there are no possible way to move forward, vertex 3 will have a higher post number than anyone in vertex 2's treeline However, since we are confined to only u and v's that together create an edge, this behavior will not come into play. Instead, we can only consider where there is u, v that share an edge and are explored through DFS.

Then, by the given information, post(u) $<$ post(v) which will look like $[_?[_?|||]_u]_v$ where the ?'s represent unknown pre values (a vertex with a lower pre number is the ancestor. Then u have a lower pre number than v. Then formulation look like $[_u[_v|||]_?]_?$ However, backtracking after finding the n^{th} element, since in order to backtrack to u, we need to first backtrack through we have a contradiction, it must be the case that $[_v[_u|||]_u]_v$ which makes v and ancestor of u. **QED**