(a) Show that in an undirected graph, $\sum_{u \in V} d(u) = 2|E|$.
Let $x =$ the sum of all edges attached at all $u \in V$
. That is $x = d(u_1) + d(u_2) + d(u_3) + ... + d(u_n)$

Then the average number of edges for given node $u = \frac{x}{n}$
However, because every node $u \exists$ another node $v|u$ must share a common edge with v, we end up double counting, the average number of edges for u is actually
$\frac{1}{2} * \frac{x}{n}$

Hence, by our previous equation, $\frac{1}{2}x = \frac{1}{2}d(u_1) + \frac{1}{2}d(u_2) + ... \frac{1}{2}d(u_n)$.

Therefore, since $\frac{1}{2} \sum_{u \in V} d(u)$, $\sum_{u \in V} = d(u) = 2|E|$
Q.E.D.

(b) Use part (a) to show that in an undirected graph,
there must be an even number of vertices whose degree is odd.
Let $\{d(u_1), d(u_2), d(u_3), d(u_n)\}$
represent the degrees of all the vertices n $\in V$
Then let's assume there is an odd number of odd degree vertices such that the average looks like $\{(2k+1)_1, (2k+1)_2, ..., (2k+1)_{2t+1}\}$ where $d = 2k+1$, and $2t+1$ represents some odd number
Then we can match each vertex with degree $2k+1$ to another vertex with degree $2k+1$.
However, since the vertices with odd degrees in the form $2k+1$ will always be $+1$ or -1 off from an even number vertex in the form $2k$, the odd number vertices can only will be missing 1 or having 1 too many connections with even vertices.
Hence, there must be matching pairs of odd degree vertices in an undirected graph

(c) This statement cannot be proven for a directed graph. Take a 2 node graph with a single directional edge going from one to the other. If you count the indegree, one would be 0 and the other 1. Next counting the outdegree, one would be 1 and the other 0
Hence, the above theorum will not hold for directed graphs