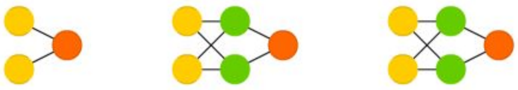
Double descent

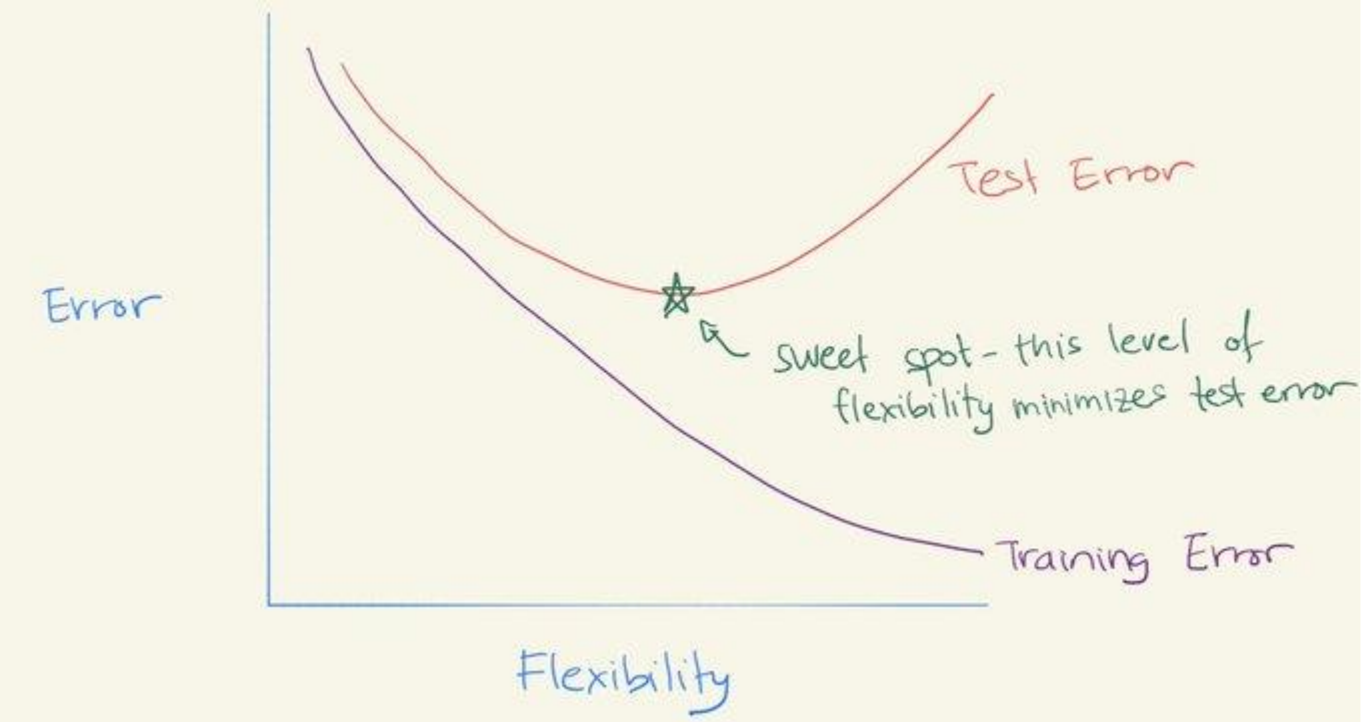
Is the bias-variance trade-off still valid in the era of deep learning?

Filippo Biscarini
Senior Scientist
CNR, Milan (Italy)

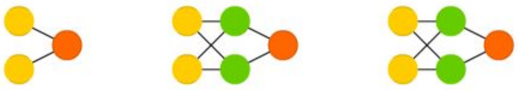
Nelson Nazzicari
Research fellow
CREA, Lodi (Italy)



Bias-variance trade-off

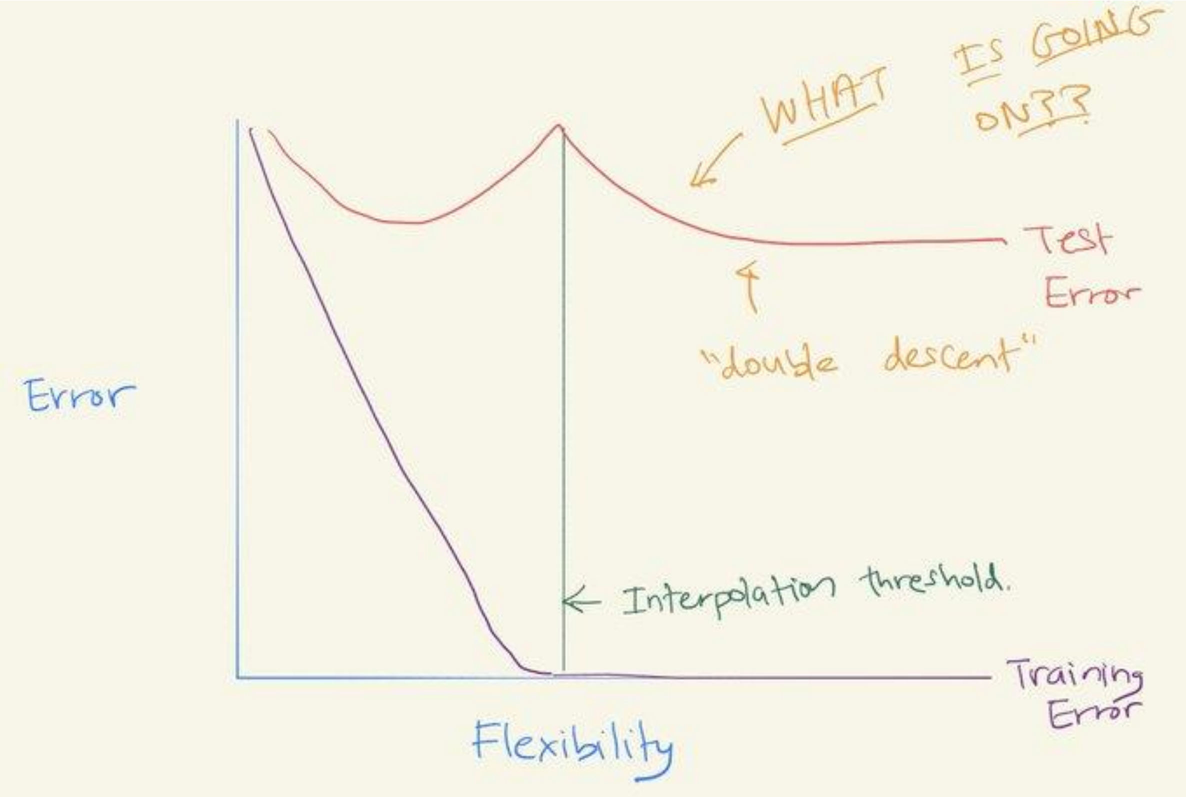


From Daniela Witten



- Err = var + $bias^2$ + irred_err
- fine-tuning of hyperparameters is essentially trading around variance and bias
- the aim is to get the lowest possible prediction error (in test data)
- flexibility is the same as model complexity

Double descent

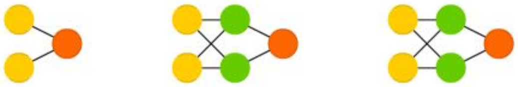


<u>double descent</u>

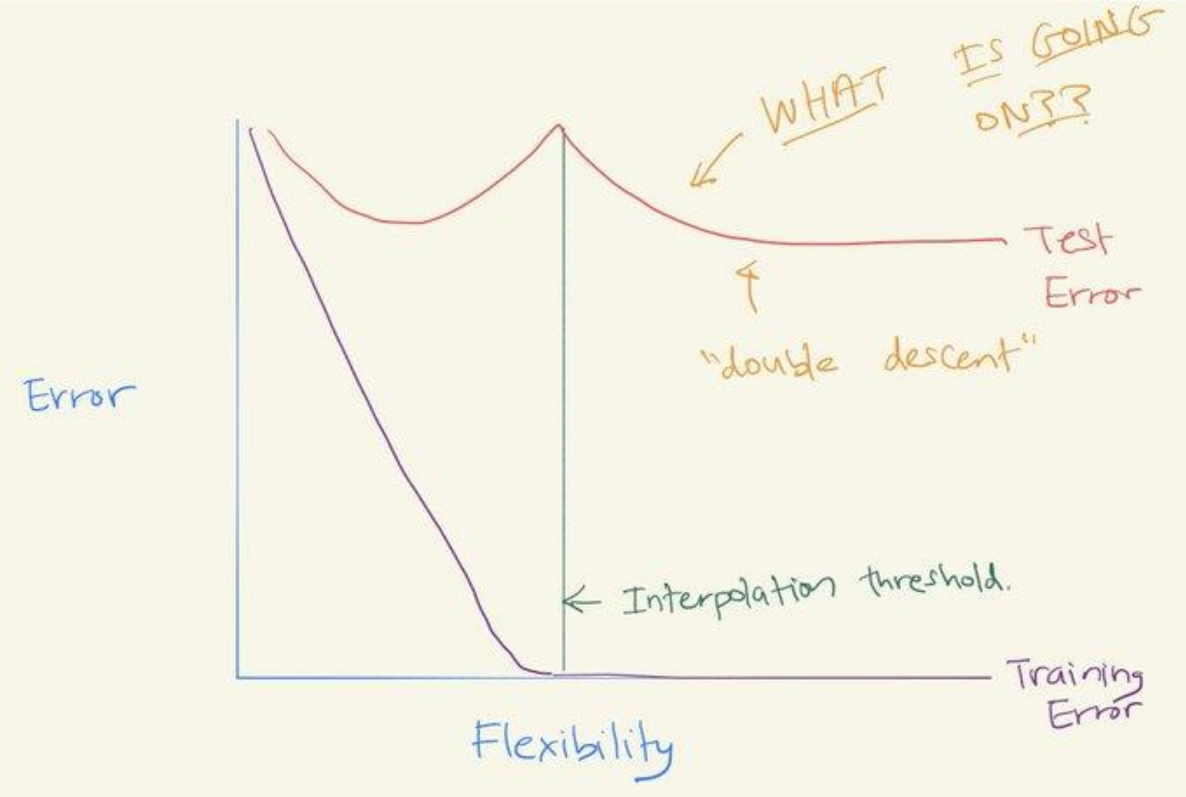
deep learning → increasingly complex models that give a second descent of the test error

remember: DL models can have millions of parameters

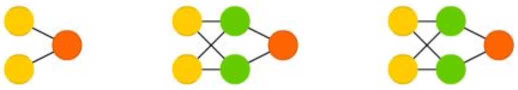
From Daniela Witten



Double descent



From Daniela Witten



<u>double descent</u>

deep learning → increasingly complex models that give a second descent of the test error

remember: DL models can have millions of parameters

<u>"scientific" questions</u>

is the bias-variance trade-off not true?

is deep learning 'magic'?

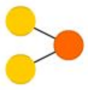
A simplified insight

polynomial regression

$$y = \beta_0 + \beta_{i1} X_i + \beta_{i2} x_i^2 + \beta_{i3} x_i^3 + \ldots + \beta_{id} x_i^d + \epsilon$$

- many parameters “p”
- highly non-linear functions

similar to deep learning



A simplified insight

polynomial regression

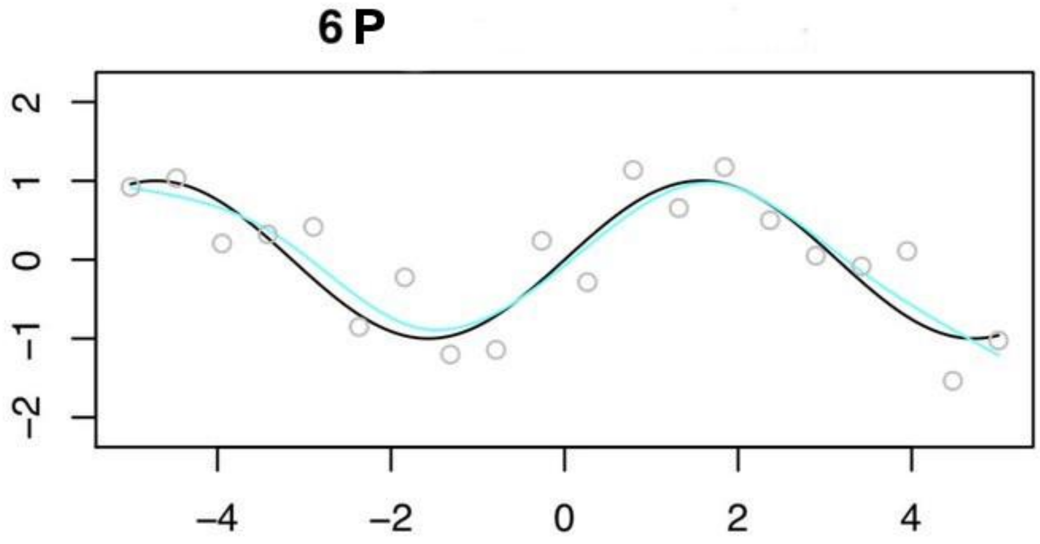
$$y = \beta_0 + \beta_{i1} X_i + \beta_{i2} x_i^2 + \beta_{i3} x_i^3 + \ldots + \beta_{id} x_i^d + \epsilon$$

- n = 20 (sample size)
- increasing n. of parameters p
- solve by least square



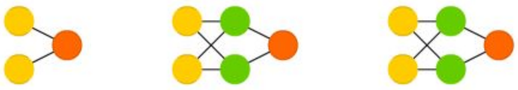
A simplified insight

polynomial regression: n = 20, p = 6



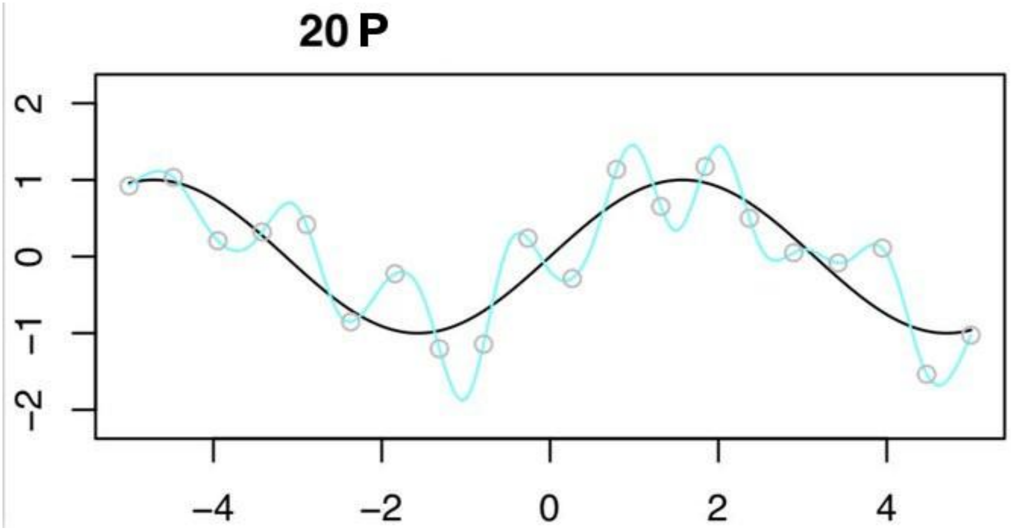
good fit! (black line: true function; blue line: fitted function)

From Daniela Witten



A simplified insight

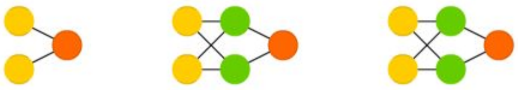
polynomial regression: n = 20, p = 20



From Daniela Witten

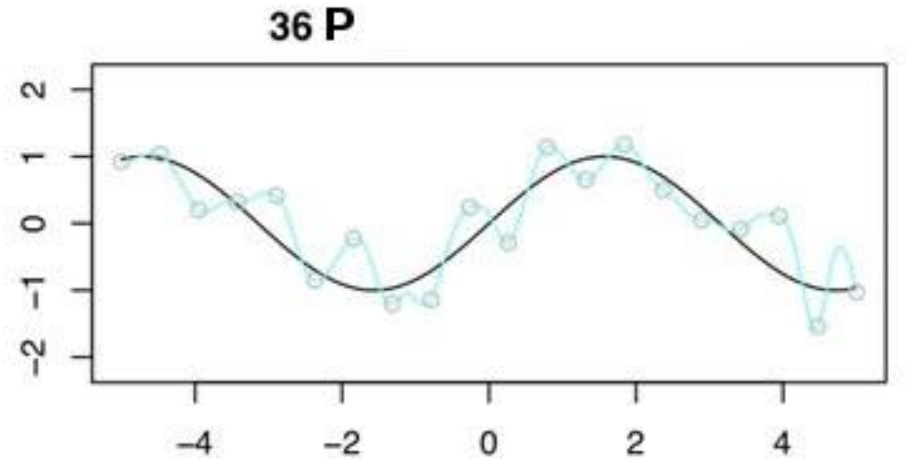
- only one least-square fit
- **zero training error**
- no generalization → **very high test error**
- complex model, with high variance and low bias

bias-variance trade-off



A simplified insight

polynomial regression: n = 20, p = 36 → p > n

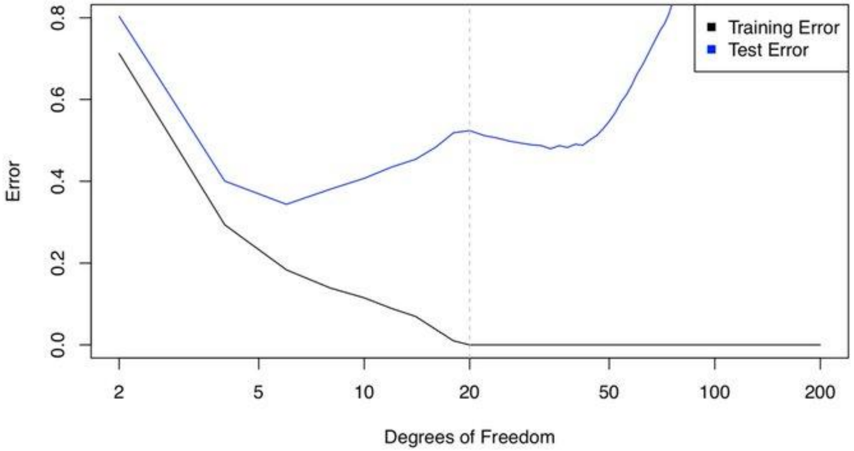


looks bad, but not worse than model with p = 20

From Daniela Witten

A simplified insight

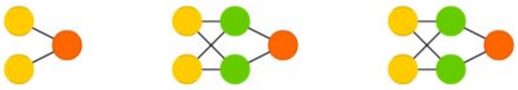
polynomial regression: n = 20, p = 36 → p > n



From Daniela Witten

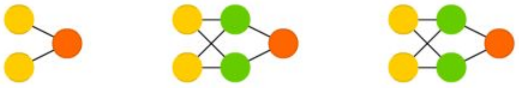
- increasing p, beyond n
- double descent: test error decreases briefly when p > n (then goes up again)

is the bias-variance trade-off not true?



What is going on?

- n = p → **only one fit**, very wiggly (zero training error, like KNN with k=1)
- p > n → **many possible fits** (no unique solution)
- model solvers / optimisers will pick (one of) the **least wiggly among such curves** (which is usually less wiggly than least square fit when n = p)
- the **mere number of parameters "p"** probably **isn't always the right quantity for model flexibility**: picking the "least wiggly" fit implies that polynomial regression with 36 p is "less flexible" than the model with 20 p



To recap

- **double descent is a real thing** that happens
- it is not magic!
- the **bias-variance trade-off still holds** and helps explain double descent

All this came from a discussion led by **Daniela Witten**:
https://www.biostat.washington.edu/people/daniela-witten
Check her material for more details and a less simplistic explanation