MODEL PREDICTIVE CONTROL

DATA-DRIVEN MPC

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COURSE STRUCTURE

- ✔ Linear model predictive control (MPC)
- ✔ Linear time-varying and nonlinear MPC
- ✔ MPC computations: quadratic programming (QP), explicit MPC
- ✔ Hybrid MPC
- ✔ Stochastic MPC
- Data-driven MPC

MATLAB Toolboxes:

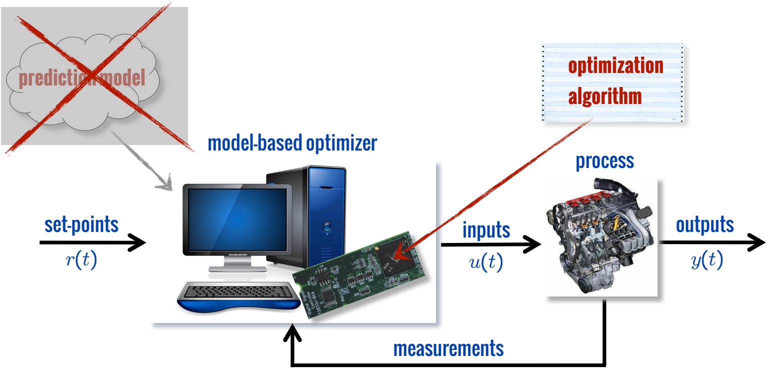
- **MPC Toolbox** (linear/explicit/parameter-varying MPC)
- **Hybrid Toolbox** (explicit MPC, hybrid systems)

Course page:

`http://cse.lab.imtlucca.it/~bemporad/mpc_course.html`

DATA DRIVEN MPC

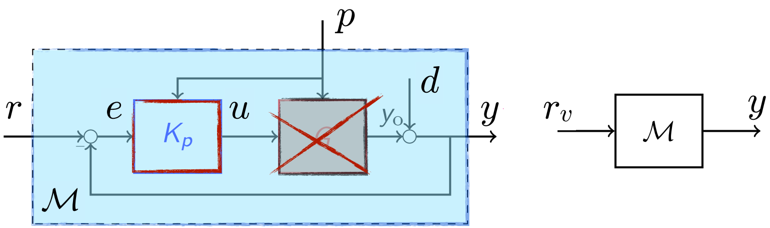
DATA-DRIVEN MPC



- Can we design an MPC controller **without** first identifying a model of the **open-loop process** ?

DATA-DRIVEN DIRECT CONTROLLER SYNTHESIS

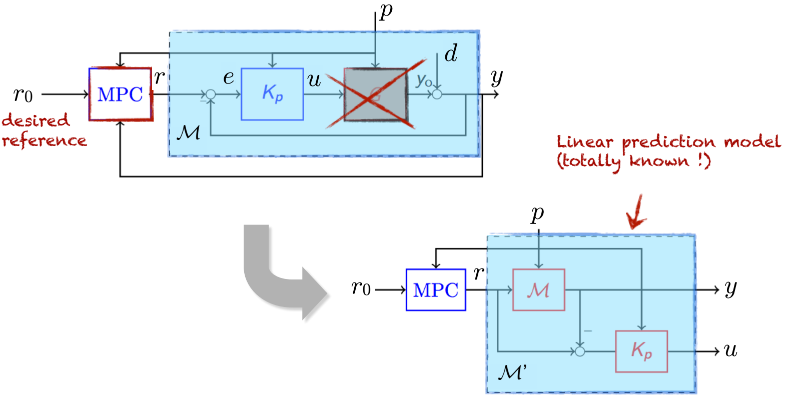
(Campi, Lecchini, Savaresi, 2002) (Formentin et al., 2015)



- Collect a set of **data** $\{u(t), y(t), p(t)\}$, $t = 1, \ldots, N$
- Specify a **desired closed-loop linear model** $\mathcal{M}$ from r to y
- Compute $r_v(t) = \mathcal{M}^{\#} y(t)$ from **pseudo-inverse model** $\mathcal{M}^{\#}$ of $\mathcal{M}$
- **Identify** linear (LPV) model K_p from $e_v = r_v - y$ (virtual tracking error) to u

- Design a linear MPC (**reference governor**) to generate the reference r

(Bemporad, Mosca, 1994) (Gilbert, Kolmanovsky, Tan, 1994)



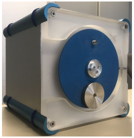
- MPC designed to handle input/output **constraints** and improve **performance**

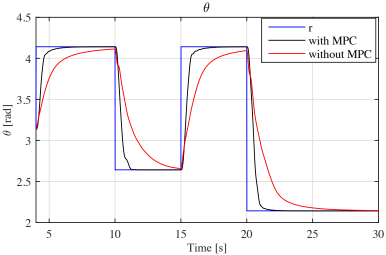
(Piga, Formentin, Bemporad, 2017)

DATA-DRIVEN MPC - AN EXAMPLE

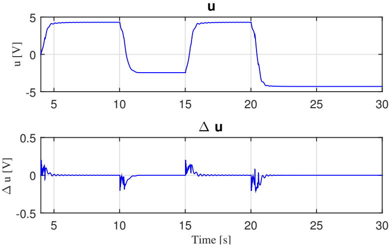
- Experimental results: MPC handles soft constraints on u, Δu and y

  (motor equipment by courtesy of TU Delft)





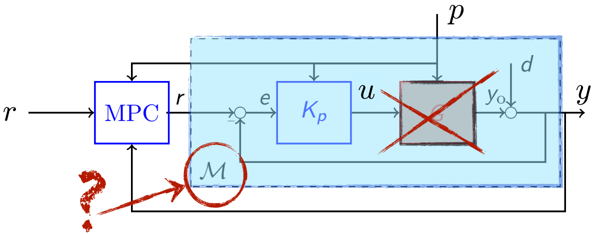
desired tracking performance achieved



constraints on input increments satisfied

No open-loop process model is identified to design the MPC controller!

- **Question:** How to choose the reference model $\mathcal{M}$?



- Can we choose $\mathcal{M}$ from data so that K_p is an **optimal controller** ?

OPTIMAL DATA-DRIVEN MPC

(Selvi, Piga, Bemporad, 2018)

- **Idea:** parameterize desired closed-loop model $\mathcal{M}(\theta)$ and optimize

$$\min_{\theta} J(\theta) = \frac{1}{N}\sum_{t=0}^{N-1} \underbrace{W_y(r(t) - y_p(\theta,t))^2 + W_{\Delta u}\Delta u_p^2(\theta,t)}_{\text{performance index}} + \underbrace{W_{\rm fit}(u(t) - u_v(\theta,t))^2}_{\text{identification error}}$$

- Evaluating $J(\theta)$ requires synthesizing $K_p(\theta)$ from data and simulating the nominal model and control law

$$y_p(\theta,t) = \mathcal{M}(\theta)r(t) \qquad u_p(\theta,t) = K_p(\theta)(r(t) - y_p(\theta,t))$$

$$\Delta u_p(\theta,t) = u_p(\theta,t) - u_p(\theta,t-1)$$

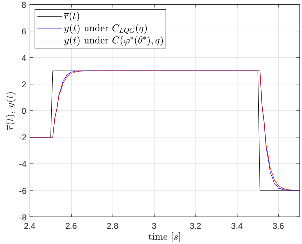
- Optimal θ obtained by solving a **(non-convex) nonlinear programming** problem

(Selvi, Piga, Bemporad, 2018)

- **Results:** **linear** process

$$G(z) = \frac{z - 0.4}{z^2 + 0.15z - 0.325}$$

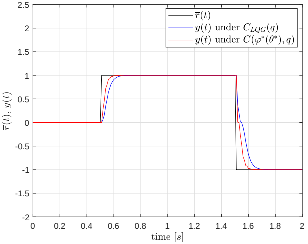
The data-driven controller is **only 1.3% worse** than model-based LQR



- **Results:** **nonlinear (Wiener)** process

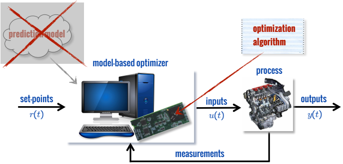
$$\begin{aligned} y_L(t) &= G(z)u(t) \\ y(t) &= |y_L(t)| \arctan(y_L(t)) \end{aligned}$$

The data-driven controller is **24% better** than LQR based on identified open-loop model !



ONGOING RESEARCH ON LEARNING MPC FROM DATA

- **Goal**: learn an MPC controller **without** a prediction model, that optimizes a given index



- **Q-learning**: optimize parameters of Q-function defining the MPC law from data. Parameters can also include model coeffs, but not necessarily

  (Zanon, Gros, Bemporad, submitted ECC'19)

- **Policy gradient methods**: learn optimal policy coefficients directly from data using stochastic gradient descent (Ferrarotti, Bemporad, submitted ECC'19)

- **Lessons learned** so far: if chosen model/policy structure does not include real plant/optimal policy
  - optimal policy **learned from data** can be better than **model-based** optimal policy
  - when open-loop model is used as a tuning parameter, **learned model** can be quite different from best **open-loop model** that can be identified from the same data