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# Modeling and numerical simulations of dendritic crystal growth

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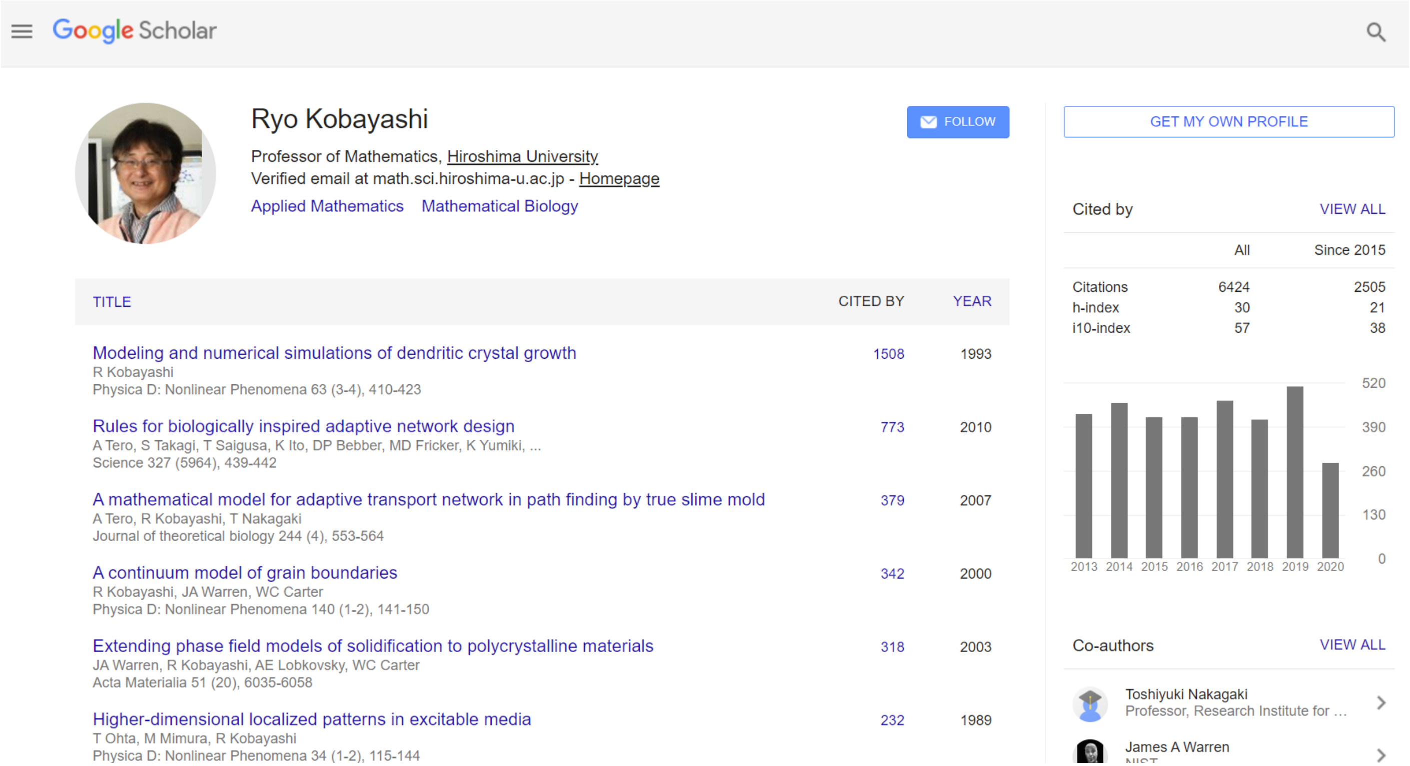


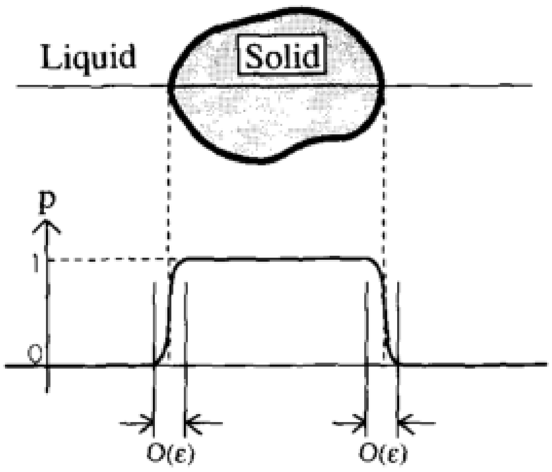
晶体生长是一种自然界中自发形成的现象。如雪花，它们形成于接近均匀的环境，却有各种各样非常美丽和复杂的枝晶形状，枝晶结构也常见于金属的凝固或过饱和溶液中的结晶。

晶体的存在或平衡，或生长，本视频中使用到的理论认为，其通过使表面能最小来趋近于平衡状态。

因此，如果表面能是各向同性的，则晶体为球形；如果存在各向异性，则为多边形。

$$F(\varphi,m)=\boxed{\int_V \frac{1}{2}\varepsilon^2\,|\nabla\varphi|^2}+\boxed{f(\varphi,m)dv}$$

能量梯度项　　局部自由能项




各向异性梯度能量系数: $\varepsilon=\bar{\varepsilon}\sigma(\theta)$

$$\sigma(\theta)=1+\delta\cos\left(j\left(\theta-\theta_o\right)\right)$$

$$f(\varphi,m)=\frac{1}{4}\varphi^4-\left(\frac{1}{2}-\frac{1}{3}m\right)\varphi^3+\left(\frac{1}{4}-\frac{1}{2}m\right)\varphi^2$$

初始偏移角: $\theta=\tan^{-1}\left(\frac{\partial\varphi/\partial y}{\partial\varphi/\partial x}\right)$

界面驱动力: $m(T)=\left(\frac{\alpha}{\pi}\right)\tan^{-1}\left[\gamma\left(T_{eq}-T\right)\right]$

演化过程假设: $\tau\frac{\partial\varphi}{\partial t}=-\frac{\delta F}{\delta\phi}$

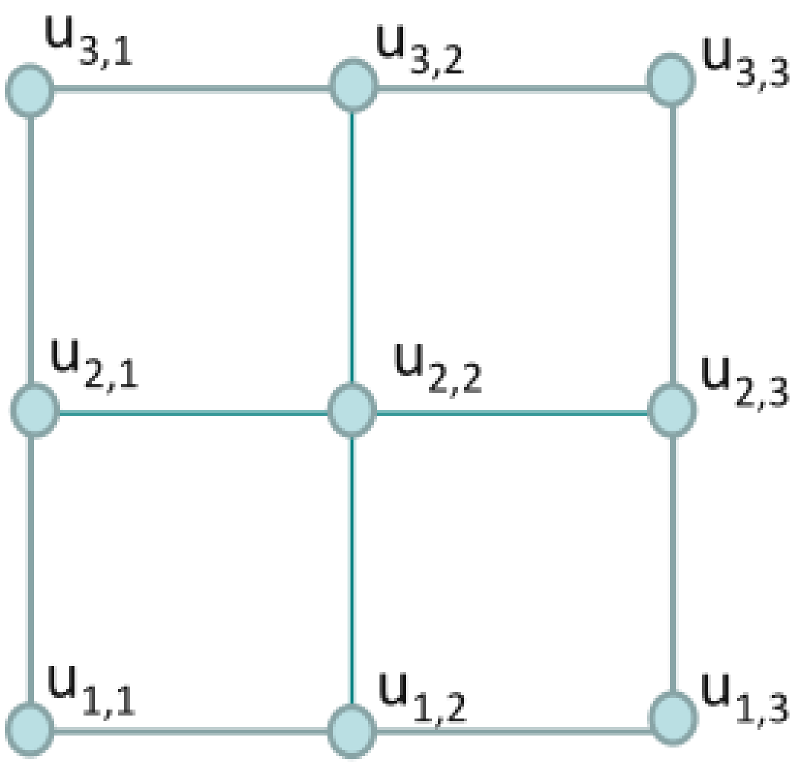
相演化离散方程:

$$\tau\frac{\partial\varphi}{\partial t}=\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial\varepsilon}{\partial\theta}\frac{\partial\varphi}{\partial x}\right)-\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial\varepsilon}{\partial\theta}\frac{\partial\varphi}{\partial y}\right)+\nabla\cdot\left(\varepsilon^2\nabla\varphi\right)+\varphi(1-\varphi)\left(\varphi-\frac{1}{2}+m\right)$$

温度演化离散方程:

$$\frac{\partial T}{\partial t}=\nabla^2T+\kappa\frac{\partial\varphi}{\partial t}$$

$$\left(\nabla^2 u\right)_{i,j} = \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{h^2}$$



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tau = 0.0003; %时间演化系数
epsilonb = 0.01; %各向异性梯度能量系数平均值0.01
kappa = 1.8; %无量纲潜热，正比于潜热，反比于冷却强度1.8
delta = 0.02; %各向异性强度0.02
aniso = 6.0; %j：各向异性模数4&6
alpha = 0.9; %过冷系数0.9
gamma = 10.0; %温差放大系数10.0
teq = 1.0; %平衡温度1.0
theta0 = 0.2; %初始偏移角0.2
seed = 5.0; %定义晶核种子大小5.0
```