



Modeling and numerical simulations of dendritic crystal growth

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晶体生长是一种自然界中自发形成的现象。如雪花，它们形成于接近均匀的环境，却有各种各样非常美丽和复杂的枝晶形状，枝晶结构也常见于金属的凝固或过饱和溶液中的结晶。

晶体的存在或平衡，或生长，本视频中使用到的理论认为，其通过使表面能最小来趋近于平衡状态。

因此，如果表面能是各向同性的，则晶体为球形；如果存在各向异性，则为多边形。

$$F(\varphi, m) = \int_V \frac{1}{2} \varepsilon^2 |\nabla \varphi|^2 + f(\varphi, m) dv$$

能量梯度项 局部自由能项

各向异性梯度能量系数: $\varepsilon = \overline{\varepsilon} \sigma(\theta)$

$$\sigma(\theta) = 1 + \delta \cos(j(\theta - \theta_o))$$

$$f(\varphi, m) = \frac{1}{4} \varphi^4 - \left(\frac{1}{2} - \frac{1}{3} m \right) \varphi^3 + \left(\frac{1}{4} - \frac{1}{2} m \right) \varphi^2$$

初始偏移角: $\theta = \tan^{-1} \left(\frac{\partial \varphi / \partial y}{\partial \varphi / \partial x} \right)$

界面驱动力: $m(T) = \left(\frac{\alpha}{\pi} \right) \tan^{-1} \left[\gamma (T_{eq} - T) \right]$

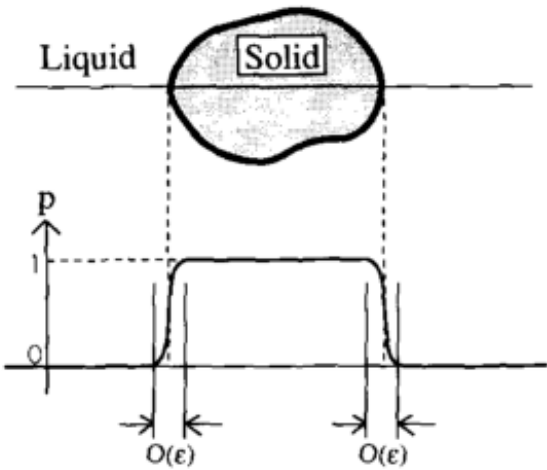
演化过程假设: $\tau \frac{\partial \varphi}{\partial t} = - \frac{\delta F}{\delta \phi}$

相演化离散方程:

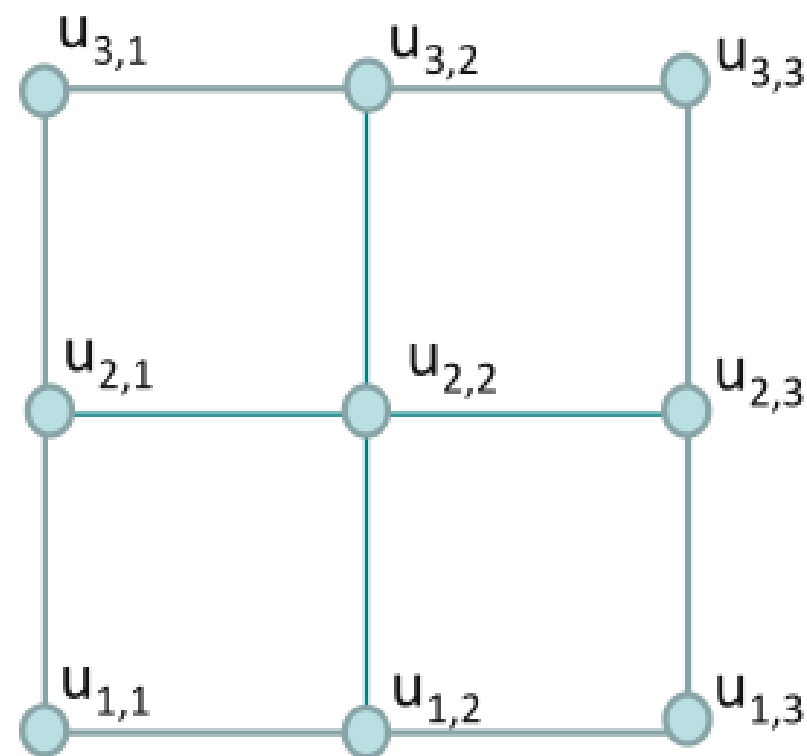
温度演化离散方程:

$$\tau \frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial y} \left(\varepsilon \frac{\partial \varepsilon}{\partial \theta} \frac{\partial \varphi}{\partial x} \right) - \frac{\partial}{\partial x} \left(\varepsilon \frac{\partial \varepsilon}{\partial \theta} \frac{\partial \varphi}{\partial y} \right) + \nabla \cdot (\varepsilon^2 \nabla \varphi) + \varphi(1 - \varphi) \left(\varphi - \frac{1}{2} + m \right)$$

$$\frac{\partial T}{\partial t} = \nabla^2 T + \kappa \frac{\partial \varphi}{\partial t}$$



$$(\nabla^2 u)_{i,j} = \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{h^2}$$



```
tau = 0.0003; %时间演化系数
epsilonb = 0.01; %各向异性梯度能量系数平均值0.01
kappa = 1.8; %无量纲潜热, 正比于潜热, 反比于冷却强度1.8
delta = 0.02; %各向异性强度0.02
aniso = 6.0; %j: 各向异性模数4&6
alpha = 0.9; %过冷系数0.9
gamma = 10.0; %温差放大系数10.0
teq = 1.0; %平衡温度1.0
theta0 = 0.2; %初始偏移角0.2
seed = 5.0; %定义晶核种子大小5.0
```