# Evapotranspiration estimation models

## Introduction

The estimation of evapotranspiration and water masses evaporation plays a significant role in the compilation of water mass balance, in the assessment of water needs and in other applications of hydrological interest. The examined methodologies concern the indirect calculation, by means of hydrometeorological variables, of monthly time step that can also be applied to daily time step. The applied methodologies at “Hydrognomon”, classified under the requirement of number of series, are the following:

Analytical models (Penman, for the evaporation estimation and Penman-Monteith, for the evapotranspiration estimation) that require four hydrometeological variables (mean temperature, relevant humidity, wind speed and sunshine duration).

* Empirical evapotranspiration estimation models (Thornthwaite, Hargreaves και Blaney-Criddle) that require temperature time series.

Special methodology for the extension and completion of evaporation or evapotranspiration time series that require temperature time series and a certain sample of calculated values by the Penman or the Penman-Monteith method.

## The Penman / Penman-Monteith models

The Penman water surface evaporation estimation model as well as the modification of that model by Monteith (Penman-Monteith model) for the estimation of potential evapotranspiration, are based on relations of physics and produce the most accurate results regarding other methodologies. The main disadvantage of these methods is the number of hydrometeorological variables that they demand. More specifically, individual time series are required for:

Mean temperature *Ta*, °C

* Relevant humidity *U*, %
* Relevant sunshine *n*/*N*, %

Wind speed u, m/s.

These time series have a monthly time step. However, daily time step can be used for the daily evaporation or evapotranspiration estimation, though producing less reliable results. As we will see later on, as far as the relevant sunshine, the user can insert the sunshine duration in minutes or alternatively, the measured amount of short waves clear radiation *Sn,m*.

### Calculation of natural variables

In order to estimate the evaporation or evapotranspiration, certain variables need to be calculated first, in the specific order presented below.

#### Water vapor saturation pressure *e*\*, hPa

Given the mean air temperature, it can be calculated by the formula:

 1.1

#### Slope of water vapor saturation curve *Δ*, hPa/K

Given the mean air temperature and the calculated water vapor saturation pressure *e*\*, it can be calculated by the formula:

*Δ* 1.2

#### Latent heat of evaporation *λ*, kJ/kg

For the calculation of *λ*, the temperature *Ts* (in °C) on the water’s surfaceis needed. It is assumed that this is equal to the air temperature (*Ts*=*Ta*). *λ* is given by the formula:

*λ* = 2501 - 2.361 *Ts* 1.3

#### Psychrometric coefficient *γ*, hPa/K

It can be derived from the formula:

 1.4

where the *λ* is calculated as mentioned above, *cp* is the specific air heat at constant pressure (default value 1.013 kJ/kg/°C), *ε* is the molecular weight ratio of water to dry air (considered to have a value of 0.622) and *p* is the atmospheric pressure, where a mean value in relation to the altitude is used according to the formula:

*p* =1013.25 (1-2.256×10-5*z*)5.256 1.5

where, *z* is the altitude in m.

#### Day numeration, *J*

When applying a daily time step,a number between 1 (1 January) and 365 (December 31) can be simply used for the day numbering. In casemonthly time step, a representative value can be used for each month that derives from the formula:

*J* = *J*0+(*μ*+1)%2-1 1.6

where *μ* is the number of days of the month, *J*0 the number of the first day of the month and % the integer division symbol.

#### Sun declination *δ*, rad

This variable representsthe latitude where the sun’s rays are perpendicular to the earth’s surface during the sun’s culmination:

 1.7

#### Sunset solar angle *ωs*, rad

Using the calculated *δ* and the region’s latitude *φ*, the sunset solar angle can be derived from the formula:

*ωs* = cos-1(-tan *φ* tan *δ*) 1.8

Usage of this equation is limited to geographic latitudes |*φ*|<66.5°.

#### Astronomical day’s duration *N*, h

Using the calculated *ωs*, the duration of the astronomical day derives from the formula:

 1.9

#### Extraterrestrial solar radiation *S*0, kJ/m²/d

Using the calculated *ωs*, and sun declination *δ*, the extraterrestrial solar radiation derives from the formula:

 1.10

where *td* is the mean day duration (86400 s), *Is* the solar constant (1.367 kW/m²) and *dr* is the eccentricity derived from the formula:

 1.11

#### Sunshine duration *n*, h

The user has the following alternatives concerning the insertion of the sunshine’s representative amount:

Sunshine percentage *n*/*N*

* Total sunshine duration in minutes (min) (monthly sunshine duration should be divided by the sum of the days of the month *μ*).

Measured radiation *Sn*,*m*.

As long as the issue is to calculate the sunshine duration in hours (h) the above three amounts are estimated by the following ways:

Percentage’s multiplication by the astronomical day’s duration *N*

* Day’s duration divided by 60 in order to convert it in hours

Application of the Relation 1.12.

 1.12

where the indexes *as* και *bs* are examined below.

#### Atmospheric absorption coefficient (Prescott) *fs*

The coefficient *f*s results from the formula:

*fs* = *as* + *bs* *n*/*N* 1.13

The *as*, *bs* are considered to have the default values *as*=0.25, *bs*=0.50. As for the sunshine duration, the calculated value from the previous paragraph is used.

#### Net shortwave radiation *Sn*, kJ/m²/d

It is calculated by the relation:

*Sn* = (1-*a*) *fsS*0 1.14

As long as the sunshine duration is calculated from equation 1.12, *Sn* will differ from the measured value *Sn*,*m* by (1-*a*). That is because the formula 1.12 derives from the Prescott equation resolved to *n*. *fs* is obtained from the formula 1.14 by setting the albedo value to zero (as long as there is a radiation measuring device with practically zero reflectivity).

The coefficient *a* is called albedo and is defined as the ratio of the reflected radiation from the surface to the incident radiation upon it. It will be inserted by the user. However, the following default values should be providedregarding the application of the Penman method, where the evaporation rate is calculated, the typical value should be *a*=0.08 and regarding the application of the Penman-Monteith method, the typical value should be *a*=0.25 whic is a typical value for crops.

#### Nebulosity impact coefficient *fL*

It derives from the following formula:

*fL* = *aL* + *bL* *n*/*N* 1.15

where the default values for *aL*, *bL* *aL*=0.10 και *bL*=0.90 are used.

#### Net emission ability *εn*

It derives from the Brunt formula:

 1.16

where *e* is the vapor pressure measured in hPa and derives from the equation:

*e* = *U e*\* 1.17

where *U* is the relative humidity percentage.

The dimensionless parameters *ae* and *be* are considered to be *ae*=0.56 και *be*=0.08 for the Penman method and *ae*=0.34 και *be*=0.044 for the Penman-Monteith method.

#### Net longwave radiation *Ln*, kJ/m²/d

It derives from the equation:

*Ln* = *εn* *fL* *σ* (*Ta* + 273)4 1.18

where σ is the Stefan-Boltzmann constant *σ*=4.9×10-6 kJ/(m²K4d) and *Ta* is the mean air temperature expressed in °C.

#### Total net radiation energy at the earth surface *Rn*, kJ/m²/d

It is the difference between the net shortwave radiation and the net longwave radiation:

*Rn* = *Sn* – *Ln* 1.19

#### Typical day of the month

In cases where a specific day of the year is required, e.g. the extraterrestrial solar radiation *S0* is not always given. Such cases are when the calculation is done on a monthly scale and requires a typical day of the month in order to determine the desired variable. One approach is through the use of the mean day of the month, i.e. adding to the fisrt day the number of the half days of the month. A more accurate approach is based on the “equation of time” ( since the day duration is not linearly distributed throughout the year) and is summarized in the following table (Koutsoyiannis and Xanthopoulos, 1999, p. 173):

Table 1.1: Representative duration (in days) for each month for the calculation of the average monthly day duration and the average monthly solar radiation and for latitudes not greater than 60° (North and South).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Month** | **Day** | **Number of the day (J)** | **Month** | **Day** | **Number of the day (J)** |
| January | 18 | 18 | July | 18 | 199 |
| February | 15 | 46 | August | 17 | 229 |
| March | 16 | 75 | September | 16 | 259 |
| April | 15 | 105 | October | 16 | 289 |
| Μay | 15 | 135 | November | 14 | 318 |
| June | 11 | 162 | December | 11 | 345 |

### The Penman model

The evaporation according to the Penman model, can be calculated through the combination of the evaporation due to advection and energy balance. Finally, the mean daily evaporation (mm/d) derives from a single equation:

 1.20

If the monthly time step is used, then the above equationshould be multiplied by the number of days of the month to give the total monthly evaporation. The variables inside the Penman equation are calculated according to preceding paragraphs. In addition, the wind function *F*(*u*) and the saturation deficit *D,* are calculated according to the following relations:

*F*(*u*) = 0.13 + 0.140 *u* 1.21

where *u* is the wind speed expressed in m/s.

*D* = *e*\* (1-*U*) 1.22

where *U* is the relevant humidity percentage and *e*\* is the water vapor saturation pressure.

### The Penman – Monteith model

The Penman-Monteith model is a modification of the original Penman equation. The potential evapotranspiration (rate per day - mm/d) derives from the formula:

 1.23

The wind function *F*(*u*) is given by the formula:

 1.24

where *u* is the wind speed expressed in m/s and *Ta* the mean temperature expressed in °C. The reduced psychrometric coefficient *γ*’ derives from the relation:

*γ’* = *γ* (1+0.33 *u*) 1.25

where *γ* is the Psychrometric coefficient for the water vapor and *u* is the wind speed expressed in m/s.

### Numerical implementation - check results

The Koutsoyiannis and Xanthopoulos applications (1999, p 210) are resolved, in order to check the algorithms’ reliability of the Penman and Penman-Monteith models. The latitude *φ* is 38°45’ 'and the altitude *Η* is 145 m.

Table 1.2: Comparisons between the results of the methods Penman and Penman-Monteith. Data is derived from Koutsoyiannis and Xanthopoulos (1999, σ. 210).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Month** | ***Ta***  **°C** | ***U* %** | ***u* m/s** | ***n*/*N*** | **Penman (model)** | **Penman (data)** | **Penman – Monteith (model)** | **Penman – Monteith (data)** |
| Oct | 18.7 | 68 | 1.6 | 0.56 | 74.59 | 74.6 | 69.56 | 69.4 |
| Nov | 13.3 | 69 | 1.6 | 0.70 | 36.58 | 36.6 | 41.58 | 41.4 |
| Dec | 8.5 | 72 | 2.0 | 0.44 | 25.79 | 26.3 | 30.96 | 31.1 |
| Jan | 10.4 | 74 | 3.7 | 0.33 | 43.30 | 43.7 | 44.74 | 44.7 |
| Feb | 9.8 | 70 | 3.4 | 0.34 | 51.34 | 51.9 | 48.6 | 48.6 |
| Mar | 7.1 | 67 | 3.3 | 0.45 | 73.18 | 73.2 | 61.89 | 61.6 |
| Apr | 11.3 | 64 | 3.3 | 0.55 | 110.21 | 110.3 | 89.99 | 89.7 |
| May | 17.6 | 64 | 2.4 | 0.54 | 147.35 | 147.0 | 121.89 | 121.3 |
| June | 22.3 | 58 | 2.0 | 0.72 | 185.41 | 185.0 | 156.3 | 155.6 |
| July | 27.3 | 57 | 3.3 | 0.74 | 236.07 | 235.8 | 203.86 | 203.0 |
| Aug | 26.0 | 61 | 2.1 | 0.80 | 193.01 | 193.1 | 164.25 | 163.9 |
| Sep | 25.8 | 62 | 2.4 | 0.70 | 148.67 | 148.8 | 132.75 | 132.4 |
| Total |  | | | | **1325.50** | **1326.3** | **1166.37** | **1162** |

The differences between the calculated values ​​and the ones derived from Koutsoyiannis and Xanthopoulos (1999) are slightly different (0.1% order of magnitude). These differences are due tothe slightly modified equations.

## Thornthwaite, Hargreaves and Blaney-Criddle models

The models discussed in this section are parsimonious regarding to the amount of the required time series as only the temperature time series is sufficient. These models are based on empirical assumptions rather than the modeling of the exact physical mechanisms which describe the phenomenon and they are known as the Thornthwaite, Hargreaves and Blaney-Criddle methodologies (Koutsoyiannis and Xanthopoulos, 1999, p. 225). The Thornthwaite and Blaney-Criddle methods may be used solely to calculate monthly values of evapotranspiration, while the Hargreaves method can be used to estimate daily evapotranspiration as long as daily data for temperature is available.

### The Thornthwaite method

Although this method is the oldest one (dating back to 1948), it has the simplest computational process and therefore it is the most widely implemented. Monthly potential evapotranspiration is calculated by using the formula:

 6.26

where *Ep* is the potential evapotranspiration in mm/month, *Ta* ​​the mean monthly temperature of the month in °C, *μ* the number of the days of the month, *N* the mean astronomical duration of day in the mid of the month, *I* is the annual thermal index empirical coefficient and *a* is another empirical coefficient, which is function of *I*. The empirical coefficients can be calculated from the sample of the mean monthly temperatures of the year (*Taj*, *j* = 1 .. 12) as shown by the following relations:

 6.27

 6.28

 6.29

If the estimation of certain values for *ij* is not possible due to missing temperature values, then the user is notified that the calculation procedure is abandoned. Table 6.3 compares the calculated evapotranspiration values ​​used in the example of Koutsoyiannis and Xanthopoulos (1999, p 228); the differences between actual and calculated values are small and arise out from the use of a mean astronomical daily duration table in the solved example.

Table 6.3: Comparison of the Thornthwaite evapotranspiration estimation method results. Data taken from Koutsoyiannis and Xanthopoulos (1999, p 228).

|  |  |  |  |
| --- | --- | --- | --- |
| **Month** | ***Ta* (°C)** | ***Ep* (mm) – actual** | ***Ep* (mm) - calculated** |
| Oct | 18.70 | 69.9 | 70.17 |
| Nov | 13.30 | 33.7 | 33.69 |
| Dec | 8.50 | 15.1 | 15.07 |
| Jan | 10.40 | 22.1 | 21.97 |
| Feb | 9.80 | 19.7 | 19.62 |
| Mar | 7.10 | 13.9 | 13.81 |
| Apr | 11.30 | 33.4 | 33.39 |
| May | 17.60 | 80.9 | 80.90 |
| Jun | 22.30 | 123.0 | 123.26 |
| Jul | 27.30 | 177.9 | 178.42 |
| Aug | 26.00 | 152.9 | 153.37 |
| Sep | 25.80 | 132.7 | 132.69 |
|  | | **875.1** | **876.36** |

### The Hargreaves method

The Hargreaves method is used to estimate the evapotranspiration of a reference crop. As it was mentioned above this model can be used with monthly or daily temperature data and the value of the reference evapotransiration *Erc* is calculated by the relation (mm/d):

 6.30

where *S*0 is the extraterrestrial solar radiation measured in kJ/(m²d), *λ* is the latent heat of vaporization with a constant value *λ*=2460 kJ/kg, *Ta* is the mean temperature and *T*max, *T*min, are the maximum and minimum temperature time series, which can be given to the program as two individual time series or as the unique difference of them. Finally, if Hargreaves method is used for the estimation of daily evapotranspiration then *S*0 is calculated for each specific day, whereas if it is used for monthly estimation then *S*0 is calculated in the mid of the month as it was described in section 1.2.

### The Blaney-Criddle method

The Blaney-Criddle method has been used widely for the determination of irrigation needs. Its original formulation, as it is described by Koutsoyiannis and Xanthopoulos (1999, p. 225) that estimates the potential monthly evapotranspiration of a crop, is implemented in "Hydrognomon". In this formulation the only meteorological variable needed is the mean monthly temperature *T*a , as described by the following relation:

*Ep* = 0.254 *kc* *p* (32 + 1.8 *Ta*) 6.31

where *Ep* is the potential evapotranspiration (mm/month), *kc* crop uptake coefficient (plant growth coefficient) and *p* the percentage (%) of daylight hours of the specific month in relation to the sum of the whole year’s daylight hours. This percentage is calculated by the formula:

 6.32

where *N* the mean astronomical duration of day in the mid of the month (h) and *μ* the number of days of this specific month.

## Evapotranspiration time series extension and completion

The methodology presented here has been proposed by Koutsoyiannis and Xanthopoulos (1997, p. 222) and has been successfully used to extend evaporation samples from lakes (Efstratiadis et al., 2000) [και αλλού – and other open water surfaces]. It is described by a relatively simple analytical formulation that estimates the evaporation (or evapotranspiration) in relation to temperature *Ta*, the amount of the extraterrestrial radiation *S*0 and three coefficients a, b and c. The values ​​of these coefficients are obtained by the application of a best-fit procedure on a sample of available values ​​of evaporation or evapotranspiration, which have been estimated by Penman or Penman-Monteith method. The relation proposed is:

 6.33

where *E* is the evaporation height per day (mm/d), *Ta* the mean temperature (°C), *a*, *b* and *c* are the parameters determined by best-fit procedure and *S*0 the extraterrestrial radiation (kJ/m²/d). *S*0 is calculated on the current day for analysis on the daily time step, or in the mid of current month in the case of monthly time step (as described in the Penman methodology); and it is estimated analytically, which means that there is no need for measured data. Typical values ​​for *a*, *b* ​​and *c* are: *a*≈10-4, *b*≈0.5 and *c*≈0.02. These values ​​ however, should not be applied arbitrarily in any case, but should be adapted to local conditions using an existing, pre-calculated sample as described below.

### Best-fit procedure

The estimation method of the coefficients *a*, *b* ​​and *c* is based on an iterative least squares method, as it was modified by Balodimos (1991). If *a*,*b* and *c* are the unknown parameters and *a*0, *b*0 and *c*0 their initial, arbitrary estimates (e.g. the typical values ​​ *a*=10-4, *b*=0.5 and *c*=0.02), then the estimated vector of change for the ith computational iteration is:

 6.34

Therefore, the value of each parameter in the ith iteration becomes:

[*xi*] = [*xi-1*] + *δ*[*xi*] 6.35

We assume convergence to the final solution when the Euclidean norm of *δ*[*xi*] falls below a certain threshold *ε*, in which case the calculation process is terminated:

|*δ*[*xi*]| ≤ *ε* 6.36

This threshold is set at *ε*=0.0001. Moreover the irritation procedure will not end if *i*<3, while if *i*>10 then the initial, arbitrary values are used for each value​​. The vector *δ*[*xi*] is determined by the solution of the following linear system of equations:

[*Ν*] *δ*[*x*] = [*A*]*Τ* [*δl*]*Τ* 6.37

where [*N*] is a 3x3 matrix determined by:

[*Ν*] = [*Α*]*Τ* [*Α*] 6.38

Let *Ej*, *j*=1…*n* a sample of evaporation (or evapotranspiration) data which have been already estimated by the Penman (or Penman - Monteith) method, called here and thereafter “measured values”, and *Taj*, *j*=1…*n* the corresponding mean air temperature values. The [*A*] matrix, with dimensions n rows × 3 columns, is defined as:

 6.39

where *Ε*(*Τaj*) the analytical formulation of the parametric equation for evaporation 6.33. The partial derivatives are calculated analytically by:

 6.40

where *ai*-1, *bi*-1 και *ci*-1 the estimated parameter values by the preceding iteration.

Finally, the last variable of 6.37, the [*δl*] vector with dimension *n*, is the difference between the estimated evaporation value (as it was determined by the *a*,*b*,*c* coefficients of the last iteration) and the measured value of evaporation, that equals to:

 6.41

where *μ* is the number of days of the month corresponding to the *j* measurement for the monthly time step, or *μ*=1 for the daily time step.

Usually only few (3 or 4) iterations are required to establish the desired accuracy in order to estimate the final values ​​of *a*, *b* ​​and *c*. ]. [Με αυτήν την παραδοχή ο συντελεστής αναφέρεται στο *i*-1th βήμα, ωστόσο δεν διαφοροποιείται ιδιαίτερα μετά τον τρίτο κύκλο, έτσι υιοθετούμε την τιμή που προκύπτει για το προτελευταίο βήμα.] To determine this, a determination coefficient *δ* is easily calculated in each time step from the vector [*δl*]:

 6.43

where *w* the sum of square error:

*w* = | [*δl*] [*δl*]*T* | 6.42

and | | the Euclidean norm, while *σTa*2 is the variance of the temperature sample for the period corresponding to the measured *Ej*.

### Application of the best-fit procedure

In this section, the application of the above method in an evaporation time series which has been estimated by the Penman method, is demonstrated, in order to examine the method’s efficiency. The data used were provided by the NTUA meteorological station (latitude φ=37°58′24″) and the available sample size covered a period from February 1994 until January 2004. The resulting parametric model fit is represented in Figure 1.1.

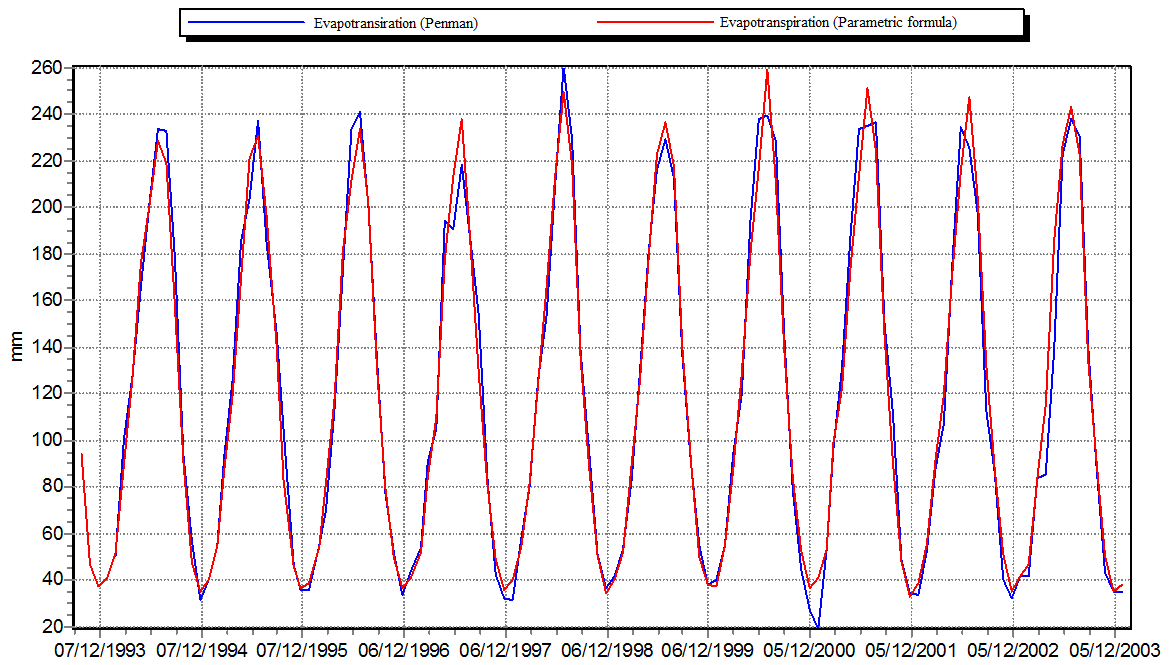


Figure 1.1: Parametric model fit in the evaporation sample of NTUA station.

The determination coefficient was *δ* = 0.977, while the values of the best-fit coefficients were *a* = 0.958×10−4, *b* = 0.519, *c* = 0.0205, which do not deviate from the typical values (i.e. ​​ *a*=10-4, *b*=0.5 and *c*=0.02).