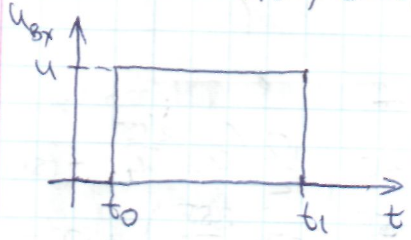
Варіант 1

2. $R = 1\,МОм$; $C = 0{,}1\,мкФ$; $U_{вх} = 1В$; $t_1 = 3\,мкс$



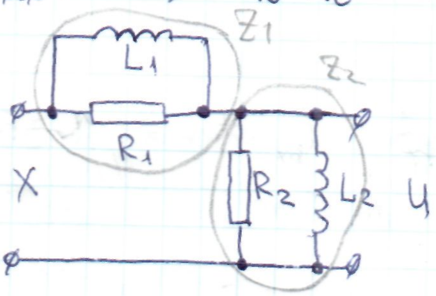
$$U_{вих} = -\frac{1}{RC}\int_{t_0}^{t_1} U_{вх}\,dt$$

$U_{вх}(t) = U$ при $t \in [t_0; t_1]$

$$U_{вих} = -\frac{1}{RC}\int_{t_0}^{t_1} U\,dt = -\frac{1}{RC}\,Ut\Big|_{t_0}^{t_1}$$

$$U_{вих}(3\,мкс) = -\frac{1}{10^{6}\cdot 10^{-7}}\cdot 3\cdot 10^{-6} = -3\cdot 10^{-5}\ В$$

3.



$$Z_1 = \frac{R_1\cdot j\omega L_1}{R_1 + j\omega L_1}$$

$$Z_2 = \frac{R_2\cdot j\omega L_2}{R_2 + j\omega L_2}$$

$$\dot{K}(j\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{\dfrac{R_2\cdot j\omega L_2}{R_2 + j\omega L_2}}{\dfrac{R_1\cdot j\omega L_1}{R_1 + j\omega L_1} + \dfrac{R_2\cdot j\omega L_2}{R_2 + j\omega L_2}} \quad (R_1 + j\omega L_1) * (R_2 + j\omega L_2) =$$

$$= \frac{R_2\cdot j\omega L_2\cdot(R_1 + j\omega L_1)}{R_1\cdot j\omega L_1\cdot(R_2 + j\omega L_2) + R_2 j\omega L_2 (R_1 + j\omega L_1)} =$$

$$= \frac{R_2 R_1 j\omega L_2 - R_2\omega^2 L_1 L_2}{R_1 R_2 j\omega L_1 - R_1\omega^2 L_1 L_2 + R_1 R_2 j\omega L_2 - R_2\omega^2 L_1 L_2} =$$

$$= \frac{R_2\omega^2 L_1 L_2 - jR_1 R_2\omega L_2}{\underbrace{\omega^2 L_1 L_2 (R_1 + R_2) - jR_1 R_2\omega(L_1 + L_2)}_{Z_n}} \left(* Z_n^* \right. =$$

$$= \frac{\left(R_2 \omega^2 L_1 L_2 - j R_1 R_2 \omega L_2\right)\left(\omega^2 L_1 L_2 (R_1+R_2) + j R_1 R_2 \omega (L_1+L_2)\right)}{\left(\omega^2 L_1 L_2 (R_1+R_2)\right)^2 + \left(R_1 R_2 \omega (L_1+L_2)\right)^2}$$

$$= \frac{R_2 \omega^4 L_1^2 L_2^2 (R_1+R_2) + j R_1 R_2^2 \omega^3 L_1 L_2 (L_1+L_2)}{\underbrace{\omega^4 L_1^2 L_2^2 (R_1+R_2)^2 + R_1^2 R_2^2 \omega^2 (L_1+L_2)^2}_{= Z_n}} +$$

$$+ \frac{-j R_1 R_2 \omega^3 L_1 L_2^2 (R_1+R_2) + R_1^2 R_2^2 \omega^2 L_2 (L_1+L_2)}{Z_n} =$$

$$= \underbrace{\frac{R_2 \omega^4 L_1^2 L_2^2 (R_1+R_2) + R_1^2 R_2^2 \omega^2 L_2 (L_1+L_2)}{Z_n}}_{Re} +$$

$$+ j \frac{R_1 R_2^2 \omega^3 L_1 L_2 (L_1+L_2) - R_1 R_2 \omega^3 L_1 L_2^2 (R_1+R_2)}{Z_n} =$$

$$= Re + j \cdot \frac{R_1 R_2^2 \omega^3 L_1^2 L_2 - R_1^2 R_2 \omega^3 L_1 L_2^2}{Z_n} = Re +$$

$$+ j \underbrace{\frac{R_1 R_2 \omega^3 L_1 L_2 (R_2 L_1 - R_1 L_2)}{Z_n}}_{Im}$$

$$|\dot{K}(j\omega)| = \text{АЧХ} = \sqrt{Re^2 + Im^2} \quad ; \quad \varphi(\omega) = \text{ФЧХ} = \operatorname{arctg} \frac{Im}{Re}$$

$$\varphi(\omega) = \operatorname{arctg} \frac{R_1 R_2 \omega^3 L_1 L_2 (R_2 L_1 - R_1 L_2)}{R_2 \omega^2 L_2 \left(\omega^2 L_1^2 L_2 (R_1+R_2) + R_1^2 R_2 (L_1+L_2)\right)}$$

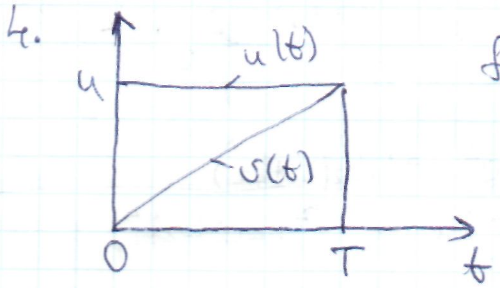
$$|\dot{K}(j\omega)| = \frac{1}{Z_n} \Big[\sqrt{R_2^2 \omega^8 L_1^4 L_2^4 (R_1+R_2)^2 + 2 R_1^2 R_2^3 \omega^6 L_1^2 L_2^3 (L_1+L_2)(R_1+R_2) +}$$

$$\overline{+ R_1^4 R_2^4 \omega^4 L_2^2 (L_1+L_2)^2 + R_1^2 R_2^4 \omega^6 L_1^4 L_2^2 - 2 R_1^3 R_2^3 \omega^6 L_1^3 L_2^3 +}$$

$$+ R_1^4 R_2^2 \omega^6 L_1^2 L_2^4$$

...

4.



$$f(t)*g(t)=\frac{1}{T}\int_{-T/2}^{T/2} f(t)\cdot g(t+\tau)\,dt$$

$$u(t)=u$$

$$v(t)=\frac{u}{T}t$$

$$u(t)\cdot v(t)=\frac{1}{T}\int_0^T u\cdot\frac{u}{T}(t+\tau)\,dt=$$

$$=\frac{u^2}{T^2}\int_0^T(t+\tau)\,dt=\frac{u^2}{T^2}\left(\frac{t^2}{2}+\tau t\right)\Bigg|_0^T=$$

$$=\frac{u^2}{T^2}\left(\frac{T^2}{2}+\tau T\right)=\frac{u^2}{2}+\frac{u^2\tau}{T}$$

5. $f_i=\cos\frac{3\pi i}{4}$, $i=0,\dots,7$ ДПФ-?

$$C_k=\frac{1}{N}\sum_{i=0}^{N-1} f_i\, e^{-ji\frac{2\pi}{N}k}, \quad k=0,\dots,N-1$$

$f_0=\cos(0)=1$

$f_1=\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

$f_2=\cos\frac{6\pi}{4}=\cos\frac{3\pi}{2}=0$

$f_3=\cos\frac{9\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$f_4=\cos\frac{12\pi}{4}=\cos 3\pi=-1$

$f_5=\cos\frac{15\pi}{4}=\cos(-\frac{\pi}{4})=\frac{\sqrt{2}}{2}$

$f_6=\cos\frac{18\pi}{4}=\cos\frac{\pi}{2}=0$

$f_7=\cos\frac{21\pi}{4}=\cos\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$

$$e^{-ji\frac{2\pi}{N}k}=\cos\left(i\frac{2\pi}{N}k\right)-j\sin\left(i\frac{2\pi}{N}k\right)$$

$k=0 \; ; \quad e^{0}=1$

$$C_0 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} - 1 + \frac{\sqrt{2}}{2} + 0 - \frac{\sqrt{2}}{2}\right) = 0$$

$k=1 \; ; \quad e_k = e^{-j\frac{\pi}{4}i}$

$$C_1 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + 0 + \frac{\sqrt{2}}{2}\left(\frac{-\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) - (-1+0) + \right.$$

$$\left. + \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) + 0 - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)\right) = \frac{1}{8}\left(1 - \frac{1}{2} + \frac{1}{2}j - \frac{1}{2} - \frac{1}{2}j + 1\right.$$

$$\left. - \frac{1}{2} + \frac{1}{2}j - \frac{1}{2} - \frac{1}{2}j\right) = 0$$

$k=2 \; ; \quad e_k = e^{-j\frac{\pi}{2}i}$

$$C_2 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}(0-j) + 0 + \frac{\sqrt{2}}{2}(0+j) - (1-0) + \frac{\sqrt{2}}{2}(0-j) + 0\right.$$

$$\left. - \frac{\sqrt{2}}{2}(0+j)\right) = 0$$

$k=3 \; ; \quad e_3 = e^{-j\frac{3\pi}{4}i}$

$$C_3 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right) + 0 + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right) - (-1+0) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) + \right.$$

$$\left. + 0 - \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right)\right) = \frac{1}{8}(4+0) = \frac{1}{2}$$

$k=4 \; ; \quad e_4 = e^{-j\pi i}$

$$C_4 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}(-1+0) + 0 + \frac{\sqrt{2}}{2}(-1+0) - (1+0) + \frac{\sqrt{2}}{2}(-1+0) + 0 - \frac{\sqrt{2}}{2}(-1+0)\right) = 0$$

$k=5 \; ; \quad e_5 = e^{-j\frac{5\pi}{4}i}$

$$C_5 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) + 0 + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) - (-1+0) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right) + \right.$$

$$\left. + 0 - \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right)\right) = \frac{1}{8}(4+0) = \frac{1}{2}$$

$k=6 \; ; \quad e_6 = e^{-j\frac{3\pi}{2}i}$

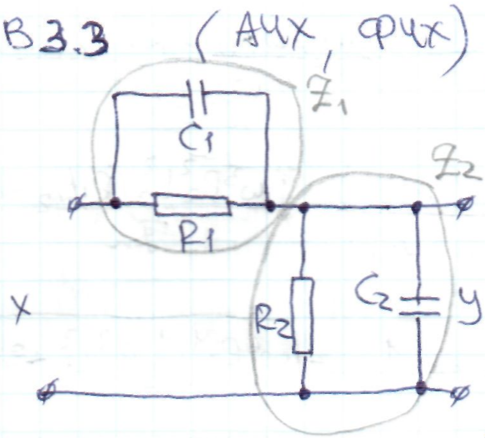
$$C_6 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}(0+j) + 0 + \frac{\sqrt{2}}{2}(0-j) - (1+0) + \frac{\sqrt{2}}{2}(0+j) + 0 - \frac{\sqrt{2}}{2}(0-j)\right) = 0$$

$k=7 \; ; \; e_7 = e^{-j\frac{7\pi}{4}}$;

$$C_7 = \frac{1}{8}\left(1 - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) + 0 + \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) - (-1+0) + \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right)\right.$$
$$\left. + 0 - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right)\right) = 0$$

$C_0 = 0$; $C_1 = 0$; $C_2 = 0$; $C_3 = \frac{1}{2}$; $C_4 = 0$; $C_5 = \frac{1}{2}$; $C_6 = 0$; $C_7 = \frac{1}{2}$

B 3.3 (АЧХ, ФЧХ)



$$Z_1 = \frac{R_1 \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \cancel{\frac{R_1}{-R_1\omega^2 C_1^2 + j\omega C_1}}$$

$$Z_2 = \frac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{R_2 j\omega C_2 + 1}$$

$$\dot{K}(j\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{R_2}{R_2 j\omega C_2 + 1}}{\frac{R_1}{R_1 j\omega C_1 + 1} + \frac{R_2}{j\omega R_2 C_2 + 1}} =$$

$$= \frac{R_2(jR_1\omega C_1 + 1)}{R_1(j\omega R_2 C_2 + 1) + R_2(R_1 j\omega C_1 + 1)} = \frac{R_2 + j\omega R_1 R_2 C_1}{(R_1 + R_2) + j\omega R_1 R_2 (C_1 + C_2)}$$

$$= \frac{(R_2 + j\omega R_1 R_2 C_1)\left[(R_1+R_2) - j\omega R_1 R_2 (C_1 + C_2)\right]}{\underbrace{R_1^2 + 2R_1R_2 + R_2^2}_{(R_1+R_2)^2} - \omega^2 R_1^2 R_2^2 (C_1+C_2)^2} =$$

$$= \frac{R_2(R_1+R_2) + \omega^2 R_1^2 R_2^2 C_1 (C_1+C_2) - j\omega R_1 R_2^2 (C_1+C_2) + (R_1+R_2) j\omega R_1 R_2 \ldots}{(R_1+R_2)^2 - \omega^2 R_1^2 R_2^2 (C_1+C_2)^2}$$

$$= \frac{R_2 R_1 + R_2^2 + \omega^2 R_1^2 R_2^2 C_1 (C_1+C_2) - j\omega R_1 R_2 (R_2 C_1 + R_2 C_2 + R_1 C_1 + R_2 C_1 \ldots}{\underbrace{(R_1+R_2)^2 - \omega^2 R_1^2 R_2^2 (C_1+C_2)^2}_{Z_n}}$$

$$АЧХ = |\dot{K}(j\omega)| = \frac{1}{Z_n}\sqrt{\left(R_2(R_1+R_2) + \omega^2 R_1^2 R_2^2 C_1 (C_1+C_2)\right)^2 +}$$

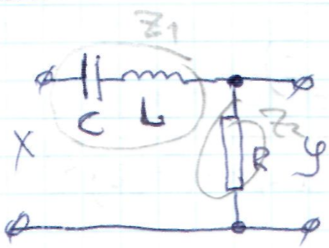
$$\overline{+\left(\omega R_1 R_2^2 (C_1+C_2) + \omega R_1 R_2 C_1 (R_1+R_2)\right)^2} =$$

$$= \frac{1}{Z_n}\sqrt{R_2^2(R_1+R_2)^2 + \cancel{2R_2^3\omega^2 R_1^2 C_1(C_1+C_2)(R_1+R_2)} + \omega^4 R_2^4 R_1^4 C_1^2 (C_1+C_2)^2}$$

$$\overline{+\,\omega^2 R_1^2 R_2^4 (C_1+C_2)^2 \cancel{-2\omega^2 R_1 R_2^3 C_1 (C_1+C_2)(R_1+R_2)} + \omega^2 R_1^2 R_2^2 C_1^2 (R_1+R_2)^2}.$$

$$\varphi_{ux} = -\operatorname{arctg}\frac{\omega R_1 R_2 (R_2 C_1 + R_2 C_2 - R_1 C_1 - R_2 C_1)}{R_2 R_1 + R_2^2 + \omega^2 R_1^2 R_2^2 \, C_1 (C_1+C_2)}$$

B4.3



$$Z_1 = \frac{1}{j\omega C} + j\omega L;$$

$$Z_2 = R$$

$$\dot{K}(j\omega) = \frac{Z_2}{Z_1+Z_2} = \frac{R}{\frac{1}{j\omega C} + j\omega L + R} = \frac{Rj\omega C}{1-\omega^2 LC + Rj\omega C} =$$

$$= \frac{jR\omega C\cdot\left((1-\omega^2 LC) - jR\omega C\right)}{(1-\omega^2 LC)^2 - \omega^2 R^2 C^2} = \frac{R^2\omega^2 C^2 + \overbrace{j\omega RC - j\omega^3 RLC}^{j\omega RC(1-\omega^2 L)}}{\underbrace{(1-\omega^2 LC)^2 - \omega^2 R^2 C^2}_{Z_n}};$$

$$A_{ux} = |\dot{K}(j\omega)| = \frac{1}{Z_n}\sqrt{R^4\omega^4 C^4 + \omega^2 R^2 C^2 (1-\omega^2 L)^2}$$

$$\varphi_{ux} = \operatorname{arctg}\frac{\omega RC(1-\omega^2 L)}{R^2\omega^2 C^2};$$