Вариант 2

2.

$$H(S) = \frac{\sqrt{2}S}{S^2 + \sqrt{2}S + 1} \qquad S = j\omega$$

$$H(j\omega) = \frac{\sqrt{2}j\omega}{(1-\omega^2) + \sqrt{2}j\omega} = \frac{\sqrt{2}j\omega(1-\omega^2) + 2\omega^2}{(1-\omega^2)^2 + 2\omega^2}$$

$$Re[H(j\omega)] = \frac{2\omega^2}{(1-\omega^2)^2 + 2\omega^2} = \frac{2\omega^2}{1+\omega^4}$$

$$Im[H(j\omega)] = \frac{\sqrt{2}\,\omega(1-\omega^2)}{1+\omega^4}$$

$$|H(j\omega)| = \sqrt{\left(\frac{2\omega^2}{1+\omega^4}\right)^2 + \left(\frac{\sqrt{2}\,\omega(1-\omega^2)}{1+\omega^4}\right)^2} =$$

$$= \frac{\sqrt{4\omega^4 + 2\omega^2 - 4\omega^4 + 2\omega^6}}{1+\omega^4} = \frac{\sqrt{2\omega^2 + 2\omega^6}}{1+\omega^4} =$$

$$= \frac{\sqrt{2\omega^2(1+\omega^4)}}{\sqrt{(1+\omega^4)^2}} = \sqrt{\frac{2\omega^2}{1+\omega^4}}$$

$$\varphi(j\omega) = -\operatorname{arctg}\left(\frac{\sqrt{2}\omega(1-\omega^2)}{2\omega^2}\right) =$$

$$= -\operatorname{arctg}\left(\frac{1-\omega^2}{\sqrt{2}\omega}\right)$$

$$\tau(j\omega) = \frac{d}{d\omega}(\varphi(\omega)) = \left(-\operatorname{arctg}\left(\frac{1-\omega^2}{\sqrt{2}\omega}\right)\right)' =$$

$$= -\left(\frac{1}{1+\left(\frac{1-\omega^2}{\sqrt{2}\omega}\right)^2} \cdot \frac{-2\sqrt{2}\omega^2 - \sqrt{2} + \sqrt{2}\omega^2}{2\omega^2}\right) =$$

Варіант 2

2.

$$\tau(j\omega) = -\left[\frac{1}{1+\left(\frac{1-\omega^2}{\sqrt{2}\omega}\right)^2} \cdot \frac{\sqrt{2}(\omega^2-2\omega^2-1)}{2\omega^2}\right] =$$

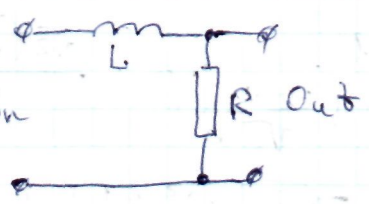
$$= -\left[\frac{1}{1+\left(\frac{1-\omega^2}{\sqrt{2}\omega}\right)^2} \cdot \frac{-\sqrt{2}(\omega^2+1)}{2\omega^2}\right] =$$

$$= -\left[\frac{-\sqrt{2}(\omega^2+1)\cdot 2\omega^2}{2\omega^2(2\omega^2+1-2\omega^2+\omega^4)}\right] =$$

$$= -\left[\frac{-\sqrt{2}(\omega^2+1)}{1+\omega^4}\right) = \frac{\sqrt{2}(\omega^2+1)}{1+\omega^4}$$

Варіант 2

3.



- рахуємо перехідну х-ку

$$1(t) = u_L(t) + u_R(t)$$

$$u_L(t) = L\frac{di_L}{dt} \quad ; \quad u_R(t) = i_L R$$

$$L\frac{di_L}{dt} + Ri_L = 1$$

$$u(t) = i_L^{одн}(t) + i_L^{з}(t)$$

$$L\frac{di_L}{dt} + Ri_L = 0 \quad ; \quad L\lambda + R = 0 \quad ; \Rightarrow \lambda = -\frac{R}{L}$$

$$i_L^{одн}(t) = A\cdot e^{-\frac{R}{L}t}$$

$$i_L^{з}(t) = B \quad ; \quad BR = 1 \; ; \Rightarrow B = \frac{1}{R}$$

$$i_L(t) = Ae^{-\frac{R}{L}t} + \frac{1}{R}$$

- нульові нач умови

$$0 = A + \frac{1}{R} \Rightarrow A = -\frac{1}{R}$$

$$i_L(t) = \frac{1}{R}\left(1 - e^{-\frac{R}{L}t}\right) \; ; \; u_{вих}(t) = u_R$$

$$g(t) = 1 - e^{-\frac{R}{L}t}$$

- $h(t) = \frac{R}{L}e^{-\frac{R}{L}t}$!!!

$t = nT$, $T = 0{,}1\,c$; $R = 2\,\Omega$, $L = 0{,}1\,H$

$$h(t) = \frac{R}{L} e^{-\frac{R}{L}t} = 20\, e^{-20t}$$

$h(n) = h(nT)$; $h(0) = 20$; $h(n) = 20e^{-2n}$

$h(1) = 20\, e^{-2}$ $\quad$ $h(4) = 20e^{-8}$

$h(2) = 20\, e^{-4}$ $\quad$ $h(5) = 20\, e^{-10}$

$h(3) = 20e^{-6}$ $\quad$ $h(6) = 20e^{-12}$ ... ~~$h(n) = 20e^{-}$~~

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n} = 20\left[1 + e^{-2} z^{-1} + e^{-4} z^{-2} + e^{-6} z^{-3} + \ldots\right] =$$

$$= 20\, \frac{1}{1 - e^{-2} z^{-1}} = \frac{20\, z}{z - e^{-2}}$$

B2._5

$$X(z) = \frac{2z^{-1}}{1 - 0{,}25z^{-2}}$$

$$X(z) = \frac{2z}{z^2 - 0{,}25}$$

$$z^2 - 0{,}25 = 0$$

$z_{1,2} = \pm 0{,}5 \qquad p_1 = -0{,}5 \qquad p_2 = 0{,}5$

$$X(z) = \frac{2z}{(z+0{,}5)(z-0{,}5)}$$

$$\frac{X(z)}{z} = \frac{\xi_1}{z+0{,}5} + \frac{\xi_2}{z-0{,}5}$$

$$\frac{X(z)(z+0{,}5)}{z} = \xi_1 + \xi_2 \frac{z+0{,}5}{z-0{,}5}\Big|_{z=-0{,}5}$$

$$\xi_1 = \frac{1}{z-0{,}5}\Big|_{z=-0{,}5} = -1$$

$$\xi_2 = \frac{1}{z+0{,}5}\Big|_{z=0{,}5} = 1$$

$$X(z) = -1\left(\frac{2z}{z+0{,}5}\right) + \left(\frac{2z}{z-0{,}5}\right)$$

$$x(n) = 2\left[(0{,}5)^n - (-0{,}5)^n\right]$$

$$z^{-1}\left(-\frac{2z}{z+0{,}5}\right) = -2(-0{,}5)^n$$

$$z^{-1}\left(\frac{2z}{z-0{,}5}\right) = 2(0{,}5)^n$$

Вариант 2

6.

$$y(n) + b_1 y(n-1) + b_2 y(n-2) = x(n)$$

$$Y(z) = X(z) - b_1 Y(z) z^{-1} - b_2 Y(z) z^{-2}$$

$$Y(z)\left(1 + b_1 z^{-1} + b_2 z^{-2}\right) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

$h(n)$:

$$H(z) = \frac{1}{1 - 0{,}5 z^{-1} - 0{,}5 z^{-2}}$$

$$H(z) = \frac{z^2}{z^2 - 0{,}5 z - 0{,}5}$$

$$z^2 - 0{,}5z - 0{,}5 = 0 \Rightarrow p_1 = 1,\ p_2 = -0{,}5$$

$$H(z) = \frac{z^2}{(z-1)(z+0{,}5)};\ \frac{H(z)}{z^2} = \frac{\xi_1}{z-1} + \frac{\xi_2}{z+0{,}5}$$

$$\frac{H(z)(z-1)}{z^2} = \xi_1 + \frac{\xi_2 (z-1)}{z^2 + 0{,}5}\Big|_{z=1}$$

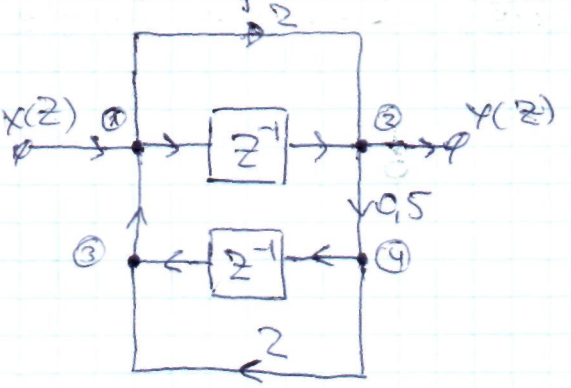
$$\xi_1 = \frac{1}{z+0{,}5}\Big|_{z=1} = \frac{1}{1{,}5} = \frac{2}{3}$$

$$\xi_2 = \frac{1}{z-1}\Big|_{z=-0{,}5} = -\frac{2}{3}$$

$$H(z) = \frac{2}{3}\left(\frac{z^2}{z-1}\right) - \frac{2}{3}\left(\frac{z^2}{z+0{,}5}\right)$$

$$h(n) = \frac{2}{3}\left[(1)^n - (-0{,}5)^n\right] = \frac{2}{3}\left(1 - (-0{,}5)^n\right)$$

Варіант 2



$y(z) = x_2(z)$

$X_1(z) = X(z) + X_3(z)$

$X_2(z) = 2X_1(z) + X_1(z)\cdot z^{-1} = Y(z)$

$X_3(z) = 2X_4(z) + X_4(z)\cdot z^{-1}$

$X_4(z) = 0{,}5\, X_2(z)$

$Y(z) = 2X_1(z) + X_1(z)\,z^{-1} = 2(X(z)+X_3(z)) +$

$+ X(z)\cdot z^{-1} + X_3(z)z^{-1} = 2X(z) + 2(2X_4(z) +$

$+ X_4(z)\cdot z^{-1}) + X(z)\,z^{-1} + 2X_4(z)\cdot z^{-1} +$

$+ X_4(z)\cdot z^{-2} = 2X(z) + X(z)\cdot z^{-1} + 2X_2(z) +$

$+ 0{,}5\, X_2(z)\cdot z^{-1} + X_2(z)\cdot z^{-1} + 0{,}5 X_2(z) z^{-2} =$

$= 2X(z) + X(z)\cdot z^{-1} + 2y(z) + 0{,}5y(z)\cdot z^{-1} +$

$+ y(z)\cdot z^{-1} + 0{,}5\, y(z)\cdot z^{-2}$

$y(z) - 2y(z) + 0{,}5\, y(z)\cdot z^{-1} - y(z)\cdot z^{-1} + 0{,}5\, y(z)\cdot z^{-2} =$

$= 2X(z) + X(z)\cdot z^{-1}$

$-y(z) - 1{,}5\, y(z)\cdot z^{-1} + 0{,}5\, y(z)\cdot z^{-2} = 2X(z) + X(z)\cdot z^{-1}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 + z^{-1}}{-1 - 1{,}5\,z^{-1} - 0{,}5\,z^{-2}}$$