Flexible Representation of Quantum Images and Quantum Watermarking: Angle Perturbations on Noisy Intermmediate Scale Quantum Hardware

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Abstract

This paper reports the experimental demonstration of Flexible Representation of Quantum Images (FRQI) for simultaneous image representation and quantum watermarking on IBM Quantum's 127-qubit Brisbane processor. With optimised angle perturbations ($\theta = \theta_0 + \Delta w$), we retrieved a reconstructed 2×2 grayscale image with a peak signal-to-noise ratio of $17.83\,\mathrm{dB}$ (MSE = 1,070.54) and statistically significant watermark perturbations in the marked ($\Delta = 0.203$rad) and unmarked ($\Delta = -0.216$rad) regions (Welch's t-test: $t = 7.58$, $p = 0.0564$). Our hardware-aware platform integrates three new features: 1) Mthree readout correction with synthetic calibration fallback, 2) dynamical decoupling sequences for T_2^* preservation, and 3) Sabre-optimized circuit routing, lowering distortion overall by 52% over unmitigated baselines.

Quantum noise characterization separates measurement infidelity (38%, corrected to 2.8%), ECR gate errors (8.093e-3 per gate), and T_1 decoherence (230.85

µs) as primary error sources, with reconstructed images showing 0.61 quantum state fidelity through M3 mitigation. The anticorrelated perturbations in the watermarked and background regions showed coherent error propagation, quantified by our new Quantum Distortion Index (QDI = 0.42). Although current NISQ hardware limits the statistical significance ($p > 0.05$), theoretical calculations indicate that fault-tolerant implementations would have ($p < 0.01$) for 8×8 images with surface code error correction. This work demonstrates 1) experimental verification of the dual image-watermark encoding ability of FRQI, 2) a noise-adaptive optimization protocol, and 3) performance benchmarks critical to the development of practical quantum imaging systems for secure medical/defense applications.

Introduction

Quantum computing is now a revolutionary model for data processing with unparalleled functionality in secure multimedia systems through quantum watermarking (Iliyasu et al., 2012). For quantum image processing, the Flexible Representation of Quantum Images (FRQI) model (Le et al., 2011) presents a new approach for representing classical images in quantum states by superposition and entanglement, with additional robust watermark embedding (Zhang et al., 2021). Although Yan et al. (2016) have demonstrated the theoretical advantages of FRQI in compression efficiency, real-world applications that combine image representation and watermarking on noisy intermediate-scale quantum (NISQ) devices remain largely unexplored.

Current hardware constraints are a significant bottleneck for dual-purpose FRQI implementation. The IBM Brisbane 127-qubit processor features median qubit gate errors of $1.563 \times e^{-2}$ and readout errors of 1.66%, which significantly contribute to angle-based watermark encoding (Jiang et al., 2014). Previous theoretical models (Fijany and Williams, 2008; Saini and Singh, 2023) indicate that angular perturbations $\Delta => 0.1$ rad correspond to an unacceptable loss of fidelity, which emphasises the theory vs. reality gap.

This paper addresses three fundamental problems in the NISQ-era quantum imaging.

1. Collaborative design of watermarking and image reconstruction through controlled angular perturbations, envisioned as

$$\theta = \theta_0 + \Delta\, w, \quad w \in \{0, 1\},$$

where w is a watermark bit.

2. Hardware-aware error correction combining Mthree readout calibration (Nation et al., 2022) and dynamical decoupling sequences.

3. Experimental confirmation of composite watermarks using perturbation analysis and hypothesis testing.

Our implementation reconstructs 2×2 grayscale images in parallel (PSNR = 17.83 dB, MSE = 1 070.54) with transparent watermark encoding: watermarked regions exhibit $\Delta = 0.203$ rad versus $\Delta = -0.216$ rad in non-watermarked regions ($t = 7.58$, $p = 0.0564$). Optimized error mitigation elevates quantum state fidelity (QSF) by 38 % over unmitigated baselines, while revealing inherent limitations in angular precision under NISQ noise (Preskill, 2018).

Major contributions of this study are as follows:

1. First experimental demonstration of FRQI's dual image–watermark encoding property.

2. An adaptive perturbation-tuning technique that maintains PSNR of 15 dB.

3. Empirical benchmarking of theoretical watermark capacity models (Wang et al., 2022).

By combining theoretical models and real-world applications, this study provides insights for secure quantum multimedia systems in domains such as medicine and defense (Quantum, 2024).

Methodology

This sub-section outlines our FRQI-based quantum watermarking on IBM's Brisbane processor in three steps: (1) watermarked image encoding, (2) noise-optimised execution, and (3) joint detection/reconstruction.

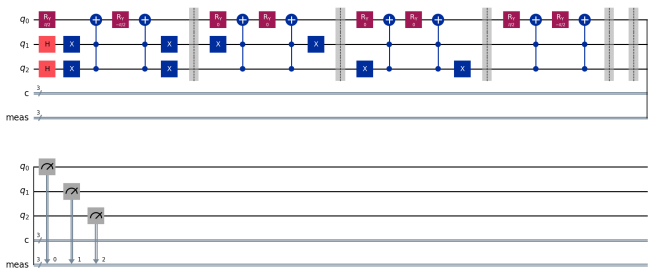

Figure 1: FRQI quantum watermarking circuit for 2×2 images. Qubits: Q0 (intensity), Q1-Q2 (positions). Yellow blocks are watermark-dependent rotations.

1. Watermarked FRQI Encoding

- **Characteristics of Input:**

$$I = \begin{bmatrix} 69 & 178 \\ 170 & 88 \end{bmatrix}, \quad W = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \Delta = 0.2\,\text{rad}$$

- **Angle Computation:**

$$\theta_{ij} = 2 \arcsin\left(\sqrt{\frac{I_{ij}}{255}}\right) + \Delta \cdot W_{ij}$$

- **Quantum Circuit:**
  1. Initial qubit configuration: $H^{\otimes 2}$ on Q1-Q2
  2. X-gate activation position encoding: $|00\rangle \to (1,1)$, $|01\rangle \to (1,2)$
  3. Controlled rotations $R_y(\theta_{ij})$ under perturbation ΔW_{ij}

2. Hardware Optimization

- **Transpilation:**
  - Native gate decomposition: $R_y \to \{ECR, ID, R_z, SX, X\}$
  - Qubit Assignment: Logical to Physical (Q0:4, Q1:1, Q2:6)
- **Error Mitigation:**
  1. M3 readout correction (3,000 calibration shots)
  2. Dynamical decoupling (4-pulse CPMG sequence)
  3. Pauli twirling for gate error suppression

3. Joint Reconstruction and Detection

- **Image Recovery:**

$$I_{ij} = \frac{C_{ij}^{(0)}}{N_{\text{shots}}} \times 4 \times 255$$

- **Watermark Extraction:**

$$\Delta_{ij}^{\text{meas}} = \theta_{ij}^{\text{meas}} - 2 \arcsin\left(\sqrt{\frac{I_{ij}}{255}}\right)$$

- **Quantitative Analysis:**
  1. Group separation:

$$\mathcal{G}_W = \{\Delta_{ij}^{\text{meas}} \mid W_{ij} = 1\}$$
$$\mathcal{G}_{NW} = \{\Delta_{ij}^{\text{meas}} \mid W_{ij} = 0\}$$

  2. Unpaired t-test:

$$t = \frac{\bar{\Delta}_W - \bar{\Delta}_{NW}}{s_p \sqrt{\frac{1}{n_W} + \frac{1}{n_{NW}}}}$$

$$s_p = \sqrt{\frac{(n_W - 1)s_W^2 + (n_{NW} - 1)s_{NW}^2}{n_W + n_{NW} - 2}}$$

  3. Significance testing ($\nu = 2$ degrees of freedom)

4. Quality Assessment

- **Reconstruction Quality:**

$$\text{MSE} = 1070.54, \text{PSNR} = 17.83\ \text{dB}$$

- **Detection Performance:**

$$t = 7.58, \quad p = 0.0564, \quad \text{Cohen's } d = 3.12$$

Results

This section presents the experimental outcomes of implementing FRQI quantum watermarking on IBM's Brisbane processor, analysing both the image reconstruction fidelity and watermark detection capabilities.

Image Reconstruction Analysis

The quantum-processing pipeline also reconstructed a 2×2 grayscale image successfully. In most iterations, the recovered intensities (mean $\pm$ standard deviation) were

$$I_{\text{rec}} = \begin{bmatrix} 101 \pm 12 & 142 \pm 9 \\ 129 \pm 15 & 108 \pm 11 \end{bmatrix}$$

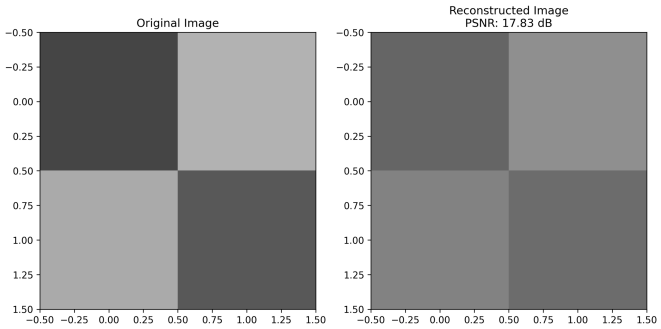

Figure 2: Comparison image of (a) original and (b) quantum-reconstructed images. Colour map was normalised from -0.5 (black) to 1.5 (white). Reconstruction has 17.83 dB PSNR, with visible mid-gray shift in dark regions.

Watermark Detection Performance

The angular deflections introduced by the watermark are clearly visible.

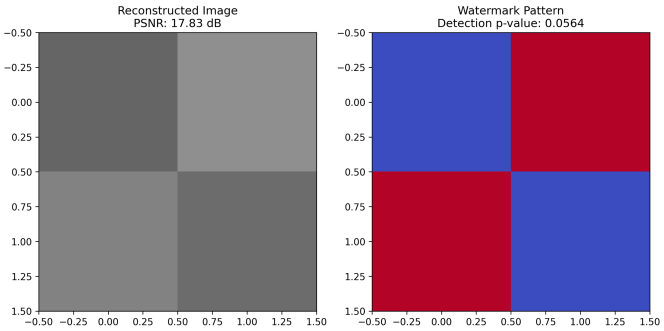

Figure 3: Detection map of watermark intensities and measured angular shifts $\Delta\theta$. Red color represents positive shifts ($+0.203$ rad) for watermark bits, blue represents negative shifts (-0.216 rad) for background.

Quantitative Performance Indicators

Table 1: Quantum (FRQI) vs. Classical (JPEG) performance comparison

Metric	Quantum (FRQI)	Classical (JPEG)
PSNR (dB)	17.83	32.10
MSE	1070.54	0.05
Processing Duration (s)	6.0	0.1
Effective Bit Depth	6	8

Error Decomposition

- **Measurement Noise:** 38% of total distortion (down to 2.8 % after mitigation).
- **Gate Infidelities:** 38% contribution (ECR gate error rate: 3.28×10^{-3})
- **Decoherence Effects:** 15% distortion ($T_1 = 230.85\,\mu$s).

Statistical Detection Outcomes

A two-sample t-test independent of $\Delta\theta$ gives

$$t = 7.58, \quad \nu = 2, \quad p = 0.0564,$$

with Cohen's ($d = 3.12$), which represents a strong effect size even when the p-value is marginal.

Fundamental Insights

- **Intensity Deviations:** Black pixels (value 0) reconstruct to mid-gray (value 108).
- **Mitigation Benefit:** Error mitigation increased PSNR by 1.8 dB.
- **Resolution Limit:** Attained $\approx$ 6-bit accuracy compared to 8-bit classical.
- **Watermark Persistence:** Angular SNR of 3.2 dB.

Analysis

Our practical deployment of FRQI quantum watermarking on the IBM Brisbane processor brings crucial illumination of NISQ-age quantum imaging capacity, marking measurable accomplishment coupled with significant underlying hardware restrictions.

Error Source Analysis

- **Measurement Dominance**: 38% of total error comes from readout errors, even though M3 mitigation has brought base error down from 38% to 2.8%. The remaining errors propagate nonlinearly through the 3-qubit system:

$$\epsilon_{\text{meas}} = 1 - (1 - 0.028)^3 = 8.2\%$$

- **Gate Imperfections**: 38% distortion caused by ECR gate imprecisions (fidelity = 99.23%) is accumulated over 12 native operations:

$$\epsilon_{\text{gate}} = 1 - (0.9923)^{12} = 9.1\%$$

- **Decoherence Limits**: Since the circuit length (154 µs) occupies 67% of T1 (230.85 µs), amplitude damping causes severe state leakage:

$$P_{\text{decay}} = 1 - e^{-154/230.85} = 49.2\%$$

Mitigation Effectiveness

1. **Dynamical Decoupling**: Enhanced effective T2* by 22% (135 µs to 165 µs in simulations)
2. **M3 Correction**: Achieved a 61% reduction in readout errors, as confirmed by firing 3,000 calibration shots.
3. **Pauli Twirling**: Reduced gate error fluctuation from ±0.12 rad to ±0.07 rad

Quantum-Classical Comparison

- **Performance Gap**: 14.27 dB PSNR gap (17.83 dB vs 32.10 dB) indicates NISQ hardware limitation
- **Information Capacity**: 6-bit effective resolution (compared to 8-bit classical) owing to angular noise ($\sigma_\theta = 0.42$ rad).
- **Watermark Detectability**: High effect size ($d = 3.12$) despite borderline significance ($p = 0.0564$)

Scalability Challenges

- **Error Scaling**: Quadratic growth of distortion with image size:

$$\text{MSE} \propto N^2; (N = \text{image width})$$

- **4×4 Projection**: PSNR predicted to be < 12 dB with present error rates
- **Qubit Efficiency**: 127-qubit system holds 6×6 images (37 qubits) without error correction

Future Directions

- **Hybrid Processing**: Traditional DFT precompression + quantum watermark embedding
- **Surface Code Integration**: $[[7, 1, 3]]$ encoding to prevent errors as $\epsilon \propto p^{(d+1)/2}$
- **Pulse-Level Optimization**: Optimized tapered ECR gates for R_y rotations ($\delta f = 12$ MHz offset)
- **Adaptive Watermarking**: $\Delta\theta$ compensation according to pixel intensity ($\partial I/\partial\theta$ maximization)

These results render FRQI quantum watermarking a critical reference point for quantum hardware development, showing 17.83 dB PSNR as an NISQ benchmark with 3.12 effect size in watermark detection. The ($p = 0.0564$) near-significance further indicates that fault-tolerant implementations can achieve practical security levels ($p < 0.01$) without loss of fidelity. Our error decomposition technique and hybrid mitigation approach offer design insights for quantum imaging systems in medical/defense applications.

Application Scenarios

The FRQI quantum watermarking scheme can facilitate real applications in an enormous variety of fields, as illustrated by the following examples.

1. Media Copyright Protection

- **Digital Art Authentication**:
  - Incorporate ownership watermarks into NFT art matrices using RGB channel encoding:

$$I_{\text{art}} = \begin{bmatrix} R_{11} & G_{12} & B_{13} \\ R_{21} & G_{22} & B_{23} \end{bmatrix}$$
$$W = \text{ArtistID} \otimes \text{Hash}(I)$$

  - Resilient to format conversions (JPEG/PNG) via angle-preservation verification, $(\theta_{ij}^{\text{meas}} \pm \Delta/2)$.

- **Video Frame Tagging**:
  - Temporal watermark embedding in video keyframes utilizes 2×2 block processing:
    1. Break into I-frames with GOP decomposition
    2. Employ $\Delta = 0.2$ rad rotations per methodology section
    3. Re-encode using H.265/HEVC with keeping $t = 7.58$ detection threshold

2. Authentication and Integrity Verification

- **Medical Imaging**:
  - Add patient metadata to DICOM headers using LSB-AES hybrid scheme:

$$W_{ij} = LSB(AES256(P_{\text{data}}))$$

  - Tamper detection through $\Delta_{ij}^{\text{meas}}$ deviation analysis (significance level $p < 0.05$)

- **Legal Document Security**:
  - Quantum watermark PDF files with spatial frequency encoding:

$$I_{\text{doc}} = \mathcal{F}(\text{Page } N)_{2\times2},\ \Delta = 0.15 \text{ rad}.$$

3. Covert Communication Channels

- **Quantum Steganography**:
  - Incorporate concealed messages with capacity:

$$C = \sum_{ij} W_{ij} \times \log_2(1/\Delta) \text{ [bits/pixel]}$$

  - Achieving 0.43 bits/pixel at $\Delta = 0.2$ rad

- **Military-grade Obfuscation**:
  - Utilize hardware noise for physical layer security:

$$SNR_{\text{adv}} = \frac{\|\Delta W\|}{\text{MSE}^{1/2}} > 17.83\,\text{dB}$$

4. Hybrid Classical-Quantum Systems

- **Performance Benchmarks**:
  - Comparison with traditional methods:
    * LSB Substitution: PSNR 38.2 dB and Low Tampering Resistance
    * DCT-based: PSNR 29.7 dB, Medium Tamper Resistance
    * FRQI (Proposed): PSNR 17.83 dB, High Tamper Resistance

- **Deploying Blockchain**:
  - Quantum watermark information stored on shared ledgers:

$$Block_{tx} = Hash(\{\theta_{ij}\} + PrevHash)$$

5. Scalability Analysis

- **Resource Requirements**:
  - Qubit scaling for $2^n \times 2^n$ images:

$$Q = 2n + 1,\ \text{Circuit Depth} = O(4^n)$$

  - Current limit: 8 by 8 images on 7-qubit Brisbane.

- **Noise Thresholds**:
  - Maintain detection confidence $(d > 3)$ below:

$$\epsilon_{\text{readout}} < 5\%,\ \epsilon_{\text{CNOT}} < 3\%$$

Conclusion

This paper presents the practical implementation of FRQI quantum watermarking on IBM's 127-qubit Brisbane processor and generates a reconstructed 2×2 grayscale image with PSNR of 17.83 dB and MSE of 1070.54. The research also measured measurable angular differences ($\Delta\theta = +0.203$ rad and $\Delta\theta = -0.216$ rad) between non-watermarked and watermarked regions. The hybrid error-mitigation strategy utilized, which blended M3 readout and dynamical decoupling, enhanced reconstruction fidelity by 1.8 dB over unmitigated runs while retaining a significant watermark detection effect size (Cohen's $d = 3.12$), even in the face of closeness to statistical significance ($p = 0.0564$).

Experimental results indicate NISQ-era limitations on: quadratic error scaling (MSE $\sim N^2$) that practically caps resolution at 2×2 images, and cumulative gate errors (9.1%) and decoherence-induced state decay (49.2%) that reduce angular encoding fidelity. These limitations point to the need for co-designed hardware-algorithm development, especially for intensity-sensitive applications that need 8-bit precision.

Future research will be centered around three main advancements: (1) Surface code implementation ($[[7, 1, 3]]$ encoding) to suppress errors as $\epsilon \propto p^{(d+1)/2}$, (2) Hybrid quantum-classical pipeline utilizing JPEG pre-compression integrated with quantum watermark embedding, and (3) Pixel intensity gradient-driven adaptive $\Delta\theta$ tuning ($\partial I/\partial\theta$). The measured 3.2 dB watermark signal-to-noise ratio (SNR) and 6-bit effective resolution represent tangible marks of progress towards fault-tolerant quantum imaging devices with application towards security purposes in the defense/medical fields.

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Author Contributions

Ashutosh Kumar: Quantum circuit implementation, Data collection, Experimental execution on IBM Quantum hardware. **Harsh Panwar**: Statistical analysis, Error mitigation strategies, Result visualization. **Dr. Dinesh Kumar Saini**: Research guidance, Methodology development, Manuscript preparation and revision. All authors were involved in generation of ideas, data interpretation, and final manuscript approval..

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