

→ Recursion

↳ function calls itself to solve smaller instance of a problem.

↳ key-concept:

(1) Recursive-case:

↳ part of function that calls itself to solve a smaller version of the original problem.

(2) Base-case

↳ stopping condition's for recursion. This prevents the function from calling itself indefinitely.

(3) Stackoverflow error

↳ without base case, recursion leads to infinite recursion, causing a stack overflow.

→ Types of recursion

↳ (1) Direct recursion: function calls itself directly, i.e. $f() \rightarrow f()$

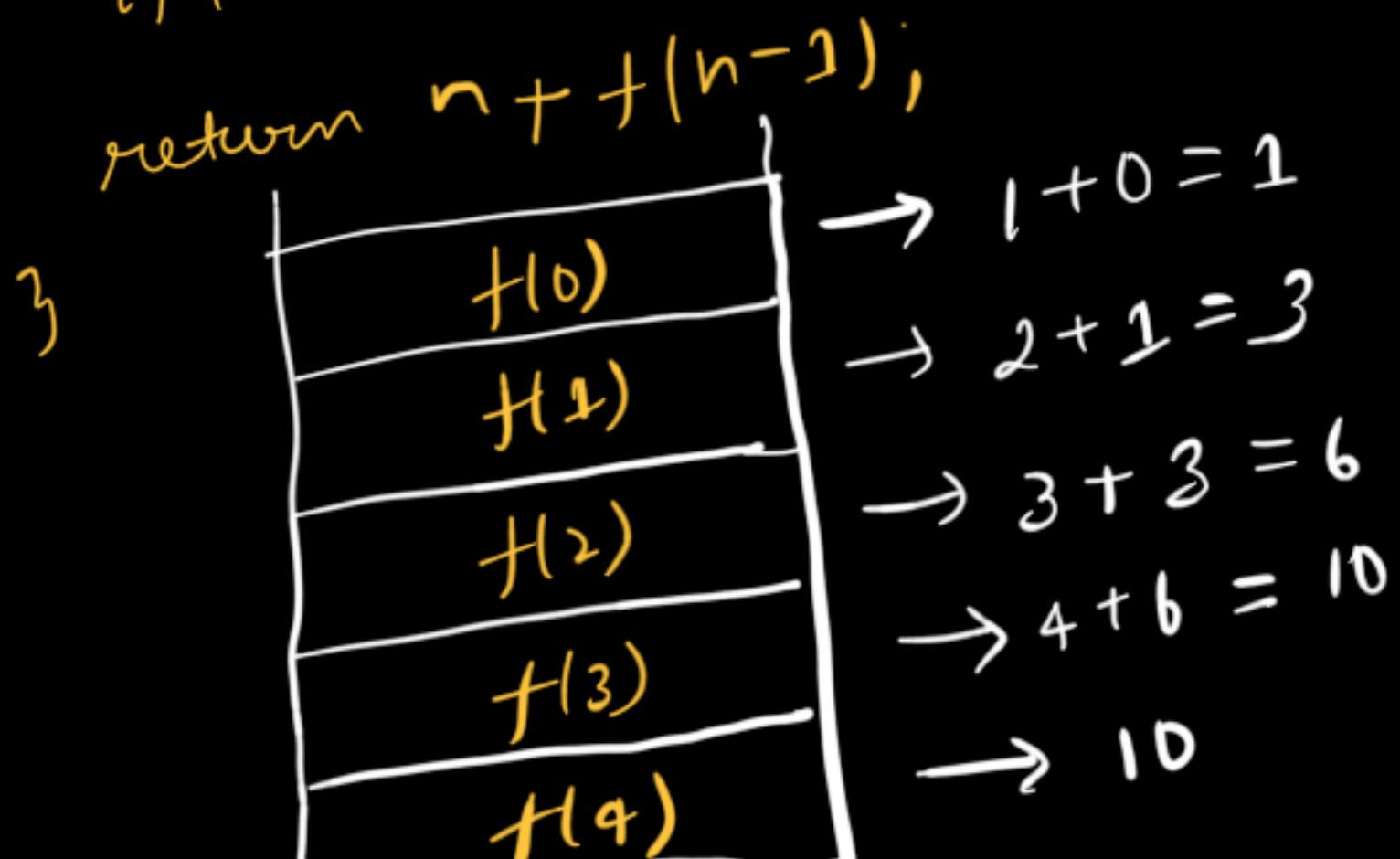
↳ (2) Indirect-recursion: function A calls $f(x)$ B, which then calls $f(x)$ A
↳ i.e. $A() \rightarrow B() \rightarrow A()$

↳ (3) Tail recursion: recursive call is the last operation in the function

↳ i.e. $f(n, acc=2) \{$
 if ($n == 0$) return acc;
 return $f(n-1, acc * n);$
 ↳ last $f(n)$ call

→ Find sum from 1 to n

$f(int n) \{$
 if ($n == 0$) return 0;
 return $n + f(n-1);$



A memory structure that stores each $f(n)$ call.
When a $f(x)$ returns, it popped off the stack.

$$f(4) \rightarrow 10$$

$$\downarrow$$

$$4 + f(3) \quad 4+6 = 10$$

$$\downarrow$$

$$3 + f(2) \quad 3+3 = 6$$

$$\downarrow$$

$$2 + f(1) = 2+2 = 3$$

$$\downarrow$$

$$1 + f(0) \rightarrow 0$$

→ Questions!

(1) Reverse Array & string

(2) check if str is palindrome

(3) digit sum & fibonaci

(4) gcd & factorial

↳ TC: $\log(\min(a,b))$

→ Subsequence

↳ contiguous / non-contiguous, but it must preserve the original order of elements.

3	1	2
---	---	---

↳ Subsequence: $\{[3], [1], [2], [3, 1], [3, 2], [1, 2], [3, 1, 2], \emptyset\}$

pick = ✓

not pick = ✗

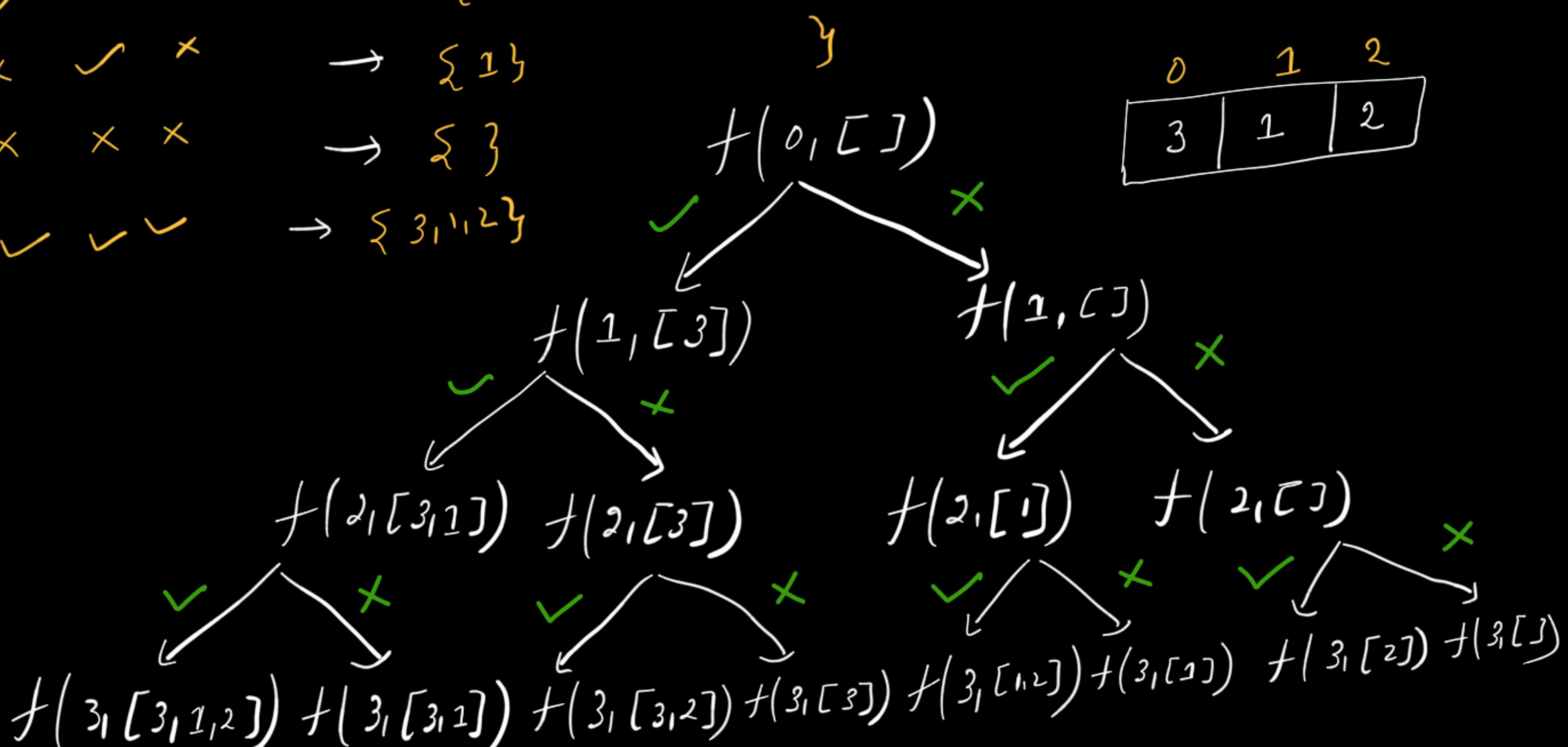
3	1	2
---	---	---

✓	✗	✓	→ {3, 2}
✓	✓	✗	→ {3, 1}
✗	✓	✓	→ {1, 2}
✗	✗	✓	→ {2}
✓	✗	✗	→ {3}
✗	✓	✗	→ {1}
✗	✗	✗	→ {}
✓	✓	✗	→ {3, 1, 2}

Pseudocode:

```
f(ind, wss) {
    Base-case
    pick (1)
    remove (2)
    not pick (3)
```

0	1	2
---	---	---



PartIV.java

```
public static void subSeq(int[] arr, int index, List<Integer> curr, List<List<Integer>> ans){
    // does not handle duplicates
    if (index >= arr.length){
        ans.add(new ArrayList<>(curr));
        return;
    }

    curr.add(arr[index]);
    subSeq(arr, index + 1, curr, ans);
    curr.remove(index: curr.size() - 1);
    subSeq(arr, index + 1, curr, ans);
}
```

Tc: O(2^n)

Sc: O(n)

→ To handle duplicate

↳ Sort the array, & before not picking up the element.

```
// skip all same adjacent element
int nextInd = index + 1;
while (nextInd < arr.length && arr[index] == arr[nextInd]) nextInd++;
```

→ Subset Sum Equal K

↳ Returns subset's whose sum Equal K

$$\boxed{1 \mid 2 \mid 3} \quad K=3$$

$$\hookrightarrow \text{Answer} = \left[[1, 2], [3] \right]$$

→ When to return the result of a recursive call when not to?

↳ Return recursive call?

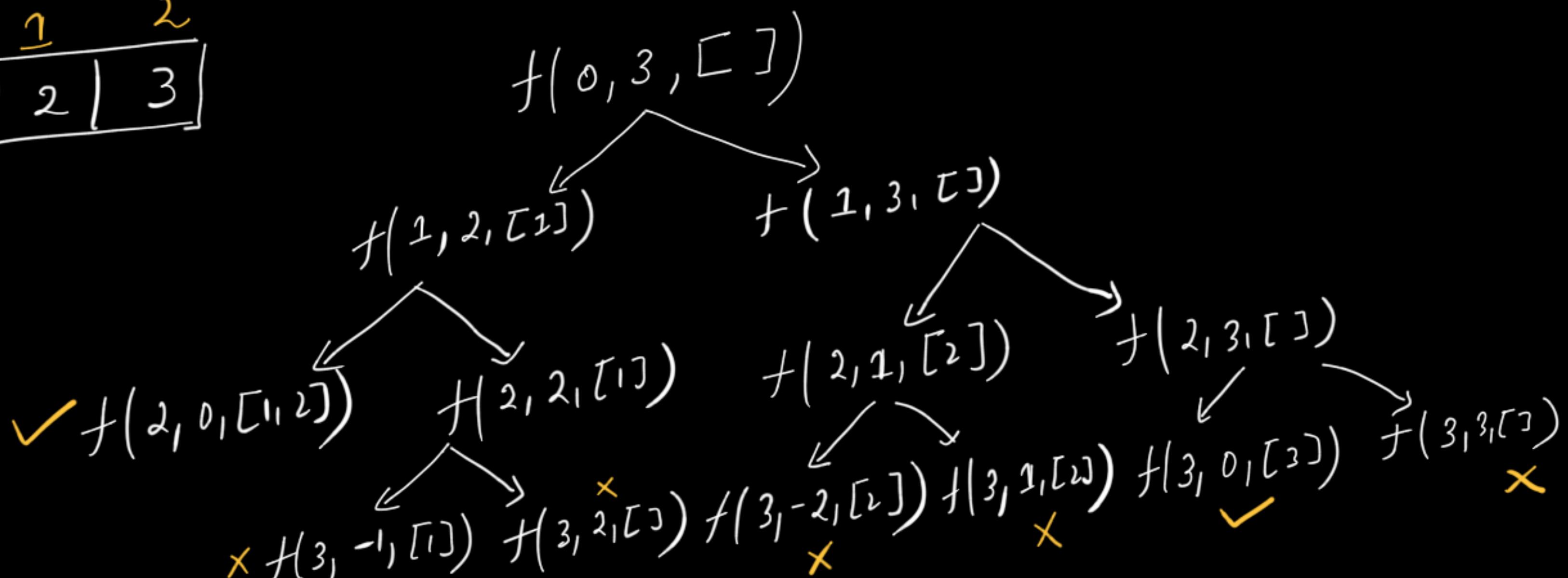
① Yes ✓

② No ✗

→ When the recursive call produces a value that need to be used, stored or returned

→ When the recursive call only perform actions (e.g: modifying a list), & we're using shared mutable state.

$$\boxed{\begin{matrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{matrix}}$$



```

PartV.java
public static void 3 usages new *
subSumEqK(int[] arr, int index, int sum, List<Integer> curr, List<List<Integer>> ans) {
    if (sum == 0) {
        ans.add(new ArrayList<>(curr));
        return;
    }
    if (index >= arr.length || sum < 0){
        return ;
    }

    curr.add(arr[index]);
    subSumEqK(arr, index + 1, sum - arr[index], curr, ans);
    curr.remove(index: curr.size() - 1);
    subSumEqK(arr, index + 1 , sum , curr , ans);
}

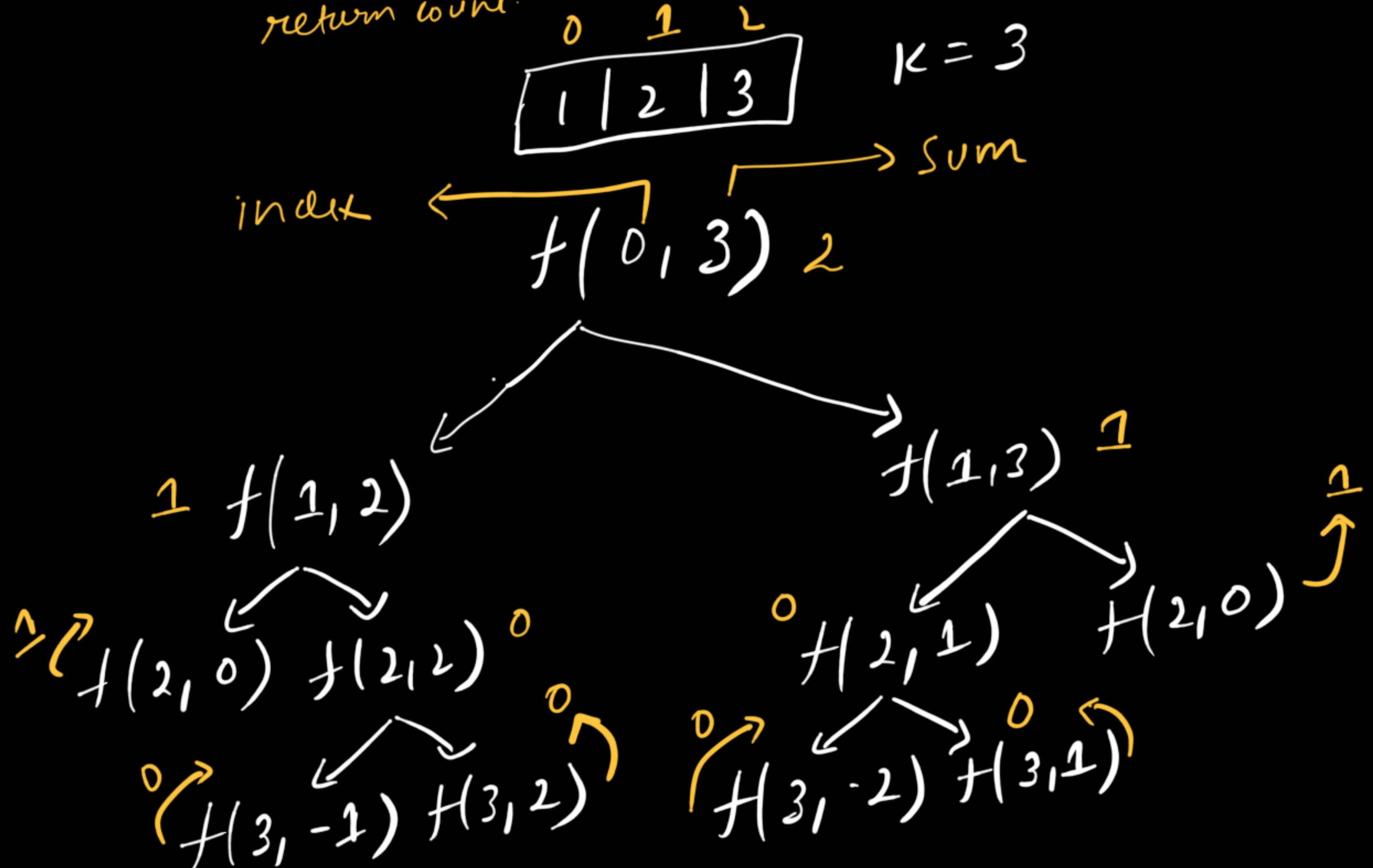
```

TC: $O(2^n) * n$ ↳ Creating copy of the list

SC: $O(2^n * n)$

Count Subset Sum Equal K

↳ same question but instead of returning subset, only return count.



```
PartV.java
public static int subSumEqKCount(int[] arr, int index, int sum) { 3 usages n
    if (sum == 0) {
        return 1;
    }
    if (index >= arr.length){ || SUM < 0
        return 0;
    }

    int left = subSumEqKCount(arr, index: index + 1, sum: sum - arr[index]);
    int right = subSumEqKCount(arr, index: index + 1 , sum);

    return left + right;
}
```

TC: $O(2^n)$

SC: $O(n)$

↓

stack
space

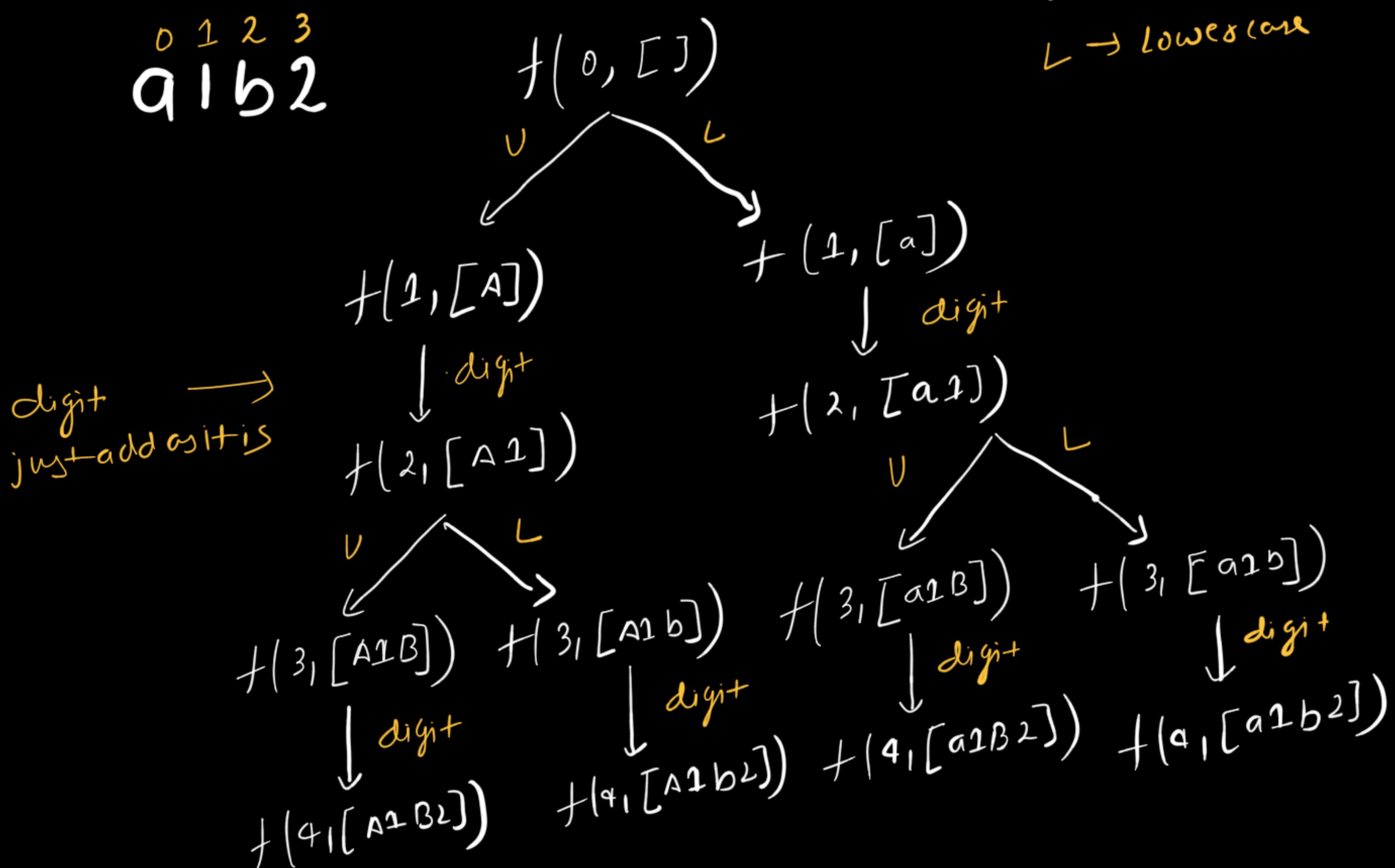
→ Letter Case permutation (Uber | Ola | Microsoft)

Given a string s , we can transform every letter individually to be lowercase or uppercase to create another string.

$$s = a1b2 \rightarrow [a1b2, A1b2, a1B2, A1B2]$$

U → uppercase

L → lowercase



```
PartVI.java
private static void letterCasePermutation(String s, int index, String curr, List<String> ans) {
    if (index >= s.length()){
        ans.add(curr);
        return;
    }
    char ch = s.charAt(index);
    if (Character.isDigit(ch)){
        letterCasePermutation(s, index: index + 1, curr: curr + ch, ans);
    }else {
        // lowercase
        char lo = Character.toLowerCase(ch);
        letterCasePermutation(s, index: index + 1, curr: curr + lo, ans);

        // uppercase
        char up = Character.toUpperCase(ch);
        letterCasePermutation(s, index: index + 1, curr: curr + up, ans);
    }
}
```

$Tc: O(2^n)$
 $Sc: O(N) + O(N)$
 ↴ For storing result

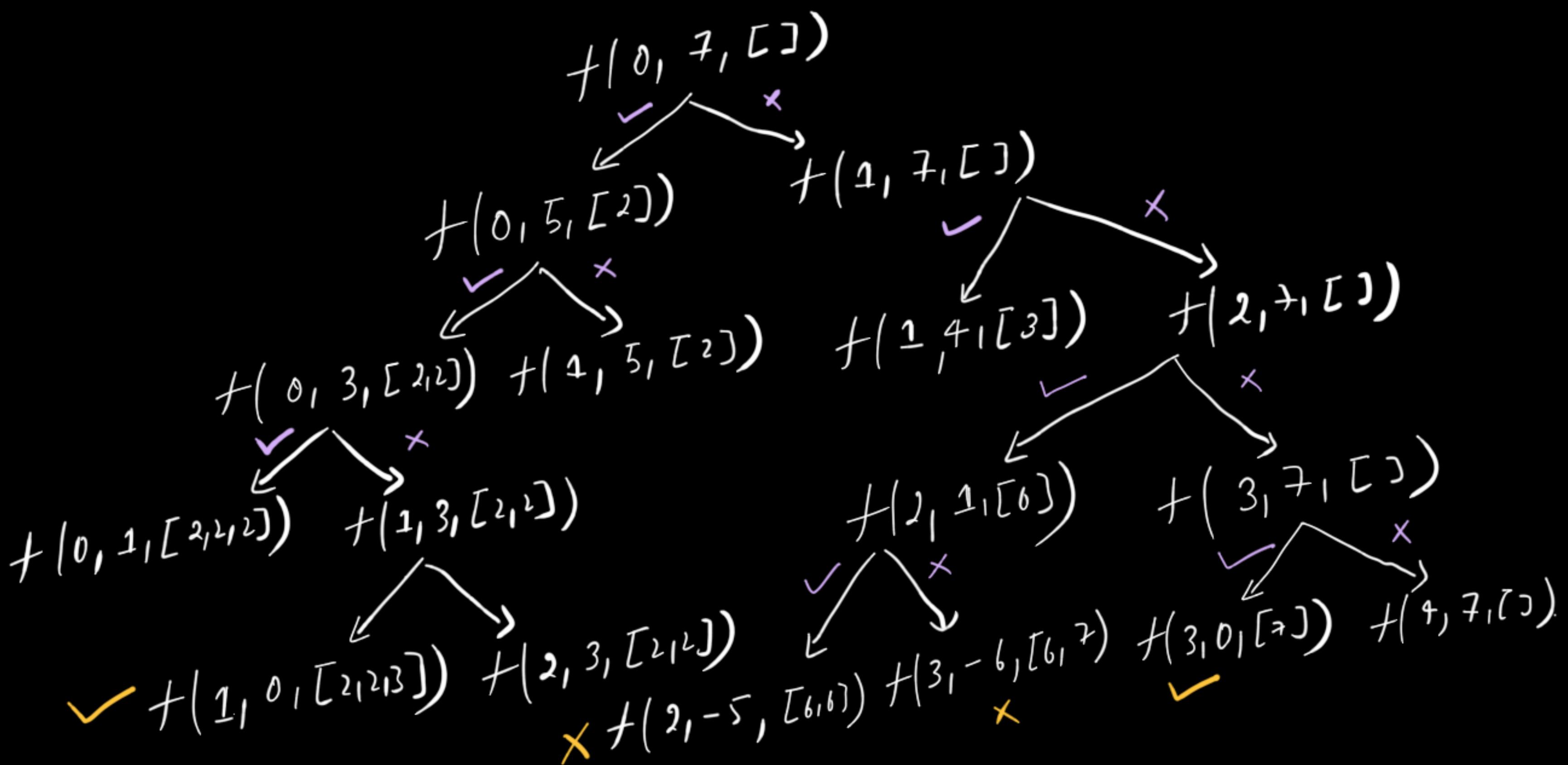
→ Combination Sum

↳ Return a list of all unique combinations, where chosen number sum up to target.

- The same number can be chosen unlimited number of times

$\begin{matrix} 0 & 1 & 2 & 3 \\ \boxed{2 \mid 3 \mid 6 \mid 7} \end{matrix}$

target = 7



→ How do we achieve unique combination?

- ① Sort the array, so the duplicate element come adjacent to each other.
- ② Whenever we stumble upon similar element while not-pick option we just keep incrementing the index i

$\begin{matrix} 2 \mid 6 \mid 7 \mid 3 \mid 3 \mid 4 \end{matrix}$

sort → $\begin{matrix} 2 \mid 3 \mid 3 \mid 4 \mid 6 \mid 7 \end{matrix}$

```
// not-pick
int nextIndex = index + 1;
while (nextIndex < arr.length &&
      arr[nextIndex] == arr[index]) {
    nextIndex++;
}
CombSum(arr, nextIndex, target, curr, ans);
```

```
PartVII.java
public static void combSum(int[] arr, int index, int target, List<Integer> curr, List<List<Integer>> ans) {
    if (target == 0) {
        ans.add(new ArrayList<>(curr));
        return;
    }

    if (index >= arr.length || target < 0) return;

    // pick
    if (arr[index] <= target) {
        curr.add(arr[index]);
        combSum(arr, index, target - arr[index], curr, ans);
        curr.remove(index, curr.size() - 1);
    }

    // remove duplicates
    int nInd = index + 1;
    while (nInd < arr.length && arr[nInd] == arr[index]) nInd++;

    // not pick
    combSum(arr, nInd, target, curr, ans);
}
```

→ Combination sum II
 ↳ similar question but, we can only choose a number once.

↳ Solution
 ↳ do index+1 while picking an element.

TC: $O(2^n \times n)$ + $n \log n$ ↳ sorting

SC: $O(n) + O(\text{ans size})$

→ Subset-Sum

subset-sum

↳ Return sum of each derived subset in a list.

gen. subset { } {1} {2} {3} {1,2} {2,3} {1,3} {1,2,3}

1	2								
2	3	0	1	2	3	3	5	9	6

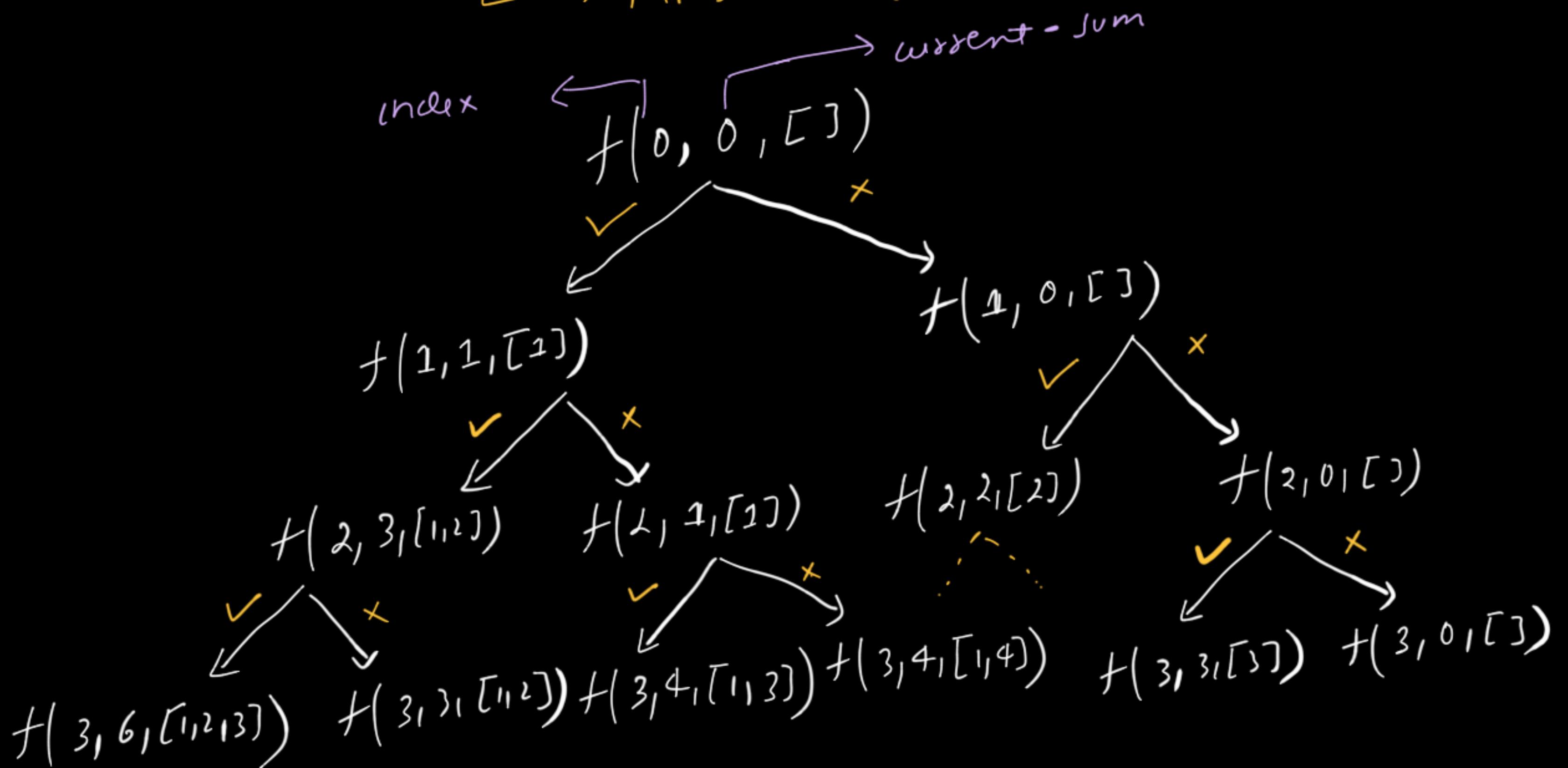
→ sum

→ Brute force - approach

route force - app

→ generate all power-set & then sum all the element
individually
 $\rightarrow O(2^n \cdot n)$ → time complexity.

Time complexity:
→ takes $\Theta(n \times n)$ → Time complexity.



```
● ● ● PartVIII.java

public static void subsetSum(int[] arr , int index , int currSum , List<Integer> ans){
    if (index >= arr.length){
        ans.add(currSum);
        return;
    }

    // pick
    subsetSum(arr, index: index + 1 , currSum: currSum + arr[index] , ans);

    // not pick
    subsetSum(arr, index: index + 1 , currSum , ans);
}
```

$$TC: O(2^n)$$

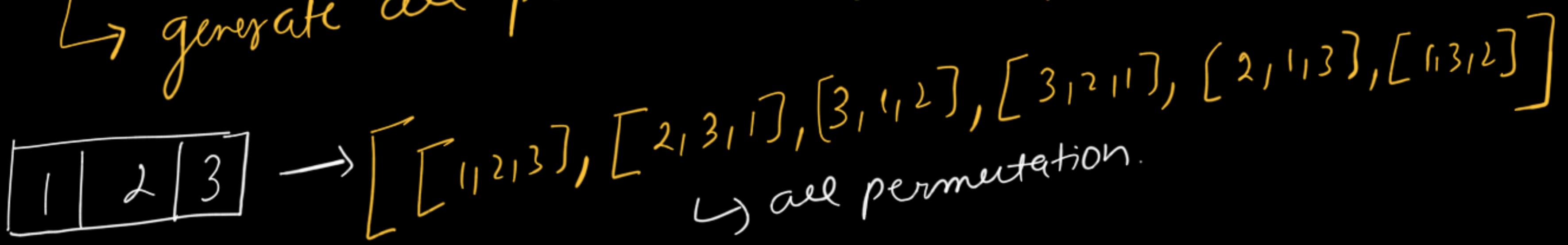
$$SC: O(n) + O(2^n)$$

↓
tagc-splice

ans list size

→ permutation

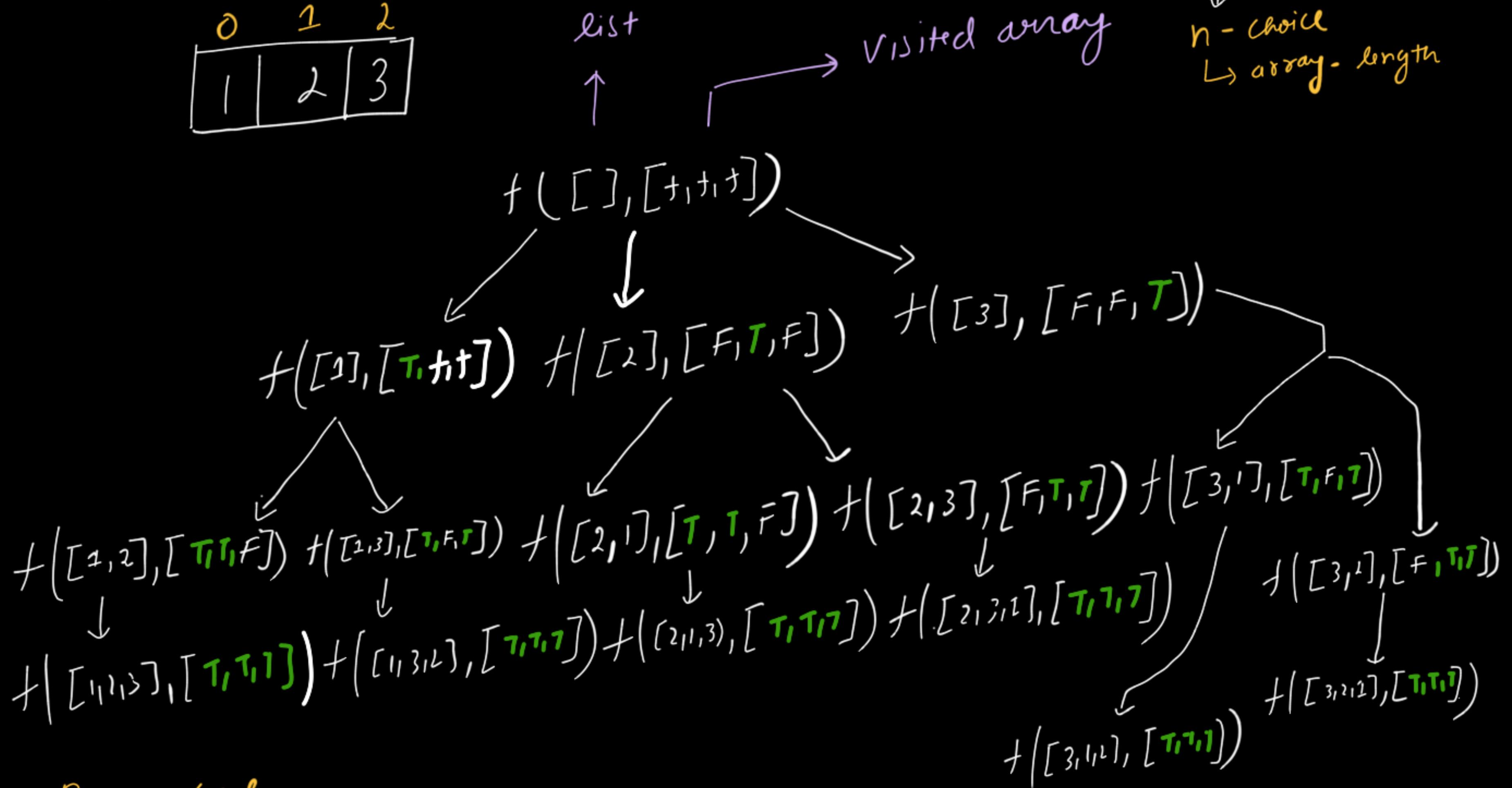
↳ generate all permutation of array / string.



→ Approach - 1

Initially, we can take any element from the array

\downarrow
n-choice
↳ array-length



Base - (are)

① list become equal to array length
↳ we have one of our permutation.

```
PartIX.java
public static void permute(int[] arr, boolean[] vis, List<Integer> curr, List<List<Integer>> ans) {
    if (arr.length == curr.size()) {
        ans.add(new ArrayList<>(curr));
        return;
    }

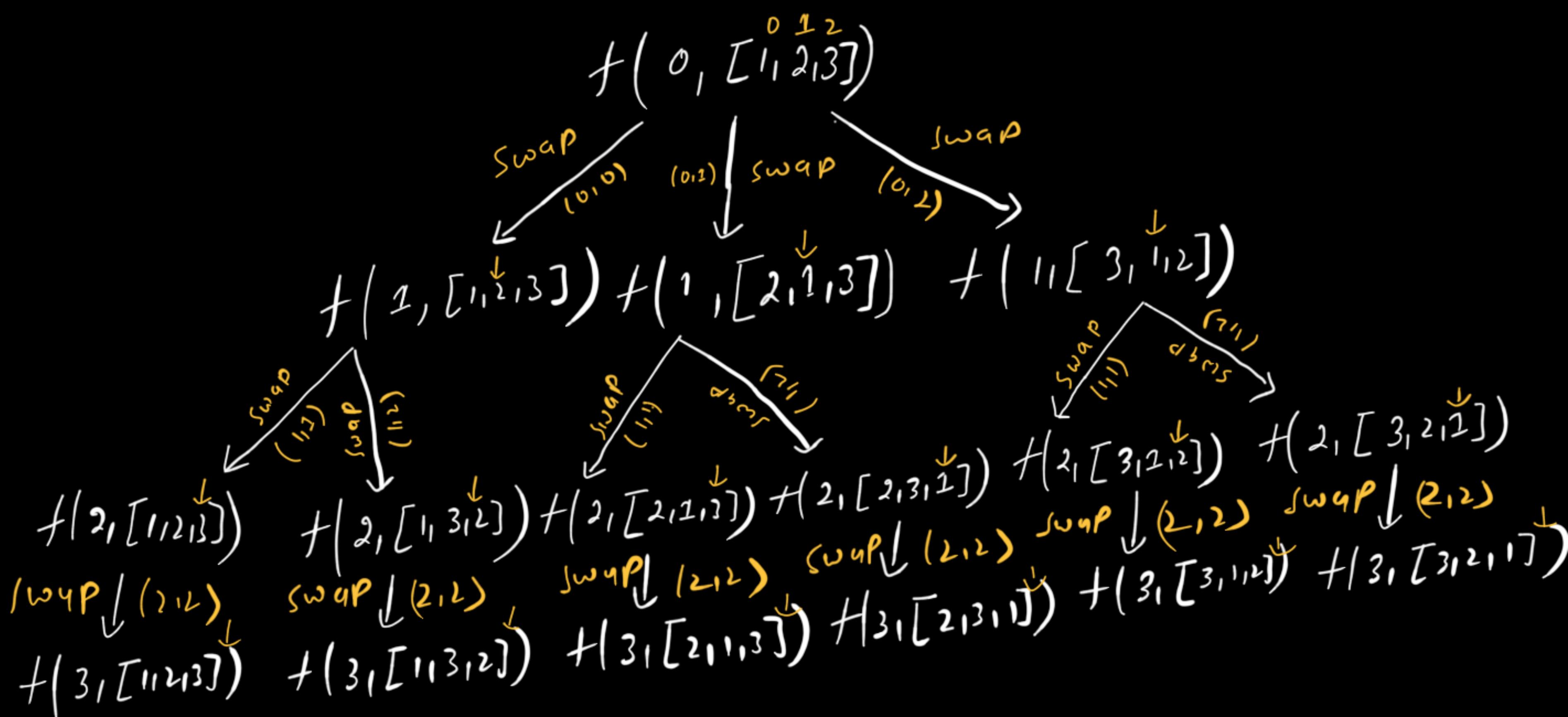
    for (int i = 0; i < arr.length; i++) {
        if (!vis[i]) {
            vis[i] = true;
            curr.add(arr[i]);
            permute(arr, vis, curr, ans);
            curr.remove(index: curr.size() - 1);
            vis[i] = false;
        }
    }
}
```

$$TC: O(n! * n)$$

$$SC: O(n! * n)$$

→ Approach - 2
 ↳ without extra visited array → swapping the element

0	1	2
1	2	3



→ Base Case

↳ whenever $\text{index} == \text{arr.length}$
 we have one of our permutation

```

PartIX.java
public static void permutation(int[] arr, int index, List<List<Integer>> ans) {
    if (index == arr.length) {
        List<Integer> curr = new ArrayList<>();
        for (int e : arr) curr.add(e);
        ans.add(curr);
        return;
    }

    for (int i = index; i < arr.length; i++) {
        swap(arr, i, index);
        permutation(arr, index + 1, ans);
        swap(arr, i, index);
    }
}

private static void swap(int[] arr, int f, int s) { 2 usages new *
    int t = arr[f];
    arr[f] = arr[s];
    arr[s] = t;
}

```

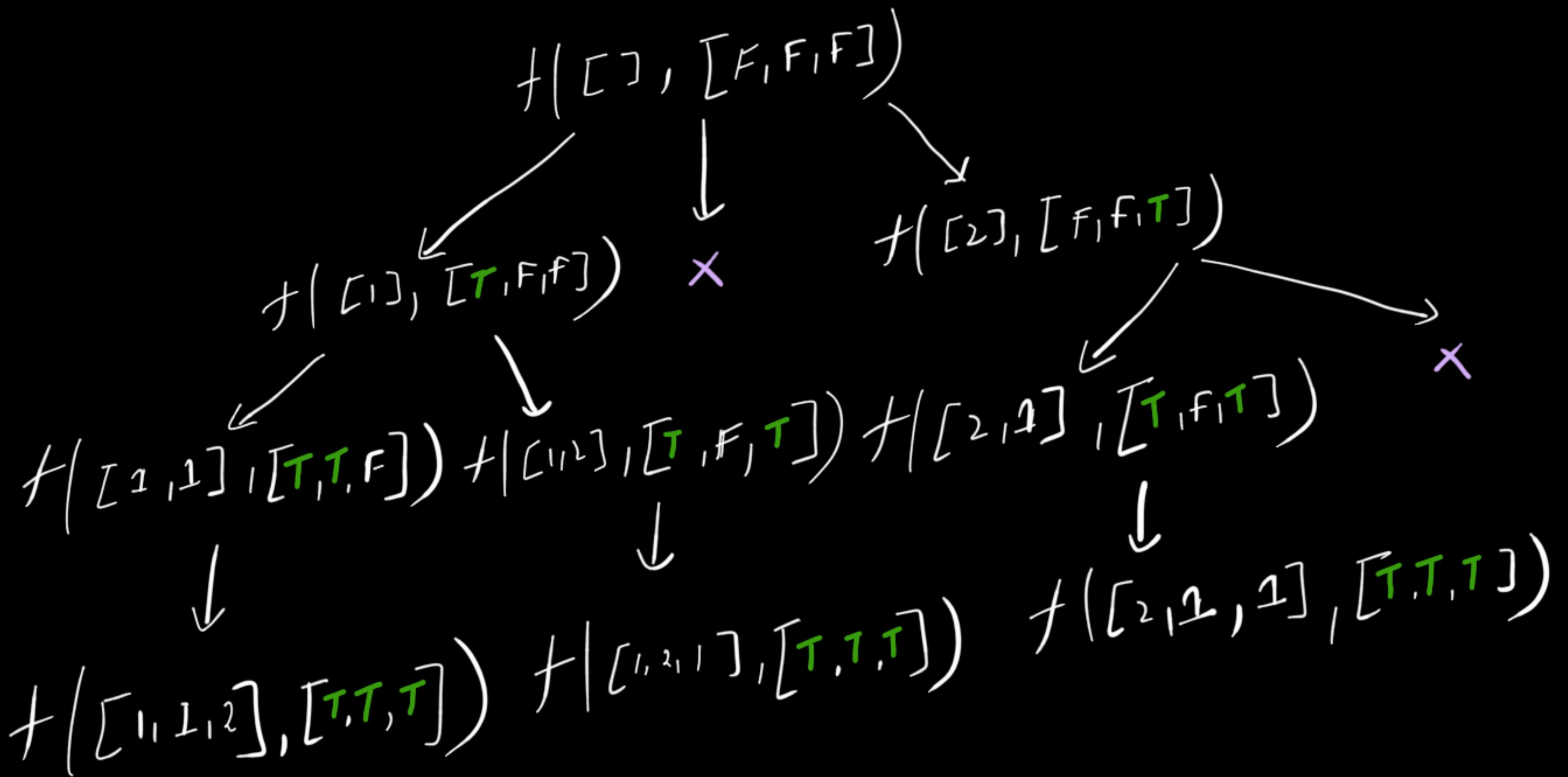
$$T.C : O(n \times n!)$$

$$S.C : O(n \times n!)$$

→ permutation II

generate unique permutation, duplicates are not allowed

0 1 2
[1 | 1 | 2]



→ To skip an element

(1) We are checking if $arr[i] == arr[i-1]$?

(2) Check if the previous same element hasn't been used in the worst recursion

path
 \downarrow
 $\neg vis[i-1]$?

To check if adjacent elements are similar or not we need to sort it before passing it to the $f()$.
 $O(n \log n)$

→ if these both condition are satisfied, then we can safely skip the duplicate.

```

PartX.java
public static void permuteUnique(int[] arr, boolean[] vis, List<Integer> curr, List<List<Integer>> ans) {
    if (arr.length == curr.size()) {
        ans.add(new ArrayList<>(curr));
        return;
    }

    for (int i = 0; i < arr.length; i++) {
        // remove duplicate
        if (i > 0 && arr[i] == arr[i - 1] && !vis[i - 1]) continue;

        if (!vis[i]) {
            vis[i] = true;
            curr.add(arr[i]);
            permuteUnique(arr, vis, curr, ans);
            curr.remove(index: curr.size() - 1);
            vis[i] = false;
        }
    }
}
  
```

↗ duplicate handling

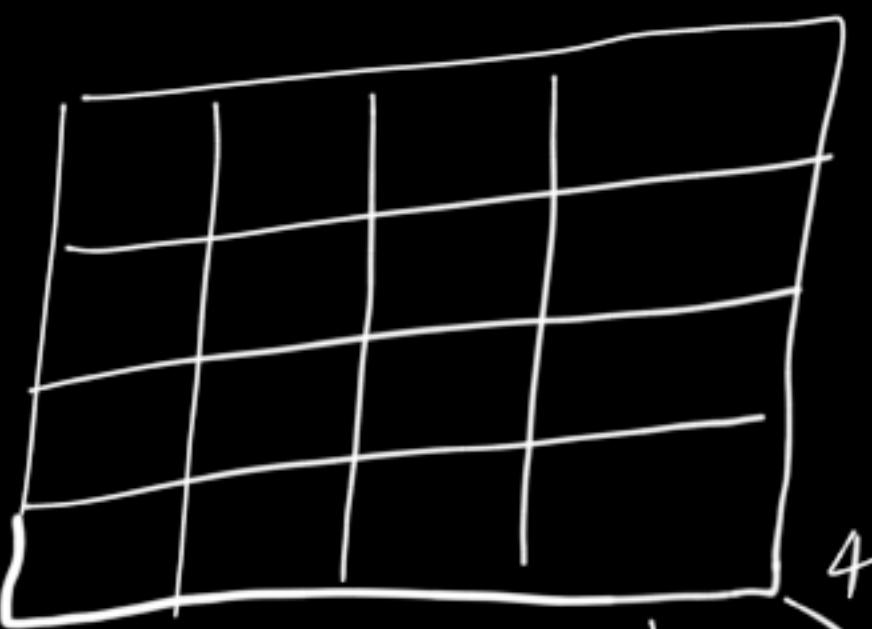
Tc: $O(n! * n)$

Sc: $O\left(\frac{n!}{f_1! \times f_2! \times \dots \times f_n!}\right)$

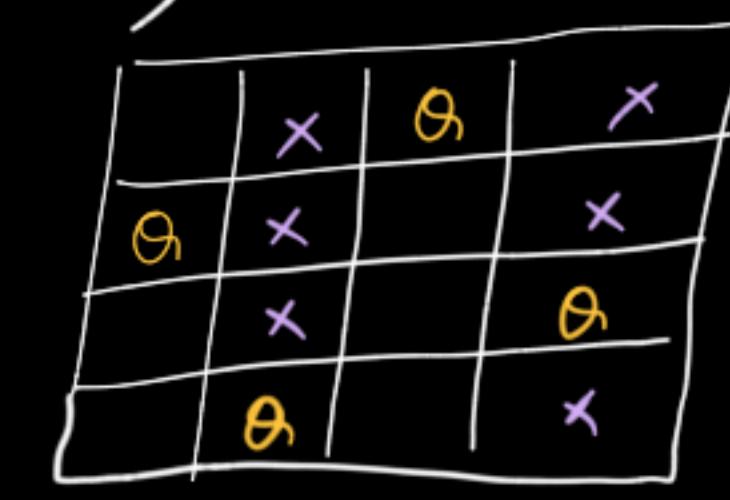
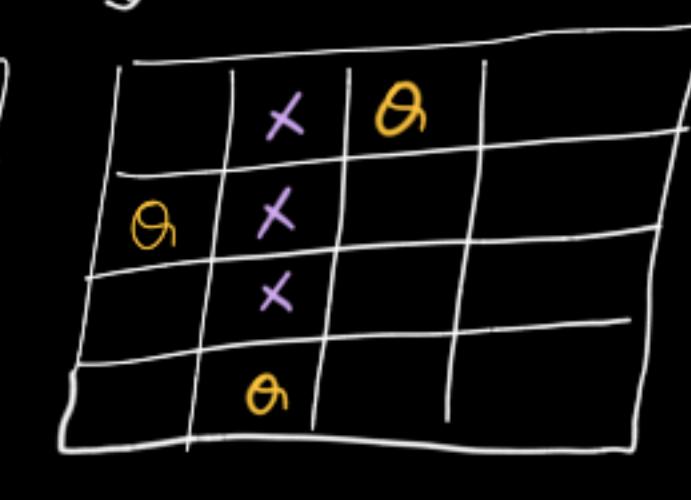
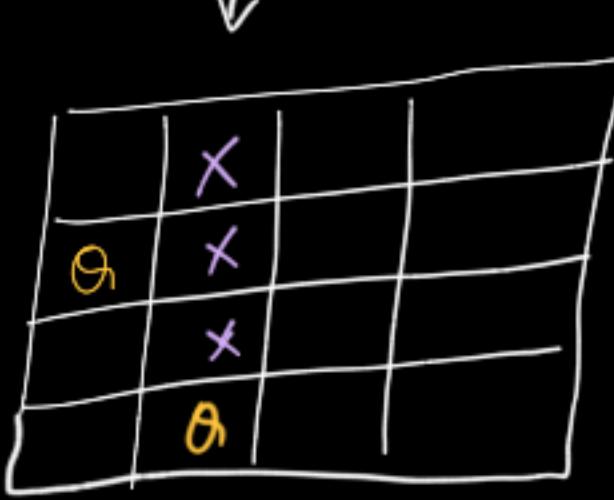
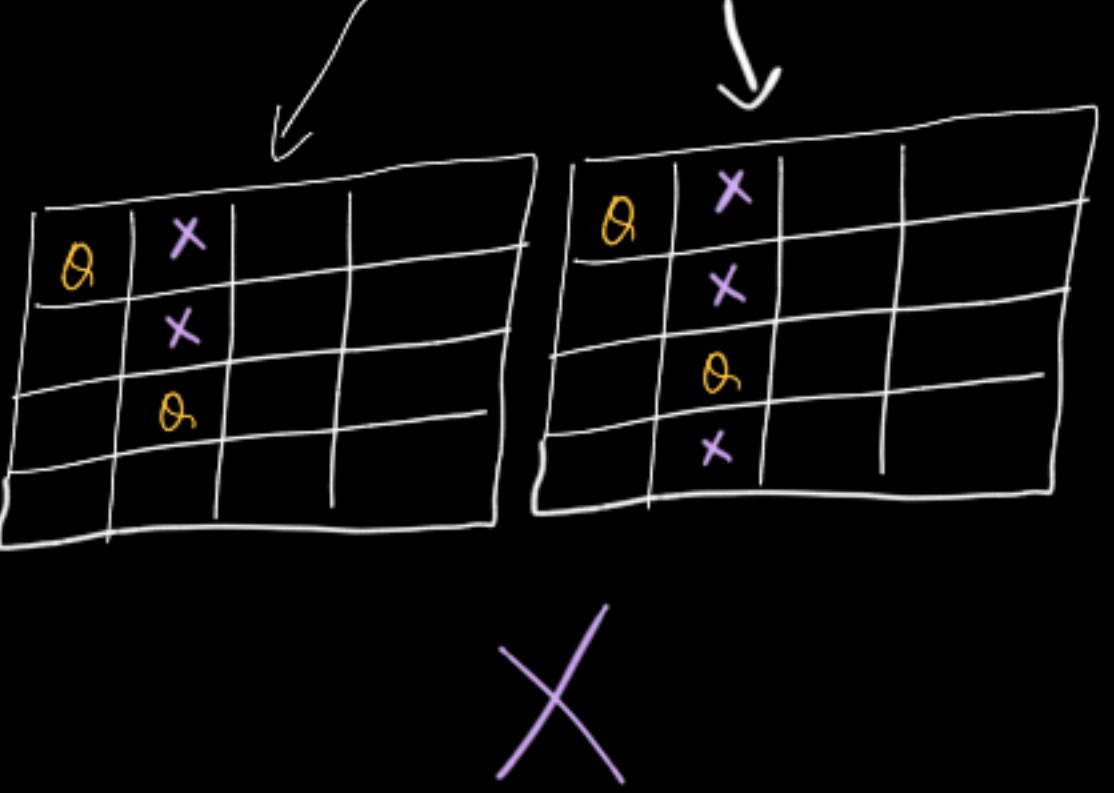
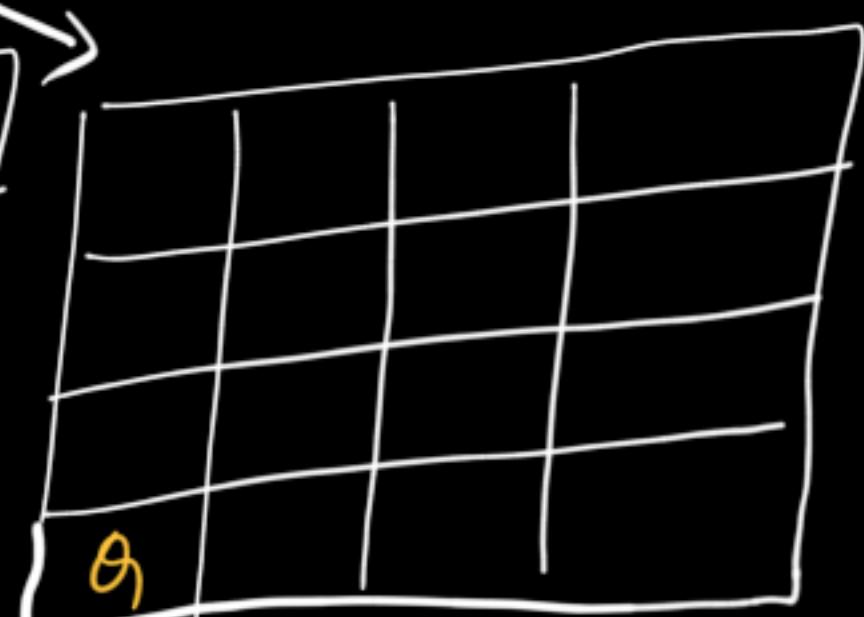
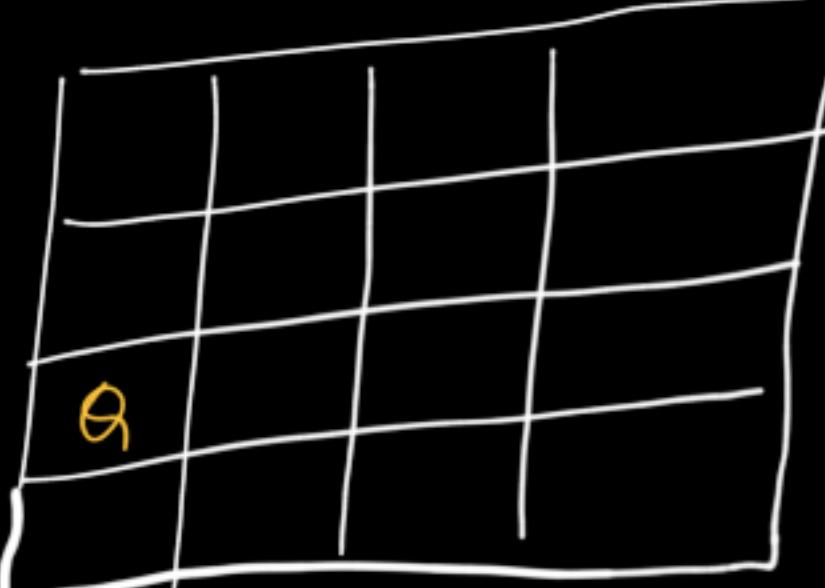
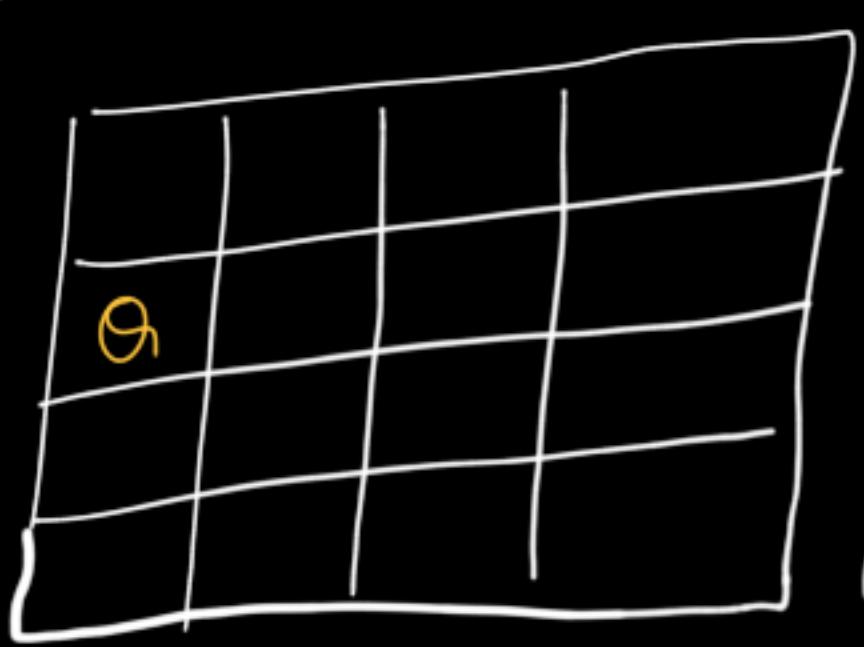
$f_i \rightarrow$ frequency of each element

$\rightarrow N$ -Queens

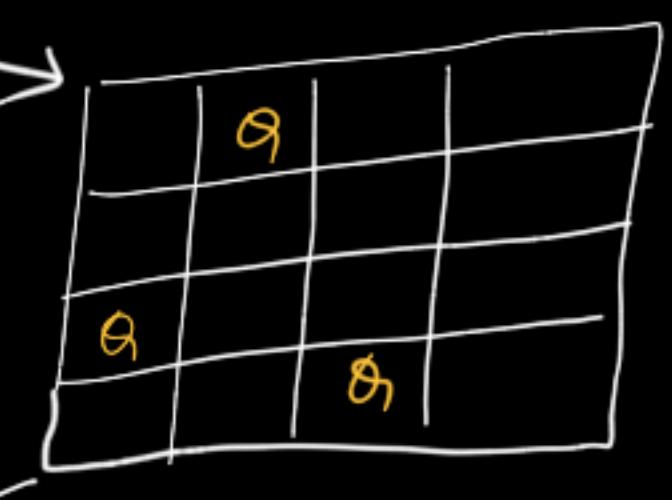
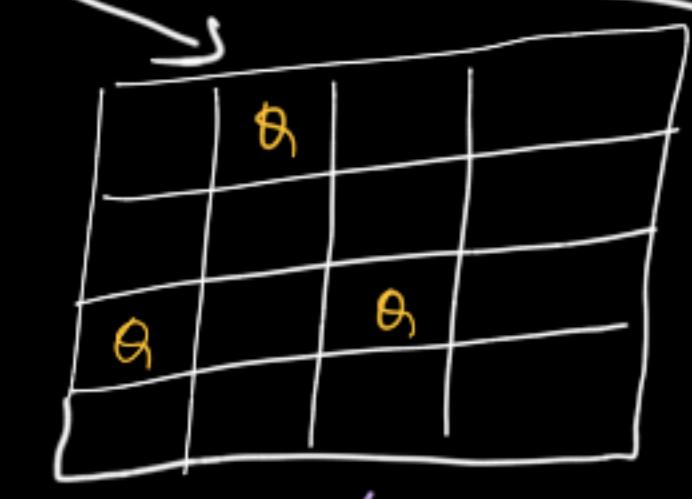
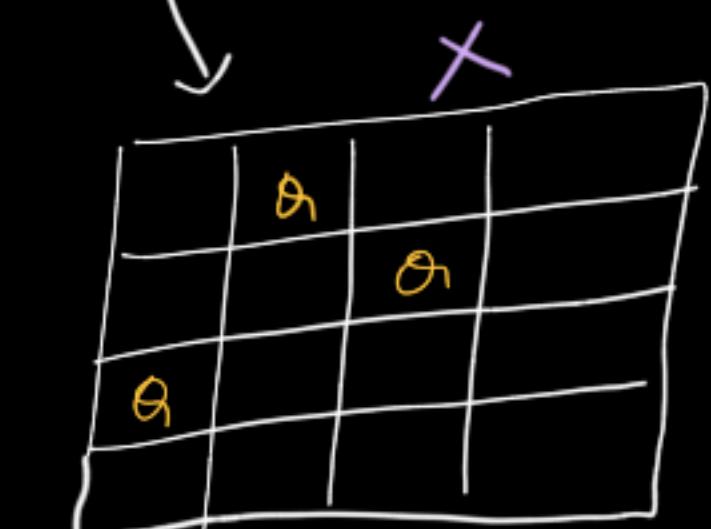
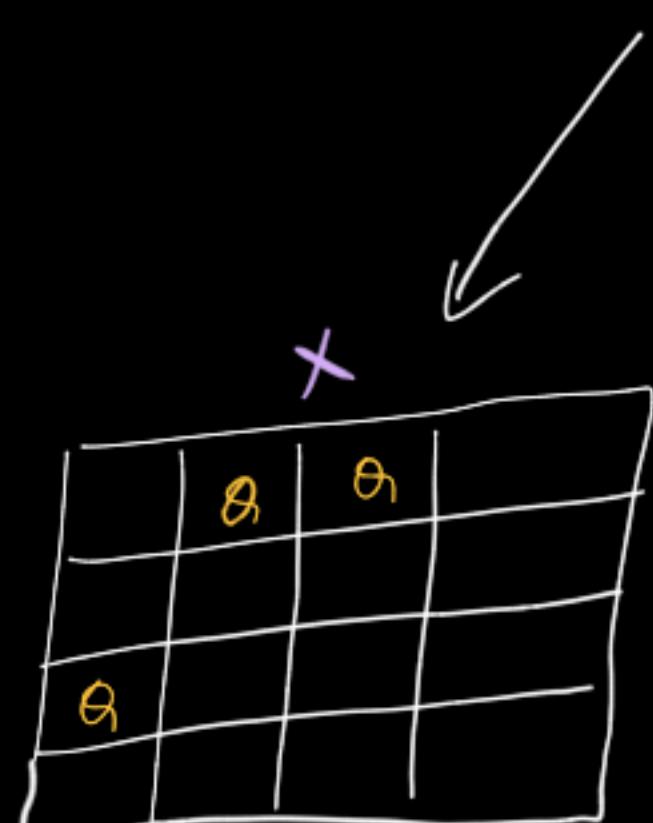
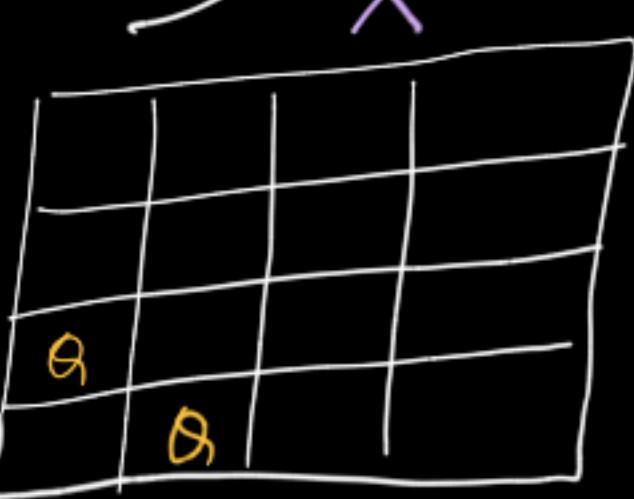
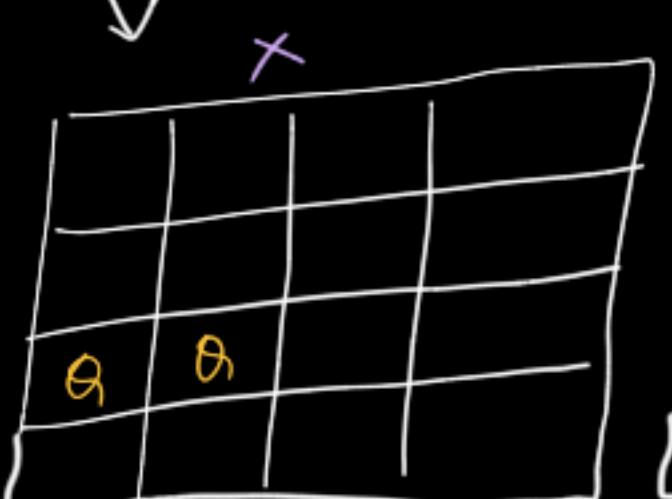
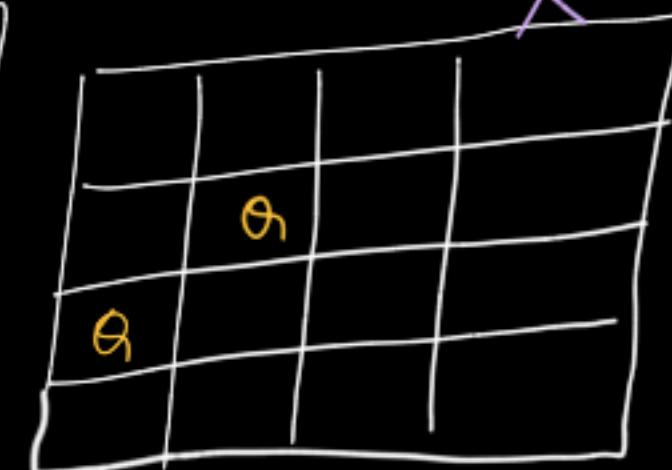
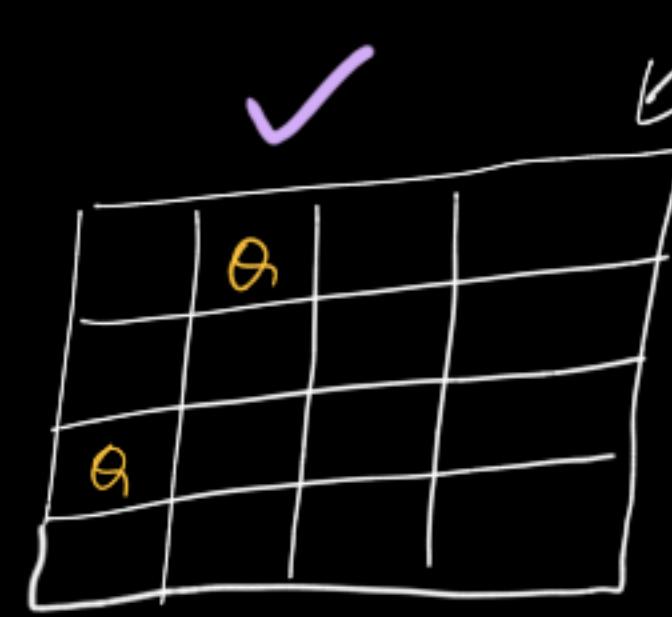
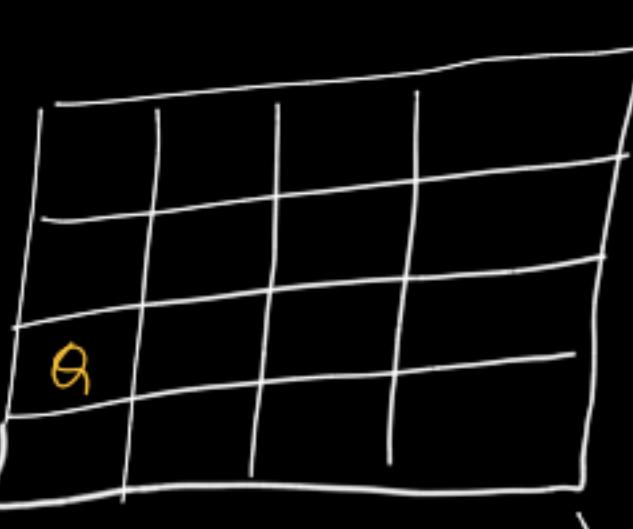
\hookrightarrow place queens on $n \times n$ chess board, such that their path do not intersect either diagonally | left, right, up or down



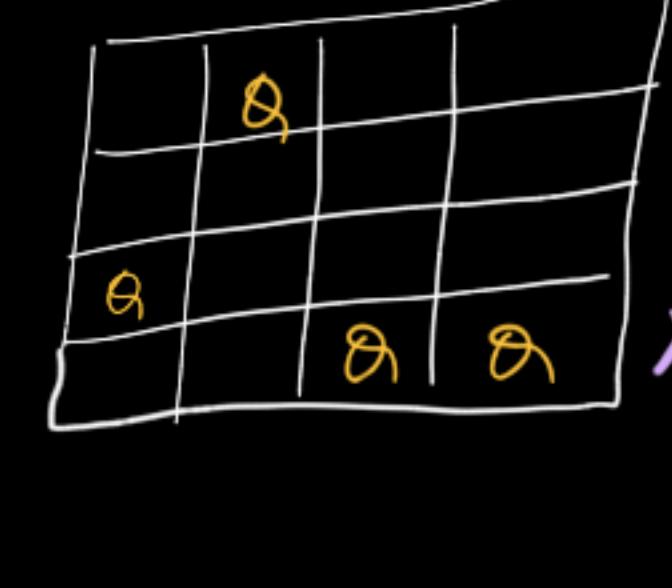
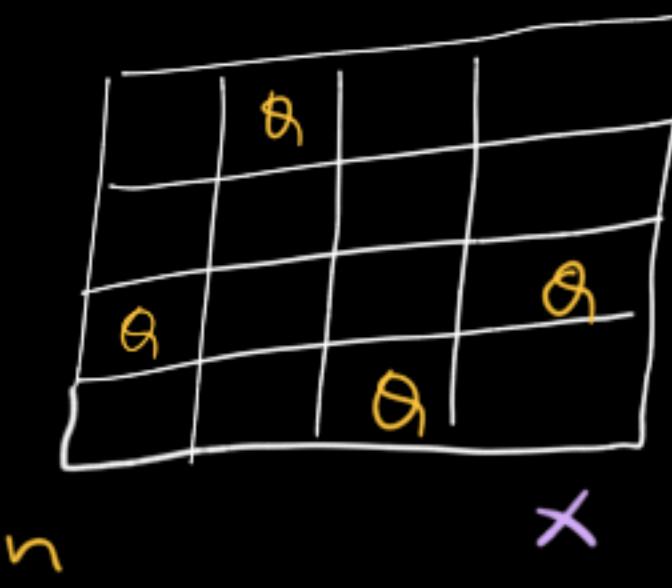
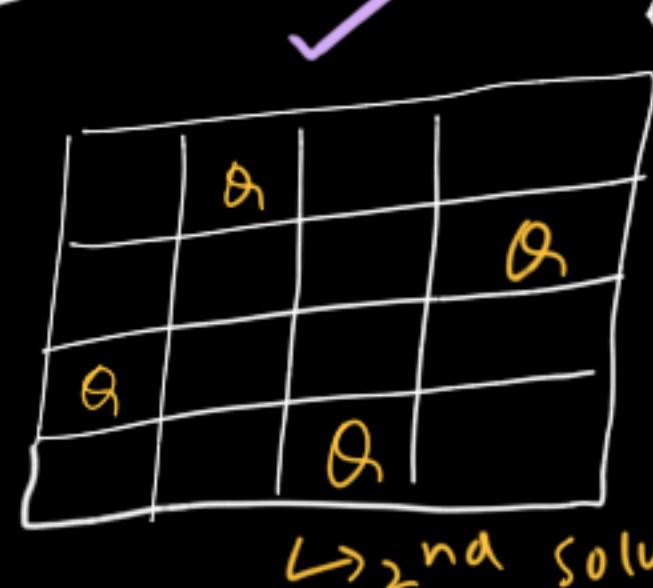
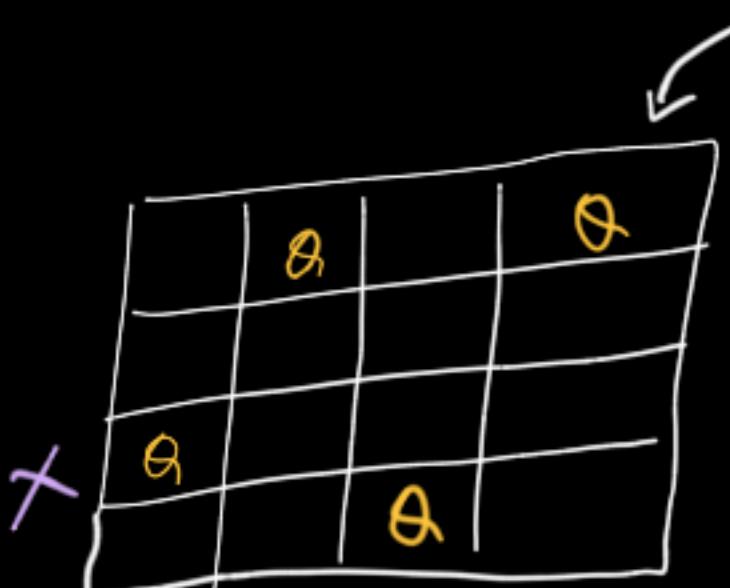
\rightarrow place Queen
row wise



$\hookrightarrow 1^{st}$ solution



✓

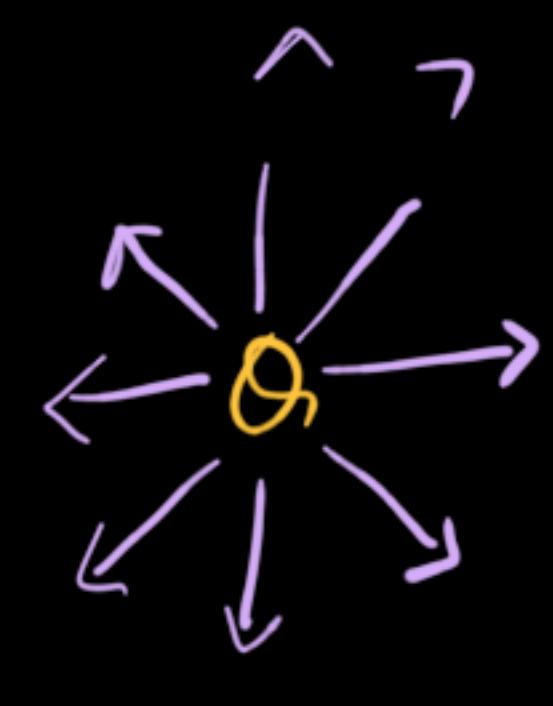
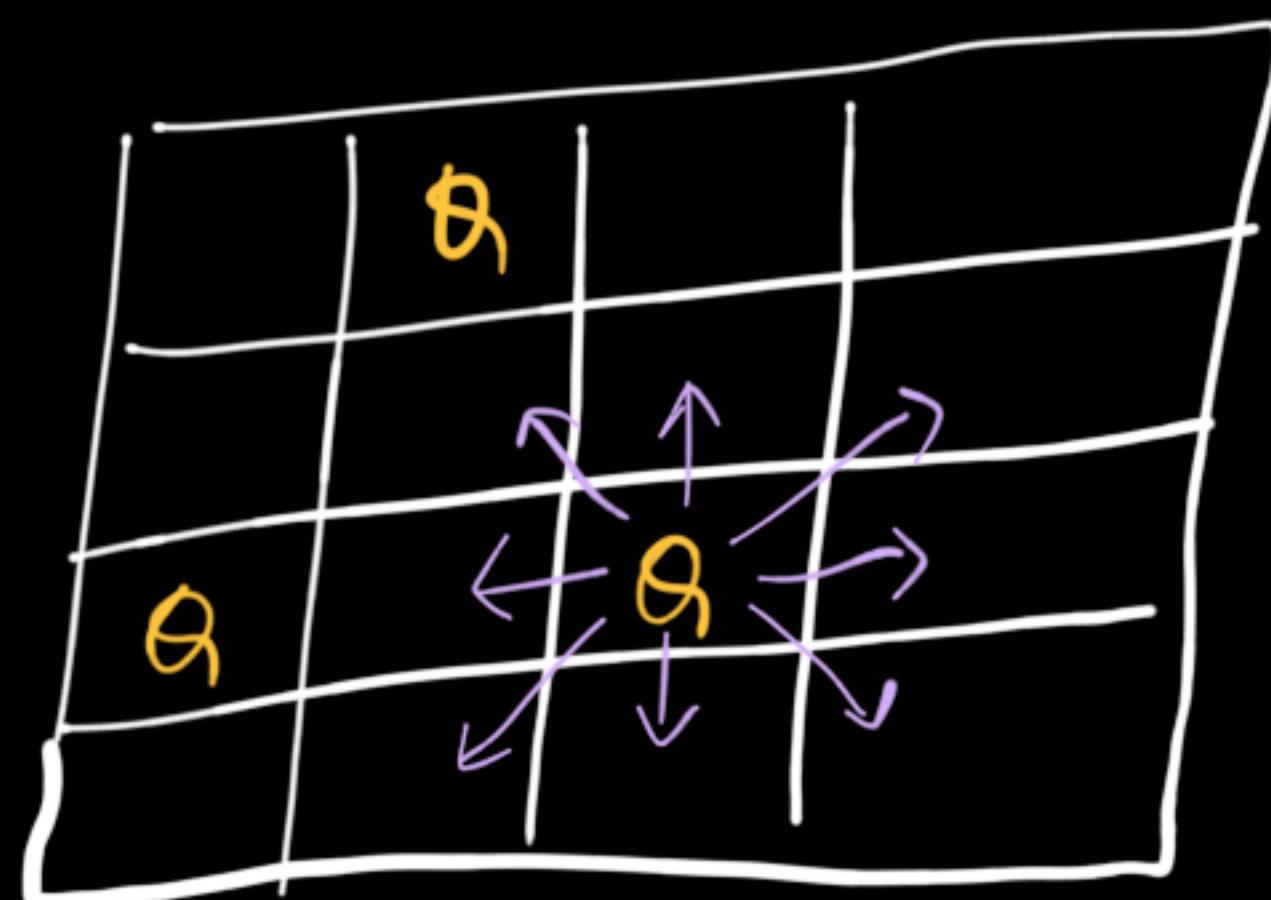


\hookrightarrow 2nd solution

✗

✗

→ How do we check if placing a queen is safe?

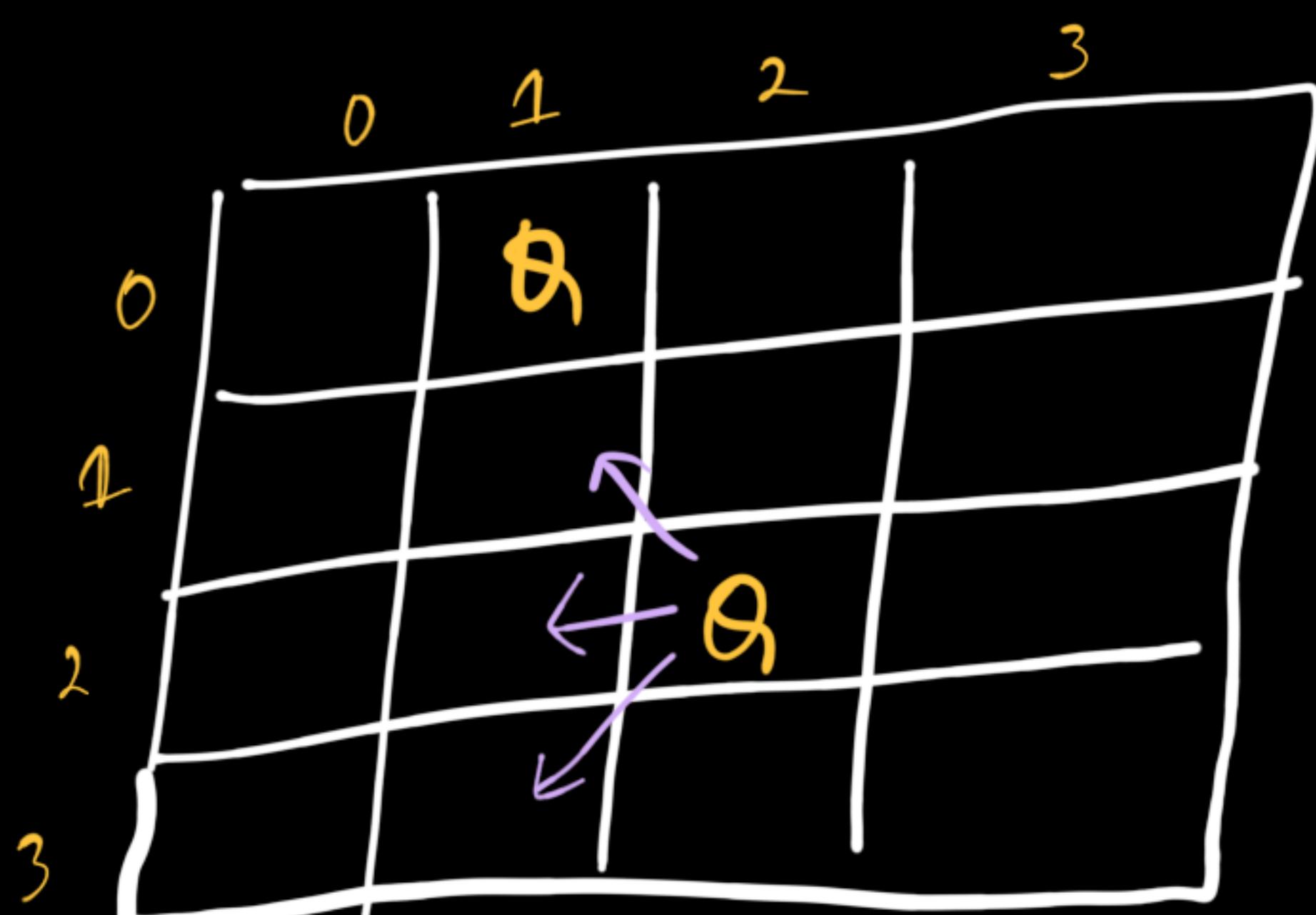
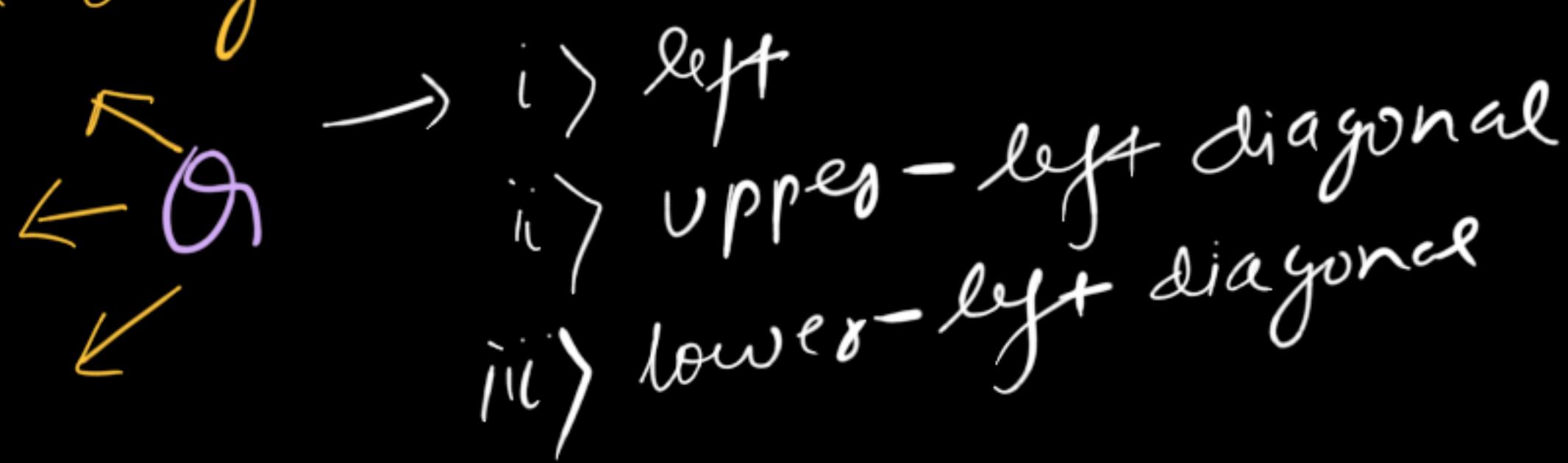


Queen can attack
in 8 directions

- We already know that, there will only be one queen per column.
↳ i.e no need to check up & down

- We also know that, there is no need to check right, because we for sure know that any queen that threatens the current position must be in a previous (left) column.

↳ i.e we only need to check for



→ For upper-left diagonal

↳ Initially: $Q(2,2)$
↓ after moving
 $Q(1,1)$

Both row & column is decreasing

→ For lower-left diagonal

↳ Initially: $Q(2,2)$
↓ after moving
 $Q(3,1)$

- row increases
- column decreases

→ For left-side

↳ Initially: $Q(2,2)$
↓ after moving
 $Q(2,1)$

$$TC: O(n! \times n)$$

$$SC: O(n) + O(N^2) + O(S \times N^2)$$

\downarrow Stack space \downarrow Board size

S: No of Valid Solution

N^2 : Each solution has n ,
each of length n

```

PartXI.java
public static List<List<String>> solve(int n) { 1 usage new *
    List<List<String>> ans = new ArrayList<>();
    Character[][] board = buildBoard(n);
    nQueen(board, col: 0, ans);
    return ans;
}

private static void nQueen(Character[][] board, int col, List<List<String>> ans) {
    if (col == board[0].length) {
        ans.add(boardToStringList(board));
        return;
    }

    for (int row = 0; row < board.length; row++) {
        if (canWePlaceSafely(board, row, col)) {
            board[row][col] = 'Q';
            nQueen(board, col: col + 1, ans);
            board[row][col] = '.';
        }
    }
}

private static boolean canWePlaceSafely(Character[][] board, int row, int col) {
    int dupRow = row;
    int dupCol = col;

    // check upper-left
    while (row >= 0 && col >= 0) {
        if (board[row][col] == 'Q') return false;
        row--;
        col--;
    }

    // check left
    row = dupRow;
    col = dupCol;
    while (col >= 0) {
        if (board[row][col] == 'Q') return false;
        col--;
    }

    // check lower-left
    col = dupCol;
    while (row < board.length && col >= 0) {
        if (board[row][col] == 'Q') return false;
        row++;
        col--;
    }

    return true;
}

private static Character[][] buildBoard(int n) { 1 usage new *
    Character[][] board = new Character[n][n];
    for (int i = 0; i < n; i++) {
        board[i] = new Character[n];
        Arrays.fill(board[i], val: '.');
    }
    return board;
}

private static List<String> boardToStringList(Character[][] board) { 1 usage new *
    List<String> res = new ArrayList<>();
    for (Character[] row : board) {
        StringBuilder sb = new StringBuilder();
        for (Character cell : row) {
            sb.append(cell);
        }
        res.add(sb.toString());
    }
    return res;
}

```

→ How can we optimize it?

From Top-left to bottom right

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

① We notice that all diagonal have same sum of $(\text{row} + \text{col})$ from top-left to bottom right.

↳ How much space do we need to store all diagonal?

↳ Range of index

↳ min: $(0, 0)$

$$\text{max: } (n-1) + (n-1) = 2n - 2$$

• All unique diagonal is

$$\rightarrow (\text{max} - \text{min}) + 1 = (2n - 2 - 0) + 1$$

$$\boxed{\text{Space} = 2n - 1}$$

boolean[] = $\boxed{F | F | \dots | F}_{(2n)}$

	0	1	2	3
0	0	-1	-2	-3
1	1	0	-1	-2
2	2	1	0	-1
3	3	2	1	0

↓
we can't use negative
index, so to make it
+ve we add n

	0	1	2	3
0	4	3	2	1
1	5	4	3	2
2	6	5	4	3
3	7	6	5	4

Range of Index

$$\rightarrow \text{min: } -(n-1) + n = 1$$

$$\text{max: } (n-1) + n = 2n - 1$$

Space needed is

$$\text{Space} = (\text{max} - \text{min}) + 1$$

$$= \left[(n-1) - [-(n-1)] \right] + 1$$

$$= \left[n-1 + n-1 \right] + 1$$

$$= [2n-2] + 1$$

$$= 2n - 1$$

boolean $\boxed{F | F | \dots | F}_{(2n)}$

→ How do we check for rows?

→ Create boolean array of size rows [board.length].

boolean[] =

F	F	F	F
---	---	---	---

```
PartXI.java
public static List<List<String>> solve(int n) {
    List<List<String>> ans = new ArrayList<>();
    Character[][] board = buildBoard(n);
    boolean[] r = new boolean[board.length];
    boolean[] tlbr = new boolean[2 * n];
    boolean[] trbl = new boolean[2 * n];
    nQueenOpt(board, col: 0, r, tlbr, trbl, ans);
    return ans;
}
```

boolean[] r = for checking rows

boolean[] tlbr = for checking top-left to bottom right diagonal.

boolean[] trbl = for checking top-right to bottom left diagonal.

```
PartXI.java
public static void nQueenOpt(Character[][] board, int col, boolean[] r, boolean[] tlbr, boolean[] trbl, List<List<String>> ans) {
    if (board.length == col) {
        ans.add(boardToStringList(board));
        return;
    }

    for (int row = 0; row < board.length; row++) {
        int topLeftBR = row + col;
        int topRightBL = row - col + board.length;

        if (r[row] || tlbr[topLeftBR] || trbl[topRightBL]) continue;

        board[row][col] = 'Q';
        r[row] = true;
        tlbr[topLeftBR] = true;
        trbl[topRightBL] = true;

        nQueenOpt(board, col: col + 1, r, tlbr, trbl, ans);
        board[row][col] = '.';
        r[row] = false;
        tlbr[topLeftBR] = false;
        trbl[topRightBL] = false;
    }
}
```

→ while visiting mark true.
→ while backtracking mark false.

TC: $O(n!)$ no. of solution. ↗
SC: $O(n) + O(2n) + O(2n) + O(n) + O(S \times N^2)$
↓ ↓ ↓ ↓ ↓
Stack space tlbr + trbl row ans list

→ Sudoku-Solver

↳

5	3		7					
6			1	9	5			
	9	8				6		
8			6					3
4		8	3					1
7			2				6	
6				2	8			
		4	1	9				5
			8			7	9	

Rule:

- 1) Each of the digit 1-9 must occur exactly once in each row & column
- 2) Each of the digit 1-9 must occur exactly once in each of the $9 \rightarrow 3 \times 3$ sub-boxes of the grid.

Solution

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

→ Each of the row & column have unique numbers

• Each of the 3×3 sub-boxes have unique numbers.

- Since we need to try all possible values of placing 1-9 digit safely. we can use recursion to try out all possible combination.

① Can we fill up a number 1-9 which is safe to place.

5	3		6	7	8	9		2
6	7	2	1	9	5	3	4	8
9	8	3	4	2	5	6	7	
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9		4	8	5	6
9	6		5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

False

True

5	3	1	6	7	8	9	?	2
6	7	2	1	9	5	3	4	8
9	8	3	4	2	5	6	7	
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9		4	8	5	6
9	6		5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

5	3	4	6	7	8	9	2	
6	7	2	1	9	5	3	4	8
9	8	3	4	2	5	6	7	
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9		4	8	5	6
9	6		5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

←

↓
Can we place 1-9 at ? ×

↓
return false

↓
This recursion call will
not have our answer

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
9	8	3	4	2	5	6	7	
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9		4	8	5	6
9	6		5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

← Try
7-9

↓ True

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9		4	8	5	6
9	6		5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

← Try 1-9

↑

↙ True

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6		5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

← Try 1-9

↙ True

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

← Try 1-9

→ How do we check if putting a new value is safe or not?

→ For rows & column, just iterate from index 0 - 8 & check
 $\text{board}[\text{row}][\text{i}] == \text{num} \text{ || } \text{board}[\text{i}][\text{col}] == \text{num}$

→ For 3×3 grid

	0	1	2	3	4	5	6	7	8
0	5	3	4	6	7	8	9	1	2
1	6	7	2	1	9	5	3	4	8
2	1	9	8	3	4	2	5	6	7
3	8	5	9	7	6	1	4	2	3
4	4	2	6	8	5	3	7	9	1
5	7	1	3	9	2	4	8	5	6
6	9	6	1	5	3	7	2	8	4
7	2	8	7	4	1	9	6	3	5
8	3	4	5	2	8	6	1	7	9

Let's suppose we are filling a safe number from 1-9 at index $(6, 2)$?



How do we check the 3×3 grid?

↳ How do we know which grid to check or from which (row, col) index?

For $(6, 2)$ index?

→ we need to know from where to start our 3×3 grid search?

$(0,0) \dots (0,2)$	$(0,3) \dots (0,5)$	$(0,6) \dots (0,8)$
$(2,0) \dots (2,2)$	$(2,3) \dots (2,5)$	$(2,6) \dots (2,8)$
$(3,0) \dots (3,2)$	$(3,3) \dots (3,5)$	$(3,6) \dots (3,8)$
$(5,0) \dots (5,2)$	$(5,3) \dots (5,5)$	$(5,6) \dots (5,8)$
$(6,0) \dots (6,2)$	$(6,3) \dots (6,5)$	$(6,6) \dots (6,8)$
$(8,0) \dots (8,2)$	$(8,3) \dots (8,5)$	$(8,6) \dots (8,8)$

9×9

Blocks of 3?

$$(8, 1) \rightarrow (6, 2)$$

$$s_R = 8/3$$

$$s_C = 1/3$$

$$(s_R, s_C) \Rightarrow \left(\frac{8}{3}, \frac{1}{3} \right) = \left(\frac{6}{3}, \frac{2}{3} \right) = (2, 0)$$

No if we multiply it by 3, we will get our start row & start column

$$3(s_R, s_C) = 3(2, 0)$$

$$\boxed{(s_R, s_C) = (6, 0)}$$

$$\rightarrow S_R = 6, \quad S_C = 0$$

$(6,0)$	$(6,1)$	$(6,2)$
$(7,0)$	$(7,1)$	$(7,2)$
$(8,0)$	$(8,1)$	$(8,2)$

0	1	2	
0	9	6	1
1	2	8	7
2	3	4	5

\rightarrow How do we access each element? from $(6,0)$ to $(8,2)$,

\hookrightarrow Flatten the 3×3 grid in 1D

$$[9, 6, 1, 2, 8, 1, 7, 1, 3, 1, 4, 1, 5]$$

0 1 2 3 4 5 6 7 8

$1 \rightarrow$ 2D index

$$0 \rightarrow (i/3, i \% 3) = (0,0)$$

$$1 \rightarrow (1/3, 1 \% 3) = (0,1)$$

$$2 \rightarrow (2/3, 2 \% 3) = (0,2)$$

$$3 \rightarrow (3/3, 3 \% 3) = (1,0)$$

→ So, to scan in single for loop?

$$sr = 3 * \lfloor \frac{i}{3} \rfloor + i \% 3;$$

$$sc = 3 * \lfloor \frac{j}{3} \rfloor + j \% 3;$$

```
PartXII.java

public static boolean sudoku(char[][] board) { 3 usages new *

    for (int row = 0; row < board.length; row++) {
        for (int col = 0; col < board[0].length; col++) {
            if (board[row][col] == '.') {

                for (char num = '1'; num <= '9'; num++) {
                    if (isValid(board, row, col, num)) {
                        board[row][col] = num;
                        if (sudoku(board)) return true;
                        board[row][col] = '.';
                    }
                }
            }
            return false;
        }
    }
    return true;
}

private static boolean isValid(char[][] board, int row, int col, char num) {

    for (int i = 0; i < 9; i++) {
        if (board[i][col] == num || board[row][i] == num) return false;
    }

    int sr = 3 * (row / 3) + i / 3;
    int sc = 3 * (col / 3) + i % 3;

    if (board[sr][sc] == num) return false;
}
return true;
}
```

$T_C : O(9^{81})$

$SC : O(N)$

\downarrow

Stack
Queue

\hookrightarrow for rows &
columns

\hookrightarrow For 3x3 grid

→ Find Unique Binary string

Given an array of strings nums containing n -unique binary strings each of length n , return a binary string of length n that does not appear in nums . Find any one.

$\text{nums} = \boxed{\begin{matrix} "111" & "011" & "001" \end{matrix}}$

↳ if we generate all binary string of length 3 → { $100, 000, 010, 110, 101$ }
↳ any one can be our answer.

$\text{set} = [\text{nums}]$

$f(" ", 3, \text{set})$

$f("0", 3, \text{set})$

$f("1", 3, \text{set})$

$f("00", 3, \text{set})$

$f("01", 3, \text{set})$ $f("10", 3, \text{set})$ $f("11", 3, \text{set})$

$f("000", 3, \text{set})$ $f("001", 3, \text{set})$ $f("100", 3, \text{set})$ $f("101", 3, \text{set})$

Base Case

① if length == 3, so
the generated
answer is not
in set,

Tc: $O(2^n)$

Sc: $O(N)$

↓
our answer

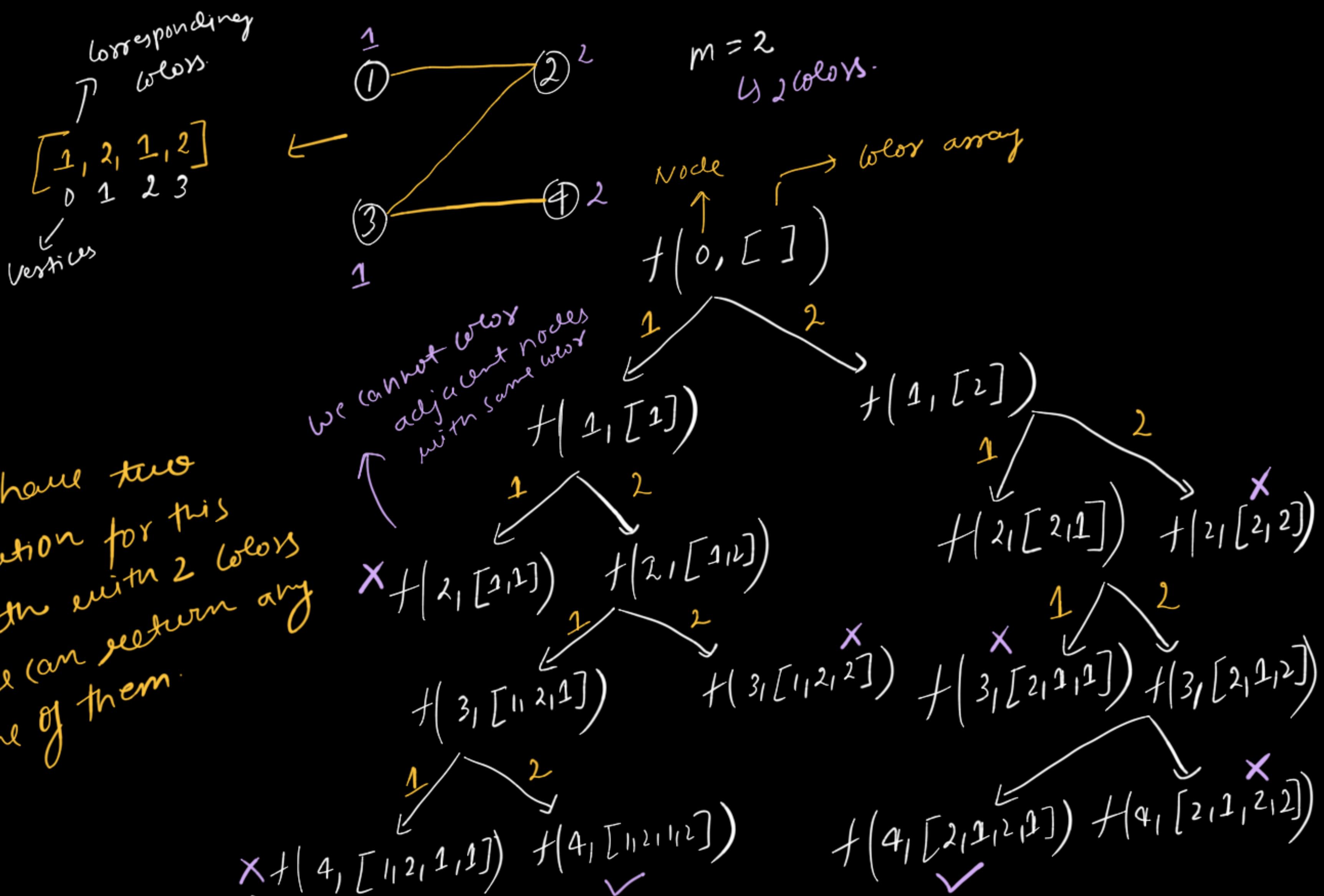
```
PartXIII.java
public static String findDifferentBinaryString(String[] nums) {
    Set<String> set = new HashSet<>();
    Collections.addAll(set, nums);
    return uniq(ans: "", nums.length, set);
}

private static String uniq(String ans, int length, Set<String> set) {
    if (ans.length() == length) {
        if (set.contains(ans)) {
            return null;
        }
        return ans;
    }
    // zero try
    String zt = uniq(ans: ans + '0', length, set);
    if (zt != null) return zt;

    // one try
    return uniq(ans: ans + '1', length, set);
}
```

M-coloring Problem

Given an undirected graph with N vertices & an Integer M , determine if it's possible to color the graph using at most M colors such that no two adjacent vertices share the same color.



TC: $O(N^m)$

SC: $O(N) + O(N \times N)$

for stack space
for building graph of vertices n .

→ Base-Case
When $node == no. of vertices$, we have one solution.

★ Same question as Leetcode Q. 1042

```

public static boolean solve(int vertices, int m, int[][] path) { 1 usage new *
    boolean[][] graph = buildGraph(vertices, path);
    int[] color = new int[vertices];

    boolean result = mColoring(node: 0, graph, color, m, vertices);

    System.out.println("Coloring: " + Arrays.toString(color));
    return result;
}

private static boolean mColoring(int node, boolean[][] graph, int[] color, int m, int vertices) {
    if (node == vertices) return true; // base case: all nodes colored

    for (int c = 1; c <= m; c++) {
        if (isSafe(node, graph, color, c)) {
            color[node] = c;
            if (mColoring(node: node + 1, graph, color, m, vertices)) return true;
            color[node] = 0; // backtrack
        }
    }

    return false;
}

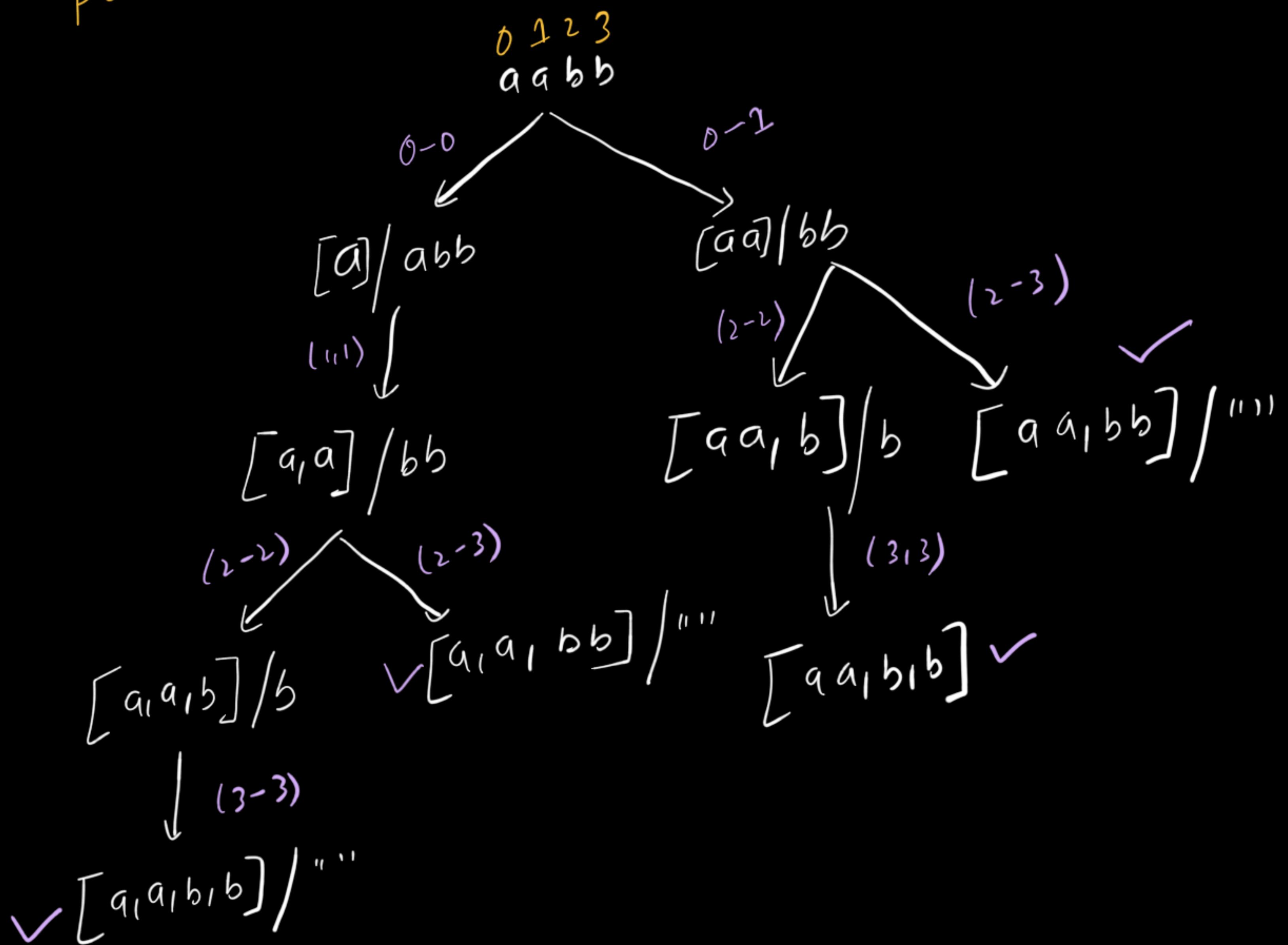
private static boolean isSafe(int node, boolean[][] graph, int[] color, int c) { 1 usage new *
    for (int i = 0; i < graph.length; i++) {
        if (graph[node][i] && color[i] == c) return false;
    }
    return true;
}

private static boolean[][] buildGraph(int n, int[][] edges) { 1 usage new *
    boolean[][] graph = new boolean[n][n];
    for (int[] e : edges) {
        int u = e[0] - 1;
        int v = e[1] - 1;
        graph[u][v] = true;
        graph[v][u] = true;
    }
    return graph;
}

```

→ Palindrome Partitioning

Given a string s , partition s such that every substring of the partition is a palindrome. Return all possible palindrome partitioning of s .



$$TC: O(N * 2^N)$$
$$SC: O(N) + O(P)$$

stencil size no. of partitions

```
PartXV.java

public static void partition(String s, int index, List<String> curr, List<List<String>> ans) {
    if (index == s.length()) { O(N) for copying the arraylist
        ans.add(new ArrayList<>(curr));
        return;
    }

    for (int i = index; i < s.length(); i++) {
        if (isPalindrome(s, index, i)) {
            curr.add(s.substring(index, i + 1));
            partition(s, index: i + 1, curr, ans);
            curr.remove(index: curr.size() - 1);
        }
    }
}

private static boolean isPalindrome(String str, int f, int s) { 1 usage new *
    while (f <= s) {
        if (str.charAt(f) != str.charAt(s)) {
            return false;
        }
        f++;
        s--;
    }
    return true;
}
```

→ Rat in a Maze problem

Given a $m \times n$ grid, rat starts at $(0,0)$ & want to reach the bottom-right cell $(n-1, n-1)$. The rat can move in four directions $\{U, D, L, R\}$. Find all possible paths.

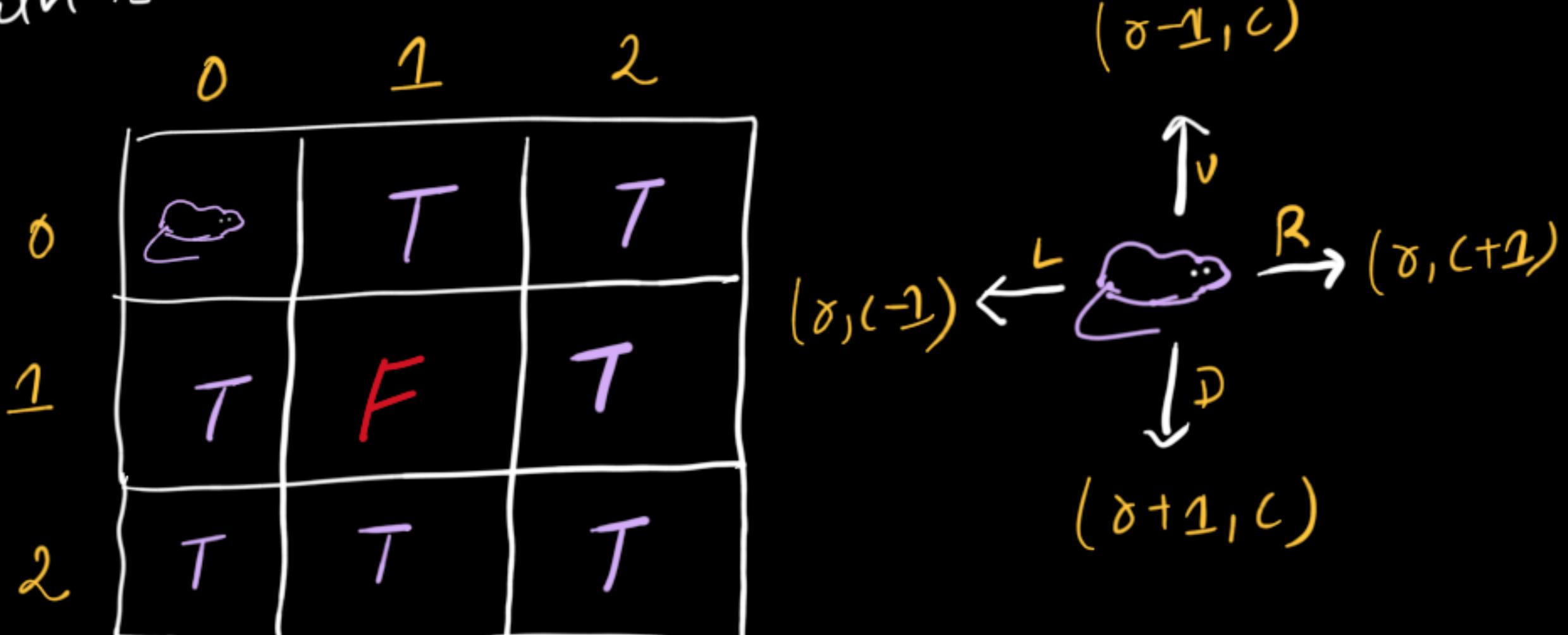
- True: Path is free to go
- False: Path is blocked

- A visited cell cannot be visited twice.

- Initially rat is at $(0,0)$

	0	1	2
0	T	T	T
1	F	F	T
2	F	F	T

Visited - cell



$f(0,0, [])$



$\times f(0,-1, []) + f(0,1, [R]) + f(-1,0, []) + f(1,0, [D])$

already visited → $\times f(0,0, []) + f(0,2, [RR]) + f(-1,1, []) + f(1,1, [RD]) \times$ marked by false

$\rightarrow \times f(0,1, [RD]) + f(0,3, [RR]) + f(-1,2, [RD]) + f(1,2, [RRD])$

Blocked cell

$\rightarrow \times f(1,1, [RRD]) + f(1,3, [RRDD]) + f(0,2, [RRD])$

→ Base-case

① Whenever $(0, c) == (n-1, n-1)$
→ we have one of our answer

✓ $f(2,2, [RRDD])$

↓
1-Solution

```

PartXVI.java
public static List<String> ratMaze() { 1 usage new *
List<String> ans = new ArrayList<>();
boolean[][] maze = new boolean[][]{
    {true, true, true},
    {true, false, true},
    {false, true, true}
};

int n = maze.length;
boolean[][] visited = new boolean[n][n];
ratMaze( row: 0, col: 0, path: "", maze, visited, ans);
return ans;
}

private static void ratMaze(int row, int col, String path, boolean[][] maze, boolean[][] visited, List<String> ans) {
int n = maze.length;

if (row == n - 1 && col == n - 1) {
    ans.add(path);
    return;
}

// Boundary, wall or visited check
if (row < 0 || col < 0 || row >= n || col >= n) return;
if (!maze[row][col] || visited[row][col]) return;

visited[row][col] = true;

// Try all directions - {L,R,U,D}
ratMaze(row, col: col - 1, path: path + 'L', maze, visited, ans); // left
ratMaze(row, col: col + 1, path: path + 'R', maze, visited, ans); // right
ratMaze( row: row - 1, col, path: path + 'U', maze, visited, ans); // up
ratMaze( row: row + 1, col, path: path + 'D', maze, visited, ans); // down

visited[row][col] = false; // backtrack
}

```

TC: $O(4^{n^2}) \rightarrow$ 4 direction to explore,
 & rat can visit every cell, there are n^2 cell

SC: $O(n^2 + k \cdot n^2)$

Recursion stack space
 ↓
 at each recursive call we explore n^2 cell in worst case

Number of valid path
 ↓
 in worst case, each of the valid path can consist of n^2 cell
 For Example

0	Y	2
0	.	V R
1	D V P	P
2	D R D	D

$\rightarrow n^2$
 current - est.

K^{th} -permutation sequence

Given n & k , return the K^{th} permutation of the sequence $[1, 2, 3, \dots, n]$.

Approach - 1

For example:

$n=3, k=3$

$\left[123, 213, 321, 132, 231, 312 \right]$

↓ sort

K^{th} -perm = 213

$\left[123, 132, 213, 231, 312, 321 \right]$

1 2 3 4 5 6

TC: $O(n! \times n) + [n^{\log n}]$

SC: $O(n! \times n)$

PartXVII.java

```

public static String getPermutation(int n, int k) { 1 usage new *
    List<String> ans = new ArrayList<>();
    StringBuilder sb = new StringBuilder();
    for (int i = 1; i <= n; i++) sb.append(i);

    permute(sb.toString().toCharArray(), index: 0, ans);
    return ans.get(k - 1);           ↳ Sort the list then
}                                     callers (K-1)

private static void permute(char[] arr, int index, List<String> ans) {
    if (index == arr.length) {
        ans.add(new String(arr));
        return;
    }

    for (int i = index; i < arr.length; i++) {
        swap(arr, i, index);
        permute(arr, index: index + 1, ans);
        swap(arr, i, index); // backtrack
    }
}

private static void swap(char[] arr, int i, int j) { 2 usages new *
    char temp = arr[i];
    arr[i] = arr[j];
    arr[j] = temp;
}

```

→ Approach - 2

$$\hookrightarrow n=4, k=17$$

For $n=4$, the set contain $[1, 2, 3, 4] \rightarrow 4! = 24$ perm
 $\text{arr} = \boxed{\begin{matrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{matrix}}$

① Initially, we can make these permutation:

$$\hookrightarrow \text{For } k=17, 1 + \{2, 3, 4\} 3! = 6 (0-5)$$

$$2 + \{1, 3, 4\} 3! = 6 (6-11)$$

$$K = K - 1 = 16 \quad 3 + \{1, 2, 4\} 3! = 6 (12-17)$$

$$\text{our answer will lie in this range} \quad 4 + \{1, 2, 3\} \frac{3! = 6}{24} (18-23)$$

• We definitely know, 3 --- will be part of our permutation

• To take 3 in our answer, we can do:-

$$\hookrightarrow \text{arr}[k/6] = \text{arr}[16] = \text{arr}[2] = 3$$

• Out of range (12-17) we need 16th seq
venue, so out of 6 permutation in this range we need,

$$\hookrightarrow k = K \cdot 1 \cdot 6 = 16 \cdot 1 \cdot 6 = 4$$

② Now, we have selected one number (3) from our permutation, remaining number are:-

$$\{0, 1, 2, 4\} \rightarrow 1 + \{2, 4\} 2! = 2 (0-1)$$

$$2 + \{1, 4\} 2! = 2 (1-2)$$

$$4 + \{1, 4\} \frac{2! = 2}{4} (2-3)$$

again we do $\rightarrow k=4$
same process

$$\bullet \text{arr}[k/2] = \text{arr}[4/2] = \text{arr}[2] = 4$$

3 4 ---

$$\bullet k = K \cdot 1 \cdot 2 = 0$$

③ Now, we have selected two numbers (3, 4) from our permutation, remaining numbers are :-

$$\{ \begin{smallmatrix} 0 & 1 \\ 1 & 2 \end{smallmatrix} \} \rightarrow 1 + \{ 2 \} \quad 1! = 1 = O(1)$$

$$\text{L) For } k=0 \quad 2 + \{ 1 \} \quad \frac{1! = 1}{2} = O(1)$$

3 4 1 -

$$\bullet \text{ arr}[k|1] = \text{arr}[0|2] = \text{arr}[0] = 1$$

$$\bullet k = 0 \% 1 = 0$$

④ We have selected four numbers (3, 4, 1, 1) from our permutation, remaining numbers are :-

1 2 → This will be our final number.

3 4 1 2 → 16th perm

TC: $O(n \times n)$

SC: $O(n)$

```
PartXVII.java
public static String getPermutationOpt(int n, int k) {
    List<Integer> list = new ArrayList<>(); O(N)
    int fact = 1;
    for (int i = 1; i < n; i++) {
        fact = fact * i;
        list.add(i);
    }
    list.add(n);
    StringBuilder ans = new StringBuilder(); O(N)
    k = k - 1;

    while (true) { O(n)
        ans.append(list.get(k / fact));
        list.remove(index: k / fact); O(n)

        if (list.isEmpty()) break;

        k = k % fact;
        fact = fact / list.size();
    }
    return ans.toString();
}
```