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The design of electric vehicle charging network

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ABSTRACT

The promotion of Electric Vehicles (EVs) has become a key measure of the governments in their attempt to reduce greenhouse gas emissions. However, range anxiety is a big barrier for drivers to choose EVs over traditional vehicles. Installing more charging stations in appropriate locations can relieve EV drivers' range anxiety. To determine the locations of public charging stations, we propose two optimization models for two different charging modes - fast and slow charging, which aim at minimizing the total cost while satisfying certain coverage goal. Instead of using discrete points, we use geometric objects to represent charging demands. Importantly, to resolve the partial coverage problem (PCP) for networks, we extend the polygon overlay method to split the demands on the road network. After applying the models to Greater Toronto and Hamilton Area (GTHA) and to Downtown Toronto, we show that the proposed models are practical and effective in determining the locations of charging stations. Moreover, they can eliminate PCP and provide much more accurate results than the complementary partial coverage method (CP).

1. Introduction

Many countries around the world are drawing up plans to electrify their transportation systems in order to reduce greenhouse gas emission and to improve air quality in urban areas. The core of such plans is to promote the adoption of Electric Vehicles (EVs). However, range anxiety is one of the primary barriers for drivers to choose EVs over traditional Internal Combustion Engine (ICE) vehicles (Eberle and von Helmolt, 2010; Pollution Probe, 2015). Installing more EV charging stations is one of the strategies that can reduce range anxiety. This leads to a facility location problem: how many charging stations do we need and where are the best locations to install those charging stations? The answer of this problem depends on many factors, including the driving ranges of EVs and the cost of charging stations.

The driving range of EVs can vary greatly by model and manufacturer. Currently, the longest EV driving range is 424 km (2014 Tesla Model S) while the shortest range is 60 km (2013 Scion iQ EV). Most EVs have ranges between 100 km and 160 km (U.S. Department of Energy, 2014).

EVs are charged through Electric Vehicle Supply Equipment (EVSE). According to Community Energy Association (2013), there are three levels of EVSEs. Level 1 EVSE, with a cost less than $1000, typically takes 10–20 h to charge. The long charging time makes Level 1 chargers suitable only for home usage. Level 2 can be used for both commercial and home charging purposes. EVs will take 4–8 h to reach a full charge. Commercial Level 2 charging equipment costs between $3500 and $6000 for

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a single cord station while residential Level 2 charging equipment is much cheaper with a cost around $1000. Level 3 EVSE, also called fast charger, provides the fastest way of charging EVs and can achieve 50% charge in 10–15 min. It's also the most expensive EVSE with its cost ranging between $60,000 and $100,000.

Home, work and public charging are three common EV charging scenarios (National Renewable Energy Laboratory, 2014). Home charging is the dominant charging scenario. At least 70% of the electricity that EVs use is charged at home (National Renewable Energy Laboratory, 2014). Work charging happens at workplace where people park their EVs during working hours. Public charging usually occurs at public places such as shopping malls, hotels, restaurants, or public parking lots. Due to different charging times required for different levels of EVSEs, Level 1 and Level 2 EVSEs are suitable for home and work charging. Level 2 and Level 3 are suitable for public charging.

In this paper, we focus on the design of a network of public charging stations. We propose to locate the Level 2 & 3 charging stations based on different standards of ranges. For Level 2 charging stations, because it usually takes hours to fully charge an EV, the EVs are often charged at the parking spaces while the drivers are conducting some other activities, e.g., shopping or dining. Therefore, the drivers will look for charging stations within walking distance of the activity. Level 3 EVSE charges much faster, requiring about 30–60 min for a complete charge. Thus, it is appropriate for mid-trip charging where the drivers usually conduct longer distance driving and expect to charge the EVs fast (Community Energy Association, 2013). In this scenario, the drivers will look for charging stations within driving distance before the battery is depleted.

Most traditional facility location models assume that the demands come from discrete points (Miller, 1996). This approach can cause error when measuring the distance between the demand and the service facility, thus affecting the result of the facility locations (Miller, 1996; Chen et al., 2013; Frade et al., 2011; Xi et al., 2013). Moreover, models using point representation suffer from the partial coverage problem (PCP) and Modifiable Areal Unit Problem (MAUP), which will be introduced in Section 3. Recent studies (Murray, 2005; Alexandris and Giannikos, 2010; Cromley et al., 2012; Wei and Murray, 2014, 2015; Yin and Mu, 2015) use polygon overlay and other modeling methods to eliminate/alleviate these problems and investigate the operational and computational costs of these methods. We assume that the sources of demands are multi-dimensional geometric objects. We model the public Level 2 charging demand using Traffic Analysis Zones (TAZ) (polygons) and the public Level 3 charging demand using links of the traffic network (lines). We apply the polygon overlay approach to TAZ, and extend the polygon overlay approach to links of traffic network. Our approach is called geometric segmentation (GS) method.

The approach proposed in this paper may be used by city planners to plan the EV public charging infrastructures, by businesses to estimate how many charging stations they need to install to fulfill their customers, or by utility companies to estimate the impact of charging loads on the electricity grid.

To identify the optimal locations for EV charging stations, we propose the GS method which is a uniform framework for both Level 2 and Level 3 charging stations. Compared to previous similar studies on the locations of EV charging stations, this paper has three major innovations (Frade et al., 2011; Liu, 2012; Lee et al., 2014). Firstly, the proposed models focus on addressing range anxiety, i.e., making sure the charging stations are accessible to the largest possible number of EVs within allowed distances. Moreover, we discuss different definitions of range anxiety for both Level 2 and Level 3 charging stations. Secondly, we use polygon overlay techniques to avoid partial coverage, which can cause models to be inaccurate. We also extend the polygon overlay approach to the case of road networks. Thirdly, the proposed approach can be applied to Level 2 and Level 3 charging stations under a uniform framework, offering a more comprehensive solution strategy than existing models.

This paper is organized as follows. Section 2 reviews the literature of charging network design. Section 3 discusses the PCP and the two major approaches to address this problem - modeling (complementary partial coverage) and data transformation (polygon overlay). Section 4 describes the framework and mathematical formulation for fast charging stations (Level 3). Section 5 describes the framework and mathematical formulation for slow charging stations (Level 2). Finally, in Section 6, we apply the proposed model for Level 3 and Level 2 to the Greater Toronto and Hamilton Area (GTHA) and Downtown Toronto respectively, and conduct numerical studies to demonstrate the effectiveness of the proposed method. The paper is concluded in Section 7.

2. Literature review

Numerous efforts have been made to tackle the EV charging station location problem. In the remainder of this paper, we refer to Level 3 as fast charging and to Level 2 as slow charging.

A large number of models have been developed for fast charging stations (Ge et al., 2011; Chen et al., 2014; Hanabusa and Horiguchi, 2011; Lee et al., 2014; Lam et al., 2014). A fast charging station serves mid-trip charging needs, so that the charging demand is usually based on the number of EVs on the road and is closely related to EV users' traveling behavior. Traffic assignment is a common tool for modeling the EV drivers' behavior (Lam and Lo, 2004). Hanabusa and Horiguchi (2011) apply the stochastic user equilibrium method to estimate the traffic flow on the road network. The goal of their model is to minimize the system's total travel time and equalize the charging load among charging stations using entropy maximization. Their model focuses on the impact of charging stations on EV driver's route choice but doesn't address the accessibility of charging stations to EVs. Chen et al. (2014) also utilize user equilibrium traffic assignment method to model the traffic flows, however, no facility location optimization model is developed to determine the optimal locations of charging stations.

None of these models address the range anxiety issue by ignoring the driving range of EVs. Lam et al. (2014) formulate the problem using a network flow model with implicit range anxiety consideration. Their model guarantees that every node on the network has at least one adjacent charging station and the charging station sub-graph remains a connected graph with the lowest cost. But their model does not consider the accumulation of demand on each charging station when determining the station's charging load. Also, their model suffers from scalability issue, which is demonstrated in their experiments.

An important stream of studies closely related to EV charging station planning involves flow-refueling location model (FRLM), which was originally developed by Kuby and Lim (2005) to find the optimal locations for refueling stations of vehicles of alternative fuels (e.g., hydrogen fuel cells or natural gas). Kuby and Lim (2005) require that refueling station can only be located on the nodes of the road networks. Demand is represented as a traffic flow along its shortest path from the origin to the destination. If the distance between any two adjacent nodes along this path is greater than the vehicle's range, then this traffic flow cannot be captured. In other words, if a good combination of nodes are chosen as charging station locations so that the traffic flow can reach every node without running out of fuels, this traffic flow is considered captured. The objective of FRLM is to maximize the captured traffic flow with a given number of charging stations. Before running the optimization model, all the feasible combinations for each traffic path should be identified, which can be computationally intensive. Many extensions of Kuby and Lim (2005) have been developed to address additional issues. Lee et al. (2014) study a stochastic model which incorporates the user equilibrium traffic assignment method endogenously into the FRLM. In this model, traffic flows sharing the same origin and destination may dynamically choose different paths instead of choosing the same path as in FRLM, which is closer to real-life situation. Capar et al. (2013) propose an arc cover-path-cover formulation which can save the intensive computation of all feasible location combinations. Chung and Kwon (2015) extend the FRLM to a multi-period optimization model, where charging stations are built along a finite discrete time horizon. A detailed comparison between the FRLM models and the proposed model will be discussed in Section 4.

Some studies are dedicated to slow charging stations (Frade et al., 2011; Xi et al., 2013; Chen et al., 2013). These models generally use regression methods to estimate charging demand of each area in the city. Frade et al.'s (2011) estimate the daytime and nighttime charging demands in each traffic zone based on employment and residence data respectively and then use an optimization model to maximize the total coverage with a given number of charging stations. Xi et al. (2013) estimate the charging need for each TAZ and the model aims at maximizing the usage of the chargers. Chen et al. (2013) also use regression analysis based on travel survey data to estimate charging demand in each TAZ and develop an optimization model to minimize the total access cost of EVs to their nearest charging stations. In Frade et al. (2011) and Chen et al. (2013) models, when the distance between a TAZ and a charging station is estimated, the centroid of a TAZ is used to approximate the zone. This approach is problematic and inaccurate in that it does not distinguish whether an area is partially covered or fully covered. This problem can be resolved by the segmentation technique introduced in Sections 3 and 4 of this paper.

Comparing the models for slow charging stations with those for fast charging stations, it is clear that models for fast charging station are based on the traffic network (lines) while models for slow charging station are based on zones (polygons). Importantly, both slow and fast charging stations should be built to tackle the range anxiety problem. Indeed, fast charging is indispensable for resolving range anxiety, since long distance drivers need fast charging solutions. Nevertheless, fast charging costs more and sets higher requirements on the capacity of the electric grid. As a result, slow charging remains a necessary charging mode at present.

Most existing models only consider either slow or fast charging stations. To the best of our knowledge, there is only one paper discussing multiple charging modes. Liu (2012) uses an ad hoc method to estimate the number of Level 1 & 2 charging posts in each residence community and parking lot based on economic and industrial data, and decides the locations of Level 3 fast charging stations according to the locations of gas stations. This ad hoc method requires less computation and is easy to implement, but the result is less reliable than those obtained through optimization models.

Table 1 is a summary of the models mentioned above. As noted in the second column, most fast charging models use traffic assignment to estimate charging demand while slow charging models estimate the demand using regression method. In the third column, with the exception of Chen et al. (2014) and Liu (2012), all the models use optimization to decide the locations of charging stations. In the fourth column, typical objectives among the models include minimizing travel time and cost, and maximizing coverage. As we can see from the fifth column, slow charging station models generally use the centroids of polygons to represent the demand location. This approach leads to PCP, which is shown in the sixth column. The last column shows that except Liu's (2012) model, all the models are designed for single mode charging.

To design an EV charging network, the proposed method should firstly be able to include both the fast charging stations (for short time charging need) and slow charging stations (for long time charging need). Secondly, given the high cost of installing public charging stations, the model should be particularly budget-sensitive. In addition to \$60,000–\$100,000 for a Level 3 charging post and \$3500–\$6000 for a commercial Level 2 charging post, one would also have to consider the cost of renting/purchasing land for the station and other construction and operation costs (Community Energy Association, 2013). Moreover, to solve the range anxiety problem with a limited budget, the charging facilities should be accessible to as many EVs as possible. Thirdly, to ensure accuracy, network/polygons should be used to estimate fast/slow charging demand and the PCP must be resolved. Last but not least, to estimate charging demand more accurately, trip generation method should be used to estimate the demand for Level 2 charging and traffic assignment should be used to estimate the demand for Level 3 charging.

Table 1
A classification of existing works.

Author(s)	Demand model	Decision model	Objective	Demand representation	Partial coverage exists	Station type
Capar et al. (2013)	Traffic assignment	Optimization	Maximize flow captured	Network	N/A	Fast charging
Chen et al. (2013)	Regression	Optimization	Minimize total access cost	Point	YES	Slow charging
Chen et al. (2014)	Traffic assignment	Ad hoc	Minimize total travel time	Network	N/A	Fast charging
Chung and Kwon (2015)	Traffic assignment	Optimization	Maximize flow captured	Network	N/A	Fast charging
Hanabusa and Horiguchi (2011)	Traffic assignment	Optimization	Minimize total travel time and equalize electric loads	Network	N/A	Fast charging
Frade et al. (2011)	Regression	Optimization	Maximize covered demand	Point	YES	Slow charging
Ge et al. (2011)	Ad hoc	Optimization	Minimize charging cost	Network	N/A	Fast charging
Kuby and Lim (2005)	Traffic assignment	Optimization	Maximize flow captured	Network	N/A	Fast charging
Xi et al. (2013)	Regression	Optimization	Maximize charging post usage	Point	YES	Slow charging
Lam et al. (2014)	Ad hoc	Optimization	Minimize cost	Network	N/A	Fast charging
Lee et al. (2014)	Traffic assignment	Optimization	Minimize network cost and trip failure	Network	N/A	Fast charging
Liu (2012)	Ad hoc	Ad hoc	Minimize number of charging stations	Polygon	N/A	Fast and slow charging

3. Partial coverage problem

It is common practice to represent demand as discrete points in many location models for the convenience of calculating the distance between demands and facilities (Miller, 1996). This approach of demand abstraction is easy to implement but too simplistic because it ignores the geometric characteristics of the demand, which could lead to problems and errors in the result. The point abstraction only allows the demand to be either fully covered or not covered by a facility and does not account for partial coverage. This may result in partial coverage for geometric objects, which is called the partial coverage problem (PCP). PCP is illustrated in Figs. 1 and 2.

A point on a network is considered covered by a charging station only if the shortest *network* distance between this point and the charging station is less than the EV driving range allowed. If all points of a link are covered by a charging station, then that link is considered to be fully covered. Otherwise, the link is only partially covered. In Fig. 1, the link EF is covered by charging station C2, link GH covered by charging station C1, and the middle link FG is not covered by any of the two charging stations. If we represent this road link by its middle point M, then road link L is not covered by any charging stations, which is inaccurate.

Likewise, a point in a zone is considered covered by a charging station if and only if the *Euclidean* distance between this point and the charging station is less than the maximum walking distance. If all points in a zone are covered by a charging station, then the zone is considered to be fully covered. This situation is illustrated in Fig. 2. The TAZ D is partially covered by charging stations C1 and C2. The left part of TAZ D is covered by charging station C2, the right part of D is covered by charging station C1, and the middle part of D is not covered by either charging station. If we use the centroid M to represent polygon D, then D is not covered at all by any charging station, which is inaccurate as well.

Aside from PCP, location models using point abstraction also suffer from MAUP, which means using different spatial units or scales for the same demand region can result in different solutions (Murray, 2005; Wong, 2009).

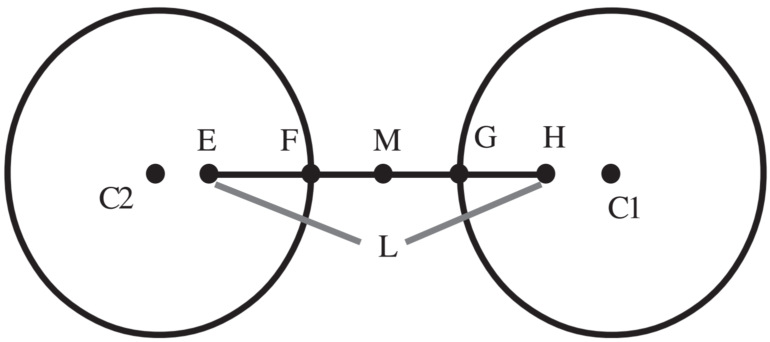

Fig. 1. Partial coverage problem for lines.

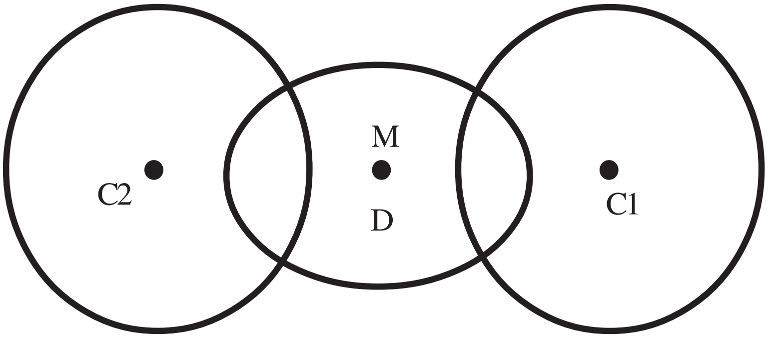

Fig. 2. Partial coverage problem for polygons.

Efforts have been made to tackle the problems caused by point simplification of demand. In Set Covering Problem (SCP), Murray (2005) develops a new SCP model using spatial objects (points, lines, polygons) to represent demand instead of point representation. In the model, complementary partial coverage (CP) by more than a given number of facilities is accounted as complete coverage. Alexandris and Giannikos (2010) extend Murray's (2005) CP approach to the Maximum Covering Location Problem (MCLP). Specifically, Alexandris and Giannikos's (2010) introduce penalty to complementary partial coverage, given that in many situations, Murray's (2005) model may lead to redundant non-complementary coverage. An example of redundant partial coverage is given in Fig. 3. Both circles cover more than half of square S but together don't cover the whole square area.

The models in Murray (2005) and Alexandris and Giannikos (2010) can improve the solution quality of SCP and MCLP. But their models are still subject to PCP and MAUP. Indeed, redundant non-complementary coverage still widely exists in their solutions, leading to more sites chosen than needed. In addition, their models cannot accurately evaluate the level of coverage.

Another approach to eliminate PCP is through transforming the representation of demand. A widely used method is polygon overlay. By overlaying the coverage area of the candidate locations against the demand region, the demand region is divided into what is called least common demand coverage units (LCDCUs) Cromley et al. (2012). LCDCUs can either be fully covered by a candidate location, or not covered at all, thus eliminating the partial coverage issue. An illustrative example can be found in Fig. 5. Wei and Murray (2014) provide mathematical proofs that by using polygon overlay approach to solve the location set covering problem (LSCP) in a continuous demand region, the resulting solution is the true minimum number of facilities. The same conclusion also applies to the maximum covering location problem (MCLP). Yin and Mu (2015) formalize this LCDCU-based demand representation in the terminology of set theory to promote its usage in the GIS community. Cromley et al. (2012) propose to use ancillary data (e.g., remote sensing image of land cover) to account for the heterogeneous distribution of demand within single demand polygons. On the other hand, Wei and Murray (2014) point out that polygon overlay requires significant operational time when splitting the demand polygons using candidate coverage and also increases the model size. Wei and Murray (2015) find that polygon overlay cannot guarantee true optimum solution in continuous space maximal coverage problem, when both the demand and candidate locations are in continuous space.

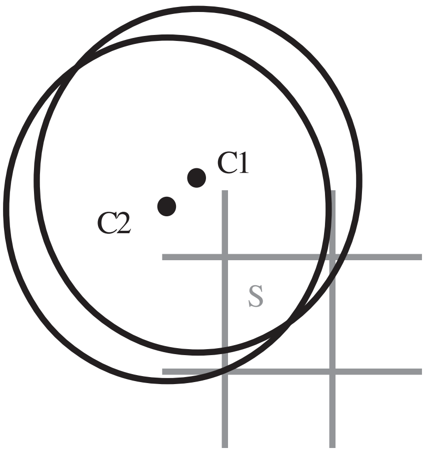

Fig. 3. Redundant non-complementary partial coverage.

We will apply the polygon overlay approach to slow charging station location problem in Section 5. We also extend the polygon overlay approach to split demand road network using each candidate location's service area for slow charging station location problem in Section 4. We refer to our general framework as geometric segmentation (GS) in later sections.

For comparison purpose, an optimization problem for EV charging stations based on the CP approach is formulated. This model resembles Murray's (2005) model, minimizing the total cost while guaranteeing the least level of coverage, and also contains penalties on partial coverage as in Alexandris and Giannikos (2010). The formulation of the modified model is as follows.

Indices:

i index for polygons split from TAZs / index for links split from road networks
j index for candidate charging station locations

Parameters:

d_{ij} the Euclidean distance from the candidate location j to the furthest point in polygon i or the network distance between the candidate location j and the furthest point on link i
w_i the number of EVs in polygon i or the volume of road link i $*$ the length of road link i multiplying the length of road link i
R the maximum walking distance that EV drivers are willing to walk from charging station to destination or the driving range of EVs with 10% battery level (we assume that EV drivers will be alerted for charging when the battery level drops to 10% of the total capacity)
c_j the cost for locating a charging station at location j
B the budget for installing charging stations
α the least percentage of coverage
β the penalty coefficient for demand partially covered by at least θ number of charging stations and not fully covered by any charging station
ω the least level of coverage that will be considered in complementary partial coverage
θ the least number of partial coverage needed to be treated as full coverage

Sets:

I set of polygons or road links
J set of all candidate charging locations
$N(i)$ set of charging locations that can cover polygon i or link $i, N(i) = \{j|d_{ij} \leqslant R\}$
$W(i)$ set of charging locations that can partially cover polygon or link i by ω
$M(j)$ set of polygons or links that are covered by charging station at $j, M(j) = \{i|d_{ij} \leqslant R\}$

Decision variables:

x_j binary; $x_j = 1$ if and only if a charging station is located in location j
y_i binary; $y_i = 1$ if and only if i is covered by at least one charging station
v_i binary; $v_i = 1$ if and only if i is partially covered by at least θ charging stations

Model CP:

$$\min \quad \sum_{j\in J} c_j x_j \tag{1}$$

$$\text{s.t.} \quad y_i \leqslant \sum_{j\in N(i)} x_j \quad \forall i \in I \tag{2}$$

$$\theta \cdot v_i \leqslant \sum_{j\in W(i)} x_j \quad \forall i \in I \tag{3}$$

$$y_i + v_i \leqslant 1 \quad \forall i \in I \tag{4}$$

$$\sum_{i\in I} w_i(y_i + \beta v_i) \geqslant \alpha \sum_{i\in I} w_i \tag{5}$$

$$x_j \in \{0,1\} \quad \forall j \in J \tag{6}$$

$$y_i, v_i \in \{0,1\} \quad \forall i \in I \tag{7}$$

The objective (1) is to minimize the total land cost for locating the charging stations. Constraint (2) guarantees that i is considered fully covered if there is at least one charging station located and can cover i. Constraint (3) ensures that i is considered partially covered in a complimentary manner if there are at least θ charging stations that each station can partially cover i at least at ω level of coverage. Constraint (4) states that i can only be either fully covered or partially covered in a complimentary manner in order to avoid redundant coverage. Constraint (5) requires that at least α level of total demand should be covered in this model. Parameter β (value no greater than 1) indicates that a demand i that is partially covered by multiple charging stations is regarded as equal to or less than complete coverage. When $\beta < 1$, it is a penalty to multiple partial coverage to reduce non-complementary partial coverage since the model cannot accurately identify if the demand is completely covered. Constraint (6) and (7) ensure the decision variables are binary. In Section 6, we will apply this model to GTHA and Downtown Toronto for fast and slow charging stations respectively, denoted by FC-CP for fast charging station and SC-CP for slow charging stations.

4. Fast charging stations

In this section, we develop a model with geometric segmentation (FC-GS) to determine the location of the fast charging stations. In previous works that are based on network data, FRLM and its extensions (Kuby and Lim, 2005; Capar et al., 2013; Lee et al., 2014; Chung and Kwon, 2015) are often used to address fast charging station location problem. Both FC-GS and FRLM use trip flows generated from traffic assignment to represent the charging demand. However, in FRLM, a traffic flow is considered as captured if a good combination of nodes are chosen as charging stations on its path so that the traffic flow can reach every node without running out of fuels, while in FC-GS, a traffic link is considered as covered if it is within a certain range (network distance) of at least one facility. FC-GS is more appropriate compared to FRLM for locating EV fast charging stations for the following reasons.

Firstly, FC-GS guarantees that EVs will have access to a nearest charging station within a given driving range while FRLM does not. According to previous research (National Renewable Energy Laboratory, 2014), home charging takes up to 70–90% of the EV charging demand. Combining this statistic with the fact that most EVs have ranges between 100 km and 160 km (U.S. Department of Energy, 2014), it's reasonable to infer that most EV owners don't often charge their EVs outside of home and thus don't use EVs for trips longer than their EVs' range. In this scenario, EV owners don't usually make trips that require them to charge in the middle of their trips. So fast charging stations act as emergency facilities for urgent unplanned charging rather than critical mid-trip refueling nodes as in FRLM. FC-GS will guarantee that EVs in most parts of the road network can have access to a nearest fast charging station within a given driving range, whereas FCLM has no such guarantee. Planners can also adjust the level of coverage in FC-GS according to their needs.

Secondly, FC-GS is more practical to implement than FRLM. In an FRLM, facilities can only be placed at the nodes on the road network. But in reality, a facility is often placed at a distance from the road (off the network link). If the facility is close enough to a road, it can be approximated as a node in FRLM. However, when a facility is placed far away from a road, it is difficult to decide if it can capture the flow on the road or not. FC-GS does not have this problem since there is no limit on the location of the charging stations. Moreover, FRLM can lead to the undesirable scenario such that fast charging stations concentrate in the denser part of the road network, and EVs on other parts of the road network may not be able to reach the charging stations when needed.

4.1. Framework

The framework of our model consists of five modules: data collection, geometric segmentation, traffic assignment, facility location optimization and charging load estimation.

Data used as model input include the road network, candidate sites for charging stations, the cost of each site, and the data needed for traffic assignment.

Geometric segmentation for the road network is particularly noteworthy as it is an extension of the polygon overlay approach. In Fig. 4, the left part of link L is covered by charging station C2, the right part of L is covered by charging station C1, and the middle part of L is not covered by any of the two charging stations. To avoid partial coverage, we can split link L into link L1, L2 and L3. After the split, L1 and L3 are fully covered by C2 and C1 separately, while L2 is not covered. Therefore,

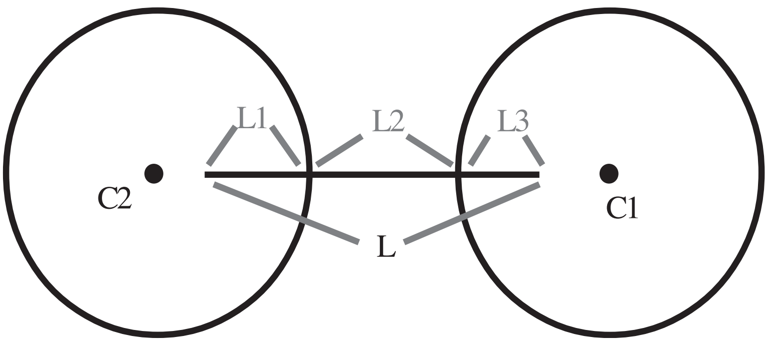

Fig. 4. Splitting partially covered link.

the original road network can be transformed to a new network after the coverage segmentation operation. In the new network, partial coverage does not exist: A link is either completely covered or completely uncovered.

Traffic assignment is a common method used to forecast traffic in transportation. It allocates travel demand (origin-destination matrices) to a road network according to assignment rules. At the end of the assignment procedure, the whole traffic network reaches equilibrium. Then the traffic volume can be calculated for each link (Lam and Lo, 2004; Sheffi, 1995).

Charging load estimation refers to how much charging demand goes to a certain charging station. This involves splitting charging demand covered by more than one charging station and summing up covered demand for each charging station. The splitting of charging demand can be achieved by using Network Analysis and Intersect function in ArcGIS, a Geographic Information System (GIS) software for mapping and spatial analysis, developed by Environmental Systems Research Institute (ESRI).

The details of the facility location optimization model for fast charging stations are presented in the next subsection.

4.2. Optimization model

Model FC-GS is a variation of the classical maximum coverage problem with an objective to minimize the total installation cost while maintaining given percentage of coverage.

Indices:

i index for road links
j index for candidate charging station locations

Parameters:

l_i the length of link i
d_{ij} the shortest network distance from i to j
w_i the traffic volume of link i
R the planned coverage range of the charging stations
c_j the cost for locating a charging station at j
B the budget limit for installing charging stations
α the least percentage of coverage

Sets:

I set of links
J set of all candidate charging locations
$N(i)$ set of charging locations that can cover link $i, N(i) = \{j|d_{ij} \leqslant R\}$
$M(j)$ set of links that are covered by the charging station located at $j, M(j) = \{i|d_{ij} \leqslant R\}$

Decision variables:

x_j binary; $x_j = 1$ if and only if a charging station is located at j
y_i binary; $y_i = 1$ if and only if road link i is covered by at least one charging station

Model FC-GS:

$$\min \quad \sum_{j \in J} c_j x_j \tag{8}$$

$$\text{s.t.} \quad y_i \leqslant \sum_{j \in N(i)} x_j \quad \forall i \in I \tag{9}$$

$$\sum_{i \in I} w_i l_i y_i \geqslant \alpha \sum_{i \in I} w_i l_i \tag{10}$$

$$x_j \in \{0, 1\} \quad \forall j \in J \tag{11}$$

$$y_i \in \{0, 1\} \quad \forall i \in I \tag{12}$$

Objective (8) is to minimize the total cost of charging stations. Constraint (9) states that i is considered covered if its demand is satisfied by at least one charging station. Constraint (10) guarantees that at least α portion of the coverage of the road network with traffic flow as the weight of each link. We use the length of each link multiplied by the link traffic volume to represent the fast charging demand. It can effectively represent the weight of each link compared to other links, and guarantee the charging stations be accessible by EVs on the most busy roads. Constraint sets (11) and (12) define that decision variables x_j and y_i are binary variables.

5. Slow charging stations

The maximum covering model for slow charging stations (SC-GS) is formulated in a similar way to the Model FC-GS for fast charging stations. The main difference is that for the slow charging model the demand for charging is based on TAZs rather than the network links that are used in the fast charging model. In other words, we assume that the slow charging demand arises from areas, instead of links.

We argue that planning for slow charging stations should be conducted based on the central districts of a city rather than on the whole metropolitan area as with fast charging stations. There are two reasons for this choice. Firstly, the use of slow charging station is for drivers to charge their EVs at their trip destinations (excluding home), thus the slow charging stations should be located in places that people are more likely to visit, such as workplaces, shopping malls, theatres and restaurants. As a result, to ensure the cost-effectiveness of the Level 2 charging stations, the less visited areas and areas without parking lots should be of low priority in slow charging station location selection. Secondly, the range requirement of slow charging is different from that of fast charging. The EV drivers will be satisfied if they could find slow charging stations within a walking distance of their activity places. In our experiment, we will use 500 meters and 300 meters as the maximum covering ranges for a slow charging station, which we assume are the possible maximum walking distances a driver might be willing to walk from a charging station to the destination, or vice versa.

5.1. Framework

The framework of the model for slow charging stations contains the five modules: data collection, trip generation, geometric segmentation, facility location optimization and charging load estimation.

Data input for the model includes TAZ file, candidate sites for charging stations, cost of each site and data needed for demand estimation.

The determination of demand consists of estimating the number of EVs that go to a certain TAZ on a daily basis. The demand can be estimated through the trip generation method in the travel forecasting process. Trip generation uses the land use and demographic information to predict the total number of trips entering or leaving a zone in the city (Meyer and Miller, 1984).

The geometric segmentation of TAZs is the same as in the polygon overlay approach. Specifically, the covering range of a charging station is a circular area with the allowed maximum walking distance as its radius. This case is illustrated in Fig. 5. The TAZ D is partially covered by charging stations C1 and C2. The left part of TAZ D is covered by charging station C2, the right part of D is covered by charging station C1, and the middle part of D is not covered by either charging station. To avoid partial coverage, we can split D into D1, D2 and D3. After the split, D2 and D3 are fully covered by C2 and C1 while D1 is not covered.

Charging load estimation for slow charging stations is similar to that for fast charging stations. The splitting of commonly covered TAZ can be achieved by dissecting the commonly covered area with Thiessen Polygon generated from the chosen charging station locations in ArcGIS.

We will present the facility location optimization model in the next subsection.

5.2. Optimization model

Indices:

i index for polygons split from TAZs
j index for candidate slow charging station locations

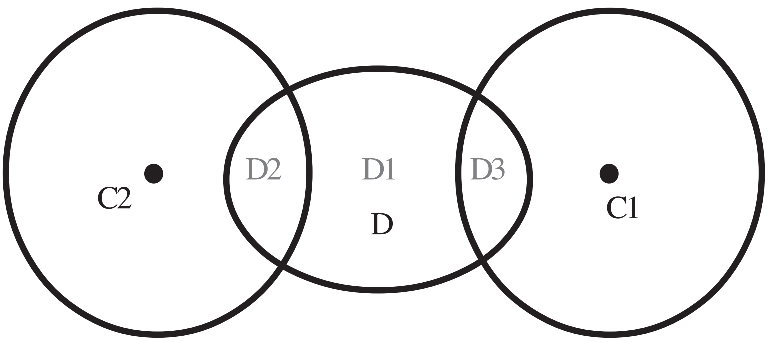

Fig. 5. Splitting partially covered TAZ.

Parameters:

d_{ij} the Euclidean distance from the candidate location j to the furthest point in polygon i
w_i the number of EVs in polygon i
R the maximum walking distance which one is willing to walk from charging station to destination, or vice versa
c_j the cost for locating a charging station at j
B the budget limit for installing charging stations
α the least percentage of coverage

Sets:

I set of polygons
J set of all candidate charging locations
$N(i)$ set of charging locations that can cover polygon $i, N(i) = \{j|d_{ij} \leqslant R\}$
$M(j)$ set of polygons that are covered by the charging station located at $j, M(j) = \{i|d_{ij} \leqslant R\}$

Decision variables:

x_j binary; $x_j = 1$ if and only if a charging station is located at j
y_i binary; $y_i = 1$ if and only if polygon i is covered by at least one charging station

Model SC-GS:

$$\min \quad \sum_{j \in J} c_j x_j \tag{13}$$

$$\text{s.t.} \quad y_i \leqslant \sum_{j \in N(i)} x_j \quad \forall i \in I \tag{14}$$

$$\sum_{i \in I} w_i y_i \geqslant \alpha \sum_{i \in I} w_i \tag{15}$$

$$x_j \in \{0, 1\} \quad \forall j \in J \tag{16}$$

$$y_i \in \{0, 1\} \quad \forall i \in I \tag{17}$$

Clearly, Model SC-GS for slow charging stations is developed in a similar way as the Model FC-GS for fast charging stations. The differences are: (1) I represents links in Model FC-GS but polygons in Model SC-GS; (2) d_{ij} represents the Euclidean distance between demand and charging station in Model SC-GS while it represents length of the shortest path between demand and charging station in Model FC-GS; (3) R is the maximum walking distance between demand and charging station in Model SC-GS while it represents the minimum driving range of EVs to be able to reach a closest charging station; (4) Model FC-GS uses the product of link length and link flow quantity as an estimation of the demand of each link, while Model SC-GS uses the number of EVs in each zone as the demand.

In summary, the major differences between fast charging and slow charging in problem formulations are: (1) slow charging need arises from EV owners conducting long time activities whereas fast charging need comes from EV driven on the road which is running out of battery, thus slow charging demand is based on area (polygons) while fast charging demand is based on roads (networks); (2) in slow charging, the number of EVs in each demand area is counted as charging demand while in fast charging, the charging demand is the EV volume on each road link multiplied by the length of each road; (3) the coverage range for slow charging station is 300–500 m (Euclidean distance), representing the maximum distance an EV driver is willing to walk while the coverage range for fast charging station is 5–20 km (network distance) determined by the remaining battery capacity.

6. Numerical experiments

6.1. Fast charging stations

Assumptions for this case study on FC models include: (1) traffic volume data on road links generated from traffic assignment can reflect the true traffic distribution on the GTHA major road links; (2) Proportions of EV among all vehicles are the same across the road networks; (3) in the CP model, two partial coverage over 50% equals is counted as a full coverage with no penalty; (4) the locations of the existing gas stations can represent the potential locations for fast charging stations; (5) EV drivers will drive to a fast charging station closest to them in terms of network distance; (6) average property price can reflect the cost of building the charging stations.

Both model FC-CP and model FC-GS are applied to the GTHA road network. The candidate locations are chosen from 657 gas stations in the GTHA area. We use the average housing price of the census tracts to which each candidate site belongs to represent the cost of locating a charging station at these sites. The original GTHA road network consists of 2511 links. We use traffic flow data generated directly from TRAFFIC, a model that was developed at the McMaster Institute for Transportation and Logistics (MITL) and has been used extensively to estimate emissions from traffic flows for several Canadian cities (Rashidi et al., 2015; Maoh and Kanaroglou, 2009; Buliung et al., 2005).

For the model FC-CP, we set $\beta = 1, \omega = 50\%, \theta = 2$, which means one road link with partial coverage over 50% from 2 stations can be regarded as full coverage without any penalty. Our experiment shows that any values of $\beta < 1$ and $\omega < 50\%$ lead to larger errors in the result or even no feasible solution. Model FC-CP for fast charging stations consists of 5679 decision variables and 7534 constraints.

For model FC-GS, we want to compare the results using range R = 5 km, 10 km, 15 km, 20 km and coverage level α = 85%, 90%, 95%, 99.9%. Using different ranges to pre-process the road network data in ArcGIS, we get 19,375 links from the original 2511 links using R = 5 km, 37,245 links for R = 10 km, 52,101 links for R = 15 km and 63,592 links for R = 20 km. The number of links increases significantly after splitting. Then the length of each new link is recalculated in ArcGIS. Table 2 shows the different model sizes using different ranges. The number of decision variables and constraints grows significantly with the increasing range. It is obvious that the size of model FC-GS increases with the range while the size of model FC-CP remains irrelevant to the range.

Using the Network Analysis module of ArcGIS, we get the coverage information from each location to each link. The models are implemented using CPLEX 12.6 API for Python and run on a Dell Latitude E5530 computer with Intel Core i7-3540 M 3.00 GHz CPU and 8 GB memory.

Table 3 shows the computational results of the model FC-CP. Table 4 shows the computational results of the model FC-GS. The solution time is the time used by the solver to solve the Mixed Integer Linear Programming (MILP) problem. Table 3 shows that there is a great discrepancy between the targeted covering level in model FC-CP and the true covering level achieved by the optimal results of model FC-CP. The true coverage is either higher or lower than the targeted coverage. This can be explained by Fig. 6, where the two circles form complementary partial coverage on the elliptical area. When the combined area of D1 and D2 is greater (smaller) than the combined area of D3 and D4, the resulting coverage will be greater (smaller) than the targeted coverage. Table 4 shows that as the range increases, fewer charging stations are needed. However, the number of charging stations increases considerably when we raise the coverage level. The number of charging stations needed for 99.9% coverage is almost twice the number of stations needed for 95% coverage. So the marginal coverage increase by a charging station will decrease as the total number of charging stations increases. By comparing these two charts, we note that the solution time is irrelevant to the model size. Obviously, for the same range and coverage level, model FC-CP requires much higher number of charging stations than model FC-GS.

We visualize the results of the two models with R = 15 km and α = 90% in Fig. 7, which compares the visualized results between model FC-GS and model FC-CP. The underlying green lines are the road network. The thickness of the line reflects the traffic volume on the road links. The blue lines are roads covered by charging stations. Both graphs show that the models tend to choose to cover roads with more traffic volume. By comparing the two graphs, it is noted that model FC-GS chooses much less charging stations than model FC-CP and the chosen stations in model FC-GS are more evenly distributed than those chosen by model FC-CP, which contains several groups of charging stations that are very close to each other. This is a result of non-complimentary partial coverage.

The results show that the model FC-GS is more effective in determining the locations of fast charging stations.

6.2. Slow charging stations

Assumptions for this case study on SC models include: (1) trip data in each TAZ generated from demand generation can reflect the true traffic destination distribution in Downtown Toronto; (2) Proportions of EV among all vehicles are the same across Toronto; (3) in the CP model, two partial coverage over 50% equals is counted as a full coverage with no penalty; (4) the locations of Point Of Interests (POI) in Toronto can represent the potential locations for slow charging stations; (5) EV drivers will go to a slow charging station closest to their places of activities within Euclidean distance 300–500 m; (6) average property price can reflect the cost of building the charging stations.

Both the model SC-GS and the model SC-CP are applied to the Downtown Toronto area. The original Downtown Toronto area consists of 64 TAZs. Using the Origin-Destination matrix we get from Traffic Model (McMaster Institute for Transportation and Logistics, 2014), we estimate how many vehicles will visit a TAZ in one day. By assuming a certain pro-

Table 2
Model sizes using different ranges (fast charging).

Model	Cov. range (km)	Dec. var. #	Cons. #
FC-GS	Before splitting	3168	2512
	5	20,032	19,376
	10	37,902	37,246
	15	52,758	52,102
	20	64,249	63,593
FC-CP	5/10/15/20	5679	7534

Table 3
Computational results of model FC-CP ($R = 15$ km).

Targeted cov.	Sol. time (min)	Station #	True cov.
85%	5.83	17	90.03%
90%	22.28	22	92.55%
95%	8.24	28	96.87%
99.9%	Infeasible	Infeasible	Infeasible

Table 4
Computational results of model FC-GS.

Cov. range (km)	Targeted cov.	Sol. time (min)	Station #
5	85%	1.1	74
	90%	0.4	111
	95%	Infeasible	Infeasible
	99.9%	Infeasible	Infeasible
10	85%	6.8	20
	90%	2.9	26
	95%	3.1	36
	99.9%	Infeasible	Infeasible
15	85%	7.1	10
	90%	5.1	12
	95%	1.4	17
	99.9%	0.4	33
20	85%	11.8	6
	90%	6.3	7
	95%	16.8	10
	99.9%	1.6	19

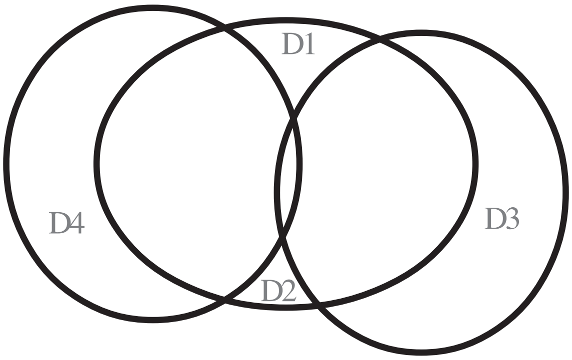

Fig. 6. Coverage gap.

portion of EVs among all vehicles, we can estimate the number of EVs that go to a TAZ on a daily basis. The 300 candidate locations are chosen from Downtown Toronto's Points of Interest (POI), including restaurants, parking lots, schools and other businesses or public institutions in downtown. Similar to Model FC, we use the average housing price of the census tracts to which each candidate site belongs to represent the cost of locating a charging station at these sites.

Model SC-CP for slow charging stations consists of 428 decision variables and 193 constraints. We set $\beta = 1, \omega = 50\%, \theta = 2$ the same way as for fast charging stations.

For model SC-GS, we compare the results using different coverage level α = 85%, 90%, 95%, 99.9%, as well. After segmentation, the TAZs are split into 7266 and 17,080 polygons by the coverage areas of the candidate locations, which are circular areas with 300 and 500 meter radius respectively. Table 5 shows the different model sizes of different models with different ranges. The weight of each polygon is the number of EVs in each polygon. When splitting TAZ to polygons, EVs in each TAZ are assigned to each polygon, too. The number of EVs assigned to each polygon is proportional to its area.

The models are implemented in the same way and run on the same environment as Section 6.1.

Table 6 shows the computational results of the model SC-CP for slow charging stations. Due to its small problem size, the SC-CP for slow charging requires no time (less than 1 s) to be solved. But the great discrepancy between true coverage and targeted coverage level also exists in slow charging. Table 7 shows the computational results of model SC-GS. We note that

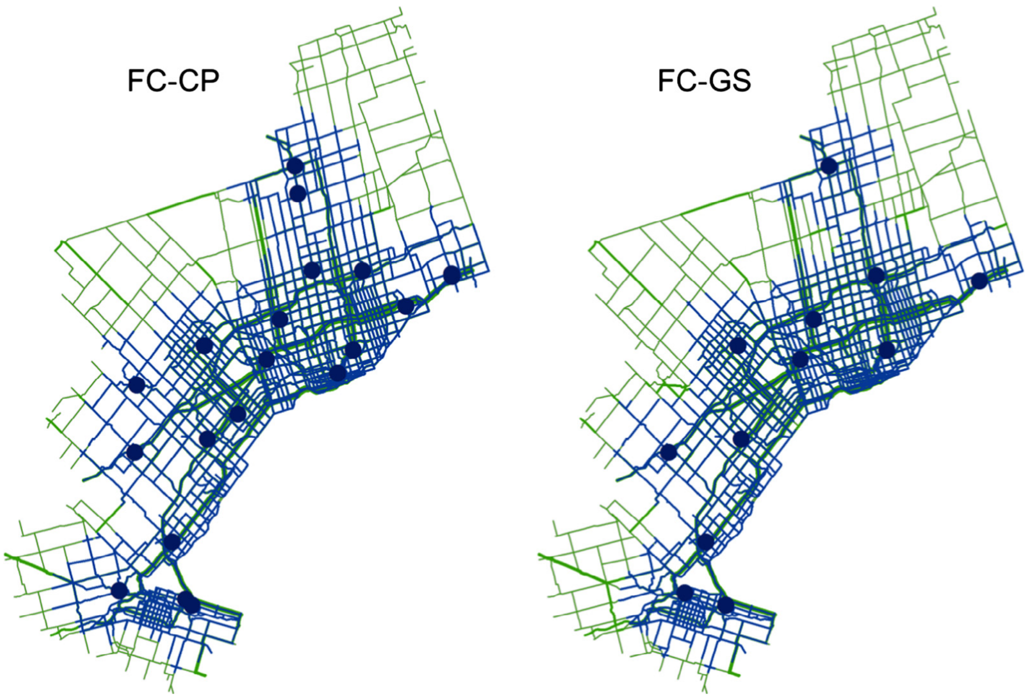

Fig. 7. Comparison of visualized results (R = 15 km, α = 90%).

Table 5
Model sizes using different ranges (slow charging).

Model	Cov. range (m)	Dec. var. #	Cons. #
SC-GS	Before splitting	364	65
	300	7566	7267
	500	17,380	17,081
SC-CP	300/500	428	193

Table 6
Computational results of model SC-CP ($R = 500$ m).

Targeted cov.	Sol. time (min)	Station #	True cov.
85%	0.004	21	85.71%
90%	0.008	24	89.87%
95%	0.006	29	92.85%
99%	0.003	32	95.60%

Table 7
Computational results of model SC-GS.

Cov. range (m)	Targeted cov.	Sol. time (min)	Station #
300	85%	27.9	37
	90%	127.1	45
	95%	69.9	54
	99%	247.8	70
500	85%	6.4	13
	90%	30.9	16
	95%	379.1	19
	99%	570.0	25

both the number of charging stations and the solution time increase significantly with the increase of the percentage of coverage. Comparing Tables 6 and 7, it is clear that model SC-CP requires more charging stations than model SC-GS.

Fig. 8 shows the comparison of visualized results between model SC-CP and model SC-GS with $R = 500$ m and $\alpha = 90\%$. The areas in the circles are covered by selected charging stations. The models tend to choose to cover TAZs with more EVs (polygons of darker colors). Also, it is evident that the result of model SC-CP suffers from serious non-complementary partial coverage problem while the charging stations selected by model SC-GS are evenly distributed and much less than charging stations selected by SC-CP. It shows that model SC-GS is more effective in determining the location of slow charging stations.

6.3. Estimating demand for each station

As we can see from the results of the experiment, facilities close to each other may have shared covered demand areas/links. How much of the demand should be assigned to each facility is a problem that needs to be addressed. We follow the rule that demand should be assigned to its nearest facility.

For fast charging stations, we use the Network Analysis module of ArcGIS to split the commonly covered demand network into different parts, and each part is assigned to one facility, ensuring that any point in the assigned link is closest to its corresponding facility compared to other facilities. In this way, we can get the non-overlapping covered demand road links for each charging station. The result is illustrated in Fig. 9.

The percentages of total charging demand allocated to each fast charging station are 15.9%, 5.4%, 11.4%, 3.7%, 4.5%, 6.5%, 9.4%, 12.0%, 14.2%, 3.1%, 5.0% and 8.9% respectively (12 stations in total).

For slow charging, we use Thiessen polygon generated from the selected charging station locations to split the shared covered demand. Thiessen polygons are polygons generated from a set of sample points on a plane. Each Thiessen polygon has a sample point inside itself. Any point inside a Thiessen polygon is closer to its sample point than any other sample point. By intersecting Thiessen polygons with the covered demand region, we split the shared covered area into several equal parts and assign them to their closest charging stations. Figs. 10 and 11 illustrate the process of using Thiessen polygons to divide the shared demand and assign the demand to each charging station. The percentages of total charging demand allocated to each slow charging station are 6.4%, 7.8%, 7.9%, 9.4%, 2.6%, 9.2%, 4.2%, 9.7%, 4.4%, 13.8%, 5.0%, 3.2%, 3.3%, 4.5%, 4.2% and 4.5% respectively (16 stations in total).

7. Conclusion

We proposed a method to locate fast and slow charging stations to address both fast and slow charging needs under the same framework. The models are designed to tackle range anxiety by minimizing the total cost while guaranteeing a given level of demand coverage. The models can also be easily adapted by limiting the budget or the number of charging stations and maximizing the demand coverage. Moreover, we use more realistic geometric objects - networks and polygons - to represent the charging demands instead of using discrete points. We also extend the polygon overlay approach to the demand segmentation on the network.

The numerical study shows that the models are practical and effective in deciding the locations of fast and slow charging stations. The result shows that the charging stations have been evenly located in the urban areas. A comparison between the results of GS and CP shows that geometric segmentation can fully eliminate the partial coverage issue and produce much

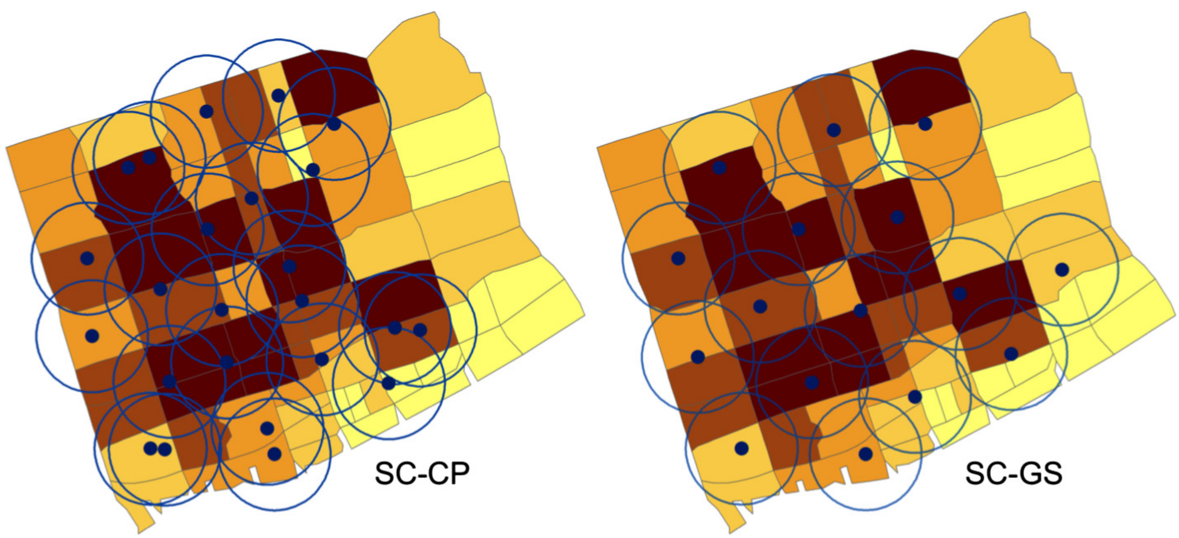

Fig. 8. Comparison of visualized results (R = 500 m, α = 90%).

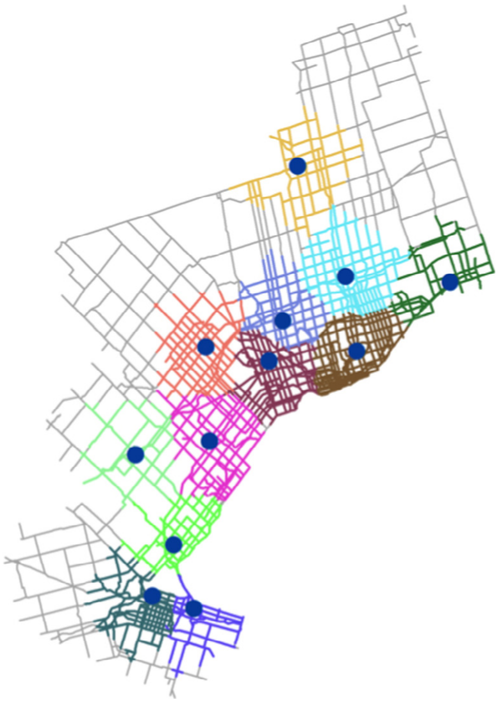

Fig. 9. Allocation of fast charging demand to stations.

more accurate and reliable result than complementary partial coverage approach. Although the segmentation of the links/polygons significantly increases the problem size, the models remain solvable within reasonable computation time.

In future research, this approach can be improved in several directions. Firstly, the capacities of the charging stations can be introduced to ensure that charging demands can be fully served by their nearby charging stations, which arise from the capacities of the electricity grid network. Secondly, we already know that EV drivers' travel behaviors affect the location decision of charging stations, but the locations of fast charging stations may affect the travel decisions of EV drivers as well. For example, long distance EV drivers may tend to choose traveling route that is close to fast charging stations and EV drivers may also choose restaurants with slow charging stations installed when making their dining decisions. So we can combine the demand estimation and facility location optimization together. Thirdly, we may implement the model within ArcGIS to automate the whole process without intermediate steps. Last but not least, heuristics can be developed to solve the models more efficiently.

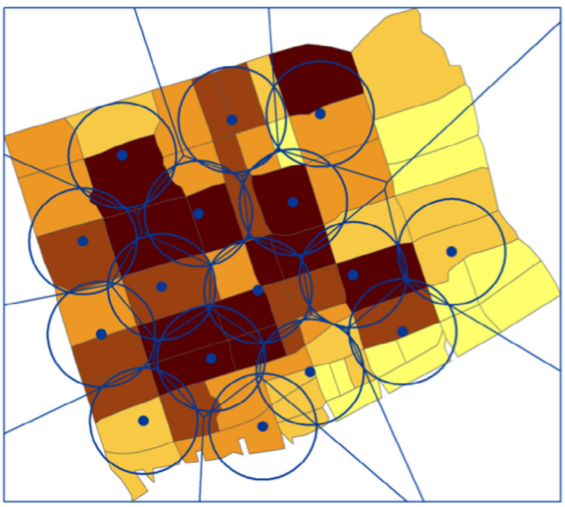

Fig. 10. Thiessen polygons splitting shared demand.

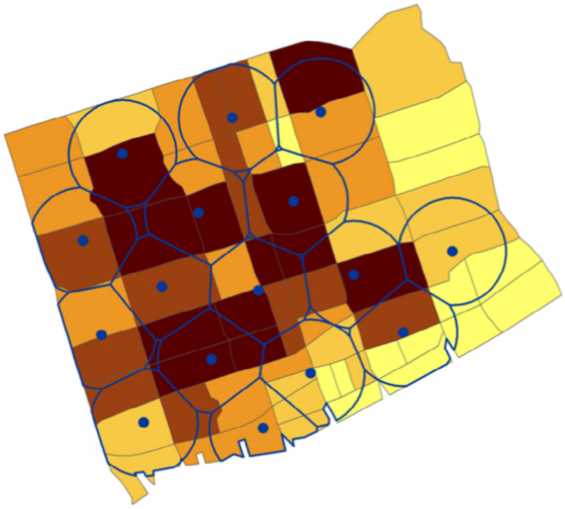

Fig. 11. Allocation of slow charging demand to stations.

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