Assignment 6

1. Determine an equation of the tangent line to the graph of the equation $\sin(x+y) - \cos y = x$ at the point $\left(0, \frac{\pi}{4}\right)$.

   When writing the equation, do not approximate the slope. You need to show that you can do this entire problem without the use of a calculator. (2 points)

   I began this problem by taking the derivative of both sides with respect to x of the equation which gave me the following:

   $$\cos(x+y)(1+\frac{dy}{dx}) + \sin(y)(\frac{dy}{dx}) = 1$$

   I then subtracted $\cos(x+y)$ to the right side of the equation. This allowed me to factor out $\frac{dy}{dx}$ out of the remaining terms on the left side.

   $$\frac{dy}{dx}(\cos(x+y) + \sin(y)) = 1 - \cos(x+y)$$

   I then divided the left side by $\cos(x+y) + \sin(y)$ which gave me the following:

   $$\frac{dy}{dx} = \frac{1-\cos(x+y)}{\cos(x+y)+\sin(y)}$$

   I was then able to plug in $\frac{\pi}{4}$ for y and 0 for x.

   $$\frac{d(\frac{\pi}{4})}{d(0)} = \frac{1-\cos(0+\frac{\pi}{4})}{\cos(0+\frac{\pi}{4})+\sin(\frac{\pi}{4})}$$

   I then simplified this further as follows:

   $$\frac{d(\frac{\pi}{4})}{d(0)} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}$$

$$\frac{d(\frac{\pi}{4})}{d(0)} = \frac{2-\sqrt{2}}{2\sqrt{2}}$$

I then used the general equation of a line $y - y' = m(x - x')$ and substituted x, y, and the slope I just found into the equation.

$$y - \frac{\pi}{4} = \frac{2-\sqrt{2}}{2\sqrt{2}}(x-0)$$

After some easy distribution and simplification I came to my final answer:

The equation of the tangent line to the graph of the equation $\sin(x+y) - \cos y = x$ at the point $\left(0, \frac{\pi}{4}\right)$ is: $\quad y = \frac{2-\sqrt{2}}{2\sqrt{2}}x + \frac{\pi}{4}$

2. Determine the exact coordinates for the four points that are the x and y intercepts of the graph of the equation $10x^2 + 5y^2 + 4xy = 110$. Explain how you determined that these points are the intercepts of the graph of the equation. (2 points)

To solve this problem, I realized that anytime $y = 0$, it will be a x-intercept and anytime $x = 0$, it will be a y-intercept.

To find x-intercepts: $10x^2 + 5(0)^2 + 4x(0) = 110$

$10x + 2 = 110$

$\sqrt{x^2} = \sqrt{11}$

$x = \pm\sqrt{11}$

The two x-intercept points are: $(-\sqrt{11}, 0)$ and $(\sqrt{11}, 0)$

To find y-intercepts:

$10(0)^2 + 5y^2 + 4(0)y = 110$

$5y^2 = 110$

$y^2 = 22$

$\sqrt{y^2} = \sqrt{22}$

$y = \pm\sqrt{22}$

The two y-intercept points are: $(0, -\sqrt{22})$ and $(0, \sqrt{22})$

3. Find the slopes of the tangent lines for each of the points from problem 2. Show your work so it is clear how you determined those slopes. Then write the equation of the tangent lines at the two points. (3 points)

I began this problem by taking the derivative of both sides with respect to x of the equation $10x^2 + 5y^2 + 4xy = 110$ which gave me the following:

$20x + 10y\frac{dy}{dx} + 4x\frac{dy}{dx} + 4y = 0$

I then subtracted $-20x$ and $-4y$ from the left side. This allowed me to be able to factor out $\frac{dy}{dx}$ from the left side.

$\frac{dy}{dx}(10y + 4x) = -20x - 4y$

After dividing by $(10y + 4x)$ I was left with my the following:

$$\frac{dy}{dx} = \frac{-20x - 4y}{10y + 4x}$$

I then used this slope to find the equations for all the tangent lines at their respective points by plugging in their x and y coordinates and solving:

Tangent line formula 1

$$\frac{d(0)}{d(-\sqrt{11})} = \frac{-20(-\sqrt{11}) - 4(0)}{10(0) + 4(-\sqrt{11})} = \frac{-20(-\sqrt{11})}{4(-\sqrt{11})} = \frac{-20}{4} = -5$$

I then plugged -5, x, and y into the the general equation of a line $y - y' = m(x - x')$ and solved. This gave me the following as my final answer:

$$y - 0 = -5(x + \sqrt{11}) = \qquad y = -5(x + \sqrt{11})$$

Tangent line formula 2

$$\frac{d(0)}{d(\sqrt{11})} = \frac{-20(\sqrt{11}) - 4(0)}{10(0) + 4(\sqrt{11})} = \frac{-20(\sqrt{11})}{4(\sqrt{11})} = \frac{-20}{4} = -5$$

I then plugged -5, x, and y into the the general equation of a line $y - y' = m(x - x')$ and solved. This gave me the following as my final answer:

$$y - 0 = -5(x - \sqrt{11}) = \qquad y = -5(x - \sqrt{11})$$

Tangent line formula 3

$$\frac{d(\sqrt{22})}{d(0)} = \frac{-20(0) - 4(\sqrt{22})}{10(\sqrt{22}) + 4(0)} = \frac{-4(\sqrt{22})}{10(\sqrt{22})} = \frac{-2}{5}$$

I then plugged $\frac{-2}{5}$, x, and y into the the general equation of a line $y - y' = m(x - x')$ and solved. This gave me the following as my final answer:

$$y - \sqrt{22} = \frac{-2}{5}(x) = \qquad y = \frac{-2}{5}x + \sqrt{22}$$

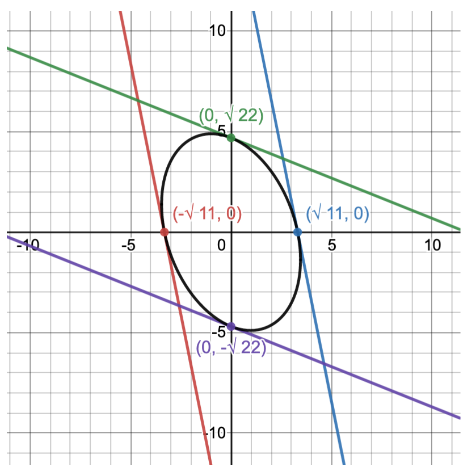
Tangent line formula 4

$$\frac{d(-\sqrt{22})}{d(0)} = \frac{-20(0) - 4(-\sqrt{22})}{10(-\sqrt{22}) + 4(0)} = \frac{-4(-\sqrt{22})}{10(-\sqrt{22})} = \frac{-2}{5}$$

I then plugged $\frac{-2}{5}$, x, and y into the the general equation of a line $y - y' = m(x - x')$ and solved. This gave me the following as my final answer:

$$y + \sqrt{22} = \frac{-2}{5}(x) = \qquad y = \frac{-2}{5}x - \sqrt{22}$$

4. Use Desmos to graph the equation, the points, and the tangent lines that you determined in problems 2 and 3. Attach your graph to the document that you turn in for this assignment. (1 point)



5. Use the method of logarithmic differentiation to find a general formula for the derivative of a function $y = f(x)^{g(x)}$. (2 points)

   I began this problem by using the natural log to bring the exponent $(g(x))$ down as follows:

$\ln(y) = g(x)\ln(f(x)$

I then took the derivative of both sides with respect to x.

$\frac{1}{y} = g'(x)\ln(f(x)) + g(x)(\frac{f'(x)}{f(x)}$

From there I re-substituted y back into the original equation and reached my final answer:

$$y' = g'(x)\ln(f(x)) + g(x) - \frac{f'(x)}{g(x)}(f(x)g(x)$$