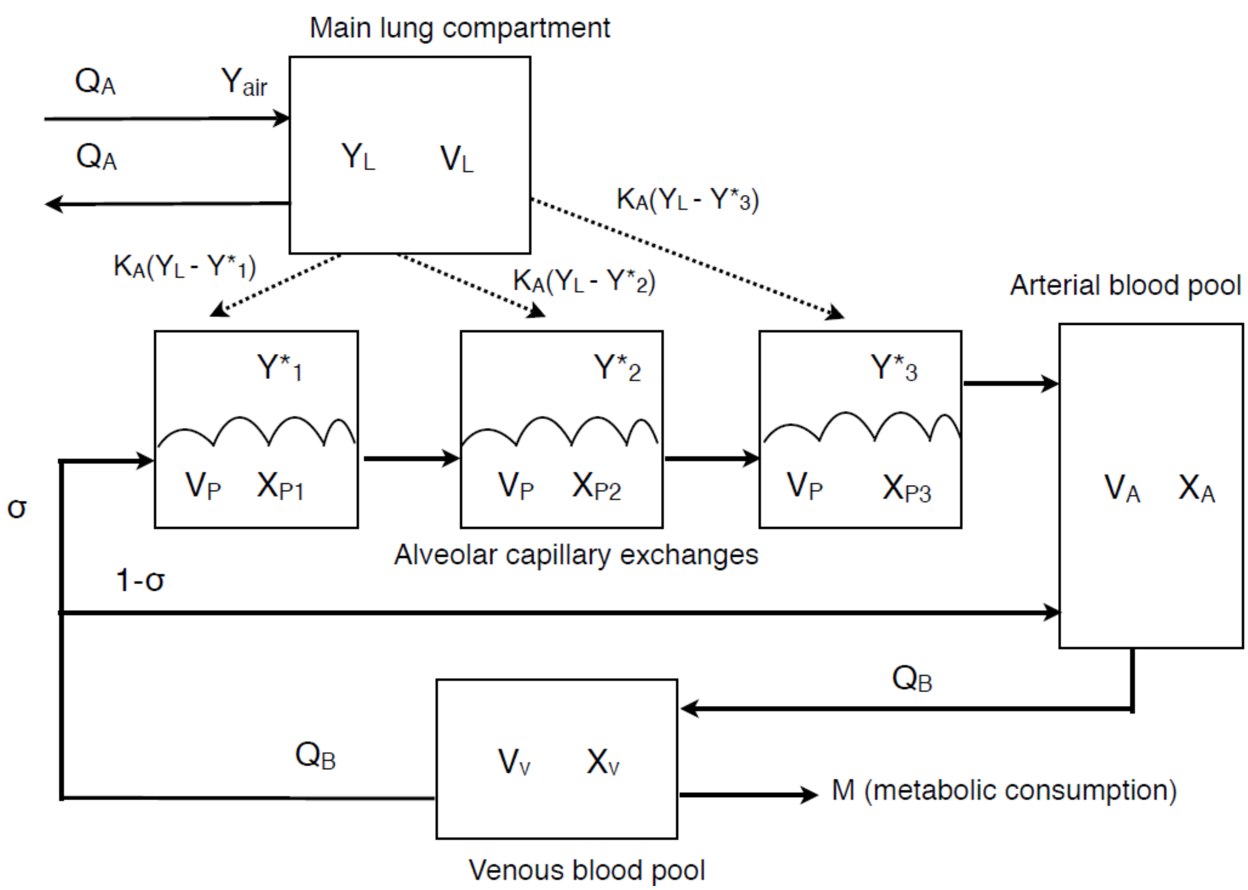
CHEN 3220 Project 1: Modeling oxygen flow in the circulatory system

Part 1 Due: Wednesday, February 26, 2014 at 11:55pm
Part 2 Due: Wednesday, March 12, 2014 at 11:55pm

The human respiratory system functions as a complex mass transfer system which supplies oxygen to the blood and removes CO_2. The major components are the lungs, where oxygen and carbon dioxide are exchanged, the circulatory system, transporting both gases, and the tissues which consume oxygen and release CO_2. Oxygen enters the lungs and is transferred by diffusion through the capillary wall into the bloodstream. Diffusion and reaction (binding with hemoglobin) take place inside the red blood cells.
In this project you are asked to develop a model of the respiratory system that will allow you to predict the steady state and dynamic behavior of oxygen concentrations in the lungs, pulmonary capillaries and venous tissue.

Description of the mathematical model

A schematic model of the process is shown in the figure below. In this model, the physical and chemical details of the membranes and of the red blood cells are ignored. Instead, we treat only the flow of oxygen into the lungs, overall mass transfer of oxygen into the blood from the alveoli, transport of oxygen from the alveolar capillaries to the main blood flow, and finally the removal of oxygen from the blood by hemoglobin for consumption throughout the body.



This model has some of the following features:

1) It allows for location dependent variation in the pulmonary capillaries, by allowing different concentrations in different capillary pools.
2) Allows for the effects of reduced diffusion across the alveolar-capillary interface, which may result from decreased area or increased thickness, modeled by changing the mass transfer coefficient for diffusion.
3) Can describe effects of blood bypassing the mass transfer surface of the lungs directly to the arterial blood.
4) Can describe hypoventilation (less than normal flow of oxygen into the lungs) due to decreased rate or volume.
5) Can describe breath-by-breath behavior, allowing for concentration changes during a breath as well as long term changes such as increased carbon dioxide inhalation.
6) Allows for variable rates of metabolic consumption and cardiac output

Each compartment or "pool" is represented as a well-stirred tank in which the concentration changes with time. The output concentration from each pool is the same as the concentration of the fluid within the pool. The total lung capacity (V_L) can change with respect to time, depending on the moment in the breath. We can model inspiration and expiration as a periodic function with both inlet and outlet periods. The longitudinal variation on the pulmonary capillaries is modeled as a series of three pools. Mass transfer is allowed to take place between the alveolar space and each of the three pulmonary pools.
The rate of mass transfer is given by the following relationship:

$$Q_j = K_A(Y_L - Y_j^*)$$

where K_A is the mass transfer coefficient, Y_L is the lung mole fraction of oxygen, Y_j^* is the gas phase volumetric fraction of oxygen that is in equilibrium with the liquid blood compositions X_{Pj}, and j refers to the pulmonary pool number (1, 2, or 3). The arterial pool is necessary to include to remix shunted and pulmonary blood and to integrate the oscillations produced in the pulmonary capillary pools. We assume that the volumes of the pulmonary pools (V_P) are all equal.
We must relate the gas phase volumetric fraction of oxygen Y_j^* with the concentration of oxygen in the blood X_{Pj} (ml oxygen per ml of blood). Assume that the equilibrium concentrations obey:

$$Y_j^* = (0.00032X_{Pj})/(0.20155 - X_{Pj}) + (11.27 - X_{Pj})(0.055X_{Pj})$$

Note that there is a discontinuity at X_{Pj} = 0.20155; since we will be operating under this limit, X_{Pj} will always be bounded at 0.20155 no matter how high the concentration of oxygen in the air goes. Assume that the addition of dissolved oxygen does not change the volume of the blood.
We will use the following variables in constructing the model. Some are parameters, and their values are given; others are time dependent variables that you will model.

Parameters:

K_A = 3050 ml air/min	Mass transfer coefficient for air.
M = 230 ml O_2/min	Metabolic O_2 consumption rate.
Q_A = 4500 ml air/min	Lung ventilation rate.
Q_B = 5500 ml blood/min	Cardiac blood output.
V_A = 1700 ml blood	Arterial blood volume.
Y_{air} = 0.2	Volume fraction of O_2 in the input flow to the lungs.
V_P = 22 ml	Pulmonary capillary blood volume (for each pool).
V_V = 40 liters	Tissue volume.
σ = 0.99	Fraction of the venous blood that goes through the lungs.
$V_L = 2700 + 260\cos(2\pi t/\tau)$ ml	Volume of the lungs, varying with time.
τ = 5 seconds	Average time between breaths.

Initial values at t = 0 of the six dependent variables:

X_A = 0.193 ml O_2/ml blood	arterial blood concentration of O_2
X_{P1} = 0.165 ml O_2/ml blood	pulmonary blood concentration of O_2 in the 1st pool
X_{P2} = 0.183 ml O_2/ml blood	pulmonary blood concentration of O_2 in the 2nd pool
X_{P3} = 0.190 ml O_2/ml blood	pulmonary blood concentration of O_2 in the 3rd pool
X_V = 0.150 ml O_2/ml blood	venous blood concentration of O_2
Y_L = 0.142 ml O_2/ml gas	lung gas volume fraction of O_2

Part 1 (Due Feb 26th at 11:55pm):

a) Set up the transient material balances for the system. Your final answer should be in terms of six differential equations with respect to time in the six dependent variables (Y_L, X_{P1}, X_{P2}, X_{P3}, X_A, and X_V). <u>Check that the units for each term in your balances match.</u>

b) Assume that at t = 0, the person holds their breath. The mass flow rates in and out of the lungs (Q_A) go to zero, and the alveolar volume is constant (assume it stops at the maximum of V_L, since one usually takes in a breath before stopping to breathe!) Write a new system of equations for this situation. Again, check units. Make sure that the statement of the differential equations is unambiguous.

Part 2 (Due March 12th at 11:55pm):

a) Solve the six ordinary differential equations from part 1a using the 4th order Runge-Kutta code. Plot the answers from t = 0 to t = 60 seconds. Describe the behavior you observe in terms of the fluctuations in concentrations.

b) Compute the average lung oxygen concentration Y_L over the time period t = 0 to t = 60 seconds, using any numerical integration technique.

c) Solve the six ordinary differential equations from part 1b (where breathing stops at time t = 0). Plot your answers. At what time would the arterial blood oxygen concentration drop to 75% of the initial value? You can calculate this by inspection of the graph, you don't need to set it up as a root finding problem (add a grid to the plot to guide you or use `xlim` and `ylim` to zoom in).

Rubric
Part 1
Methodology
- Provide a general statement of the problem
- State all assumptions

Results
- Derive the transient material balance for the system (Part 1a)
  - Answers should be in terms of $Y_L, X_{P1}, X_{P2}, X_{P3}, X_{P4}, X_A$, and X_V
  - Check to ensure that units match
- For the part when the person holds their breath (Part 1b)
  - Derive a new set of transient material balance for the system
- In general, format the differential equations in the following form
  - $\frac{dA(\vec{x})}{dt} = f(\vec{x})$ where $\vec{x}$ is a vector defining the set of variables that A depends on
- For both parts present for the solution
  - Differential equations
  - Auxiliary equations
  - Parameters
  - Dependent variables along with initial values

Appendix
- Provide an appendix indicating what each variables you used in the derivation represents

Part 2
Methodology
- Explain the theory of the 4th order Runge-Kutta method

Results
- Solve the set of differential equations from part 1a for $t \in [0, 60]$
  - Describe the observed behavior
  - Provide a plot of the solution
- Calculate the average lung oxygen concentration Y_L for $t \in [0, 60]$
  - Based on the behavior of Y_L, is the integration method you used valid?
- Solve the set of differential equations from part 1b.
  - Describe the observed behavior
  - Provide a plot of the solution
  - Determine the time it would take for the O_2 concentration in the arterial blood to drop below 75%
- Compare the solutions from part 1a with part 1b, does the change in the concentration profiles make sense?

Python Code
- Provide all Python code with proper commenting