Geomorphic Constraints on Fault Geometry and Seismic Slip Deficit in the Himalaya

Phase 1: Introduction of the problem

1. We hypothesize that Himalayan rivers show differential uplift due to tectonic forcing.
2. We aim to quantify the coupling between topography and fault geometry (infer subsurface from surface), which will (1) better understand earthquake hazard, (2) infer heterogeneity in the Himalayan fault (3) proof of concept that it is possible.
3. To do that, we need to construct our model. We will show equivalence between the structural and continuum descriptions of mountain building so we can build mountains while taking account for potentially unbalanced earthquake cycles.

Phase 2: Epistemology 1: The Seismic Cycle is 100% Balanced; Fault Geometry is Unknown

1. **Premise:** Over millions of years, the crust cannot sustain infinite elastic strain, therefore the earth must "perfectly" balance its elastic budget. All long-term uplift is dictated solely by fault geometry and conservation of mass.
2. **The Inversion:** Fix the slip deficit to zero, invert the river topography to find the fault geometry.
3. **Discussion:** Tests the limits of the purely structural viewpoint. It uses the river as a structural geology tool to "see" the crust and draw cross-sections of blind thrusts purely from geomorphic data.

Phase 3: Epistemology 2: Fault Geometry is known; The Cycle is Imbalanced.

1. **Premise:** We trust our geophysical imaging (seismic reflection, magnetotellurics) and structural balanced cross-sections more than we trust the assumption of a perfectly balanced seismic cycle.
2. **The Inversion:** Fix the fault geometry based on the geophysical literature; invert the river topography to find the spatial distribution of the slip deficit.
3. **Discussion:** Make a slip deficit map of the fault. It tells exactly where the Himalayan crust is absorbing permanent strain that is not accounted for by discrete slip on the main fault plane.

1 Phase 1

1.1 Introduction

It has been noted that many Himalayan rivers are characterized by zones of relatively high gradients that cannot be associated with differential resistance to erosion [10]. The persistent correlation between faults and zones of increased gradients strongly suggests that these steepened reaches are the result of differential uplift of the High Himalayas. Many inconsistencies appear when the correlation between river gradients and lithology is examined in detail, suggesting that rock strength cannot uniquely explain the observed channel morphology [10].

Surface processes and topography relate to tectonic uplift through a simple relationship. For example, the height of a fluvial terrace above the present river divided by its age gives the average rate at which the rock has been uplifted since that surface was abandoned [2]. More generally, if deformation is approximately steady when averaged over many seismic cycles, the cumulative rock uplift recorded by topography can be treated as proportional to the time over which that topography has integrated uplift and erosion [2, 5].

Mountainous rivers also record information regarding tectonic uplift. Because river profiles extend from upstream to downstream, they inherently provide a spatial encoding of deformation [1]. Using the stream power equation for example, the river elevation over the stream distance (slope) is an integration of rock uplift rate over space. Therefore, spatial variations in the uplift rate in the stream power equation can produce the differential uplift observed in the High Himalayas. For example, Meade [8] demonstrated that channel convexity in the Himalayas is a potential signature of earthquake-cycle imbalance. In other words, a portion of the accumulated interseismic tectonic strain is retained as permanent deformation if it is not fully recovered during coseismic release.

Recognizing these geomorphic markers of tectonic uplift is important because they offer quantitative insights into long-term seismic deficits. For example, Bilham suggested that coseismic moment release rates have lagged behind moment accumulation rates by 75% over the past 500 years [3, 4]. It may also provide an alternative method for constructing 3D fault geometry with improved accuracy in earthquake simulations in mountainous regions, where abundant and freely available satellite DEM data exist, while direct fault imaging is often difficult due to mountainous terrain and political factors.

However, the quantitative description of crustal deformation to surface signatures has historically been separated into two distinct fields: geometric kinematic models (used in structural geology) and continuum mechanics models (such as elastic dislocation theory). Avouac [2] gave a qualitative notion that the long-term average of transient earthquake cycles ultimately gives rise to structural uplift and mountain building. While other studies (e.g., Souter and Hager [11]; Dal Zilio et al. [6]) have demonstrated connections between the two models, a fundamental gap remains. A purely geometric structural models cannot account for the transient elastic strains of the earthquake cycle, and standard elastic dislocation models cannot accommodate finite, long-term mountain building. Demonstrating the equivalence between these two approaches is therefore critical because it will allow us to model long-term orogenesis while incorporating the potentially unbalanced kinematics of the earthquake cycle. Given a true cumulative rock uplift field that partitions the earthquake cycle, it then becomes possible to accurately forward-model the resulting topography using the stream power equation.

1.2 Equivalence Theorem between structural and continuum descriptions of mountain building

Consider an infinite planar fault from surface to infinite depth. Rigid hanging-wall motion equals the far-field displacement jump of a Volterra dislocation with Burgers vector $\mathbf{b}$, because both descriptions impose the same displacement discontinuity on the same surface. In linear elastostatics, the boundary-value problem for a displacement discontinuity on a surface produces a unique solution. Consequently, the resulting displacement field reduces to a uniform rigid-body translation of the hanging wall [7].

Now consider two planar fault segments meeting at an intersection point, also known as fault-bend folding in structural geology. The hanging wall is partitioned into rigid domains separated by axial surfaces. Let block i overlie fault segment i with dip angle θ_i. Assume uniform slip rate s parallel to each segment. The velocity of block i in Cartesian coordinates is:

$$\mathbf{v}_i = s \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix}. \tag{1}$$

When motion transitions from fault plane segment $i-1$ to i, the relative velocity between adjacent rigid domains is:

$$\Delta\mathbf{v}_i = \mathbf{v}_i - \mathbf{v}_{i-1} = s \begin{pmatrix} \cos\theta_i - \cos\theta_{i-1} \\ \sin\theta_i - \sin\theta_{i-1} \end{pmatrix}. \tag{2}$$

Define the intersegment dip change $\Delta\theta$ and the mean dip angle $\bar{\theta}$ as $\Delta\theta = \theta_i - \theta_{i-1}$ and $\bar{\theta} = (\theta_i + \theta_{i-1})/2$. Apply trigonometric identities to $\Delta\mathbf{v}_i$:

$$\Delta\mathbf{v}_i = 2s \sin\left(\frac{\Delta\theta}{2}\right) \begin{pmatrix} -\sin\bar{\theta} \\ \cos\bar{\theta} \end{pmatrix}. \tag{3}$$

The magnitude of the relative slip between rigid domains therefore becomes:

$$|\Delta\mathbf{v}_i| = 2s \left|\sin\left(\frac{\Delta\theta}{2}\right)\right|. \tag{4}$$

The axial surface bisects the supplementary angle of the fault bend and is orthogonal to $\bar{\theta}$. Its orientation is $\gamma = \bar{\theta} + \pi/2$, and the unit tangent vector along the axial surface is:

$$\mathbf{t}_{\text{axial}} = \begin{pmatrix} \cos\gamma \\ -\sin\gamma \end{pmatrix} = \begin{pmatrix} -\sin\bar{\theta} \\ -\cos\bar{\theta} \end{pmatrix}. \tag{5}$$

Thus,

$$\Delta\mathbf{v}_i = 2s \sin\left(\frac{\Delta\theta}{2}\right) \mathbf{t}_{\text{axial}}, \tag{6}$$

which shows that the relative motion is pure simple shear parallel to the axial surface, with zero opening or closing.

Conservation of Burgers Vectors at the Fault Bend

In Volterra dislocation theory [12], a fault is defined as a surface across which the displacement field exhibits a discontinuity of rigid-body form,

$$\Delta\mathbf{u} = \mathbf{U} + \mathbf{\Omega} \times \mathbf{r}, \tag{7}$$

where $\mathbf{U}$ is a constant translation vector and $\mathbf{\Omega}$ is a constant rotation vector. The Burgers vector $\mathbf{b}$ is the displacement jump across the surface, i.e., $\mathbf{b} = \Delta\mathbf{u}$. At a junction where multiple dislocation surfaces meet, topology imposes the closure condition $\sum \mathbf{b} = 0$. If $\sum \mathbf{b} \neq 0$, the elastic medium must accommodate the mismatch through distributed strain [7].

At a fault bend, three dislocation surfaces intersect: (1) the incoming fault segment with Burgers vector $\mathbf{b}_{\text{in}} = \mathbf{v}_{i-1}$, (2) the outgoing segment with $\mathbf{b}_{\text{out}} = \mathbf{v}_i$, and (3) the axial surface with $\mathbf{b}_{\text{axial}} = \Delta\mathbf{v}_i$. The Burgers-vector sum around a contour enclosing the junction is therefore

$$\oint d\mathbf{u} = \mathbf{b}_{\text{in}} + \mathbf{b}_{\text{axial}} - \mathbf{b}_{\text{out}} = \mathbf{v}_{i-1} + (\mathbf{v}_i - \mathbf{v}_{i-1}) - \mathbf{v}_i = 0. \tag{8}$$

The velocity triangle $\mathbf{v}_{i-1} + \Delta\mathbf{v}_i - \mathbf{v}_i = 0$ is therefore identical to the Burgers-vector closure condition $\sum \mathbf{b} = 0$. The relative velocity $\Delta\mathbf{v}_i$ is the Burgers vector required for compatibility at the fault bend. Hence, introducing the axial-surface dislocation with slip magnitude $2s\sin(\Delta\theta/2)$ eliminates internal singular stresses and reduces the solution to a piecewise rigid-body translations separated by displacement jumps. Therefore, we have established the equivalence between the structural and continuum descriptions of mountain building.

1.3 Partitioning the Earthquake Cycle via Linear Superposition

Now, let us partition the earthquake cycle using the principle of linear superposition and the equivalence theorem. Elastic rebound theory states that the permanent, long-term deformation of a fault system equals the time-averaged sum of deformation accumulated interseismically and released coseismically. In velocity form, the cycle-averaged kinematic budget is:

$$\mathbf{v}_{\text{long-term}} = \mathbf{v}_{\text{interseismic}} + \mathbf{v}_{\text{coseismic}}. \tag{9}$$

Rearranging the equation isolates the interseismic velocity field as the deficit between the long-term deformation and the coseismic release,

$$\mathbf{v}_{\text{interseismic}} = \mathbf{v}_{\text{long-term}} - \mathbf{v}_{\text{coseismic}}. \tag{10}$$

This is the slip-deficit (back-slip) statement. Classical methods assume a planar fault in a homogeneous half-space and represent the long-term motion as a uniform rigid-block translation [9]. Here, the Equivalence Theorem provides the replacement required for complex fault-bend fold because the long-term background field is equal to the kinematically balanced structural velocity.

Let $\mathcal{D}(\Sigma, s)$ denote the linear elastostatic surface velocity field generated by imposing a uniform slip rate s on a prescribed set of dislocation surfaces Σ in an elastic half-space. We assert that the true long-term secular velocity is given by the structural model, and is analytically equivalent to the elastic field generated by continuous slip on the entire geometrically compatible dislocation network,

$$\mathbf{v}_{\text{long-term}} \equiv \mathbf{v}_{\text{struct}} = \mathcal{D}(\Sigma_{\text{total}}, s), \tag{11}$$

where Σ_{total} includes the full fault geometry with the axial surfaces required for compatibility.

During earthquakes, rupture may occur on some region of the fault (may refer as to the frictionally locked seismogenic zone). Denoting that portion by $\Sigma_{\text{locked}} \subset \Sigma_{\text{total}}$, the cycle-averaged coseismic contribution becomes

$$\mathbf{v}_{\text{coseismic}} = \mathcal{D}(\Sigma_{\text{locked}}, s), \tag{12}$$

therefore, zero coseismic slip on the non-coseismic contributing portion (may refer as to the deep stable-sliding portion).

Because the governing Navier–Cauchy equations of linear elastostatics are linear [12], the mapping $\mathcal{D}$ is additive with respect to dislocation surfaces. Writing the total network as the union of locked and creeping parts,

$$\Sigma_{\text{total}} = \Sigma_{\text{locked}} \cup \Sigma_{\text{creep}}, \qquad \Sigma_{\text{locked}} \cap \Sigma_{\text{creep}} = \varnothing, \tag{13}$$

linearity implies

$$\mathcal{D}(\Sigma_{\text{total}}, s) = \mathcal{D}(\Sigma_{\text{locked}}, s) + \mathcal{D}(\Sigma_{\text{creep}}, s). \tag{14}$$

Substituting equations yields

$$\mathbf{v}_{\text{interseismic}} = \mathcal{D}(\Sigma_{\text{total}}, s) - \mathcal{D}(\Sigma_{\text{locked}}, s) = \mathcal{D}(\Sigma_{\text{creep}}, s). \tag{15}$$

Therefore, subtracting the coseismic elastic field generated by slip on the locked zone from the long-term structurally balanced field cancels the locked contribution exactly, leaving the surface velocity produced by forward slip on the deep creeping portion, also known as the interseismic velocity field.

Cycle-Averaged Rock Uplift and Geomorphic Forcing

To connect the earthquake cycle timescale (10^2–10^3 years) with geomorphic and mountain-building timescales (10^6 years) [13], we define the net tectonic forcing that drives landscape evolution. In bedrock incision models such as the stream power equation [1, 14],

$$\frac{\partial z}{\partial t} = U(x) - KA^m S^n, \tag{16}$$

topography evolves in response to a continuous rock uplift field $U(x)$. Rivers do not respond to transient elastic oscillations of individual earthquakes. Rather, they integrate the time-averaged permanent vertical motion. Thus, the uplift rate supplied to mountain building is the vertical component of the cycle-averaged velocity field.

Let s_c denote the average coseismic slip rate released on the seismogenic zone over many cycles. The cycle-averaged tectonic velocity is

$$\mathbf{v}_{\text{cycle}} = \mathcal{D}(\Sigma_{\text{creep}},\, s) + \mathcal{D}(\Sigma_{\text{locked}},\, s_c). \tag{17}$$

We now evaluate two limiting kinematic scenarios.

1. Balanced Earthquake Cycle

If the earthquake cycle is perfectly balanced, the average coseismic slip rate equals the tectonic loading rate ($s_c = s$). In that case,

$$\mathbf{v}_{\text{cycle}} = \mathcal{D}(\Sigma_{\text{creep}},\, s) + \mathcal{D}(\Sigma_{\text{locked}},\, s) = \mathcal{D}(\Sigma_{\text{total}},\, s) \equiv \mathbf{v}_{\text{struct}}. \tag{18}$$

The uplift field supplied to the stream power equation is identical to the purely geometric structural model. Topography reflects long-term fault-bend folding, and rivers incise into the structural uplift field.

2. Unbalanced Earthquake Cycle

If the locked zone releases less slip than it accumulates, a permanent slip deficit rate remains,

$$s_{\text{def}} = s - s_c. \tag{19}$$

The cycle-averaged velocity becomes

$$\begin{aligned}
\mathbf{v}_{\text{cycle}} &= \mathcal{D}(\Sigma_{\text{creep}},\, s) + \mathcal{D}(\Sigma_{\text{locked}},\, s - s_{\text{def}}) && (20)\\
&= \left[\mathcal{D}(\Sigma_{\text{creep}},\, s) + \mathcal{D}(\Sigma_{\text{locked}},\, s)\right] - \mathcal{D}(\Sigma_{\text{locked}},\, s_{\text{def}}) && (21)\\
&= \mathbf{v}_{\text{struct}} - \mathcal{D}(\Sigma_{\text{locked}},\, s_{\text{def}}). && (22)
\end{aligned}$$

This result links short-term geodetic imbalance to long-term mountain building. The uplift field is no longer purely structural. Instead, the structural field is subtracted by an elastic residual associated with unreleased slip. Because the deficit term $\mathcal{D}(\Sigma_{\text{locked}},\, s_{\text{def}})$ produces a localized residual uplift pattern, the long-term rock uplift field $U(x)$ acquires "differential uplift." ==

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