# Shutter speed ramp required for constant image brightness

Parameters used throughout:

In[18]:=
```
(* Environment brightness function parameters *)
min = 1;
max = 100;
rate = 1 / 3;

(* Seconds between end of one frame and start of next *)
pause = 0.5;

(* Timelapse parameters *)
frames = 900;
elapsed = 4 × 60 × 60; (* sec *)

(* Camera settings *)
iso = 160;
area = 0.00008658;
```

In[26]:=
```
SetOptions[{Plot, ListPlot}, ImageSize → {400, 220}];
```

Ambient brightness vs. sun zenith angle past sunset:

In[27]:=
```
BrightnessWithAngle[θ_, min_, max_, rate_] := (max - min) Exp[-rate θ] + min
```

Sun zenith angle in degrees vs. local time in hours:

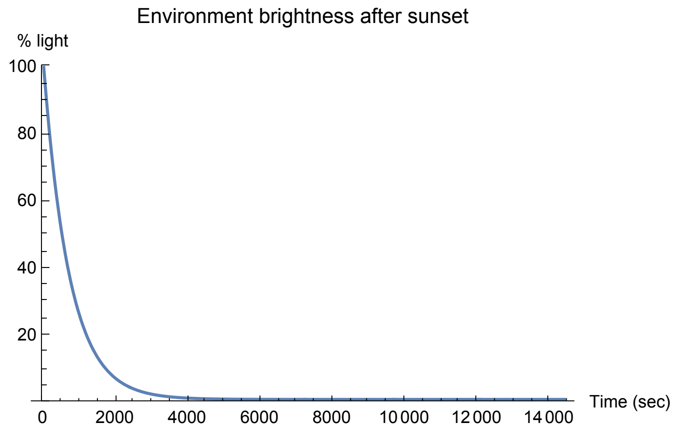
In[28]:=
```
AngleWithTime[sec_] := 360 sec / 86400 (* Assume earth is pointlike compared to distance to sun  *)
```

Ambient brightness vs. time

In[29]:=
```
BrightnessWithTime[sec_?NumericQ, min_, max_, rate_] := BrightnessWithAngle[AngleWithTime[sec], min, max, rate]
```

In[30]:=
```
Plot[BrightnessWithTime[t, min, max, rate], {t, 0, elapsed}, AxesOrigin → {0, 0}, AxesLabel → {"Time (sec)", "% light"},
 PlotLabel → "Environment brightness after sunset", LabelStyle → {GrayLevel[0]}, PlotRange → {0, 100}]
```

Out[30]=



Relative shutter speed vs. time function (continuous approximation)

In[31]:=
```
ExposureAtTimeT[T_, min_, max_, rate_, iso_, area_] :=
 x /. (FindRoot[NIntegrate[BrightnessWithTime[t, min, max, rate], {t, T, T + x}] == 1/(iso area), {x, 1}] // Quiet)
```

Compute each frame's relative exposure time

In[32]:=
```
EXPTIMES = {}; curtime = 0;
Do[curr = ExposureAtTimeT[curtime, min, max, rate, iso, area];
 AppendTo[EXPTIMES, curr];
 curtime += (curr + pause);,
 frames]
```

Check that the elementwise product of EXPTIMES and corresponding brightness (aka the image brightness) is a constant.

In[34]:=
```
CHK = {}; curfr = 1; curt = 0;
Do[
 curexp = EXPTIMES[[curfr]];
 avgbright = 1/curexp NIntegrate[BrightnessWithTime[t, min, max, rate], {t, curt, curt + curexp}];
 prod = iso area curexp avgbright;
 AppendTo[CHK, prod];
 curt += (curexp + pause);
 curfr++;,
 frames]
Print["μ = ", Mean[CHK], ", σ^2 = ", Variance[CHK]]
```

$\mu$ = 1., $\sigma^2$ = $3.08197 \times 10^{-28}$

Scale all exposure times such that the total elapsed time equals “elapsed”

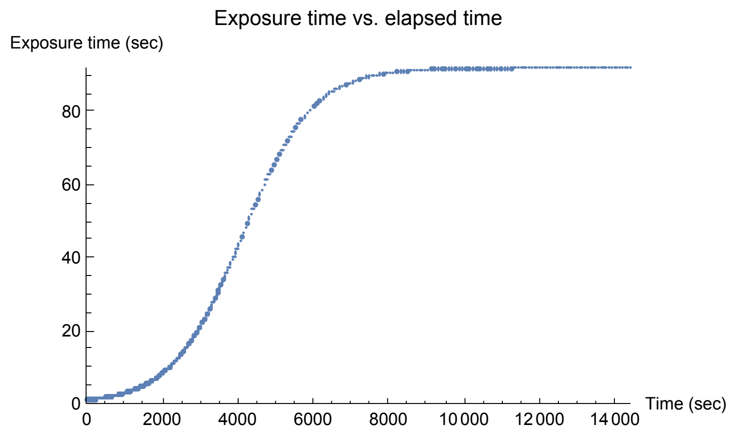
In[37]:=
```
scale = s /. Solve[(s (Total[EXPTIMES]) + (frames - 1) pause) == elapsed, s];
SCALEDEXPTIMES = scale[[1]] EXPTIMES;
```

In[39]:=

Plot scaled exposure times with elapsed time

In[40]:=
```
frtime[i_] := (SCALEDEXPTIMES[[i]] + pause)
times = Accumulate[Map[frtime, Range[1, frames]]];
ListPlot[Transpose[{times, SCALEDEXPTIMES}], AxesOrigin → {0, 0},
 AxesLabel → {"Time (sec)", "Exposure time (sec)"}, PlotLabel → "Exposure time vs. elapsed time",
 LabelStyle → {GrayLevel[0]}, PlotRange → {{0, elapsed}, {0, Max[SCALEDEXPTIMES]}}]
```

Out[42]=




Plot scaled exposure times with frame number

In[43]:=
```
ListPlot[SCALEDEXPTIMES, AxesOrigin → {1, 0},
 AxesLabel → {"Frame number", "Exposure time"}, PlotLabel → "Exposure time vs. frame number",
 LabelStyle → {GrayLevel[0]}, PlotRange → {{1, frames}, {0, Max[SCALEDEXPTIMES]}}]
```

Out[43]=