Limitations of a linear shutter ramp

Linear ramp function

In[1]:=
```
(* Exposure time as a linear function of frame number F, with initial exposure, final exposure, frame count as parameters  *)
(* Note: F is inputted in the 1:F domain but calculations are done in the 0:F domain. Input the actual frame number *)
```

$$\text{ExposureTimeAtFrameF}[F\_, \text{initExpSec}\_, \text{finExpSec}\_, \text{frames}\_] := \frac{\text{finExpSec} - \text{initExpSec}}{\text{frames}} \left(\frac{\text{frames}\,(F-1)}{\text{frames}-1}\right) + \text{initExpSec}$$

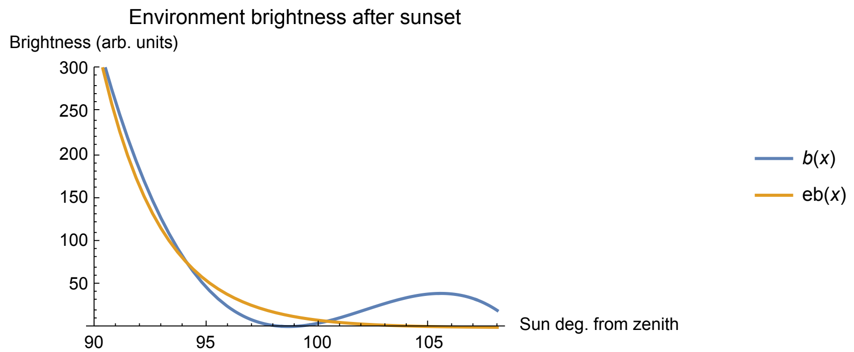
Assumed environment brightness as a function of angular distance of Sun from zenith (0°)

In[2]:=
```
(* From sunset to end of astronomical twilight *)
(* b[x] is a cubic spline constructed from ephemeris data found online *)
(* eb[x] is a decaying exponential curve that roughly matches b[x]  *)
b[x_] := -0.24170462 x^3 + 74.006284 x^2 - 7544.765 x + 256103.54 + 20;
```

$$\text{eb}[x\_] := 340\,\text{Exp}\left[-\frac{(x-90)}{\text{Exp}[1]}\right];$$

```
SetOptions[Plot, ImageSize → {400, 180}];
Plot[{b[x], eb[x]}, {x, 90, 108}, AxesOrigin → {90, 0}, PlotRange → {0, 300}, AxesLabel → {"Sun deg. from zenith", "Brightness (arb. units)"},
 PlotLabel → "Environment brightness after sunset", LabelStyle → {GrayLevel[0]}, PlotLegends → "Expressions"]
```

Out[5]=



Computations

In[6]:=
```
(* Total elapsed time is the summation of exposure times from frame A to B *)
RealWorldTimeBetweenFramesAandB[A_, B_, pause_, initExpSec_, finExpSec_, frames_] :=
 Sum[(ExposureTimeAtFrameF[k, initExpSec, finExpSec, frames] + pause), {k, A, B}]
```

$$\sum_{k=A}^{B}\left(\text{ExposureTimeAtFrameF}[k,\ \text{initExpSec},\ \text{finExpSec},\ \text{frames}]+\text{pause}\right)$$

In[7]:=
```
(* Brightness of environment as a function of frame number *)
SceneBrightnessAtFrameF[F_, sunsetTimeSec_, initExpSec_, finExpSec_, frames_, pause_] := {  eb[x]  90 ≤ x ≤ 108
                                                                                          {  0      True          /.
  {x → (((18 RealWorldTimeBetweenFramesAandB[1, F, pause, initExpSec, finExpSec, frames] - sunsetTimeSec) / (4320 + sunsetTimeSec)) + 90)}
```

In[8]:=
```
(* Image brightness = (area) * (iso speed) * (exposure time) * (scene brightness) *)
ImageBrightnessAtFrameF[F_, sunsetTimeSec_, initExpSec_, finExpSec_, frames_, pause_, isoSpeed_, apertureArea_] := (apertureArea) (isoSpeed)
  (SceneBrightnessAtFrameF[F, sunsetTimeSec, initExpSec, finExpSec, frames, pause]) (ExposureTimeAtFrameF[F, initExpSec, finExpSec, frames])
```

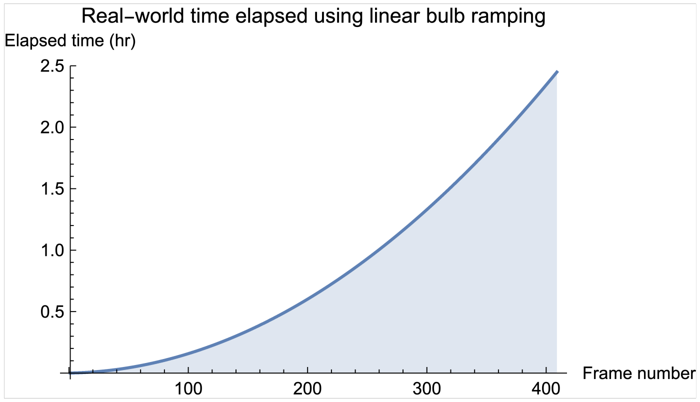
Parameters from 3-26-19 timelapse, 7:27 PM to 9:55 PM

In[9]:=
```
(* number of frames *)
fr = 409;

(* initial exposure, final exposure (sec) *)
init = 0.1; fin = 42;

(* time between end of one frame and beginning of next (sec) *)
p = 0.5;

(* sensor sensitivity and aperture area in mm^2 *)
iso = 100;
area = 0.004751;
```

Plot 1: The ratio of successive exposure times grows

In[14]:=
```
plot1 = Show[DiscretePlot[1/3600 RealWorldTimeBetweenFramesAandB[1, k, p, init, fin, fr], {k, 1, fr}],
  AxesLabel → {"Frame number", "Elapsed time (hr)"}, PlotLabel → "Real-world time elapsed using linear bulb ramping", LabelStyle → {GrayLevel[0]}]
```

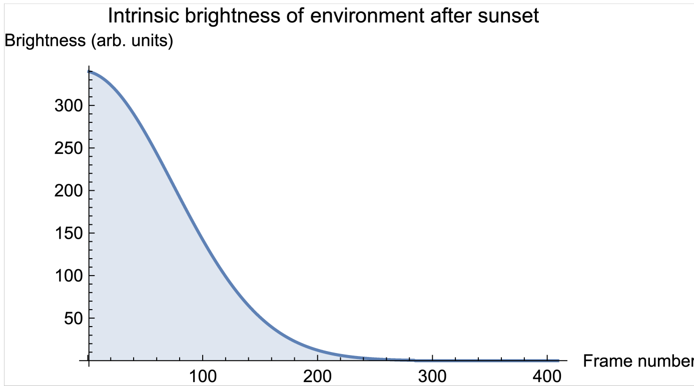
Out[14]=



Plot 2: How bright we expect the environment to be with each frame

In[15]:=
```
plot2 = Show[DiscretePlot[SceneBrightnessAtFrameF[k, 0, init, fin, fr, p], {k, 1, fr}],
  AxesLabel → {"Frame number", "Brightness (arb. units)"}, PlotLabel → "Intrinsic brightness of environment after sunset", LabelStyle → {GrayLevel[0]}]
```

Out[15]=



Plot 3: How bright we expect each frame to be when using a linear ramp

In[16]:=

```
(* What we expect the brightness of each frame to be when we use a linear ramp *)
plot3 = Show[DiscretePlot[ImageBrightnessAtFrameF[k, 0, init, fin, fr, p, iso, area], {k, 1, fr}],
  AxesLabel → {"Frame number", "Brightness (arb. units)"}, PlotLabel → "Brightness of recorded images", LabelStyle → {GrayLevel[0]}]
```

Out[16]=