Maxwell tensor calculation

Find the force at a general point on the XZ plane due to charges +q at <0, a, 0> and -q at <0, -a, 0>.

Field point:

```
rfield = {x, 0, z};
```

Source points:

```
rsource1 = {0, a, 0};
```

```
rsource2 = {0, -a, 0};
```

Script r for each:

```
rvec1 = rfield - rsource1
```

$\{x, -a, z\}$

```
rvec2 = rfield - rsource2
```

$\{x, a, z\}$

Length of each script r:

```
rmag1 = √rvec1.rvec1
```

$\sqrt{a^2 + x^2 + z^2}$

```
rmag2 = √rvec2.rvec2
```

$\sqrt{a^2 + x^2 + z^2}$

Coulomb's constant:

$$k = \frac{1}{4\,\pi\,\epsilon 0};$$

Individual electric fields at <x, 0, z> :

$$e1 = k\,q\,\frac{rvec1}{rmag1^3}$$

$$\left\{\frac{q\,x}{4\,\pi\,\left(a^2 + x^2 + z^2\right)^{3/2}\,\epsilon 0},\ -\frac{a\,q}{4\,\pi\,\left(a^2 + x^2 + z^2\right)^{3/2}\,\epsilon 0},\ \frac{q\,z}{4\,\pi\,\left(a^2 + x^2 + z^2\right)^{3/2}\,\epsilon 0}\right\}$$

```
e2 = k (-q) rvec2/rmag2^3
```

$$\left\{-\frac{q\,x}{4\,\pi\,\left(a^2+x^2+z^2\right)^{3/2}\,\epsilon 0},\ -\frac{a\,q}{4\,\pi\,\left(a^2+x^2+z^2\right)^{3/2}\,\epsilon 0},\ -\frac{q\,z}{4\,\pi\,\left(a^2+x^2+z^2\right)^{3/2}\,\epsilon 0}\right\}$$

Total electric field:

```
e = e1 + e2
```

$$\left\{0,\ -\frac{a\,q}{2\,\pi\,\left(a^2+x^2+z^2\right)^{3/2}\,\epsilon 0},\ 0\right\}$$

Squared magnitude of the electric field:

```
esquared = Simplify[e.e]
```

$$\frac{a^2\,q^2}{4\,\pi^2\,\left(a^2+x^2+z^2\right)^3\,\epsilon 0^2}$$

Components of the Maxwell stress tensor (B = 0):

```
txx = Simplify[ϵ0 e[[1]]^2 - ϵ0/2 esquared]
```

$$-\frac{a^2\,q^2}{8\,\pi^2\,\left(a^2+x^2+z^2\right)^3\,\epsilon 0}$$

```
tyy = Simplify[ϵ0 e[[2]]^2 - ϵ0/2 esquared]
```

$$\frac{a^2\,q^2}{8\,\pi^2\,\left(a^2+x^2+z^2\right)^3\,\epsilon 0}$$

```
tzz = Simplify[ϵ0 e[[3]]^2 - ϵ0/2 esquared]
```

$$-\frac{a^2\,q^2}{8\,\pi^2\,\left(a^2+x^2+z^2\right)^3\,\epsilon 0}$$

```
txy = Simplify[ϵ0 e[[1]] e[[2]]]
```

0

```
txz = Simplify[ϵ0 e[[1]] e[[3]]]
```

0

```
tyz = Simplify[ϵ0 e[[2]] e[[3]]]
```

0

```
tyx = txy
```

0

```
tzx = txz
```

0

```
tzy = tyz
```

0

The tensor:

$$t = \begin{pmatrix} txx & txy & txz \\ tyx & tyy & tyz \\ tzx & tzy & tzz \end{pmatrix}; \text{MatrixForm}[t]$$

$$\begin{pmatrix} -\frac{a^2 q^2}{8 \pi^2 \left(a^2+x^2+z^2\right)^3 \epsilon 0} & 0 & 0 \\ 0 & \frac{a^2 q^2}{8 \pi^2 \left(a^2+x^2+z^2\right)^3 \epsilon 0} & 0 \\ 0 & 0 & -\frac{a^2 q^2}{8 \pi^2 \left(a^2+x^2+z^2\right)^3 \epsilon 0} \end{pmatrix}$$

Tensor dotted into the X-Z plane's (positive) normal vector:

```
tdotn = t.{0, 1, 0}
```

$$\left\{0, \frac{a^2 q^2}{8 \pi^2 \left(a^2 + x^2 + z^2\right)^3 \epsilon 0}, 0\right\}$$

Force found by integrating $\overleftrightarrow{T} \cdot \hat{n}$ over the plane:

$$\text{Assuming}\left[a \in \text{Reals}, \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{tdotn}\, dx\, dz\right]$$

$$\left\{0, \frac{q^2}{16 a^2 \pi \epsilon 0}, 0\right\}$$