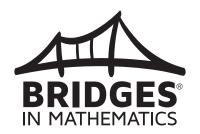


Teachers Guide

GRADE 4 – UNIT 3 – MODULE 2



Module 2

Comparing, Composing & Decomposing Fractions & Mixed Numbers

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Unit 3 Module 2

Module 2

Comparing, Composing & Decomposing Fractions & Mixed Numbers

Overview

In this module, the geoboard is assigned a value of 1. Students name fractional parts of the geoboard and describe the parts' relationships to one another. Their observations are then extended into comparing fractions with unlike numerators and denominators, and adding fractions with like denominators. The last three sessions in the module feature an extended problem-solving opportunity followed by a math forum, as well as two new Work Places that provide practice with composing and decomposing fractions.

Planner

Session & Work Places Introduced	PI	PS	MF	WP	Α	нс	DP
Session 1 Exploring Fractions on the Geoboard This session begins with a quick checkpoint on equivalent fractions. Then students learn how to represent fractions on the geoboard. Students name fractional parts of the geoboard and describe the parts' relationships to one another.	•				•		•
Session 2 Last Equation Wins Students play, first as a class and then in pairs, a game that provides practice with decomposing fractions represented on a geoboard and recording those decompositions with equations. Toward the end of the session, students solve addition problems involving fractions with like denominators.	•					•	•
Session 3 Comparing, Adding & Subtracting Fractions Students work together as a class to create a chart of equivalent fractions for ¼, ½, and ¾. Students make observations about the chart and the equivalent fractions, and then they use those fractions as benchmarks with which to make comparisons among fractions with like and unlike denominators. Finally, they practice adding and subtracting fractions with like denominators.	•						•
Session 4 Dozens of Eggs Today, students add fractions using the egg carton model. The teacher then introduces a related Work Place game by playing it with the class. Students spend any time remaining in the session visiting Work Places, including the new one.	•			•		•	•
Work Place 3A Dozens of Eggs Players take turns drawing from a deck of fraction cards, modeling the designated fraction on an Egg Carton Diagram with colored tiles and string, and recording their results. Players take turns until one person has filled all four egg cartons on his or her record sheet and written a matching addition equation that equals 1 whole for each carton. The first player to fill all four egg cartons wins.							
Session 5 How Many Candy Bars? Mrs. Wiggens is bringing the dessert treats for the annual class picnic, and needs help figuring out how many candy bars she'll have to buy if she gives each student ¾ of a bar. The trouble is, she's not quite sure how many of her fourth graders will be attending the picnic. Students add fractions on an open number line and track their results on a ratio table, working together to help Mrs. Wiggens initially, and then complete the work on their own or in pairs.	•			•			•
Session 6 Racing Fractions Students discuss solutions and strategies for the candy bar problem from the previous session in a math forum. During the discussion, the class talks about adding fractions to mixed numbers and multiplying a fraction by a whole number. Then the teacher introduces the equivalent fraction game, Racing Fractions, which will become a Work Place in later sessions.			•	•		•	•
Work Place 3B Racing Fractions Players draw from the Racing Fractions Cards deck and move game markers along the Racing Fractions Game Board, which shows fraction number lines for halves, thirds, fourths, fifths, sixths, eighths, and tenths. Each player has a game marker on each line and may move one or more markers in a single turn to equal the fraction on the card drawn. Players may also move backward on a turn. The first player to move her markers to 1 on all of the number lines is the winner.							

PI – Problems & Investigations, PS – Problem String, MF – Math Forum, WP – Work Place, A – Assessment, HC – Home Connection, DP – Daily Practice

Materials Preparation

Each session includes a complete list of the materials you'll need to conduct the session, as well as notes about any preparation you'll need to do in advance. If you would like to prepare materials ahead of time for the entire module, you can use this to-do list.

Task		Done
Copies	Run copies of Teacher Masters T1–T11 according to the instructions at the top of each master.	
	Run a single display copy of Student Book pages 98–99, 101–102, 104, and 108.	
	If students do not have their own Student Books, run a class set of Student Book pages 97–111.	
	If students do not have their own Home Connections books, run a class set of the assignments for this module using pages 55–60.	
Work Place Preparation	Prepare the materials for Work Places 3A & 3B using the lists of materials on the Work Place Guides (Teacher Masters T6 & T9).	
	Before Session 4, prepare student Work Place folders with the Unit 3 Work Place Log.	
Charts	Before Session 1, on a piece of chart paper titled "Geoboard Regions," outline five columns titled Region A, Region B, Region C, Region D, and Region E.	
	Before Session 3, using a copy of the Geoboards Teacher Master, draw ½, ¼, and ¾, each on a separate geoboard; then cut them out. Divide a piece of chart paper into 4 columns and 4 rows and label them as shown below.	
Special Items	Before Session 4, cut 10" pieces of heavy string or yarn, 6 per student. (You will also need to cut 6 pieces per student pair for Work Place 3A, introduced in that session. See the Work Place Guide 3A materials list.)	

(Halves	Fourths	Eighths	Sixteenths
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	1/2			
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Please see this module's Resources section of the Bridges Educator site for a collection of resources you can use with students to supplement your instruction.

Session 1

Exploring Fractions on the Geoboard

Summary

This session begins with a quick checkpoint on equivalent fractions. Then students learn how to represent fractions on the geoboard. Students name fractional parts of the geoboard and describe the parts' relationships to one another.

Skills & Concepts

- Use a visual model to explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ (4.NF.1)
- Use a visual model to generate and recognize equivalent fractions (4.NF.1)
- Write an equation showing a fraction a/b as the sum of a number of the unit fraction 1/b (4.NF.3)
- Express a fraction as the sum of other fractions with the same denominator in more than one way and write equations to show those decompositions (4.NF.3b)
- Add fractions with like denominators (supports 4.NF)
- Write an equation showing that a fraction a/b is the product of $a \times 1/b$ (4.NF.4a)
- Multiply a fraction by a whole number (4.NF.4b)
- · Reason abstractly and quantitatively (4.MP.2)
- Model with mathematics (4.MP.4)

Materials

Copies	Kit Materials	Classroom Materials				
Assessment Equivalent Fraction	ns					
TM T1 Equivalent Fractions Checkpoint		student-made fraction kits egg carton fraction materials				
Problems & Investigations Ex	ploring Fractions on the Geoboa	rd				
TM T2 Geoboard Area of One	geoboards with geobands (class set, plus 1 for display)	student math journals chart paper				
Daily Practice						
SB 97 Pizza Party Planning						

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master Copy instructions are located at the top of each teacher master.

Preparation

On a piece of chart paper titled "Geoboard Regions," outline five columns titled "Region A, Region B, Region C, Region D, Region E." See step 9 in the lesson for an example.

Unit 3 Module 2 Session 1

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

area* region unit*



Assessment

Equivalent Fractions Checkpoint

- 1 Introduce today's activities.
 - Let students know that they will take a quick assessment to show what they've learned about equivalent fractions.
 - After that, they will investigate fractions using a new model—the geoboard.
- 2 Display the Equivalent Fraction Checkpoint and give students a minute to look it over and ask any questions. Then have them start work.
 - Let students know that they can use the egg carton fraction materials and/or the fraction kits they made a few days ago during the assessment, and tell them how to access these materials.
 - Encourage students to read each question carefully and remind them they can ask you
 for help reading any of the questions.
 - While students work, walk around the room to make observations and answer questions.
 - Give students 10–15 minutes to do the checkpoint. As this is not a timed test, if you have students who do not finish the checkpoint in the time allotted, give them a chance to finish later on.
- 3 Collect students' checkpoints.
 - See the Grade 4 Assessment Guide for scoring and intervention suggestions.

Q

Problems & Investigations

Exploring Fractions on the Geoboard

- 4 Explain that today students will investigate fractions with the geoboard.
- Display the Geoboard Area of One Teacher Master and invite students to think quietly about, then share in pairs their observations about the six regions shown. After pairs have had a minute to discuss, ask volunteers to share with the class.

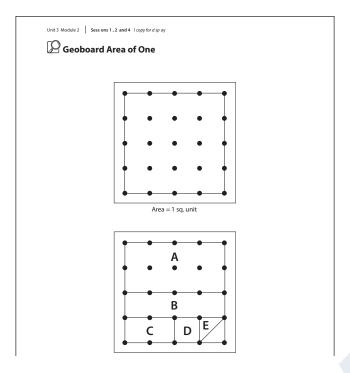
Tell students that for today's session the largest square on the geoboard has an area of 1 unit.

Students The parts are all different sizes.

The parts go from bigger at the top to smaller at the bottom.

You could fit some of the smaller ones into the bigger ones.

A is the biggest and E is the smallest.



- Give each student a geoboard and bag of geobands. Have students use the geobands to divide their geoboards into the six regions shown on the display. Students will use the letter names when referring to regions, but they will not write the names on their geoboards.
- Ask students to think quietly, then talk in pairs, about the area of region A. Listen for evidence that students relate the part (the region) to the whole (the large square).

Teacher Let's take a look at the region labeled with the letter A.

What is the area of region A?

Freddie Region A has an area of 8.

Teacher What makes you say that?

Freddie Because there are 8 little squares inside it, and each square is worth one... oh, wait... you said the whole thing has an area of 1! Let me think. That means A is worth ½.

Teacher I like the way you corrected your own thinking. What makes you think it's now worth ½?

Freddie Because two A's will cover the whole thing. And there are 16 little squares inside of the whole big square and A covers 8, or half of them.

Teacher What other fraction name is that, besides ½?

Monica Oh—%16. That's why Freddie said 8 earlier. It's 8 out of 16 little squares.

Teacher I am going to add what you two said to a chart up here on the board. I am also going to record how A relates to the whole large square.

8 Record students' thinking about region A on the chart you prepared earlier.

Ask students to consider the area of region B. Give them a few moments to think privately, and then invite students to share their thoughts while you record them on the chart.

Dario B is smaller than A. You can fit four of them on the large square, so that's ¼.

Lola I'm not sure how you got that. I'm pretty sure it's 4 out of 16.

Teacher Can anyone help make things clearer?

Ethan I think they're both right. It's $\frac{1}{4}$ and $\frac{4}{4}$ out of 16. B is $\frac{1}{4}$ of the big square because you can fit $\frac{4}{4}$ of them inside it. But if you look at the 16 little squares, $\frac{4}{16}$ of them are inside the B part, so that's $\frac{4}{16}$ out of 16. One is $\frac{1}{16}$, so B has $\frac{1}{16}$, $\frac{2}{16}$, $\frac{4}{16}$.

Dario We're both right, because ¼ and ¾16 are just different ways of looking at it!

Teacher On our chart, I am going to write both $\frac{1}{4}$ and $\frac{4}{16}$. I am also going to write $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$ and $4 \times \frac{1}{16}$ to model what Ethan said while he was counting them for us.

4	Region A	Region B	Region C	Region D	Region E
	\bullet $\frac{1}{2}$ of a large square	•			
	• 8 out of 16 little squares, or $\frac{8}{16}$	• 4 out of 16 little squares, or $\frac{4}{16}$			
	$ \bullet \frac{8}{16} + \frac{8}{16} = \frac{16}{16} = 1 $	$\bullet \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16}$			
	• $2 \times \frac{8}{16} = \frac{16}{16} = 1$	$\bullet \ \ + \ \times \frac{1}{16} = \frac{4}{16}$	6		

- 10 Repeat step 9 for regions C, D, and E.
- 11 Then, tell students they are going to examine the regions' relationships to one another. Model for students by thinking aloud about the relationship between regions A and B.

Teacher We've listed fractional names for each region in relation to the whole geoboard, but now we're going to shift our thinking a bit. Let's consider instead how the regions relate to each other. For instance, when I look at regions A and B, I could say that B is half of A. Who thinks they can explain what I mean?

Chin B is worth 4 little square and A is worth 8 little squares, and 4 is half of 8.

Teacher So, how could I describe that in an equation?

Pilar You could write that $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. That would mean that 2 of the B pieces are worth the A piece.

Teacher I'll add that to the chart along with a sentence that says "B is half of A." I'm going to also add $2 \times \frac{1}{4} = \frac{1}{2}$.

Give students about 10 minutes to turn and talk with a partner about other relationships they can find between regions and record them in their journals.

Have students record the relationships in their journals with both words and equations.

SUPPORT/ELL Pair students with a classmate who can describe the relationships between regions. Suggest that students build models of the regions on geoboards to compare them. Use sentence frames for example, "The area of this region is ____".

- 13 Gather the class together to share students' discoveries.
 - Record the relationships on the chart with both words and equations.
 - Encourage students to refer to the whole when describing fraction names.

SUPPORT/ELL As students share, model the relationships on a geoboard and clarify that when we talk about regions, we are talking about the area of each region as a fraction of the whole geoboard.

	\nearrow				
4	Region A	Region B	Region C	Region D	Region E
	\bullet $\frac{1}{2}$ of a large square	\bullet $\frac{1}{4}$ of a large square	\bullet $\frac{1}{8}$ of a large square	\bullet $\frac{1}{16}$ of a large square	\bullet $\frac{1}{32}$ of a large square
	• 8 out of 16 little squares, or $\frac{8}{16}$	• 4 out of 16 little squares, or $\frac{4}{16}$	• 2 out of 16 little squares, or $\frac{2}{16}$	• I out of 16 little squares	$\bullet \frac{1}{2}$ of one of the little squares
	$\bullet \frac{8}{16} + \frac{8}{16} = \frac{16}{16} = 1$	$\bullet \ \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16}$	$\bullet \frac{1}{16} + \frac{1}{16} = \frac{2}{16}$	• region B equals 4 Ds $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{16} = \frac{1}{16}$	
	• $2 \times \frac{8}{16} = \frac{16}{16} = 1$		• $2 \times \frac{1}{16} = \frac{2}{16}$	• region C equals 2 Ds	
	• region A equals 4 Cs $\frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8}$		• region C is half of B $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$	$\frac{1}{16} + \frac{1}{16} = \frac{2}{16} = \frac{1}{8}$ $\bullet 6 \times \frac{1}{16} = $	
			• region C is twice D $\frac{1}{4} = 2 \times \frac{1}{8}$		
4					

If a relationship is stated that students cannot agree upon after brief discussion, record it on a sticky note and place it on the chart to revisit later in the module.

CHALLENGE Ask students to find ways to prove or disprove the statements on sticky notes. Have them model the relationships with labeled sketches, words, or numbers.

- 14 Close the session by inviting students to tell one observation shared today that was particularly interesting or confusing.
 - Note confusing statements to inform your work with students during the next session.
 - Save the Geoboard Area of One Teacher Master and chart for use in future sessions in this module.

Extension

The class will probably not have time during the session to explore all the possible relationships shown on the geoboard. Leave the chart hanging so students can continue to consider it, and place stacks of sticky notes nearby. When students have time, they can record their thoughts on sticky notes and attach them to the poster.



Daily Practice

The optional Pizza Party Planning Student Book page provides additional opportunities to apply the following skills:

- Use a visual model to explain why a fraction a/b is equivalent to a fraction $n \times a/n \times b$ (4.NF.1)
- Use a visual model to generate and recognize equivalent fractions (4.NF.1)
- Add fractions with like denominators (supports 4.NF)
- Solve story problems involving addition of fractions referring to the same whole and with like denominators (4.NF.3d)
- Express a measurement in a larger unit in terms of a smaller unit within the same system of measurement (e.g., convert from liters to milliliters) (4.MD.1)
- Multiply a 2-digit whole number by 10 and by 100 (supports 4.NBT)



Session 2

Last Equation Wins

Summary

Students play, first as a class and then in pairs, a game that provides practice with decomposing fractions represented on a geoboard and recording those decompositions with equations. Toward the end of the session, the teacher invites students to solve addition problems involving fractions with like denominators. Finally, the teacher introduces and assigns the Brownie Dessert Home Connection.

Skills & Concepts

- Write an equation showing a fraction a/b as the sum of a number of the unit fraction 1/b (4.NF.3)
- Express a fraction as the sum of other fractions with the same denominator in more than one way and write equations to show those decompositions (4.NF.3b)
- Add fractions with like denominators (supports 4.NF)
- Demonstrate an understanding that a fraction a/b is the product of $a \times 1/b$ (4.NF.4a)
- Solve problems involving the addition of fractions with like denominators (4.NF.3d)
- Make sense of problems and persevere in solving them (4.MP.1)
- Look for and express regularity in repeated reasoning (4.MP.8)

Materials

Copies	Kit Materials	Classroom Materials						
Problems & Investigations Last Equation Wins								
TM T3 Geoboards SB 98-99* Last Equation Wins	spinner overlays (1 per student pair, plus 1 for display) geoboards and bands (1 per student, plus 1 for display)	Geoboard Area of One (from Session 1) Geoboard Regions chart (from Session 1)						
Home Connection								
HC 55-56 Brownie Dessert								
Daily Practice								
SB 100 What's the Share?								

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master Copy instructions are located at the top of each teacher master.

Preparation

When it comes time for students to play Last Equation Wins in pairs, you'll want partners to be working at roughly the same level of comfort with fractions. Spend some time before this session deciding how you will pair students.

Unit 3 Module 2 Session 2

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

denominator*
equivalent fractions*
numerator*
product*
sum or total*

^{*} Run 1 copy of these pages for display.

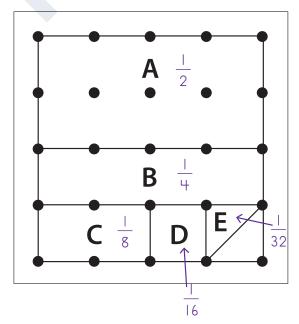


Problems & Investigations

Last Equation Wins

- Review the Geoboard Area of One Teacher Master and the Geoboard Regions chart from the previous session.
 - Ask students to review what fraction of the geoboard is represented by each region and label the teacher master if you haven't already.
 - Invite them to refer to the Geoboard Regions chart they created in the previous session.
 - Briefly review some of the addition and multiplication equations they wrote to represent each region as the sum or product of other numbers.

	\nearrow				
4	Region A	Region B	Region C	Region D	Region E
	• $\frac{1}{2}$ of a large square	• ¹ / ₄ of a large square	\bullet $\frac{1}{8}$ of a large square	• 1/16 of a large square	\bullet $\frac{1}{32}$ of a large square
	• 8 out of 16 little squares, or 8 16	• 4 out of 16 little squares, or $\frac{4}{14}$	• 2 out of 16 little squares, or $\frac{2}{16}$	• I out of 16 little squares	$\bullet \frac{1}{2}$ of one of the little squares
		16	$\bullet \frac{1}{16} + \frac{1}{16} = \frac{2}{16}$	• region B equals 4 Ds	• 4 Es makes 1 C
	• $2 \times \frac{8}{16} = \frac{16}{16} = 1$	$\bullet \ \ + \ \times \frac{1}{16} = \frac{4}{16}$	• $2 \times \frac{1}{16} = \frac{2}{16}$	$\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$ • region C equals 2 Ds	
	• region A equals 4 Cs $\frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8}$	• region B equals 8 Es	• region C is half of B $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$		02 X 32 T
		1 32	• region C is twice D	$\bullet \ 6 \times \frac{1}{16} = \ $	
			$\frac{1}{4} = 2 \times \frac{1}{8}$		

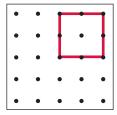


- 2 Display your copy of the Last Equation Wins Student Book page, and explain that today students are going to play a game that will give them more practice thinking about different ways to write equations for fractions.
- 3 Briefly explain how the game is played.
 - Players take turns spinning the spinner to see who goes first.
 - The first player spins the spinner two times and records the number from each spin. Players work together to record a fraction in which the smaller number is the numerator and the larger number is the denominator.
 - The first player records the fraction below the geoboard for Round 1 and then draws the fraction on the geoboard, confirming with the other player that this is indeed a representation of the fraction.
 - Players take turns writing unique equations for the fraction.
 - » Each equation must show the fraction as either the sum of other fractions with the same denominator or as the product of a whole number and a fraction with the same denominator.
 - » Equations with the same numbers, but in a different order, are considered the same because of the commutative property of addition and multiplication.
 - The partner who is able to write the last unique equation wins.
 - » If both partners have written five equations, players can choose to call it a draw and spin for a new fraction, or they can continue writing equations as long as it interests them.
- 4 Give each student a geoboard and a set of geobands. Begin the game by spinning for a fraction and then recording and modeling it on the geoboard, with input from students.

There are a variety of ways to model each fraction on the geoboard, with the exception of those fractions that are equal to 1. Invite students to work in pairs to find a few ways to model the fraction on their geoboards, and settle on the model that seems like it will be most helpful as students decompose the fraction. Remind students that they can look at the Geoboard Area of One Teacher Master from the previous session for ideas, but that they are not confined to those representations.

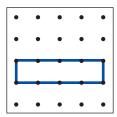
Teacher How did you show $\frac{4}{16}$? Jayden, can you bring your geoboard up and show us what you did?

Jayden I looked at the picture from yesterday and I remembered that the part D is $\frac{1}{16}$ of the whole. There are 16 of those little squares on the geoboard, so each one is $\frac{1}{16}$ of the whole. So I outlined 4 squares on my board like this.



Teacher Did someone do it differently?

Mei I looked at the chart we made yesterday and I saw that we talked about how region B was 1/16 because it's 4 of the little squares, so I made a picture of region B on my geoboard.



Rosa Wait, I'm confused. We just said today that region B was \(\frac{1}{4} \).

Mei Well, it's both, remember?

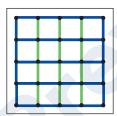
Rosa I don't get how it can be both. It's just one fraction.

Teacher Let's pause a moment. I'd like everyone to use their geoboards and talk in pairs. Can this part of the geoboard be both ¼ and ⅙6? How can you tell? You might divide one partner's geoboard into fourths and the other's into sixteenths and put one on top of the other and talk about what you see.... Who can come tell us what you figured out?

Dominick We just made region B on our boards, and we counted that there were 4 of the little squares inside it. So we were sure that it can be ¼ and ½ at the same time. There are ½ inside ½.

Rosa We did what you said. I divided my board up into the 16 small squares and my partner divided her board up into 4 rectangles that look like region B.

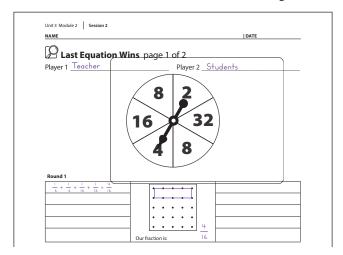
Mei Then we put mine on hers, and look... If you look at one of the fourths, you can see it's the same as 4 of the sixteenths.



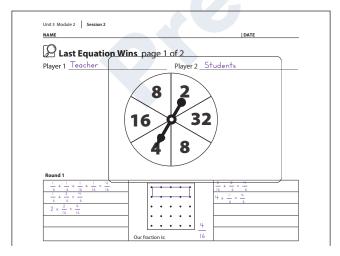
Rosa Wow, so it's the same part of the whole, but you can call it different things.

Teacher We call \$\frac{4}{16}\$ and \$\frac{1}{4}\$ equivalent fractions. That means that they name the same fraction of the whole. \$\frac{4}{16}\$ of the geoboard is equal to \$\frac{1}{4}\$ of the geoboard.

Write the first equation, and ask students to use their geoboards to prove to each other that the equation is true. Remember that the fractions in the equation must have the same denominator as the original fraction.



- When it is the students' turn, give them time to think silently and then talk in pairs about some equations they could write for this fraction. Ask them to use their geoboards to prove their thinking to each other.
- 7 Call on a student to write an equation on the students' side.
- 8 Then ask pairs of students to use their geoboards to make certain that the equation makes sense.
- 9 Continue taking turns until either you or the students cannot write another equation, or you have written five equations each.





Games give students a wonderful opportunity to make sense of problems and persevere in solving them. As they persevere to write as many equations as possible, students are deepening their understanding and developing skills that will help them compute fluently with fractions.

10 Teacher Can you think of any other addition or multiplication equations to write for this fraction that use only sixteenths?

Ramona No way. We got all the addition and all the multiplication.

Holly What about $\frac{3}{16} + \frac{1}{16}$?

André That's the same as $\frac{1}{16} + \frac{3}{16}$. I think we lost this round!

At this point, you can either continue to play as a class or have students play in pairs. You might also have most students work in pairs while you play as a small group with students who seem to need more support.

- Have students locate the Last Equation Wins Student Book page in their books.
- Point out that there is a second page, allowing them to play 6 rounds if they have enough time.
- Explain that in this game, the players share a record sheet, as you have just done with the class, so one of the students in each pair can put their book away for now.
- Let students know that if they would prefer to draw rather than build on the geoboards
 they can use the Geoboards Teacher Master. Place the copies in a central location
 where students can get them if they like.

SUPPORT. Work in a small group with students who seem to need extra support. Instead of spinning for each fraction, present an accessible fraction (e.g. 4/8) to the group and focus on adding and multiplying with the unit fraction (in this example, the unit fraction would be 1/8).

CHALLENGE You can make this game more challenging by modifying the rules in the following ways. An example is provided with each suggested modification.

- » Try to be the first partner to write four equations, each of which uses a different basic operation. ($\frac{1}{16} + \frac{3}{16} = \frac{4}{16}$, $\frac{8}{16} \frac{4}{16} = \frac{4}{16}$, $4 \times \frac{1}{16} = \frac{4}{16}$, $\frac{8}{16} \div 2 = \frac{4}{16}$)
- » Write equations that use fractions with different denominators. ($\frac{1}{8} + \frac{2}{16} = \frac{4}{16}$)
- » Write equations that show equivalent fractions. $(\frac{4}{16} = \frac{1}{4})$
- » Write equations without referring to the geoboard model.

CHALLENGE You can provide more of a challenge to students by asking them to generate a list of all of the possible fractions they could get using two spins on this spinner and then considering the following the questions. (You might suggest that students create a table with spin 1 represented along the side, spin 2 represented along the top, and all the resulting fractions on the grid, as shown.)

	Spin 2									
		2	4	ŀ	8		8	1	6	32
	2	2/2	2/	4	2/8		2/8	2/	, 16	2/32
_	4	2/4	4/	4	4/8	(4/8	4/	, 16	4/32
Spin	8	2/8	4/	8	8/8		8/8	8/	, 16	8/32
0,	8	2/8	4/	8	8/8	>	8/8	8/	, 16	8/32
	16	² /16	4/-	16	8/16		8/16	¹⁶ / ₁₆		16/32
	32	2/32	4/	32	8/32		8/32	16,	⁄ ₃₂	32/32
ed	equal to 1 equal to ½ equal to ¼					equal to	1/8	equ	ual to 1/16	

- » What fractions come up most frequently?
- » What fractions come up least frequently?
- » Which of those fractions are equal to 1? (to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$)
- » What do you notice about each collection of equivalent fractions? Do you notice any patterns?
- » Can you add to each collection of equivalent fractions, using fractions with other denominators? How did you know what fractions to write?
- » If we had two 4s on the spinner instead of two 8s, how would the results be different?

12 As students play, clarify the following points as needed.

- All fractions in the equations should have the same denominator as the original fraction.
- Students can use addition or multiplication in their equations.

- If students get a fraction that is identical or equivalent to one they have already worked with, they can spin for a new fraction. (If students don't recognize the fraction as equivalent to one they have worked with previously, it is probably a good use of their time to continue working with that fraction.)
- If each partner has written 5 equations, they can continue writing equations for the fraction if they're interested, or they can call the round a draw and move on to a new round.
- Students should prove their thinking to each other using either the geoboard itself or copies of the Geoboards Teacher Master.
- When there are about 10 minutes left in your math period, ask students to finish writing the equations they are working on and put away their Student Books, while you pass out a copy of the Geoboards Teacher Master for each student.
- 14 Spend the rest of the session exploring addition of fractions with like denominators.
 - Explain that you saw someone write the following expression while playing the game:
 ½8 + ½8 + ½8
 - Ask students to write the expression on a copy of the Geoboards Teacher Master and determine what fraction the student was playing for.
 - Ask them to use the geoboards on the teacher master as needed and to write their final answer as an equation.
- Repeat this exercise as time allows, inviting students to work in pairs first and then share as a group. Select from the following examples, or make up your own, based on students' needs and comfort level.

$$1/16 + 1/16 + 1/16 + 1/16$$
 $3/16 + 1/16$
 $4 \times 1/16$
 $4 \times 1/8$
 $1/8 + 1/8 + 1/8 + 1/8$

CHALLENGE Invite students who complete each problem quickly to write additional equations for the fraction, using fractions with different denominators and any of the four basic operations. For example, for 4/8, students might write the following kinds of equations:

$$\frac{1}{2} = \frac{4}{8}$$
 $\frac{1}{4} + \frac{1}{4} = \frac{4}{8}$
 $(2 \times \frac{1}{8}) + (2 \times \frac{1}{8}) = \frac{4}{8}$
 $4 \div 8 = \frac{4}{8}$
 $\frac{8}{8} \div 2 = \frac{4}{8}$
 $\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$

16 Close the session by letting students know that tomorrow they will work more with adding, as well as subtracting, fractions. Collect students' papers if you want to review them prior to Session 3.



Home Connection

- Introduce and assign the Brownie Dessert Home Connection, which provides more practice with the following skills:
 - Use a visual model to explain why a fraction a/b is equivalent to a fraction $n \times a/n \times b$ (4.NF.1)
 - Use a visual model to generate and recognize equivalent fractions (4.NF.1)Add fractions with like denominators (supports 4.NF)
 - Solve story problems involving addition of fractions referring to the same whole and with like denominators (4.NF.3d)



Daily Practice

The optional What's the Share? Student Book page provides additional opportunities to apply the following skills:

- Find all factor pairs for a whole number between 1 and 100 (4.OA.4)
- Determine whether a whole number between 1 and 100 is prime or composite (4.OA.4)
- Use a visual model to explain why a fraction a/b is equivalent to a fraction $n \times a/n \times b$ (4.NF.1)
- Recognize equivalent fractions (4.NF.1)
- Use the symbols >, =, and < to record comparisons of two fractions with different numerators and different denominators (4.NF.2)
- Write an equation to show a fraction as the sum of other fractions with the same denominator (4.NF.3b)

Session 3

Comparing, Adding & Subtracting Fractions

Summary

Students work together as a class to create a chart of equivalent fractions for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. Students make observations about the chart and the equivalent fractions, and then they use those fractions as benchmarks with which to make comparisons among fractions with like and unlike denominators. Finally, they practice adding and subtracting fractions with like denominators.

Skills & Concepts

- Use a visual model to explain why a fraction a/b is equivalent to a fraction $n \times a/n \times b$ (4.NF.1)
- · Recognize equivalent fractions (4.NF.1)
- Compare two fractions with different numerators and different denominators and explain why one must be greater than or less than the other (4.NF.2)
- Explain subtraction of fractions as separating parts referring to the same whole (4.NF.3a)
- Add and subtract fractions and mixed numbers with like denominators (4.NF.3c)
- Model with mathematics (4.MP.4)
- Look for and express regularity in repeated reasoning (4.MP.8)

Materials

Copies	Kit Materials	Classroom Materials				
Problems & Investigations Comparing, Adding & Subtracting Fractions						
TM T3 Geoboards SB 101–102* Comparing, Adding & Subtracting Fractions	geoboards and bands (class set, plus 1 for display)	chart paper (see Preparation) glue or tape scissors				
Daily Practice						
SB 103 Adding & Subtracting Fractions						

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master Copy instructions are located at the top of each teacher master.

Preparation

Using a copy of the Geoboards Teacher Master, draw ½, ¼, and ¾, each on a separate geoboard and cut them out. Divide a piece of chart paper into 4 columns and 4 rows, and label them as shown.

Halves	Fourths	Eighths	Sixteenths
naives	Fourins	Eignins	Sixteenins
1/2			
	3/4		

Unit 3 Module 2 Session 3

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

denominator*
equivalent fractions*
numerator*

^{*} Run 1 copy of these pages for display.



Problems & Investigations

Comparing, Adding & Subtracting Fractions

- Begin the session by calling students' attention to the chart you prepared. Give them some time to think quietly about what they notice, and then have them share observations and ideas, first in pairs and then as a group.
- Explain that today they will think of different names for each of these fractions and then use that information to compare other fractions.
- 3 Begin by generating two equivalent fractions for ¼, one in eighths and another in sixteenths, together as a class. For each equivalent fraction, draw a new geoboard model together with the class and then have a student volunteer cut out the model and glue or tape it to the chart in the column where it belongs (see graphic).

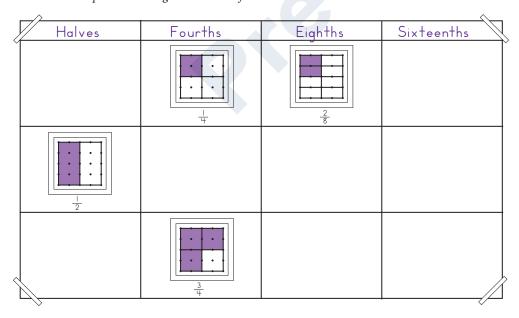
Teacher Let's start by thinking of some fractions that are equal to ¼. Does anyone have another fraction name for this part of the geoboard?

Darnell You could also call it 2/8.

Teacher Can you come on up and draw \(^2\)% on a geoboard to prove that % is equal to 4? I'd like you to first show the whole divided into eighths, and then color in 2/8.

Darnell OK, here are 8 equal parts. Each of these little rectangles is an eighth. So I'll just color in 2 of them.

Teacher Thanks. Could you please label that 2/8 and cut it out and put it in the eighths column of the ¼ row?



- Complete the chart by drawing and labeling the missing fractions for each row.
 - Give students time to work in pairs with copies of the Geoboards Teacher Master to draw and label equivalent fractions for ½ and ¾ (and ¼ if you did not complete the row together as a class).

- As you circulate around the room, you might find it useful and even necessary to invite pairs to focus on specific fractions. For example, you might say to a pair of students, "I haven't seen anyone show ¾ using sixteenths. Can you find a way to do it?"
- Reconvene and ask students to volunteer their equivalent fractions, working in one row at a time and placing the fractions in each row in order by denominator.
- 5 When the chart is complete, ask students to think silently about what they notice about the fractions in each row and column. Do they see any patterns? Students might notice the following patterns:
 - For all equivalent fractions, the numerators and denominators double as you move from left to right across each row.
 - In each column, the denominators stay the same, but the numerators change. The numerators change by a constant amount.

	_^			
/	Halves	Fourths	Eighths	Sixteenths
~			2 8	4 16
	1/2	2 4	ψ ₈	8 16
<		3/4	68	12/16



in Action 4.MP.8 Asking students to

describe patterns they notice on the chart invites them to look for and express regularity in repeated reasoning. In their search for regularity, students will make observations related to equivalent fractions and adding fractions with like denominators.

Shawn I noticed that the numbers get bigger as you go across the row.

Teacher Hmm, I thought that all the fractions in each row were equal. Can you point to what you're seeing and tell us about it?

Shawn Well, yeah, the fractions are the same size. But look. The numerators and denominators get bigger and bigger. So you start here with 1 over 4. And then 2 over 8, and 4 over 16.

Veronica Hey! They're doubling.

Shawn What do you mean?

Veronica Here. So 1 over 4 and then 2 over 8. You double the 1 to get 2 and then the 4 to get 8.

Shawn Oh yeah, it does that to get to 4 over 8 too. So 2 doubled is 4 and 8 doubled is 16.

Teacher Let's pause for a moment. Turn to your partner and see if you can explain to each other what's going on. Take a look at the other rows on our chart as well, and see if you notice anything else. What if you skip over a column, like to go from ¼ to 4/16? What's happening there?

6 After spending some time discussing the patterns students notice on the chart, ask them to turn to the Comparing, Adding & Subtracting Fractions Student Book pages.

- 7 Review the problems on the page, do one or two of the comparison problems together as a class, and then give students about 20 minutes to work independently or in pairs on the pages.
 - While students work, circulate around the room and make note of which problems are most interesting and challenging for your students. Plan to revisit them as a group at the end of this session.
 - Encourage students to refer to the chart you created at the beginning of the session as needed to make comparisons.
 - Encourage students to use copies of the Geoboards Teacher Master to solve the addition and subtraction problems.

SUPPORT Gather a small group of students who are struggling to work with you.

CHALLENGE Invite students to work on their own and solve as many of the challenge problems on the second page as they're able to in the time available.

- 8 Reconvene the entire class when you have about 15 minutes left in the period. Spend the time talking about some of the more challenging problems together as a group.
 - When discussing comparison problems, be sure to have students share strategies that involve using comparisons to the landmarks 14, 1/2, and 3/4, as well as strategies that involve thinking in terms of a common denominator. For example, a student might conclude that ¼ is less than % because % is clearly more than ½ (4%), and ¼ is less than ½. Another student, however, might see that 1/4 is equal to 2/8 and must, therefore, be less than 5/8.
 - When discussing comparisons, be sure students understand that they are not comparing numerators with numerators and denominators with denominators, but two entire fractions that both refer to the same whole. For example, ¾ is greater than ¾, even though 5 is greater than 3 and 8 is greater than 4. Note with students that the comparisons they're making are valid only when each fraction refers to the same whole.
 - Be sure to have students explore an addition problem and a subtraction problem that involve mixed numbers.
 - Be sure to have students model the addition and subtraction problems on geoboards or the Geoboards Teacher Master to demonstrate why, when the two fractions share a common denominator, students can simply find the sum or difference of the numerators and keep the denominator the same to find the sum or difference of the two fractions (e.g., $\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$ and $\frac{6}{8} - \frac{2}{8} = \frac{4}{8}$).



Daily Practice

The optional Adding & Subtracting Fractions Student Book page provides additional opportunities to apply the following skill:

• Solve story problems involving addition and subtraction of fractions referring to the same whole and with like denominators (4.NF.3d)

Session 4

Dozens of Eggs

Summary

Today, students add fractions using the egg carton model. The teacher then introduces a related Work Place game by playing it with the class. Students spend any time remaining in the session visiting Work Places, including the new one. At the end of the session, the teacher introduces and assigns the Planning a Garden Home Connection.

Skills & Concepts

- Use a visual model to explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ (4.NF.1)
- Use a visual model to generate and recognize equivalent fractions (4.NF.1)
- Explain addition of fractions as joining parts referring to the same whole (4.NF.3a)
- Express a fraction as the sum of other fractions with the same denominator in more than one way (4.NF.3b)
- · Write an equation to show a fraction as the sum of other fractions with the same denominator (4.NF.3b)
- Model with mathematics (4.MP.4)
- Look for and make use of structure (4.MP.7)

Materials

Copies	Kit Materials	Classroom Materials			
Problems & Investigations Combining Egg Carton Fractions					
TM T4 Egg Carton Diagram	colored tiles (12 per student)	heavy string or yarn, 10" lengths, 6 pieces per student			
Work Places Introducing Work Place 3A Dozens of Eggs					
TM T5 Unit 3 Work Place Log TM T6 Work Place Guide 3A Dozens of Eggs TM T7 3A Dozens of Eggs Record Sheet SB 104* Introducing Dozens of Eggs SB 105–106** Work Place Instructions 3A Dozens of Eggs	Dozens of Eggs Fraction Cards (1 deck) 12 colored tiles 6 base ten linear pieces game markers	students' Work Place folders (see Preparation) colored pencils in several different colors (class set)			
Work Places in Use					
 2A What's Missing? Bingo (introduced in Unit 2, Module 1, Session 4) 2B Division Capture (introduced in Unit 2, Module 2, Session 1) 2C Moolah on My Mind (introduced in Unit 2, Module 3, Session 4) 2D Remainders Win (introduced in Unit 2, Module 4, Session 3) 2E More or Less Multiplication (introduced in Unit 2, Module 4, Session 4) 3A Dozens of Eggs (introduced in this session) 					
Home Connection					
HC 57-58 Planning a Garden					
Daily Practice					
SB 107 Egg Carton Fractions					

HC - Home Connection, SB - Student Book, TM - Teacher Master

Copy instructions are located at the top of each teacher master.

* Run 1 copy of the page for display.

21

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

denominator* equal* equation* equivalent fractions* numerator* sum or total* twelfths

^{**} Run 1 copy of this page and store it for use by the teacher and other adult helpers during Work Place time.

Preparation

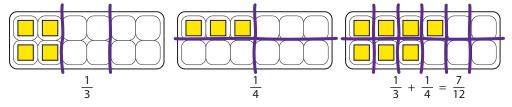
- Remove the Unit 1 Work Place Log from the front of each student's Work Place folder, and
 replace it with a copy of the Unit 3 Work Place Log, stapled at all four corners. Leave the
 Unit 2 Work Place Log stapled to the back of each folder. This will allow students to keep
 track of the number of times they have visited the Unit 2 Work Places that will remain in
 use during Unit 3, and also track their progress through the new Work Places as they're
 introduced, starting today.
- In today's session, you'll introduce Work Place 3A Dozens of Eggs. Before this session, you should review the Work Place Guide, as well as the Work Place Instructions. Run a class set of the Dozens of Eggs Record Sheet and store the copies in the Work Place 3A Dozens of Eggs tray, along with the 4 decks of Dozens of Eggs Fraction Cards you'll find in your Bridges Kit. This activity replaces Work Place 1F, to keep the total number of Work Places steady at six.
- Write a list of Work Places from which students can choose today. You can just write the numbers (2A–3A) or write out the full names if you prefer. (See the Work Places in Use row of the Materials Chart for the complete list of Work Places in use today.)



Problems & Investigations

Combining Egg Carton Fractions

- Open the session by letting students know they are going do some more thinking about adding fractions, learn a new Work Place game, and then visit Work Places in pairs.
- 2 Distribute the materials students will need for the first part of the session and have them pair up.
 - Give each student a copy of the Egg Carton Diagram Teacher Master, 12 colored tiles, and 6 pieces of heavy string or yarn.
 - Have the students pair up so they have 2 copies of the Egg Carton Diagram with which to work.
- 3 Pose the first problem and have students work with their partners to solve it.
 - Write $\frac{1}{3}$ and $\frac{1}{4}$ on the board, and ask each pair to show $\frac{1}{3}$ on one of the egg cartons and $\frac{1}{4}$ on the other.
 - Then ask pairs to combine the tiles from both sheets onto one.
 - Ask them to share observations about the results and suggest ways to name this fraction.



Students It's like half a carton and one more.

We got an odd number of eggs. It's a hard one.

It's 7 out of 12, so it must be seven-twelfths.

Ask students to share observations or questions about what happened when they added ¼ and ¼.

If students do not mention that the problem began with thirds and fourths and ended up with twelfths, make a comment to invite further discourse and reflection.

Teacher How did we start with a third here and a fourth here and end up with twelfths over here? Does that seem right? Talk with the person next to you about why we ended up with twelfths.

Ebony We got 7 eggs in all. The only way you can even say a fraction for 7 eggs is 7/12 because you can't make that number into thirds, or fourths, or even sixths.

Armando I think we got twelfths because we added a fourth and a third together. If we just added a fourth plus a fourth, or a third plus a third, it would be easy.

Bobbie I think it's funny because 3 and 4 both go into 12.

5 After some discussion have student pairs combine the fractional amounts shown below, one combination at a time.

Each time, ask students to talk first in pairs and then as a group about the methods for building and combining the two fractions, as well as their observations about the results. Have them express their totals in twelfths and any other fraction names that make sense to them.

> % and ¾ [11/12] 3/12 and 3/6 [9/12 or 3/4] 1/3 and 1/2 [11/12 or 5/6]



Work Places

Introducing Work Place 3A Dozens of Eggs

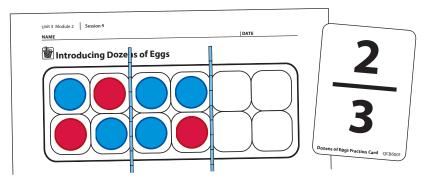
- Introduce the game Dozens of Eggs.
 - Display your copy of Introducing Dozens of Eggs Student Book page, and show students a few of the Dozens of Eggs Fraction Cards.
 - Explain that the game will help them practice combining fractions.
 - Have students find the Introducing Dozens of Eggs Student Book page in their books, and get out their colored pencils in preparation for learning this new game.
 - Let the students know that they will record the results for both teams—you and the class—during today's demonstration game.
- 7 Briefly summarize the game before playing it with the class.

Players take turns drawing from the deck of fraction cards, modeling the designated fraction on an Egg Carton Diagram with colored tiles and string, and recording the results. Players take turns until one person has filled all four egg cartons on his or her record sheet and written a matching addition equation that equals 1 whole for each carton. The first player to fill all four egg cartons wins.

- 8 Shuffle the deck of fraction cards, place them in a stack face-down near your display area, and begin the game by taking the first turn.
 - Draw a card from the deck and read it aloud. Then use the plastic game markers and base ten linear pieces to build a model of the fraction in the large egg carton diagram at the top of the sheet.



The Assessment Guide includes a Work Places Differentiation Chart for each unit. If you like, you can use these charts to make notes about which students need support or challenge with the skills featured in each Work Place. Suggestions for differentiating a particular Work Place activity are included on the Work Place Guide.



- Have students talk with their partners to see if they agree that you have modeled the fraction accurately, and invite two or three volunteers to explain how they know.
- When there is general agreement, use a single color to sketch the eggs (but not the subdivision lines) in one of your smaller egg carton diagrams on the lower part of the sheet, and have the students do the same on their sheets.
- Work with student input to record the results of your first turn in twelfths.

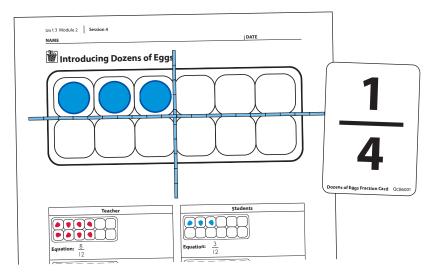
Teacher In this game, we're going to use a different color to record the eggs we get on each turn. We're also going to write all the fractions we get as twelfths so we can add them more easily. Talk with the person next to you about this. How would we write ½ as a fraction that has 12 in the denominator?

Monica I think it's %12 because each egg is like a twelfth.

Abe If you use more of the divider line things, you can put the carton into twelfths. Can I show?

Carlos Another thing I noticed is that if you double both of the numbers on $\frac{2}{3}$, it's $\frac{4}{6}$, and then double that, and you get $\frac{8}{12}$.

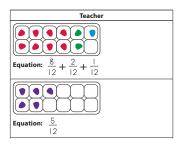
- 9 Now clear away the game markers and linear pieces and have students take their turn.
 - Call up a volunteer to draw a card for the class and build the designated fraction at the top of the sheet.
 - When the rest of the students agree that the fraction has been built accurately, have them sketch the eggs in one of the cartons on their side of the sheet and record their results in twelfths.

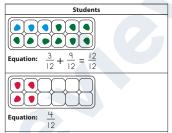


10 Continue to take turns with the students until one team has filled in all four egg cartons.

Refer to the Work Place Instructions 3A Dozens of Eggs Student Book page as needed.

- Sketch the results of each turn within a single egg carton diagram using a different color and have students do the same on their sheets. Work with the class to write each fraction as a number of twelfths.
- Make sure students understand that the fraction on the card they draw must be recorded in one egg carton and cannot be split into two cartons. For example, if you have a carton with 4 empty spaces and you draw a card that says 2/4, you cannot use 4 of the eggs in one carton and 2 in another. You can, however, start a new carton.
- If one team or the other draws a card for a fraction that cannot be placed in one of their 4 egg cartons, they lose that turn.
- 11 Pose questions like the following to promote discussion of fraction skills and concepts while you play.
 - » Can you tell how many twelfths there will be in the fraction on the card your classmate just drew for your team?
 - » How many more twelfths do you need to fill this egg carton? Is there a single fraction card you could draw that would give you that many twelfths? How do you know?
 - » Which team is ahead, and by how much?





Teacher Who's ahead?

Students We are, because we've already filled one whole egg carton.

We got really lucky on our second turn, when we got ¾. That was worth 9 twelfths!

Actually, I think we're tied because both teams have $\%_{12}$ left until they fill up 2 whole cartons. You still have 1/12 to fill on your first carton, and then 7/12 more to go on your second carton.

But it's our turn next, so I think we can get ahead.

Teacher Is there one fraction card you could draw that would make it possible for you to finish filling your second egg carton?

Willie Yeah! If we get %12!

Teacher I can tell you that there's no card in the deck for $%_{12}$, but there are a couple of other cards with fractions that are equivalent to $%_{12}$.

Ann What are they?

Teacher I'll give you a hint. One of them has a 6 in the denominator.

Roberta It's sixths....oh, I know! It must be 4% because that's the same as 8/12. I hope we get that card!

When the game is finished, ask students to turn to a partner and summarize the directions for Dozens of Eggs. Then let them know this game will be available during Work Places for the next several weeks.



Work Places

- If time allows, have students find a partner, get their Work Place folders and pencils, and choose one of the available Work Place games or activities.
 - Note with students that you have removed the Unit 1 Work Place Log from the front of their folders, and replaced it with a Work Place Log for Unit 3. Point out that the Unit 2 Work Place Log is still stapled to the back of their folders, and all the Work Places from the previous unit are available, along with the new game you introduced today.
- Close the session. 14

Have students clean up and put away materials.



Home Connection

- Introduce and assign the Planning a Garden Home Connection, which 15 provides more practice with the following skill:
 - Solve story problems involving addition and subtraction of fractions referring to the same whole and with like denominators (4.NF.3d)



Daily Practice

The optional Egg Carton Fractions Student Book page provides additional opportunities to apply the following skills:

- Use a visual model to demonstrate why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times a)$ $(n \times b)$ (4.NF.1)
- Use a visual model to generate equivalent fractions (4.NF.1)
- Use the symbols >, =, and < to record comparisons of two fractions with different numerators and different denominators (4.NF.2)

Session 5

How Many Candy Bars?

Module 2 Session 5

Unit 3

Summary

Mrs. Wiggens is bringing the dessert treats for the annual class picnic and needs help figuring out how many candy bars she'll have to buy if she gives each student ¾ of a bar. The trouble is, she's not quite sure how many of her fourth graders will be attending the picnic. Students add fractions on an open number line and track their results on a ratio table, working together to help Mrs. Wiggens initially, and then complete the work on their own or in pairs. As they finish the assignment, which will be revisited during a math forum next session, students go to Work Places.

Skills & Concepts

- · Convert a fraction to a mixed number (supports 4.NF)
- Explain addition of fractions as joining parts referring to the same whole (4.NF.3a)
- Add fractions and mixed numbers with like denominators (4.NF.3c)
- Solve story problems involving addition of fractions referring to the same whole and with like denominators (4.NF.3d)
- Make sense of problems and persevere in solving them (4.MP.1)
- · Look for and express regularity in repeated reasoning (4.MP.8)

Materials

Copies	Kit Materials	Classroom Materials			
Problems & Investigations How Many Candy Bars?					
SB 108* How Many Candy Bars? TM T8 How Many Candy Bars Forum Planner		student math journals space on the whiteboard or chart paper			
Work Places in Use					
2A What's Missing? Bingo (introduced in Unit 2, Module 1, Session 4) 2B Division Capture (introduced in Unit 2, Module 2, Session 1) 2C Moolah on My Mind (introduced in Unit 2, Module 3, Session 4) 2D Remainders Win (introduced in Unit 2, Module 4, Session 3) 2E More or Less Multiplication (introduced in Unit 2, Module 4, Session 4) 3A Dozens of Eggs (introduced Unit 2, Module 2, Session 4)					
Daily Practice					
SB 110 Fractions & Mixed Numbers					

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master Copy instructions are located at the top of each teacher master.

Preparation

- Write a list of Work Places from which students can choose today. You can just write the numbers (2A–3A) or write out the full names if you prefer. See the Work Places in Use row of the chart above for the complete list of Work Places in use today.
- Read Session 6 to see how students might share their work from today's session. Before tomorrow's forum, use the How Many Candy Bars Forum Planner to help select students to share their work.

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Problems & Investigations

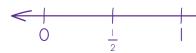
How Many Candy Bars?

- Set the stage for today's session.
 - Let students know that they are going to tackle a problem that involves adding fractions today. They will work together at first and then complete the work on their own or with a partner.
 - As they finish, they will go to Work Places.
 - Have students get out their math journals and Student Books. Ask them to find the next available page in their journal, label it with the date, and title it "How Many Candy Bars?"
- 2 Introduce today's problem by explaining that Mrs. Wiggens, a fourth grade teacher, hosts a class picnic every year. She usually brings the dessert, and has decided to give each student 34 of a candy bar this year.

Teacher Mrs. Wiggens wants to bring dessert treats for the class picnic. She has decided to buy candy bars and give each student 3/4 of a bar. Her students are checking with their parents to see if they can come. Meanwhile, since she isn't sure yet how many students are coming, Mrs. Wiggens wants to figure out the different amounts of candy bars she might have to buy depending how many students attend. She needs our help.

- Ask students to turn and talk to a partner about how Mrs. Wiggens could 3 find and keep track of the candy bar information.
- Display the How Many Candy Bars? Student Book page, and ask students 4 to find the page in their own books.
 - Introduce the chart on the page.
 - Note with the class that the chart includes space to enter information about the number of candy bars and the number of students.
 - Work on the first few entries together. Use an open number line to model the addition of each ¾ of a candy bar, and ask students to work along with you in their journals.

Teacher I'm going to draw an open number line to use in modeling this situation and I'd like you to do the same in your math journals. Let's start by marking and labeling 0, ½, and 1 on the line.



Teacher Where should we mark the line to show ¾ of a whole candy bar for 1 student?

Quinlan It's exactly halfway between 1/2 and 1, because one-half is the same as two-fourths, and then you need to add another fourth on.



Math Practices in Action 4.MP.1

By working together first, you provide scaffolding to help students begin to make sense of the problem and persevere in solving it. Students can continue to use the number line model if they like when they begin working independently. They can also choose to use models and strategies of their own.



Teacher So, how do we figure out how many candy bars Mrs. Wiggens would need for 2 students?

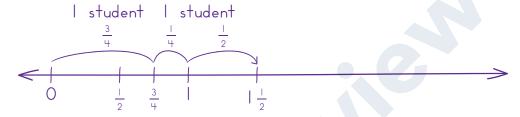
Students Add ¾ to that ¾.

Just take another hop of ¾ on the line.

Teacher All of you, please use your lines to model and solve $\frac{3}{4} + \frac{3}{4}$. (Waits for students to mark and label their number lines.) What did you get? How many candy bars for 2 students?

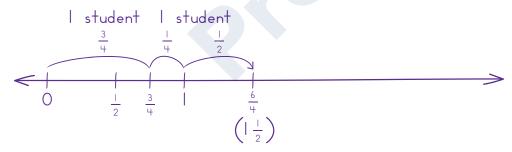
Students It's 1 1/2.

I already knew the answer in my head, but I found a good way to do it on the line. Can I show? You can add ¼ first, and that takes you to 1. Then, if you add another ½ to make ¾, it's 1 ½.



Teacher Did anyone figure this a different way?

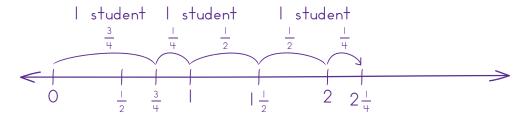
Lin I just took 2 hops of $\frac{3}{4}$. That made $\frac{9}{4}$ in all, and I know that's $1\frac{1}{2}$ because $\frac{4}{4}$ make 1, and then $\frac{2}{4}$ more makes another half.



Teacher Now how about for 3 students? How do we add another ¾? Everyone, please try that on your line, and share your strategy with the person sitting next to you.

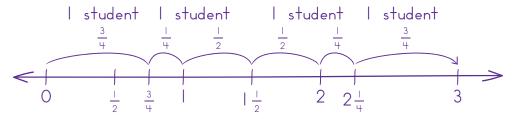
Trevor So, I was at $1\frac{1}{2}$. Then I jumped another half, and that got me to 2, and then I just added the last fourth to get up to $2\frac{1}{4}$.

Donna I got the same thing.

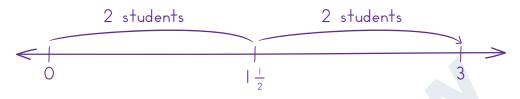


Teacher Let's talk about 4 students now. How many candy bars? Take a few moments to figure it out, and then let's hear some ideas.

Eduardo We were at $2\frac{1}{4}$, right? So I just added on another $\frac{3}{4}$, and that's 3 in all. It's 3 candy bars for 4 kids.



Cora I got 3, but I did it a different way. I knew that for 2 students it was $1\frac{1}{2}$ candy bars, so I just doubled that to get 3 candy bars for 4 students, like this.



- Have students record the first four answers on the chart in their Student Books if they haven't already, and then fill in the rest of the chart on their own or with a partner.
 - Explain that you'll revisit Mrs. Wiggens's situation as a class next session during math forum, but for now students need to complete the assignment on their own or with a partner.
 - Let students know that they can model the fraction addition on an open number line, but they don't have to if they have alternative strategies. In either case, they should continue to use the journal page they've labeled to do their figuring. (Note with them that they need to describe their strategies on the worksheet when they've completed the chart.)
 - Encourage students to search for patterns as they work.
- 6 Circulate to provide support and look for students to share their thinking in a math forum in the next session. As you circulate, use your copy of the How Many Candy Bars Forum Planner Teacher Master to make notes about the strategies students are using.

SUPPORT. If students struggle to repeatedly add 3/4, help them use a number line to model the addition.

CHALLENGE Encourage students to come up with a general rule for determining the number of candy bars needed for any number of fourth graders.

As students finish the assignment, have them meet with other classmates to share and compare their answers.

Encourage them to take responsibility for resolving any differences by re-examining the problem together.

Extension

Have students develop a chart for Mrs. Wiggens to use that shows the number of candy bars needed if each student gets ½ of a candy bar instead of ¾.



Work Places

- 8 When students have shared their work with at least one classmate, have them get their Work Place folders and choose a Work Place to use quietly. **SUPPORT** Suggest specific Work Places for struggling students to work on critical skills. **CHALLENGE** Encourage students to think about the strategies they use and share their thinking. Encourage students to generalize what happens in certain Work Places.
- 9 Close the session by reminding students that tomorrow you will discuss their solutions and strategies during a math forum.



Daily Practice

The optional Fractions & Mixed Numbers Student Book page provides additional opportunities to apply the following skills:

- Convert a mixed number to a fraction and vice versa (supports 4.NF)
- Use a visual model to generate equivalent fractions (4.NF.1)



Session 6

Racing Fractions

Summary

Students discuss solutions and strategies for the candy bar problem from the previous session in a math forum. During the discussion, the class talks about adding fractions to mixed numbers and multiplying a fraction by a whole number. Then the teacher introduces the equivalent fraction game, Racing Fractions, which will become a Work Place in later sessions. At the end of the session, the teacher introduces and assigns the Fractions & More Fractions Home Connection.

Skills & Concepts

- Use a visual model to generate and recognize equivalent fractions (4.NF.1)
- Explain addition of fractions as joining parts referring to the same whole (4.NF.3a)
- Express a fraction as the sum of other fractions with the same denominator in more than one way (4.NF.3b)
- · Add and subtract fractions and mixed numbers with like denominators (4.NF.3c)
- Demonstrate an understanding that a fraction a/b is a multiple of the unit fraction 1/b (4.NF.4a)
- Write an equation showing that a fraction a/b is the product of $a \times 1/b$ (4.NF.4a)
- Multiply a fraction by a whole number (4.NF.3b)
- Demonstrate an understanding that any multiple of a/b is also a multiple of the unit fraction 1/b (4.NF.3b)
- Make sense of problems and persevere in solving them (4.MP.1)
- Construct viable arguments and critique the reasoning of others (4.MP.3)

Materials

Copies	Kit Materials	Classroom Materials			
Math Forum How Many Candy Bars?					
SB 108* How Many Candy Bars?		student math journals			
Work Places Introducing Work Place 38	Racing Fractions				
TM T9 Work Place Guide 3B Racing Fractions TM T10 3B Racing Fractions Record Sheet TM T11 3B Racing Fractions Game Board SB 110** Work Place Instructions 3B Racing Fractions	 game markers (7 red and 7 blue) Racing Fractions Cards (1 deck) 				
Home Connection					
HC 59–60 Fractions & More Fractions					
Daily Practice					
SB 111 Understanding Fractions & Mixed Numbers					

HC – Home Connection, **SB** – Student Book, **TM** – Teacher Master

Copy instructions are located at the top of each teacher master.

* Run 1 copy of this page for display.

Unit 3 Module 2 Session 6

Vocabulary

An asterisk [*] identifies those terms for which Word Resource Cards are available.

denominator*
equivalent fraction*
improper fraction*
mixed number*
numerator*
pattern*
ratio table*
whole

33

^{**} Run 1 copy of this page and store it for use by the teacher and other adult helpers during Work Place time.

Preparation

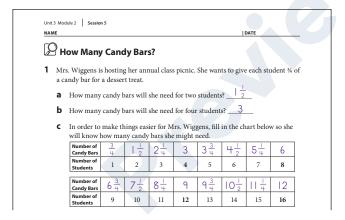
In today's session, you'll introduce Work Place 3B Racing Fractions, which takes the place of Work Place 2A What's Missing? Bingo. Before this session, you should review the Work Place Guide, as well as the Work Place Instructions. Make copies of the 3B Racing Fractions Record Sheet and the 3B Racing Fractions Game Board Teacher Master as directed at the top of each master. Pull out one copy of each for use during this session and store the rest in the Work Place 3B Racing Fractions tray, along with the 4 decks of Racing Fraction Cards from your Bridges kit.



Math Forum

How Many Candy Bars?

- 1 Let students know that you're going to hold a math forum to discuss the problem they worked on yesterday, and then you're going to teach them a new Work Place game.
- 2 Display your copy of the How Many Candy Bars? Student Book page from the previous session and have students take out their completed pages, as well as their math journals.
 - Review the problem with the class.
 - · Work with input to fill in the first two rows of the chart.



3 Ask students to discuss, first in pairs and then as a whole class, any patterns they can find in the numbers on the chart so far.

Patterns students might comment on include, but are not limited to the following:

- The number of students increases by 1 each time, while the number of candy bars increases by 3/4.
- Although most of the answers involve mixed numbers, there is a repeating pattern in the fractions that accompany the whole numbers: ¾, ½, ¼, and no fraction at all; ¾, ½, ¼, and no fraction at all.
- Every fourth box in the row for the number of candy bars features a whole number.
- In looking at every fourth entry on the chart, the number of candy bars increases by 3, while the number of students increases by 4 (e.g., 3 bars, 4 students; 6 bars, 8 students; 9 bars, 12 students; 12 bars, 16 students).

You might quickly sketch a ratio table on the board and enter just the ratios that involve whole numbers to make this pattern visible to all the students. Students may also be invited to make predictions based on this pattern.

Number of Candy Bars	3	6	9	12	
Number of Students	4	8	12	16	

Students Oh my gosh! I never noticed that yesterday!

The candy bars go by 3s and the students go by 4s.

You know what's really weird? If those were fractions, they'd all be the same as $\frac{3}{4}$. Look— $\frac{6}{8}$, $\frac{9}{12}$, and $\frac{12}{16}$ are all the same as $\frac{3}{4}$!

Teacher Can you extend this pattern? What numbers will we see the next time there's a whole number of candy bars?

Students It would be 15 on the top and 20 on the bottom—15 bars for 20 kids.

And the one after that would be 18 and 24—18 bars for 24 kids.

- 4 Explain that as you were watching students work last session, you noticed that they used different strategies to figure out the numbers of candy bars. Now you're going to invite several pairs of students to share their strategies in a math forum.
- After students share, work with the class to come up with a general rule for determining the number of candy bars needed for any number of students.

During this part of the discussion, emphasize the connections between unit fractions, common fractions and multiplying a fraction times a whole number. For example:

- $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4} = \frac{3}{4}$ (3 one-fourths is three fourths)
- $5 \times \frac{3}{4} = 5 \times (3 \times \frac{1}{4}) = 15 \times \frac{1}{4} = \frac{15}{4}$ (5 times $\frac{3}{4}$ is just like 5 times (3 groups of $\frac{1}{4}$))

Teacher So, if you were Mrs. Wiggens, how would you find the number of candy bars you need for any number of students? Is there a general rule that you could use?

Carlos No matter how many students, each one of them will always need ¾ of a candy bar. So, if you had 5 students, then you need 5 of those ¾s.

Teacher Five 3/4s? How many one-fourths is that?

Georgia It's like 5 groups of ¾s. And each ¾ is 3 one-fourths. So 5 groups of 3 groups of ¼.

Helen And 5 groups of 3 groups is 15. So, you have 15 one-fourths.

Teacher I am going to write what you just said: $5 \times \frac{3}{4} = 5 \times 3 \times \frac{1}{4} = 15 \times \frac{1}{4}$. [...] How do we write $15 \times \frac{1}{4}$?

Carlos 15 fourths.

Teacher OK, let's add that to our equation: $5 \times \frac{3}{4} = 5 \times 3 \times \frac{1}{4} = 15 \times \frac{1}{4} = \frac{15}{4}$. If we had 15 one-fourth candy bars, how many whole bars would that be?

Georgia Since 12/4 is 3 candy bars, it's 3 and 3/4 candy bars. Almost 4.

Helen I thought about 16/4 because that's 4 candy bars. This is just 1/4 short, so it's 3 3/4.



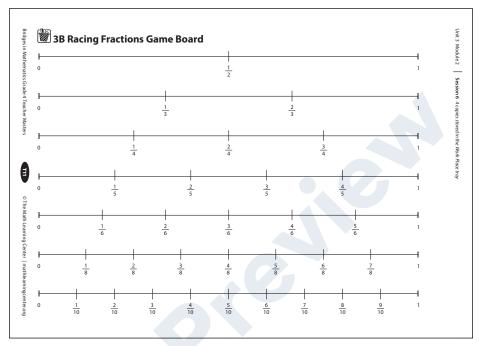
Work Places

Introducing Work Place 3B Racing Fractions

Let students know that you are going to teach them how to play a new Work Place game that will help them think flexibly about fractions, find equivalent fractions, and add fractions. First, however, you want them to take a close look at the game board.

The game board itself is a powerful visual; virtually a guide to equivalent fractions, so don't skip this step.

• Display a copy of the Racing Fractions Game Board Teacher Master, and give students a few moments to examine it quietly.



- Ask students to share, first in pairs and then as a whole group, any observations they can make about the board. Observations may include but are certainly not limited to the following:
 - » There are 7 number lines on the game board, and they all go from 0 to 1.
 - » Each number line is divided into smaller and smaller parts.
 - The parts keep getting smaller, but the denominators of the fractions keep getting larger. (Here, you might want to ask students to explain why this is the case.)
 - » If you look down all the lines, you can see some equivalent fractions. For example, ½, ¾, ¾, 48, and 5/10 all land exactly halfway along their lines. (If you lay a ruler perpendicular to the set of 7 lines along a set of equivalent fractions such as those just listed, it will help all the students see and understand.)
 - » You can see that fifths and tenths relate to one another because $\frac{1}{5}$ and $\frac{2}{10}$ land on the same place along their lines. Same with 2/5 and 4/10, 3/5 and 6/10, and so on.
 - » There are equivalent fractions on the lines for thirds and sixths; also halves, fourths, and eighths.

7 Briefly summarize the game before playing against the class.

Players draw from a deck of Racing Fractions Cards and move game markers along the Racing Fractions Game Board, which shows fraction number lines for halves, thirds, fourths, fifths, sixths, eighths, and tenths. Each player has a game marker on each line and may move one or more markers in a single turn to equal the fraction on the card drawn. Players may also move backward on a turn. The first player to move his or her markers to 1 on all of the number lines is the winner.

Play a game of Racing Fractions against the class. Use your copy of the Work Place Instructions 3B Racing Fractions Student Book page as needed.

Pose questions like the following to promote flexible thinking and strategy development while you play:

- What are some possible moves for this card?
- Which move will help you the most?
- How can you check to see if the moves you made add up to the fraction on the card you chose?
- When would you want to move backward? Why?
- 9 Ask students to turn to a partner to summarize the directions for the Racing Fractions Work Place.
- 10 Close the session.
 - Let students know this game will be available during Work Places for several more weeks.

Home Connection

- 11 Introduce and assign the Fractions & More Fractions Home Connection, which provides more practice with the following skills:
 - Order fractions (including mixed numbers) on number lines (supports 4.NF)
 - Use visual models to generate equivalent fractions (4.NF.1)
 - Express a fraction as the sum of other fractions with the same denominator in more than one way (4.NF.3b)
 - Write an equation to show a fraction as the sum of other fractions with the same denominator (4.NF.3b)
 - Solve story problems involving addition of fractions referring to the same whole and with like denominators (4.NF.3d)
 - Multiply a fraction by a whole number (4.NF.4b)

Daily Practice

The optional Understanding Fractions & Mixed Numbers Student Book page provides additional opportunities to apply the following skills:

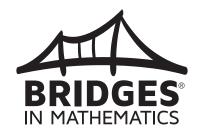
- Convert a mixed number to a fraction (supports 4.NF)
- Convert a fraction to a mixed number (supports 4.NF)
- Create a visual representation of a mixed number or improper fraction (supports 4.NF)





Teacher Masters

GRADE 4 – UNIT 3 – MODULE 2

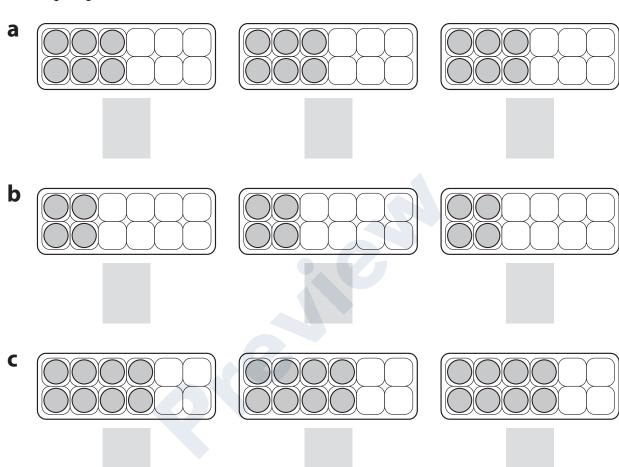


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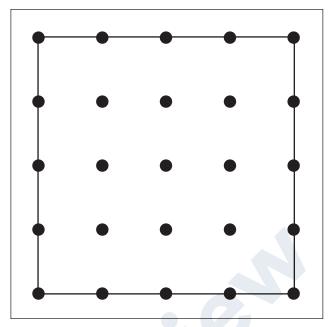
Equivalent Fractions Checkpoint

Write three equivalent fractions to show what part of the egg cartons in each row is filled. Draw lines on the egg cartons in each row to show how you divided them into equal parts.

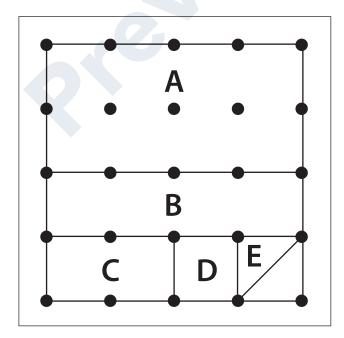


- LaTonya says that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ can all be worth the same amount.
 - Do you agree with her? _____ a
 - Use labeled sketches to explain your thinking.

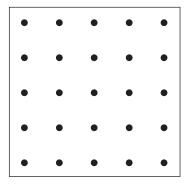
Geoboard Area of One

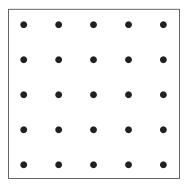


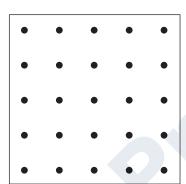
Area = 1 sq. unit

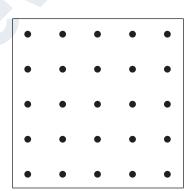


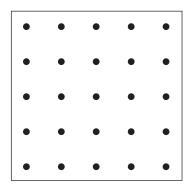
Geoboards

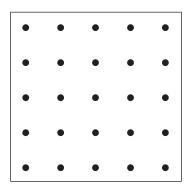


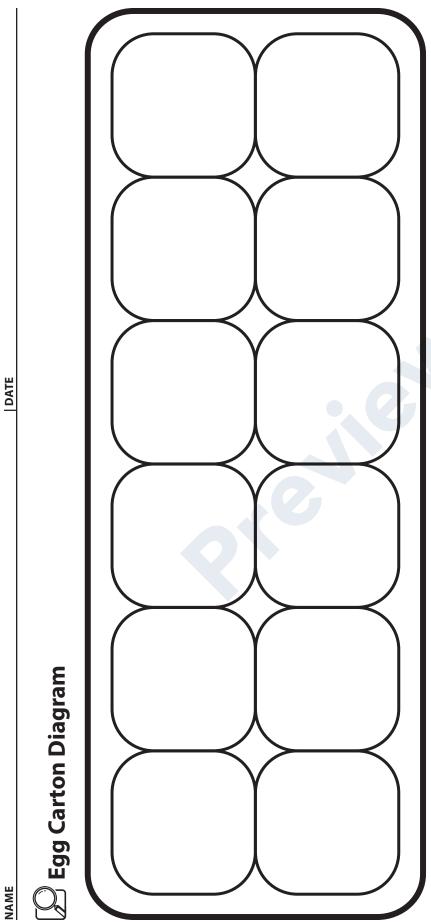








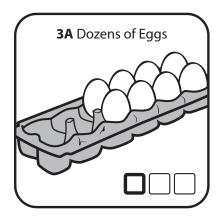


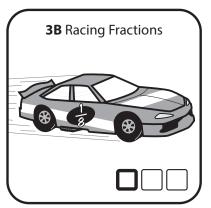


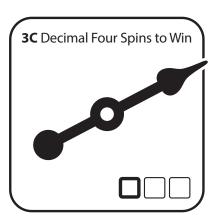
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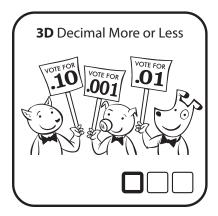


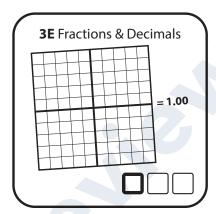
Unit 3 Work Place Log



















Work Place Guide 3A Dozens of Eggs

Summary

Players take turns drawing from a deck of fraction cards, modeling the designated fraction on an Egg Carton Diagram with colored tiles and string, and recording their results. Players take turns until one person has filled all four egg cartons on his or her record sheet and written a matching addition equation that equals 1 whole for each carton. The first player to fill all four egg cartons wins.

Skills & Concepts

- Use visual models to recognize equivalent fractions (4.NF.1)
- Explain addition of fractions as joining parts referring to the same whole (4.NF.3a)
- · Express a fraction as the sum of other fractions with the same denominator in more than one way (4.NF.3b)
- Write an equation to show a fraction as the sum of other fractions with the same denominator (4.NF.3b)

Materials

Copies	Kit Materials	Classroom Materials
TM T6 Work Place Guide 3A Dozens of Eggs	3 decks Dozens of Eggs Fraction Cards	36 colored tiles 18 pieces of heavy string or yarn, 10" long
TM T7 3A Dozens of Eggs Record Sheet		• crayons
TM T4 Egg Carton Diagram		
SB 107-108 Work Place Instructions 3A Dozens of Eggs	0.00	

Assessment & Differentiation

If you see that	Differentiate	Example
A student struggles to build a model of the fraction drawn.	SUPPORT Use strings to model cutting the egg carton into pieces and have the student name the fractional parts.	"I see your card is ¾. How many equal parts do we need? Lets use the pieces of string to split the egg carton into that many equal pieces. Show me how you could split the 12 egg compartments into 4 equal groups."
Students are unable to determine whether the fraction drawn can fit into one of their cartons.	the fraction that is left in each diagram and any equivalent fractions they can determine before they try to fill the carton with the fraction drawn.	
One or more students struggles to play the game due to a lack of comfort with fractions, the egg carton model, or the process of building and recording each time.	SUPPORT Have students play game variation A with classmates who are more comfortable with the game. SUPPORT Gather a small group of these students during Work Places and have them play as a team against you.	
One or more students easily build the designated fractions, fill the egg diagrams, and record equations to match without difficulty.	CHALLENGE Have students play using game variation B.	

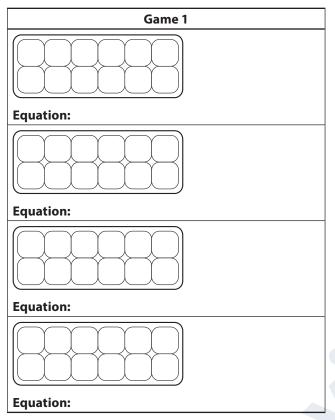
English-Language Learners Use the following adaptations to support the ELL students in your classroom.

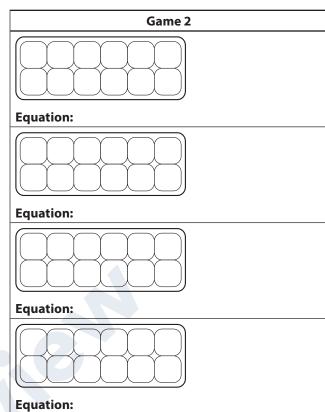
- Have ELL students observe other students playing the game before playing it themselves.
- Pair each ELL student with a supportive partner (an English-speaking student or another ELL student with more command of English) who can offer support and explain the instructions while they play.
- Play the game with the ELL students yourself. Model how to play and put emphasis on how to model the fraction on the egg carton.
- Once students understand the game, help them demonstrate their strategies and verbalize them.

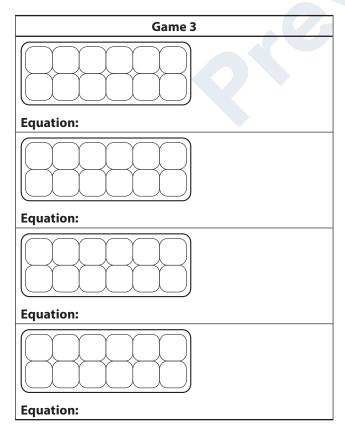
DATE

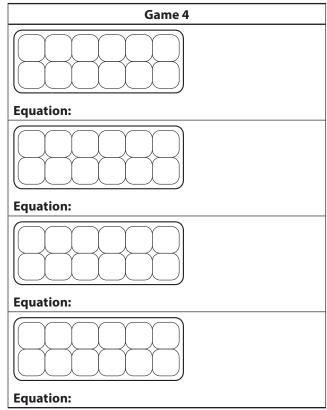


3A Dozens of Eggs Record Sheet











How Many Candy Bars Forum Planner

Use this planner to make a record of the strategies you see students using to solve problems during Session 5. Prior to Session 6, use the third column to indicate the order in which you plan to have students share during the forum.

How many candy bars if each student gets 3/4?

Strategy	Student Names and Notes	Order of Sharing in Forum
Adding ¾ Add ¾ to get each new figure, breaking it apart into ½ + ¼ when it is advantageous to do so.		
Doubling or multiplying Double ¾ to make 1 ½ candy bars for two students, double again to make three candy bars for four students, and so on.		
Using a ratio Use known information to find more answers by keeping the ratio the same. For example, students know that three students get 2 ¼ bars and four students get 3 bars, so they add 2 ¼ + 3 = 5 ¼ to find the number of bars for seven students.		

T8



Work Place Guide 3B Racing Fractions

Summary

Players draw from the Racing Fractions Cards deck and move game markers along the Racing Fractions Game Board, which shows fraction number lines for halves, thirds, fourths, fifths, sixths, eighths, and tenths. Each player has a game marker on each line and may move one or more markers in a single turn to equal the fraction on the card drawn. Players may also move backward on a turn. The first player to move her markers to 1 on all of the number lines is the winner.

Skills & Concepts

- Recognize equivalent fractions (4.NF.1)
- Explain addition of fractions as joining parts referring to the same whole (4.NF.3a)
- Express a fraction as the sum of other fractions with the same denominator in more than one way (4.NF.3b)
- Add and subtract fractions with like denominators (supports 4.NF)
- Add and subtract fractions with unlike denominators, including mixed numbers (5.NF.1)

Materials

Copies	Kit Materials	Classroom Materials
TM T9 Work Place Guide 3B Racing Fractions	21 red and 21 blue game markers (7 of each per student pair)	
TM T10	3 decks of Racing Fractions Cards	
3B Racing Fractions Record Sheet		
TM T11		
3B Racing Fractions Game Board		
SB 112 Work Place Instructions 3B Racing Fractions		

Assessment & Differentiation

If you see that	Differentiate	Example
One or more students are uncertain when moving game markers.	SUPPORT Gather a small group to work together as a team against you. Each time you take a turn, share your thinking, and ask students to do the same.	
	rather than competitively with classmates who are working more easily with fractions.	
A student always tries to move only one game marker exactly the value of the card.	SUPPORT Ask the student to find two or three possible moves for each card. Then ask the student which possibility is the best move.	"You have the % card. What are some different ways you could move %? What fractions add up to %?"
One or more students are readily making correct moves for any card. CHALLENGE Ask students questions that prompt them to make generalizations and extend their		"Which cards are the most helpful to draw at the beginning of the game?"
	thinking about the game.	"Since the cards have all of the fractions with denominators 2, 3, 4, 5, 6, 8, 10, what are the fewest possible moves you could make to win?"
		"When would you want to move backward? Why?"
One or more students are developing strategies for playing the game.	CHALLENGE Partway through a game, ask students which cards would or would not work at this point. Challenge them to communicate their reasoning clearly.	

English-Language Learners Use the following adaptations to support the ELL students in your classroom.

- Post Word Resource Cards for important vocabulary such as equivalent fractions, numerator, and denominator.
- · Pair ELL students with supportive partners who can explain the game, including others who speak the same language.
- Play a demonstration game, focusing on different possibilities for each fraction card selected. Emphasize developing game strategies.

NAME DATE

3B	Racing	Era
JD	Racing	гrа

Player 1	Player 2
i layer i	i layer Z

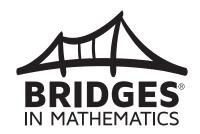
Use the chart below to record your work with Racing Fractions. Write the fraction on the fraction card in the first column. Write an equation that represents your moves in the second column. The first one has been filled in for you as an example.

	Player 1	Player 2		
Fraction	Equation	Fraction	Equation	
$\frac{3}{4}$	$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$	$\frac{4}{5}$	$\frac{1}{2} + \frac{1}{5} + \frac{1}{10} = \frac{4}{5}$	
		0.0		
	16			



Student Book

GRADE 4 – UNIT 3 – MODULE 2



DATE



Pizza Party Planning

A fourth grade class won a pizza party for collecting the most paper for recycling in their school contest. Medium pizzas were cut into 8 slices, and large pizzas were cut into 12 slices.

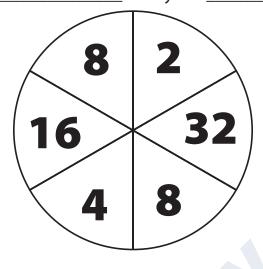
- Mariah ate 2 slices of a large pizza. What fraction of the pizza did she eat? Draw a sketch to show your thinking.
- Carlos said that Mariah ate of $\frac{1}{6}$ a pizza. Tell why you agree or disagree.
- Mariah's table seats 4 students. Each student ate 2 slices of a large pizza. Write an equation that shows what fraction of a pizza was eaten at Mariah's table.
- Tony ate 3 slices of a medium pizza. His friend, Connor, ate 4 slices of the same pizza.
 - Write two different fractions to describe how much pizza Connor ate.
 - What fraction of the pizza did the boys eat together?
- Lionel's table group drank $1\frac{1}{2}$ liters of juice with their pizza. How many milliliters did they drink? Show your work.
- Complete the problems.

$$\frac{100}{\times 8,500}$$

$$\frac{\times 40}{240}$$

Last Equation Wins page 1 of 2

Player 1 _____ Player 2 _____



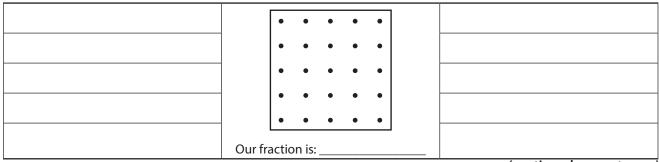
Round 1

	•	•	•	•	•	
	•	•	•	•	•	
	•	•	•	•	•	
					•	
Our fr	actio	n is:				

Round 2

	•	•	•	•	•		
	•	•	•	•	•		
	•	•	•	•	•		
		•	•	•	•		
Our fra		n is:					

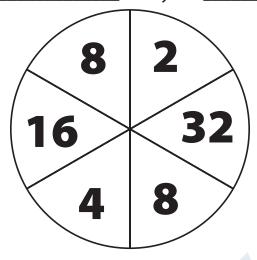
Round 3



(continued on next page)

Last Equation Wins page 2 of 2

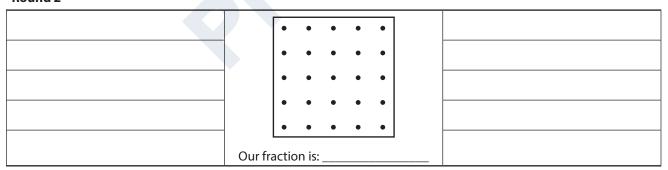
Player 1 _____ Player 2 _____



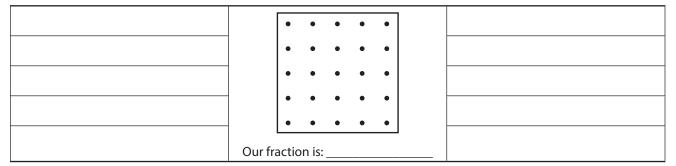
Round 1

Our fraction is:	

Round 2

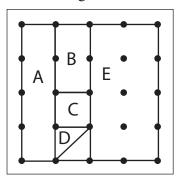


Round 3



What's the Share?

If the area of the largest square on the geoboard is 1, what is the area of each region?



Α	В	C	D	E

2 Write four statements and matching fraction equations that compare two regions.

	1 0
Statements	Equations
ex A is half of E	$2 \times \frac{1}{4} = \frac{1}{2}$

3 Fill in the blank with the correct relational symbol: <, >, or =.

- List all the factor pairs for the number 32.
- List three prime numbers greater than 20.



Comparing, Adding & Subtracting Fractions page 1 of 2

Use the symbols >, =, or < to compare each pair of fractions.

$\frac{3}{8}$ $\frac{2}{8}$	$\frac{3}{8}$ $\frac{1}{4}$	$\frac{3}{8}$ $\frac{7}{16}$	$\frac{7}{16}$ $\frac{1}{4}$
$\frac{1}{4}$ $\frac{3}{4}$	$\frac{4}{16}$ $\frac{2}{16}$	$\frac{4}{16}$ $\frac{5}{8}$	$\frac{3}{4}$ $\frac{5}{8}$

Find each sum.

$\frac{1}{4} + \frac{3}{4} =$	$\frac{2}{4} + \frac{1}{4} =$	$\frac{2}{4} + \frac{3}{4} =$
$1\frac{1}{4} + \frac{1}{4} =$	$\frac{2}{8} + \frac{5}{8} =$	$\frac{5}{8} + \frac{6}{8} =$

Find each difference.

$\frac{3}{4} - \frac{1}{4} =$	$1\frac{1}{4} - \frac{1}{4} =$	$1\frac{1}{4} - \frac{3}{4} =$
$\frac{6}{8} - \frac{2}{8} =$	$1\frac{3}{8} - \frac{2}{8} =$	$1\frac{3}{8} - \frac{4}{8} =$

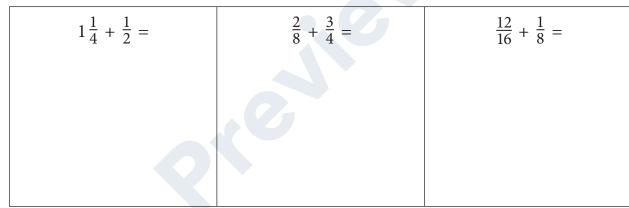
Comparing, Adding & Subtracting Fractions page 2 of 2

4 CHALLENGE Write as many equivalent fractions as you can for each fraction shown below.

а	$\frac{1}{8}$	
b	$\frac{1}{4}$	
c	<u>2</u> 3	

d Describe how you can write equivalent fractions for any fraction.

5 CHALLENGE Find each sum.



6 CHALLENGE Find each difference.

$\frac{3}{4} - \frac{3}{8} =$	$\frac{5}{16} - \frac{1}{4} =$
	$\frac{3}{4} - \frac{3}{8} =$

NAME



Adding & Subtracting Fractions

Ariel got a new box of 8 crayons and a set of 10 markers for her birthday. Use this information as you solve each problem below. Use numbers, labeled sketches, or words to show your thinking.

- Ariel used 5 crayons to make a thank-you card. What fraction of the box did she use?
- Ariel gave her brother 4 crayons. What fraction does she have left out of her box of 8?
- After she gave some crayons to her brother, Ariel's dog ate 2 of her crayons.
 - Now what fraction does Ariel have left of her original box of 8 crayons? a
 - What fraction of the crayons went to Ariel's brother and her dog?
- Ariel took 6 markers out of her marker set. What fraction of the markers are left in the set?
- Two of Ariel's markers are green, 2 are red, and 3 are blue. What fraction of the markers are

green? _____

red?

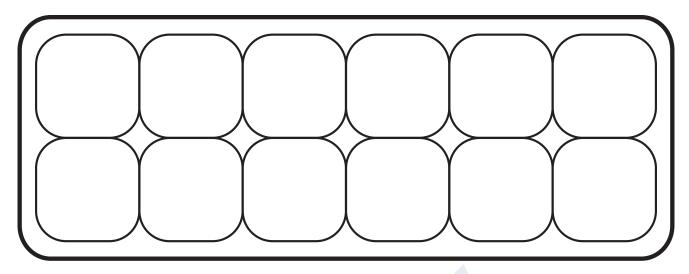
blue?

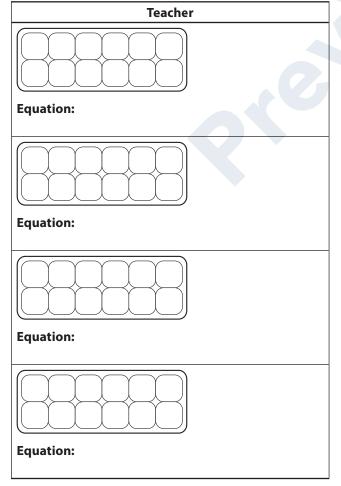
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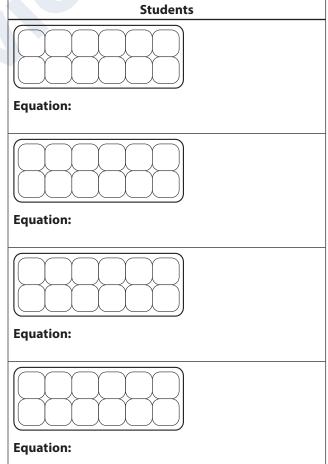
NAME DATE



Introducing Dozens of Eggs







NAME DATE

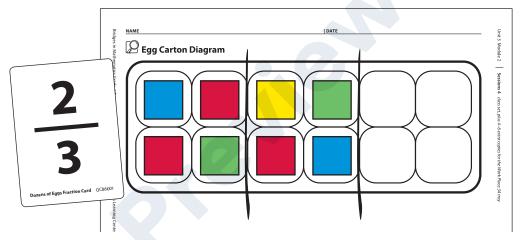


Work Place Instructions 3A Dozens of Eggs page 1 of 2

Each pair of players needs:

- · 2 Dozens of Eggs Record Sheets
- 1 deck of Dozens of Eggs Fraction Cards
- 1 Egg Carton Diagram
- · 6 pieces of string or yarn
- · 12 colored tiles
- · colored pencils or crayons
- Players shuffle the fraction cards and lay them face-down in a stack. Each player draws one card. The player with the larger fraction goes first. The cards just drawn go at the bottom of the stack.

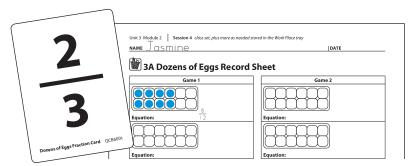
 Players may build fractions on the Egg Carton Diagram if needed to determine which fraction is larger.
- Player 1 draws a card from the top of the deck, reads the fraction out loud, and uses string and colored tiles to build a model of the fraction on the Egg Carton Diagram. Player 2 checks Player 1's work.



Jasmine Wow! I got a really big fraction on my first turn. So I'm going to divide the egg carton into 3 equal parts, and fill 2 of them, like this.

Sara I agree that $\frac{2}{3}$ of the egg carton is 8 eggs, because I know that there are 4 eggs in one-third of a carton.

3 Player 1 draws circles to represent that number of eggs in one of the diagrams on her record sheet and records that number of twelfths as a fraction on the sheet.



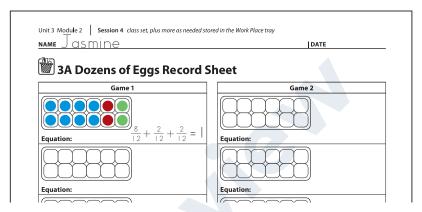
Jasmine I have to change $\frac{2}{3}$ into twelfths, but that's easy, because each egg is one-twelfth of the carton, so I got $\frac{8}{12}$ on my first turn. I only need 4 more twelfths to fill this carton.

(continued on next page)

NAME DATE

Work Place Instructions 3A Dozens of Eggs page 2 of 2

- 4 Player 1 empties the egg carton diagram and puts the card in a discard stack. Then Player 2 takes a turn.
- **5** Players continue to take turns until one person has filled in all four cartons on the record sheet. Players should use a different color to record each new turn.
 - When all the cards in the deck have been used, shuffle the deck and use it again.
- **6** On each turn, players must put all of the eggs in one carton. However, players may begin to fill another carton before the first is completely filled.
- 7 If the fraction drawn does not fit into one of the cartons, the player misses that turn.
- **8** When a carton is filled, the player writes an equation by inserting plus signs between the fractions for that carton and showing them equal to 1 whole.



9 The winner is the first player to fill all four cartons on his record sheet. If Player 1 is the first to fill all four cartons, Player 2 may take one last turn.

Game Variations

- A Players work together to fill all four cartons on a single record sheet rather than playing against each other.
- Players begin with all four cartons filled, by drawing 12 circles in each of the cartons and writing $\frac{12}{12}$ at the start of each equation line. Then each player subtracts the fractions that are written on the cards they get, crossing out that many eggs and subtracting that many twelfths. Players must subtract the entire fraction from one carton rather than splitting the fraction between two or more cartons. The winner is the first player to get rid of all the eggs from all four cartons.

NAME

DATE



Egg Carton Fractions

Solve the following multiplication and division problems. They might help you think about the egg cartons in problem 2.

$$12 \div 2 =$$
 $12 \div 3 =$ $12 \div 4 =$ $12 \div 6 =$

$$12 \div 3 =$$

$$12 \div 4 =$$

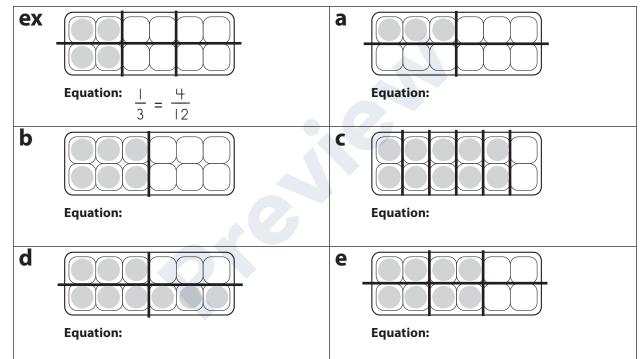
$$12 \div 6 =$$

$$6 \times 3 =$$

$$4 \times 2 =$$

$$3 \times 3 =$$

Write a fraction to show the amount of each egg carton that is filled with eggs. (The cartons are divided into equal parts for you.) Then write an equivalent fraction with 12 in the denominator.



Use the symbols >, =, or < to compare each pair of fractions.

ex	$\frac{1}{4}$	<	$\frac{1}{2}$	ex	<u>1</u> 2	>	$\frac{1}{3}$	a	$\frac{4}{6}$	<u>2</u> 3
b	$\frac{1}{3}$		$\frac{1}{4}$	C	$\frac{3}{4}$		<u>5</u>	d	$\frac{1}{3}$	3/4
е	1/2		<u>2</u>	f	<u>2</u> 3		$\frac{3}{4}$	g	<u>2</u>	1/3

How Many Candy Bars?

- Mrs. Wiggens is hosting her annual class picnic. She wants to give each student ¾ of a candy bar for a dessert treat.
 - How many candy bars will she need for two students? _____ a
 - b How many candy bars will she need for four students? ____
 - C In order to make things easier for Mrs. Wiggens, fill in the chart below so she will know how many candy bars she might need.

		-)						
Number of Candy Bars								
Number of Students	1	2	3	4	5	6	7	8
Number of Candy Bars								
Number of Students	9	10	11	12	13	14	15	16
Number of Candy Bars								
Number of Students	17	18	19	20	21	22	23	24
Number of Candy Bars								
Number of Students	25	26	27	28	29	30	31	32

How did you get your answers on the chart above? Use numbers, words, or labeled sketches to describe your strategy.

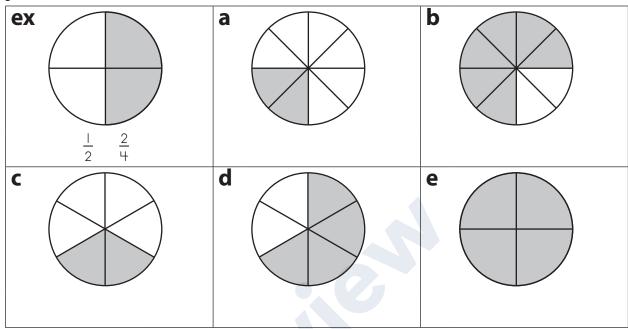
CHALLENGE What kind of general rule could you give Mrs. Wiggens to know how many candy bars to get no matter how many students she has?

| DATE



Fractions & Mixed Numbers

The circles below are divided into equal parts. Write two fractions to show what part of each circle is filled in.



The circles below are divided into equal parts. Write a fraction and a mixed number to show how many circles are filled in.

	Fraction	Mixed Number		Fraction	Mixed Number
ex	$\frac{3}{2}$	$\frac{1}{2}$	a		
b			c ————————————————————————————————————		

Fill in the missing fractions or mixed numbers.

							Challenge	
Fractions	<u>5</u> 2	9/2	$\frac{9}{4}$	$\frac{14}{4}$			<u>62</u> 3	
Mixed Numbers	2 1/2				$3\frac{1}{2}$	$2\frac{3}{4}$		$30\frac{1}{3}$

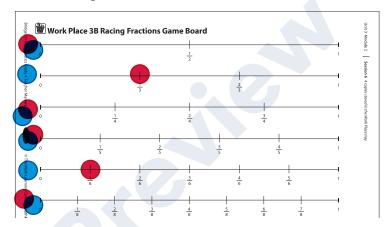


Work Place Instructions 3B Racing Fractions

Each pair of players needs:

- 1 Racing Fractions Record Sheet to share
- 1 Racing Fractions Game Board
- 7 red game markers
- 7 blue game markers
- · 1 deck of Racing Fraction Cards
- 1 Players decide who will play with the red game markers, and who will play with the blue markers. Then both players place one of their game markers at the beginning of each number line on the game board.
- **2** Players shuffle the fraction cards and lay them face-down in a stack. Each player draws one card. The player with the larger fraction goes first. Players put the cards just drawn at the bottom of the stack.
- **3** Player 1 draws a new card and moves one or more game markers the distance shown on the card.

Player 1 I got $\frac{3}{6}$. That's the same as $\frac{1}{2}$, so I could go $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, or $\frac{5}{10}$. Hmm...or I could do $\frac{1}{3}$ and $\frac{1}{6}$. I remember those make $\frac{1}{2}$ from when we looked at egg carton fractions. I think I'll do that.



- Player 1 records the fraction in his Fraction column on the record sheet and writes the fraction or equation that describes how the game markers were moved in his Equation column.

 (If the player selected $\frac{1}{2}$ and moved $\frac{1}{2}$, he would write $\frac{1}{2}$. If the player selected $\frac{1}{2}$ and moved one marker to $\frac{1}{3}$ and another marker to $\frac{1}{6}$, he would write $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.)
- **5** Player 2 checks first player's work on the record sheet. Player 1 tries again if an error was made.
- **6** Then Player 2 draws a fractions card and takes a turn. Player 1 checks the second player's work.
- Players continue to take turns, record moves, and check each other's work until one player's game markers are all on 1. If Player 1 is the first to land on 1, Player 2 may take one last turn.

 If a player cannot find a possible move for a card he has drawn, the player loses the turn.

 Players may also move game markers backward. For example, if a player selects \(\frac{1}{3} \), she can move one marker up \(\frac{1}{2} \)

Game Variations

A Play cooperatively. Players can work together and help each other finish the track in a certain time period.

and another back $\frac{1}{6}$. The sum or the difference of the moves still needs to equal the value on the fraction card.

B Double the length of each track by taping 2 copies of the Racing Fractions Game Board Teacher Master together, writing a 1 in front of every fraction on the second sheet, and changing the 1 at the end of each track on the second sheet to a 2.



Understanding Fractions & Mixed Numbers

Sketch and label a picture that represents $1\frac{1}{2}$.

Answer each question below:

- How many halves are in $\frac{3}{2}$?
- **b** How many fourths are in $\frac{7}{4}$?
- How many thirds are in $\frac{8}{3}$? _____
- **d** How many fifths are in $\frac{6}{5}$?
- What do you notice about problems a-d?

Write each fraction as a mixed number. Make a drawing, if needed.

- $\frac{5}{2} = \underline{\hspace{1cm}}$
- **b** $\frac{7}{6} =$ _____
- c $\frac{4}{3} =$ _____
- **d** $\frac{12}{8} =$

4 Write each mixed number as a fraction. Make a drawing, if needed.

- **a** $1\frac{2}{3} =$ _____
- **b** $1\frac{3}{5} =$ _____
- **c** $2\frac{1}{4} =$ _____
- **d** $3\frac{1}{2} =$ _____



Home Connections

GRADE 4 – UNIT 3 – MODULE 2



DATE



Brownie Dessert page 1 of 2

A fourth grade class earned a brownie dessert party for having the highest attendance in one grading period. Small pans of brownies were cut into 9 pieces, and large pans were cut into 16 pieces.

Tori ate 2 brownies from a small pan. What fraction of the brownies in that pan did she eat? Draw a sketch to show your thinking.

Holly ate 1 more brownie than Tori from the same small pan. Write two equivalent fractions that describe how much Holly ate.

Henry's table group seats 5 students. Each student ate 2 brownies from a large pan. Write an equation that shows what fraction of a large pan of brownies was eaten at Henry's table.

- April ate 1 brownie from a large pan, and her friend, Christina, ate 4 brownies from the same pan.
 - Write two fractions to tell how much of the large pan of brownies Christina ate. a
 - What fraction of a large pan of brownies did the girls eat together?

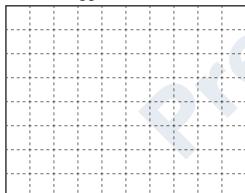
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Brownie Dessert page 2 of 2

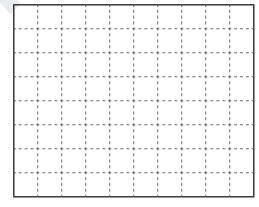
1 Freddy had 2 of the brownies from a large pan. His friend said he ate $\frac{1}{8}$ of the brownies in that pan. Tell why you agree or disagree.

2 CHALLENGE In an 18-egg carton, $\frac{1}{3}$ equals 6 eggs. Use the grids below to help you imagine and draw cartons where:

a $\frac{1}{2}$ is 9 eggs.



b $\frac{3}{8}$ is 18 eggs.

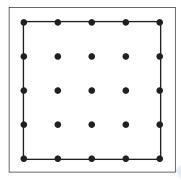




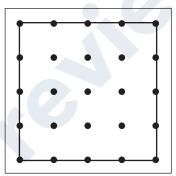
Planning a Garden page 1 of 2

The Brown family is trying to decide how to plan their garden for the vegetables they want to grow. Use the geoboard model to design a garden that fits each description. Label every area to show where each vegetable will be planted.

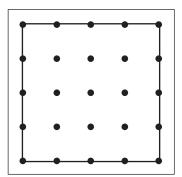
The Browns could plant $\frac{1}{2}$ tomatoes, $\frac{1}{4}$ squash, and $\frac{1}{4}$ lettuce.



They could plant $\frac{1}{4}$ tomatoes, $\frac{1}{4}$ squash, $\frac{1}{4}$ lettuce, $\frac{1}{8}$ peppers, and $\frac{1}{8}$ cabbage.



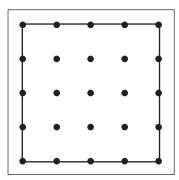
The Brown family might plant $\frac{1}{8}$ tomatoes, $\frac{1}{8}$ cabbage, and $\frac{1}{8}$ peppers. If they do, what fraction of their garden will be unplanted?



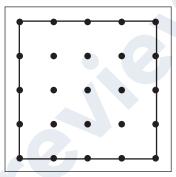
NAME DATE

Planning a Garden page 2 of 2

1 If the Browns plant $\frac{3}{16}$ tomatoes, $\frac{1}{4}$ cabbage, and $\frac{2}{8}$ peppers, what fraction of their garden will be unplanted?

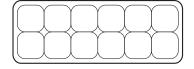


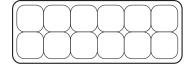
2 CHALLENGE Create a plan for a garden that has room for 5 different vegetables. Label the vegetables in the garden and write a equation to represent the model.

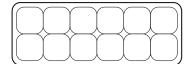


Fractions & More Fractions page 1 of 2

Ethan used an egg carton model to add fractions. Draw eggs in the cartons to show and solve the problem. Then fill in the blank to show the answer.

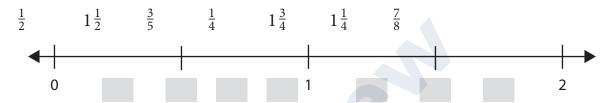






$$\frac{1}{6} =$$

Put the following numbers in order on the number line below.



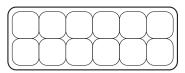
- Maria is writing as many different addition and multiplication equations as she can for $2\frac{2}{8}$. Her rule is that all the fractions in each equation must have a denominator of 8.
 - Here are the equations Maria has written so for. Fill in the bubble beside each equation that is true.
 - $2\frac{2}{8} = 1 + 1 + \frac{2}{8}$
 - $\bigcirc 2\frac{2}{8} = \frac{8}{8} + \frac{10}{8}$

 - \bigcirc 18 × $\frac{1}{8}$ = 2 $\frac{2}{8}$
 - **b** Write at least four more addition or multiplication equations for $2\frac{2}{8}$ in which all the fractions have a denominator of 8.

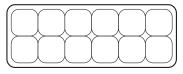
NAME DATE

Fractions & More Fractions page 2 of 2

4 Calvin and Leah are playing a game that has them draw fraction cards to add up to numbers that fill a 12-egg carton. Calvin had $\frac{1}{3}$ of his egg carton full when he chose a card with $\frac{8}{12}$ on it. He says he will fill his egg carton. Do you agree or disagree? Why? Use a labeled sketch in the egg carton diagram below to help explain your answer.



Leah had $\frac{4}{6}$ of her egg carton full when she chose the $\frac{5}{12}$ card. Can she fit $\frac{5}{12}$ in this egg carton? Why or why not? Use a labeled sketch in the egg carton diagram below to help explain your answer.



- **6 CHALLENGE** Imagine you are playing the game with egg cartons that hold 18 eggs, and the fraction cards refer to 18 eggs instead of 12 eggs. (For example, if you draw the $\frac{1}{2}$ card, that means half of 18, not half of 12.)
 - If you have $\frac{2}{3}$ of your first 18-egg carton full, how many more eggs will fit in that carton? What fraction card will you need to draw to fill the first carton exactly?
 - **b** You have $\frac{1}{3}$ of your second 18-egg carton full when you select the $\frac{5}{6}$ card. Can you use this card to place more eggs in the second carton, or will you have to use your third carton instead?